Exercise 3.6

Consider the following function:

$$f(x) = 4x^4 - 32x^3 + 89x^2 - 95x + 29$$

First, draw the graph of the function $f(x)$ in the domain $(-10, 10)$.

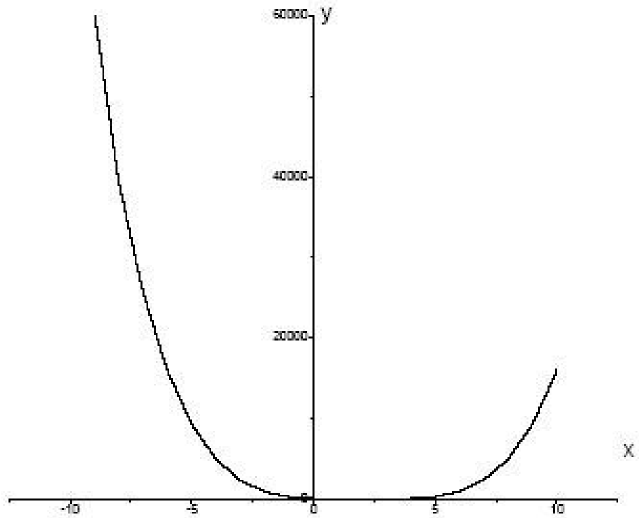


FIGURE – 1

In the interval (0, 4), there may be some details because on the large scale it is looking like straight line.

So take the viewing rectangle [0, 4] by [-6, 10] in figure 2 and [2.4, 2.7] by [3.96, 4.04].

Now, find all the intervals where graph is increasing or decreasing by sketching the curve of $f'(x)$.

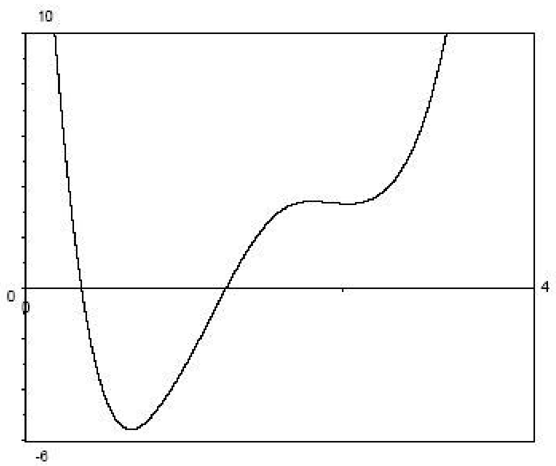


FIGURE – 2

The first derivative of the function $f(x)$ is,

$$f'(x) = 16x^3 - 96x^2 + 178x - 95$$

Draw the graph of f'.

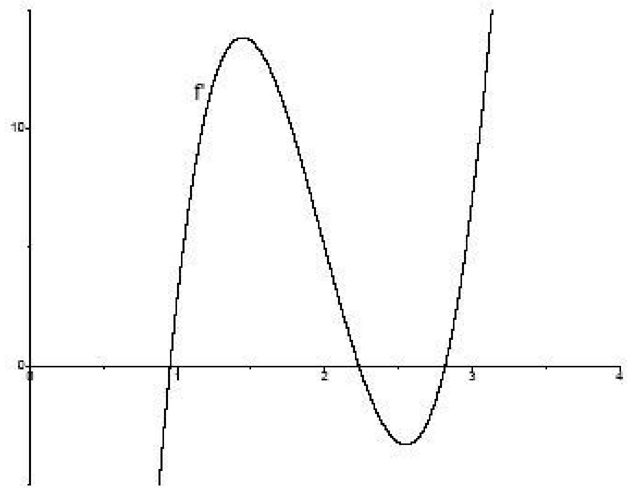


FIGURE – 3

Since $f'(x) < 0$ on the intervals $(-\infty, 0.92)$ and $(2.5, 2.58)$, the function $f(x)$ is decreasing on $\boxed{(-\infty, 0.92) \text{ and } (2.5, 2.58)}$.

Since $f'(x) > 0$ on the intervals $(0.92, 2.5)$ and $(2.58, \infty)$, the function $f(x)$ is decreasing on $\boxed{(0.92, 2.5) \text{ and } (2.58, \infty)}$.

Thus, the local maximum of the function is given by,

$\boxed{f(2.5) \approx 4}$.

The local minimum of the function is given by,

$$\boxed{\begin{array}{l} f(0.92) \approx -5.12 \\ f(2.58) \approx 3.998 \end{array}}$$

Find the second derivative of the function $f(x)$.

$f'' = 48x^2 - 192x + 178$

Now, draw the graph of $f''(x)$ in the interval $[0,4]$.

Estimate the intervals in which graph has downward on upward concavity and get the points of inflections as (Figure 4),

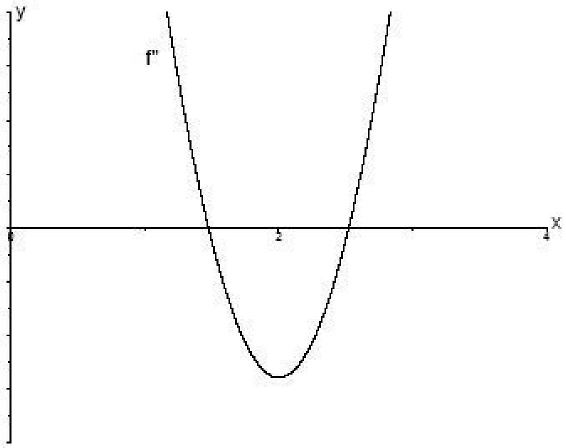


FIGURE – 4

Since $f'' < 0$ on the interval $(1.46, 2.54)$, so *f* is concave downward on $\boxed{(1.46, 2.54)}$.

Since $f'' > 0$ on the interval $(-\infty, 1.46)$ and $(2.54, \infty)$, so *f* is concave upward on $\boxed{(-\infty, 1.46) \text{ and } (2.54, \infty)}$.

So, the inflection points of the function are $\boxed{(1.46, -1.40) \text{ and } (2.54, 3.999)}$.

Thus, figure 5 is the final graph of the function $f(x)$ which shows all the details.

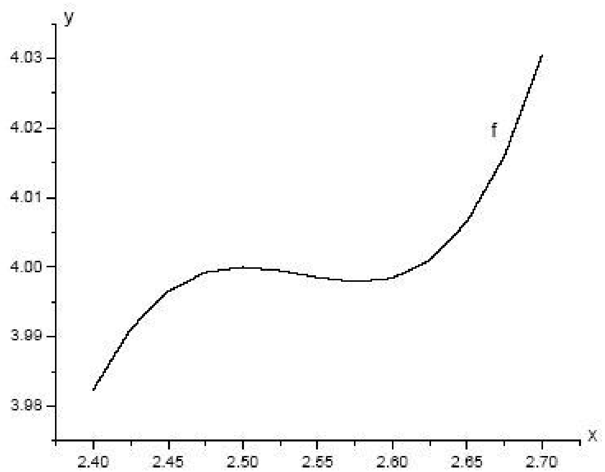


FIGURE - 5

Chapter 3 Applications of Differentiation Exercise 3.6. 2E

Consider the following curve

$f(x) = x^6 - 15x^5 + 75x^4 - 125x^3 - x$

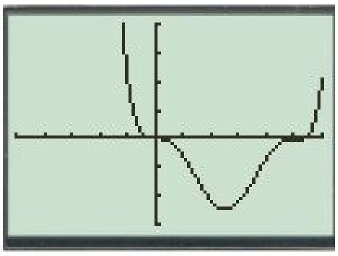
Estimate the intervals of increase and decrease, extreme values, intervals of concavity, and inflection points by using the graphs of f' and f'':

Sketch the graph of $f(x) = x^6 - 15x^5 + 75x^4 - 125x^3 - x$ is as follows:

Enter the equation into Y1in the equation editor Y=.

First set the window as shown in figure

Now click on the GRAPH button to get the graph.



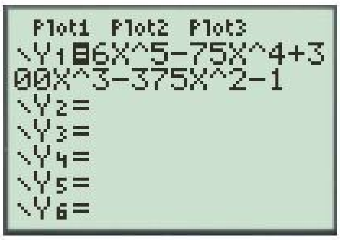
Find f':

$f(x) = x^6 - 15x^5 + 75x^4 - 125x^3 - x$

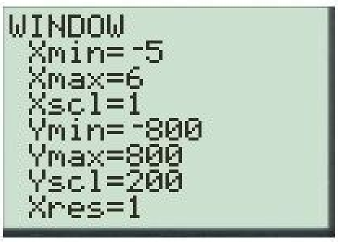
$f'(x) = 6x^5 - 75x^4 + 300x^3 - 375x^2 - 1$

Sketch the graph of $f'(x) = 6x^5 - 75x^4 + 300x^3 - 375x^2 - 1$ is as follows:

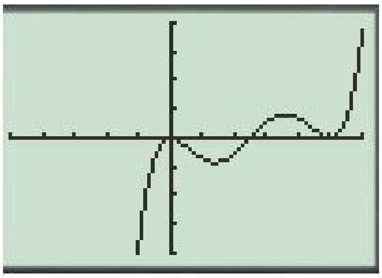
Enter the equation into Y1in the equation editor Y=.



First set the window as shown in figure

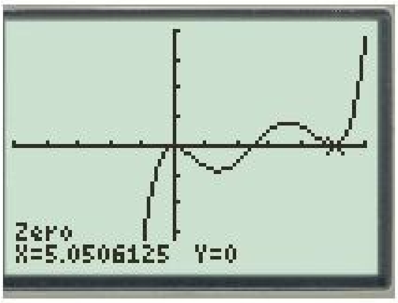


Now click on the $\boxed{\text{GRAPH}}$ button to get the graph.



Now press $\boxed{\text{2nd}}\boxed{\text{TRACE}}\boxed{\nabla}\boxed{\text{Zero}}$ and then press ENTER.

Now hit the $\boxed{\text{ENTER}}$ button 3 times to find *x*-value



From the graph of $f\,'$observe that $f\,'(x)>0$ on the interval $[2.5,5.1]$ **and** $[5.1,\infty]$, so f is increasing on $\boxed{(2.5,5.1)\text{ and }(5.1,\infty)}$ and $f\,'(x)<0$ on the interval $[-\infty,0]$ and $[0,2.5]$, so f is decreasing on $\boxed{(-\infty,0)\text{ and }(0,2.5)}$.

From the graph of $f\,'$observe that $f\,'(x)$changes from negative to positive at $x=2.5$

Put $x=2.5$ in $f(x)=x^6-15x^5+75x^4-125x^3-x$ it becomes

$$\begin{aligned}f(x)&=(2.5)^6-15(2.5)^5+75(2.5)^4-125(2.5)^3-(2.5)\\&=-246.64\end{aligned}$$

The local minimum value is $\boxed{-246.64}$

From the graph of $f\,'$observe that $f\,'(x)$changes from positive to negative at $x=4.94$

Put $x=4.94$ in $f(x)=x^6-15x^5+75x^4-125x^3-x$ it becomes

$$\begin{aligned}f(4.94)&=(4.94)^6-15(4.94)^5+75(2.5)^4-125(4.94)^3-(4.94)\\&=-4.97\end{aligned}$$

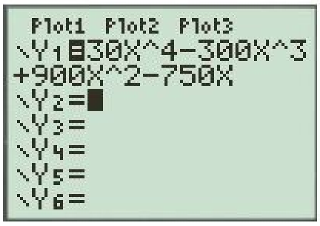
The local minimum value is $\boxed{-4.97}$

Find f'':

$$f(x) = x^6 - 15x^5 + 75x^4 - 125x^3 - x$$
$$f'(x) = 6x^5 - 75x^4 + 300x^3 - 375x^2 - 1$$
$$f''(x) = 30x^4 - 300x^3 + 900x^2 - 750x$$

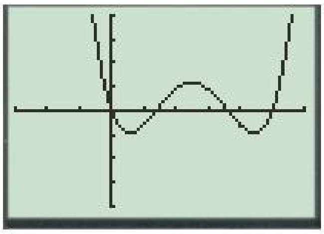
Sketch the graph of $f''(x) = 30x^4 - 300x^3 + 900x^2 - 750x$ is as follows:

Enter the equation into Y1in the equation editor $\boxed{Y=}$.



First set the window as shown in figure

Now click on the $\boxed{\text{GRAPH}}$ button to get the graph.



Observe that $f'' > 0$ and that f is concave up on $\boxed{(-\infty, 0), (1.38, 3.62), (5, \infty)}$ and that $f'' < 0$ and *f* is concave down $\boxed{(0, 1.38) \text{ and } (3.62, 5)}$.

Now find the inflection points:

$$f(x) = x^6 - 15x^5 + 75x^4 - 125x^3 - x$$
$$f(0) = (0)^6 - 15(0)^5 + 75(0)^4 - 125(0)^3 - (0)$$
$$= 0$$

Put $x = 1.38$ in $f(x)$, it becomes

$$f(1.38) = (1.38)^6 - 15(1.38)^5 + 75(1.38)^4 - 125(1.38)^3 - (1.38)$$
$$= -126.1$$

Put $x = 3.62$ in $f(x)$, it becomes

$$f(3.62) = (3.62)^6 - 15(3.62)^5 + 75(3.62)^4 - 125(3.62)^3 - (3.62)$$
$$= -128.3$$

Put $x = 5$ in $f(x)$, it becomes

$$f(5) = (5)^6 - 15(5)^5 + 75(5)^4 - 125(5)^3 - (5)$$
$$= -5$$

The inflection points are $\boxed{(0,0)(1.38,-126.1)(3.62,-128.3)(5,-5)}$.

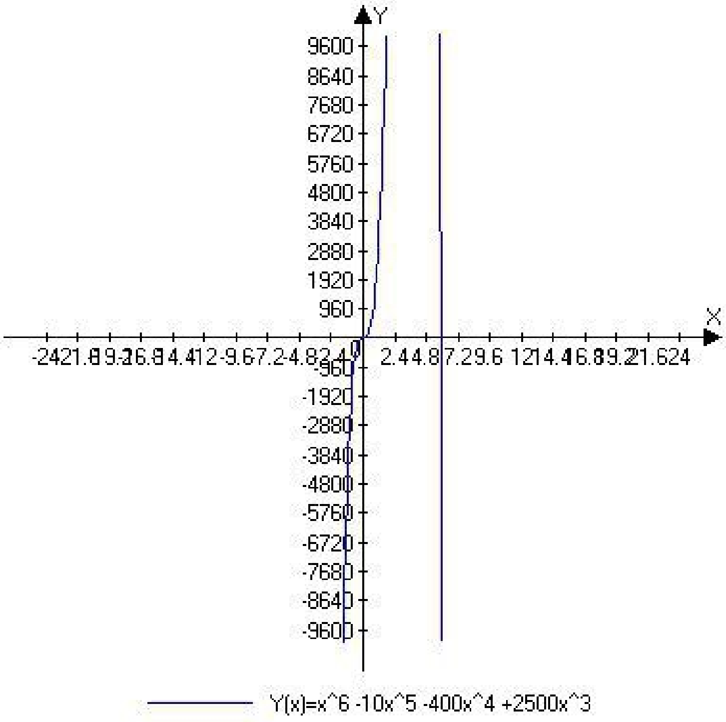
Chapter 3 Applications of Differentiation Exercise 3.6. 3E

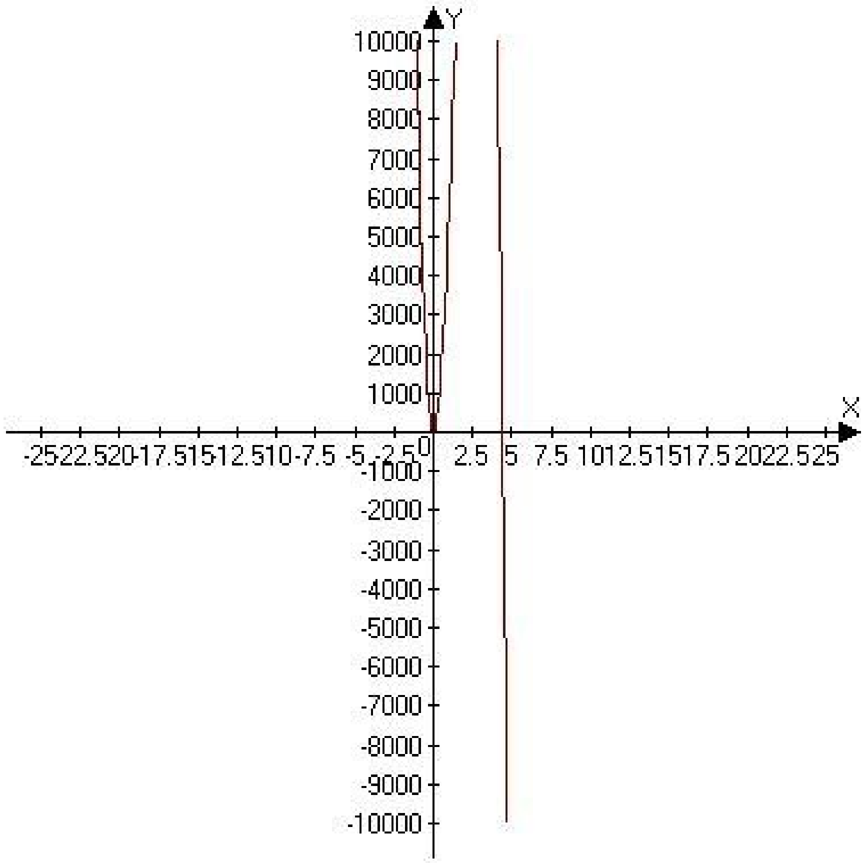
$f(x) = x^6 - 10x^5 - 400x^4 + 2500x^3$

$f'(x) = 6x^5 - 50x^4 - 1600x^3 + 7500x^2$

$f''(x) = 30x^4 - 200x^3 - 4800x^2 + 15000x$

we graph these functions and follow the nature of the function.




Y
X
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9000
8000
7000
6000
5000
4000
3000
2000
1000
0
-1000
-2000
-3000
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-5000
-6000
-7000
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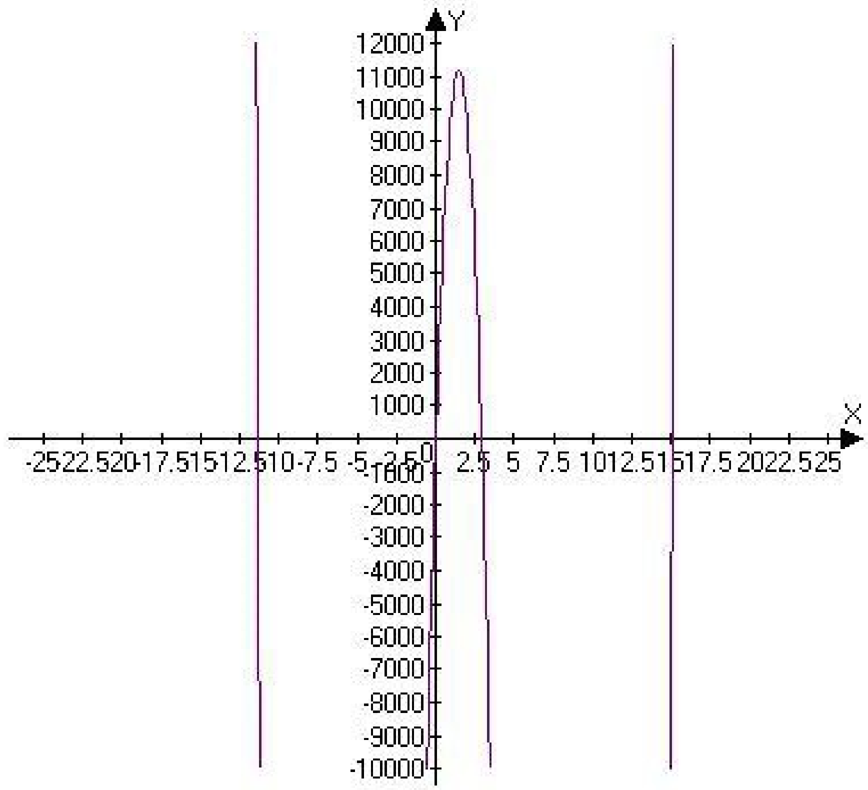
-9000
-10000
-2522.520-17.51512.510-7.5 -5 -2.5
2.5 5 7.5 1012.51517.5 2022.525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-4000
-5000
-6000
-7000
-8000
-9000
-10000
-2522.520-17.51512.510-7.5 -5 -2.5
2.5 5 7.5 1012.51517.5 2022.525

the functions f , f ' , f " are drawn in the order from top.

clearly the given function f is defined for every real number.

so, domain is R.

from f ' , we follow that f ' > 0 when x is in (-15,4.4) ,(18.93, ∞)

while f ' < 0 when x is in (-∞ , -15) , (4.4 , 18.93).

we know that when f ' > 0, f is increasing and f ' < 0 ==> f is decreasing on that given interval.

so, f is increasing on (-15,4.4) ,(18.93, ∞) and decreasing on (-∞ , -15) , (4.4 , 18.93)

further, f ' (x) = 0 ==> x is a critical number .

