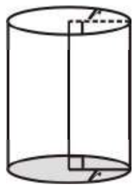


Fig 11.26
(This is a right circular cylinder)

Did you notice the following:

The cylinder has congruent circular faces that are parallel to each other (Fig 11.26). Observe that the line segment joining the center of circular faces is perpendicular to the base. Such cylinders are known as **right circular cylinders**. We are only going to study this type of cylinders, though there are other types of cylinders as well (Fig 11.27).

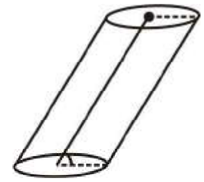
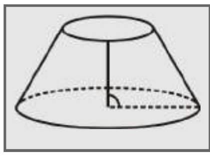


Fig 11.27
(This is not a right circular cylinder)

THINK, DISCUSS AND WRITE



Why is it incorrect to call the solid shown here a cylinder?

11.7 Surface Area of Cube, Cuboid and Cylinder

Imran, Monica and Jaspal are painting a cuboidal, cubical and a cylindrical box respectively of same height (Fig 11.28).

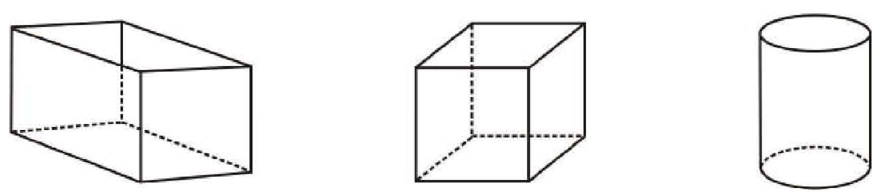


Fig 11.28

They try to determine who has painted more area. Hari suggested that finding the surface area of each box would help them find it out.

To find the total surface area, find the area of each face and then add. The surface area of a solid is the sum of the areas of its faces. To clarify further, we take each shape one by one.

11.7.1 Cuboid

Suppose you cut open a cuboidal box and lay it flat (Fig 11.29). We can see a net as shown below (Fig 11.30).

Write the dimension of each side. You know that a cuboid has three pairs of identical faces. What expression can you use to find the area of each face?

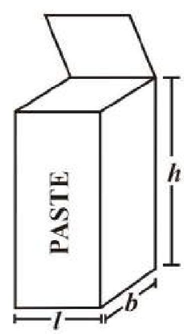


Fig 11.29

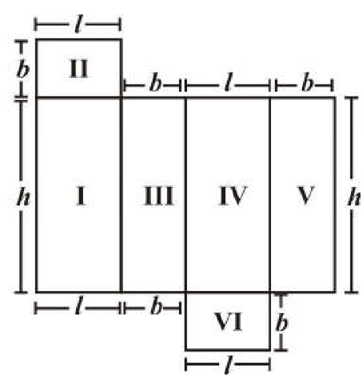


Fig 11.30

Find the total area of all the faces of the box. We see that the total surface area of a cuboid is area I + area II + area III + area IV +area V + area VI

$= h \times l + b \times l + b \times h + l \times h + b \times h + l \times b$

So total surface area = $2\,(h \times l + b \times h + b \times l) = 2(lb + bh + hl)$

where h, l and b are the height, length and width of the cuboid respectively.

Suppose the height, length and width of the box shown above are 20 cm, 15 cm and 10 cm respectively.

Then the total surface area = $2\,(20 \times 15 + 20 \times 10 + 10 \times 15)$

$= 2\,(300 + 200 + 150) = 1300\ \text{m}^2$.

TRY THESE

Find the total surface area of the following cuboids (Fig 11.31):

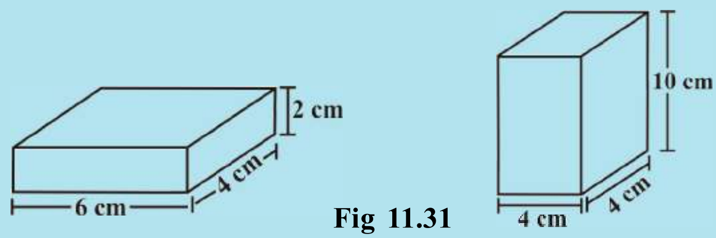


Fig 11.31

- The side walls (the faces excluding the top and bottom) make the lateral surface area of the cuboid. For example, the total area of all the four walls of the cuboidal room in which you are sitting is the lateral surface area of this room (Fig 11.32). Hence, the lateral surface area of a cuboid is given by $2(h \times l + b \times h)$ or $2h\,(l + b)$.

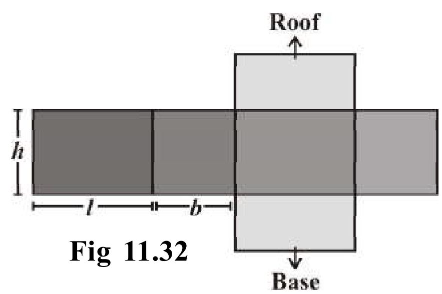


Fig 11.32

DO THIS

(i) Cover the lateral surface of a cuboidal duster (which your teacher uses in the class room) using a strip of brown sheet of paper, such that it just fits around the surface. Remove the paper. Measure the area of the paper. Is it the lateral surface area of the duster?

(ii) Measure length, width and height of your classroom and find
  (a) the total surface area of the room, ignoring the area of windows and doors.
  (b) the lateral surface area of this room.
  (c) the total area of the room which is to be white washed.

THINK, DISCUSS AND WRITE

1. Can we say that the total surface area of cuboid = lateral surface area + 2 × area of base?
2. If we interchange the lengths of the base and the height of a cuboid (Fig 11.33(i)) to get another cuboid (Fig 11.33(ii)), will its lateral surface area change?

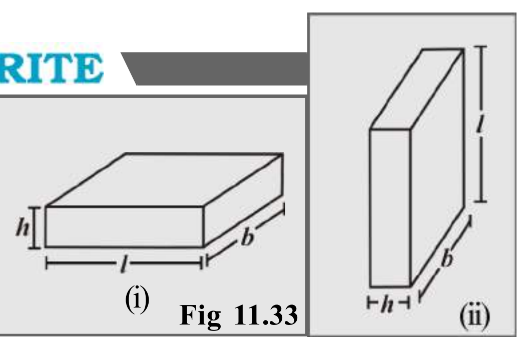


Fig 11.33

11.7.2 Cube

DO THIS

Draw the pattern shown on a squared paper and cut it out [Fig 11.34(i)]. (You know that this pattern is a net of a cube. Fold it along the lines [Fig 11.34(ii)] and tape the edges to form a cube [Fig 11.34(iii)].

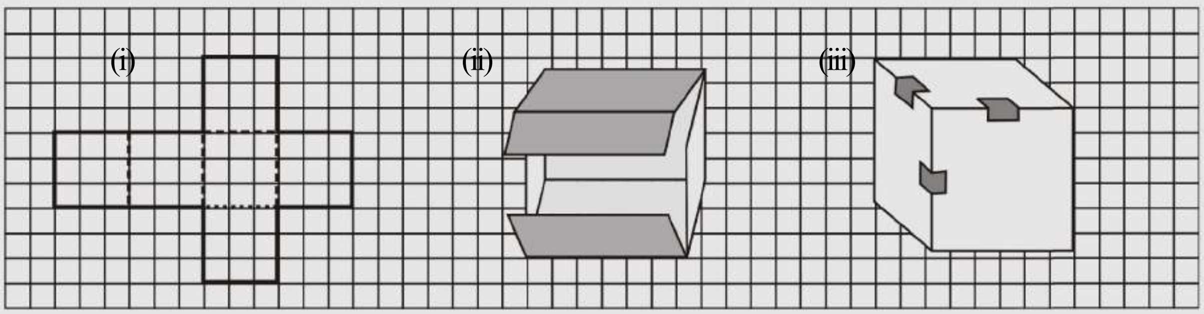


Fig 11.34

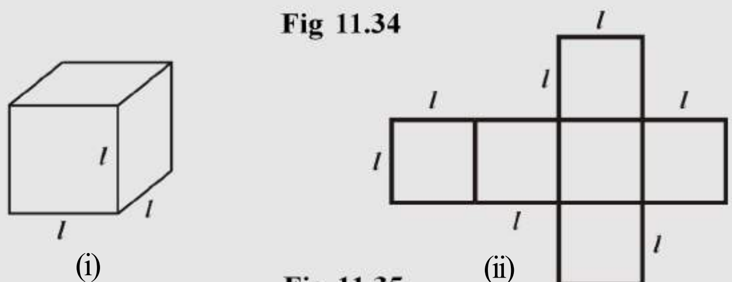


Fig 11.35

(a) What is the length, width and height of the cube? Observe that all the faces of a cube are square in shape. This makes length, height and width of a cube equal (Fig 11.35(i)).

(b) Write the area of each of the faces. Are they equal?

(c) Write the total surface area of this cube.

(d) If each side of the cube is l, what will be the area of each face? (Fig 11.35(ii)).

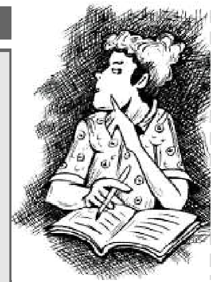
Can we say that the total surface area of a cube of side l is $6l^2$?

TRY THESE

Find the surface area of cube A and lateral surface area of cube B (Fig 11.36).

Fig 11.36

THINK, DISCUSS AND WRITE



(i) Two cubes each with side b are joined to form a cuboid (Fig 11.37). What is the surface area of this cuboid? Is it $12b^2$? Is the surface area of cuboid formed by joining three such cubes, $18b^2$? Why?

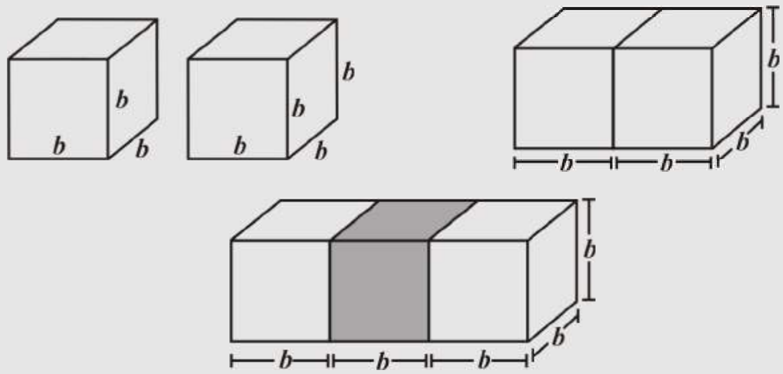


Fig 11.37

(ii) How will you arrange 12 cubes of equal length to form a cuboid of smallest surface area?

(iii) After the surface area of a cube is painted, the cube is cut into 64 smaller cubes of same dimensions (Fig 11.38). How many have no face painted? 1 face painted? 2 faces painted? 3 faces painted?

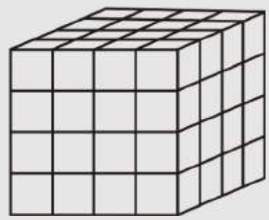


Fig 11.38

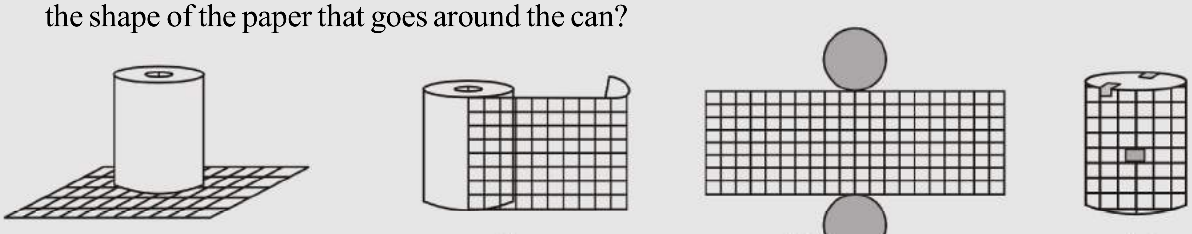
11.7.3 Cylinders

Most of the cylinders we observe are right circular cylinders. For example, a tin, round pillars, tube lights, water pipes etc.

DO THIS

(i) Take a cylindrical can or box and trace the base of the can on graph paper and cut it [Fig 11.39(i)]. Take another graph paper in such a way that its width is equal to the height of the can. Wrap the strip around the can such that it just fits around the can (remove the excess paper) [Fig 11.39(ii)].

Tape the pieces [Fig 11.39(iii)] together to form a cylinder [Fig 11.39(iv)]. What is the shape of the paper that goes around the can?



(i) (ii) (iii) (iv)

Fig 11.39

Of course it is rectangular in shape. When you tape the parts of this cylinder together, the length of the rectangular strip is equal to the circumference of the circle. Record the radius (r) of the circular base, length (l) and width (h) of the rectangular strip. Is $2\pi r$ = length of the strip. Check if the area of rectangular strip is $2\pi rh$. Count how many square units of the squared paper are used to form the cylinder. Check if this count is approximately equal to $2\pi r(r + h)$.

(ii) We can deduce the relation $2\pi r(r + h)$ as the surface area of a cylinder in another way. Imagine cutting up a cylinder as shown below (Fig 11.40).

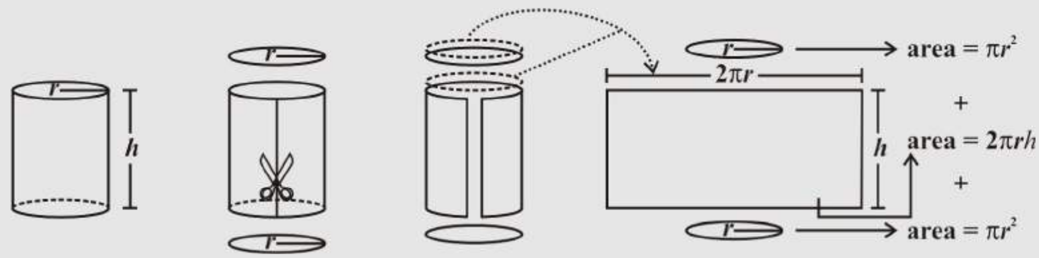


Fig 11.40

Note: We take π to be $\frac{22}{7}$ unless otherwise stated.

The lateral (or curved) surface area of a cylinder is $2\pi rh$.

The total surface area of a cylinder $= \pi r^2 + 2\pi rh + \pi r^2$

$= 2\pi r^2 + 2\pi rh$ or $2\pi r(r + h)$

TRY THESE

Find total surface area of the following cylinders (Fig 11.41)

Fig 11.41

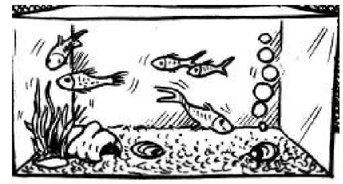
THINK, DISCUSS AND WRITE

Note that lateral surface area of a cylinder is the circumference of base × height of cylinder. Can we write lateral surface area of a cuboid as perimeter of base × height of cuboid?

Example 4: An aquarium is in the form of a cuboid whose external measures are 80 cm × 30 cm × 40 cm. The base, side faces and back face are to be covered with a coloured paper. Find the area of the paper needed?

Solution: The length of the aquarium = l = 80 cm

Width of the aquarium = b = 30 cm



Height of the aquarium = h = 40 cm

Area of the base = $l \times b = 80 \times 30 = 2400 \text{ cm}^2$

Area of the side face = $b \times h = 30 \times 40 = 1200 \text{ cm}^2$

Area of the back face = $l \times h = 80 \times 40 = 3200 \text{ cm}^2$

Required area = Area of the base + area of the back face + (2 × area of a side face)

$= 2400 + 3200 + (2 \times 1200) = 8000 \text{ cm}^2$

Hence the area of the coloured paper required is 8000 cm^2.

