9. घातांक तथा करणी
(INDICES & SURDS)

आवश्यक तथ्य एवं सूत्र

1. घातांक के नियम :

(*i*) $a^m \times a^n = a^{m+n}$ (*ii*) $\frac{a^m}{a^n} = a^{m-n}$ (*iii*) $(a^m)^n = a^{mn}$

(*iv*) $(ab)^n = (a^n \times b^n)$ (*v*) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ (*vi*) $a^0 = 1$

2. करणी : माना a एक परिमेय संख्या है तथा n एक धनपूर्णांक, ऐसा है कि :

$a^{\frac{1}{n}} = \sqrt[n]{a}$ अपरिमेय है.

तब, $\sqrt[n]{a}$ को क्रम n की करणी कहा जाता है.

3. करणी के नियम :

(*i*) $\sqrt[n]{a} = a^{\frac{1}{n}}$ (*ii*) $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$ (*iii*) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

(*iv*) $\left(\sqrt[n]{a}\right)^n = a$ (*v*) $\left(\sqrt[n]{a}\right)^m = \left(a^{\frac{1}{n}}\right)^m = a^{\frac{m}{n}}$.

साधित उदाहरण

उदाहरण 1. सरल कीजिए : (*i*) $(243)^{\frac{3}{5}}$ (*ii*) $\left(\frac{1}{16}\right)^{\frac{-5}{4}}$ (*iii*) $(32)^{\frac{-1}{5}}$

हल : (*i*) $(243)^{\frac{3}{5}} = (3^5)^{\frac{3}{5}} = 3^{\left(5\times\frac{3}{5}\right)} = 3^3 = 27.$

(*ii*) $\left(\frac{1}{16}\right)^{\frac{-5}{4}} = (16)^{\frac{5}{4}} = (2^4)^{\frac{5}{4}} = 2^{\left(4\times\frac{5}{4}\right)} = 2^5 = 32.$

(*iii*) $(32)^{\frac{-1}{5}} = \left(\frac{1}{32}\right)^{\frac{1}{5}} = \left\{\left(\frac{1}{2}\right)^5\right\}^{\frac{1}{5}} = \left(\frac{1}{2}\right)^{\left(5\times\frac{1}{5}\right)} = \left(\frac{1}{2}\right)^1 = \frac{1}{2}.$

उदाहरण 2. सरल कीजिए : $\left(\frac{x^a}{x^b}\right)^{(a^2+ab+b^2)} \cdot \left(\frac{x^b}{x^c}\right)^{(b^2+bc+c^2)} \cdot \left(\frac{x^c}{x^a}\right)^{(c^2+ca+a^2)}$

हल : दिया गया व्यंजक

$= x^{(a-b)(a^2+ab+b^2)} \cdot x^{(b-c)(b^2+bc+c^2)} \cdot x^{(c-a)(c^2+ca+a^2)}$

$= x^{(a^3-b^3)} x^{(b^3-c^3)} \cdot x^{(c^3-a^3)} = x^{(a^3-b^3+b^3-c^3+c^3-a^3)}$

$= x^0 = 1.$

उदाहरण 3. सरल कीजिए : $\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} \cdot \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \cdot \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}}$.

हल : दिया गया व्यंजक $= \left(x^{a-b}\right)^{\frac{1}{ab}} \cdot \left(x^{b-c}\right)^{\frac{1}{bc}} \cdot \left(x^{c-a}\right)^{\frac{1}{ca}} = x^{(a-b)\cdot\frac{1}{ab}} \cdot x^{(b-c)\cdot\frac{1}{bc}} \cdot x^{(c-a)\cdot\frac{1}{ca}}$

$$= x^{\left(\frac{1}{b}-\frac{1}{a}\right)} \cdot x^{\left(\frac{1}{c}-\frac{1}{b}\right)} \cdot x^{\left(\frac{1}{a}-\frac{1}{c}\right)} = x^{\left(\frac{1}{b}-\frac{1}{a}+\frac{1}{c}-\frac{1}{b}+\frac{1}{a}-\frac{1}{c}\right)} = x^0 = 1.$$

उदाहरण 4. यदि $3^x - 3^{x-1} = 18$ हो, तो $x^x = ?$

हल : $3^x - 3^{x-1} = 18 \Rightarrow 3^{x-1}(3-1) = 18 \Rightarrow 3^{x-1} = \frac{18}{2} = 9 = 3^2 \Rightarrow x-1 = 2 \Rightarrow x = 3.$

$\therefore\ x^x = 3^3 = 27.$

उदाहरण 5. $\frac{(243)^{\frac{n}{5}} \times 3^{2n+1}}{9^n \times 3^{n-1}} = ?$

हल : दिया गया व्यंजक $= \frac{\left(3^5\right)^{\frac{n}{5}} \times 3^{2n+1}}{\left(3^2\right)^n \times 3^{n-1}} = \frac{3^{\left(5\times\frac{n}{5}\right)} \times 3^{2n+1}}{3^{2n} \times 3^{n-1}} = \frac{3^n \times 3^{2n+1}}{3^{2n} \times 3^{n-1}}$

$$= \frac{3^{(n+2n+1)}}{3^{(2n+n-1)}} = \frac{3^{(3n+1)}}{3^{(3n-1)}} = 3^{(3n+1)-(3n-1)} = 3^2 = 9.$$

उदाहरण 6. यदि $x = y^a$, $y = z^b$ तथा $z = x^c$ हो तो सिद्ध करो कि $abc = 1$.

