9 LINES AND ANGLES

CHAPTER

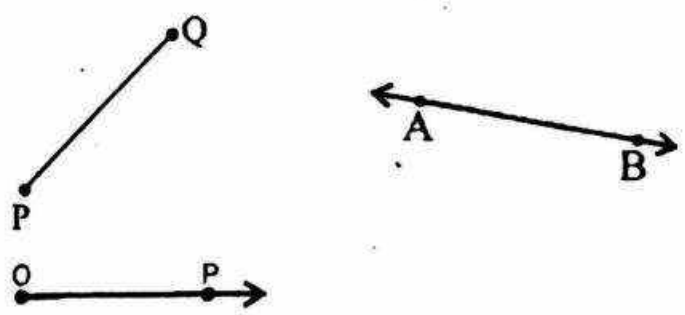
INTRODUCTION:

- A line is made up of an infinite number of points and it has only length.

  i.e. A line has no end points on either side.

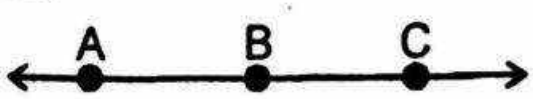
  A **line segment** has two end points.

  A **ray** has one end point.



line segment line ray

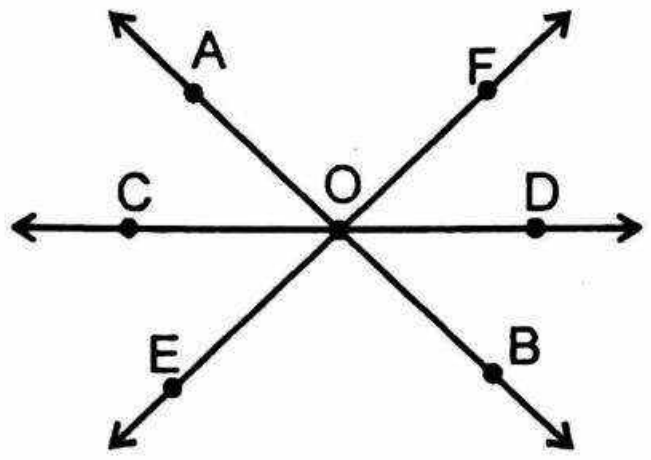
- A line segment PQ is generally denoted by $\overline{PQ}$
- A line AB is denoted by $\overleftrightarrow{AB}$
- And the ray OP is denoted by $\overrightarrow{OP}$
- **Collinear points-** Three or more than three points are said to be collinear if there is a line which contains them all.



Here A,B and C are collinear points

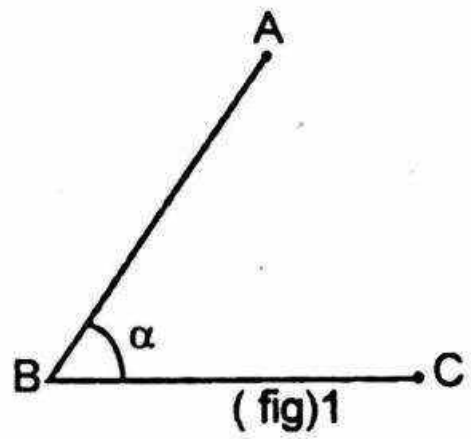
- **Concurrent lines-** Three or more than three lines are said to be concurrent if there is a point which lies on all of them.

AB, CD and EF are concurrent lines.

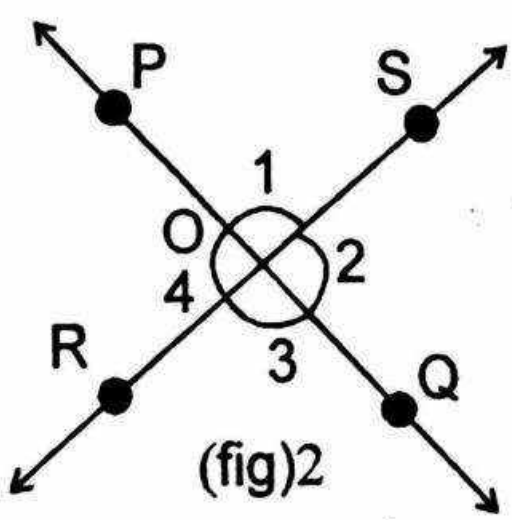


Recall that an "angle" is formed when lines or line segments meet.

E.g.



(fig)1

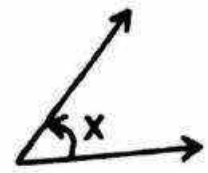


(fig)2

in figure (1) $\angle\alpha$ fomed by AB and BC
in figure (2) $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$ are formed by PQ and RS

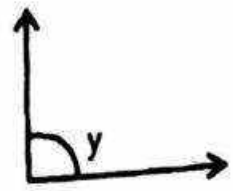
Related Angles

- **Acute angles -**



$0° < x < 90°$

- **Right Angle -**



$y = 90°$

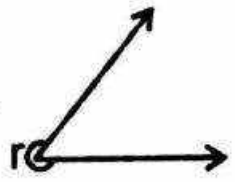
- **Obtuse Angle -**

$180° < z < 360°$

- **Straight Angle -**

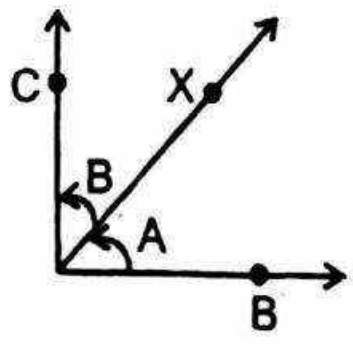
$t = 180°$

- **Reflex Angle -**



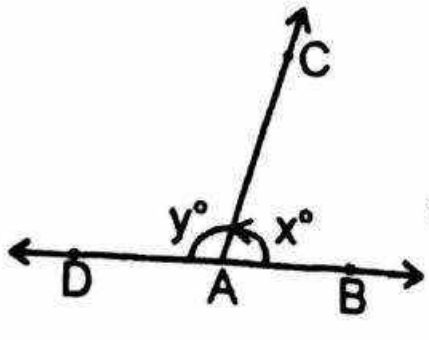
$180° < r < 360°$

- **Complementary Angles** - When the sum of the measures of the two angles is 90°, the angles are called "Complementary Angles".



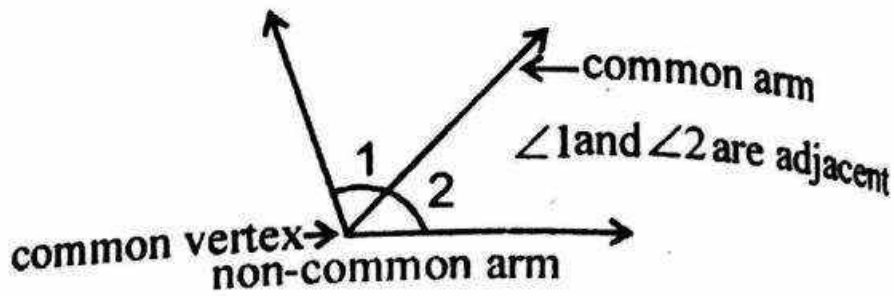
i.e. $\angle A + \angle B = 90°$ or $\angle A = \angle 90° - \angle B$ - complement of $\angle A$

- **Supplementary Angles** - Sum of two angles which are supplementary is 180°



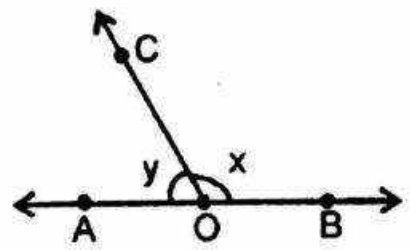
sum of two angles which are supplementary is 180°. *i.e.* $\angle x + \angle y = 180°$

- **Adjacent Angles**- Two angles are said to be adjacent if



1. They have a common vertex;
2. They have a common arm; and
3. The non-common arms are on either side of the common arm.