Example 5: The internal measures of a cuboidal room are 12 m × 8 m × 4 m. Find the total cost of whitewashing all four walls of a room, if the cost of white washing is ₹ 5 per m^2. What will be the cost of white washing if the ceiling of the room is also whitewashed.

Solution: Let the length of the room = l = 12 m

Width of the room = b = 8 m

Height of the room = h = 4 m

Area of the four walls of the room = Perimeter of the base × Height of the room

$= 2\,(l + b) \times h = 2\,(12 + 8) \times 4$

$= 2 \times 20 \times 4 = 160 \text{ m}^2$.

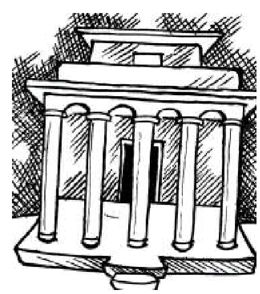
Cost of white washing per m^2 = ₹ 5

Hence the total cost of white washing four walls of the room = ₹ (160 × 5) = ₹ 800

Area of ceiling is $12 \times 8 = 96 \text{ m}^2$

Cost of white washing the ceiling = ₹ (96 × 5) = ₹ 480

So the total cost of white washing = ₹ (800 + 480) = ₹ 1280



Example 6: In a building there are 24 cylindrical pillars. The radius of each pillar is 28 cm and height is 4 m. Find the total cost of painting the curved surface area of all pillars at the rate of ₹ 8 per m^2.

Solution: Radius of cylindrical pillar, r = 28 cm = 0.28 m

height, h = 4 m

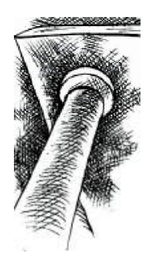
curved surface area of a cylinder = $2\pi rh$

curved surface area of a pillar = $2 \times \frac{22}{7} \times 0.28 \times 4 = 7.04 \text{ m}^2$

curved surface area of 24 such pillar = $7.04 \times 24 = 168.96 \text{ m}^2$

cost of painting an area of 1 m^2 = ₹ 8

Therefore, cost of painting 1689.6 m^2 = 168.96 × 8 = ₹ 1351.68



Example 7: Find the height of a cylinder whose radius is 7 cm and the total surface area is 968 cm^2.

Solution: Let height of the cylinder = h, radius = r = 7cm

Total surface area = $2\pi r\,(h + r)$

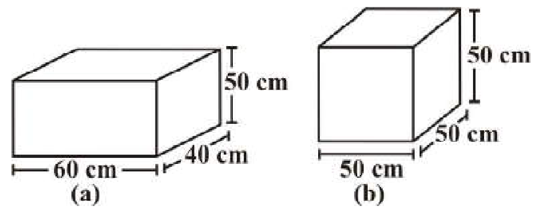
i.e., $$2 \times \frac{22}{7} \times 7 \times (7 + h) = 968$$

$$h = 15 \text{ cm}$$

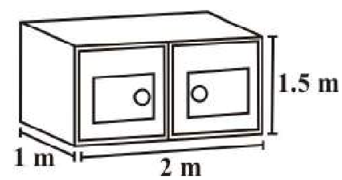
Hence, the height of the cylinder is 15 cm.

EXERCISE 11.3

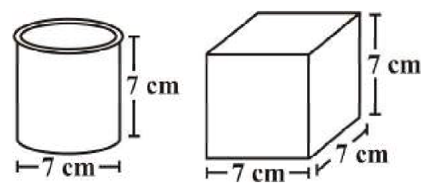
1. There are two cuboidal boxes as shown in the adjoining figure. Which box requires the lesser amount of material to make?



2. A suitcase with measures 80 cm × 48 cm × 24 cm is to be covered with a tarpaulin cloth. How many metres of tarpaulin of width 96 cm is required to cover 100 such suitcases?
3. Find the side of a cube whose surface area is 600 cm^2.

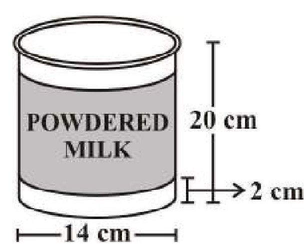


4. Rukhsar painted the outside of the cabinet of measure 1 m × 2 m × 1.5 m. How much surface area did she cover if she painted all except the bottom of the cabinet.
5. Daniel is painting the walls and ceiling of a cuboidal hall with length, breadth and height of 15 m, 10 m and 7 m respectively. From each can of paint 100 m^2 of area is painted. How many cans of paint will she need to paint the room?



6. Describe how the two figures at the right are alike and how they are different. Which box has larger lateral surface area?
7. A closed cylindrical tank of radius 7 m and height 3 m is made from a sheet of metal. How much sheet of metal is required?
8. The lateral surface area of a hollow cylinder is 4224 cm^2. It is cut along its height and formed a rectangular sheet of width 33 cm. Find the perimeter of rectangular sheet?

9. A road roller takes 750 complete revolutions to move once over to level a road. Find the area of the road if the diameter of a road roller is 84 cm and length is 1 m.
10. A company packages its milk powder in cylindrical container whose base has a diameter of 14 cm and height 20 cm. Company places a label around the surface of the container (as shown in the figure). If the label is placed 2 cm from top and bottom, what is the area of the label.



11.8 Volume of Cube, Cuboid and Cylinder

Amount of space occupied by a three dimensional object is called its **volume**. Try to compare the volume of objects surrounding you. For example, volume of a room is greater than the volume of an almirah kept inside it. Similarly, volume of your pencil box is greater than the volume of the pen and the eraser kept inside it.

Can you measure volume of either of these objects?

Remember, we use square units to find the area of a region. Here we will use cubic units to find the volume of a solid, as cube is the most convenient solid shape (just as square is the most convenient shape to measure area of a region).

For finding the area we divide the region into square units, similarly, to find the volume of a solid we need to divide it into cubical units.

Observe that the volume of each of the adjoining solids is 8 cubic units (Fig 11.42).

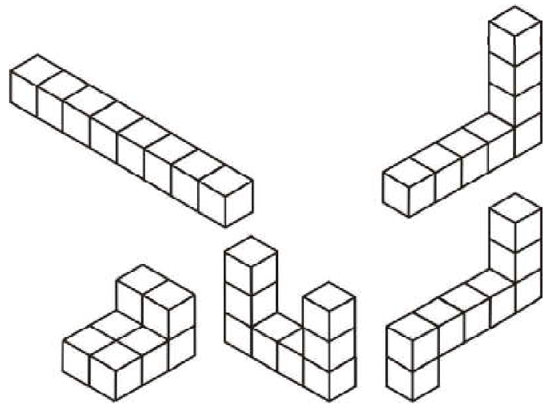


Fig 11.42

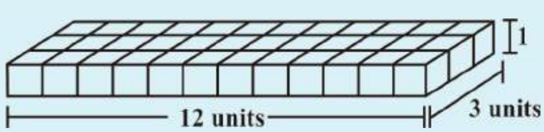
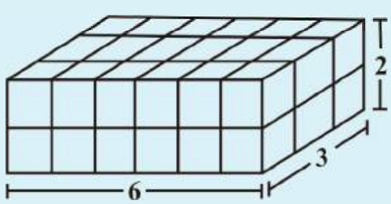
We can say that the volume of a solid is measured by counting the number of unit cubes it contains. Cubic units which we generally use to measure volume are

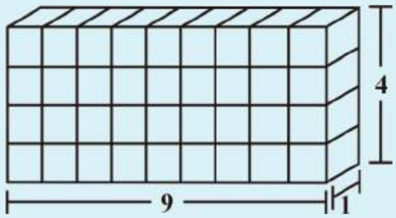
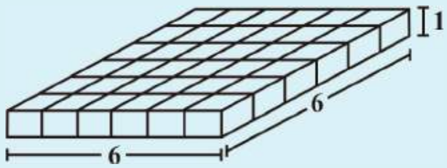
$$1 \text{ cubic cm} = 1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm} = 1 \text{ cm}^3$$
$$= 10 \text{ mm} \times 10 \text{ mm} \times 10 \text{ mm} = \ldots\ldots\ldots\ldots\ldots \text{ mm}^3$$
$$1 \text{ cubic m} = 1 \text{ m} \times 1 \text{ m} \times 1 \text{ m} = 1 \text{ m}^3$$
$$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{ cm}^3$$
$$1 \text{ cubic mm} = 1 \text{ mm} \times 1 \text{ mm} \times 1 \text{ mm} = 1 \text{ mm}^3$$
$$= 0.1 \text{ cm} \times 0.1 \text{ cm} \times 0.1 \text{ cm} = \ldots\ldots\ldots\ldots\ldots\ldots \text{ cm}^3$$

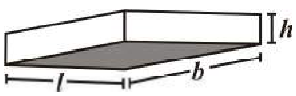
We now find some expressions to find volume of a cuboid, cube and cylinder. Let us take each solid one by one.

11.8.1 Cuboid

Take 36 cubes of equal size (i.e., length of each cube is same). Arrange them to form a cuboid. You can arrange them in many ways. Observe the following table and fill in the blanks.

	cuboid	length	breadth	height	$l \times b \times h = V$
(i)		12	3	1	$12 \times 3 \times 1 = 36$
(ii)		...	...	...	...

(iii)		...	...	...	...
(iv)		...	...	...	...



What do you observe?

Since we have used 36 cubes to form these cuboids, volume of each cuboid is 36 cubic units. Also volume of each cuboid is equal to the product of length, breadth and height of the cuboid. From the above example we can say volume of cuboid $= l \times b \times h$. Since $l \times b$ is the area of its base we can also say that,

Volume of cuboid = area of the base × height

DO THIS

Take a sheet of paper. Measure its area. Pile up such sheets of paper of same size to make a cuboid (Fig 11.43). Measure the height of this pile. Find the volume of the cuboid by finding the product of the area of the sheet and the height of this pile of sheets.

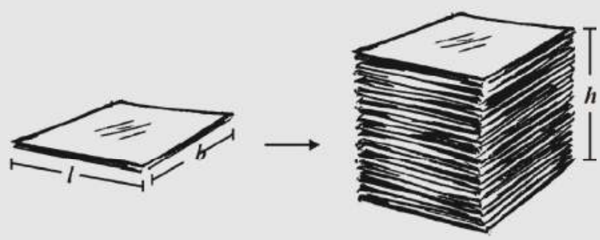


Fig 11.43

This activity illustrates the idea that volume of a solid can be deduced by this method also (if the base and top of the solid are congruent and parallel to each other and its edges are perpendicular to the base). Can you think of such objects whose volume can be found by using this method?

TRY THESE

Find the volume of the following cuboids (Fig 11.44).

(i)

Fig 11.44

11.8.2 Cube

The cube is a special case of a cuboid, where $l = b = h$.
Hence, volume of cube $= l \times l \times l = l^3$

TRY THESE

Find the volume of the following cubes

(a) with a side 4 cm (b) with a side 1.5 m

DO THIS

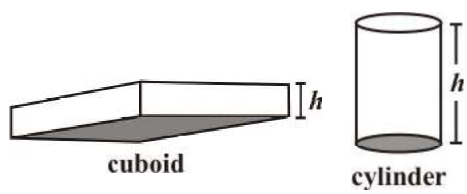
Arrange 64 cubes of equal size in as many ways as you can to form a cuboid. Find the surface area of each arrangement. Can solid shapes of same volume have same surface area?

THINK, DISCUSS AND WRITE

A company sells biscuits. For packing purpose they are using cuboidal boxes: box A $\rightarrow$ 3 cm × 8 cm × 20 cm, box B $\rightarrow$ 4 cm × 12 cm × 10 cm. What size of the box will be economical for the company? Why? Can you suggest any other size (dimensions) which has the same volume but is more economical than these?

11.8.3 Cylinder

We know that volume of a cuboid can be found by finding the product of area of base and its height. Can we find the volume of a cylinder in the same way?



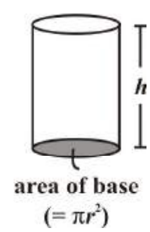
Just like cuboid, cylinder has got a top and a base which are congruent and parallel to each other. Its lateral surface is also perpendicular to the base, just like cuboid.

So the Volume of a cuboid = area of base × height

$= l \times b \times h = lbh$

Volume of cylinder = area of base × height

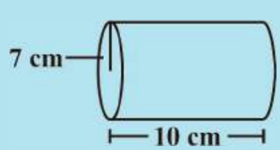
$= \pi r^2 \times h = \pi r^2 h$



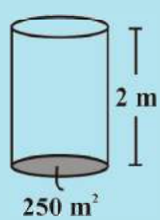
TRY THESE

Find the volume of the following cylinders.

(i)



(ii)



11.9 Volume and Capacity

There is not much difference between these two words.

(a) Volume refers to the amount of space occupied by an object.

(b) Capacity refers to the quantity that a container holds.

Note: If a water tin holds 100 cm^3 of water then the capacity of the water tin is 100 cm^3.

Capacity is also measured in terms of litres. The relation between litre and cm^3 is, 1 mL = 1 cm^3,1 L = 1000 cm^3. Thus, 1 m^3 = 1000000 cm^3 = 1000 L.

Example 8: Find the height of a cuboid whose volume is 275 cm^3 and base area is 25 cm^2.

Solution: Volume of a cuboid = Base area × Height

$$\text{Hence height of the cuboid} = \frac{\text{Volume of cuboid}}{\text{Base area}}$$

$$= \frac{275}{25} = 11 \text{ cm}$$

Height of the cuboid is 11 cm.

Example 9: A godown is in the form of a cuboid of measures 60 m × 40 m × 30 m. How many cuboidal boxes can be stored in it if the volume of one box is 0.8 m^3 ?

Solution: Volume of one box = 0.8 m^3

Volume of godown = 60 × 40 × 30 = 72000 m^3

$$\text{Number of boxes that can be stored in the godown} = \frac{\text{Volume of the godown}}{\text{Volume of one box}}$$

$$= \frac{60 \times 40 \times 30}{0.8} = 90{,}000$$

Hence the number of cuboidal boxes that can be stored in the godown is 90,000.

Example 10: A rectangular paper of width 14 cm is rolled along its width and a cylinder of radius 20 cm is formed. Find the volume of the cylinder (Fig 11.45). (Take $\frac{22}{7}$ for π)

Solution: A cylinder is formed by rolling a rectangle about its width. Hence the width of the paper becomes height and radius of the cylinder is 20 cm.

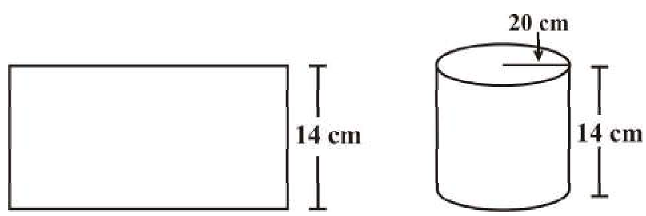


Fig 11.45

Height of the cylinder = h = 14 cm

Radius = r = 20 cm

$$\text{Volume of the cylinder} = V = \pi r^2 h$$

$$= \frac{22}{7} \times 20 \times 20 \times 14 = 17600 \text{ cm}^3$$

Hence, the volume of the cylinder is 17600 cm^3.

Example 11: A rectangular piece of paper 11 cm × 4 cm is folded without overlapping to make a cylinder of height 4 cm. Find the volume of the cylinder.