हल : $z = x^c = \left(y^a\right)^c = y^{ac} = \left(z^b\right)^{ac} = z^{abc}.$

$\therefore\ z^{abc} = z^1 \Rightarrow abc = 1.$

उदाहरण 7. यदि $abc = 1$ हो, तो $\left\{\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}}\right\} = ?$

हल : दिया गया व्यंजक $= \frac{1}{1+a+b^{-1}} + \frac{b^{-1}}{b^{-1}+1+b^{-1}c^{-1}} + \frac{a}{a+ac+1}$

$$= \frac{1}{1+a+b^{-1}} + \frac{b^{-1}}{1+a+b^{-1}} + \frac{a}{1+a+b^{-1}} \left[\because b^{-1}c^{-1} = a,\ ac = b^{-1}\right]$$

$$= \frac{1+a+b^{-1}}{1+a+b^{-1}} = 1.$$

उदाहरण 8. $\sqrt{2}$ तथा $\sqrt[3]{3}$ में से कौन-सा बड़ा है ?

हल : दी गई करणियाँ हैं $2^{\frac{1}{2}}$ तथा $3^{\frac{1}{3}}$. अब 2 तथा 3 का ल०स० = 6.

$2^{\frac{1}{2}} = \left(2^3\right)^{\frac{1}{6}} = 8^{\frac{1}{6}}$ तथा $3^{\frac{1}{3}} = \left(3^2\right)^{\frac{1}{6}} = 9^{\frac{1}{6}}.$

स्पष्ट है कि $9^{\frac{1}{6}} > 8^{\frac{1}{6}}$. अत: $3^{\frac{1}{3}} > 2^{\frac{1}{2}} \Rightarrow \sqrt[3]{3} > \sqrt{2}.$

उदाहरण 9. $\sqrt[4]{6}, \sqrt{2}$ तथा $\sqrt[3]{4}$ में से सबसे बड़ी संख्या कौन-सी है ?

हल : दी गई करणियाँ हैं $6^{\frac{1}{4}}, 2^{\frac{1}{2}}$ तथा $4^{\frac{1}{3}}$. अब 4, 2, 3 का ल०स० = 12.

$6^{\frac{1}{4}} = (6^3)^{\frac{1}{12}} = (216)^{\frac{1}{12}}$, $2^{\frac{1}{2}} = (2^6)^{\frac{1}{12}} = (64)^{\frac{1}{12}}$ तथा $4^{\frac{1}{3}} = (4^4)^{\frac{1}{12}} = (256)^{\frac{1}{12}}$.

स्पष्ट है कि $(256)^{\frac{1}{12}} > (216)^{\frac{1}{12}} > (64)^{\frac{1}{12}} \Rightarrow 4^{\frac{1}{3}} > 6^{\frac{1}{4}} > 2^{\frac{1}{2}}$

अत: $4^{\frac{1}{3}}$ इन सबमें सबसे बड़ा है.

प्रश्नमाला 9

निम्नलिखित प्रश्नों में से प्रत्येक में ठीक उत्तर को चिन्हांकित (✓) कीजिए :

1. $8^{\frac{2}{3}} = ?$

(a) $3\frac{1}{3}$ (b) 4 (c) $5\frac{1}{2}$ (d) $21\frac{1}{3}$

2. $16^{\frac{3}{4}} = ?$

(a) $2\sqrt{2}$ (b) $4\sqrt{2}$ (c) 8 (d) 16

3. $(\sqrt{8})^{\frac{1}{3}} = ?$

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) 2 (d) 4

4. $(36)^{\frac{1}{6}} = ?$

(a) 1 (b) 6 (c) $\sqrt{6}$ (d) $\sqrt[3]{6}$

5. $(16)^{1.75} = ?$

(a) 64 (b) $64\sqrt{2}$ (c) 128 (d) $128\sqrt{2}$

6. $(1000)^9 \div 10^{24} = ?$ **(बैंक पी०ओ० 2006)**

(a) 100 (b) 1000 (c) 10000 (d) 10 (e) इनमें से कोई नहीं

7. $\left(\frac{81}{169}\right)^{-\frac{1}{2}} = ?$

(a) $\frac{3}{169}$ (b) $\frac{9}{169}$ (c) $\frac{9}{13}$ (d) $\frac{13}{9}$

8. $\left(\frac{32}{243}\right)^{\frac{-4}{5}} = ?$

(a) $\frac{16}{81}$ (b) $\frac{81}{16}$ (c) $\frac{4}{9}$ (d) $\frac{9}{4}$

9. $\left(\frac{-1}{343}\right)^{\frac{-2}{3}} = ?$

(a) $\frac{-1}{49}$ (b) $\frac{1}{49}$ (c) -49 (d) 49

10. $\frac{1}{(216)^{\frac{-2}{3}}} + \frac{1}{(256)^{\frac{-3}{4}}} + \frac{1}{(243)^{\frac{-1}{5}}} = ?$ **(एम०बी०ए० 2006)**

(a) 103 (b) 105 (c) 107 (d) इनमें से कोई नहीं

11. यदि $\sqrt{2^n} = 64$ हो, तो $n = ?$ **(रेलवे, 2006)**

(a) 2 (b) 4 (c) 6 (d) 12

12. यदि $\sqrt{4^n} = 1024$ हो, तो $n = ?$ **(रेलवे, 2006)**

(a) 5 (b) 8 (c) 10 (d) 12

13. $\frac{3^{n+2} - 3^{n+1}}{3^{n+4} + 3^{n+1}} = ?$

(a) $\frac{1}{5}$ (b) $\frac{1}{10}$ (c) $\frac{1}{14}$ (d) $\frac{1}{28}$

14. $\left\{(2^4)^{\frac{1}{2}}\right\}^{?} = 256$ **(बैंक पी०ओ० 2007)**

(a) 1 (b) 2 (c) 4 (d) 8 (e) इनमें से कोई नहीं

15. $\frac{(\sqrt{5} - \sqrt{3})}{(\sqrt{5} + \sqrt{3})} = ?$ **(एम०बी०ए० 2007)**

