ISC SEMESTER 2 EXAMINATION
SAMPLE PAPER - 3
MATHEMATICS

Maximum Marks: 40

Time allowed: One and a half hour

*Candidates are allowed an additional **10 minutes** for **only** reading the paper.*

*They must **Not** start writing during this time.*

The Question Paper consists of three sections A, B and C.

*Candidates are required to attempt all questions from **Section A** and all questions **EITHER** from **Section B** **OR** **Section C***

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

Mathematical tables and graph papers are provided.

Section-A

Question 1.

Choose the correct option for the following questions.

(i) If I = $\int_{0.2}^{3.5} [x]\, dx$, where $[x]$ is a greatest integer function, then I is equal to :

(a) 0 (b) 2.6 (c) 4.3 (d) 4.5

(ii) A and B are two events such that P(A) = 0.25 and P(B) = 0.50. The probability of both happening together is 0.14. The probability of both A and B not happening is :

(a) 0.39 (b) 0.25 (c) 0.11 (d) none of these

(iii) Given $\int a^x\, dx = \frac{a^x}{f(x)} + c$. Then $f(x)$ satisfying the equation if :

(a) $f(x) = 1$ (b) $f(x) = x$ (c) $f(x) = \log_e a$ (d) $f(x) = \log_a e$

(iv) The general Answer of the differential equation $\frac{y\, dx - x\, dy}{y} = 0$ is :

(a) $y = x$ (b) $y = mx$ (c) $y = cx$ (d) $y = x + c$

(v) The probability of A of hitting a target is $\frac{4}{5}$ and that of B is $\frac{2}{3}$. They both fire at a target, the probability that only A hits the target is :

(a) $\frac{8}{15}$ (b) $\frac{1}{15}$ (c) $\frac{4}{15}$ (d) 0

(vi) $\int x \sec^2 x dx$ =

(a) $x \cot x + \log |\cos x| + c$ (b) $x \tan x + \log |\cos x| + c$

(c) $x \tan x - \log |\cos x| + c$ (d) None of these

Question 2.

Evaluate : $\int \frac{x^4 + x^2 + 1}{x^2 + x + 1}\, dx$.

OR

Evaluate : $\int_0^{\frac{\pi}{2}} \log \tan x \, dx$

Question 3.

Solve : $\sqrt{a + x}\, dy + x\, dx = 0.$

OR

Solve $\frac{dy}{dx} = 1 - xy + y - x.$

Question 4.

Solve : $\int \frac{\sqrt{x}}{a^3 - x^3}\, dx.$

Question 5.

A speaks truth in 60% of the cases, while B in 40% of the cases. In what percent of cases are they likely to contradict each other in stating the same fact?

OR

A bag contains 5 white and 3 black balls. Four balls are successively drawn out without replacement. What is the probability that they are alternatively of different colours?

Question 6.

Evaluate : $\int_0^{\pi} \frac{x}{1 + \sin x}\, dx.$

Question 7.

A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}\cdot$ The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$, if he comes by train, bus and scooter respectively, but he comes by other means of transport then he will not be late. When he arrives, he is late. What is the probability that he comes by train ?

Question 8.

Evaluate : $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}\, dx.$

OR

Evaluate : $\int \frac{dx}{\sqrt{\sin^3 x \sin(x + \alpha)}}$

Section-B

Question 9.

Choose the correct option for the following questions.

(i) The equation of the plane which cuts equal intercepts of unit length on the coordinate axes is :

(a) $x + y + z = 1$ (b) $x + y + z = 0$ (c) $x + y - z = 1$ (d) $x + y + z = 2$

(ii) The equation of the plane passing through (2, – 1, 1) and parallel to the plane $3x + 2y - z = 7$ is :

(a) $3x + 2y - z = 7$ (b) $3x + 2y - z = 3$ (c) $2x + 3y - z = 7$ (d) $2x + 3y - z = 3$

Question 10.

Find the equation of a plane, which is at a distance of 3 units from the origin and perpendicular to a line

$$\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-1}{2}.$$

Write the equation in **(i)** Cartesian form, **(ii)** Normal form.

Question 11.

Find the area of the region bounded by the curve $x = 4y - y^2$ and the Y-axis.