Solution: Length of the paper becomes the perimeter of the base of the cylinder and width becomes height.

Let radius of the cylinder = r and height = h

$$\text{Perimeter of the base of the cylinder} = 2\pi r = 11$$

or $$2 \times \frac{22}{7} \times r = 11$$

Therefore, $$r = \frac{7}{4} \text{ cm}$$

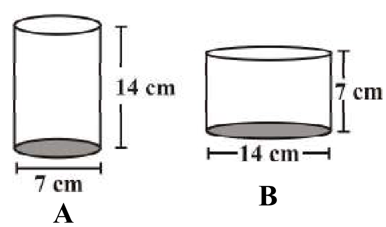
$$\text{Volume of the cylinder} = V = \pi r^2 h$$

$$= \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 4 \text{ cm}^3 = 38.5 \text{ cm}^3.$$

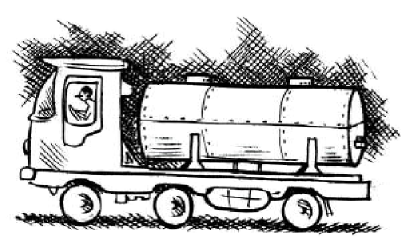
Hence the volume of the cylinder is 38.5 cm^3.

EXERCISE 11.4

1. Given a cylindrical tank, in which situation will you find surface area and in which situation volume.
   (a) To find how much it can hold.
   (b) Number of cement bags required to plaster it.
   (c) To find the number of smaller tanks that can be filled with water from it.
2. Diameter of cylinder A is 7 cm, and the height is 14 cm. Diameter of cylinder B is 14 cm and height is 7 cm. Without doing any calculations can you suggest whose volume is greater? Verify it by finding the volume of both the cylinders. Check whether the cylinder with greater volume also has greater surface area?

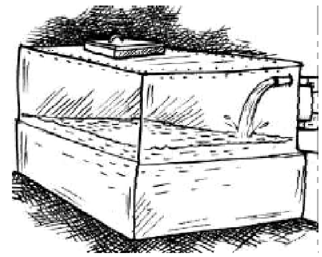


3. Find the height of a cuboid whose base area is 180 cm^2 and volume is 900 cm^3?
4. A cuboid is of dimensions 60 cm × 54 cm × 30 cm. How many small cubes with side 6 cm can be placed in the given cuboid?
5. Find the height of the cylinder whose volume is 1.54 m^3 and diameter of the base is 140 cm ?
6. A milk tank is in the form of cylinder whose radius is 1.5 m and length is 7 m. Find the quantity of milk in litres that can be stored in the tank?



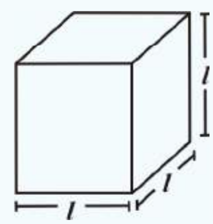
7. If each edge of a cube is doubled,
   (i) how many times will its surface area increase?
   (ii) how many times will its volume increase?

8. Water is pouring into a cubiodal reservoir at the rate of 60 litres per minute. If the volume of reservoir is 108 m^3, find the number of hours it will take to fill the reservoir.

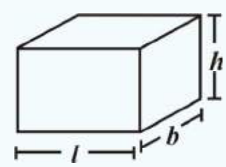


WHAT HAVE WE DISCUSSED?

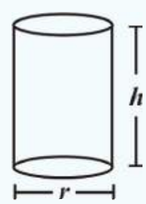
1. Area of
   (i) a trapezium = half of the sum of the lengths of parallel sides × perpendicular distance between them.
   (ii) a rhombus = half the product of its diagonals.
2. **Surface area** of a solid is the sum of the areas of its faces.



3. Surface area of
   a cuboid = $2(lb + bh + hl)$
   a cube = $6l^2$
   a cylinder = $2\pi r(r + h)$
4. Amount of region occupied by a solid is called its **volume**.



5. Volume of
   a cuboid = $l \times b \times h$
   a cube = l^3
   a cylinder = $\pi r^2 h$

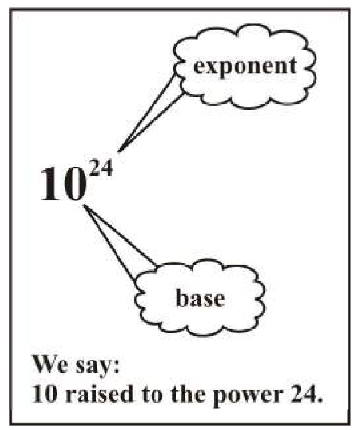


6. (i) 1 cm^3 = 1 mL
   (ii) 1L = 1000 cm^3
   (iii) 1 m^3 = 1000000 cm^3 = 1000L

CHAPTER 12

Exponents and Powers

12.1 Introduction



Do you know?

Mass of earth is 5,970,000,000,000, 000, 000, 000, 000 kg. We have already learnt in earlier class how to write such large numbers more conveniently using exponents, as, 5.97×10^{24} kg.

We read 10^{24} as 10 raised to the power 24.

We know $\quad 2^5 = 2 \times 2 \times 2 \times 2 \times 2$

and $\quad 2^m = 2 \times 2 \times 2 \times 2 \times \ldots \times 2 \times 2 \ldots$ (m times)

Let us now find what is 2^{-2} is equal to?

Exponent is a negative integer.

12.2 Powers with Negative Exponents

You know that,

$$10^2 = 10 \times 10 = 100$$

$$10^1 = 10 = \frac{100}{10}$$

$$10^0 = 1 = \frac{10}{10}$$

$$10^{-1} = ?$$

As the exponent decreases by1, the value becomes one-tenth of the previous value.

Continuing the above pattern we get, $10^{-1} = \frac{1}{10}$

Similarly

$$10^{-2} = \frac{1}{10} \div 10 = \frac{1}{10} \times \frac{1}{10} = \frac{1}{100} = \frac{1}{10^2}$$

$$10^{-3} = \frac{1}{100} \div 10 = \frac{1}{100} \times \frac{1}{10} = \frac{1}{1000} = \frac{1}{10^3}$$

What is 10^{-10} equal to?

Now consider the following.

$$3^3 = 3 \times 3 \times 3 = 27$$

$$3^2 = 3 \times 3 = 9 = \frac{27}{3}$$

$$3^1 = 3 = \frac{9}{3}$$

$$3^0 = 1 = \frac{3}{3}$$

The previous number is divided by the base 3.

So looking at the above pattern, we say

$$3^{-1} = 1 \div 3 = \frac{1}{3}$$

$$3^{-2} = \frac{1}{3} \div 3 = \frac{1}{3 \times 3} = \frac{1}{3^2}$$

$$3^{-3} = \frac{1}{3^2} \div 3 = \frac{1}{3^2} \times \frac{1}{3} = \frac{1}{3^3}$$

You can now find the value of 2^{-2} in a similar manner.

We have, $10^{-2} = \frac{1}{10^2}$ or $10^2 = \frac{1}{10^{-2}}$

$10^{-3} = \frac{1}{10^3}$ or $10^3 = \frac{1}{10^{-3}}$

$3^{-2} = \frac{1}{3^2}$ or $3^2 = \frac{1}{3^{-2}}$ etc.

In general, we can say that for any non-zero integer a, $a^{-m} = \frac{1}{a^m}$, where m is a positive integer. a^{-m} is the multiplicative inverse of a^m.

TRY THESE

Find the multiplicative inverse of the following.

(i) 2^{-4} (ii) 10^{-5} (iii) 7^{-2} (iv) 5^{-3} (v) 10^{-100}

We learnt how to write numbers like 1425 in expanded form using exponents as $1 \times 10^3 + 4 \times 10^2 + 2 \times 10^1 + 5 \times 10^0$.

Let us see how to express 1425.36 in expanded form in a similar way.

We have $1425.36 = 1 \times 1000 + 4 \times 100 + 2 \times 10 + 5 \times 1 + \frac{3}{10} + \frac{6}{100}$

$$= 1 \times 10^3 + 4 \times 10^2 + 2 \times 10 + 5 \times 1 + 3 \times 10^{-1} + 6 \times 10^{-2}$$

$10^{-1} = \frac{1}{10}$, $10^{-2} = \frac{1}{10^2} = \frac{1}{100}$

TRY THESE

Expand the following numbers using exponents.

(i) 1025.63 (ii) 1256.249

12.3 Laws of Exponents

We have learnt that for any non-zero integer a, $a^m \times a^n = a^{m+n}$, where m and n are natural numbers. Does this law also hold if the exponents are negative? Let us explore.

(i) We know that $2^{-3} = \frac{1}{2^3}$ and $2^{-2} = \frac{1}{2^2}$

$a^{-m} = \frac{1}{a^m}$ for any non-zero integer a.

Therefore, $2^{-3} \times 2^{-2} = \frac{1}{2^3} \times \frac{1}{2^2} = \frac{1}{2^3 \times 2^2} = \frac{1}{2^{3+2}} = 2^{-5}$

–5 is the sum of two exponents – 3 and – 2

(ii) Take $(-3)^{-4} \times (-3)^{-3}$

$$(-3)^{-4} \times (-3)^{-3} = \frac{1}{(-3)^4} \times \frac{1}{(-3)^3}$$

$$= \frac{1}{(-3)^4 \times (-3)^3} = \frac{1}{(-3)^{4+3}} = (-3)^{-7}$$

$(-4) + (-3) = -7$

(iii) Now consider $5^{-2} \times 5^4$

$$5^{-2} \times 5^4 = \frac{1}{5^2} \times 5^4 = \frac{5^4}{5^2} = 5^{4-2} = 5^{(2)}$$

$(-2) + 4 = 2$

In Class VII, you have learnt that for any non-zero integer a, $\frac{a^m}{a^n} = a^{m-n}$, where m and n are natural numbers and $m > n$.

(iv) Now consider $(-5)^{-4} \times (-5)^2$

$$(-5)^{-4} \times (-5)^2 = \frac{1}{(-5)^4} \times (-5)^2 = \frac{(-5)^2}{(-5)^4} = \frac{1}{(-5)^4 \times (-5)^{-2}}$$

$$= \frac{1}{(-5)^{4-2}} = (-5)^{-(2)}$$

$(-4) + 2 = -2$

In general, we can say that for any non-zero integer a,
$a^m \times a^n = a^{m+n}$, where m and n are integers.

TRY THESE

Simplify and write in exponential form.

(i) $(-2)^{-3} \times (-2)^{-4}$ (ii) $p^3 \times p^{-10}$ (iii) $3^2 \times 3^{-5} \times 3^6$

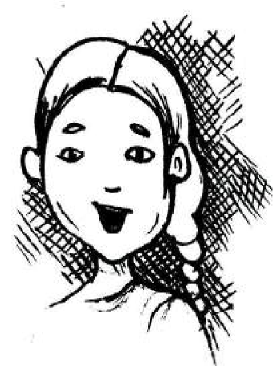
On the same lines you can verify the following laws of exponents, where a and b are non zero integers and m, n are any integers.

(i) $\frac{a^m}{a^n} = a^{m-n}$ (ii) $(a^m)^n = a^{mn}$ (iii) $a^m \times b^m = (ab)^m$

(iv) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$ (v) $a^0 = 1$

These laws you have studied in Class VII for positive exponents only.

Let us solve some examples using the above Laws of Exponents.



Example 1: Find the value of

(i) 2^{-3} (ii) $\frac{1}{3^{-2}}$

Solution:

(i) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ (ii) $\frac{1}{3^{-2}} = 3^2 = 3 \times 3 = 9$

Example 2: Simplify

(i) $(-4)^5 \times (-4)^{-10}$ (ii) $2^5 \div 2^{-6}$

Solution:

(i) $(-4)^5 \times (-4)^{-10} = (-4)^{(5-10)} = (-4)^{-5} = \frac{1}{(-4)^5}$ $\left(a^m \times a^n = a^{m+n},\ a^{-m} = \frac{1}{a^m}\right)$

(ii) $2^5 \div 2^{-6} = 2^{5-(-6)} = 2^{11}$ $(a^m \div a^n = a^{m-n})$

Example 3: Express 4^{-3} as a power with the base 2.

Solution: We have, $4 = 2 \times 2 = 2^2$

Therefore, $(4)^{-3} = (2 \times 2)^{-3} = (2^2)^{-3} = 2^{2 \times (-3)} = 2^{-6}$ $[(a^m)^n = a^{mn}]$

Example 4: Simplify and write the answer in the exponential form.

(i) $(2^5 \div 2^8)^5 \times 2^{-5}$ (ii) $(-4)^{-3} \times (5)^{-3} \times (-5)^{-3}$

(iii) $\frac{1}{8} \times (3)^{-3}$ (iv) $(-3)^4 \times \left(\frac{5}{3}\right)^4$

Solution:

(i) $(2^5 \div 2^8)^5 \times 2^{-5} = (2^{5-8})^5 \times 2^{-5} = (2^{-3})^5 \times 2^{-5} = 2^{-15-5} = 2^{-20} = \frac{1}{2^{20}}$

(ii) $(-4)^{-3} \times (5)^{-3} \times (-5)^{-3} = [(-4) \times 5 \times (-5)]^{-3} = [100]^{-3} = \frac{1}{100^3}$

[using the law $a^m \times b^m = (ab)^m$, $a^{-m} = \frac{1}{a^m}$]

(iii) $\frac{1}{8} \times (3)^{-3} = \frac{1}{2^3} \times (3)^{-3} = 2^{-3} \times 3^{-3} = (2 \times 3)^{-3} = 6^{-3} = \frac{1}{6^3}$

(iv) $(-3)^4 \times \left(\frac{5}{3}\right)^4 = (-1 \times 3)^4 \times \frac{5^4}{3^4} = (-1)^4 \times 3^4 \times \frac{5^4}{3^4}$

$= (-1)^4 \times 5^4 = 5^4$ $[(-1)^4 = 1]$

Example 5: Find m so that $(-3)^{m+1} \times (-3)^5 = (-3)^7$

Solution: $(-3)^{m+1} \times (-3)^5 = (-3)^7$

$(-3)^{m+1+5} = (-3)^7$

$(-3)^{m+6} = (-3)^7$

On both the sides powers have the same base different from 1 and – 1, so their exponents must be equal.

Therefore, $m + 6 = 7$

or $m = 7 - 6 = 1$

$a^n = 1$ only if $n = 0$. This will work for any a except $a = 1$ or $a = -1$. For $a = 1$, $1^1 = 1^2 = 1^3 = 1^{-2} = ... = 1$ or $(1)^n = 1$ for infinitely many n.
For $a = -1$,
$(-1)^0 = (-1)^2 = (-1)^4 = (-1)^{-2} = ... = 1$ or $(-1)^p = 1$ for any even integer p.

Example 6: Find the value of $\left(\frac{2}{3}\right)^{-2}$.