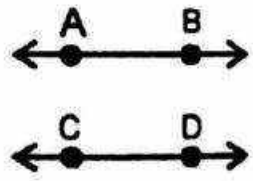
- **Linear pair**- A linear pair is a pair of adjacent angles whose non-common sides are opposite rays.



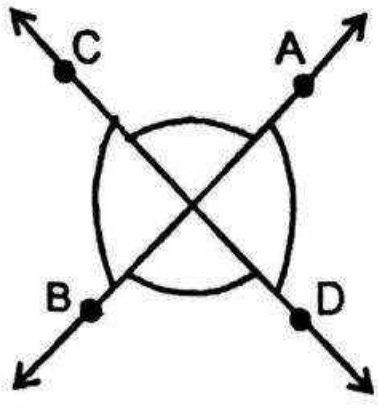
$\angle x$, $\angle y$ and A linear pair and $\angle x + \angle y = 180°$

i.e. linear pair Angles are supplementary.

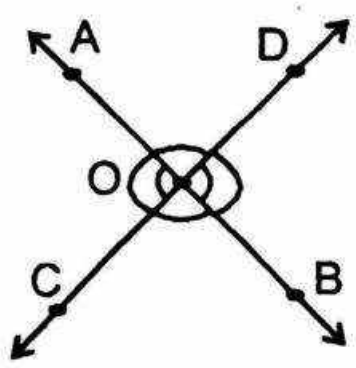
- **Parallel Lines**- If two lines have no point in common they are said to be Parellel lines.



- **Intersecting lines**- If two lines have a point in common, they are said to be intersecting lines. Two lines can intersect at the most at one point.

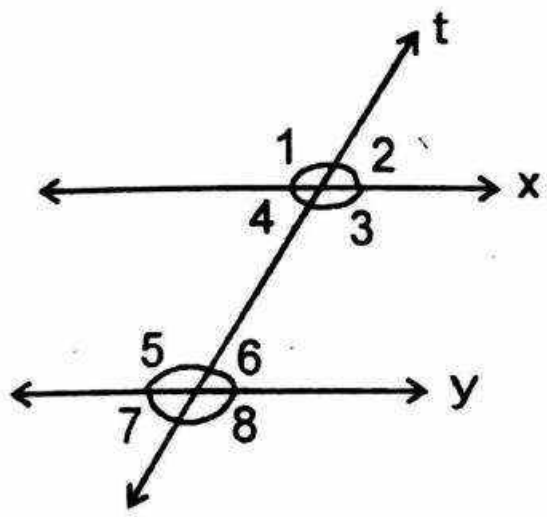


Vertically opposite Angles-



$\angle AOD$ $\angle BOC$ } are vertically opposite angles
$\angle AOC$ $\angle BOD$ }

Angles made by a transversal line -



Interior Angles- $\angle 3, \angle 4, \angle 5, \angle 6$
Exterior Angles- $\angle 1, \angle 2, \angle 7, \angle 8$

Pairs of corresponding angles- $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 4$ and $\angle 7$, $\angle 3$ and $\angle 8$
Pairs of alternate interior angles- $\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 6$
Pairs of alternate exterior angles- $\angle 1$ and $\angle 8$, $\angle 2$ and $\angle 7$
Pairs of interior angles on the same side of the transversal- $\angle 3$ and $\angle 6$, $\angle 4$ and $\angle 5$

→ If two parallel lines are interested by a transversal then-

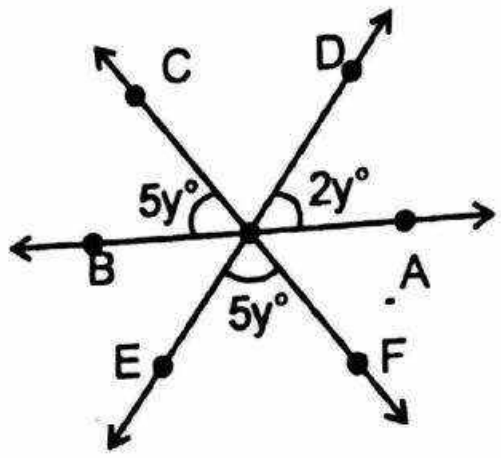
1. Each pair of corresponding angles are eqal.
2. Each pair of alternate interior are equal.
3. Interior angles on the same side of the transversal are supplementary.

→ **Checking for parallel lines-** If two lines are parallel, then you know that a transversal gives rise to
pairs of equal corresponding angles
pairs of equal alternate interior angles and
interior angles on the same side of the transversal being SUPPLEMENTARY.

Exercise
LEVEL - 1

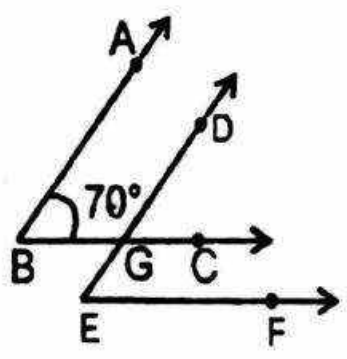
1. Find the measures of an angle which is complement of itself.
(a) 40° (b) 30°
(c) 45° (d) 50°

2. Determine the value of y.



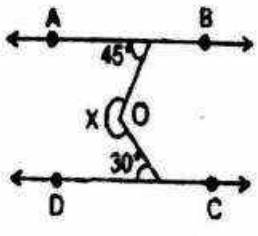
(a) 17° (b) 15°
(c) 20° (d) 25°

3. In the given figure, the arms of two angles are parallel. If $\angle ABC=70°$, then find - $\angle DGC$ and $\angle DEF$



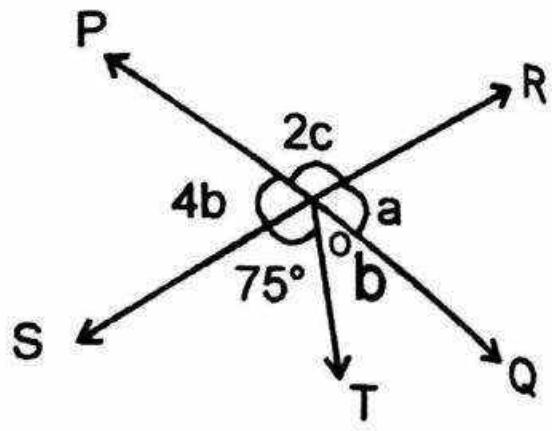
(a) 70°, 60°
(b) 60°, 50°
(c) 70°,70°
(d) None of these