Section-C

Question 12.

Choose the correct option for the following questions.

(i) Regression analysis was applied between sales (y) and advertising (x) across all the branches of a major international corporation. The following regression function was obtained.

$$y = 5000 + 7.25x$$

If the advertising budgets of two branches of the corporation differ by ₹ 30,000, then what will be the predicted differences in their sales?

(a) ₹ 2,17,500 (b) ₹ 2,22,500 (c) ₹ 5,000 (d) ₹ 7.25

(ii) If the regression coefficients for the variables x and y are $b_{yx} = \frac{4}{5}$ and $b_{xy} = \frac{1}{5}$, then the correlation coefficient between x and y is :

(a) 4 (b) 1 (c) $\frac{-2}{5}$ (d) $\frac{2}{5}$

Question 13.

Following table relate to demand and price of a commodity.

Demand	20	22	24	26	28	30	32	34	36	38
Price ₹/kg	10	12	16	18	20	20	22	24	24	24

Calculate the regression coefficent X on Y.

Question 14.

A manufacturer makes two types of toy A and B. Three machines are needed for this purpose and the time (in minutes) required for each toy on the machines is given below :

Type of Toy	Machines			Profit in ₹
	I	II	III	
A	12 min.	18 min.	6 min.	7.50
B	6 min.	0 min.	9 min.	5.00
Time available	6 hrs	6 hrs	6 hrs	

Each machine is available for a maximum 6 hr per day. If the profit on each toy of type A is ₹ 7.50 and that on each toy of type B is ₹ 5.00, show that 15 toys of type A and 30 toys of type B should be manufactured in a day to get maximum profit.

Answers

Section-A

Answer 1.

(i) (d) 4.5

Explanation :

Since $[x]$ is a greatest integer function, So

$$f(x) = \begin{cases} 0, & \text{if } 0.2 \le x < 1 \\ 1, & \text{if } 1 \le x < 2 \\ 2, & \text{if } 2 \le x < 3 \\ 3, & \text{if } 3 \le x \le 3.5 \end{cases}$$

$$\therefore \quad I = \int_{0.2}^{3.5} [x] = \int_{0.2}^{1} 0\,dx + \int_{1}^{2} 1\,dx + \int_{2}^{3} 2\,dx + \int_{3}^{3.5} 3\,dx$$

$$\Rightarrow \quad I = 0 + [x]_1^2 + 2[x]_2^3 + 3[x]_3^{3.5}$$

$$\Rightarrow \quad I = (2-1) + 2 \times 1 + 3 \times 0.5$$

$$I = 4.5$$

(ii) (a) 0.39

Explanation :

Given, $P(A) = 0.25, P(B) = 0.50$ and $P(A \cap B) = 0.14$

We know,
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.25 + 0.50 - 0.14$$
$$= 0.61$$

Required probability $= P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B) = 1 - 0.61 = 0.39.$

(iii) (c) $f(x) = \log_e a$

Explanation :

We have
$$\int a^x\, dx = \int e^{\log a^x}$$
$$= \int e^{x \log a}\, dx$$
$$= \int e^{x \log a}\, dx$$

Let $u = x \log a \Rightarrow du = \log a\, dx$

$$= \int e^u \frac{du}{\log a}$$
$$= \frac{1}{\log a}\int e^u\, du$$
$$= \frac{1}{\log a} e^u = \frac{1}{\log a} e^{x \log a} + c$$
$$= \frac{e^{\log a^x}}{\log a} + c$$
$$= \frac{a^x}{\log a} + c$$

(iv) (c) $y = cx$

Explanation :

Given, $\frac{y dx - x dy}{y} = 0$

$\Rightarrow$ $y\, dx - x\, dy = 0$

$\Rightarrow$ $\frac{dy}{y} = \frac{dx}{x}$

$\Rightarrow$ $\int \frac{dy}{y} = \int \frac{dx}{x}$

$\Rightarrow$ $\log |y| = \log |x| + \log |c|$

$\Rightarrow$ $\log |y| = \log |cx|$

$\therefore$ $y = cx.$

(v) (c) $\frac{4}{15}$

Explanation :