∴ the critical numbers of f are -15 ,4.4 , 18.93.

also, f "(4.4) < 0 says f has local maximum at 4.4 which is 53.8

f " (x) > 0 at -15 , 18.93 .so, local minimum exist at these points and the local minima are

f(-15) = -9,700,000. , f(18.93) = -12,700,000

f " (x) = 0 when x is - 11.34 , -8 , 0 , 2.92 , 15.08 .

futher , when x isin (-11.34 , -8) , (0 , 2.92) , (15.08 , ∞) , f "(x) > 0. so, in these intervals the function has concavity upwards .

when x is in (-8, 0),(2.92 , 15.08) , f "(x) < 0 . so, f has downward concavity in these intervals.

to find the points of inflection, we find the images of the points - 11.34 , -8 , 0 , 2.92 , 15.08 in f .

∴ the points of inflection are (0,0) , (-11.34 , -6,250,000) , (2.92, 31,800) ,

(15.08 , -8,150,000).

Chapter 3 Applications of Differentiation Exercise 3.6. 4E

$$f\left(x\right) = \frac{x^2 - 1}{40x^3 + x + 1} \quad f'\left(x\right) = \frac{121x^2 - 40x^4 + 2x + 1}{\left(40x^3 + x + 1\right)^2}$$

$$f''\left(x\right) = \frac{\left\{\left(1600x^6 + 80x^4 + 80x^3 + x^2 + 2x + 1\right)\left(242x - 160x^3 + 2\right)\right\} - 2\left\{\left(4800x^5 - 1600x^7 + 40x^4 + \right.\right.}{\left(40x^3 + x + 1\right)^4}$$

the given function doesnot exist when the denominator is zero.

the denominator is zero when x = -0.264001093.

so, the domain of f is R -{ -0.264001093}

f ' (x) = 0 when x = -1.73334 , 1.74978

so, these are the critical numbers of the given function.

now, f " (-1.73334) = 0.0095844892 > 0.

so, the function f has local minimum at -1.73334 and the minimum value = f(-1.73334) =

similarly, f " (1.74978) = -0.0091647597 < 0

so, f has local maximum at 1.74978 and the maximum is f(1.74978) =

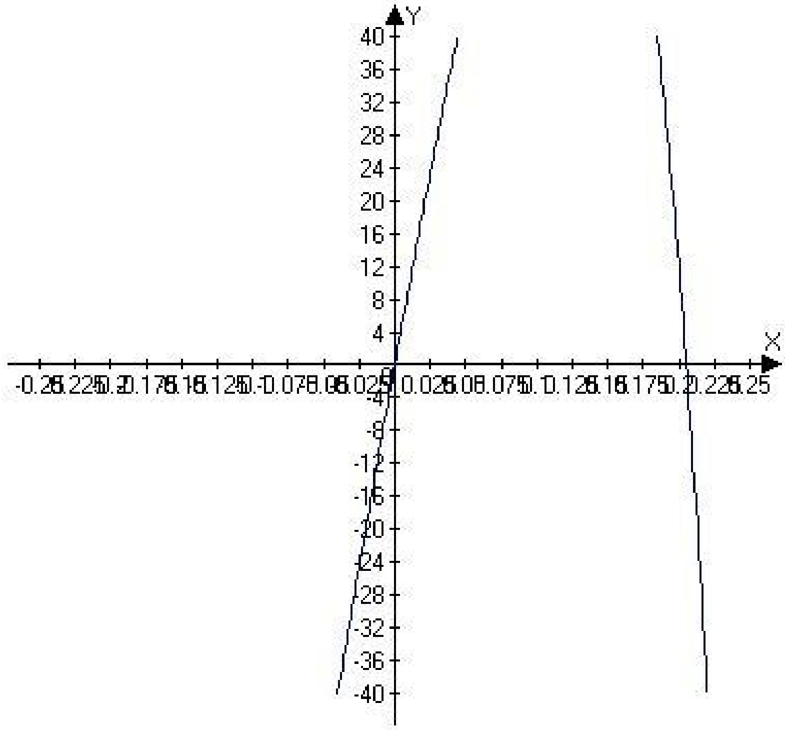
further, f " (x) =0 when x = 2.465998.

when x is in (0 ,0.12537128), (0.17235123, 0.2081987602) , f " (x) > 0 . so, f has concavity upwards on (0 ,0.12537128), (0.17235123, 0.2081987602) and

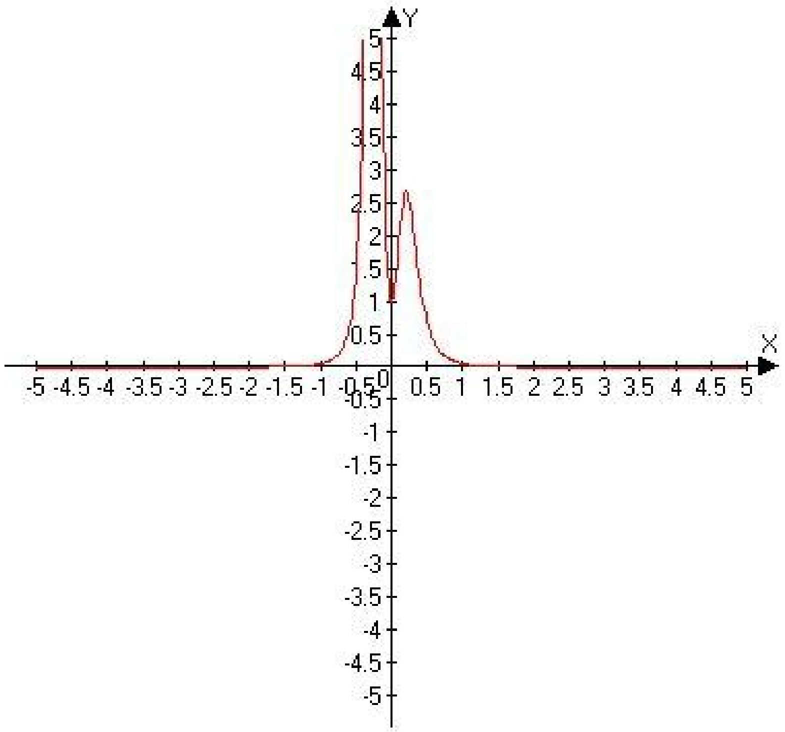
when x is in (-∞ , 0) , (0.12537128 , 0.17235123) ,(0.2081987602, ∞) , f " (x) < 0 . so, f has concavity down wards on (-∞ , 0) , (0.12537128 , 0.17235123) ,

(0.2081987602, ∞) .

now, the graphs of these functions are as follows to confirm the above details :



)x^4+80x^3+x^2+2x+1)(242x-160x^3+2)-2(4800x^5-1600x^7+40x^4+161x^3+123x^2



Y(x)=(121x^2-40x^4+2x+1)/(40x^3+x+1)^2

Chapter 3 Applications of Differentiation Exercise 3.6. 5E

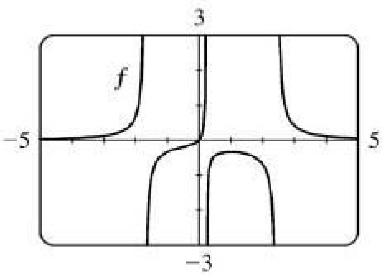
First lets find our 1st and 2nd derivatives;

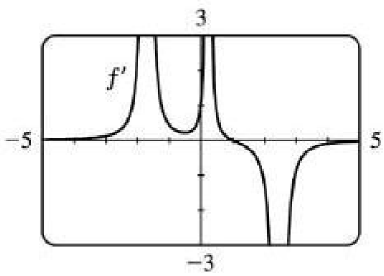
$$f(x) = \frac{x}{x^3 - x^2 - 4x + 1}$$

$$f'(x) = \frac{-2x^3 + x^2 + 1}{(x^3 - x^2 - 4x + 1)^2}$$

$$f''(x) = \frac{2(3x^5 - 3x^4 + 5x^3 - 6x^2 + 3x + 4)}{(x^3 - x^2 - 4x + 1)^3}$$

Now lets look at the graphs of the derivatives;





We estimate that the graph of f that y=0 is a horizontal asymptote, and there are three vertical asymptotes at x= -1.7, x=0.24, and x=2.46

From the graph of f ', we estimate that f is increasing on (-∞, -1.7), (-1.7, 0.24), and

(0.24, 1).

f is decreasing on (1,2.46) and (2.46,∞). There is also a local maximum at $f(1) = -\frac{1}{3}$

From the graph of f ", we estimate that f is concave up on (-∞,-1.7), (-0.506,0.24), and (2.46,∞)

and that f is concave down on (-1.7,-0.506) and (0.24,2.46).

There is also an inflection point at (-0.506,-0.192)

Chapter 3 Applications of Differentiation Exercise 3.6. 6E

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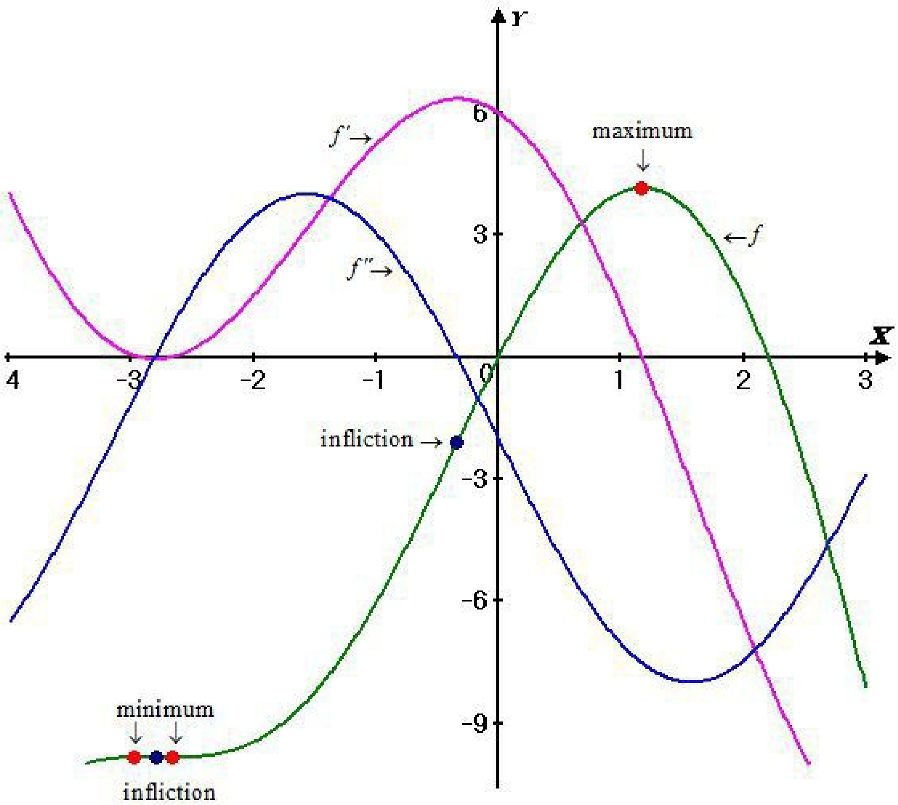
Verify these using the graphs of f, f', f''



From the above graphs

$f'(x)=0 \Rightarrow x=-2.94,-2.66,1.17$

$f'(x)>0$ for $(-5,-2.94)\cup(-2.66,1.17)$ and $f'(x)<0$ for $(-2.94,-2.66)\cup(1.17,3)$

Local maximum occurs at $\boxed{x=-2.94,1.17}$

Local minimum occurs at $\boxed{x=-2.66}$

Local maximum values are $\boxed{f(-2.94)=-9.84, f(1.17)=4.15}$

Local minimum value is $\boxed{f(-2.66)=-9.85}$

Here $f''(x)=0 \Rightarrow x=-2.8,-0.34$

And $f''(x)>0$ for $(-2.8,-0.34)$

$f''(x)<0$ for $(-5,-2.8)\cup(-0.34,3)$

Hence $f(x)$ is concave up on $\boxed{(-2.8,-0.34)}$

And concave down on $\boxed{(-5,-2.8)\cup(-0.34,3)}$

Hence the inflection points are $\boxed{(-2.8,-9.85),(-0.34,-2.12)}$.

Chapter 3 Applications of Differentiation Exercise 3.6. 7E

Curve sketching is done by making use of different characteristics such as determining the domain, range, asymptotes, intercepts, interval of increase or decrease and concave.

Based on the above measure, the curve is sketched.

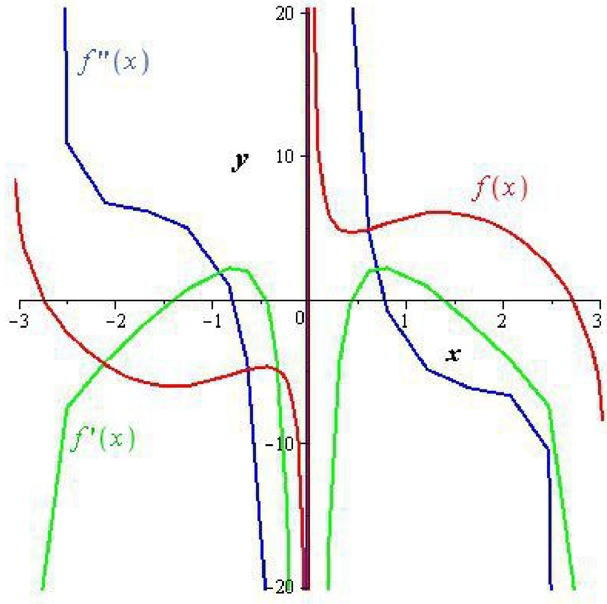
Consider the function:

$f(x) = 6\sin x + \cot x, \qquad -\pi \le x \le \pi$

Determine the first and second derivative of the above function:

$$f(x) = 6\sin x + \cot x$$
$$f'(x) = 6\cos x - \operatorname{cosec}^2 x$$
$$f''(x) = -6\sin x - 2\operatorname{cosec} x\left(-(\operatorname{cosec} x)(\cot x)\right)$$
$$= 2\operatorname{cosec}^2 x \cot x - 6\sin x$$

Consider the graph of the function as shown below:



Observe the graph to obtain the results shown below:

The function $f(x)$ increases on the interval $(-1.40,-0.44),(0.44,1.40)$ because $f'(x)>0$ for the interval.

The function $f(x)$ decreases on the interval $(-\pi,-1.40),(-0.44,0),(0.44),(1.40,\pi)$ because $f'(x)<0$ for the interval.

Consider the condition shown below:

$$x = -0.44$$
$$f'(x) = 0$$

Also the sign of the derivative changes from positive to negative at the above point.

So, the local minimum value of the function is:

$f(-0.44) \approx -4.68$

Consider the condition shown below:

$$x = 1.40$$
$$f'(x) = 0$$

The sign $f'(x)$ changes from positive to negative at the above point.

So, the local maximum value of the function is:

$f(1.40) \approx 6.09$

$f(0.44) \approx 5.22$

Consider the value of the second derivative over the interval $(-\pi,0.77),(0,0.77)$:

$f''(x)>0$

So, the function is concave upward on the interval $(-\pi,-0.77),(0,0.77)$.

Consider the value of the second derivative over the interval $(-0.77,0),(0.77,\pi)$:

$f''(x)<0$

So, the function is concave down on $(-0.77,0),(0.77,\pi)$.

The inflection points are $(-0.77,-5.22),(0.77,5.22)$.

Chapter 3 Applications of Differentiation Exercise 3.6. 8E

$$f(x)=\frac{\sin x}{x},\quad -2\pi\le x\le 2\pi$$

f ' (x) = cosx / 1

f " (x) = - sinx

observe that f exists at 0 also, the domain of f is [-2π , 2π]

$$f'(x)=0\Rightarrow x=\frac{-3\pi}{2},\ \frac{-\pi}{2},\ \frac{\pi}{2},\ \frac{3\pi}{2}$$

so, these are the critical points of f.

f '(x) < 0 when x is in $\left(\frac{-3\pi}{2},\frac{-\pi}{2}\right),\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$.

so, f is decreasing on $\left(\frac{-3\pi}{2},\frac{-\pi}{2}\right),\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$

f ' (x) > 0 when x is in $\left(-2\pi,\frac{-3\pi}{2}\right),\left(\frac{-\pi}{2},\frac{\pi}{2}\right),\left(\frac{3\pi}{2},2\pi\right)$

so, f is increasing on $\left(-2\pi,\frac{-3\pi}{2}\right),\left(\frac{-\pi}{2},\frac{\pi}{2}\right),\left(\frac{3\pi}{2},2\pi\right)$.

substituting the critical values in f " , we have

$$f''\left(\frac{-3\pi}{2}\right) = -1, f''\left(\frac{-\pi}{2}\right) = 1, f''\left(\frac{\pi}{2}\right) = -1, f''\left(\frac{3\pi}{2}\right) = 1$$

so, f has local maxima at $\frac{-3\pi}{2}, \frac{\pi}{2}$ and local minima at $\frac{-\pi}{2}, \frac{3\pi}{2}$

and the local maxima for f are 0.63636 and the local minimum is -0.21212

f " (x) = 0 when x = -2π , -π , π, 2π

so, the points of infliction are (-π , -0.3333) , (π, -0.3333).

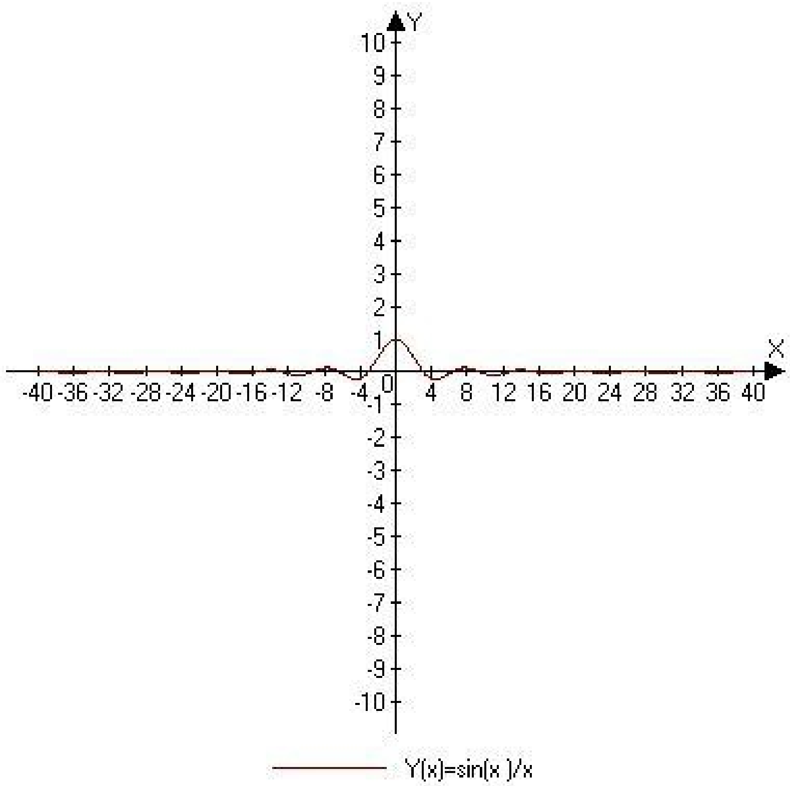
observe that when x is in (-2π , -π) and (0 , π) , f"(x) < 0.

so, f has concavity downwards in these intervals.

on the other hand , when x is in (-π , 0) , (π , 2π) , f "(x) > 0.

so, f has concavity upwards in these intervals.

we see these facts in the following graphs in the order f , f ' , f ".