4. In the given figure, AB||CD, then X is equal to :



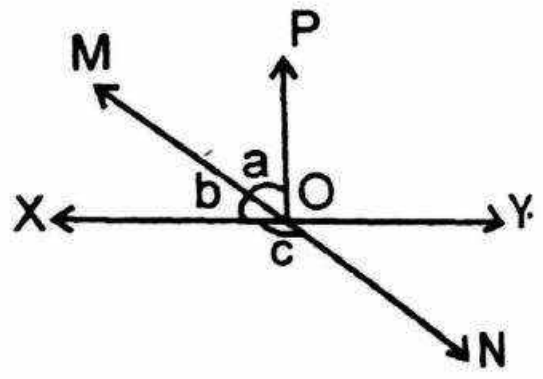
(a) 290° (b) 300°
(c) 280° (d) 285°

5. In the given two straight lines PQ and RS intersect each other at 'O'. If $\angle SOT = 75°$, find the vallue of a, b and c:



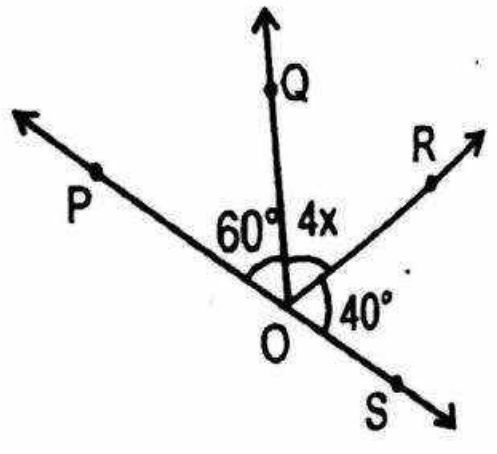
(a) a = 84°, b = 21°, c = 48°
(b) a = 48°, b = 20°, c = 50°
(c) a = 72°, b = 24°, c = 54°
(d) a = 64°, b = 28°, c = 45°

6. In the given figure XY and MN intersect at O. If POY = 90° and a:b = 2 : 3, then find c :



(a) 113° (b) 54°
(c) 126° (d) 48°

7. In the given figure POS is a line, find x :

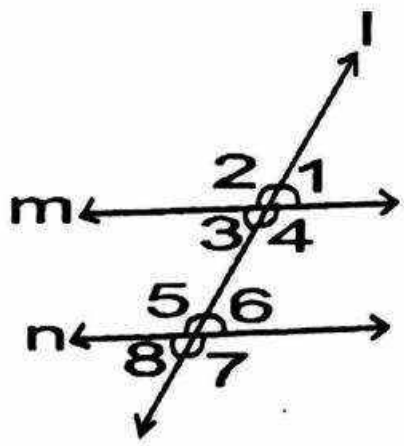


(a) 20° (b) 80°
(c) 10° (d) 100°

8. Which of the following statements are true :
   (a) Angles forming a linear pair are supplementary.
   (b) If two adjacent angles are equal, then measures of each angle will only be 90°.
   (c) Angles forming a linear pair can both be acute angles.
   (d) If angles forming a linear pair are equal, then each of these angles is of measure 90°.
   (1) Only a (2) a, b and d
   (3) c and d (4) a and d

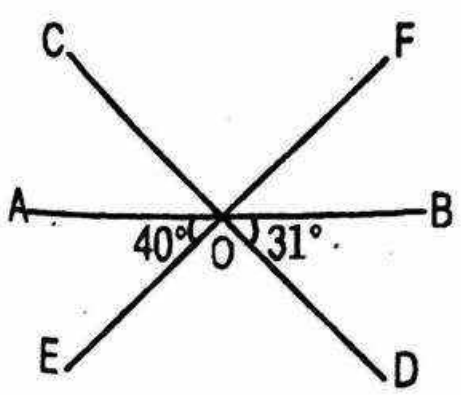
9. If one angle of a linear pair is acute, then its other angle will be :
   (a) acute (b) obtuse
   (c) right angle
   (d) None of these

10. In the given figure m||n and ∠1=65°, find ∠5 and ∠8 :-



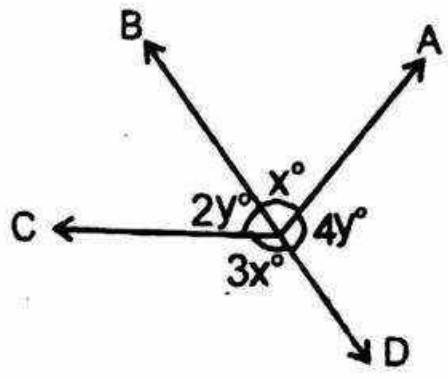
   (a) 125°, 55° (b) 115°, 65°
   (c) 105° , 75°
   (d) None of these

11. In the following figure find the value of ∠BOC :



   (a) 101° (b) 149°
   (c) 71° (d) 140°

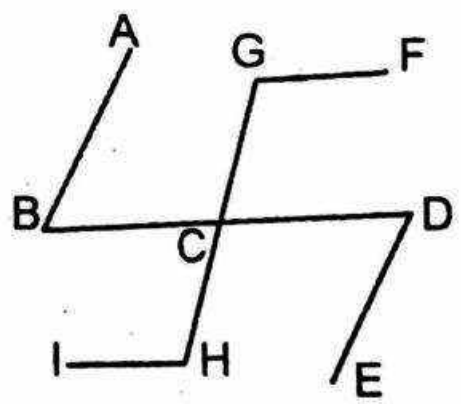
12. Find y, if $x° = 36°$, as per the given diagram :



   (a) 36° (b) 16°
   (c) 12° (d) 42°

13. AB is a straight line and O is a point on AB, if line OC is drawn not conciding with OA or OB, then ∠AOC and ∠BOC are :
   (a) equal
   (b) complementary
   (c) supplementary
   (d) together equal to 100

14. In the given diagram AB||GH||DE and GF||BD||HI, ∠FGC = 80°. Find the value of ∠CHI :

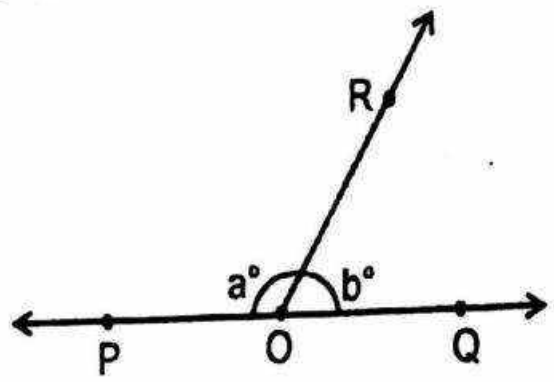


   (a) 80° (b) 120°
   (c) 100° (d) 160°

LEVEL - 2

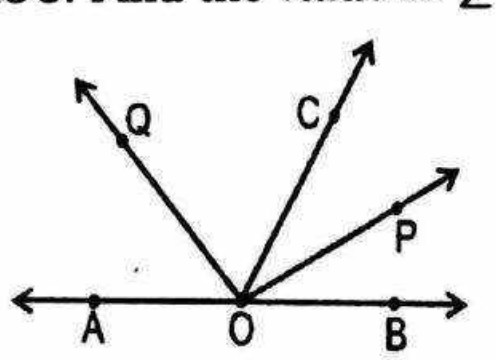
1. An angle is equal to one-third of its supplement. Find its measure :
(a) 45° (b) 50°
(c) 55°
(d) None of these