Given, $P(A) = \frac{4}{5}$ and $P(B) = \frac{2}{3}$

Probability that only A hits the target $= P(A).P(\overline{B})$

$$= \frac{4}{5}\left(1 - \frac{2}{3}\right)$$
$$= \frac{4}{5} \times \frac{1}{3}$$
$$= \frac{4}{15}.$$

(vi) (b) $x \tan x + \log |\cos x| + c$

Explanation :

$$\int x \sec^2 x \, dx = \mathrm{I}$$

Using "ILATE", 1st function is 'x' and 2nd function is "$\sec^2 x$"

$$\mathrm{I} = x\int \sec^2 x \, dx - \int\left[\frac{d}{dx}(x) \times \int \sec^2 x \, dx\right]dx$$

$$= x \tan x - \int \tan x \, dx$$

$$= x \tan x - \log |\sec x| + c$$

$$= x \tan x + \log |\cos x| + c$$

Answer 2.

Let,
$$\mathrm{I} = \int \frac{x^4 + x^2 + 1}{x^2 + x + 1} dx$$

$$= \int \frac{x^4 + 2x^2 + 1 - x^2}{x^2 + x + 1} dx$$

$$= \int \frac{(x^2 + 1)^2 - (x)^2}{x^2 + x + 1} dx$$

$$= \int \frac{(x^2 + x + 1)(x^2 + 1 - x)}{(x^2 + x + 1)} dx$$

$$= \int (x^2 + 1 - x) dx$$

$$= \int x^2 dx + \int 1 dx - \int x \, dx$$

$$= \frac{x^3}{3} + x - \frac{x^2}{2} + c$$

Ans.

OR

Let,
$$\mathrm{I} = \int_0^{\frac{\pi}{2}} \log \tan x \, dx \quad \text{..(i)}$$

$$= \int_0^{\frac{\pi}{2}} \log \tan\left(\frac{\pi}{2} - x\right) dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)dx\right)$$

$$= \int_0^{\frac{\pi}{2}} \log \cot x \, dx \quad \text{...(ii)}$$

On adding equations (i) and (ii), we get,

$$2\mathrm{I} = \int_0^{\frac{\pi}{2}} \log \tan x \, dx + \int_0^{\frac{\pi}{2}} \log \cot x \, dx$$

$$\Rightarrow \quad 2\mathrm{I} = \int_0^{\frac{\pi}{2}} \log \tan x . \cot x \, dx = \int_0^{\frac{\pi}{2}} \log 1 \, dx \qquad \left(\because \tan x = \frac{1}{\cot x}\right)$$

$$= \int_0^{\frac{\pi}{2}} 0 \, dx$$

$$\therefore \quad \mathrm{I} = 0$$

Ans.

Answer 3.

We have,
$$\sqrt{a + x}\, dy + x \, dx = 0$$

$$\sqrt{a + x}\, dy = - x \, dx$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-x}{\sqrt{a + x}}$$

$$\Rightarrow \quad dy = \frac{-x}{\sqrt{a+x}}\,dx$$

On integrating both sides, we get

$$\int dy = -\int \frac{x+a-a}{\sqrt{a+x}}\,dx$$

$$\Rightarrow \quad y = -\int \frac{x+a}{\sqrt{a+x}}\,dx + \int \frac{a}{\sqrt{a+x}}\,dx$$

$$\Rightarrow \quad y = -\int \sqrt{a+x}\,dx + a\int \frac{1}{\sqrt{a+x}}\,dx$$

$$\Rightarrow \quad y = -\frac{(a+x)^{3/2}}{3/2} + \frac{a(a+x)^{1/2}}{1/2} + c$$

$$\Rightarrow \quad y = \frac{-2}{3}(a+x)^{3/2} + 2a(a+x)^{1/2} + c$$

$$\Rightarrow \quad y = \frac{2}{3}(a+x)^{1/2}\{-(a+x)+3a\} + c$$

$$\Rightarrow \quad y = \frac{2}{3}\sqrt{a+x}\,(2a-x) + c.$$ **Ans.**

OR

Given : $\dfrac{dy}{dx} = 1 - xy + y - x$

$= (1-x) + y(1-x)$

$$\Rightarrow \quad \frac{dy}{dx} = (1-x)(1+y)$$

$$\Rightarrow \quad \frac{1}{1+y}dy = (1-x).dx$$

Integrating both sides,

$$\int \frac{1}{1+y}dy = \int (1-x)\,dx$$

$$\Rightarrow \quad \log(1+y) = x - \frac{x^2}{2} + c$$ **Ans.**

Answer 4.