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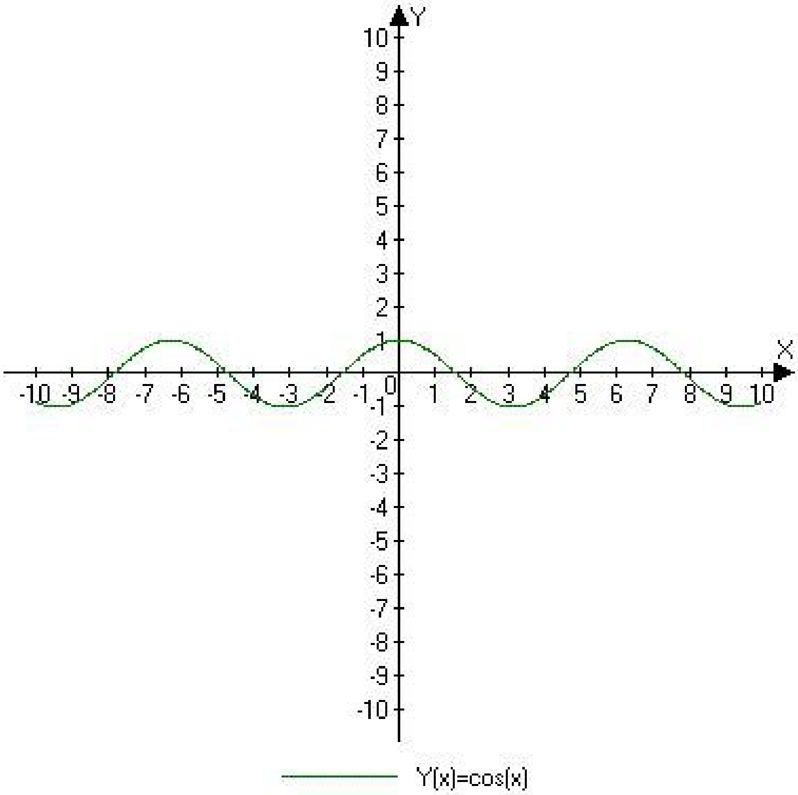
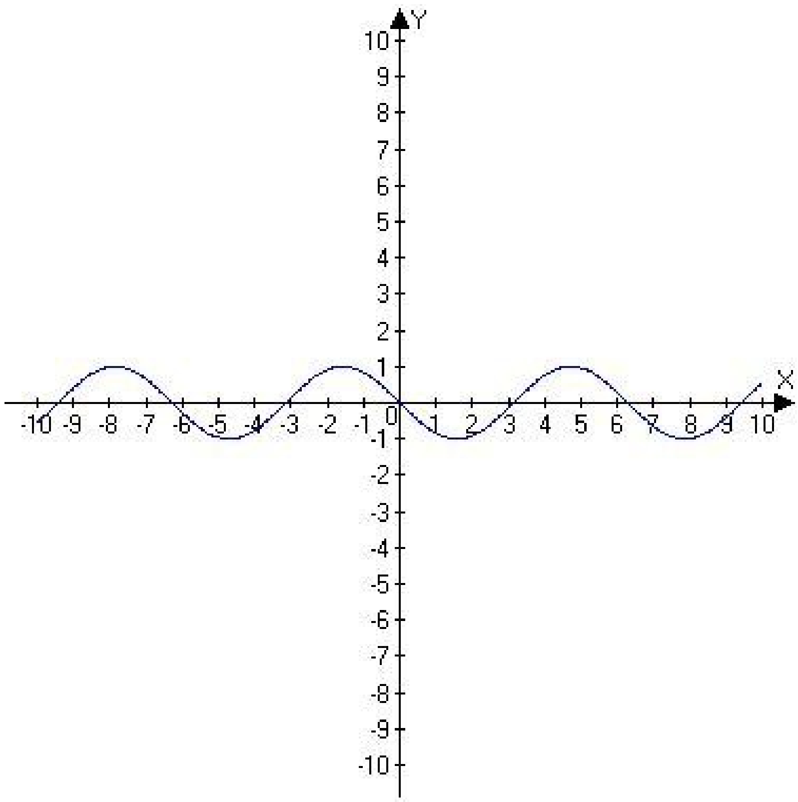

Y
X
Y(x)=cos(x)

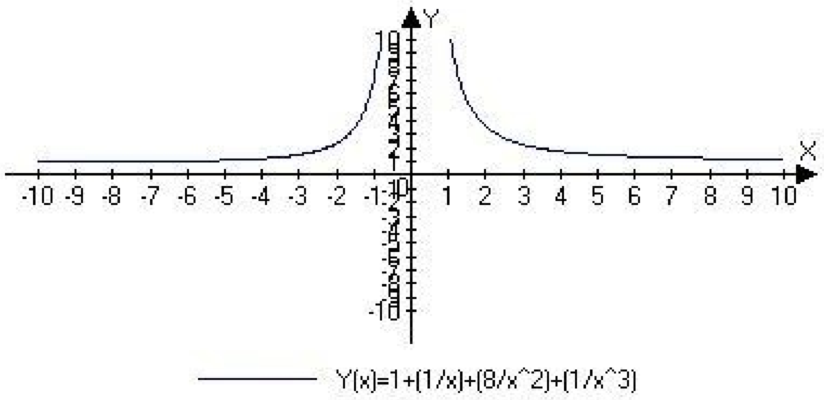

Y
X

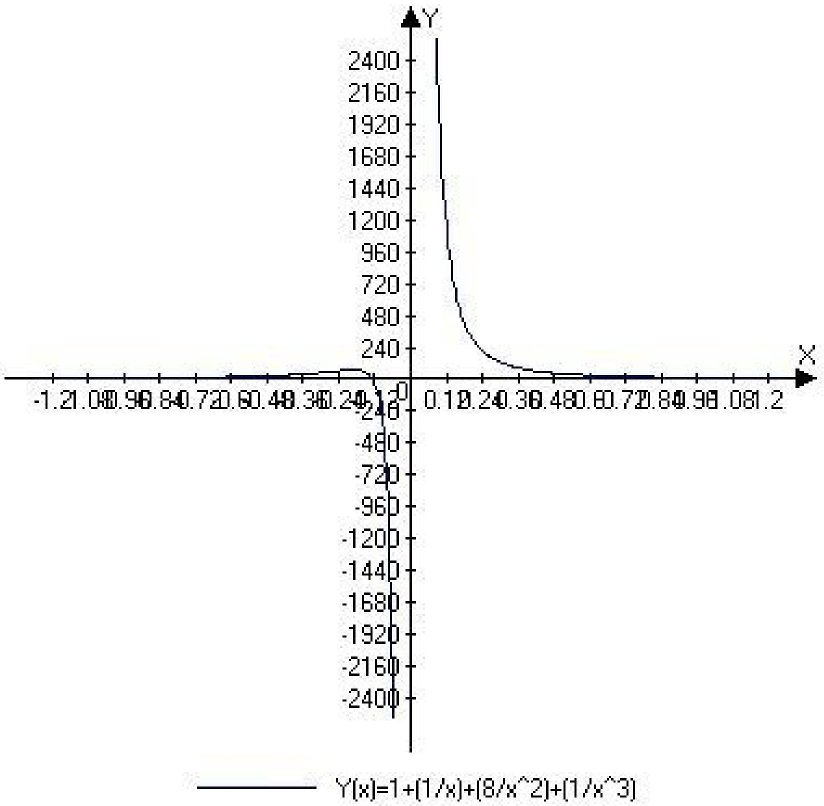
>

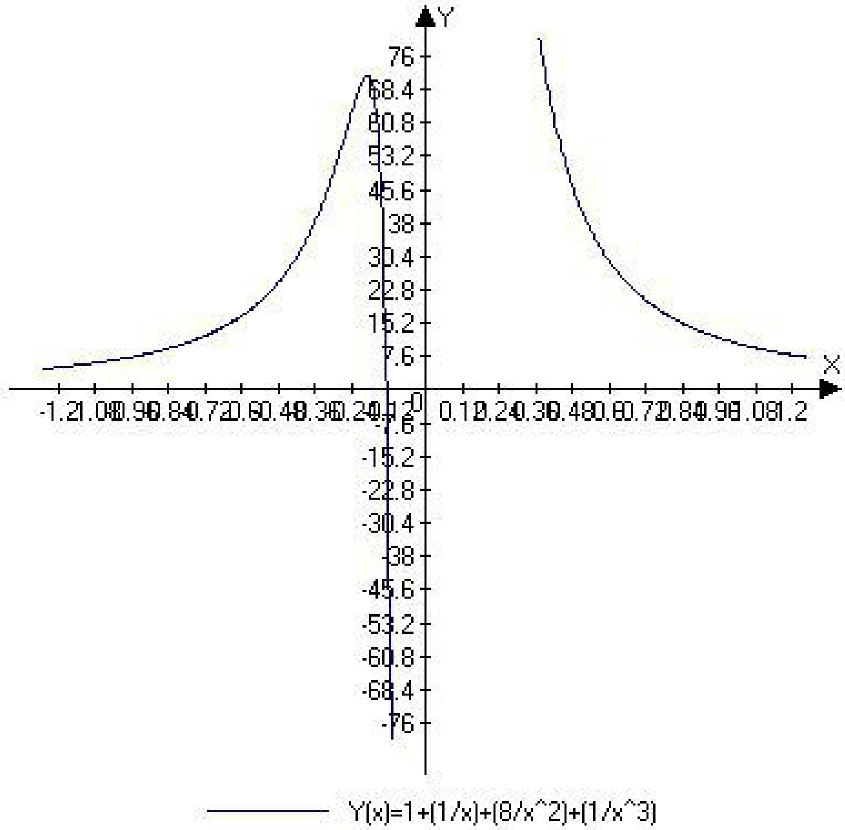
Chapter 3 Applications of Differentiation Exercise 3.6. 9E

$$f\left(x\right) = 1 + \frac{1}{x} + \frac{8}{x^2} + \frac{1}{x^3}$$

graph of the function is as follows :







all the above three graphs refer the given function itself.

from these graphs we follow that the function is increasing on (-15.8102 , -0.1897),

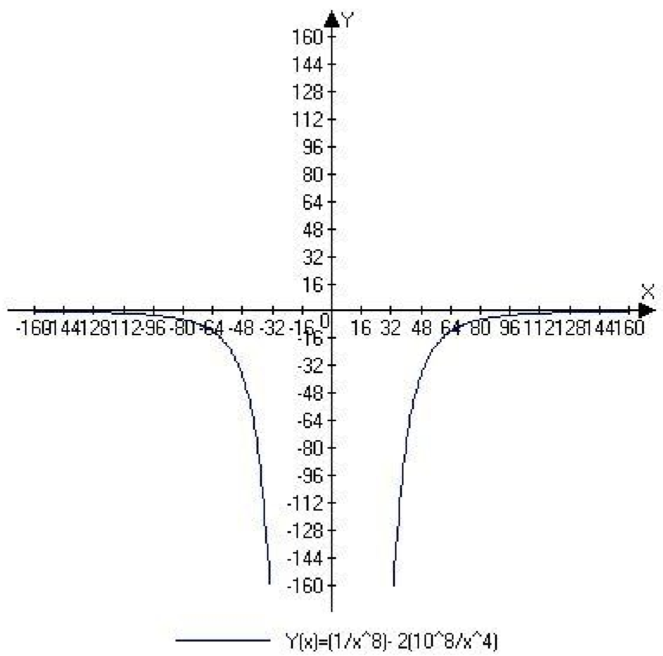
decreasing on (- ∞ , -15.8102) ,(-0.1897 , 0), (0 , ∞).

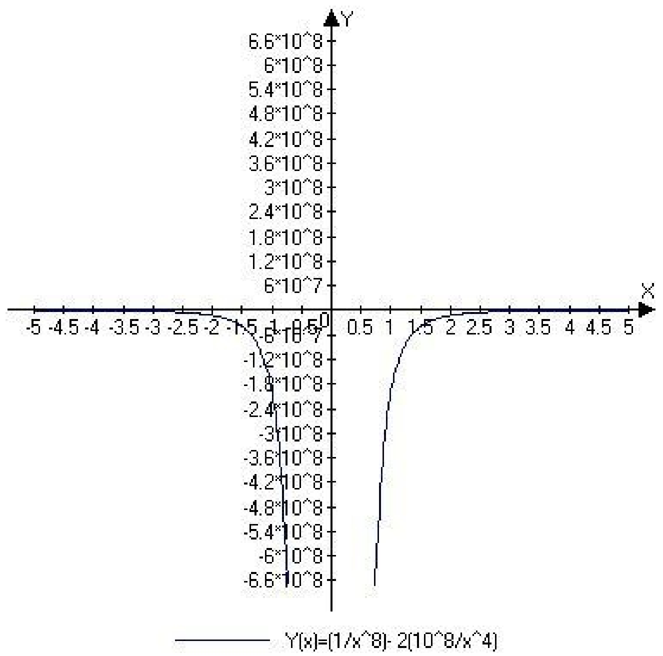
the curve has concavity upwards on (-23.747 , -0.25265), (0 , ∞)

while the concavity downwards on (- ∞ , -23.747) , (-0.25265 , 0).

$$f\left(x\right)=\frac{1}{x^8}-\frac{2*10^8}{x^4}$$

we graph this function and find out the details there of :





from the graph we follow that the graph is increasing on (0 , ∞) , decreasing on (-∞ , 0) .

the graph has maximum at (-∞ , 0) and (∞ , 0) and minimum at (0 , -∞).

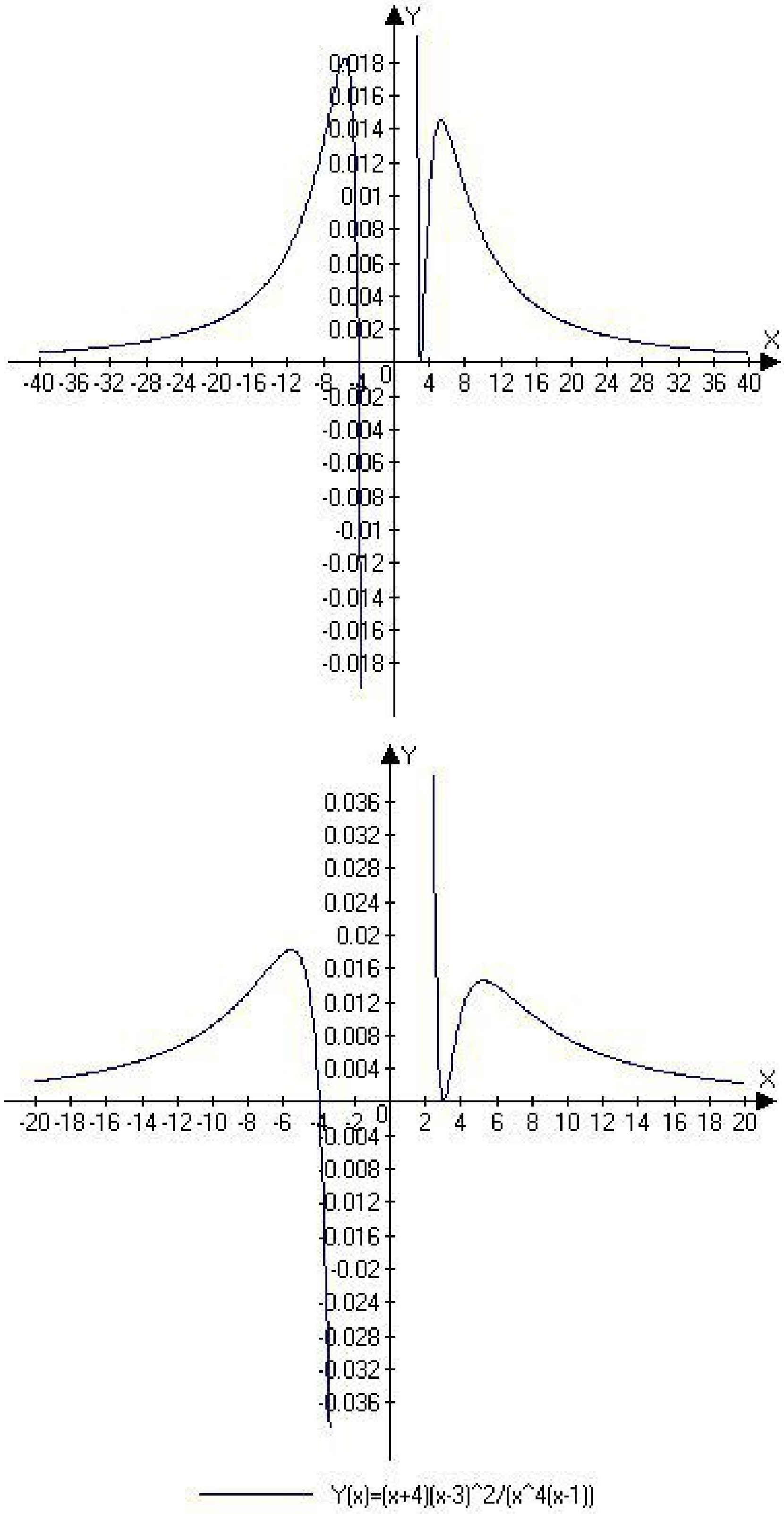
in other words the function has horizontal assymptotes at y = 0 and verticle assymptotes at x = 0.

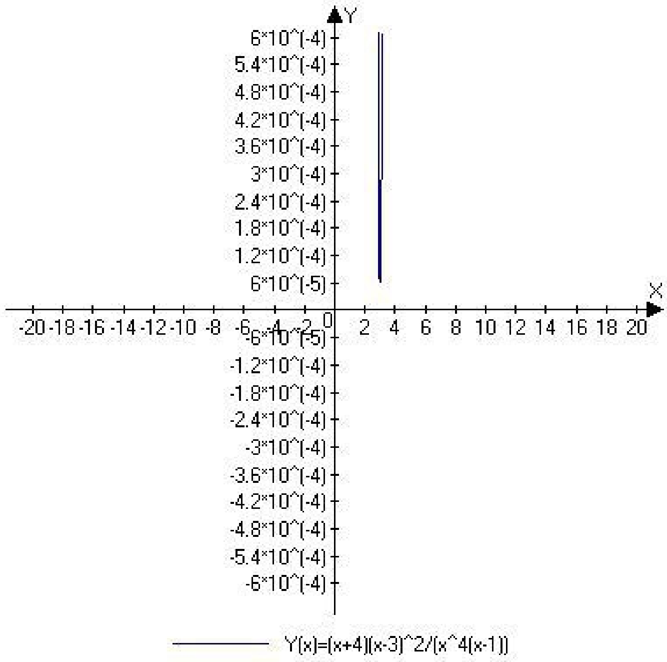
the graph has concavity down wards only on (- ∞ , 0) and (0 , ∞).