(a) 1 (b) 2 (c) 3 (d) इनमें से कोई नहीं

16. यदि $\frac{9^n \times 3^5 \times (27)^3}{3 \times (81)^4} = 27$ हो, तो $n = ?$

(a) 0 (b) 2 (c) 3 (d) 4

17. $8^3 \times 8^2 \times 8^{-5} = ?$ **(रेलवे, 2006)**

(a) 0 (b) 1 (c) 8 (d) इनमें से कोई नहीं

18. $\sqrt[3]{5}$ किस घात की करणी है ? **(रेलवे, 2006)**

(a) 2 (b) 3 (c) 5 (d) इनमें से कोई नहीं

19. यदि $(27)^{\frac{2}{3}} \times (81)^{-\frac{1}{2}} = 3^n$ हो, तो $n = ?$

(a) 1 (b) 0 (c) 27 (d) 81

20. $\left(\frac{1}{216}\right)^{-\frac{2}{3}} \div \left(\frac{1}{27}\right)^{-\frac{4}{3}} = ?$

(a) $\frac{3}{4}$ (b) $\frac{2}{3}$ (c) $\frac{4}{9}$ (d) $\frac{1}{8}$

21. $\left(2^3+2^2+2^{-2}+2^{-3}\right)=?$

(a) $\frac{99}{8}$ (b) $\frac{99}{16}$ (c) $\frac{97}{8}$ (d) 6

22. $(42\times229)\div(9261)^{\frac{1}{3}}=?$ **(बैंक पी०ओ० 2007)**

(a) 448 (b) 452 (c) 456 (d) 458 (e) इनमें से कोई नहीं

23. $\frac{(5)^{25}\times(125)^{25}}{(256)^{10}\times(256)^{15}}=?$

(a) $\frac{25}{16}$ (b) $\frac{25}{2}$ (c) $\frac{5}{4}$ (d) $\frac{\sqrt{5}}{2}$

24. यदि $\left(\sqrt{3}\right)^5\times9^2=3^{\alpha}\times3\sqrt{3}$ हो, तो $\alpha=?$

(a) 2 (b) 3 (c) 4 (d) 5

25. यदि $2^x\times8^{\frac{1}{5}}=2^{\frac{1}{5}}$ हो, तो $x=?$

(a) $\frac{1}{5}$ (b) $\frac{-1}{5}$ (c) $\frac{2}{5}$ (d) $\frac{-2}{5}$

26. यदि $5\sqrt{5}\times5^3\div5^{-3/2}=5^{a+2}$ हो, तो $a=?$

(a) 4 (b) 5 (c) 6 (d) 8

27. $\left(x^{b+c}\right)^{(b-c)}.\left(x^{c+a}\right)^{(c-a)}.\left(x^{a+b}\right)^{(a-b)}=?$ **(रेलवे, 2006)**

(a) 0 (b) 1 (c) x (d) $x^{(a^2+b^2+c^2)}$

28. $\left(\frac{x^a}{x^b}\right)^{(a+b)}.\left(\frac{x^b}{x^c}\right)^{(b+c)}.\left(\frac{x^c}{x^a}\right)^{(c+a)}=?$ **(एम०बी०ए० 2004)**

(a) 0 (b) 1 (c) x^{abc} (d) $x^{(a+b+c)}$

29. $\frac{1}{1+a^{n-m}}+\frac{1}{1+a^{m-n}}=?$ **(एम०बी०ए० 2003)**

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) a^{m+n}

30. $\frac{1}{1+x^{(b-a)}+x^{(c-a)}}+\frac{1}{1+x^{(a-b)}+x^{(c-b)}}+\frac{1}{1+x^{(b-c)}+x^{(a-c)}}=?$

(a) 0 (b) 1 (c) x^{a-b-c} (d) इनमें से कोई नहीं

31. $\left(\frac{x^b}{x^c}\right)^{(b+c-a)}.\left(\frac{x^c}{x^a}\right)^{(c+a-b)}.\left(\frac{x^a}{x^b}\right)^{(a+b-c)}$

(a) 1 (b) x^{abc} (c) $x^{(a+b+c)}$ (d) $x^{(ab+bc+ca)}$

32. यदि $\left(\frac{a}{b}\right)^{x-1}=\left(\frac{b}{a}\right)^{x-3}$ हो, तो $x=?$ **(एम०बी०ए० 2003)**

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) $\frac{7}{2}$

33. यदि $2^{2x+4} = 16^x$ हो, तो $x^3 = ?$ **(रेलवे, 2006)**

(a) 8 (b) 16 (c) 27 (d) 64

34. यदि $5^{5x+5} = 1$ हो, तो $x = ?$ **(एस०एस०सी० 2003)**

(a) -1 (b) $\frac{-4}{5}$ (c) 0 (d) 1

35. यदि $3^{x+3} + 7 = 250$ हो, तो $x = ?$

(a) 1 (b) 2 (c) 3 (d) 5

36. यदि $a^{2x+2} = 1$, जहाँ a एक धनात्मक वास्तविक संख्या है तथा $a \neq 1$, तब $x = ?$ **(एस०एस०सी० 2007)**