$$\int \frac{\sqrt{x}}{a^3 - x^3}\,dx$$

Let $x^{3/2} = t$

$$\Rightarrow \quad \frac{3}{2}x^{1/2}\,dx = dt$$

$$\Rightarrow \quad \sqrt{x}\,dx = \frac{2}{3}\,dt$$

$$\therefore \quad \int \frac{\sqrt{x}}{a^3 - x^3}\,dx = \int \frac{\frac{2}{3}}{a^3 - t^2}\,dt = \frac{2}{3}\int \frac{dt}{(a^{3/2})^2 - t^2}$$

$$= \frac{2}{3}\cdot\frac{1}{2a^{3/2}}\log\left|\frac{a^{3/2}+t}{a^{3/2}-t}\right| + c$$

$$= \frac{1}{3a^{3/2}}\log\left|\frac{a^{3/2}+x^{3/2}}{a^{3/2}-x^{3/2}}\right| + c.$$ **Ans.**

Answer 5.

Let X and Y be the events that A and B speak truth, respectively.

$$\therefore \quad P(X) = 60\% = \frac{60}{100}$$

and $$P(Y) = 40\% = \frac{40}{100}$$

Now, A and B will contradict each other if one is speaking truth and the other one is lying.

$\therefore$ Required probability = $P(X)P(\bar{Y}) + P(\bar{X})P(Y)$

$$= \frac{60}{100}\times\left(1-\frac{40}{100}\right)+\left(1-\frac{60}{100}\right)\left(\frac{40}{100}\right)$$

$$= \frac{60}{100}\times\frac{60}{100}+\frac{40}{100}\times\frac{40}{100}$$

$$= \frac{36}{100}+\frac{16}{100}$$

$$= \frac{52}{100}$$

Hence, there is 52% chances that they will contradict each other while stating the same fact. **Ans.**

OR

Let W_i denotes the event of drawing a white ball in i^{th} drawn and B_i denotes the event of drawing a black ball in i^{th} drawn, where $i = 1, 2, 3, 4$.

$\therefore$ Required probability = $P[(W_1 \cap B_2 \cap W_3 \cap B_4)] \cup P[(B_1 \cap W_2 \cap B_3 \cap W_4)]$

$$\therefore \quad = P(W_1)\, P\left(\frac{B_2}{W_1}\right) P\left(\frac{W_3}{W_1 \cap B_2}\right) P\left(\frac{B_4}{W_1 \cap B_2 \cap W_3}\right)$$

$$+ P(B_1)\, P\left(\frac{W_2}{B_1}\right) P\left(\frac{B_3}{B_1 \cap W_2}\right) P\left(\frac{W_4}{B_1 \cap W_2 \cap B_3}\right)$$

$$= \frac{5}{8}\times\frac{3}{7}\times\frac{4}{6}\times\frac{2}{5}+\frac{3}{8}\times\frac{5}{7}\times\frac{2}{6}\times\frac{4}{5}$$

$$= \frac{1}{14}+\frac{1}{14}$$

$\therefore$ Required probability $= \frac{1}{7}$ **Ans.**

Answer 6.

Let, $$I = \int_0^{\pi} \frac{x}{1+\sin x}\,dx \qquad \ldots\text{(i)}$$

$$\Rightarrow \quad I = \int_0^{\pi} \frac{\pi - x}{1+\sin(\pi - x)}\,dx \qquad \left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$$\Rightarrow \quad I = \int_0^{\pi} \frac{\pi - x}{1+\sin x}\,dx$$

$$\Rightarrow \quad I = \int_0^{\pi} \frac{\pi}{1+\sin x} - \int_0^{\pi} \frac{x}{1+\sin x}\,dx \qquad \ldots\text{(ii)}$$