2. In figure ∠POR and ∠QOR form a linear pair. If a-b = 80, find the value of 'a' and 'b' :



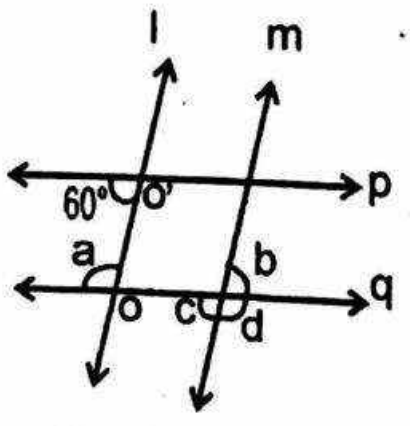
(a) a = 125°, b = 55°
(b) a = 110°, b = 70°
(c) a = 130°, b = 50°
(d) a = 75°, b = 105°

3. In figure OP bisects ∠BOC and OQ, ∠AOC. Find the value of ∠POQ :



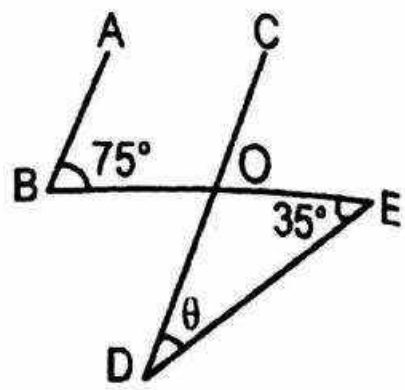
(a) 60° (b) 75°
(c) 90°
(d) None of these

4. Lines l||m, p||q. Find a, b, c, d :



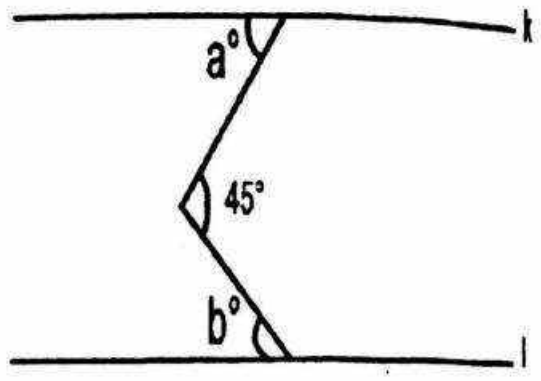
(a) a =120°, b = 60°, c = 60°, d = 120°
(b) a =120°, b = 120°, c = 60°, d = 120°
(c) a = 120°, b = 60°, c = 120°,d = 60°
(d) None of these

5. In figure, AB||CD. Find θ :



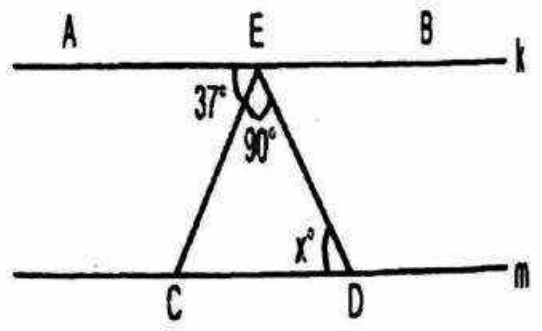
(a) 30° (b) 35°
(c) 40° (d) 45°

7. In the figure below, lines K and L are parallel . The value of a° + b° is :



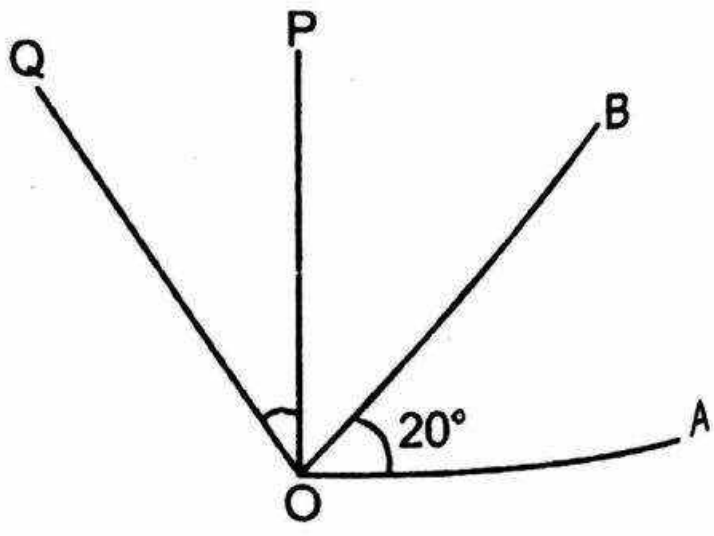
(a) 45° (b) 180°
(c) 180° (d) 360°

8. In the figure below, if AB||CD and CE⊥ED, then the value of x is :



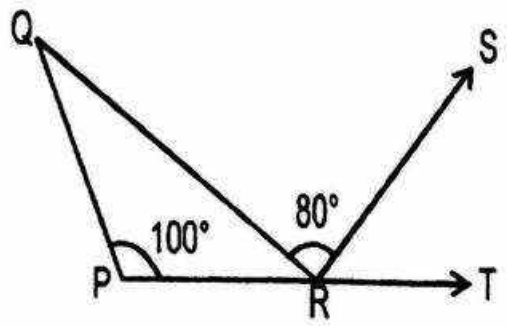
(a) 53° (b) 63°
(c) 37° (d) 45°

9. In the figure, OP⊥OA and OQ⊥OB. Find ∠POQ if ∠AOB = 20°



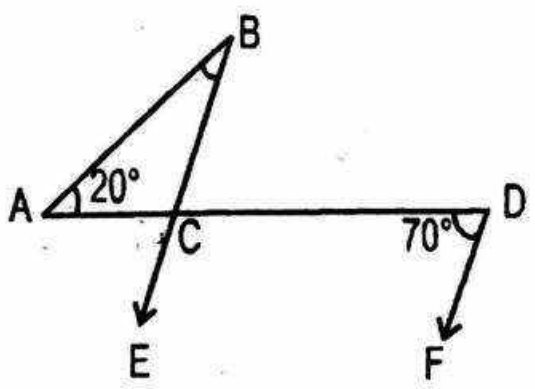
(a) 20° (b) 30°
(c) 40°
(d) None of these

10. In the figure $\angle PRQ = \angle SRT$. If $\angle QPR = 100°$ and $\angle QRS = 80°$, Find $\angle PQR$



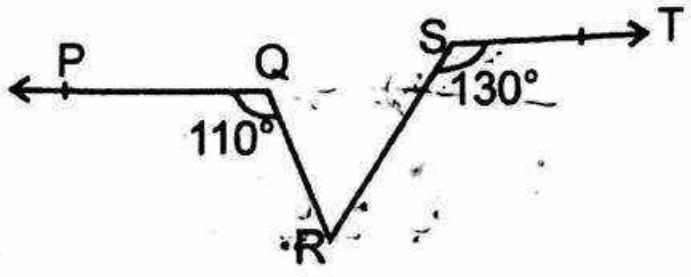
(a) 20° (b) 30°
(c) 40° (d) 60°

11. From the given figure, find $\angle ABC$, if BE||DE



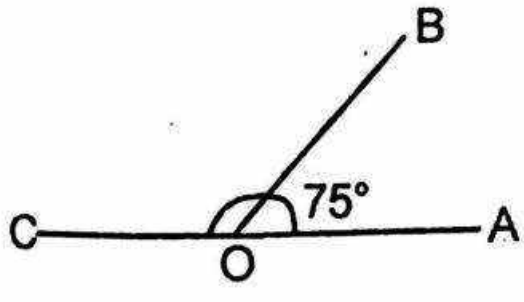
(a) 50° (b) 40°
(c) 35°
(d) None of these