Chapter 3 Applications of Differentiation Exercise 3.6. 11E

$$f\left(x\right) = \frac{(x+4)(x-3)^2}{x^4\left(x-1\right)} = \frac{x^3\left(1+\frac{4}{x}\right)\left(1-\frac{3}{x}\right)^2}{x^5\left(1-\frac{1}{x}\right)} = \frac{\left(1+\frac{4}{x}\right)\left(1-\frac{3}{x}\right)^2}{x^2\left(1-\frac{1}{x}\right)}$$

the graph of this function is as follows :





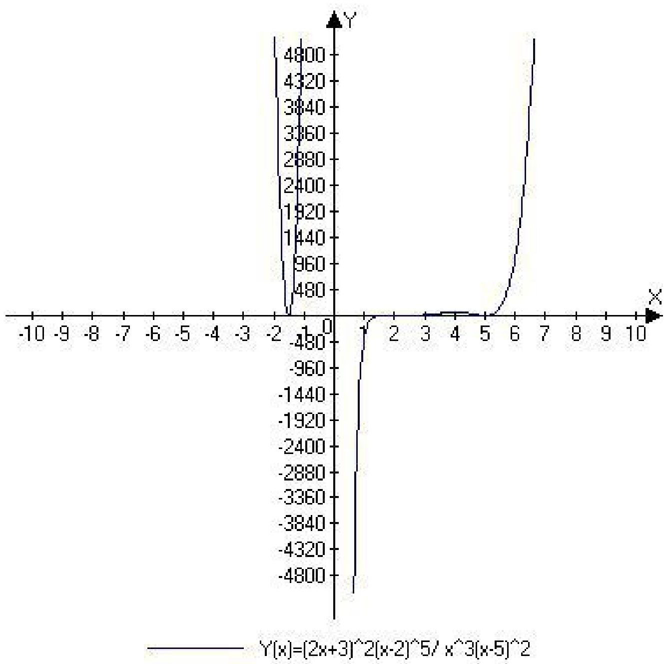
all the three diagrams show the function and see that the curve is not touching x axis at its minimum while the local minimum is (3.02 , 6.4663*10-5) approximately (3,0)

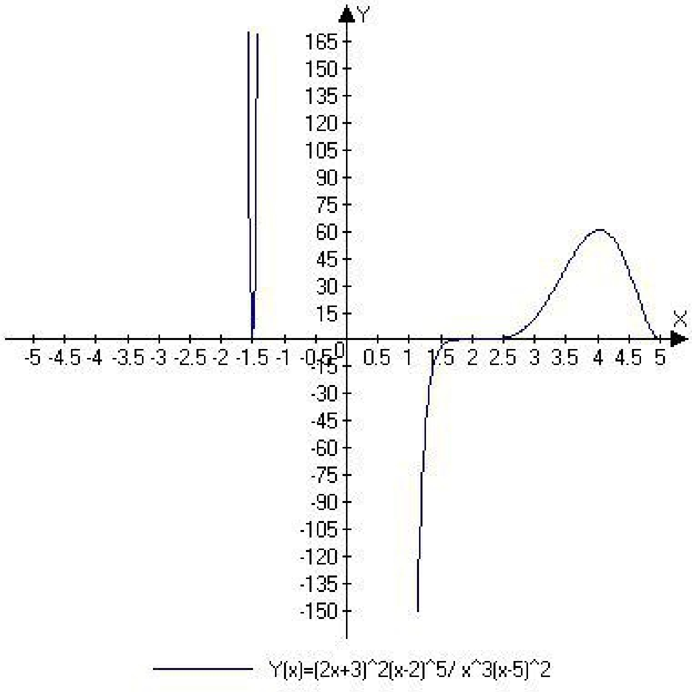
while the local maximum is from the first two figures f(-5.6) = 0.018, f (0.820 = -281.5 , f(5.2) = 0.0145.

Chapter 3 Applications of Differentiation Exercise 3.6. 12E

$$f\left(x\right) = \frac{(2x+3)^2(x-2)^5}{x^3(x-5)^2} = \frac{x^7\left(2+\frac{3}{x}\right)^2\left(1-\frac{2}{x}\right)^5}{x^5\left(1-\frac{5}{x}\right)^2} = \frac{x^2\left(2+\frac{3}{x}\right)^2\left(1-\frac{2}{x}\right)^5}{\left(1-\frac{5}{x}\right)^2}$$

the graph of this function is as follows from which we decide the nature of the function .





note that both the graphs belong to the same function and the second one is a close view of the curve from which we follow that the local maximum is (4.047 , 60.4) while the local minimum is (-1.51 , 9).

Chapter 3 Applications of Differentiation Exercise 3.6. 13E

$$f\left(x\right) = \frac{x^2(x+1)^3}{(x-2)^2(x-4)^4}$$

the first and second derivatives using computer algebraic system are

$$f'\left(x\right) = -\frac{4x^2(x+1)^3}{(x-4)^5(x-2)^2} - \frac{2x^2(x+1)^3}{(x-4)^4(x-2)^3} + \frac{2x(x+1)^3}{(x-4)^4(x-2)^2} + \frac{3x^2(x+1)^2}{(x-4)^4(x-2)^2}$$

$$= -\frac{x(x+1)^2\left(x^3+18x^2-44x-16\right)}{(x-2)^3(x-4)^5}$$

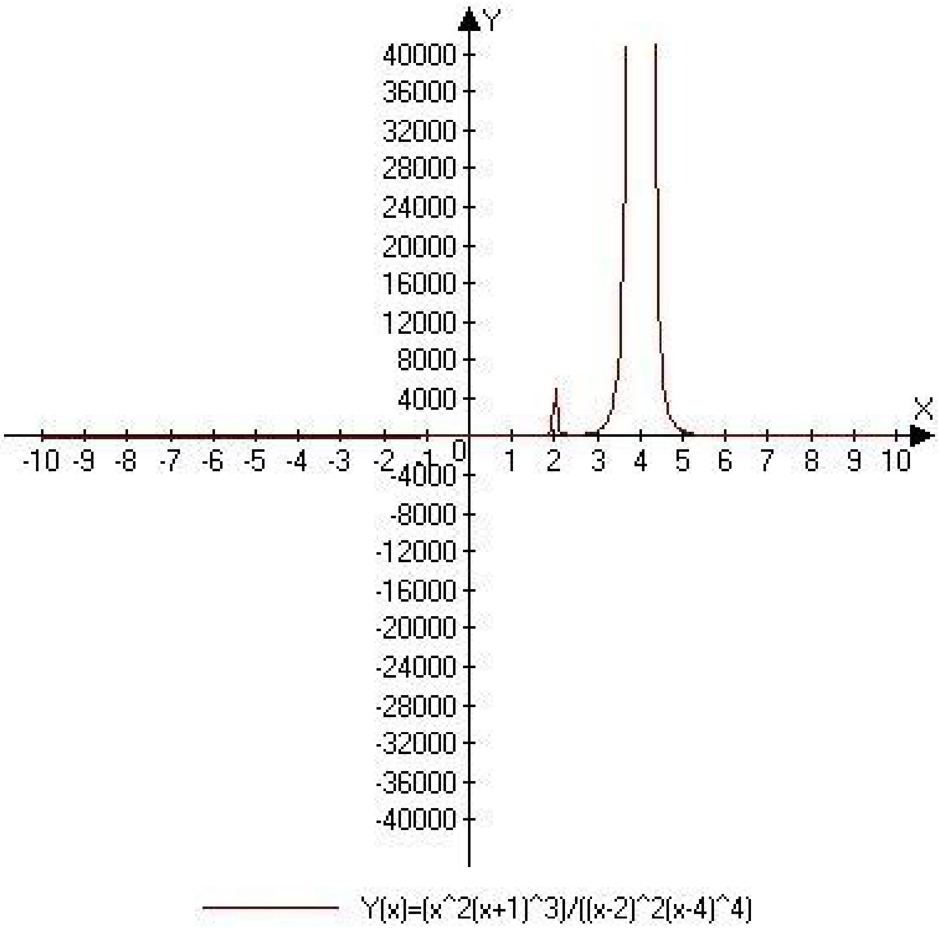
$$f''\left(x\right) = -\frac{x\left(3x^2+36x-44\right)\left(x+1\right)^2}{(x-4)^5(x-2)^3} + \frac{5x\left(x^3+18x^2-44x-16\right)(x+1)^2}{(x-4)^6(x-2)^3} +$$

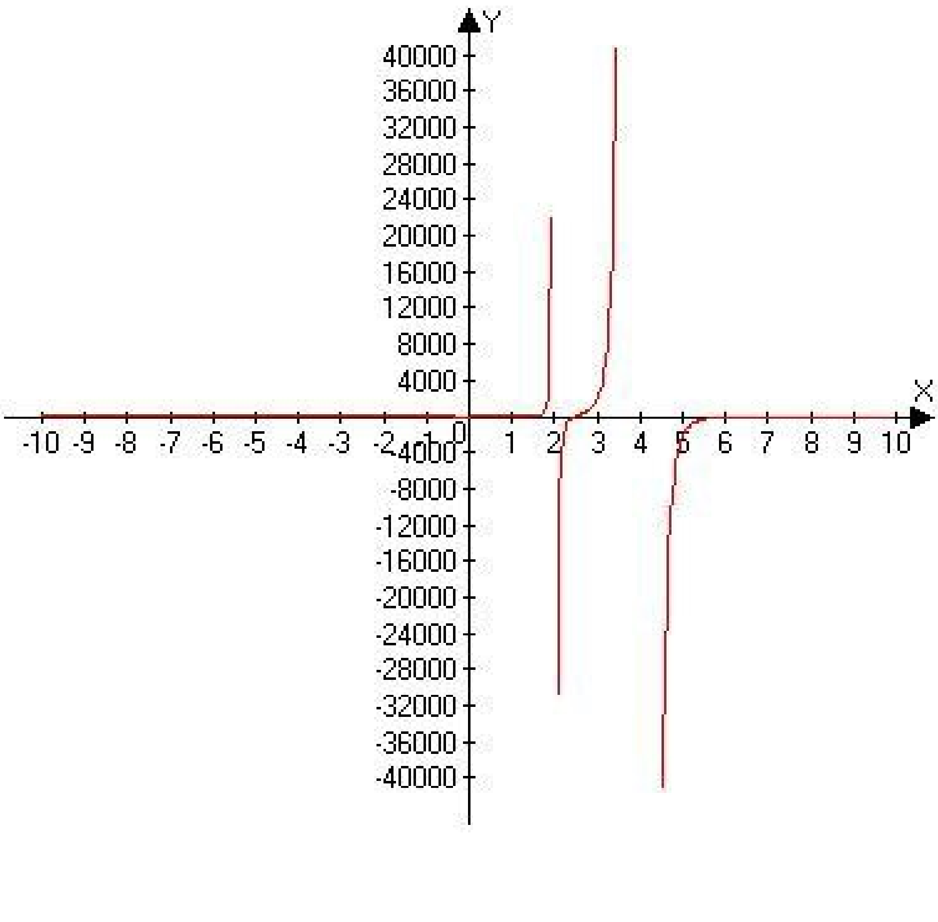
$$\frac{3x\left(x^3+18x^2-44x-16\right)\left(x+1\right)^2}{(x-4)^5(x-2)^4} - \frac{\left(x^3+18x^2-44x-16\right)(x+1)^2}{(x-4)^5(x-2)^3}$$

$$-\frac{2x\left(x^3+18x^2-44x-16\right)\left(x+1\right)}{(x-4)^5(x-2)^3}$$

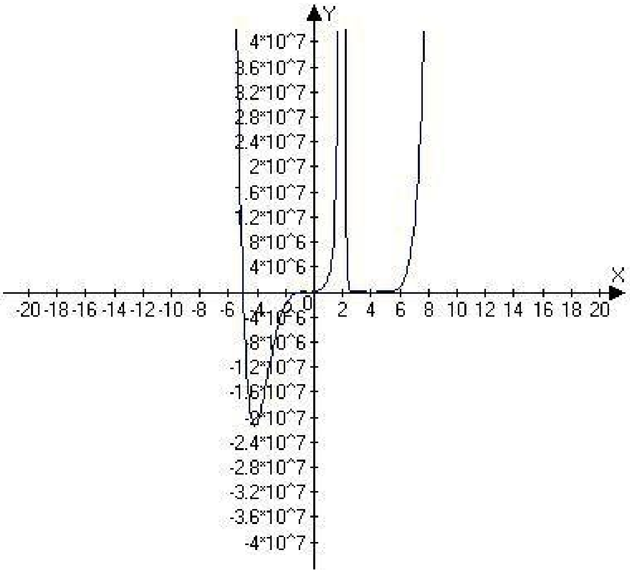
after simplification, we get $f''\left(x\right) = \frac{2\left\{\left(x+1\right)\left(x^6+36x^5+6x^4-628x^3+684x^2+672x+64\right)\right\}}{(x-2)^4(x-4)^6}$

now, we draw the graphs of these functions to find f ' (x) = 0 , < 0 and > 0 to identify the critical points , intervals of increasing , decreasing , local maximum, minimum,concavity upwards and downwards and points of infliction.


Y
X
Y(x)=(x^2(x+1)^3)/((x-2)^2(x-4)^4)


Y
X

x(x+1)^3+x^2*3*(x+1)^2)*(x-2)^2*(x-4)^4-x^2*(x+1)^3*(2*(x-2)*(x-4)^4+(x-2)^2*4*(x-4)



from the above graphs we observe that the graph has concavity upwards in (-35.3, -5) ,(-1 , -0.5) , (-0. 1 , 2) , (2 , 4) ,(4 , ∞).

the concavity down wards is (- ∞ , -35.3) ,(-5.0 , -1) ,(-0.5 , -0.1)

the points of infliction are (-35.3 , -0.015) , (-5, -0.005) , (-1 , 0) ,(-0.5, 0.00001),

(-0.1 , 0.0000066).

Chapter 3 Applications of Differentiation Exercise 3.6. 14E

$$f\left(x\right) = \frac{(2x+3)^2(x-2)^5}{x^3(x-5)^2}$$

using the computer algebraic system we find f ' and f " the judge the nature of the function.

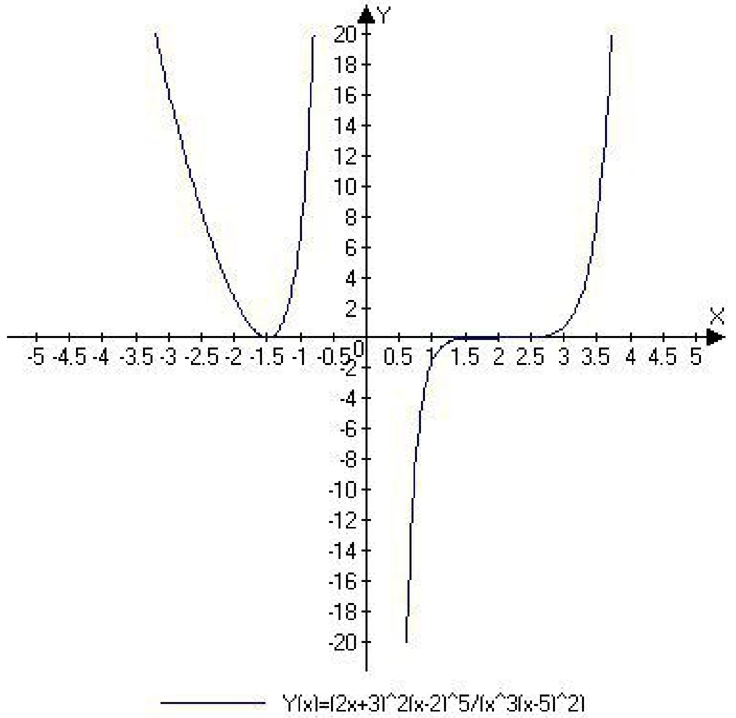
$$f'\left(x\right) = \frac{2\left(x-5\right)(2x+3)^2(x-2)^5}{x^3} - \frac{3(x-5)^2(2x+3)^2(x-2)^5}{x^4} + \frac{4(x-5)^2(2x+3)(x-2)^5}{x^3} + \frac{5(x-5)^2(2x+3)^2(x-2)^4}{x^3}$$

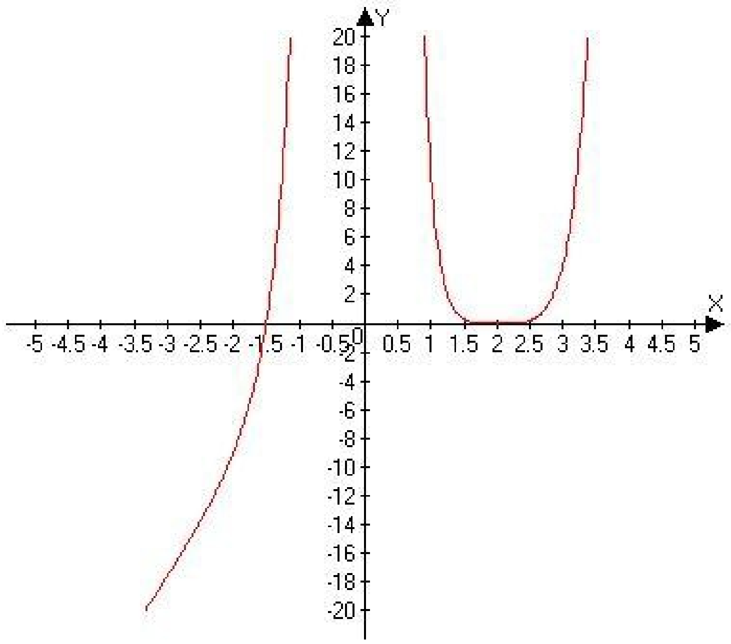
$$f''\left(x\right) = -\frac{15(2x+3)^2(x-2)^6}{x^4} + \frac{20\left(2x+3\right)(x-2)^6}{x^3} + 32\frac{(2x+3)^2(x-2)^5}{x^3} - 12\frac{\left(x-5\right)(2x+3)^2(x-2)^5}{x^4} + 12\frac{(x-5)^2(2x+3)^2(x-2)^5}{x^5} +$$

$$16\frac{\left(x-5\right)\left(2x+3\right)(x-2)^5}{x^3} - 24\frac{(x-5)^2\left(2x+3\right)(x-2)^5}{x^4} + 8\frac{(x-5)^2(x-2)^5}{x^3}$$

$$+10\frac{\left(x-5\right)(2x+3)^2(x-2)^4}{x^3} - 15\frac{(x-5)^2(2x+3)^2(x-2)^4}{x^3} - 15\frac{(x-5)^2(2x+3)^2(x-2)^4}{x^4} + 20\frac{(x-5)^2(2x+3)(x-2)^4}{x^3}$$

we now graph the functions f , f ' , f '' to judge the nature of the function f.





·x+3)*2*(x-2)^5+(2*x+3)^2*5*(x-2)^4)*x^3*(x-5)^2-(2*x+3)^2*(x-2)^5*(3*x^2*(x-5)^2+x^·

the above graphs show f and f ' .

f has local minimum at (-1.51 , 0.00583) and has no local maximum.

the concavity downwards on (0,2) while the concav ity upwards on (- ∞ , -0.55) , (2, ∞)

the points of infliction are (1.83 , -7.23*10-5) , (2.81 , 1.302*10 -4)

Chapter 3 Applications of Differentiation Exercise 3.6. 15E

Let us consider the $f(x)=\dfrac{x^3+5x^2+1}{x^4+x^3-x^2+2}$

Use CAS, estimate the intervals of increase and decrease, extreme values and inflection points

Using Maple CAS, the first derivative of this function is

$$> \frac{d}{dx}\left(\frac{(x^3+5\cdot x^2+1)}{x^4+x^3-x^2+2}\right)=$$

$$\frac{3x^2+10x}{x^4+x^3-x^2+2}-\frac{(x^3+5x^2+1)(4x^3+3x^2-2x)}{(x^4+x^3-x^2+2)^2}$$

simplify

$$-\frac{x(x^5+10x^4+6x^3-3x+4x^2-22)}{(x^4+x^3-x^2+2)^2}$$

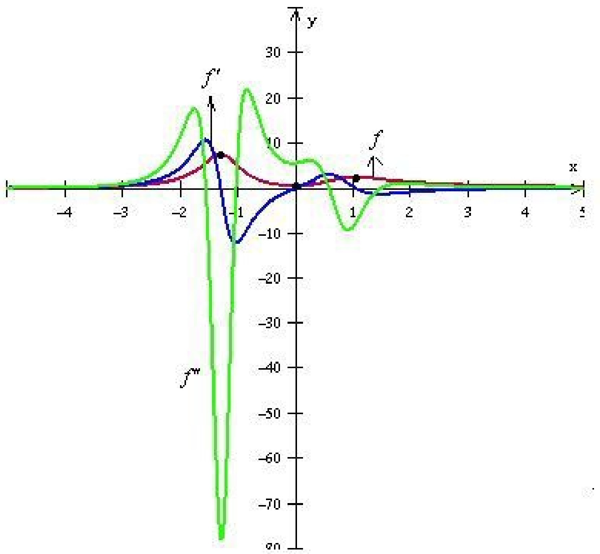
Similarly, the second derivative of the function is

$$> \frac{d}{dx}\left(-\frac{(x^6+10\cdot x^5+6\cdot x^4+4\cdot x^3-3\cdot x^2-22\cdot x)}{(x^4+x^3-x^2+2)^2}\right)$$

$$-\frac{6x^5+50x^4+24x^3+12x^2-6x-22}{(x^4+x^3-x^2+2)^2}+\frac{1}{(x^4+x^3-x^2+2)^3}(2(x^6+10x^5+6x^4+4x^3-3x^2-22x)(4x^3+3x^2-2x))$$

simplify $$\frac{1}{(x^4+x^3-x^2+2)^3}(2(x^9+15x^8+18x^7-9x^5+21x^6-135x^4-76x^3+21x^2+6x+22))$$

Graphs of f, f', f'' are



From the graph it is clear that

$f'(x)=0 \Rightarrow x=-1.29, 0, 1.05$

$f'(x)>0$ for $(-\infty,-1.29)\cup(0,1.05)$ and $f'(x)<0$ for $(-1.29,0)\cup(1.05,\infty)$

Local maximum occurs at $x=-1.29, 1.05$

Local minimum occurs at $x=0$

Local maximum values are $\boxed{f(-1.29)=7.49, f(1.05)=2.35}$

Local minimum value is $\boxed{f(0)=0.5}$.