(a) -2 (b) -1 (c) 0 (d) 1

37. यदि $5^{x+3} = (25)^{3x-4}$ हो, तो $x = ?$

(a) $\frac{5}{11}$ (b) $\frac{13}{5}$ (c) $\frac{11}{3}$ (d) $\frac{11}{5}$

38. यदि $2^{2x-1} = \frac{1}{8^{(x-3)}}$ हो, तो $x = ?$

(a) 3 (b) 2 (c) -2 (d) 0

39. यदि $2^{x-1} + 2^{x+1} = 320$ हो, तो $x = ?$

(a) 6 (b) 8 (c) 5 (d) 7

40. यदि $2^{x+4} - 2^{x+2} = 3$ हो, तो $x = ?$

(a) 0 (b) 2 (c) -1 (d) -2

41. यदि $\sqrt[3]{32} = 2^x$ हो, तो $x = ?$

(a) 5 (b) 3 (c) $\frac{5}{3}$ (d) $\frac{3}{5}$

42. $\sqrt{a^{-1}b} \cdot \sqrt{b^{-1}c} \cdot \sqrt{c^{-1}a} = ?$

(a) abc (b) $\sqrt{abc}$ (c) $\frac{1}{abc}$ (d) 1

43. यदि $x = y^a$, $y = z^b$ तथा $z = x^c$ हो, तो $abc = ?$ **(एस०एस०सी० 2005)**

(a) 1 (b) 2 (c) 3 (d) 4

44. यदि $a^x = b^y = c^z$ तथा $b^2 = ac$ हो, तो $y = ?$

(a) $\frac{xz}{x+z}$ (b) $\frac{xz}{2(x-z)}$ (c) $\frac{xz}{2(z-x)}$ (d) $\frac{2xz}{(x+z)}$

45. यदि $2^x = 3^y = 6^{-z}$ हो, तो $\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = ?$

(a) 0 (b) 1 (c) $\frac{3}{2}$ (d) $\frac{-1}{2}$

46. यदि $2^x = 4^y = 8^z$ हो तथा $\left(\frac{1}{2x} + \frac{1}{4y} + \frac{1}{6z}\right) = \frac{24}{7}$ हो, तो $z = ?$

(a) $\frac{7}{16}$ (b) $\frac{7}{32}$ (c) $\frac{7}{48}$ (d) $\frac{7}{64}$

47. $27^{\frac{5}{2}} \times 9^{\frac{3}{2}} \div 3^7 = 3^?$ **(बैंक पी०ओ० 2004)**

(a) $\frac{7}{2}$ (b) $\frac{11}{2}$ (c) $\frac{21}{2}$ (d) 14 (e) इनमें से कोई नहीं

48. $(256)^{0.16} \times (16)^{0.18} = ?$ **(एस०एस०सी० 2007)**

(a) 4 (b) 16 (c) 64 (d) 256.25

49. $10^{7.5} \times 5^{2.5} \times 2^{2.5} = 10^?$ **(बैंक पी०ओ० 2005)**

(a) 9.5 (b) 10 (c) 11.5 (d) 12.5 (e) इनमें से कोई नहीं

50. $8^{4.2} \times 64^{2.1} \times 7^{8.4} \times 56^{3.5} = (56)^?$ **(बैंक पी०ओ० 2005)**

(a) 9.8 (b) 11.9 (c) 12.6 (d) 18.2 (e) इनमें से कोई नहीं

51. $9^{8.6} \times 8^{3.9} \times 72^{4.4} \times 9^{3.9} \times 8^{8.6} = 72^?$ **(बैंक पी०ओ० 2005)**

(a) 15.1 (b) 17.9 (c) 20.9 (d) 29.4 (e) इनमें से कोई नहीं

52. $\sqrt[4]{(625)^3} = ?$ **(एम०बी०ए० 2005)**

(a) 25 (b) 125 (c) 3125 (d) इनमें से कोई नहीं

53. $\sqrt[4]{10000} = (100)^?$ **(बैंक पी०ओ० 2007)**

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) 2 (e) इनमें से कोई नहीं

54. यदि $x = (7 - 4\sqrt{3})$ हो, तो $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) = ?$ **(एस०एस०सी० 2005)**

(a) 1 (b) 2 (c) 3 (d) 4

55. यदि $x = (3 + 2\sqrt{2})$ हो, तो $\left(x^4 + \frac{1}{x^4}\right) = ?$ **(एम०बी०ए० 2007)**

(a) 1024 (b) 1154 (c) 1734 (d) इनमें से कोई नहीं

56. $\sqrt{2}, \sqrt[6]{3}, \sqrt[3]{4}, \sqrt[4]{5}$ में से सबसे बड़ा कौन-सा है ? **(एस०एस०सी० 2005)**

(a) $\sqrt{2}$ (b) $\sqrt[3]{4}$ (c) $\sqrt[4]{5}$ (d) $\sqrt[6]{3}$

57. $\sqrt{2}, \sqrt[3]{4}, \sqrt[4]{6}$ को चढ़ते क्रम में लिखने पर : **(एम०बी०ए० 2007)**

(a) $\sqrt{2} < \sqrt[3]{4} < \sqrt[4]{6}$ (b) $\sqrt[4]{6} < \sqrt{2} < \sqrt[3]{4}$

(c) $\sqrt[4]{6} < \sqrt[3]{4} < \sqrt{2}$ (d) $\sqrt{2} < \sqrt[4]{6} < \sqrt[3]{4}$