12. In the figure, if PQ||ST, $\angle PQR = 110°$ and $\angle RST = 130°$, find $\angle QRS$.



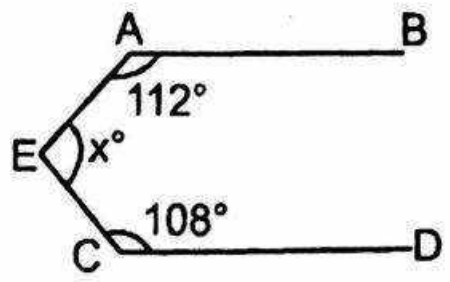
(a) 40° (b) 50°
(c) 60° (d) 70°

13. Given that $\angle AOB = 75°$ and $\angle BOC = 105°$ then which statement is true:



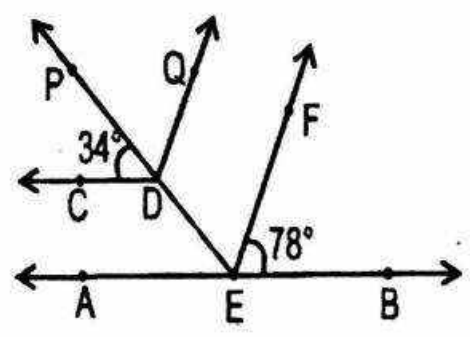
(a) $AB \perp OC$
(b) $OC \perp OA$
(c) C, O and A are in line
(d) None of these

14. In the figure, AB||CD, the value of x is :



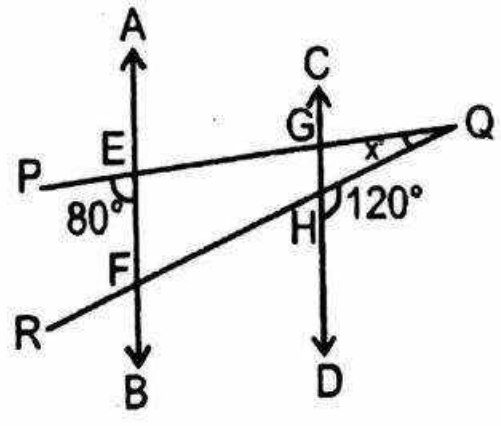
(a) 220° (b) 140°
(c) 150°
(d) none of these

15. In the figure, AB||CD and EF||DQ, the value of $\angle PDQ$ is :



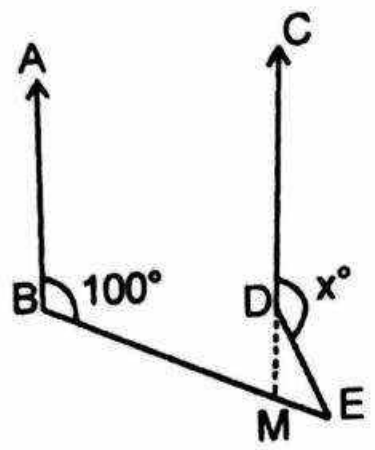
(a) 68° (b) 78°
(c) 56°
(d) None of these

16. In the given figure AB||CD, given that $\angle PEB = 80°$, $\angle QHD = 120°$ and $\angle PQR = X°$, find the value of x :



(a) 40° (b) 20°
(c) 100° (d) 30°

17. In the given figure AB||CD, $\angle ABE = 100°, \angle MED = 25°$. Find $\angle CDE$:



(a) 125° (b) 55°
(c) 65° (d) 75°

LEVEL III

1. If a straight line L makes an angle θ ($\theta > 90°$) with the positive direction of x-axis, then the acute angle made by a straight line L_1, perpendicular to L, with the y-axis is :

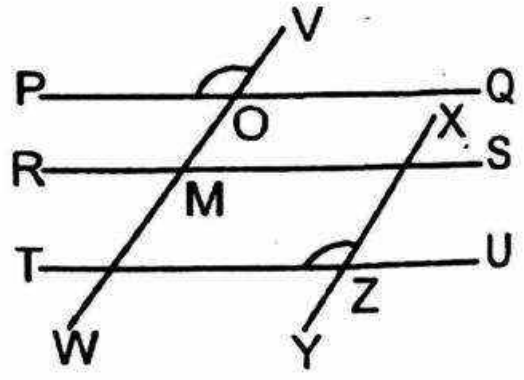
(a) $\frac{\pi}{2}+\theta$ (b) $\frac{\pi}{2}-\theta$

(c) $\pi+\theta$ (d) $\pi-\theta$

2. A,O,B, are three points on a line segment and C is a point not lying on AOB. If $\angle AOC = 40°$ and OX, OY are the internal and external bisectors of $\angle AOC$and $\angle BOC$ respectively, then $\angle BOY$ is :