On adding equations (i) and (ii), we get

$$2I = \int_0^{\pi} \frac{\pi}{1+\sin x}\,dx$$

$$\Rightarrow \quad 2I = \int_0^{\frac{\pi}{2}} \frac{\pi}{1+\sin x}\,dx + \int_0^{\frac{\pi}{2}} \frac{\pi}{1+\sin(\pi - x)}\,dx \quad \left[\because \int_0^{2a} f(x)\,dx = \int_0^{a} \{f(x)+f(2a-x)\}\,dx\right]$$

$$\Rightarrow \quad 2I = \pi\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x} + \pi\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x}$$

$$\Rightarrow \quad 2I = 2\pi\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x}$$

$$\Rightarrow \quad I = \pi\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x} \times \frac{1-\sin x}{1-\sin x}\, dx$$

$$\Rightarrow \quad I = \pi\left[\int_0^{\frac{\pi}{2}} \frac{1}{\cos^2 x}\, dx - \int_0^{\frac{\pi}{2}} \frac{\sin x}{\cos^2 x}\, dx\right]$$

$$\Rightarrow \quad I = \pi\left[\int_0^{\frac{\pi}{2}} \sec^2 dx - \int_0^{\frac{\pi}{2}} \sec x \tan x dx\right]$$

$$\Rightarrow \quad I = \pi\left[\tan x - \sec x\right]_0^{\frac{\pi}{2}} = \pi\left[\frac{\sin x - 1}{\cos x}\right]_0^{\frac{\pi}{2}}$$

$$\Rightarrow \quad I = -\pi\left[\frac{1-\sin x}{\cos x}\cdot\frac{1+\sin x}{1+\sin x}\right]_0^{\frac{\pi}{2}} = -\pi\left[\frac{\cos x}{1+\sin x}\right]_0^{\frac{\pi}{2}}$$

$$\Rightarrow \quad I = -\pi\left[\frac{0}{2} - \frac{1}{1+0}\right] = -\pi(-1) = \pi$$

$$\therefore \quad \int_0^{\pi} \frac{x}{1+\sin x}\, dx = \pi.$$ **Ans.**

Answer 7.

Let E_1, E_2, E_3 and E_4 be the events that the doctor visits the patient by train, bus, scooter and by other transport respectively.

$$\therefore \quad P(E_1) = \frac{3}{10}, P(E_2) = \frac{1}{5}, P(E_3) = \frac{1}{10} \text{ and } P(E_4) = \frac{2}{5}$$

Let A be the event that the doctor is late.

$$\therefore \quad P(A/E_1) = \frac{1}{4}, P(A/E_2) = \frac{1}{3}, P(A/E_3) = \frac{1}{12}, P(A/E_4) = 0$$

$\therefore$ Required probability

$$P(E_1/A) = \frac{P(E_1)\times P(A/E_1)}{P(E_1)\times P(A/E_1)+P(E_2)\times P(A/E_2)+P(E_3)\times P(A/E_3)+P(E_4)\times P(A/E_4)}$$

$$= \frac{\frac{3}{10}\times\frac{1}{4}}{\frac{3}{10}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{3}+\frac{1}{10}\times\frac{1}{12}+\frac{2}{5}\times 0}$$

$$= \frac{\frac{3}{40}}{\frac{3}{40}+\frac{1}{15}+\frac{1}{120}+0}$$

$$= \frac{\frac{3}{40}}{\frac{9+8+1}{120}}$$

$$\Rightarrow \quad P(E_1/A) = \frac{3}{40}\times\frac{120}{18} = \frac{1}{2}$$

$\therefore$ Probability he being late when he comes by train $= \frac{1}{2}\cdot$ **Ans.**

Answer 8.