Also, $f''(x)=0 \Rightarrow x=-1.55,-1.03,0.6,1.48$

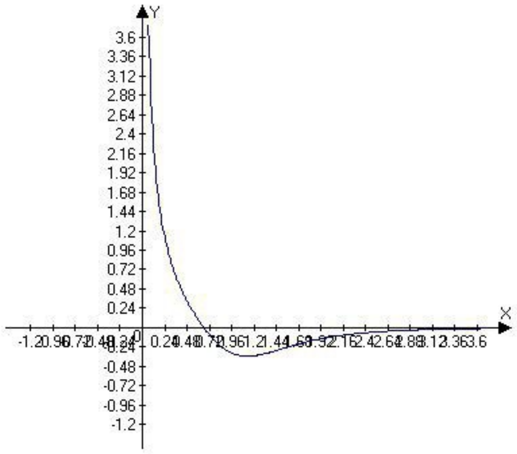
By the rule of signs, we get $f''(x)>0$ for $(-\infty,-1.55)\cup(-1.03,0.6)\cup(1.48,\infty)$

$f''(x)<0$ for $(-1.55,-1.03)\cup(0.6,1.48)$

Therefore, $f(x)$ is concave up on $\boxed{(-\infty,-1.55)\cup(-1.03,0.6)\cup(1.48,\infty)}$

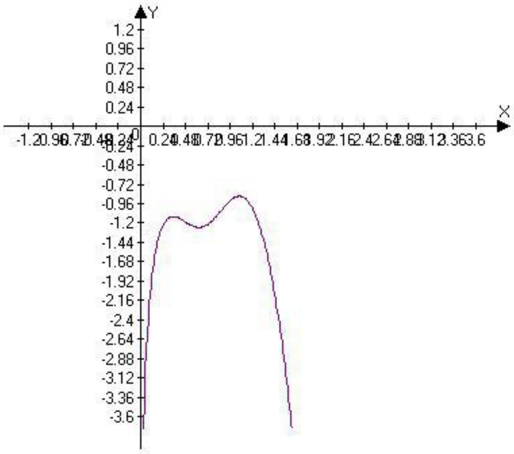
And concave down on $\boxed{(-1.55,-1.03)\cup(0.6,1.48)}$

Hence the inflection points are $\boxed{(-1.55,5.64),(-1.03,5.36),(0.6,1.52),(1.48,1.94)}$



Y(x)=

Y(x)=(2/(3(x^(2/3)(x^4+x+1))))-(x^(2/3)(4x^3+1))/(x^4+x+1)^2



3))/(x^4+x+1)^2)+((2(4x^3+1)^2(x^(2/3))/(x^4+x+1)^3))-((4(4x^3+1)/(3(x^4+x+1)^2(x^

the above functions are f , f ' , f " in the order from above.

we easily see that f ' = 0 at x = 0.661.

i.e. f has the critical point at x = 0.661.

f ' > 0 when x when is in (0, 0.661) .==> f is increasing on (0 , 0.661).

so, f has the local minimum at 0 and the minimum value is 0 it self.

f ' < 0 on (0.661 , ∞) . so, f is decreasing on this interval.

so, f attains its maximum value at 0.661 and its maximum value is f(0.661) = 0.4352.

also, we observe that f " < 0 for all x in the domain.

∴ f has concavity down wards only and so there are no points of infliction .

3))/(x^4+x+1)^2)+((2(4x^3+1)^2(x^(2/3))/(x^4+x+1)^3))-((4(4x^3+1)/(3(x^4+x+1)^2(x^

the above functions are f , f ' , f " in the order from above.

we easily see that f ' = 0 at x = 0.661.

i.e. f has the critical point at x = 0.661.

f ' > 0 when x when is in (0, 0.661) .==> f is increasing on (0 , 0.661).

so, f has the local minimum at 0 and the minimum value is 0 it self.

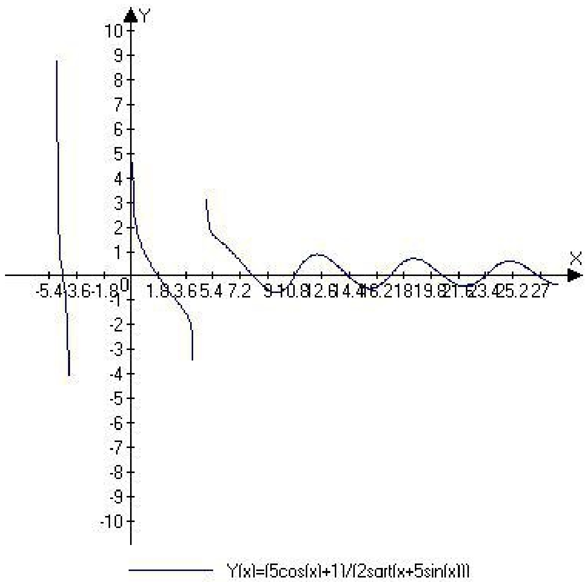
f ' < 0 on (0.661 , ∞) . so, f is decreasing on this interval.

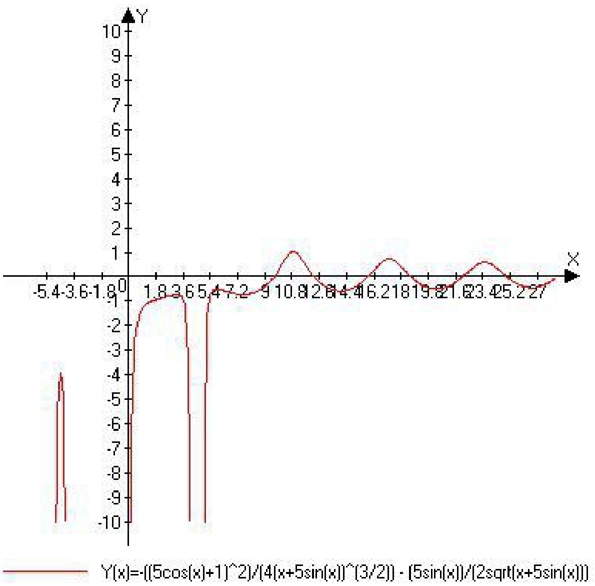
so, f attains its maximum value at 0.661 and its maximum value is f(0.661) = 0.4352.

also, we observe that f " < 0 for all x in the domain.

∴ f has concavity down wards only and so there are no points of infliction .

Chapter 3 Applications of Differentiation Exercise 3.6. 17E





the above curves denote f , f ' , f " in the order from the above.

we can easily see that f is increasing on (-4.91 , -4.51) , (0, 1.77) ,(4.91, 8.06) ,

(10.79 , 14.34), (17.08,20)

and decreasing on (-4.51 , -4.10) ,(1.77 , 4.10) , (8.06, 10.79) , (14.34 , 17.08) .

f has local maximum f(-4.51) = 0.62 , f(1.77) = 2.58 , f(8.06) 3.6 , f(14.34) = 4.39.

f has local minimum f(10.79) = 2.43 , f(17.08) = 3.49 , :

f has concavity upwards on 9.6 , 12.25) , (15.81 , 18.65)

while the concavity downwards on (-4.91, -4.1) ,(0 , 4.1) , (4.91, 9.6) ,(12.25, 15.81), (18.65,20).

the points of infliction are (9.6 , 2.95) , (12.25,3.27) ,(15.81,3.91) ,(18.65,4.2).

Chapter 3 Applications of Differentiation Exercise 3.6. 18E

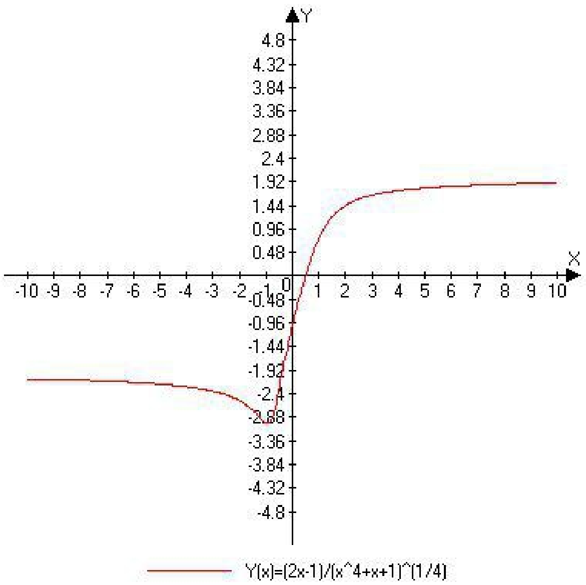
$$f\left(x\right) = \frac{2x-1}{\sqrt[4]{x^4+x+1}}$$

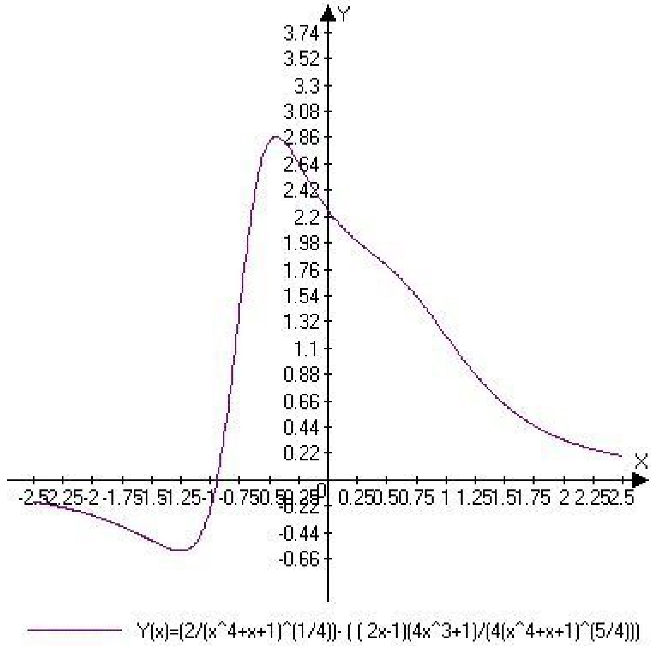
we find f ' , f " using computer algebraic system :

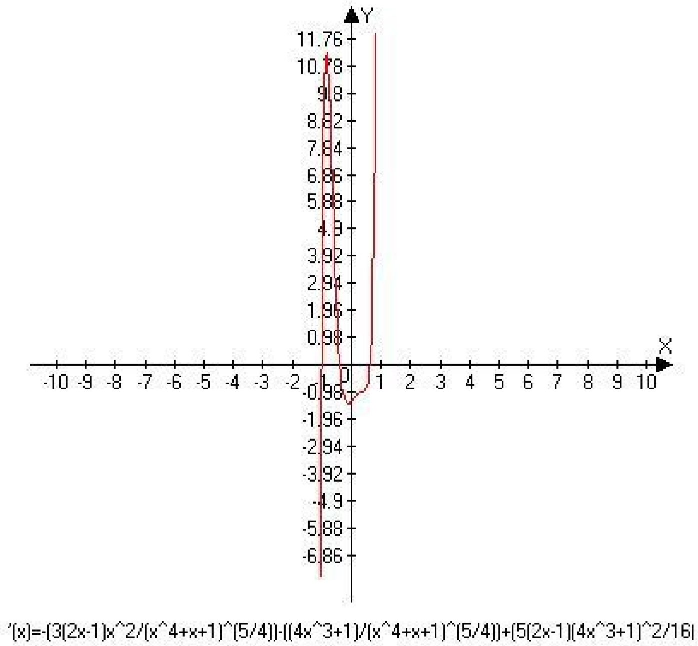
$$f'(x) = \frac{2}{\sqrt[4]{x^4+x+1}} - \frac{(2x-1)(4x^3+1)}{4(x^4+x+1)^{\frac{5}{4}}}$$

$$f''(x) = -\frac{3(2x-1)x^2}{(x^4+x+1)^{5/4}} - \frac{4x^3+1}{(x^4+x+1)^{5/4}} + \frac{5(2x-1)(4x^3+1)^2}{16(x^4+x+1)^{9/4}}$$

we now draw the graphs of these functions and decide the nature of the given function f.







the abovegraphs are f , f ' , f '' in the order.

we follow from these graphs that f has the critical point at -0.938 .

f ' < 0 on (- ∞ , -0.938) , f '> 0 on (-0.938 , ∞).

so, f has local minimum at -0.938 and is equal to -3.027.

also observe that f has no local maximum .

f '' = 0 at x = -1.02 , -0.38 , 0.63.

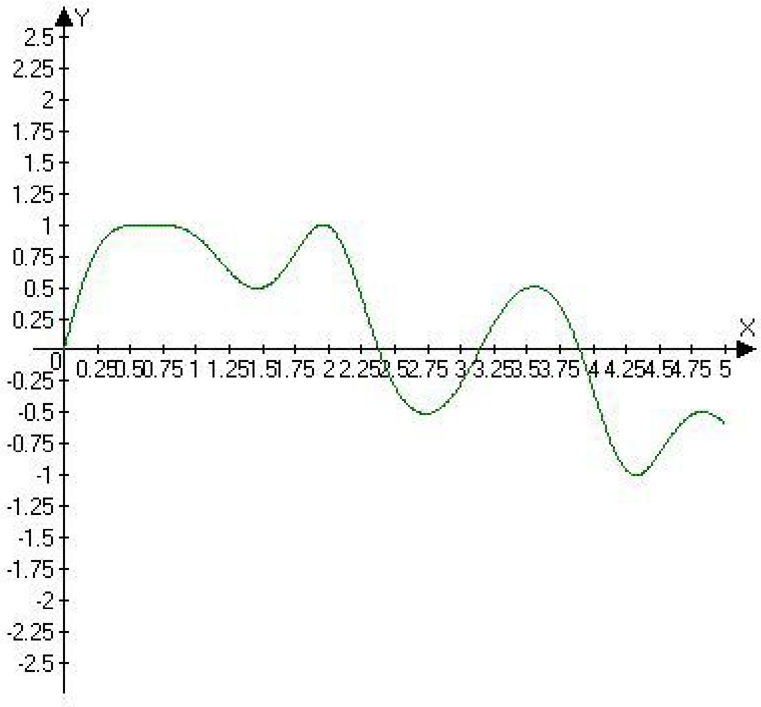
so, the points of infliction are (-1.02 , -3) , (-0.38 , -1.930 ,(0.63,0).

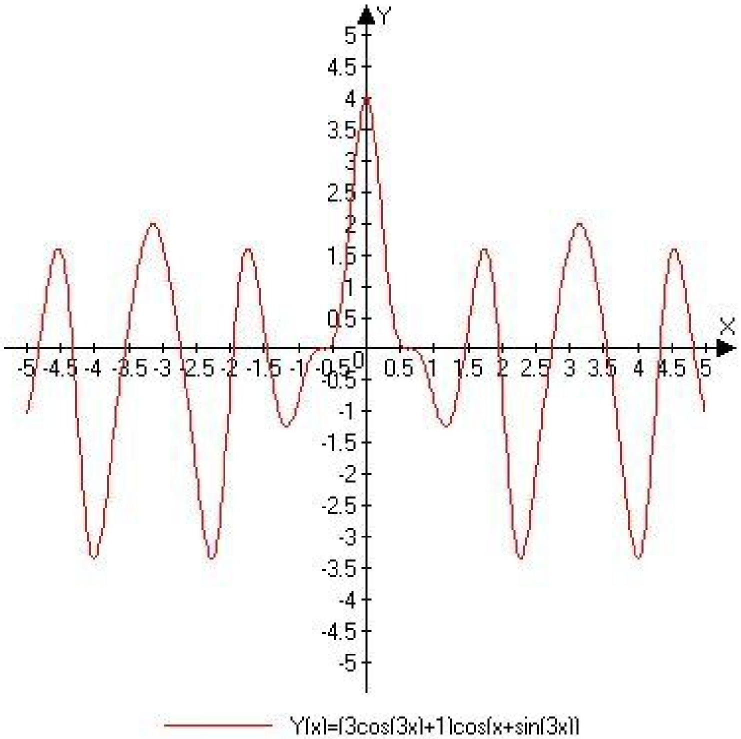
also, f has concavity downwards on (- ∞ , -1.02) , (0.63,∞) and concavity upwards on

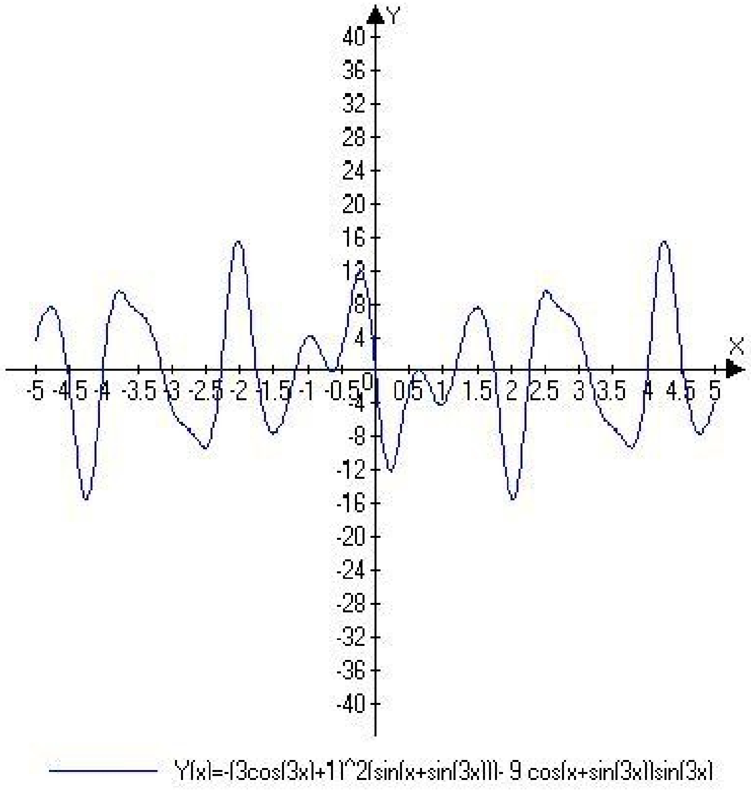
(-1.02 , 0.63).

f(x) = sin(x + sin 3x)

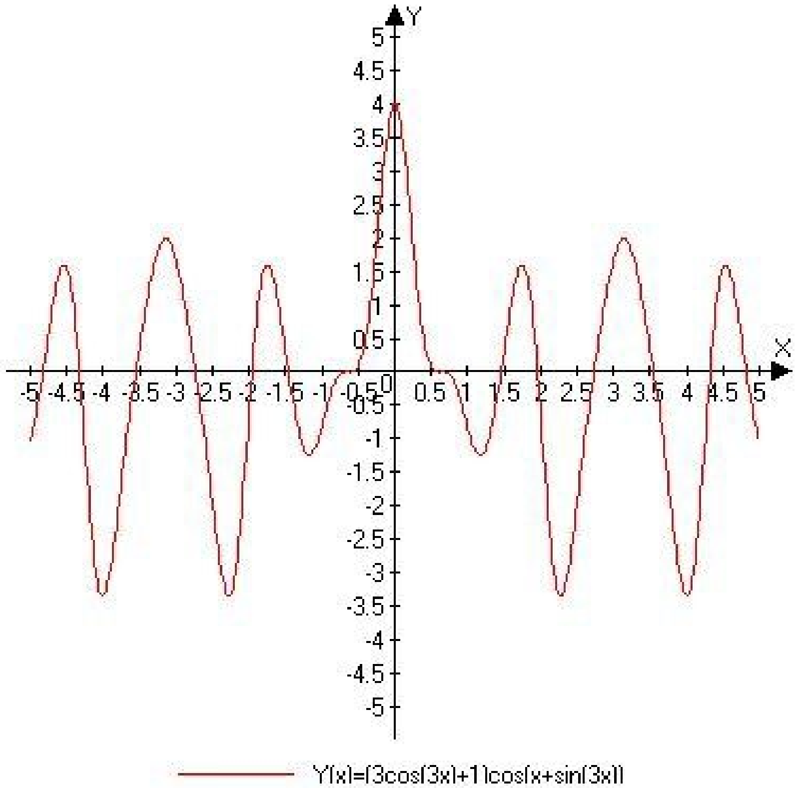
we first graph f , f ' , f '' in the viewing rectangle [0,π] , [-2π , 2π] , observe keenly to identify the number of maximum and minimum values of f.