Solution: $\left(\frac{2}{3}\right)^{-2} = \frac{2^{-2}}{3^{-2}} = \frac{3^2}{2^2} = \frac{9}{4}$

$$\left(\frac{2}{3}\right)^{-2} = \frac{2^{-2}}{3^{-2}} = \frac{3^2}{2^2} = \left(\frac{3}{2}\right)^2$$

In general, $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$

Example 7: Simplify (i) $\left\{\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2}$

(ii) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-5}$

Solution:

(i) $$\left\{\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2} = \left\{\frac{1^{-2}}{3^{-2}} - \frac{1^{-3}}{2^{-3}}\right\} \div \frac{1^{-2}}{4^{-2}}$$

$$= \left\{\frac{3^2}{1^2} - \frac{2^3}{1^3}\right\} \div \frac{4^2}{1^2} = \{9 - 8\} \div 16 = \frac{1}{16}$$

(ii) $$\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-5} = \frac{5^{-7}}{8^{-7}} \times \frac{8^{-5}}{5^{-5}} = \frac{5^{-7}}{5^{-5}} \times \frac{8^{-5}}{8^{-7}} = 5^{(-7)-(-5)} \times 8^{(-5)-(-7)}$$

$$= 5^{-2} \times 8^2 = \frac{8^2}{5^2} = \frac{64}{25}$$

EXERCISE 12.1

1. Evaluate.

(i) 3^{-2} (ii) $(-4)^{-2}$ (iii) $\left(\frac{1}{2}\right)^{-5}$

2. Simplify and express the result in power notation with positive exponent.

(i) $(-4)^5 \div (-4)^8$ (ii) $\left(\frac{1}{2^3}\right)^2$

(iii) $(-3)^4 \times \left(\frac{5}{3}\right)^4$ (iv) $(3^{-7} \div 3^{-10}) \times 3^{-5}$ (v) $2^{-3} \times (-7)^{-3}$

3. Find the value of.

(i) $(3^\circ + 4^{-1}) \times 2^2$ (ii) $(2^{-1} \times 4^{-1}) \div 2^{-2}$ (iii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

(iv) $(3^{-1} + 4^{-1} + 5^{-1})^0$ (v) $\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2$

4. Evaluate (i) $\frac{8^{-1} \times 5^3}{2^{-4}}$ (ii) $(5^{-1} \times 2^{-1}) \times 6^{-1}$

5. Find the value of m for which $5^m \div 5^{-3} = 5^5$.

6. Evaluate (i) $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$ (ii) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4}$

7. Simplify.

(i) $\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}}$ $(t \neq 0)$ (ii) $\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

12.4 Use of Exponents to Express Small Numbers in Standard Form

Observe the following facts.

1. The distance from the Earth to the Sun is 149,600,000,000 m.
2. The speed of light is 300,000,000 m/sec.
3. Thickness of Class VII Mathematics book is 20 mm.
4. The average diameter of a Red Blood Cell is 0.000007 mm.
5. The thickness of human hair is in the range of 0.005 cm to 0.01 cm.
6. The distance of moon from the Earth is 384, 467, 000 m (approx).
7. The size of a plant cell is 0.00001275 m.
8. Average radius of the Sun is 695000 km.
9. Mass of propellant in a space shuttle solid rocket booster is 503600 kg.
10. Thickness of a piece of paper is 0.0016 cm.
11. Diameter of a wire on a computer chip is 0.000003 m.
12. The height of Mount Everest is 8848 m.

Observe that there are few numbers which we can read like 2 cm, 8848 m, 6,95,000 km. There are some large numbers like 150,000,000,000 m and some very small numbers like 0.000007 m.

Identify very large and very small numbers from the above facts and write them in the adjacent table:

Very large numbers	Very small numbers
150,000,000,000 m	0.000007 m
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---------------	---------------
---------------	---------------
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We have learnt how to express very large numbers in standard form in the previous class.

For example: $150{,}000{,}000{,}000 = 1.5 \times 10^{11}$

Now, let us try to express 0.000007 m in standard form.

$$0.000007 = \frac{7}{1000000} = \frac{7}{10^6} = 7 \times 10^{-6}$$

$0.000007 \text{ m} = 7 \times 10^{-6} \text{ m}$

150000000000. Decimal is moved 11 places to the left.

0.000007 decimal is moved 6 places to the right.

Similarly, consider the thickness of a piece of paper which is 0.0016 cm.

$$0.0016 = \frac{16}{10000} = \frac{1.6 \times 10}{10^4} = 1.6 \times 10 \times 10^{-4}$$

$= 1.6 \times 10^{-3}$

Therefore, we can say thickness of paper is 1.6×10^{-3} cm.

Again notice

0.0016 decimal is moved 3 places to the right.

TRY THESE

1. Write the following numbers in standard form.
   (i) 0.000000564 (ii) 0.0000021 (iii) 21600000 (iv) 15240000
2. Write all the facts given in the standard form.

12.4.1 Comparing very large and very small numbers

The diameter of the Sun is 1.4×10^9 m and the diameter of the Earth is 1.2756×10^7 m. Suppose you want to compare the diameter of the Earth, with the diameter of the Sun.

Diameter of the Sun $= 1.4 \times 10^9$ m

Diameter of the earth $= 1.2756 \times 10^7$ m

Therefore $\dfrac{1.4 \times 10^9}{1.2756 \times 10^7} = \dfrac{1.4 \times 10^{9-7}}{1.2756} = \dfrac{1.4 \times 100}{1.2756}$ which is approximately 100

So, the diameter of the Sun is about 100 times the diameter of the earth.

Let us compare the size of a Red Blood cell which is 0.000007 m to that of a plant cell which is 0.00001275 m.

Size of Red Blood cell $= 0.000007 \text{ m} = 7 \times 10^{-6}$ m

Size of plant cell $= 0.00001275 = 1.275 \times 10^{-5}$ m

Therefore, $\dfrac{7 \times 10^{-6}}{1.275 \times 10^{-5}} = \dfrac{7 \times 10^{-6-(-5)}}{1.275} = \dfrac{7 \times 10^{-1}}{1.275} = \dfrac{0.7}{1.275} = \dfrac{0.7}{1.3} = \dfrac{1}{2}$ (approx.)

So a red blood cell is half of plant cell in size.

Mass of earth is 5.97×10^{24} kg and mass of moon is 7.35×10^{22} kg. What is the total mass?

Total mass $= 5.97 \times 10^{24} \text{ kg} + 7.35 \times 10^{22}$ kg.

$= 5.97 \times 100 \times 10^{22} + 7.35 \times 10^{22}$

$= 597 \times 10^{22} + 7.35 \times 10^{22}$

$= (597 + 7.35) \times 10^{22}$

$= 604.35 \times 10^{22}$ kg.

When we have to add numbers in standard form, we convert them into numbers with the same exponents.

The distance between Sun and Earth is 1.496×10^{11}m and the distance between Earth and Moon is 3.84×10^8m.

During solar eclipse moon comes in between Earth and Sun.

At that time what is the distance between Moon and Sun.

Distance between Sun and Earth $= 1.496 \times 10^{11}$m

Distance between Earth and Moon $= 3.84 \times 10^{8}$m

Distance between Sun and Moon $= 1.496 \times 10^{11} - 3.84 \times 10^{8}$

$= 1.496 \times 1000 \times 10^{8} - 3.84 \times 10^{8}$

$= (1496 - 3.84) \times 10^{8}$ m $= 1492.16 \times 10^{8}$ m

Example 8: Express the following numbers in standard form.

(i) 0.000035 (ii) 4050000

Solution: (i) $0.000035 = 3.5 \times 10^{-5}$ (ii) $4050000 = 4.05 \times 10^{6}$

Example 9: Express the following numbers in usual form.

(i) 3.52×10^{5} (ii) 7.54×10^{-4} (iii) 3×10^{-5}

Solution:

(i) $3.52 \times 10^{5} = 3.52 \times 100000 = 352000$

(ii) $7.54 \times 10^{-4} = \frac{7.54}{10^{4}} = \frac{7.54}{10000} = 0.000754$

(iii) $3 \times 10^{-5} = \frac{3}{10^{5}} = \frac{3}{100000} = 0.00003$

Again we need to convert numbers in standard form into a numbers with the same exponents.

EXERCISE 12.2

1. Express the following numbers in standard form.
   (i) 0.0000000000085 (ii) 0.00000000000942
   (iii) 6020000000000000 (iv) 0.00000000837
   (v) 31860000000
2. Express the following numbers in usual form.
   (i) 3.02×10^{-6} (ii) 4.5×10^{4} (iii) 3×10^{-8}
   (iv) 1.0001×10^{9} (v) 5.8×10^{12} (vi) 3.61492×10^{6}
3. Express the number appearing in the following statements in standard form.
   (i) 1 micron is equal to $\frac{1}{1000000}$ m.
   (ii) Charge of an electron is 0.000,000,000,000,000,000,16 coulomb.
   (iii) Size of a bacteria is 0.0000005 m
   (iv) Size of a plant cell is 0.00001275 m
   (v) Thickness of a thick paper is 0.07 mm
4. In a stack there are 5 books each of thickness 20mm and 5 paper sheets each of thickness 0.016 mm. What is the total thickness of the stack.

WHAT HAVE WE DISCUSSED?

1. Numbers with negative exponents obey the following laws of exponents.
   (a) $a^{m} \times a^{n} = a^{m+n}$ (b) $a^{m} \div a^{n} = a^{m-n}$ (c) $(a^{m})^{n} = a^{mn}$
   (d) $a^{m} \times b^{m} = (ab)^{m}$ (e) $a^{0} = 1$ (f) $\frac{a^{m}}{b^{m}} = \left(\frac{a}{b}\right)^{m}$
2. Very small numbers can be expressed in standard form using negative exponents.

CHAPTER 13

Direct and Inverse Proportions

13.1 Introduction

Mohan prepares tea for himself and his sister. He uses 300 mL of water, 2 spoons of sugar, 1 spoon of tea leaves and 50 mL of milk. How much quantity of each item will he need, if he has to make tea for five persons?

If two students take 20 minutes to arrange chairs for an assembly, then how much time would five students take to do the same job?

We come across many such situations in our day-to-day life, where we need to see variation in one quantity bringing in variation in the other quantity.

For example:

(i) If the number of articles purchased increases, the total cost also increases.

(ii) More the money deposited in a bank, more is the interest earned.

(iii) As the speed of a vehicle increases, the time taken to cover the same distance decreases.

(iv) For a given job, more the number of workers, less will be the time taken to complete the work.

Observe that change in one quantity leads to change in the other quantity.

Write five more such situations where change in one quantity leads to change in another quantity.

How do we find out the quantity of each item needed by Mohan? Or, the time five students take to complete the job?

To answer such questions, we now study some concepts of variation.

13.2 Direct Proportion

If the cost of 1 kg of sugar is ₹ 18, then what would be the cost of 3 kg sugar? It is ₹ 54.

Similarly, we can find the cost of 5 kg or 8 kg of sugar. Study the following table.

Weight of sugar (in kg)	1	3	5	6	8	10
Cost (in Rs)	18	54	90	...	...	...

Observe that as weight of sugar increases, cost also increases in such a manner that their ratio remains constant.

Take one more example. Suppose a car uses 4 litres of petrol to travel a distance of 60 km. How far will it travel using 12 litres? The answer is 180 km. How did we calculate it? Since petrol consumed in the second instance is 12 litres, i.e., three times of 4 litres, the distance travelled will also be three times of 60 km. In other words, when the petrol consumption becomes three-fold, the distance travelled is also three fold the previous one. Let the consumption of petrol be x litres and the corresponding distance travelled be y km . Now, complete the following table:

Petrol in litres (x)	4	8	12	15	20	25
Distance in km (y)	60	...	180	...	...	...

We find that as the value of x increases, value of y also increases in such a way that the ratio $\frac{x}{y}$ does not change; it remains constant (say k). In this case, it is $\frac{1}{15}$ (check it!).

We say that ***x* and *y* are in direct proportion**, **if** $\frac{x}{y} = k$ **or** $x = ky$.

In this example, $\frac{4}{60} = \frac{12}{180}$, where 4 and 12 are the quantities of petrol consumed in litres (x) and 60 and 180 are the distances (y) in km. So when x and y are in **direct proportion**, we can write $\frac{x_1}{y_1} = \frac{x_2}{y_2}$. [$y_1$, y_2 are values of y corresponding to the values x_1, x_2 of x respectively]

The consumption of petrol and the distance travelled by a car is a case of direct proportion. Similarly, the total amount spent and the number of articles purchased is also an example of direct proportion.

Think of a few more examples for direct proportion. Check whether Mohan [in the initial example] will take 750 mL of water, 5 spoons of sugar, $2\frac{1}{2}$ spoons of tea leaves and 125 mL of milk to prepare tea for five persons! Let us try to understand further the concept of direct proportion through the following activities.

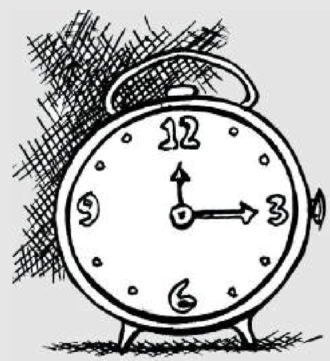
DO THIS

(i)
- Take a clock and fix its minute hand at 12.
- Record the angle turned through by the minute hand from its original position and the time that has passed, in the following table:

Time Passed (T) (in minutes)	(T_1) 15	(T_2) 30	(T_3) 45	(T_4) 60
Angle turned (A) (in degree)	(A_1) 90	(A_2) ...	(A_3) ...	(A_4) ...
$\frac{T}{A}$	...	...	...	...

What do you observe about T and A? Do they increase together?

Is $\frac{T}{A}$ same every time?



Is the angle turned through by the minute hand directly proportional to the time that has passed? Yes!

From the above table, you can also see

$T_1 : T_2 = A_1 : A_2$, because

$T_1 : T_2 = 15 : 30 \quad = 1:2$

$A_1 : A_2 = 90 : 180 \quad = 1:2$

Check if $T_2 : T_3 = A_2 : A_3$ and $T_3 : T_4 = A_3 : A_4$

You can repeat this activity by choosing your own time interval.

(ii) Ask your friend to fill the following table and find the ratio of his age to the corresponding age of his mother.

	Age five years ago	**Present age**	**Age after five years**
Friend's age (F)			
Mother's age (M)			
$\frac{F}{M}$			

What do you observe?

Do F and M increase (or decrease) together? Is $\frac{F}{M}$ same every time? No!

You can repeat this activity with other friends and write down your observations.

Thus, variables increasing (or decreasing) together need not always be in direct proportion. For example:

(i) physical changes in human beings occur with time but not necessarily in a predetermined ratio.

(ii) changes in weight and height among individuals are not in any known proportion and

(iii) there is no direct relationship or ratio between the height of a tree and the number of leaves growing on its branches. Think of some more similar examples.

TRY THESE

1. Observe the following tables and find if x and y are directly proportional.

(i)

x	20	17	14	11	8	5	2
y	40	34	28	22	16	10	4

(ii)

x	6	10	14	18	22	26	30
y	4	8	12	16	20	24	28

(iii)

x	5	8	12	15	18	20
y	15	24	36	60	72	100

2. Principal = ₹ 1000, Rate = 8% per annum. Fill in the following table and find which type of interest (simple or compound) changes in direct proportion with time period.

$$\frac{P \times r \times t}{100}$$

$$P\left(1+\frac{r}{100}\right)^t - P$$

Time period	1 year	2 years	3 years
Simple Interest (in ₹)			
Compound Interest (in ₹)			

THINK, DISCUSS AND WRITE

If we fix time period and the rate of interest, simple interest changes proportionally with principal. Would there be a similar relationship for compound interest? Why?

Let us consider some solved examples where we would use the concept of direct proportion.

Example 1: The cost of 5 metres of a particular quality of cloth is ₹ 210. Tabulate the cost of 2, 4, 10 and 13 metres of cloth of the same type.

Solution: Suppose the length of cloth is x metres and its cost, in ₹, is y.

x	2	4	5	10	13
y	y_2	y_3	210	y_4	y_5

As the length of cloth increases, cost of the cloth also increases in the same ratio. It is a case of direct proportion.