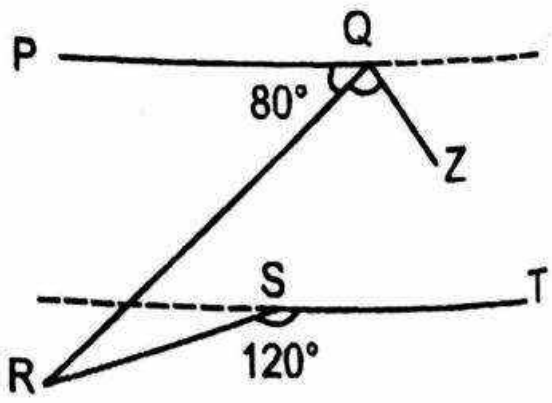
(a) 70° (b) 80°
(c) 72° (d) 68°

3. In the given figure, $\angle XZT = 130°$, PQ, RS, and TU are parallel. VW||XY. Find $\angle VOP$:



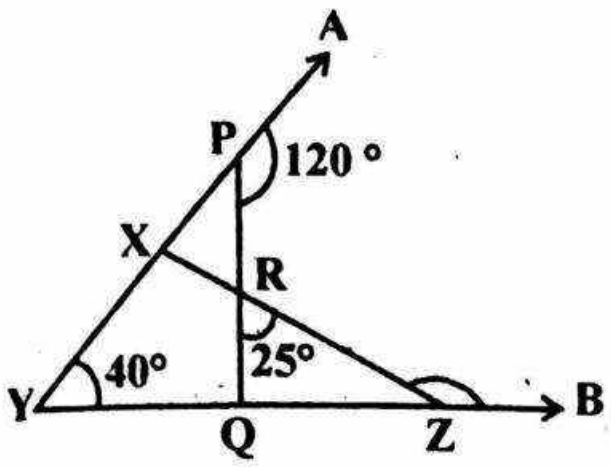
(a) 125° (b) 130°
(c) 120° (d) 135°

4. From the following figure, find if $\angle RQZ = 2\angle QRS$ and PQ||ST :



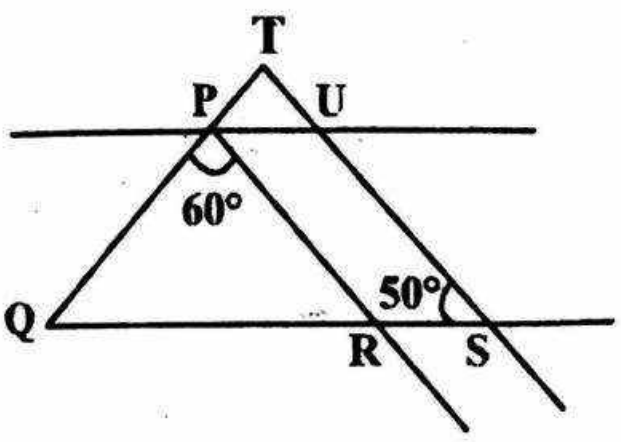
(a) 20° (b) 30°
(c) 40° (d) 60°

5. From the following figure, find $\angle BZX$:



(a) 80° (b) 90°
(c) 110° (d) 125°

6. In the given figure, PR||TS and PU||RS. Find $\angle TPU$:

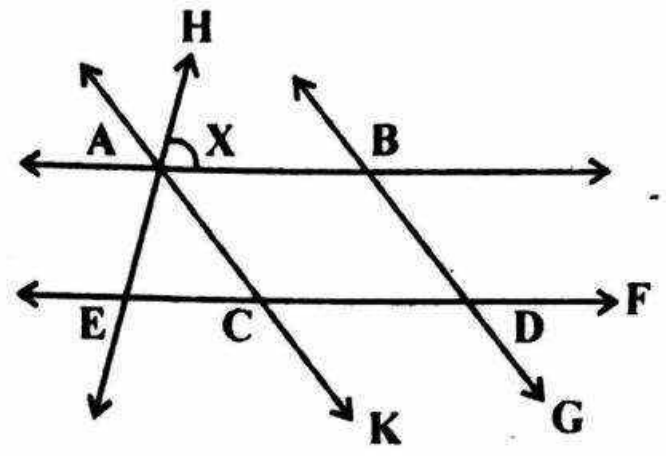


(a) 60° (b) 70°
(c) 80° (d) 100°

7. The angle between lines L and M measures 35°. If line M is rotated 45° counter clockwise about point P to line M^1 what is the angle in degrees between lines L and M^1 :

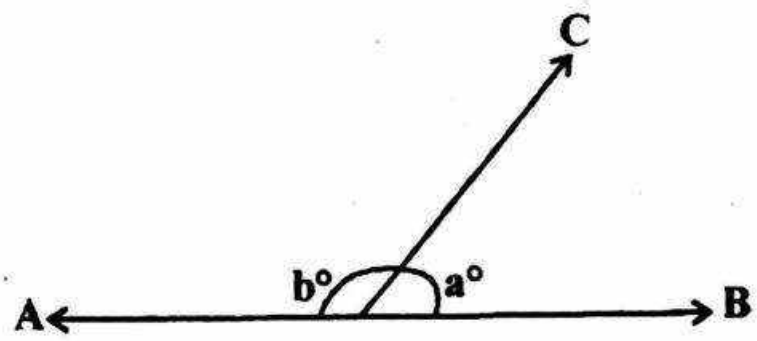
(a) 90° (b) 80°
(c) 75° (d) 60°

8. In the given figure, AB||CD and AC||BD and AC||BD. If $\angle EAC = 40°$, $\angle FDG = 55°$, $\angle HAB = x$, then the value of x is :



(a) 95° (b) 70°
(c) 35° (d) 85°

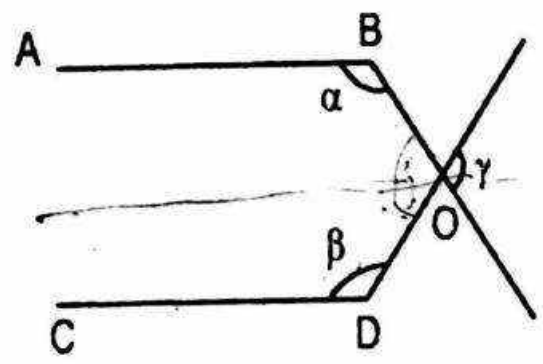
9. In the given figure, $\angle a$ is greater than one-sixth of right angle, then :



(a) $b > 165°$ (b) $b < 165°$
(c) $b \leq 165°$ (d) $b \geq 165°$

10. If AB || CD, then find the value of $\alpha+\beta+\gamma$:

(a) 180° (b) 270°
(c) 360° (d) 90°



Hints and Solutions:

LEVEL-1

1.(c) Let the measur of angle = x°
measure of its complement = x°

$\therefore$ $x^\circ + x^\circ = 90^\circ \Rightarrow x^\circ = 45^\circ$

2.(b) $\angle COD = \angle EOF = 5Y^\circ$ (vertically opposite angle)

$\therefore$ $\angle AOD + \angle DOC + \angle COB = 180^\circ$

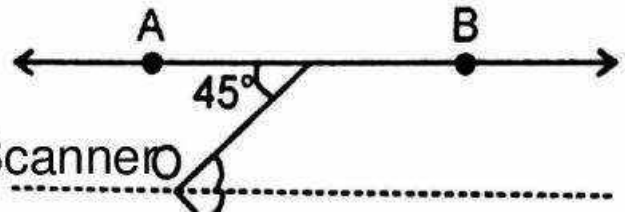
$\Rightarrow$ $2y^\circ + 5y^\circ + 5y^\circ = 180^\circ$

$\Rightarrow$ $12y^\circ = 180^\circ$

$y = 15^\circ$

3.(c) $\angle DGC = \angle DEF = \angle ABC = 70^\circ$
(corresponding angles)

4.(d)



$= 4 \times 21$

$= 84^\circ$

$a + 2c = 180^\circ$

$\Rightarrow$ $2c = 180^\circ - 84^\circ$

$= 96^\circ$

$\Rightarrow$ $c = 48^\circ$

$\therefore$ $a = 84^\circ$, $b = 21^\circ$, $c = 48^\circ$

6.(c) $\angle POQ = 90^\circ$
by line property - $\angle a + \angle b = 90^\circ$
$a : b = 2 : 3$
So, $\angle a = 36^\circ$ and $\angle b = 54^\circ$
Here MN is a line so

$\Rightarrow$ $54^\circ + \angle C = 180^\circ$

$\Rightarrow$ $\angle C = 126^\circ$

7.(a) by line property,
$60 + 4x + 40 = 180^\circ$

$\Rightarrow$ $4x = 80^\circ$

$\Rightarrow$ $x = 20^\circ$

9.(b) Let $X < 90^\circ$ and other angle = y

LEVEL-2

1.(a) Let the measure of angle = $x°$
$\therefore$ its supplement = $(180 - x)°$

$$x = \frac{1}{3}(180 - x) \Rightarrow x = 45°$$

2.(c) $\angle POR = a°$ and $\angle QOR = b°$ form a linear pair
therefore, $a + b = 180°$ _____(1)
$a - b = 80°$ _____(2) given
(1) + (2) $2a = 260° \Rightarrow a = 130°$
$\therefore$ form (1) $b = 180° - a$
$= 180° - 130°$
$= 50°$
$\therefore$ $a = 130°$ and $b = 50°$

3.(c) $\angle BOC = 2\angle POC$
$\because$ OP bisects $\angle BOC$
$\angle AOC = 2\angle QOC$
$\because$ OQ bisects $\angle AOC$
Since, ray OC stands on line AB.
Therefore,
$\angle AOC + \angle BOC = 180°$
$\Rightarrow 2\angle QOC + 2\angle POC = 180°$
$\Rightarrow \angle QOC + \angle POC = 90°$
$\Rightarrow \angle POQ = 90°$