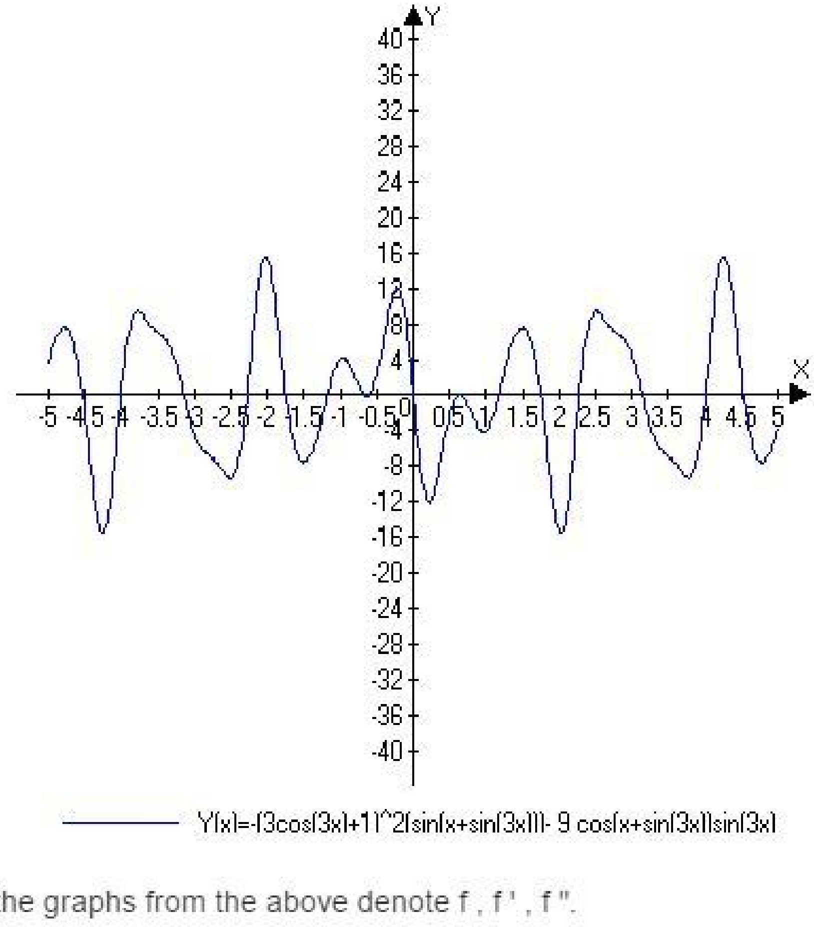






the graphs from the above denote f , f ' , f ''.





the graphs from the above denote f , f ' , f ''.

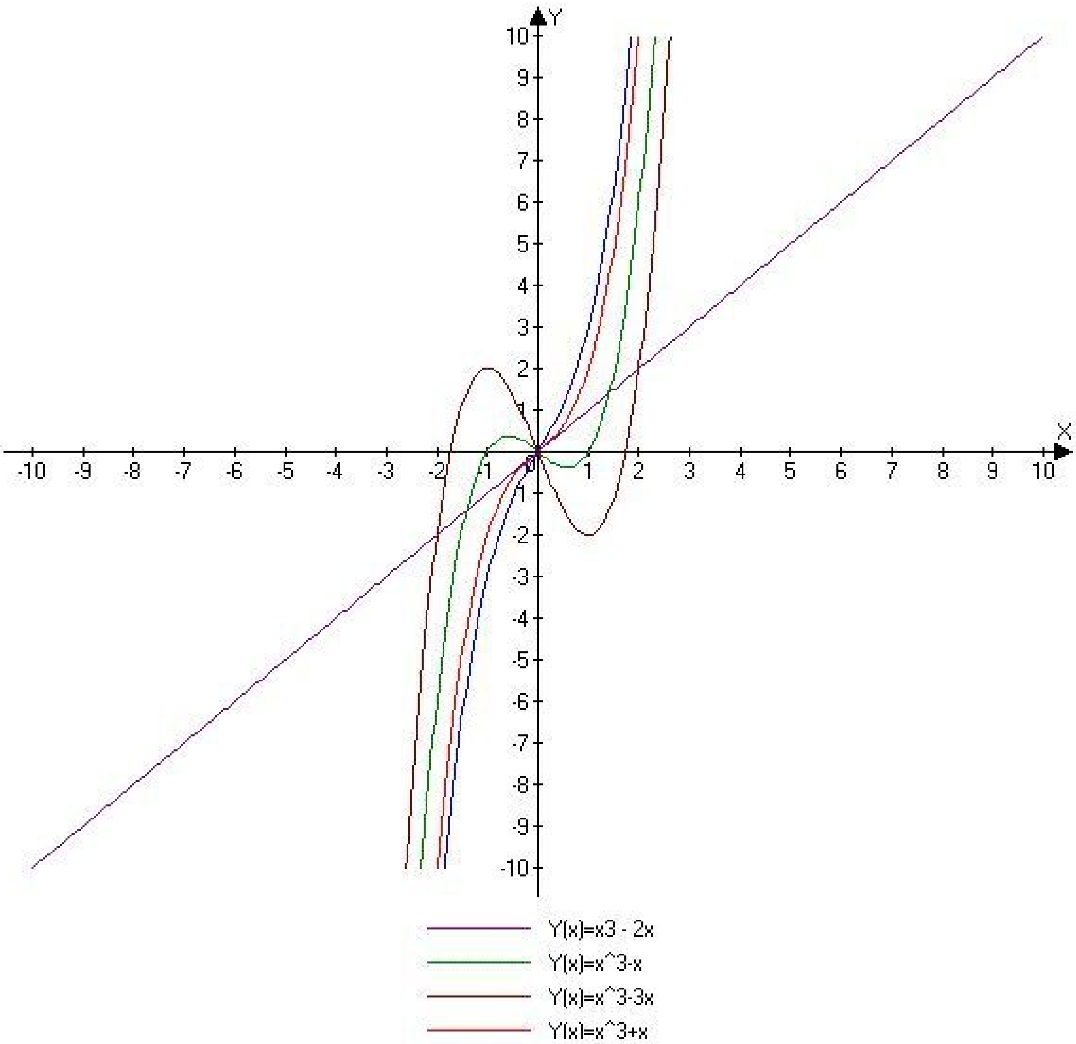
Chapter 3 Applications of Differentiation Exercise 3.6.20E

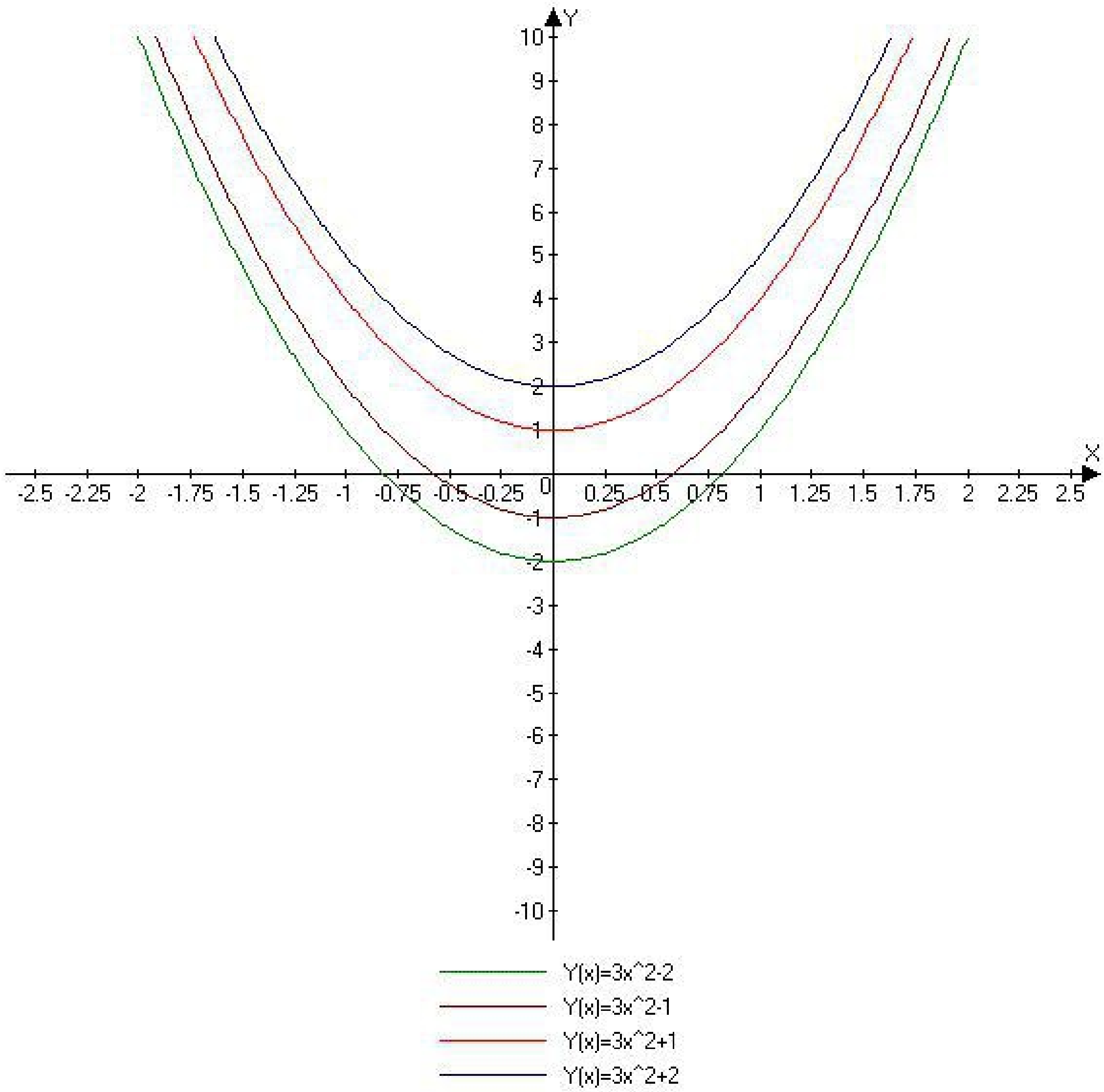
f(x) = x3+ cx

f ' (x) = 3x 2 + c

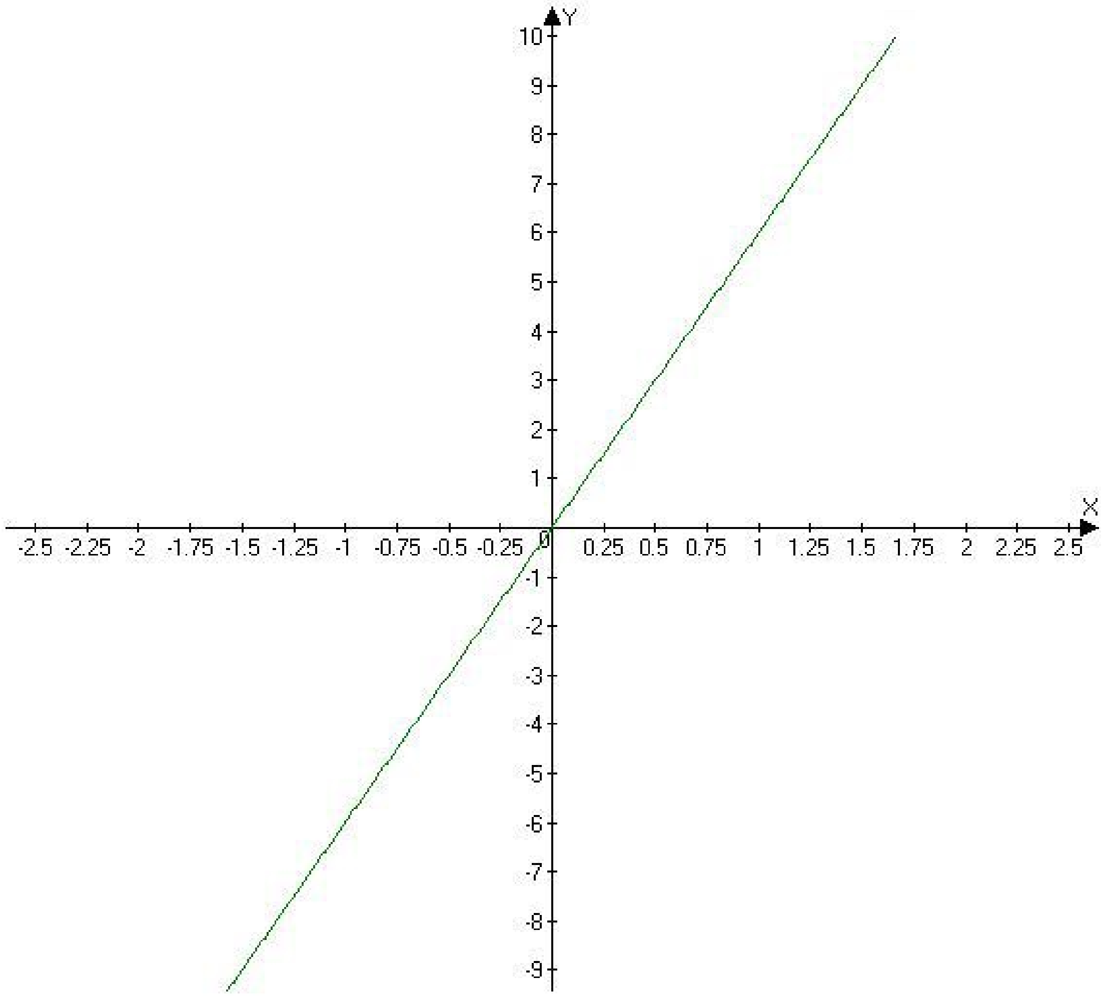
f " (x) = 6x

putting c = -2,-1,0,1,2 we draw the graphs for f , f ' , f " in the order and decide how c influences the maximum , minimum and points of infliction.





Y
X
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
-2.5 -2.25 -2 -1.75 -1.5 -1.25 -1 -0.75 -0.5 -0.25
0.25 0.5 0.75 1 1.25 1.5 1.75 2 2.25 2.5
Y(x)=3x^2-2
Y(x)=3x^2-1
Y(x)=3x^2+1
Y(x)=3x^2+2



when c < 0 , the curve has two points of infliction and when c > 0, it has only one point of infliction.

when c < 0 , f has two critical points and when c > 0 f has no critical point.

i,.e. f has local minimum and maximum when c < 0 and for c > 0 , f has no maximum and minimum values.

further, f has no assymptotes .

Chapter 3 Applications of Differentiation Exercise 3.6. 21E

Given that

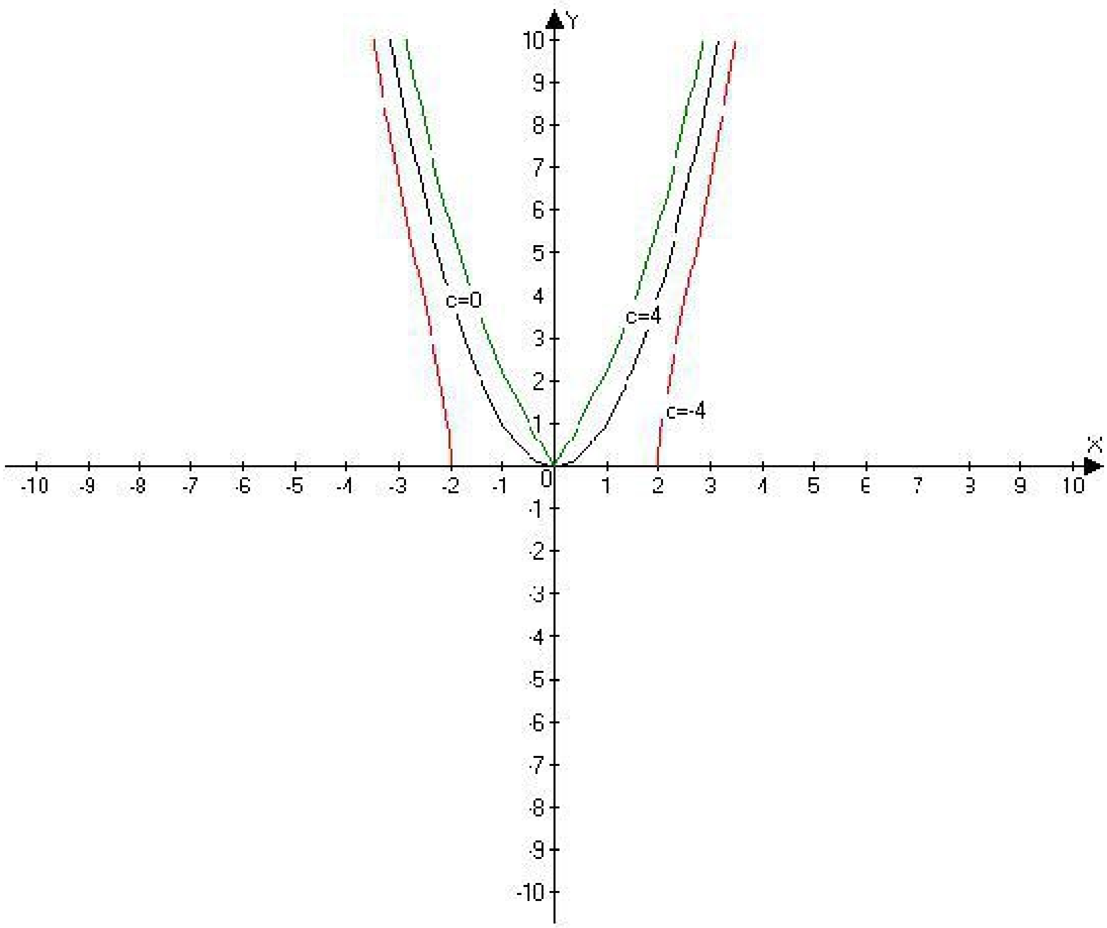
$$f(x) = \sqrt{x^4 + cx^2}$$

For $c \geq 0$, there is an absolute minimum at the origin. There are bi other maxima or minima

The more negative c become, the farther the two inflection points move from the origin.