उत्तरमाला

1. (b)	2. (c)	3. (a)	4. (d)	5. (c)	6. (b)	7. (d)	8. (b)	9. (d)	10. (a)
11. (d)	12. (c)	13. (c)	14. (c)	15. (d)	16. (c)	17. (b)	18. (b)	19. (b)	20. (c)
21. (a)	22. (d)	23. (c)	24. (d)	25. (d)	26. (a)	27. (b)	28. (b)	29. (c)	30. (b)
31. (a)	32. (c)	33. (a)	34. (a)	35. (b)	36. (b)	37. (d)	38. (b)	39. (d)	40. (d)
41. (c)	42. (d)	43. (a)	44. (d)	45. (a)	46. (c)	47. (a)	48. (a)	49. (b)	50. (b)
51. (e)	52. (b)	53. (a)	54. (d)	55. (b)	56. (b)	57. (a)			

दिये गये प्रश्नों के हल

1. $(8)^{\frac{2}{3}} = (2^3)^{\frac{2}{3}} = 2^{\left(3\times\frac{2}{3}\right)} = 2^2 = 4.$

2. $(16)^{\frac{3}{4}} = (2^4)^{\frac{3}{4}} = 2^{\left(4\times\frac{3}{4}\right)} = 2^3 = 8.$

3. $(\sqrt{8})^{\frac{1}{3}} = \left(8^{\frac{1}{2}}\right)^{\frac{1}{3}} = 8^{\left(\frac{1}{2}\times\frac{1}{3}\right)} = (2^3)^{\frac{1}{6}} = 2^{\left(3\times\frac{1}{6}\right)} = 2^{\frac{1}{2}} = \sqrt{2}.$

4. $(36)^{\frac{1}{6}} = (6^2)^{\frac{1}{6}} = 6^{\left(2\times\frac{1}{6}\right)} = 6^{\frac{1}{3}} = \sqrt[3]{6}.$

5. $(16)^{1.75} = (16)^{\frac{175}{100}} = (2^4)^{\frac{7}{4}} = 2^{\left(4\times\frac{7}{4}\right)} = 2^7 = 128.$

6. $\dfrac{(1000)^9}{10^{24}} = \dfrac{(10^3)^9}{10^{24}} = \dfrac{10^{(3\times9)}}{10^{24}} = \dfrac{10^{27}}{10^{24}} = 10^{(27-24)} = 10^3 = 1000.$

7. $\left(\dfrac{81}{169}\right)^{-\frac{1}{2}} = \left(\dfrac{169}{81}\right)^{\frac{1}{2}} = \dfrac{(169)^{\frac{1}{2}}}{(81)^{\frac{1}{2}}} = \dfrac{(13^2)^{\frac{1}{2}}}{(9^2)^{\frac{1}{2}}} = \dfrac{13^{\left(2\times\frac{1}{2}\right)}}{9^{\left(2\times\frac{1}{2}\right)}} = \dfrac{13^1}{9^1} = \dfrac{13}{9}.$

8. $\left(\dfrac{32}{243}\right)^{\frac{-4}{5}} = \left(\dfrac{243}{32}\right)^{\frac{4}{5}} = \dfrac{(243)^{4/5}}{(32)^{4/5}} = \dfrac{(3^5)^{\frac{4}{5}}}{(2^5)^{\frac{4}{5}}} = \dfrac{3^{\left(5\times\frac{4}{5}\right)}}{2^{\left(5\times\frac{4}{5}\right)}} = \dfrac{3^4}{2^4} = \dfrac{81}{16}.$

9. $\left(\dfrac{-1}{343}\right)^{-\frac{2}{3}} = (-343)^{\frac{2}{3}} = \left\{(-7)^3\right\}^{\frac{2}{3}} = (-7)^{\left(3\times\frac{2}{3}\right)} = (-7)^2 = 49.$

10. दिया गया व्यंजक $= (216)^{\frac{2}{3}} + (256)^{\frac{3}{4}} + (243)^{\frac{1}{5}} = (6^3)^{\frac{2}{3}} + (4^4)^{\frac{3}{4}} + (3^5)^{\frac{1}{5}} = 6^{\left(3\times\frac{2}{3}\right)} + 4^{\left(4\times\frac{3}{4}\right)} + 3^{\left(5\times\frac{1}{5}\right)}$

$= (6^2 + 4^3 + 3^1) = (36 + 64 + 3) = 103.$

11. $\sqrt{2^n} = 64 \Rightarrow (2^n)^{\frac{1}{2}} = 2^6 \Rightarrow 2^{n/2} = 2^6 \Rightarrow \dfrac{n}{2} = 6 \Rightarrow n = 12.$

12. $\sqrt{4^n} = 1024 \Rightarrow (4^n)^{\frac{1}{2}} = 4^5 \Rightarrow 4^{n/2} = 4^5 \Rightarrow \dfrac{n}{2} = 5 \Rightarrow n = 10.$

13. दिया गया व्यंजक $= \dfrac{3^{n+1}(3-1)}{3^{n+1}(3^3+1)} = \dfrac{2}{28} = \dfrac{1}{14}.$

14. माना $\left\{(2^4)^{\frac{1}{2}}\right\}^x = 256.$ तब, $\left\{2^{\left(4\times\frac{1}{2}\right)}\right\}^x = 2^8 \Rightarrow (2^2)^x = 2^8$

$\therefore\ 2^{2x} = 2^8 \Rightarrow 2x = 8 \Rightarrow x = 4.$

15. $\dfrac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})} = \dfrac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})} \times \dfrac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})} = \dfrac{(\sqrt{5}-\sqrt{3})^2}{(5-3)} = \dfrac{5+3-2\times\sqrt{5}\times\sqrt{3}}{2}$