Let, $$I = \int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx \quad \ldots\text{(i)}$$

$$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2}-x\right)\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}{\sin^4\left(\frac{\pi}{2}-x\right)+\cos^4\left(\frac{\pi}{2}-x\right)} dx \qquad \left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2}-x\right)\cos x \sin x}{\cos^4 x + \sin^4 x} dx$$

$$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \frac{\pi}{2} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx - \int_0^{\frac{\pi}{2}} \frac{x \cos x \sin x}{\sin^4 x + \cos^4 x} dx \quad \ldots\text{(ii)}$$

On adding equations (i) and (ii), we get

$$2I = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx$$

$$\Rightarrow \quad 2I = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{\frac{\sin x \cos x}{\cos^4 x}}{\frac{\sin^4 x}{\cos^4 x} + \frac{\cos^4 x}{\cos^4 x}} dx \qquad [\text{Divide by } \cos^4 x]$$

$$\Rightarrow \quad 2I = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{\tan x \sec^2 x}{\tan^4 x + 1} dx$$

$\therefore$ Let, $\tan^2 x = t$

$\therefore$ $2\tan x \sec^2 x\, dx = dt$

$\Rightarrow$ $\tan x \sec^2 x\, dx = \frac{dt}{2}$

$$\therefore \quad 2I = \frac{\pi}{4} \int_0^{\infty} \frac{dt}{t^2 + 1}$$

$$\Rightarrow \quad 2I = \frac{\pi}{4} \left[\tan^{-1} t\right]_0^{\infty}$$

$$\Rightarrow \quad 2I = \frac{\pi}{4} [\tan^{-1} \infty - \tan^{-1} 0]$$

$$\Rightarrow \quad 2I = \frac{\pi}{4} \times \frac{\pi}{2}$$

$$\therefore \quad I = \frac{\pi}{16}$$

$$\therefore \quad \int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} = \frac{\pi}{16}$$ **Ans.**

OR

Let $$I = \int \frac{dx}{\sqrt{\sin^3 x \sin(x+\alpha)}}$$

$$\Rightarrow \quad I = \int \frac{dx}{\sqrt{\sin^3 x(\sin x \cos\alpha + \cos x \sin\alpha)}}$$

$$\Rightarrow \quad I = \int \frac{dx}{\sqrt{\sin^4 x \sin\alpha\left(\frac{\cos\alpha}{\sin\alpha} + \frac{\cos x}{\sin x}\right)}}$$

$$\Rightarrow \quad I = \int \frac{dx}{\sin^2 x\sqrt{\sin\alpha(\cot\alpha+\cot x)}}$$

$$\Rightarrow \quad I = \int \frac{\operatorname{cosec}^2 x dx}{\sqrt{\sin\alpha}\sqrt{\cot\alpha+\cot x}}$$

Let $\cot\alpha + \cot x = t^2$

$$\Rightarrow \quad -\operatorname{cosec}^2 x = \frac{2tdt}{dx}$$

$$\Rightarrow \quad \operatorname{cosec}^2 x\, dx = -2t\, dt$$

$$\therefore \quad I = \int \frac{-2t\, dt}{\sqrt{\sin\alpha}\,\sqrt{t^2}}$$

$$\Rightarrow \quad I = \frac{-2}{\sqrt{\sin\alpha}}\int dt$$

$$\Rightarrow \quad I = \frac{-2t}{\sqrt{\sin\alpha}} + c$$

$$\Rightarrow \quad I = \frac{-2\sqrt{\cot\alpha+\cot x}}{\sqrt{\sin\alpha}} + c$$

$$\Rightarrow \quad I = -2\sqrt{\frac{\cot\alpha+\cot x}{\sin\alpha}} + c$$

$$\Rightarrow \quad I = -2\sqrt{\frac{\frac{\cos\alpha}{\sin\alpha}+\frac{\cos x}{\sin x}}{\sin\alpha}} + c$$

$$\Rightarrow \quad I = -2\sqrt{\frac{\cos\alpha\sin x+\cos x\sin\alpha}{\sin^2\alpha\sin x}} + c$$

$$\Rightarrow \quad I = \frac{-2}{\sin\alpha}\sqrt{\frac{\sin(x+\alpha)}{\sin x}} + c$$

$$\Rightarrow \quad I = -2\operatorname{cosec}\alpha\sqrt{\frac{\sin(x+\alpha)}{\sin x}} + c$$ **Ans.**

Section-B

Answer 9.

(i) (a) $x + y + z = 1$.

Explanation :

We know equation of the plane which cut the intercepts a, b and c respectively on coordinate axes is :

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1 \qquad \text{[Given } a = b = c = 1\text{]}$$

$$\therefore \quad x + y + z = 1.$$

(ii) (b) $3x + 2y - z = 3$.