$c = 0$ is a transitional value

The graph of the function is



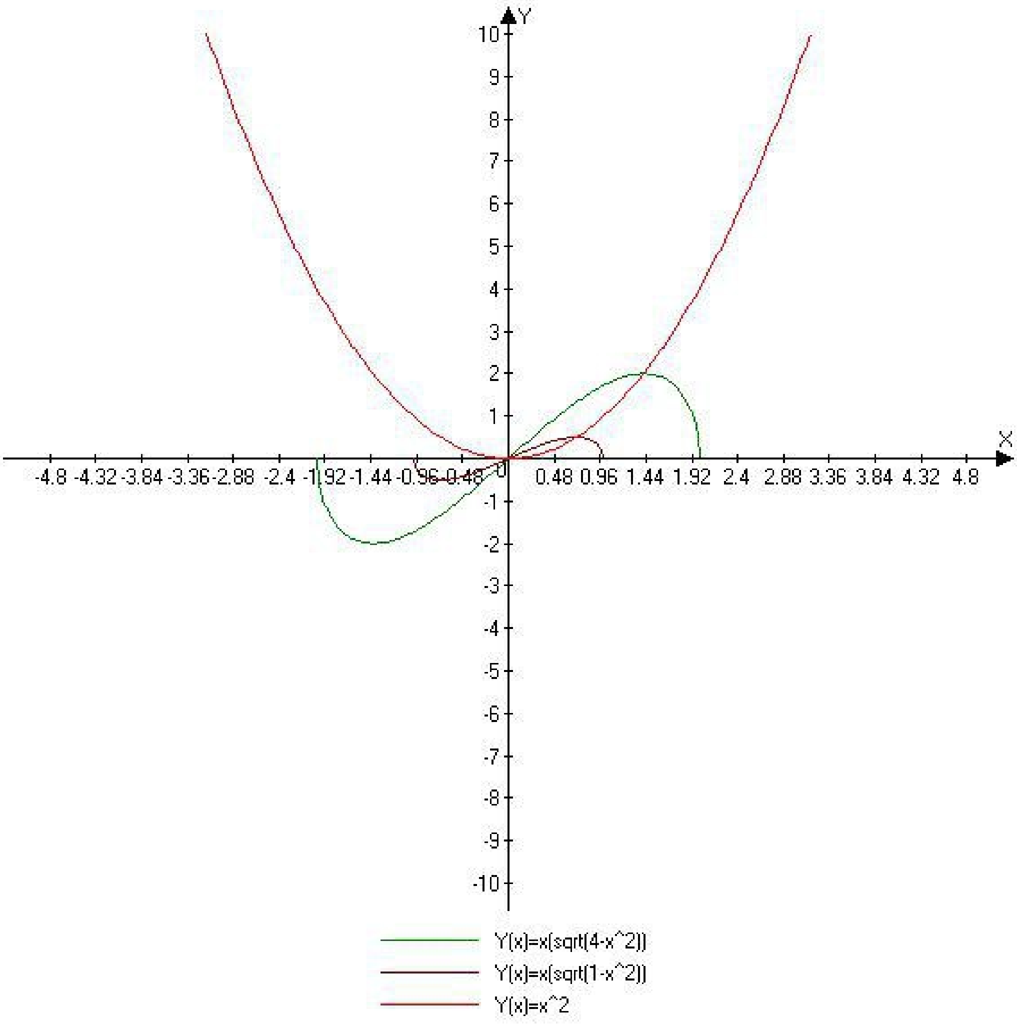
Chapter 3 Applications of Differentiation Exercise 3.6. 22E

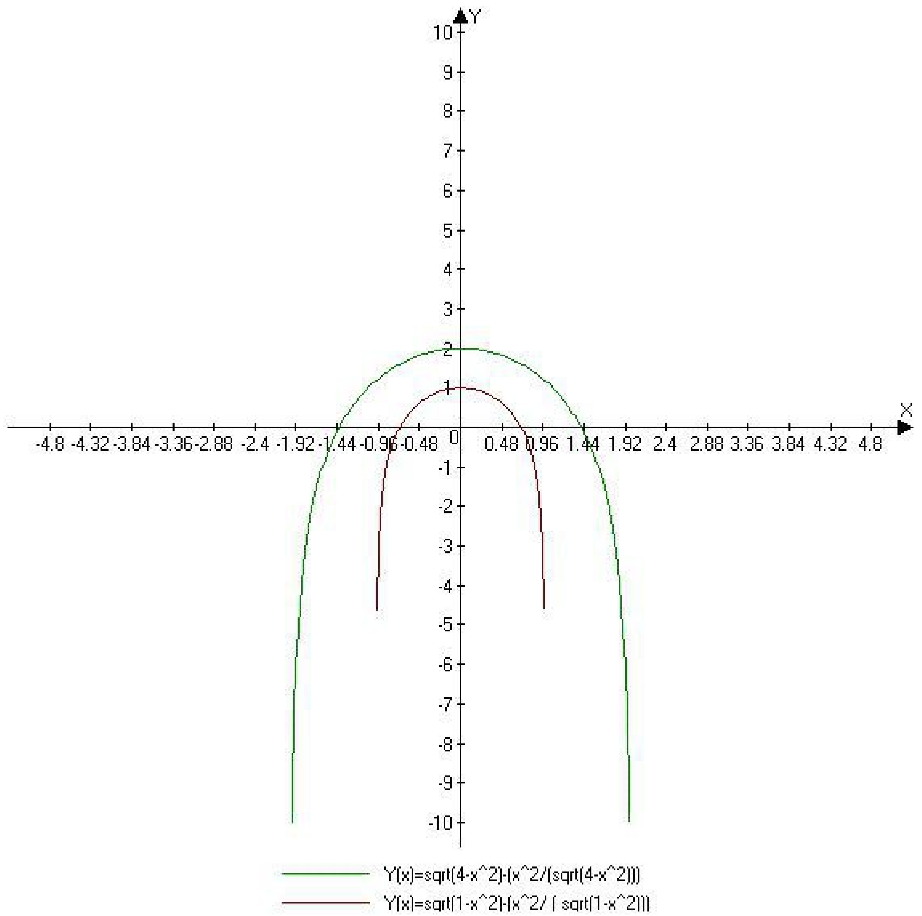
$$f(x) = x\sqrt{c^2 - x^2}$$

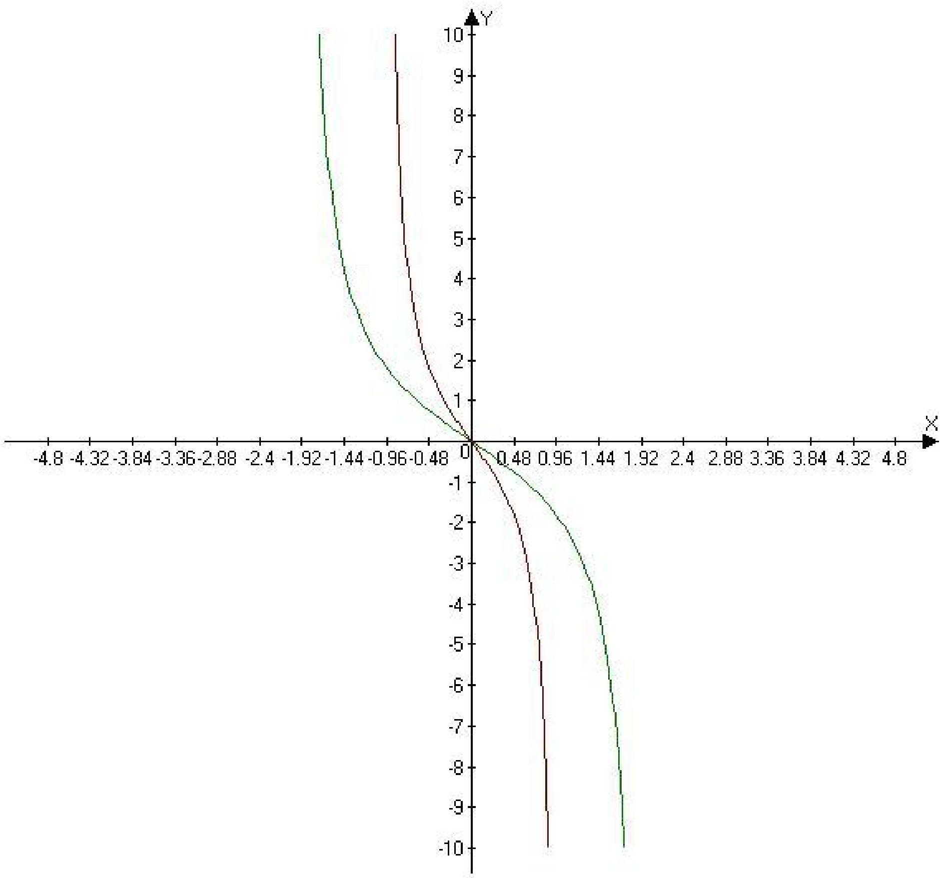
$$f'(x) = \sqrt{c^2 - x^2} - \frac{x^2}{\sqrt{c^2 - x^2}}$$

$$f''(x) = -\frac{x^3}{(c^2 - x^2)^{\frac{3}{2}}} - \frac{3x}{\sqrt{c^2 - x^2}}$$

we first graph the above in the order f , f ' , f" by putting c = -2 , -1 , 1, 2 and decide the influence of the variation of c in the function f.







observe that the positive or negitive signs of c is not going to influence the curve or nature of the function.when c = 0 , the curve represents a parabola while c is not zero , the function is an odd function and so, it has two points in which one is maximum and the other is minimum.

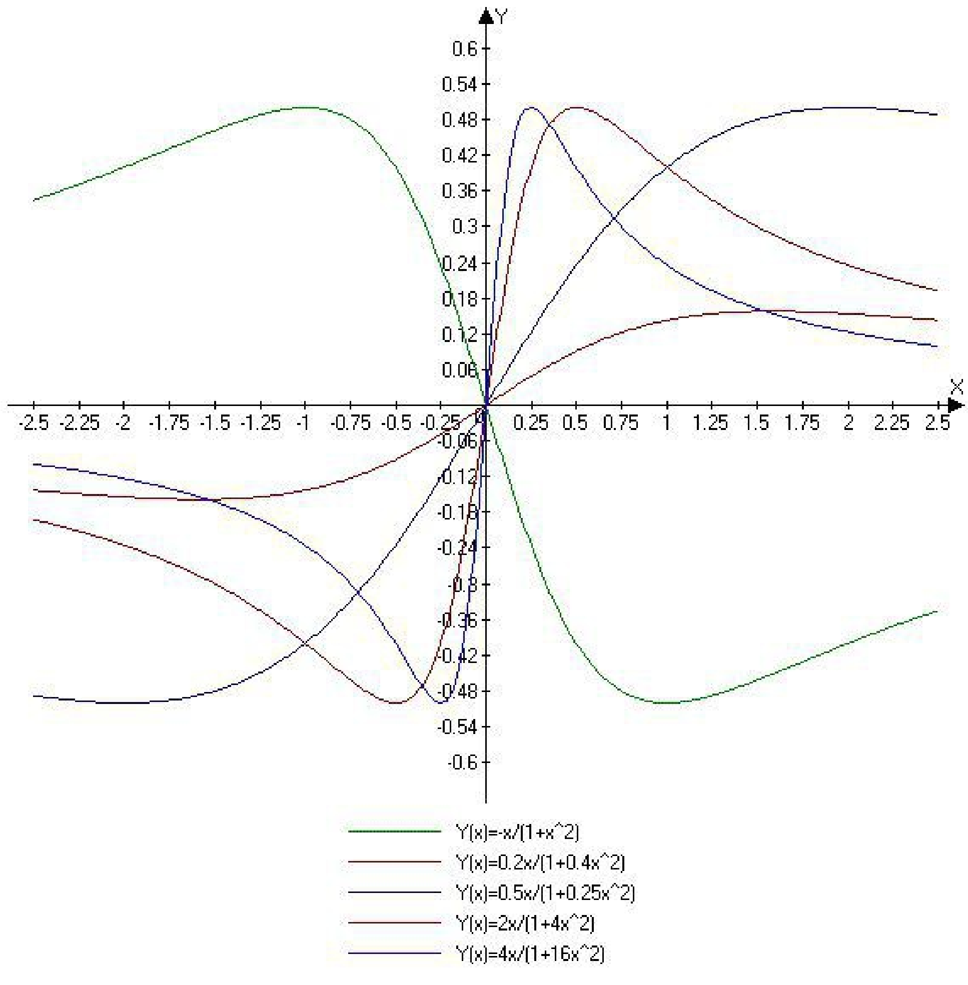
for each c , the function has two critical points and f has only one point of infliction.

$$f(x) = \frac{cx}{1+c^2x^2},$$

$$f'(x) = \frac{c}{1+c^2x^2} - \frac{2c^3x^2}{(1+c^2x^2)^2}$$

$$f''(x) = \frac{8c^3x^5}{(1+c^2x^2)^3} - \frac{6c^3x}{(1+c^2x^2)^2}$$

putting c = -1 , 0.2,0.5,1,2,4 in f , f ' , f '' we get the following graphs for the functions :



Chapter 3 Applications of Differentiation Exercise 3.6. 24E

$$f\left(x\right) = \frac{1}{\left(1-x^2\right)^2 + cx^2}$$

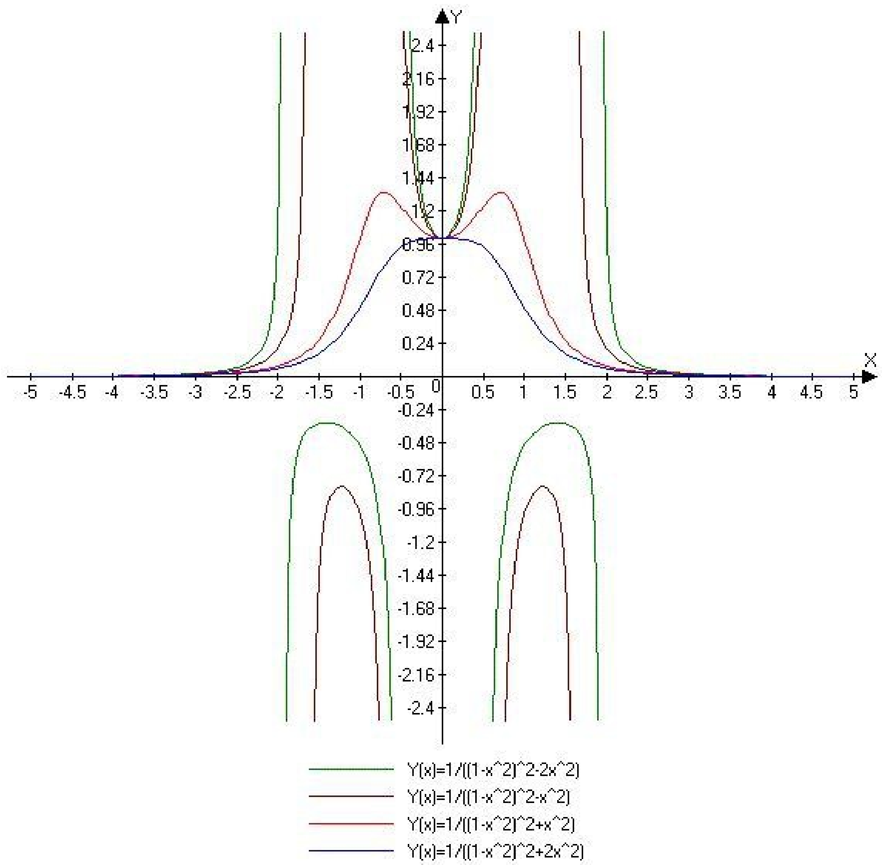
$$f'\left(x\right) = \frac{4x\left(1-x^2\right) - 2cx}{\left(\left(1-x^2\right)^2 + cx^2\right)^2}$$

f "(x) =

$$\frac{-8x^2 - 2c + 4(1-x^2)}{\left(c x^2 + (1-x^2)^2\right)^2} - \frac{2\,(2cx - 4x(1-x^2))\,(4x(1-x^2) - 2cx)}{\left(c x^2 + (1-x^2)^2\right)^3}$$

we substitute c = -2,-1 , 0.5 , 1, 2 to study the nature of the function and how different values of c influence it.:

the graphs in the order f , f ' , f " are as follows :



for every value of c , x axis is the horizontal assymptote.

when c < 0 , the graph has 3 intervals of concavity upwards and two intervals of concavity down wards . two equal local maximmum and one local minimum value 4 verticle assymptotes.

when c > 0 , the function hs two equal local maximum and one local minimum values.and as c increases , the curve slowly receeds to have only one local maximum and no minimum value . ultimately , the curve coincides with x axis.

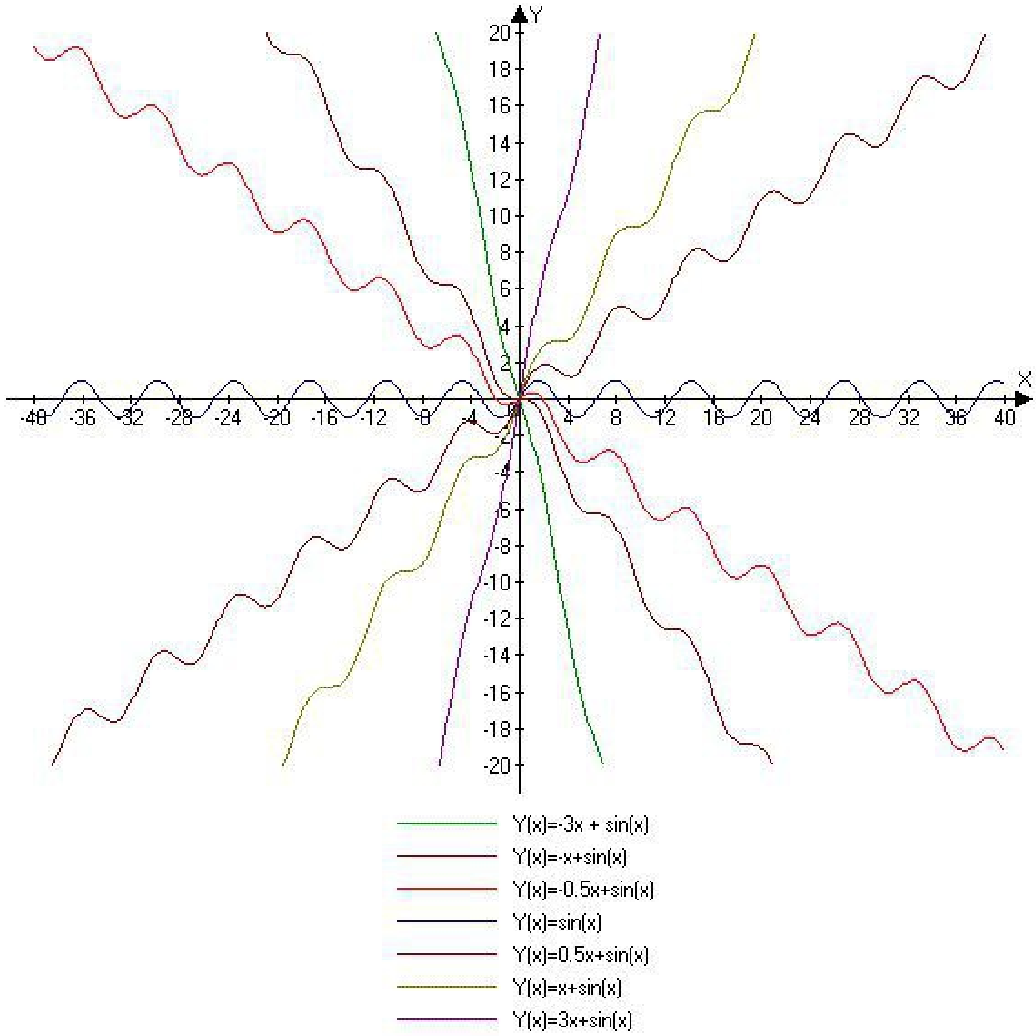
also, when c > 0 , the function has no veritcle assymptotes .

but all the curves are symmetric about origin.

Chapter 3 Applications of Differentiation Exercise 3.6. 25E

f(x) = cx + sin x , f ' (x) = c + cos x , f " (x) = - sinx

putting c = -3, -1 , -0.5,0,0.5,1,3 the graph is as follows :



for c in (- 1, 1) , the curve has local maximum and minimum values and for c lies away from (-1, 1) ,I they does not.

the function increases for c $\geq$ 1 and creases for c $\leq$1.

for c in (- 1, 1) , the curve has local maximum and minimum values and for c lies away from (-1, 1) ,I they does not.

the function increases for c $\geq$ 1 and creases for c $\leq$1.

i.e. when c greater than or equal to 1 , the curve exists in the first and third quadrants and for less than or equal to 1, the curve lies in the 2nd and 4th quadrants.