$= \dfrac{8-2\sqrt{15}}{2} = \dfrac{2(4-\sqrt{15})}{2} = (4-\sqrt{15}).$

16. $\dfrac{(3^2)^n \times 3^5 \times (3^3)^3}{3\times(3^4)^4} = 3^3 \Rightarrow \dfrac{3^{2n}\times 3^5 \times 3^9}{3\times 3^{16}} = 3^3 \Rightarrow \dfrac{3^{2n}\times 3^{14}}{3^{17}} = 3^3$

$$\Rightarrow 3^{2n}\times 3^{14}=3^{17}\times 3^{3}=3^{20}\Rightarrow 3^{2n}=\frac{3^{20}}{3^{14}}=3^{(20-14)}=3^{6}$$

$$\Rightarrow 2n=6\Rightarrow n=3.$$

17. $8^3\times 8^2\times 8^{-5}=8^{(3+2-5)}=8^0=1.$

18. स्पष्ट है कि $\sqrt[3]{5}$ की घात 3 है.

19. $(3^3)^{\frac{2}{3}}\times(3^4)^{-\frac{1}{2}}=3^n\Rightarrow 3^{\left(3\times\frac{2}{3}\right)}\times 3^{4\times\left(\frac{-1}{2}\right)}=3^n$

$$\Rightarrow 3^2\times 3^{-2}=3^n\Rightarrow 3^{2+(-2)}=3^n\Rightarrow 3^n=3^0\Rightarrow n=0.$$

20. दिया गया व्यंजक $=(216)^{\frac{2}{3}}\div(27)^{4/3}=(6^3)^{\frac{2}{3}}\div(3^3)^{4/3}$

$$=6^{\left(3\times\frac{2}{3}\right)}\div 3^{\left(3\times\frac{4}{3}\right)}=6^2\div 3^4=\frac{36}{81}=\frac{4}{9}.$$

21. दिया गया व्यंजक $=\left(8+4+\frac{1}{2^2}+\frac{1}{2^3}\right)=\left(12+\frac{1}{4}+\frac{1}{8}\right)=\frac{(96+2+1)}{8}=\frac{99}{8}.$

22. $9261=3^3\times 7^3=(3\times 7)^3=(21)^3.$

3	9261
3	3087
3	1029
7	343
7	49
	7

$\therefore (9261)^{\frac{1}{3}}=21.$

दिया गया व्यंजक $=\frac{42\times 229}{21}=458.$

23. दिया गया व्यंजक $=\frac{(5)^{.25}\times(5^3)^{.25}}{(256)^{(.10+.15)}}=\frac{5^{.25}\times 5^{.75}}{(256)^{.25}}=\frac{5^{(.25+.75)}}{(4^4)^{\frac{1}{4}}}=\frac{5^1}{4^{\left(4\times\frac{1}{4}\right)}}=\frac{5^1}{4^1}=\frac{5}{4}.$

24. $\left(3^{\frac{1}{2}}\right)^5\times(3^2)^2=3^{\alpha}\times 3\times 3^{\frac{1}{2}}\Rightarrow 3^{\frac{5}{2}}\times 3^4=3^{\alpha}\times 3^{\left(1+\frac{1}{2}\right)}\Rightarrow 3^{\left(\frac{5}{2}+4\right)}=3^{\alpha}\times 3^{\frac{3}{2}}$

$$\Rightarrow 3^{\alpha}=\frac{3^{13/2}}{3^{3/2}}=3^{\left(\frac{13}{2}-\frac{3}{2}\right)}=3^5\Rightarrow \alpha=5.$$

25. $2^x\times(2^3)^{\frac{1}{5}}=2^{\frac{1}{5}}\Rightarrow 2^x\times 2^{\frac{3}{5}}=2^{\frac{1}{5}}\Rightarrow 2^x=\frac{2^{1/5}}{2^{3/5}}=2^{\left(\frac{1}{5}-\frac{3}{5}\right)}=2^{-2/5}\Rightarrow x=\frac{-2}{5}.$

26. $5\times 5^{\frac{1}{2}}\times 5^3\div 5^{-\frac{3}{2}}=5^{a+2}\Rightarrow\frac{5^{\left(1+\frac{1}{2}+3\right)}}{5^{-3/2}}=5^{a+2}\Rightarrow 5^{\left(\frac{9}{2}+\frac{3}{2}\right)}=5^{a+2}\Rightarrow 5^6=5^{a+2}\Rightarrow a+2=6\Rightarrow a=4.$

27. दिया गया व्यंजक $=x^{(b+c)(b-c)}.x^{(c+a)(c-a)}.x^{(a+b)(a-b)}$

$$=x^{(b^2-c^2)}.x^{(c^2-a^2)}.x^{(a^2-b^2)}=x^{(b^2-c^2+c^2-a^2+a^2-b^2)}=x^0=1.$$

28. दिया गया व्यंजक $=x^{(a-b)(a+b)}.x^{(b-c)(b+c)}.x^{(c-a)(c+a)}$

$$=x^{(a^2-b^2)}.x^{(b^2-c^2)}.x^{(c^2-a^2)}=x^{(a^2-b^2+b^2-c^2+c^2-a^2)}=x^0=1.$$

29. दिया गया व्यंजक $=\frac{1}{1+\frac{a^n}{a^m}}+\frac{1}{1+\frac{a^m}{a^n}}=\frac{a^m}{(a^m+a^n)}+\frac{a^n}{(a^m+a^n)}=\frac{(a^m+a^n)}{(a^m+a^n)}=1.$

30. दिया गया व्यंजक $= \frac{1}{1+\frac{x^b}{x^a}+\frac{x^c}{x^a}} + \frac{1}{1+\frac{x^a}{x^b}+\frac{x^c}{x^b}} + \frac{1}{1+\frac{x^b}{x^c}+\frac{x^a}{x^c}}$