We make use of the relation of type $\frac{x_1}{y_1} = \frac{x_2}{y_2}$

(i) Here $x_1 = 5$, $y_1 = 210$ and $x_2 = 2$

Therefore, $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ gives $\frac{5}{210} = \frac{2}{y_2}$ or $5y_2 = 2 \times 210$ or $y_2 = \frac{2 \times 210}{5} = 84$

(ii) If $x_3 = 4$, then $\frac{5}{210} = \frac{4}{y_3}$ or $5y_3 = 4 \times 210$ or $y_3 = \frac{4 \times 210}{5} = 168$

[Can we use $\frac{x_2}{y_2} = \frac{x_3}{y_3}$ here? Try!]

(iii) If $x_4 = 10$, then $\frac{5}{210} = \frac{10}{y_4}$ or $y_4 = \frac{10 \times 210}{5} = 420$

(iv) If $x_5 = 13$, then $\frac{5}{210} = \frac{13}{y_5}$ or $y_5 = \frac{13 \times 210}{5} = 546$

$\left[\text{Note that here we can also use } \frac{2}{84} \text{ or } \frac{4}{168} \text{ or } \frac{10}{420} \text{ in the place of } \frac{5}{210}\right]$

Example 2: An electric pole, 14 metres high, casts a shadow of 10 metres. Find the height of a tree that casts a shadow of 15 metres under similar conditions.

Solution: Let the height of the tree be x metres. We form a table as shown below:

height of the object (in metres)	14	x
length of the shadow (in metres)	10	15

Note that more the height of an object, the more would be the length of its shadow.

Hence, this is a case of direct proportion. That is, $\frac{x_1}{y_1} = \frac{x_2}{y_2}$

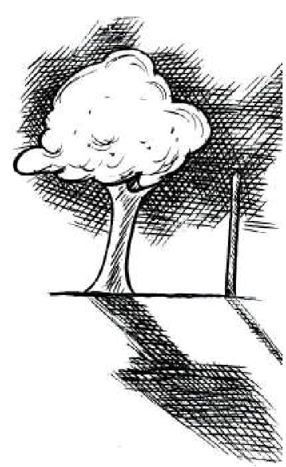
We have $\frac{14}{10} = \frac{x}{15}$ (Why?)

or $\frac{14}{10} \times 15 = x$

or $\frac{14 \times 3}{2} = x$

So $21 = x$

Thus, height of the tree is 21 metres.



Alternately, we can write $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ as $\frac{x_1}{x_2} = \frac{y_1}{y_2}$

so $\quad x_1 : x_2 = y_1 : y_2$

or $\quad 14 : x = 10 : 15$

Therefore, $\quad 10 \times x = 15 \times 14$

or $\quad x = \frac{15 \times 14}{10} = 21$

Example 3: If the weight of 12 sheets of thick paper is 40 grams, how many sheets of the same paper would weigh $2\frac{1}{2}$ kilograms?

Solution:

Let the number of sheets which weigh $2\frac{1}{2}$ kg be x. We put the above information in the form of a table as shown below:

Number of sheets	12	x
Weight of sheets (in grams)	40	2500

1 kilogram = 1000 grams

$2\frac{1}{2}$ kilograms = 2500 grams

More the number of sheets, the more would their weight be. So, the number of sheets and their weights are directly proportional to each other.

So, $\quad \frac{12}{40} = \frac{x}{2500}$

or $\quad \frac{12 \times 2500}{40} = x$

or $\quad 750 = x$

Thus, the required number of sheets of paper = 750.

Alternate method:

Two quantities x and y which vary in direct proportion have the relation $x = ky$ or $\frac{x}{y} = k$

Here, $\quad k = \frac{\text{number of sheets}}{\text{weight of sheets in grams}} = \frac{12}{40} = \frac{3}{10}$

Now x is the number of sheets of the paper which weigh $2\frac{1}{2}$ kg [2500 g].

Using the relation $x = ky$, $x = \frac{3}{10} \times 2500 = 750$

Thus, 750 sheets of paper would weigh $2\frac{1}{2}$ kg.

Example 4: A train is moving at a uniform speed of 75 km/hour.

(i) How far will it travel in 20 minutes?

(ii) Find the time required to cover a distance of 250 km.

Solution: Let the distance travelled (in km) in 20 minutes be x and time taken (in minutes) to cover 250 km be y.

1 hour = 60 minutes

Distance travelled (in km)	75	x	250
Time taken (in minutes)	60	20	y

Since the speed is uniform, therefore, the distance covered would be directly proportional to time.

(i) We have $\frac{75}{60} = \frac{x}{20}$

or $\frac{75}{60} \times 20 = x$

or $x = 25$

So, the train will cover a distance of 25 km in 20 minutes.

(ii) Also, $\frac{75}{60} = \frac{250}{y}$

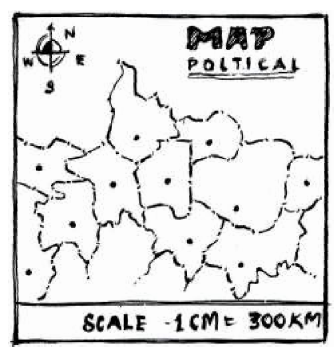
or $y = \frac{250 \times 60}{75} = 200$ minutes or 3 hours 20 minutes.

Therefore, 3 hours 20 minutes will be required to cover a distance of 250 kilometres.

Alternatively, when x is known, then one can determine y from the relation $\frac{x}{20} = \frac{250}{y}$.

> You know that a map is a miniature representation of a very large region. A scale is usually given at the bottom of the map. The scale shows a relationship between actual length and the length represented on the map. The scale of the map is thus the ratio of the distance between two points on the map to the actual distance between two points on the large region.
>
> For example, if 1 cm on the map represents 8 km of actual distance [i.e., the scale is 1 cm : 8 km or 1 : 800,000] then 2 cm on the same map will represent 16 km. Hence, we can say that scale of a map is based on the concept of direct proportion.

Example 5: The scale of a map is given as 1:30000000. Two cities are 4 cm apart on the map. Find the actual distance between them.



Solution: Let the map distance be x cm and actual distance be y cm, then

$$1:30000000 = x : y$$

or $$\frac{1}{3 \times 10^7} = \frac{x}{y}$$

Since $x = 4$ so, $$\frac{1}{3 \times 10^7} = \frac{4}{y}$$

or $$y = 4 \times 3 \times 10^7 = 12 \times 10^7 \text{ cm} = 1200 \text{ km}.$$

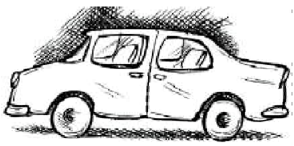
Thus, two cities, which are 4 cm apart on the map, are actually 1200 km away from each other.

DO THIS

Take a map of your State. Note the scale used there. Using a ruler, measure the "map distance" between any two cities. Calculate the actual distance between them.

EXERCISE 13.1

1. Following are the car parking charges near a railway station upto



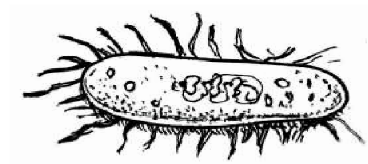
4 hours	₹ 60
8 hours	₹ 100
12 hours	₹ 140
24 hours	₹ 180

Check if the parking charges are in direct proportion to the parking time.

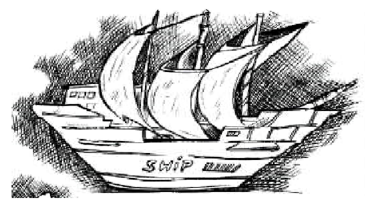
2. A mixture of paint is prepared by mixing 1 part of red pigments with 8 parts of base. In the following table, find the parts of base that need to be added.

Parts of red pigment	1	4	7	12	20
Parts of base	8	...	...	...	...

3. In Question 2 above, if 1 part of a red pigment requires 75 mL of base, how much red pigment should we mix with 1800 mL of base?
4. A machine in a soft drink factory fills 840 bottles in six hours. How many bottles will it fill in five hours?



5. A photograph of a bacteria enlarged 50,000 times attains a length of 5 cm as shown in the diagram. What is the *actual* length of the bacteria? If the photograph is enlarged 20,000 times only, what would be its enlarged length?



6. In a model of a ship, the mast is 9 cm high, while the mast of the actual ship is 12 m high. If the length of the ship is 28 m, how long is the model ship?
7. Suppose 2 kg of sugar contains 9×10^6 crystals. How many sugar crystals are there in (i) 5 kg of sugar? (ii) 1.2 kg of sugar?
8. Rashmi has a road map with a scale of 1 cm representing 18 km. She drives on a road for 72 km. What would be her distance covered in the map?
9. A 5 m 60 cm high vertical pole casts a shadow 3 m 20 cm long. Find at the same time (i) the length of the shadow cast by another pole 10 m 50 cm high (ii) the height of a pole which casts a shadow 5m long.
10. A loaded truck travels 14 km in 25 minutes. If the speed remains the same, how far can it travel in 5 hours?

DO THIS

1. On a squared paper, draw five squares of different sides. Write the following information in a tabular form.

	Square-1	**Square-2**	**Square-3**	**Square-4**	**Square-5**
Length of a side (L)					
Perimeter (P)					
$\frac{L}{P}$					

Area (A)					
$\frac{L}{A}$					

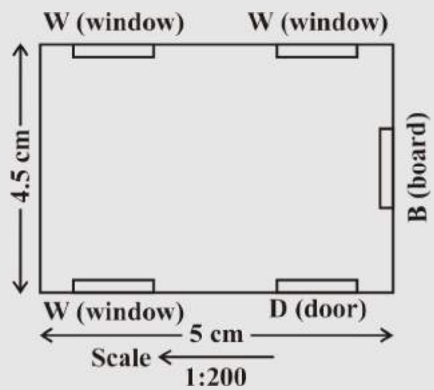
Find whether the length of a side is in direct proportion to:

(a) the perimeter of the square.

(b) the area of the square.

2. The following ingredients are required to make halwa for 5 persons:
Suji/Rawa = 250 g, Sugar = 300 g,
Ghee = 200 g, Water = 500 mL.
Using the concept of proportion, estimate the changes in the quantity of ingredients, to prepare halwa for your class.

3. Choose a scale and make a map of your classroom, showing windows, doors, blackboard etc. (An example is given here).



THINK, DISCUSS AND WRITE

Take a few problems discussed so far under 'direct variation'. Do you think that they can be solved by 'unitary method'?

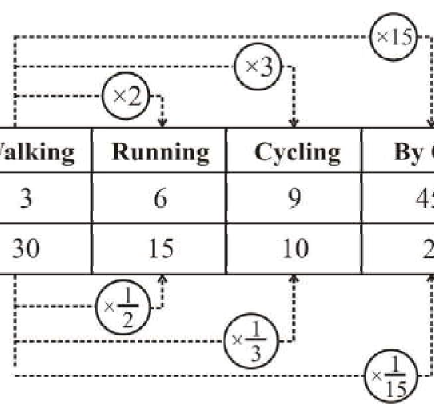
13.3 Inverse Proportion

Two quantities may change in such a manner that if one quantity increases, the other quantity decreases and vice versa. For example, as the number of workers increases, time taken to finish the job decreases. Similarly, if we increase the speed, the time taken to cover a given distance decreases.

To understand this, let us look into the following situation.

Zaheeda can go to her school in four different ways. She can walk, run, cycle or go by car. Study the following table.

	Walking	Running	Cycling	By Car
Speed in km/hour	3	6	9	45
Time taken (in minutes)	30	15	10	2



Observe that as the speed increases, time taken to cover the same distance decreases.

As Zaheeda doubles her speed by running, time reduces to half. As she increases her speed to three times by cycling, time decreases to one third. Similarly, as she increases her speed to 15 times, time decreases to one fifteenth. (Or, in other words the ratio by which time decreases is inverse of the ratio by which the corresponding speed increases). Can we say that speed and time change inversely in proportion?

> Multiplicative inverse of a number is its reciprocal. Thus, $\frac{1}{2}$ is the inverse of 2 and vice versa. (Note that $2 \times \frac{1}{2} = \frac{1}{2} \times 2 = 1$).

Let us consider another example. A school wants to spend ₹ 6000 on mathematics textbooks. How many books could be bought at ₹ 40 each? Clearly 150 books can be bought. If the price of a textbook is more than ₹ 40, then the number of books which could be purchased with the same amount of money would be less than 150. Observe the following table.

Price of each book (in ₹)	40	50	60	75	80	100
Number of books that can be bought	150	120	100	80	75	60

What do you observe? You will appreciate that as the price of the books increases, the number of books that can be bought, keeping the fund constant, will decrease.

Ratio by which the price of books increases when going from 40 to 50 is 4 : 5, and the ratio by which the corresponding number of books decreases from 150 to 120 is 5 : 4. This means that the two ratios are inverses of each other.

Notice that the product of the corresponding values of the two quantities is constant; that is, $40 \times 150 = 50 \times 120 = 6000$.

If we represent the price of one book as x and the number of books bought as y, then as x increases y decreases and vice-versa. It is important to note that the product xy remains constant. We say that x varies inversely with y and y varies inversely with x. *Thus two quantities x and y are said to vary in inverse proportion, if there exists a relation of the type $xy = k$ between them, k being a constant. If y_1, y_2 are the values of y corresponding to the values x_1, x_2 of x respectively then $x_1y_1 = x_2y_2$ (= k), or $\frac{x_1}{x_2} = \frac{y_2}{y_1}$.*

We say that x and y are in **inverse proportion**.

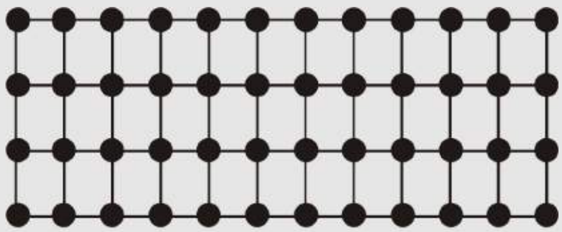
Hence, in this example, cost of a book and number of books purchased in a fixed amount are inversely proportional. Similarly, speed of a vehicle and the time taken to cover a fixed distance changes in inverse proportion.

Think of more such examples of pairs of quantities that vary in inverse proportion. You may now have a look at the furniture – arranging problem, stated in the introductory part of this chapter.

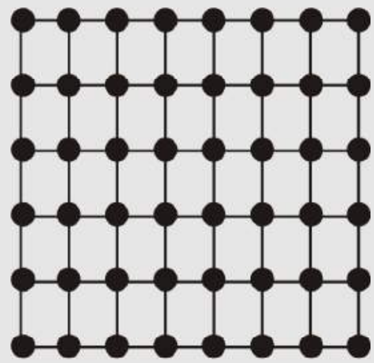
Here is an activity for better understanding of the inverse proportion.

DO THIS

Take a squared paper and arrange 48 counters on it in different number of rows as shown below.



4 Rows, 12 columns



6 Rows, 8 columns

Number of Rows (R)	(R_1) 2	(R_2) 3	(R_3) 4	(R_4) 6	(R_5) 8
Number of Columns (C)	(C_1) ...	(C_2) ...	(C_3) 12	(C_4) 8	(C_5) ...

What do you observe? As R increases, C decreases.

(i) Is $R_1 : R_2 = C_2 : C_1$? (ii) Is $R_3 : R_4 = C_4 : C_3$?

(iii) Are R and C inversely proportional to each other?

Try this activity with 36 counters.

TRY THESE

Observe the following tables and find which pair of variables (here x and y) are in inverse proportion.