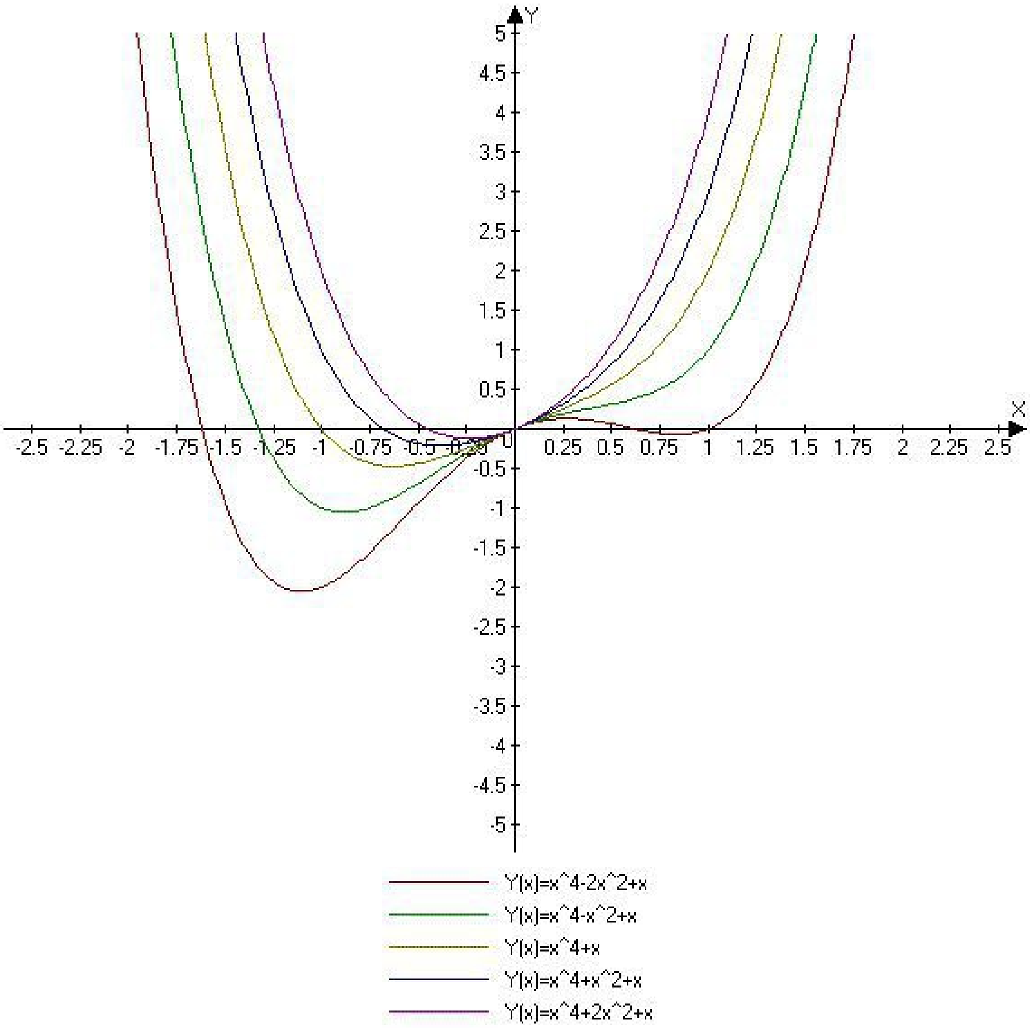
as c changes, the points of infliction move vertically but not horizontally.

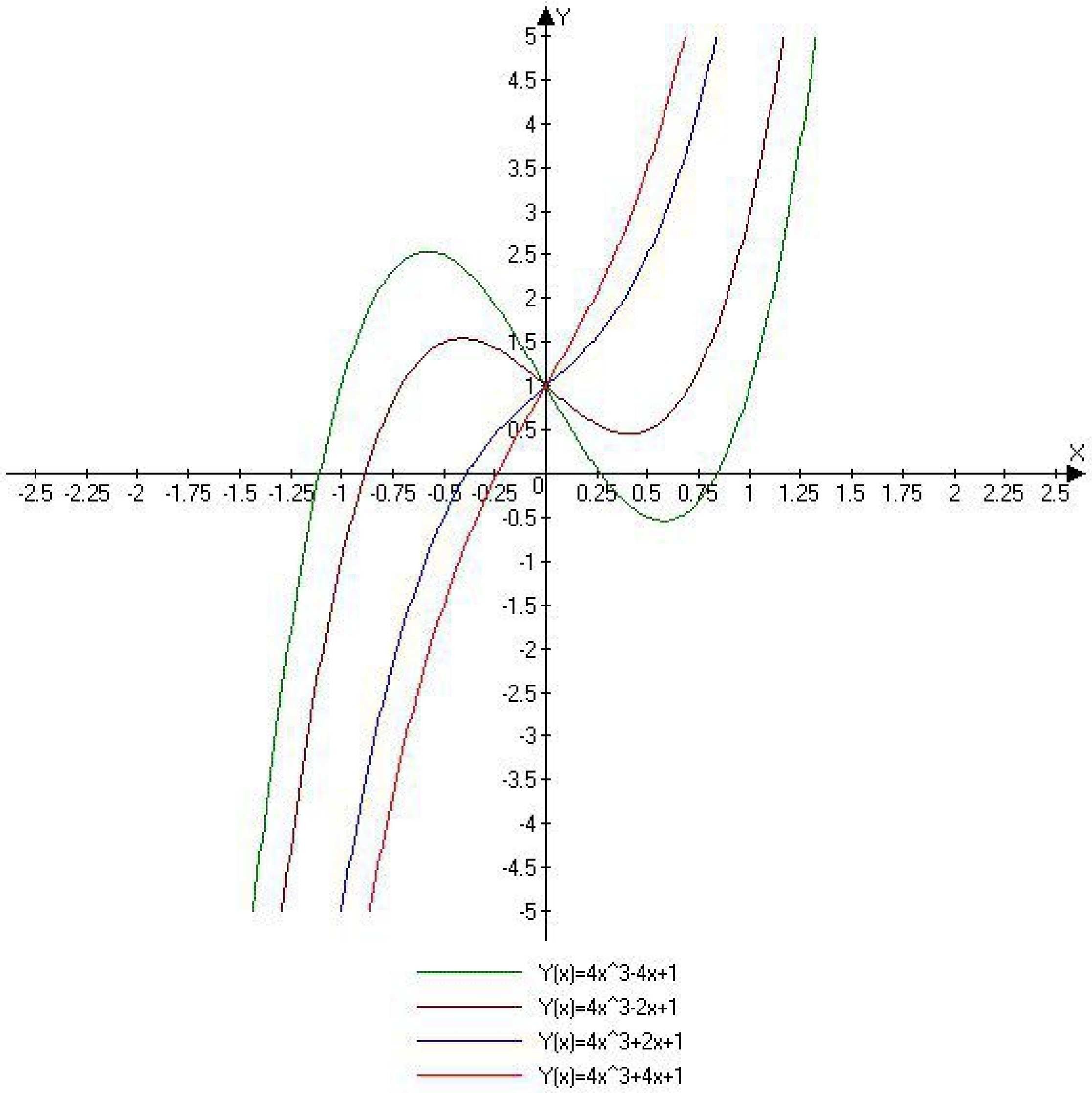
Chapter 3 Applications of Differentiation Exercise 3.6. 26E

f(x) = x4 + cx2+x

f ' (x) = 4x3 + 2cx + 1 , f " (x) = 12x2 + 2c

we first graph f for c = -2,-1,0,1,2 and decide the nature of f.





observe that for the negitive values of c , the function has two points of infliction and one critica point while for the positive values of c , f has one point of infliction and one critical point.

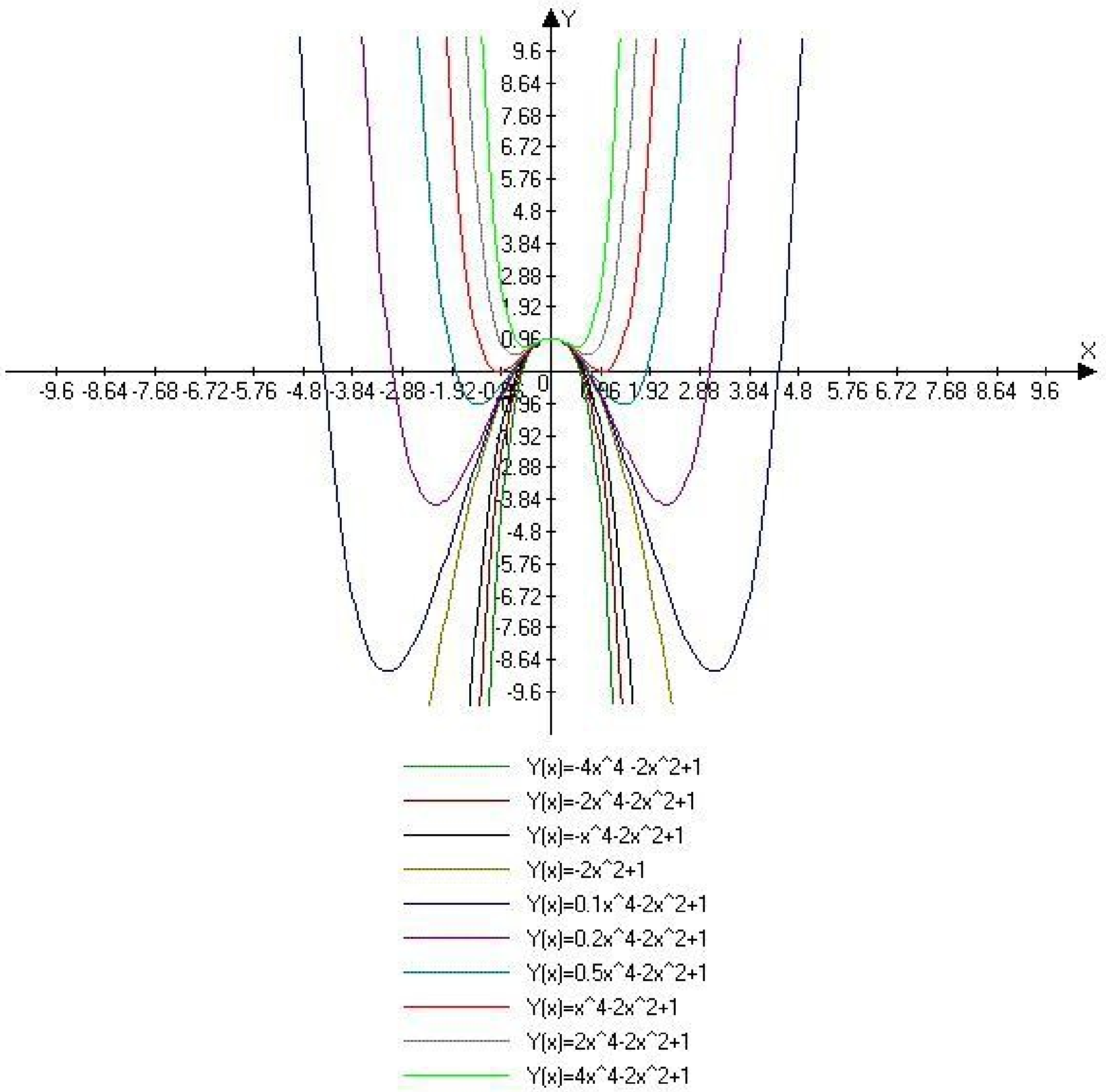
the transitional value of c is 0 when the number of critical points change and another critical point is x = -1.5 which is observed from the f ' graph in what shows the green color graph has more no. of negitive values than the other .

so, when c = -1.5, f ' touches x axis and goes up while when c = -2 , f ' goes down x axis to show more number of critical numbers.

Chapter 3 Applications of Differentiation Exercise 3.6. 27E

f(x) = cx4 -2x2 + 1

we put c = -4 ,-2,-1 ,0, , 0.1, 0 .2 , 0.5 , 1 , 4 and check for what values of c , the function has local minimum and for what values of c , the function has no minimum values.



observe that the graph is a down ward parabola when c is negitive.

so, in this case , f has no minimum value.

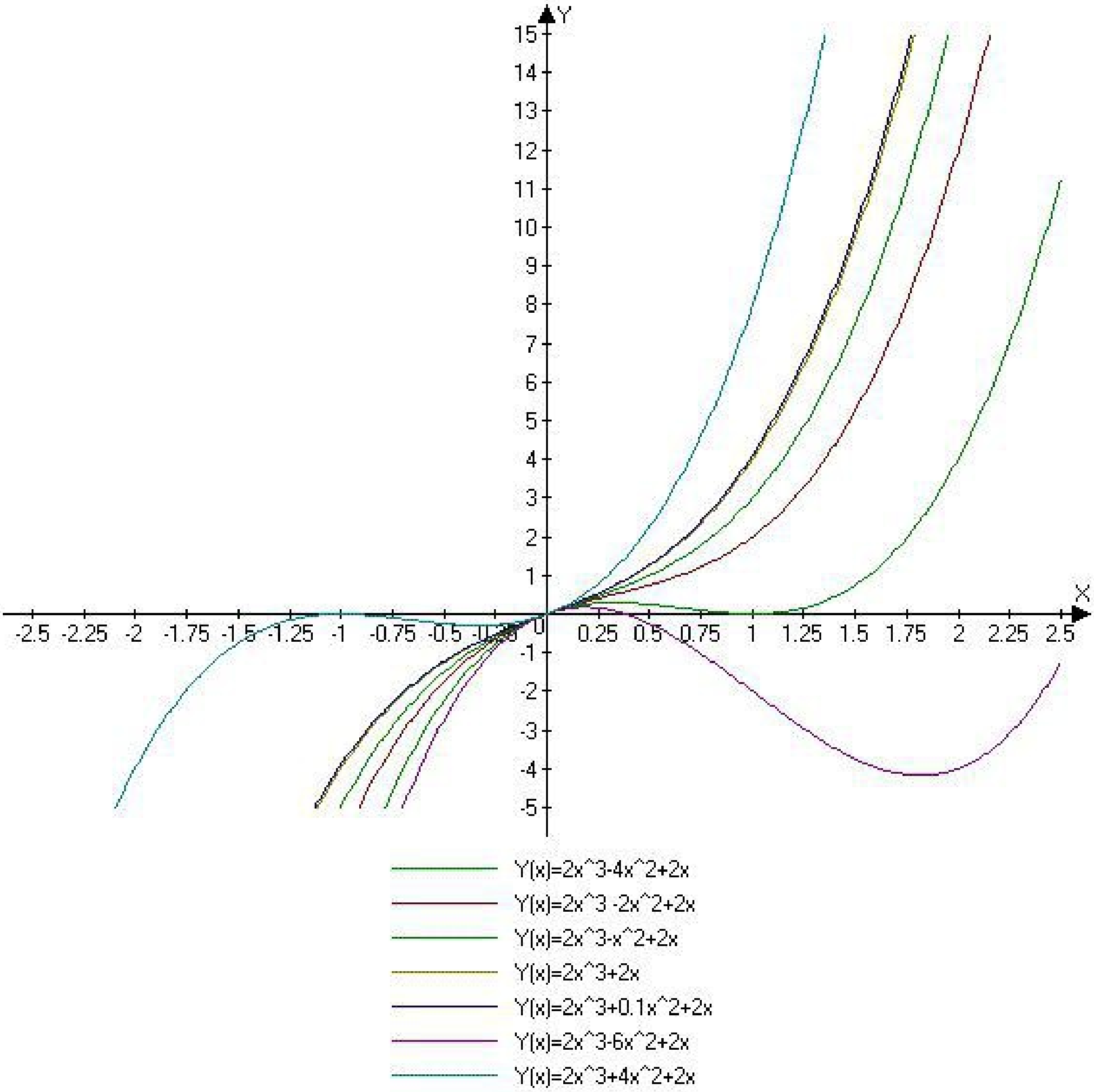
but starting from c = 0.1 , f started going up symmetric about y = 1 , posses 2 minimum values .

from this , we conclude that for positive values of c , f has local minimum values.

(b) the above graph is considered and observed that for the negitive values of c , f has maximum value while for positive value of c , f has minimum value.

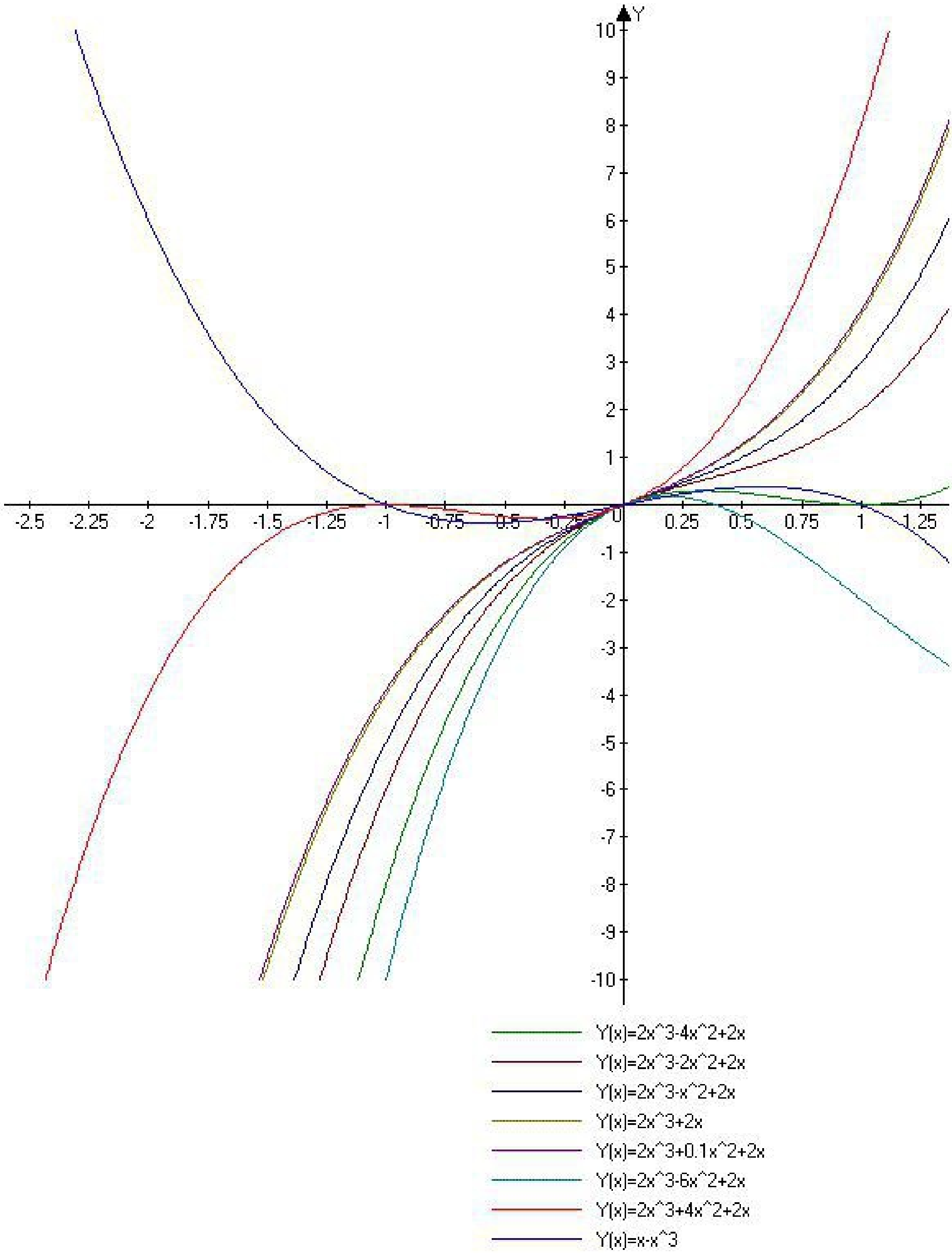
f(x) = 2x 3+cx2 + 2x

(a) we consider c = -4 , -2, =1 , 0 , 0.2 , 0.5 , 1, 4 and graph this function to determine what values of c alllows the function to possess the maximum and minimum values .



for c away from (-4,4) , the function has minimum and maximum values.

the functions whose c lies in (-4 ,4) will be passing through origin but having two points of infliction.



observe that for different values of c , the minimum values of the curves lie on the curve

y = x - x3