$= \frac{x^a}{(x^a+x^b+x^c)} + \frac{x^b}{(x^a+x^b+x^c)} + \frac{x^c}{(x^a+x^b+x^c)} = \frac{(x^a+x^b+x^c)}{(x^a+x^b+x^c)} = 1.$

31. दिया गया व्यंजक $= x^{(b-c)(b+c-a)} \cdot x^{(c-a)(c+a-b)} \cdot x^{(a-b)(a+b-c)}$

$= x^{(b^2-c^2+c^2-a^2+a^2-b^2)} \cdot x^{-a(b-c)-b(c-a)-c(a-b)} = x^0 \cdot x^0 = 1.$

32. $\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{x-3} = \left(\frac{a}{b}\right)^{3-x} \Rightarrow x-1 = 3-x \Rightarrow 2x = 4 \Rightarrow x = 2.$

33. $2^{2x+4} = 16^x = (2^4)^x = 2^{4x} \Rightarrow 2x+4 = 4x \Rightarrow 2x = 4 \Rightarrow x = 2.$

$\therefore\ x^3 = 2^3 = 8.$

34. $5^{5x+5} = 1 = 5^0 \Rightarrow 5x+5 = 0 \Rightarrow 5x = -5 \Rightarrow x = -1.$

35. $3^{x+3} + 7 = 250 \Rightarrow 3^{x+3} = 243 = 3^5 \Rightarrow x+3 = 5 \Rightarrow x = 2.$

36. $a^{2x+2} = 1 = a^0 \Rightarrow 2x+2 = 0 \Rightarrow 2x = -2 \Rightarrow x = -1.$

37. $5^{x+3} = (5^2)^{3x-4} = 5^{2(3x-4)} = 5^{(6x-8)} \Rightarrow x+3 = 6x-8 \Rightarrow 5x = 11 \Rightarrow x = \frac{11}{5}.$

38. $2^{2x-1} = 8^{3-x} = (2^3)^{3-x} = 2^{9-3x} \Rightarrow 2x-1 = 9-3x \Rightarrow 5x = 10 \Rightarrow x = 2.$

39. $2^{x-1}(1+2^2) = 320 \Rightarrow 2^{x-1} \times 5 = 320 \Rightarrow 2^{x-1} = \frac{320}{5} = 64 = 2^6 \Rightarrow x-1 = 6 \Rightarrow x = 7.$

40. $2^{x+2}(2^2-1) = 3 \Rightarrow 2^{x+2} \times 3 = 3 \Rightarrow 2^{x+2} = 1 = 2^0 \Rightarrow x+2 = 0 \Rightarrow x = -2.$

41. $(32)^{\frac{1}{3}} = 2^x \Rightarrow (2^5)^{\frac{1}{3}} = 2^x \Rightarrow 2^{\left(5\times\frac{1}{3}\right)} = 2^x \Rightarrow x = \frac{5}{3}.$

42. दिया गया व्यंजक $= \sqrt{\frac{b}{a}} \cdot \sqrt{\frac{c}{b}} \cdot \sqrt{\frac{a}{c}} = \frac{\sqrt{b}}{\sqrt{a}} \times \frac{\sqrt{c}}{\sqrt{b}} \times \frac{\sqrt{a}}{\sqrt{c}} = 1.$

43. $z = x^c = (y^a)^c = y^{ac} = (z^b)^{ac} = z^{abc} \Rightarrow abc = 1.$

44. माना $a^x = b^y = c^z = k.$ तब, $a = k^{\frac{1}{x}}, b = k^{\frac{1}{y}}, c = k^{\frac{1}{z}}.$

$b^2 = ac \Rightarrow \left(k^{\frac{1}{y}}\right)^2 = k^{\frac{1}{x}} \times k^{\frac{1}{z}} \Rightarrow k^{(2/y)} = k^{\left(\frac{1}{x}+\frac{1}{z}\right)} \Rightarrow \frac{2}{y} = \left(\frac{1}{x}+\frac{1}{z}\right) = \frac{z+x}{xz} \Rightarrow y = \frac{2xz}{(x+z)}.$

45. $2^x = 3^y = 6^{-z} = k \Rightarrow 2 = k^{\frac{1}{x}}, 3 = k^{\frac{1}{y}}, 6 = k^{\frac{-1}{z}}$

अब, $2\times 3 = 6 \Rightarrow k^{\frac{1}{x}} \times k^{\frac{1}{y}} = k^{\frac{-1}{z}} \Rightarrow k^{\left(\frac{1}{x}+\frac{1}{y}\right)} = k^{\left(\frac{-1}{z}\right)} \Rightarrow \frac{1}{x}+\frac{1}{y} = \frac{-1}{z} \Rightarrow \frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 0.$

46. $2^x = 4^y = 8^z \Rightarrow 2^x = 2^{2y} = 2^{3z} \Rightarrow x = 2y = 3z.$

$\frac{1}{2x}+\frac{1}{4y}+\frac{1}{6z} = \frac{24}{7} \Rightarrow \frac{1}{6z}+\frac{1}{6z}+\frac{1}{6z} = \frac{24}{7} \Rightarrow \frac{3}{6z} = \frac{24}{7} \Rightarrow z = \frac{7\times 3}{6\times 24} = \frac{21}{144} = \frac{7}{48}.$