(i)

x	50	40	30	20
y	5	6	7	8

(ii)

x	100	200	300	400
y	60	30	20	15

(iii)

x	90	60	45	30	20	5
y	10	15	20	25	30	35

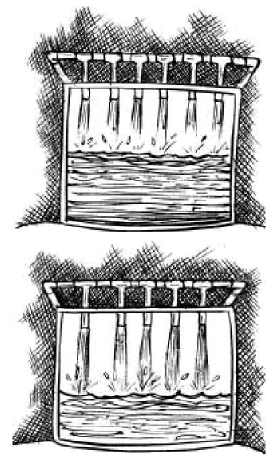
Let us consider some examples where we use the concept of inverse proportion.

When two quantities x and y are in direct proportion (or vary directly) they are also written as $x \propto y$.

When two quantities x and y are in inverse proportion (or vary inversely) they are also written as $x \propto \frac{1}{y}$.

Example 7: 6 pipes are required to fill a tank in 1 hour 20 minutes. How long will it take if only 5 pipes of the same type are used?

Solution:



Let the desired time to fill the tank be x minutes. Thus, we have the following table.

Number of pipes	6	5
Time (in minutes)	80	x

Lesser the number of pipes, more will be the time required by it to fill the tank. So, this is a case of inverse proportion.

Hence, $80 \times 6 = x \times 5$ $[x_1 y_1 = x_2 y_2]$

or $\frac{80 \times 6}{5} = x$

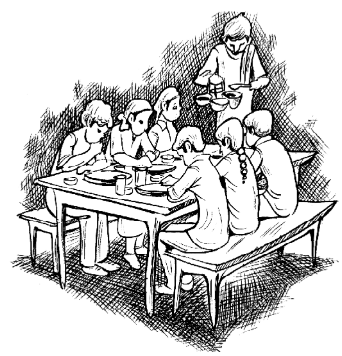
or $x = 96$

Thus, time taken to fill the tank by 5 pipes is 96 minutes or 1 hour 36 minutes.

Example 8: There are 100 students in a hostel. Food provision for them is for 20 days. How long will these provisions last, if 25 more students join the group?

Solution: Suppose the provisions last for y days when the number of students is 125. We have the following table.

Number of students	100	125
Number of days	20	y



Note that more the number of students, the sooner would the provisions exhaust. Therefore, this is a case of inverse proportion.

So, $100 \times 20 = 125 \times y$

or $\frac{100 \times 20}{125} = y$ or $16 = y$

Thus, the provisions will last for 16 days, if 25 more students join the hostel.

Alternately, we can write $x_1 y_1 = x_2 y_2$ as $\frac{x_1}{x_2} = \frac{y_2}{y_1}$.

That is, $x_1 : x_2 = y_2 : y_1$

or $100 : 125 = y : 20$

or $y = \frac{100 \times 20}{125} = 16$

Example 9: If 15 workers can build a wall in 48 hours, how many workers will be required to do the same work in 30 hours?

Solution:

Let the number of workers employed to build the wall in 30 hours be y.

We have the following table.

Number of hours	48	30
Number of workers	15	y

Obviously more the number of workers, faster will they build the wall.
So, the number of hours and number of workers vary in inverse proportion.

So $\quad 48 \times 15 = 30 \times y$

Therefore, $\quad \frac{48 \times 15}{30} = y \quad$ or $\quad y = 24$

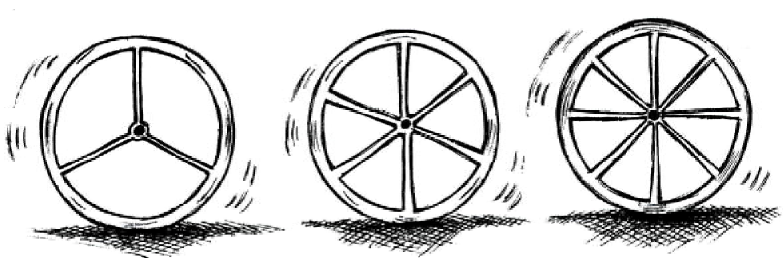
i.e., to finish the work in 30 hours, 24 workers are required.

EXERCISE 13.2

1. Which of the following are in inverse proportion?
   (i) The number of workers on a job and the time to complete the job.
   (ii) The time taken for a journey and the distance travelled in a uniform speed.
   (iii) Area of cultivated land and the crop harvested.
   (iv) The time taken for a fixed journey and the speed of the vehicle.
   (v) The population of a country and the area of land per person.
2. In a Television game show, the prize money of ₹ 1,00,000 is to be divided equally amongst the winners. Complete the following table and find whether the prize money given to an individual winner is directly or inversely proportional to the number of winners?

Number of winners	1	2	4	5	8	10	20
Prize for each winner (in ₹)	1,00,000	50,000	...	...	...	...	...

3. Rehman is making a wheel using spokes. He wants to fix equal spokes in such a way that the angles between any pair of consecutive spokes are equal. Help him by completing the following table.



Number of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90°	60°	...	...	...

(i) Are the number of spokes and the angles formed between the pairs of consecutive spokes in inverse proportion?

(ii) Calculate the angle between a pair of consecutive spokes on a wheel with 15 spokes.

(iii) How many spokes would be needed, if the angle between a pair of consecutive spokes is 40°?

4. If a box of sweets is divided among 24 children, they will get 5 sweets each. How many would each get, if the number of the children is reduced by 4?

5. A farmer has enough food to feed 20 animals in his cattle for 6 days. How long would the food last if there were 10 more animals in his cattle?

6. A contractor estimates that 3 persons could rewire Jasminder's house in 4 days. If, he uses 4 persons instead of three, how long should they take to complete the job?

7. A batch of bottles were packed in 25 boxes with 12 bottles in each box. If the same batch is packed using 20 bottles in each box, how many boxes would be filled?

8. A factory requires 42 machines to produce a given number of articles in 63 days. How many machines would be required to produce the same number of articles in 54 days?

9. A car takes 2 hours to reach a destination by travelling at the speed of 60 km/h. How long will it take when the car travels at the speed of 80 km/h?

10. Two persons could fit new windows in a house in 3 days.

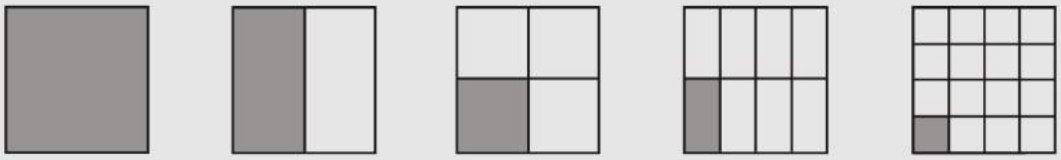
(i) One of the persons fell ill before the work started. How long would the job take now?

(ii) How many persons would be needed to fit the windows in one day?

11. A school has 8 periods a day each of 45 minutes duration. How long would each period be, if the school has 9 periods a day, assuming the number of school hours to be the same?

DO THIS

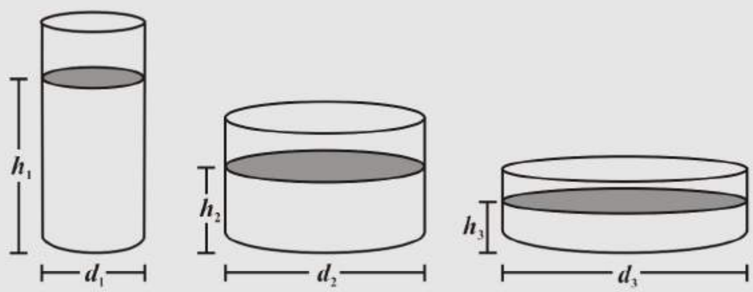
1. Take a sheet of paper. Fold it as shown in the figure. Count the number of parts and the area of a part in each case.



Tabulate your observations and discuss with your friends. Is it a case of inverse proportion? Why?

Number of parts	1	2	4	8	16
Area of each part	area of the paper	$\frac{1}{2}$ the area of the paper	...	...	...

2. Take a few containers of different sizes with circular bases. Fill the same amount of water in each container. Note the diameter of each container and the respective height at which the water level stands. Tabulate your observations. Is it a case of inverse proportion?



Diameter of container (in cm)			
Height of water level (in cm)			

WHAT HAVE WE DISCUSSED?

1. Two quantities x and y are said to be in **direct proportion** if they increase (decrease) together in such a manner that the ratio of their corresponding values remains constant. That is if $\frac{x}{y} = k$ [k is a positive number], then x and y are said to vary directly. In such a case if y_1, y_2 are the values of y corresponding to the values x_1, x_2 of x respectively then $\frac{x_1}{y_1} = \frac{x_2}{y_2}$.

2. Two quantities x and y are said to be in **inverse proportion** if an increase in x causes a proportional decrease in y (and vice-versa) in such a manner that the product of their corresponding values remains constant. That is, if $xy = k$, then x and y are said to vary inversely. In this case if y_1, y_2 are the values of y corresponding to the values x_1, x_2 of x respectively then $x_1 y_1 = x_2 y_2$ or $\frac{x_1}{x_2} = \frac{y_2}{y_1}$.

CHAPTER 14

Factorisation

14.1 Introduction

14.1.1 Factors of natural numbers

You will remember what you learnt about factors in Class VI. Let us take a natural number, say 30, and write it as a product of other natural numbers, say

$$30 = 2 \times 15$$
$$= 3 \times 10 = 5 \times 6$$

Thus, 1, 2, 3, 5, 6, 10, 15 and 30 are the factors of 30. Of these, 2, 3 and 5 are the prime factors of 30 (Why?)

A number written as a product of prime factors is said to be in the prime factor form; for example, 30 written as $2 \times 3 \times 5$ is in the prime factor form.

> We know that 30 can also be written as
> $30 = 1 \times 30$
> Thus, 1 and 30 are also factors of 30. You will notice that 1 is a factor of any number. For example, $101 = 1 \times 101$. However, when we write a number as a product of factors, we shall not write 1 as a factor, unless it is specially required.

The prime factor form of 70 is $2 \times 5 \times 7$.

The prime factor form of 90 is $2 \times 3 \times 3 \times 5$, and so on.

Similarly, we can express algebraic expressions as products of their factors. This is what we shall learn to do in this chapter.

14.1.2 Factors of algebraic expressions

We have seen in Class VII that in algebraic expressions, terms are formed as products of factors. For example, in the algebraic expression $5xy + 3x$ the term $5xy$ has been formed by the factors 5, x and y, i.e.,

$$5xy = 5 \times x \times y$$

Observe that the factors 5, x and y of $5xy$ cannot further be expressed as a product of factors. We may say that 5, x and y are 'prime' factors of $5xy$. In algebraic expressions, we use the word 'irreducible' in place of 'prime'. We say that $5 \times x \times y$ is the irreducible form of $5xy$. Note $5 \times (xy)$ is not an irreducible form of $5xy$, since the factor xy can be further expressed as a product of x and y, i.e., $xy = x \times y$.

> Note 1 is a factor of $5xy$, since
> $5xy = 1 \times 5 \times x \times y$
> In fact, 1 is a factor of every term. As in the case of natural numbers, unless it is specially required, we do not show 1 as a separate factor of any term.

Next consider the expression $3x\,(x+2)$. It can be written as a product of factors. 3, x and $(x + 2)$

$$3x(x + 2) = 3\times x\times\left(x+2\right)$$

The factors 3, x and $(x + 2)$ are irreducible factors of $3x\,(x+2)$.

Similarly, the expression $10x\,(x+2)\,(y+3)$ is expressed in its irreducible factor form as $10x\,(x + 2)\,(y + 3) = 2\times 5\times x\times\left(x+2\right)\times\left(y+3\right)$.

14.2 What is Factorisation?

When we factorise an algebraic expression, we write it as a product of factors. These factors may be numbers, algebraic variables or algebraic expressions.

Expressions like $3xy$, $5x^2y$, $2x\,(y+2)$, $5\,(y+1)\,(x+2)$ are already in factor form. Their factors can be just read off from them, as we already know.

On the other hand consider expressions like $2x+4$, $3x+3y$, x^2+5x, x^2+5x+6. It is not obvious what their factors are. We need to develop systematic methods to factorise these expressions, i.e., to find their factors. This is what we shall do now.

14.2.1 Method of common factors

- We begin with a simple example: Factorise $2x+4$.

  We shall write each term as a product of irreducible factors;

$$2x = 2\times x$$
$$4 = 2\times 2$$

Hence $\qquad 2x + 4 = (2\times x) + (2\times 2)$

Notice that factor 2 is common to both the terms.

Observe, by distributive law

$$2\times(x + 2) = (2\times x) + (2\times 2)$$

Therefore, we can write

$$2x + 4 = 2\times(x + 2) = 2\,(x + 2)$$

Thus, the expression $2x+4$ is the same as $2\,(x+2)$. Now we can read off its factors: they are 2 and $(x+2)$. These factors are irreducible.

Next, factorise $5xy+10x$.

The irreducible factor forms of $5xy$ and $10x$ are respectively,

$$5xy = 5\times x\times y$$
$$10x = 2\times 5\times x$$

Observe that the two terms have 5 and x as common factors. Now,

$$5xy + 10x = (5\times x\times y) + (5\times x\times 2)$$
$$= (5x\times y) + (5x\times 2)$$

We combine the two terms using the distributive law,

$$(5x\times y) + (5x\times 2) = 5x\times(y + 2)$$

Therefore, $5xy+10x = 5\,x\,(y+2)$. (This is the desired factor form.)

Example 1: Factorise $12a^2b + 15ab^2$

Solution: We have $12a^2b = 2 \times 2 \times 3 \times a \times a \times b$

$15ab^2 = 3 \times 5 \times a \times b \times b$

The two terms have 3, a and b as common factors.

Therefore, $12a^2b + 15ab^2 = (3 \times a \times b \times 2 \times 2 \times a) + (3 \times a \times b \times 5 \times b)$

$= 3 \times a \times b \times [(2 \times 2 \times a) + (5 \times b)]$ (combining the terms)

$= 3ab \times (4a + 5b)$

$= 3ab\,(4a + 5b)$ (required factor form)

Example 2: Factorise $10x^2 - 18x^3 + 14x^4$

Solution:

$10x^2 = 2 \times 5 \times x \times x$

$18x^3 = 2 \times 3 \times 3 \times x \times x \times x$

$14x^4 = 2 \times 7 \times x \times x \times x \times x$

The common factors of the three terms are 2, x and x.

Therefore, $10x^2 - 18x^3 + 14x^4 = (2 \times x \times x \times 5) - (2 \times x \times x \times 3 \times 3 \times x) + (2 \times x \times x \times 7 \times x \times x)$

$= 2 \times x \times x \times [(5 - (3 \times 3 \times x) + (7 \times x \times x)]$ (combining the three terms)

$= 2x^2 \times (5 - 9x + 7x^2) = 2x^2(7x^2 - 9x + 5)$

Do you notice that the factor form of an expression has only one term?

TRY THESE

Factorise: (i) $12x + 36$ (ii) $22y - 33z$ (iii) $14pq + 35pqr$

14.2.2 Factorisation by regrouping terms

Look at the expression $2xy + 2y + 3x + 3$. You will notice that the first two terms have common factors 2 and y and the last two terms have a common factor 3. But there is no single factor common to all the terms. How shall we proceed?

Let us write $(2xy + 2y)$ in the factor form:

$2xy + 2y = (2 \times x \times y) + (2 \times y)$

$= (2 \times y \times x) + (2 \times y \times 1)$

$= (2y \times x) + (2y \times 1) = 2y\,(x + 1)$

Similarly, $3x + 3 = (3 \times x) + (3 \times 1)$

$= 3 \times (x + 1) = 3\,(x + 1)$

Note, we need to show 1 as a factor here. Why?