4.(a) $60° + a = 180° \Rightarrow a = 120°$
$\therefore \angle loq = 180° - a = 60°$
$\therefore \angle c = \angle loq = 60°$ (Alternate angles)
and $\angle b = \angle c = 60°$
(Vertically opposite angles)
and $\angle d = 180° - \angle c = 120°$
$\therefore$ $a = 120°$, $b = 60°$, $c = 60°$, $d = 120°$

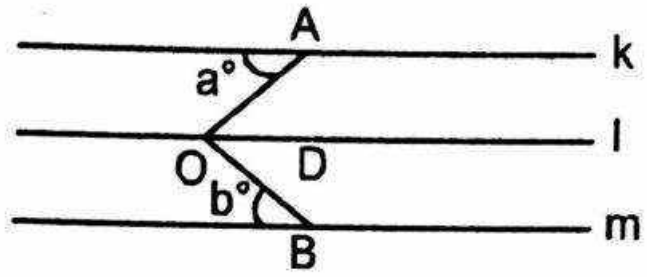
5.(c) $\angle COE = \angle ABE = 75°$
(corresponding angles)
$\therefore \angle DOE = 180° - \angle COE = 180° - 75°$
$= 105°$

in ΔDOE $-105° + 35° + \theta = 180°$

$\Rightarrow \theta = 180° - 140° = 40°$

6.(b) Let $y = 3p$ and $z = 7p$
y and z are atlernative angles
$\therefore \angle y + \angle z = 180° \Rightarrow 3p + 7p$
$= 180° \Rightarrow p = 18°$
$\therefore y = 3p = 54°$
$\because \angle y$ and $\angle x$ are consecutive interior angles
$\therefore \angle y + \angle x = 180°$
$\Rightarrow \angle x = 180° - 54°$
$= 126°$

7.(a)



$\angle BOA = 45°$
$\because$ k || l || m
$\therefore \angle DOB = b°$ and $\angle AOD = a°$
$\therefore a° + b° = \angle AOB = 45°$

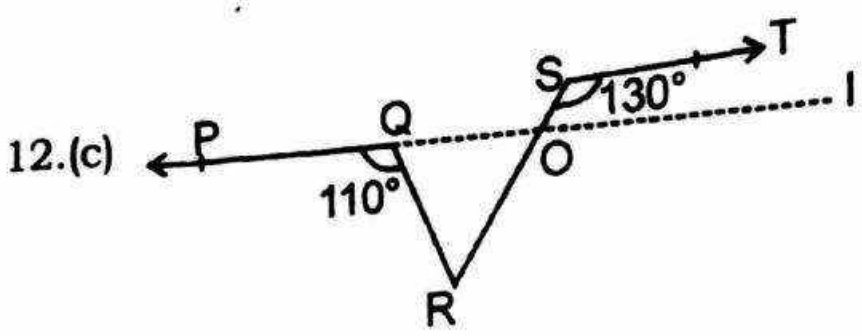
8.(a) $\angle AEC = \angle CED + \angle DEB = 180°$
$\Rightarrow 37° + 90° + \angle DEB = 180°$
$\Rightarrow \angle DEB = 180° - 127°$
$= 53°$
$\because$ EB || CD
$\therefore \angle DEB = \angle EDC = 53°$

9.(a) $\angle BOP = 90° - \angle AOB$
$= 90° - 70°$
$= 20°$
$\therefore \angle POQ = 90° - \angle BOP$
$= 90° - 70°$
$= 20°$

10.(b) $\angle PRQ + \angle QRS + \angle SRT = 180°$
$\therefore \angle PRQ + 80° + \angle PRQ = 180°$
$(\because \angle PRQ = \angle SRT)$
$\Rightarrow \angle PRQ = 50°$
$\Rightarrow \angle PQR = 180° - \angle QPR - \angle PRR$
$= 180° - 100° - 50°$
$= 30°$

11.(a) $\because$ BE || DF
$\therefore \angle ACE = \angle CDF = 70°$
$\therefore \angle ACB = 180° - \angle ACE$
$= 110°$
$\therefore \angle ABC = 180° - 20° - 110°$
$= 50°$

12.(c)



draw a line QI, then

$\angle RQO = 180° - 110° = 70°$

$\angle ROI = 130°$ (corresponding)

So, $\angle ROQ = 180° - 130° = 50°$

Now in ΔQRO

$\Rightarrow \angle RQO + \angle ROQ + \angle QRO = 180°$

$\Rightarrow \angle QRO = 180° - (70° + 50°)$

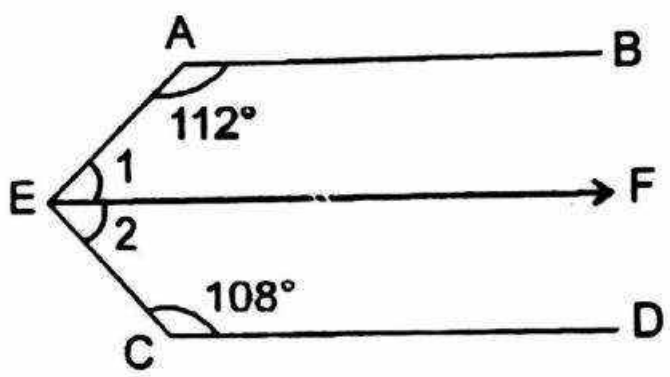
$= 60° = \angle QRS$

13.(c) $\angle AOB + \angle BOC$

$= 75 + 105$

$= 180°$

14.(b)



Draw EF parallel to both AB and CD

$\because$ AB || EF and AE transversal cuts them at A and E respectively.

$\therefore \angle BAE + \angle FEA = 180°$

$\Rightarrow 112° + \angle 1 = 180°$

$\angle 1 = 68°$

again EF || CD and transversal cuts them at C and E.

$\therefore \angle FEC + \angle ECD = 180°$

$\Rightarrow \angle 2 + 108° = 180°$

$\Rightarrow \angle 2 = 72°$

Now, $x = \angle 1 + \angle 2$

$\Rightarrow x = 72° + 68° = 140°$

15.(a) CD || AB $\therefore \angle AED = \angle PDC = 34°$

$\therefore \angle DEF = 180° - 78° - 34°$

$= 68°$

$\because$ QD || EF

$\therefore \angle PDQ = \angle DEF = 68°$ (corresponding)

16.(b) $\angle PGH = 80° \Rightarrow \angle QGH = 100°$

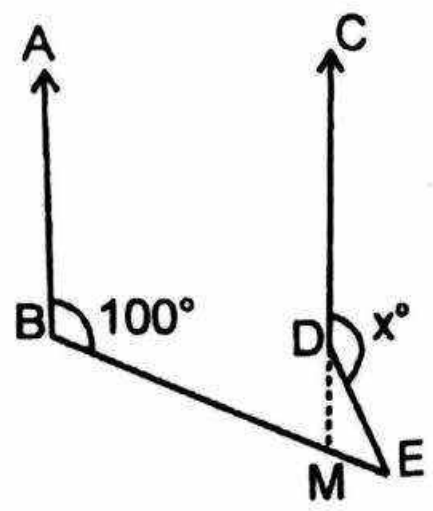
$\angle QHD = 120° \Rightarrow \angle CHQ = 60°$