47. $\frac{(3^3)^{\frac{5}{2}} \times (3^2)^{\frac{3}{2}}}{3^7} = 3^x \Rightarrow \frac{3^{\frac{15}{2}} \times 3^{\left(2\times\frac{3}{2}\right)}}{3^7} = 3^x \Rightarrow \frac{3^{\frac{15}{2}} \times 3^3}{3^7} = 3^x \Rightarrow 3^x = \frac{3^{\left(\frac{15}{2}+3\right)}}{3^7}$

$$\Rightarrow 3^x = 3^{\left(\frac{21}{2}-7\right)} = 3^{7/2} \Rightarrow x = \frac{7}{2}.$$

48. दिया गया व्यंजक $= (4^4)^{.16} \times (4^2)^{.18} = 4^{(4\times.16)} \times 4^{(2\times.18)} = 4^{.64} \times 4^{.36} = 4^{(.64+.36)} = 4^1 = 4.$

49. $10^{7.5} \times (5\times 2)^{2.5} = 10^x \Rightarrow 10^{7.5} \times 10^{2.5} = 10^x \Rightarrow 10^{(7.5+2.5)} = 10^x \Rightarrow 10^x = 10^{10} \Rightarrow x = 10.$

50. $8^{4.2} \times (8^2)^{2.1} \times 7^{8.4} \times 56^{3.5} = (56)^x \Rightarrow 8^{4.2+4.2} \times 7^{8.4} \times 56^{3.5} = (56)^x$

$$\Rightarrow (8\times 7)^{8.4} \times 56^{3.5} = (56)^x \Rightarrow (56)^{(8.4+3.5)} = (56)^x \Rightarrow x = 11.9.$$

51. $9^{(8.6+3.9)} \times 8^{(3.9+8.6)} \times 72^{4.4} = 72^x \Rightarrow 9^{12.5} \times 8^{12.5} \times 72^{4.4} = 72^x \Rightarrow 72^x = (9\times 8)^{12.5} \times 72^{4.4}$

$$\Rightarrow 72^x = (72)^{(12.5+4.4)} = (72)^{(16.9)} \Rightarrow x = 16.9.$$

52. $\sqrt[4]{(625)^3} = (625)^{\frac{3}{4}} = (5^4)^{\frac{3}{4}} = 5^{\left(4\times\frac{3}{4}\right)} = 5^3 = 125.$

53. माना $(10000)^{\frac{1}{4}} = (100)^x$. तब, $(10^4)^{\frac{1}{4}} = (10^2)^x$.

$$\therefore 10^{2x} = 10^{\left(4\times\frac{1}{4}\right)} = 10^1 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}.$$

54. $\frac{1}{x} = \frac{1}{(7-4\sqrt{3})} \times \frac{(7+4\sqrt{3})}{(7+4\sqrt{3})} = \frac{(7+4\sqrt{3})}{(49-48)} = (7+4\sqrt{3}).$

$$\therefore \left(x+\frac{1}{x}\right) = (7-4\sqrt{3}) + (7+4\sqrt{3}) = 14 \Rightarrow x + \frac{1}{x} + 2 = 14 + 2 = 16$$

$$\Rightarrow \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 = 4^2 \Rightarrow \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) = 4.$$

55. $x = (3+2\sqrt{2}) \Rightarrow \frac{1}{x} = \frac{1}{(3+2\sqrt{2})} \times \frac{(3-2\sqrt{2})}{(3-2\sqrt{2})} = \frac{(3-2\sqrt{2})}{(9-8)} = (3-2\sqrt{2}).$

$$\therefore x + \frac{1}{x} = 6 \Rightarrow \left(x+\frac{1}{x}\right)^2 = 6^2 \Rightarrow x^2 + \frac{1}{x^2} + 2 = 36 \Rightarrow x^2 + \frac{1}{x^2} = 34$$

$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = (34)^2 = 1156 \Rightarrow \left(x^4 + \frac{1}{x^4} + 2\right) = 1156 \Rightarrow \left(x^4 + \frac{1}{x^4}\right) = 1154.$$

56. दी गई करणी हैं $2^{\frac{1}{2}}$, $3^{\frac{1}{6}}$, $4^{\frac{1}{3}}$, $5^{\frac{1}{4}}$.

2, 6, 3, 4 का ल०स० = 12.

$2^{\frac{1}{2}} = (2^6)^{\frac{1}{12}} = (64)^{\frac{1}{12}}$, $3^{\frac{1}{6}} = (3^2)^{\frac{1}{12}} = (9)^{\frac{1}{12}}$, $4^{\frac{1}{3}} = (4^4)^{\frac{1}{12}} = (256)^{\frac{1}{12}}$, $5^{\frac{1}{4}} = (5^3)^{\frac{1}{12}} = (125)^{\frac{1}{12}}$

इनमें सबसे बड़ा है $(256)^{\frac{1}{12}}$ अर्थात $\sqrt[3]{4}$.

57. दी गई करणी हैं $2^{\frac{1}{2}}$, $4^{\frac{1}{3}}$, $6^{\frac{1}{4}}$ तथा 2, 3, 4 का ल०स० = 12.

$2^{\frac{1}{2}} = (2^6)^{\frac{1}{12}} = (64)^{\frac{1}{12}}$, $4^{\frac{1}{3}} = (4^4)^{\frac{1}{12}} = (256)^{\frac{1}{12}}$ तथा $6^{\frac{1}{4}} = (6^3)^{\frac{1}{12}} = (216)^{\frac{1}{12}}$.

स्पष्ट है कि $(64)^{\frac{1}{12}} < (216)^{\frac{1}{12}} < (256)^{\frac{1}{12}}$ अर्थात $\sqrt{2} < \sqrt[3]{4} < \sqrt[4]{6}$.