Hence, $2xy + 2y + 3x + 3 = 2y\,(x + 1) + 3\,(x + 1)$

Observe, now we have a common factor $(x + 1)$ in both the terms on the right hand side. Combining the two terms,

$$2xy + 2y + 3x + 3 = 2y\,(x + 1) + 3\,(x + 1) = (x + 1)\,(2y + 3)$$

The expression $2xy + 2y + 3x + 3$ is now in the form of a product of factors. Its factors are $(x + 1)$ and $(2y + 3)$. Note, these factors are irreducible.

What is regrouping?

Suppose, the above expression was given as $2xy + 3 + 2y + 3x$; then it will not be easy to see the factorisation. Rearranging the expression, as $2xy + 2y + 3x + 3$, allows us to form groups $(2xy + 2y)$ and $(3x + 3)$ leading to factorisation. This is regrouping.

Regrouping may be possible in more than one ways. Suppose, we regroup the expression as: $2xy + 3x + 2y + 3$. This will also lead to factors. Let us try:

$$2xy + 3x + 2y + 3 = 2 \times x \times y + 3 \times x + 2 \times y + 3$$
$$= x \times (2y + 3) + 1 \times (2y + 3)$$
$$= (2y + 3)(x + 1)$$

The factors are the same (as they have to be), although they appear in different order.

Example 3: Factorise $6xy - 4y + 6 - 9x$.

Solution:

Step 1 Check if there is a common factor among all terms. There is none.

Step 2 Think of grouping. Notice that first two terms have a common factor $2y$;

$$6xy - 4y = 2y\,(3x - 2) \qquad \text{(a)}$$

What about the last two terms? Observe them. If you change their order to $-9x + 6$, the factor $(3x - 2)$ will come out;

$$-9x + 6 = -3\,(3x) + 3\,(2)$$
$$= -3\,(3x - 2) \qquad \text{(b)}$$

Step 3 Putting (a) and (b) together,

$$6xy - 4y + 6 - 9x = 6xy - 4y - 9x + 6$$
$$= 2y\,(3x - 2) - 3\,(3x - 2)$$
$$= (3x - 2)(2y - 3)$$

The factors of $(6xy - 4y + 6 - 9x)$ are $(3x - 2)$ and $(2y - 3)$.

EXERCISE 14.1

1. Find the common factors of the given terms.
   (i) $12x$, 36 (ii) $2y$, $22xy$ (iii) $14\,pq$, $28p^2q^2$
   (iv) $2x$, $3x^2$, 4 (v) $6\,abc$, $24ab^2$, $12\,a^2b$
   (vi) $16\,x^3$, $-4x^2$, $32x$ (vii) $10\,pq$, $20qr$, $30rp$
   (viii) $3x^2\,y^3$, $10x^3\,y^2$, $6\,x^2\,y^2z$
2. Factorise the following expressions.
   (i) $7x - 42$ (ii) $6p - 12q$ (iii) $7a^2 + 14a$
   (iv) $-16\,z + 20\,z^3$ (v) $20\,l^2\,m + 30\,a\,l\,m$
   (vi) $5\,x^2\,y - 15\,xy^2$ (vii) $10\,a^2 - 15\,b^2 + 20\,c^2$
   (viii) $-4\,a^2 + 4\,ab - 4\,ca$ (ix) $x^2\,y\,z + x\,y^2z + x\,y\,z^2$
   (x) $a\,x^2\,y + b\,x\,y^2 + c\,x\,y\,z$
3. Factorise.
   (i) $x^2 + x\,y + 8x + 8y$ (ii) $15\,xy - 6x + 5y - 2$

(iii) $ax + bx - ay - by$ (iv) $15\,pq + 15 + 9q + 25p$

(v) $z - 7 + 7\,x\,y - x\,y\,z$

14.2.3 Factorisation using identities

We know that

$$(a + b)^2 = a^2 + 2ab + b^2 \qquad \text{(I)}$$

$$(a - b)^2 = a^2 - 2ab + b^2 \qquad \text{(II)}$$

$$(a + b)\,(a - b) = a^2 - b^2 \qquad \text{(III)}$$

The following solved examples illustrate how to use these identities for factorisation. What we do is to observe the given expression. If it has a form that fits the right hand side of one of the identities, then the expression corresponding to the left hand side of the identity gives the desired factorisation.

Example 4: Factorise $x^2 + 8x + 16$

Solution: Observe the expression; it has three terms. Therefore, it does not fit Identity III. Also, it's first and third terms are perfect squares with a positive sign before the middle term. So, it is of the form $a^2 + 2ab + b^2$ where $a = x$ and $b = 4$

such that $a^2 + 2ab + b^2 = x^2 + 2\,(x)\,(4) + 4^2$

$= x^2 + 8x + 16$

Since $a^2 + 2ab + b^2 = (a + b)^2$,

by comparison $x^2 + 8x + 16 = (x + 4)^2$ (the required factorisation)

Example 5: Factorise $4y^2 - 12y + 9$

Solution: Observe $4y^2 = (2y)^2$, $9 = 3^2$ and $12y = 2 \times 3 \times (2y)$

Therefore, $4y^2 - 12y + 9 = (2y)^2 - 2 \times 3 \times (2y) + (3)^2$

$= (2y - 3)^2$ (required factorisation)

Observe here the given expression is of the form $a^2 - 2ab + b^2$. Where $a = 2y$, and $b = 3$ with $2ab = 2 \times 2y \times 3 = 12y$.

Example 6: Factorise $49p^2 - 36$

Solution: There are two terms; both are squares and the second is negative. The expression is of the form $(a^2 - b^2)$. Identity III is applicable here;

$49p^2 - 36 = (7p)^2 - (6)^2$

$= (7p - 6)\,(7p + 6)$ (required factorisation)

Example 7: Factorise $a^2 - 2ab + b^2 - c^2$

Solution: The first three terms of the given expression form $(a - b)^2$. The fourth term is a square. So the expression can be reduced to a difference of two squares.

Thus, $a^2 - 2ab + b^2 - c^2 = (a - b)^2 - c^2$ (Applying Identity II)

$= [(a - b) - c)\,((a - b) + c)]$ (Applying Identity III)

$= (a - b - c)\,(a - b + c)$ (required factorisation)

Notice, how we applied two identities one after the other to obtain the required factorisation.

Example 8: Factorise $m^4 - 256$

Solution: We note $m^4 = (m^2)^2$ and $256 = (16)^2$

Thus, the given expression fits Identity III.

Therefore, $m^4 - 256 = (m^2)^2 - (16)^2$

$= (m^2 - 16)\ (m^2 + 16)$ [(using Identity (III)]

Now, $(m^2 + 16)$ cannot be factorised further, but $(m^2 - 16)$ is factorisable again as per Identity III.

$$m^2 - 16 = m^2 - 4^2$$
$$= (m - 4)\ (m + 4)$$

Therefore, $m^4 - 256 = (m - 4)\ (m + 4)\ (m^2 + 16)$

14.2.4 Factors of the form (x + a) (x + b)

Let us now discuss how we can factorise expressions in one variable, like $x^2 + 5x + 6$, $y^2 - 7y + 12$, $z^2 - 4z - 12$, $3m^2 + 9m + 6$, etc. Observe that these expressions are not of the type $(a + b)^2$ or $(a - b)^2$, i.e., they are not perfect squares. For example, in $x^2 + 5x + 6$, the term 6 is not a perfect square. These expressions obviously also do not fit the type $(a^2 - b^2)$ either.

They, however, seem to be of the type $x^2 + (a + b)\,x + a\,b$. We may therefore, try to use Identity IV studied in the last chapter to factorise these expressions:

$$(x + a)\ (x + b) = x^2 + (a + b)\ x + ab \qquad \text{(IV)}$$

For that we have to look at the coefficients of x and the constant term. Let us see how it is done in the following example.

Example 9: Factorise $x^2 + 5x + 6$

Solution: If we compare the R.H.S. of Identity (IV) with $x^2 + 5x + 6$, we find $ab = 6$, and $a + b = 5$. From this, we must obtain a and b. The factors then will be $(x + a)$ and $(x + b)$.

If $a\,b = 6$, it means that a and b are factors of 6. Let us try $a = 6$, $b = 1$. For these values $a + b = 7$, and not 5, So this choice is not right.

Let us try $a = 2$, $b = 3$. For this $a + b = 5$ exactly as required.

The factorised form of this given expression is then $(x + 2)\ (x + 3)$.

> In general, for factorising an algebraic expression of the type $x^2 + px + q$, we find two factors a and b of q (i.e., the constant term) such that
>
> $$ab = q \quad \text{and} \quad a + b = p$$
>
> Then, the expression becomes $x^2 + (a + b)\ x + ab$
>
> or $x^2 + ax + bx + ab$
>
> or $x(x + a) + b(x + a)$
>
> or $(x + a)\ (x + b)$ which are the required factors.

Example 10: Find the factors of $y^2 - 7y + 12$.

Solution: We note $12 = 3 \times 4$ and $3 + 4 = 7$. Therefore,

$$y^2 - 7y + 12 = y^2 - 3y - 4y + 12$$
$$= y\ (y - 3) - 4\ (y - 3) = (y - 3)\ (y - 4)$$

Note, this time we did not compare the expression with that in Identity (IV) to identify a and b. After sufficient practice you may not need to compare the given expressions for their factorisation with the expressions in the identities; instead you can proceed directly as we did above.

Example 11: Obtain the factors of $z^2 - 4z - 12$.

Solution: Here $ab = -12$; this means one of a and b is negative. Further, $a + b = -4$, this means the one with larger numerical value is negative. We try $a = -4$, $b = 3$; but this will not work, since $a + b = -1$. Next possible values are $a = -6$, $b = 2$, so that $a + b = -4$ as required.

Hence,
$$z^2 - 4z - 12 = z^2 - 6z + 2z - 12$$
$$= z(z - 6) + 2(z - 6)$$
$$= (z - 6)(z + 2)$$

Example 12: Find the factors of $3m^2 + 9m + 6$.

Solution: We notice that 3 is a common factor of all the terms.

Therefore, $3m^2 + 9m + 6 = 3(m^2 + 3m + 2)$

Now, $m^2 + 3m + 2 = m^2 + m + 2m + 2$ (as $2 = 1 \times 2$)
$$= m(m + 1) + 2(m + 1)$$
$$= (m + 1)(m + 2)$$

Therefore, $3m^2 + 9m + 6 = 3(m + 1)(m + 2)$

EXERCISE 14.2

1. Factorise the following expressions.
   (i) $a^2 + 8a + 16$ (ii) $p^2 - 10p + 25$ (iii) $25m^2 + 30m + 9$
   (iv) $49y^2 + 84yz + 36z^2$ (v) $4x^2 - 8x + 4$
   (vi) $121b^2 - 88bc + 16c^2$
   (vii) $(l + m)^2 - 4lm$ (Hint: Expand $(l + m)^2$ first)
   (viii) $a^4 + 2a^2b^2 + b^4$
2. Factorise.
   (i) $4p^2 - 9q^2$ (ii) $63a^2 - 112b^2$ (iii) $49x^2 - 36$
   (iv) $16x^5 - 144x^3$ (v) $(l + m)^2 - (l - m)^2$
   (vi) $9x^2y^2 - 16$ (vii) $(x^2 - 2xy + y^2) - z^2$
   (viii) $25a^2 - 4b^2 + 28bc - 49c^2$
3. Factorise the expressions.
   (i) $ax^2 + bx$ (ii) $7p^2 + 21q^2$ (iii) $2x^3 + 2xy^2 + 2xz^2$
   (iv) $am^2 + bm^2 + bn^2 + an^2$ (v) $(lm + l) + m + 1$
   (vi) $y(y + z) + 9(y + z)$ (vii) $5y^2 - 20y - 8z + 2yz$
   (viii) $10ab + 4a + 5b + 2$ (ix) $6xy - 4y + 6 - 9x$

4. Factorise.

(i) $a^4 - b^4$ (ii) $p^4 - 81$ (iii) $x^4 - (y + z)^4$

(iv) $x^4 - (x - z)^4$ (v) $a^4 - 2a^2b^2 + b^4$

5. Factorise the following expressions.

(i) $p^2 + 6p + 8$ (ii) $q^2 - 10q + 21$ (iii) $p^2 + 6p - 16$

14.3 Division of Algebraic Expressions

We have learnt how to add and subtract algebraic expressions. We also know how to multiply two expressions. We have not however, looked at division of one algebraic expression by another. This is what we wish to do in this section.

We recall that division is the inverse operation of multiplication. Thus, $7 \times 8 = 56$ gives $56 \div 8 = 7$ or $56 \div 7 = 8$.

We may similarly follow the division of algebraic expressions. For example,

(i) $2x \times 3x^2 = 6x^3$

Therefore, $6x^3 \div 2x = 3x^2$

and also, $6x^3 \div 3x^2 = 2x$.

(ii) $5x\,(x + 4) = 5x^2 + 20x$

Therefore, $(5x^2 + 20x) \div 5x = x + 4$

and also $(5x^2 + 20x) \div (x + 4) = 5x$.

We shall now look closely at how the division of one expression by another can be carried out. To begin with we shall consider the division of a monomial by another monomial.

14.3.1 Division of a monomial by another monomial

Consider $6x^3 \div 2x$

We may write $2x$ and $6x^3$ in irreducible factor forms,

$$2x = 2 \times x$$

$$6x^3 = 2 \times 3 \times x \times x \times x$$

Now we group factors of $6x^3$ to separate $2x$,

$$6x^3 = 2 \times x \times (3 \times x \times x) = (2x) \times (3x^2)$$

Therefore, $6x^3 \div 2x = 3x^2$.

A shorter way to depict cancellation of common factors is as we do in division of numbers:

$$77 \div 7 = \frac{77}{7} = \frac{7 \times 11}{7} = 11$$

Similarly, $6x^3 \div 2x = \dfrac{6x^3}{2x}$

$$= \frac{2 \times 3 \times x \times x \times x}{2 \times x} = 3 \times x \times x = 3x^2$$

Example 13: Do the following divisions.

(i) $-20x^4 \div 10x^2$ (ii) $7x^2y^2z^2 \div 14xyz$

Solution:

(i) $-20x^4 = -2 \times 2 \times 5 \times x \times x \times x \times x$

$10x^2 = 2 \times 5 \times x \times x$

Therefore, $(-20x^4) \div 10x^2 = \dfrac{-2 \times 2 \times 5 \times x \times x \times x \times x}{2 \times 5 \times x \times x} = -2 \times x \times x = -2x^2$

(ii) $7x^2y^2z^2 \div 14xyz \quad = \dfrac{7 \times x \times x \times y \times y \times z \times z}{2 \times 7 \times x \times y \times z}$

$$= \frac{x \times y \times z}{2} = \frac{1}{2}xyz$$

TRY THESE

Divide.

(i) $24xy^2z^3$ by $6yz^2$ (ii) $63a^2b^4c^6$ by $7a^2b^2c^3$

14.3.2 Division of a polynomial by a monomial

Let us consider the division of the trinomial $4y^3 + 5y^2 + 6y$ by the monomial $2y$.

$$4y^3 + 5y^2 + 6y = (2 \times 2 \times y \times y \times y) + (5 \times y \times y) + (2 \times 3 \times y)$$

(Here, we expressed each term of the polynomial in factor form) we find that $2 \times y$ is common in each term. Therefore, separating $2 \times y$ from each term. We get

$$4y^3 + 5y^2 + 6y = 2 \times y \times (2 \times y \times y) + 2 \times y \times \left(\frac{5}{2} \times y\right) + 2 \times y \times 3$$

$$= 2y\,(2y^2) + 2y\left(\frac{5}{2}y\right) + 2y\,(3)$$

$$= 2y\left(2y^2 + \frac{5}{2}y + 3\right)$$ (The common factor $2y$ is shown separately.