Explanation :

Plane is passing through the point $(2, -1, 1)$ and parallel to the plane $3x + 2y - z = 7$.

Equation of the plane passing through (x_1, y_1, z_1) and parallel to plane $ax + by + cz = d$ is

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

$$\Rightarrow \quad 3(x - 2) + 2(y + 1) + (-1)(z - 1) = 0$$

$$\Rightarrow \quad 3x - 6 + 2y + 2 - z + 1 = 0$$

$$\Rightarrow \quad 3x + 2y - z = 3.$$

Answer 10.

Given equation of line is

$$\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-1}{2}$$

Since, required plane is perpendicular to the given line.

So, direction ratios of normal to the required plane are (1, 2, 2).

Direction cosines of the normal $= \left(\frac{1}{\sqrt{1^2+2^2+2^2}}, \frac{2}{\sqrt{1^2+2^2+2^2}}, \frac{2}{\sqrt{1^2+2^2+2^2}}\right) = \left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$

(i) Equation of the plane in Cartesian form :

$$ax + by + cz = p$$

$$\Rightarrow \quad \frac{1}{3}x + \frac{2}{3}y + \frac{2}{3}z = 3$$

$$\Rightarrow \quad x + 2y + 2z = 9$$ **Ans.**

(ii) Equation of the plane in normal form :

$$r \;\; \hat{n} = p$$

$$\Rightarrow \quad r \;\; \frac{1}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k} \;\; = 3$$

$$\Rightarrow \quad \vec{r}.(\hat{i} + 2\hat{j} + 2\hat{k}) = 9$$ **Ans.**

Answer 11.

Given : $\quad x = 4y - y^2$

$\Rightarrow \quad y^2 - 4y = -x$

$\Rightarrow \quad y^2 - 4y + 4 = -x + 4$

$\Rightarrow \quad (y - 2)^2 = -(x - 4)$

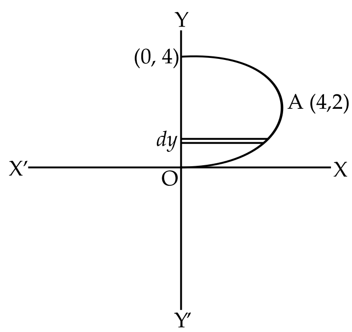
Given curve represents a left parabola with vertex A(4, 2) and intersect the Y-axis at origin and (0, 4).

$$\text{Required area} = \int_0^4 x\,dy = \int_0^4 (4y - y^2)\,dy$$

$$= \left[\frac{4y^2}{2} - \frac{y^3}{3}\right]_0^4$$

$$= \left[2y^2 - \frac{y^3}{3}\right]_0^4$$

$$= 32 - \frac{64}{3}$$



$$\text{Required area} = \frac{32}{3}\text{ unit}^2.$$ **Ans.**

Section-C

Answer 12.

(i) (a) ₹ 2,17,500.

Explanation :

Advertising budget of two branches differ by ₹ 30,000 then difference in their sales

$$= ₹\ 7.25 \times 30{,}000$$
$$= ₹\ 2{,}17{,}500.$$

(ii) (d) $\frac{2}{5}$

Explanation :

We know, Correlation coefficient (r) $= \pm\sqrt{b_{yx} \times b_{xy}}$

$$= \pm\sqrt{\frac{4}{5} \times \frac{1}{5}}$$

$$= \pm\frac{2}{5}$$

$\because$ $b_{yx} > 0, b_{xy} > 0$

$\therefore$ Correlation coefficient > 0

Hence, $r = \frac{2}{5}$

Answer 13.