$\therefore \angle x + 100° + 60° = 180° \Rightarrow x = 20°$

17.(a) Extend CD to M, then $\angle DME =$

$\angle ABE = 100°$

$\angle MED = 25°$



$\therefore \angle MDE = 180° - (100° + 25°)$

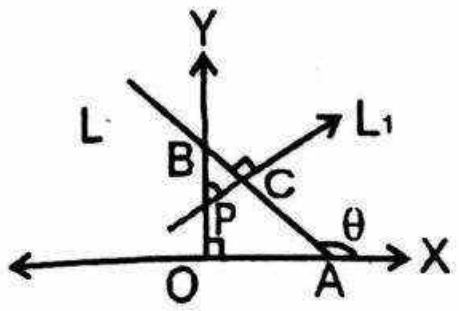
$= 55°$

$\therefore \angle CDE = 180° - 55°$

$= 125°$

LEVEL-3

1.(d)



$\angle BCP = 180° - \angle BCL_1 = 90°$

in Δ BOA,

$\angle O = 90°$, $\angle A = 180° - \theta$

$\therefore \angle OBA = 180° - (90° + 180° - \theta)$

$= \theta - 90$

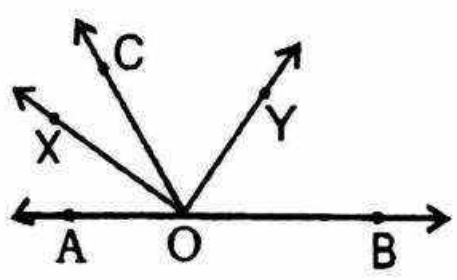
Now in Δ BPC,

$\angle BPC = 180° - (90° + \theta - 90°)$

$= 180° - \theta$

$= \pi - \theta$

2.(a)



$\because \angle AOC = 40°$

$\therefore \angle BOY = \frac{1}{2}\angle BOC = \frac{1}{2} \times 140 = 70°$

3.(b) RS || TU

$\therefore \angle XNR = \angle XZT = 130°$ (corresponding agles)

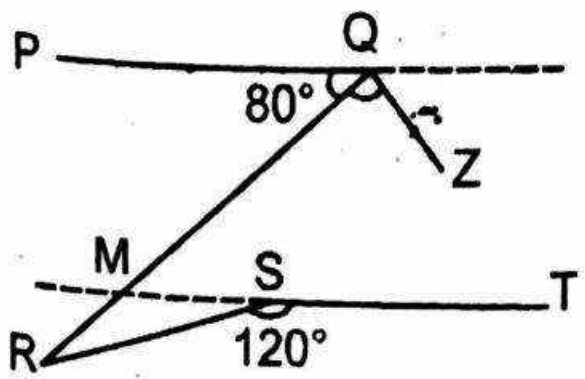
$\therefore$ VW || XY

$\therefore \angle OMR = \angle XNR = 130°$ (correspondig angles)

$\because$ PQ || RS

$\therefore \angle VOP = \angle OMR = 130°$ (corresponding angles)

4.(c)



$\angle MSR = 180° - \angle RST$

$= 180° - 120°$

$= 60°$

$\because$ PQ || ST

$\therefore \angle QMS = \angle PQR = 80°$ (alternate angles)

$\therefore \angle RMS = 180° - \angle QMS = 100°$

$\angle RMS + \angle MSR + \angle SRM = 180°$

$\Rightarrow \angle SRM = 180° - 100° - 60° = 20°$

$\because \angle RQZ = 2\angle QRS = 2\angle SRM$

$\therefore \angle RQZ = 2 \times 20° = 40°$

5.(d) $\angle PQY = 180° - \angle PYQ - \angle YPQ$

$= 180° - 40° - (180° - 120°) = 80°$

$\therefore \angle RQZ = 180° - \angle PQY$

$= 180° - 80°$

$= 100°$

$\therefore \angle RZQ = 180° - 25° - 100°$

$= 55°$

$\therefore \angle BZX = 180° - \angle RZQ$

$= 180° - 55°$

$= 125°$

6.(b) $\because$ PR || TS

$\therefore \angle PRQ = \angle USR = 50°$

In ΔPQR :

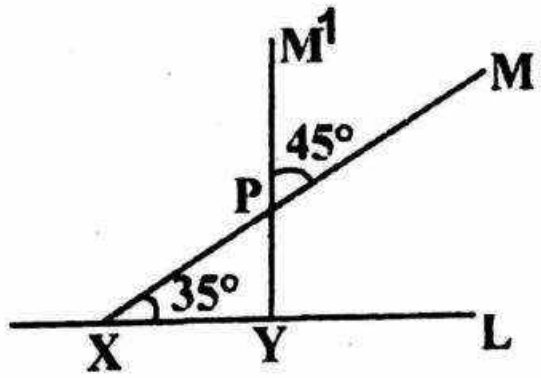
$\angle PQR = 180° - (50° + 60°)$

$= 70°$

$\therefore \angle TPU = \angle PQR = 70°$

[$\because$ PU || RS || QS]

7.(b)



$\angle PYX = 180° - 35° - \angle XPY$

$= 180° - 35° - 45°$

$= 100°$

$\therefore \angle PYL = 180° - \angle PYX$

$= 80°$

8.(d) $\angle DCK = \angle FDG$
$= 55°$(corresponding)

$\therefore \angle ACE = \angle DCK$
$= 55°$ (vertically opposite)

So, $\angle AEC = 180° - (40° + 55°)$
$= 85°$

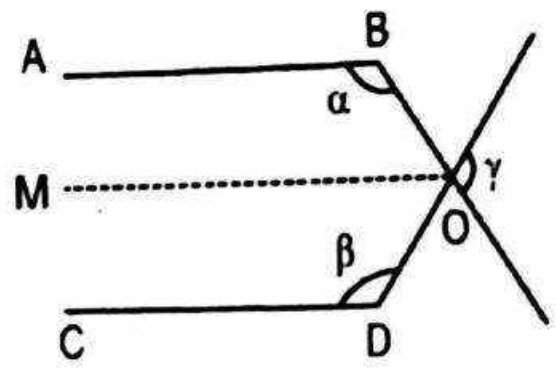
$\therefore \angle HAB = \angle AEC$
$= 85°$ (corresponding)

Hence, $x = 85°$

9.(b) $a > \frac{90°}{6} \Rightarrow a > 15°$

$a + b = 180° \Rightarrow b < 165°$ ($\because a > 15°$)

10.(c) Draw OM || AB || CD



$\because$ AB || OM

$\therefore \angle BOM = 180° - \alpha$

Also,

$\because$ OM || CD

$\therefore \angle DOM = 180° - \beta$

$\therefore \gamma = \angle BOD$

$\Rightarrow \gamma = \angle BOM + \angle DOM$

$\Rightarrow \gamma = 180° - \alpha + 180° - \beta$

$\Rightarrow \alpha + \beta + \gamma = 360°$

Answer-Key

LEVEL - 1

1. (c)	2. (b)	3. (c)
4. (d)	5. (a)	6. (c)
7. (a)	8. (d)	9. (b)
10. (b)	11. (b)	12. (a)
13. (c)	14. (a)	

LEVEL - 2

1. (a)	2. (c)	3. (c)
4. (a)	5. (c)	6. (b)
7. (a)	8. (a)	9. (a)
10. (b)	11. (a)	12. (c)
13. (c)	14. (b)	15. (a)
16. (b)	17. (a)	

LEVEL - 3

1. (d)	2. (a)	3. (b)
4. (c)	5. (d)	6. (b)
7. (b)	8. (d)	9. (b)
10. (c)		