Demand in kg (x)	Deviation from A = 30 (dx)	Price in ₹ (y)	Deviation from B = 20 (dy)	Squared deviations $(dy)^2$	Product of deviation ($dx.dy$)
20	– 10	10	– 10	100	100
22	– 08	12	– 08	64	64
24	– 06	16	– 04	16	24
26	– 04	18	– 02	04	08
28	– 02	20	00	00	00
30	0	20	00	00	00
32	02	22	02	04	04
34	04	24	04	16	16
36	06	24	04	16	24
38	08	24	04	16	32
	$\Sigma dx = -10$		$\Sigma dy = -10$	$\Sigma(dy)^2 = 236$	$\Sigma dxdy = 272$

Here, N = 10

$\therefore$ Regression coefficient X on Y,

$$b_{xy} = \frac{N\Sigma dx.dy - (\Sigma dx)(\Sigma dy)}{N.\Sigma(dy)^2 - (\Sigma dy)^2}$$

$$\Rightarrow \quad b_{xy} = \frac{10 \times 272 - (-10)(-10)}{10 \times 236 - (-10)^2}$$

$$= \frac{2720 - 100}{2360 - 100}$$

$$\Rightarrow \quad b_{xy} = \frac{2620}{2260} = 1.15$$

Ans.

Answer 14.

Let manufacturer makes x number of toys of type A and y number of toys of type B.

$\therefore$ Maximise $Z = 7.50x + 5y$

Subject to constraints :

$$12x + 6y \leq 360 \text{ or } 2x + y \leq 60 \quad ...(i)$$

$$18x + 0y \leq 360 \text{ or } x \leq 20 \quad ...(ii)$$

$$6x + 9y \leq 360 \text{ or } 2x + 3y \leq 120 \quad ...(iii)$$

$$x \geq 0, y \geq 0$$

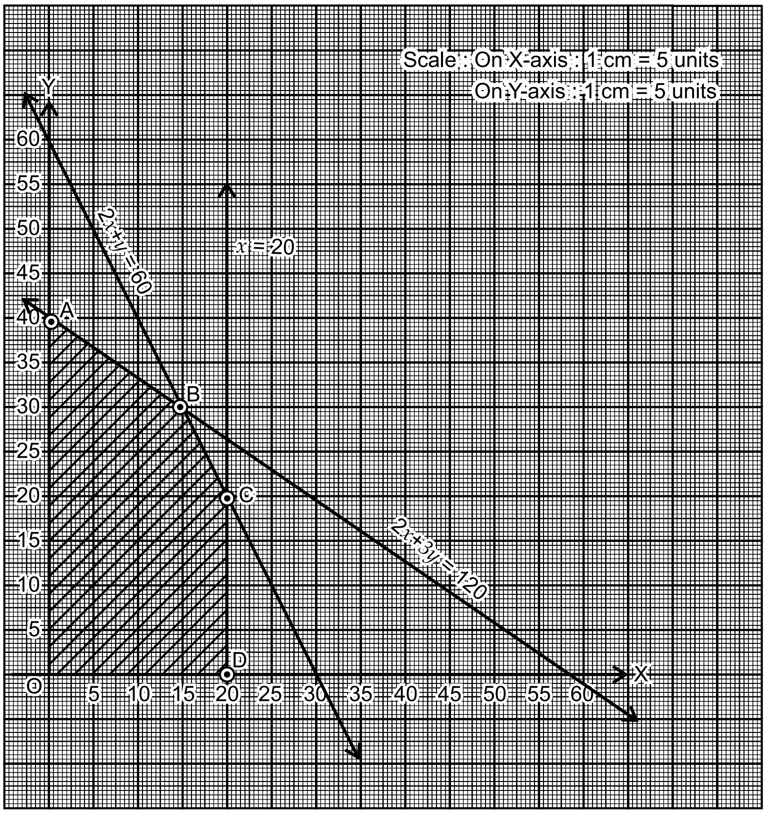
Consider,

$2x + y = 60$

x	0	30	20
y	60	0	20

$2x + 3y = 120$

x	0	60	30
y	40	0	20



OABCD is the required feasible region.

Coordinates	Maximise $Z = 7.5x + 5y$
O(0,0)	$Z = 0 + 0 = 0$
A(0, 40)	$Z = 7.5 \times 0 + 5 \times 40 =$ ₹ 200
B(15, 30)	$Z = 7.5 \times 15 + 5 \times 30 =$ ₹ 262.5
C(20, 20)	$Z = 7.5 \times 20 + 5 \times 20 =$ ₹ 250
D(20, 0)	$Z = 7.5 \times 20 + 5 \times 0 =$ ₹ 150

Hence, 15 toys of type A and 30 toys of type B should be manufactured in a day to get maximum profit of ₹ 262.5.

Hence Proved.

❑❑