Therefore, $(4y^3 + 5y^2 + 6y) \div 2y$

$$= \frac{4y^3 + 5y^2 + 6y}{2y} = \frac{2y\,(2y^2 + \frac{5}{2}y + 3)}{2y} = 2y^2 + \frac{5}{2}y + 3$$

Alternatively, we could divide each term of the trinomial by the monomial using the cancellation method.

Here, we divide each term of the polynomial in the numerator by the monomial in the denominator.

$$(4y^3 + 5y^2 + 6y) \div 2y = \frac{4y^3 + 5y^2 + 6y}{2y}$$

$$= \frac{4y^3}{2y} + \frac{5y^2}{2y} + \frac{6y}{2y} = 2y^2 + \frac{5}{2}y + 3$$

Example 14: Divide $24(x^2yz + xy^2z + xyz^2)$ by $8xyz$ using both the methods.

Solution: $24\,(x^2yz + xy^2z + xyz^2)$

$= 2 \times 2 \times 2 \times 3 \times [(x \times x \times y \times z) + (x \times y \times y \times z) + (x \times y \times z \times z)]$

$= 2 \times 2 \times 2 \times 3 \times x \times y \times z \times (x + y + z) = 8 \times 3 \times xyz \times (x + y + z)$ (By taking out the common factor)

Therefore, $24\,(x^2yz + xy^2z + xyz^2) \div 8xyz$

$$= \frac{8 \times 3 \times xyz \times (x + y + z)}{8 \times xyz} = 3 \times (x + y + z) = 3\,(x + y + z)$$

Alternately, $24(x^2yz + xy^2z + xyz^2) \div 8xyz = \dfrac{24x^2yz}{8xyz} + \dfrac{24xy^2z}{8xyz} + \dfrac{24xyz^2}{8xyz}$

$= 3x + 3y + 3z = 3(x + y + z)$

14.4 Division of Algebraic Expressions Continued (Polynomial ÷ Polynomial)

- Consider $(7x^2 + 14x) \div (x + 2)$

  We shall factorise $(7x^2 + 14x)$ first to check and match factors with the denominator:

$$7x^2 + 14x = (7 \times x \times x) + (2 \times 7 \times x)$$

$$= 7 \times x \times (x + 2) = 7x(x + 2)$$

Now $(7x^2 + 14x) \div (x + 2) = \dfrac{7x^2 + 14x}{x + 2}$

$= \dfrac{7x(x + 2)}{x + 2} = 7x$ (Cancelling the factor $(x + 2)$)

> Will it help here to divide each term of the numerator by the binomial in the denominator?

Example 15: Divide $44(x^4 - 5x^3 - 24x^2)$ by $11x(x - 8)$

Solution: Factorising $44(x^4 - 5x^3 - 24x^2)$, we get

$$44(x^4 - 5x^3 - 24x^2) = 2 \times 2 \times 11 \times x^2(x^2 - 5x - 24)$$

(taking the common factor x^2 out of the bracket)

$= 2 \times 2 \times 11 \times x^2(x^2 - 8x + 3x - 24)$

$= 2 \times 2 \times 11 \times x^2 [x(x - 8) + 3(x - 8)]$

$= 2 \times 2 \times 11 \times x^2 (x + 3)(x - 8)$

Therefore, $44(x^4 - 5x^3 - 24x^2) \div 11x(x - 8)$

$$= \frac{2 \times 2 \times 11 \times x \times x \times (x + 3) \times (x - 8)}{11 \times x \times (x - 8)}$$

$= 2 \times 2 \times x(x + 3) = 4x(x + 3)$

> We cancel the factors 11, x and $(x - 8)$ common to both the numerator and denominator.

Example 16: Divide $z(5z^2 - 80)$ by $5z(z + 4)$

Solution: Dividend $= z(5z^2 - 80)$

$= z[(5 \times z^2) - (5 \times 16)]$

$= z \times 5 \times (z^2 - 16)$

$= 5z \times (z + 4)(z - 4)$ [using the identity $a^2 - b^2 = (a + b)(a - b)$]

Thus, $z(5z^2 - 80) \div 5z(z + 4) = \dfrac{5z(z - 4)(z + 4)}{5z(z + 4)} = (z - 4)$

EXERCISE 14.3

1. Carry out the following divisions.

(i) $28x^4 \div 56x$ (ii) $-36y^3 \div 9y^2$ (iii) $66pq^2r^3 \div 11qr^2$

(iv) $34x^3y^3z^3 \div 51xy^2z^3$ (v) $12a^8b^8 \div (-6a^6b^4)$

2. Divide the given polynomial by the given monomial.

(i) $(5x^2 - 6x) \div 3x$ (ii) $(3y^8 - 4y^6 + 5y^4) \div y^4$

(iii) $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2$ (iv) $(x^3 + 2x^2 + 3x) \div 2x$

(v) $(p^3q^6 - p^6q^3) \div p^3q^3$

3. Work out the following divisions.

(i) $(10x - 25) \div 5$ (ii) $(10x - 25) \div (2x - 5)$

(iii) $10y(6y + 21) \div 5(2y + 7)$ (iv) $9x^2y^2(3z - 24) \div 27xy(z - 8)$

(v) $96abc(3a - 12)(5b - 30) \div 144(a - 4)(b - 6)$

4. Divide as directed.

(i) $5(2x + 1)(3x + 5) \div (2x + 1)$ (ii) $26xy(x + 5)(y - 4) \div 13x(y - 4)$

(iii) $52pqr(p + q)(q + r)(r + p) \div 104pq(q + r)(r + p)$

(iv) $20(y + 4)(y^2 + 5y + 3) \div 5(y + 4)$ (v) $x(x + 1)(x + 2)(x + 3) \div x(x + 1)$

5. Factorise the expressions and divide them as directed.

(i) $(y^2 + 7y + 10) \div (y + 5)$ (ii) $(m^2 - 14m - 32) \div (m + 2)$

(iii) $(5p^2 - 25p + 20) \div (p - 1)$ (iv) $4yz(z^2 + 6z - 16) \div 2y(z + 8)$

(v) $5pq(p^2 - q^2) \div 2p(p + q)$

(vi) $12xy(9x^2 - 16y^2) \div 4xy(3x + 4y)$ (vii) $39y^3(50y^2 - 98) \div 26y^2(5y + 7)$

14.5 Can you Find the Error?

Task 1 While solving an equation, Sarita does the following.

$$3x + x + 5x = 72$$

Therefore $\qquad 8x = 72$

and so, $\qquad x = \frac{72}{8} = 9$

Where has she gone wrong? Find the correct answer.

Coefficient 1 of a term is usually not shown. But while adding like terms, we include it in the sum.

Task 2 Appu did the following:

For $x = -3$, $5x = 5 - 3 = 2$

Is his procedure correct? If not, correct it.

Remember to make use of brackets, while substituting a negative value.

Task 3 Namrata and Salma have done the multiplication of algebraic expressions in the following manner.

Remember, when you multiply the expression enclosed in a bracket by a constant (or a variable) outside, each term of the expression has to be multiplied by the constant (or the variable).

	Namrata	**Salma**
(a)	$3(x - 4) = 3x - 4$	$3(x - 4) = 3x - 12$

(b) $(2x)^2 = 2x^2$ $\qquad$ $(2x)^2 = 4x^2$

> Remember, when you square a monomial, the numerical coefficient and each factor has to be squared.

> Make sure, before applying any formula, whether the formula is really applicable.

(c) $(2a - 3)(a + 2)$ $\qquad$ $(2a - 3)(a + 2)$

$= 2a^2 - 6$ $\qquad$ $= 2a^2 + a - 6$

(d) $(x + 8)^2 = x^2 + 64$ $\qquad$ $(x + 8)^2$

$= x^2 + 16x + 64$

(e) $(x - 5)^2 = x^2 - 25$ $\qquad$ $(x - 5)^2 = x^2 - 10x + 25$

Is the multiplication done by both Namrata and Salma correct? Give reasons for your answer.

Task 4 Joseph does a division as : $\dfrac{a+5}{5} = a + 1$

> While dividing a polynomial by a monomial, we divide each term of the polynomial in the numerator by the monomial in the denominator.

His friend Sirish has done the same division as: $\dfrac{a+5}{5} = a$

And his other friend Suman does it this way: $\dfrac{a+5}{5} = \dfrac{a}{5} + 1$

Who has done the division correctly? Who has done incorrectly? Why?

Some fun!

Atul always thinks differently. He asks Sumathi teacher, "If what you say is true, then why do I get the right answer for $\dfrac{64}{16} = \dfrac{4}{1} = 4$?" The teacher explains, " This is so because 64 happens to be 16 × 4; $\dfrac{64}{16} = \dfrac{16 \times 4}{16 \times 1} = \dfrac{4}{1}$. In reality, we cancel a factor of 16 and not 6, as you can see. In fact, 6 is not a factor of either 64 or of 16." The teacher adds further, "Also, $\dfrac{664}{166} = \dfrac{4}{1}, \dfrac{6664}{1666} = \dfrac{4}{1}$, and so on". Isn't that interesting? Can you help Atul to find some other examples like $\dfrac{64}{16}$?

EXERCISE 14.4

Find and correct the errors in the following mathematical statements.

1. $4(x - 5) = 4x - 5$
2. $x(3x + 2) = 3x^2 + 2$
3. $2x + 3y = 5xy$
4. $x + 2x + 3x = 5x$
5. $5y + 2y + y - 7y = 0$
6. $3x + 2x = 5x^2$
7. $(2x)^2 + 4(2x) + 7 = 2x^2 + 8x + 7$
8. $(2x)^2 + 5x = 4x + 5x = 9x$
9. $(3x + 2)^2 = 3x^2 + 6x + 4$

10. Substituting $x = -3$ in

(*a*) $x^2 + 5x + 4$ gives $(-3)^2 + 5(-3) + 4 = 9 + 2 + 4 = 15$

(*b*) $x^2 - 5x + 4$ gives $(-3)^2 - 5(-3) + 4 = 9 - 15 + 4 = -2$

(*c*) $x^2 + 5x$ gives $(-3)^2 + 5(-3) = -9 - 15 = -24$

11. $(y - 3)^2 = y^2 - 9$ **12.** $(z + 5)^2 = z^2 + 25$

13. $(2a + 3b)(a - b) = 2a^2 - 3b^2$ **14.** $(a + 4)(a + 2) = a^2 + 8$

15. $(a - 4)(a - 2) = a^2 - 8$ **16.** $\frac{3x^2}{3x^2} = 0$

17. $\frac{3x^2 + 1}{3x^2} = 1 + 1 = 2$ **18.** $\frac{3x}{3x + 2} = \frac{1}{2}$ **19.** $\frac{3}{4x + 3} = \frac{1}{4x}$

20. $\frac{4x + 5}{4x} = 5$ **21.** $\frac{7x + 5}{5} = 7x$

WHAT HAVE WE DISCUSSED?

1. When we factorise an expression, we write it as a product of factors. These factors may be numbers, algebraic variables or algebraic expressions.
2. An irreducible factor is a factor which cannot be expressed further as a product of factors.
3. A systematic way of factorising an expression is the common factor method. It consists of three steps: (i) Write each term of the expression as a product of irreducible factors (ii) Look for and separate the common factors and (iii) Combine the remaining factors in each term in accordance with the distributive law.
4. Sometimes, all the terms in a given expression do not have a common factor; but the terms can be grouped in such a way that all the terms in each group have a common factor. When we do this, there emerges a common factor across all the groups leading to the required factorisation of the expression. This is the method of regrouping.
5. In factorisation by regrouping, we should remember that any regrouping (i.e., rearrangement) of the terms in the given expression may not lead to factorisation. We must observe the expression and come out with the desired regrouping by trial and error.
6. A number of expressions to be factorised are of the form or can be put into the form : $a^2 + 2ab + b^2$, $a^2 - 2ab + b^2$, $a^2 - b^2$ and $x^2 + (a + b) + ab$. These expressions can be easily factorised using Identities I, II, III and IV, given in Chapter 9,

$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$
$$a^2 - b^2 = (a + b)(a - b)$$
$$x^2 + (a + b)x + ab = (x + a)(x + b)$$

7. In expressions which have factors of the type $(x + a)(x + b)$, remember the numerical term gives ab. Its factors, a and b, should be so chosen that their sum, with signs taken care of, is the coefficient of x.
8. We know that in the case of numbers, division is the inverse of multiplication. This idea is applicable also to the division of algebraic expressions.

9. In the case of division of a polynomial by a monomial, we may carry out the division either by dividing each term of the polynomial by the monomial or by the common factor method.
10. In the case of division of a polynomial by a polynomial, we cannot proceed by dividing each term in the dividend polynomial by the divisor polynomial. Instead, we factorise both the polynomials and cancel their common factors.
11. In the case of divisions of algebraic expressions that we studied in this chapter, we have
    Dividend = Divisor × Quotient.
    In general, however, the relation is
    Dividend = Divisor × Quotient + Remainder
    Thus, we have considered in the present chapter only those divisions in which the remainder is zero.
12. There are many errors students commonly make when solving algebra exercises. You should avoid making such errors.

Introduction to Graphs

CHAPTER 15

15.1 Introduction

Have you seen graphs in the newspapers, television, magazines, books etc.? The purpose of the graph is to show numerical facts in visual form so that they can be understood quickly, easily and clearly. Thus graphs are visual representations of data collected. Data can also be presented in the form of a table; however a graphical presentation is easier to understand. This is true in particular when there is **a trend** or **comparison** to be shown. We have already seen some types of graphs. Let us quickly recall them here.

15.1.1 A Bar graph

A bar graph is used to show comparison among categories. It may consist of two or more parallel vertical (or horizontal) bars (rectangles).

The bar graph in Fig 15.1 shows Anu's mathematics marks in the three terminal examinations. It helps you to compare her performance easily. She has shown good progress.

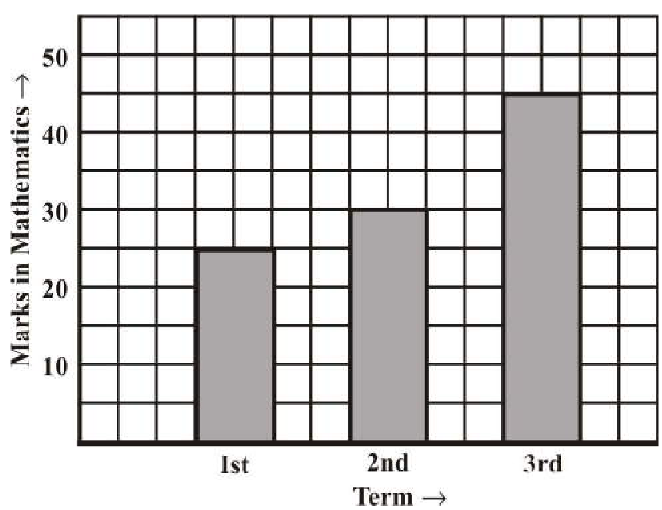


Fig 15.1

Bar graphs can also have double bars as in Fig 15.2. This graph gives a comparative account of sales (in ₹) of various fruits over a two-day period. How is Fig 15.2 different from Fig 15.1? Discuss with your friends.

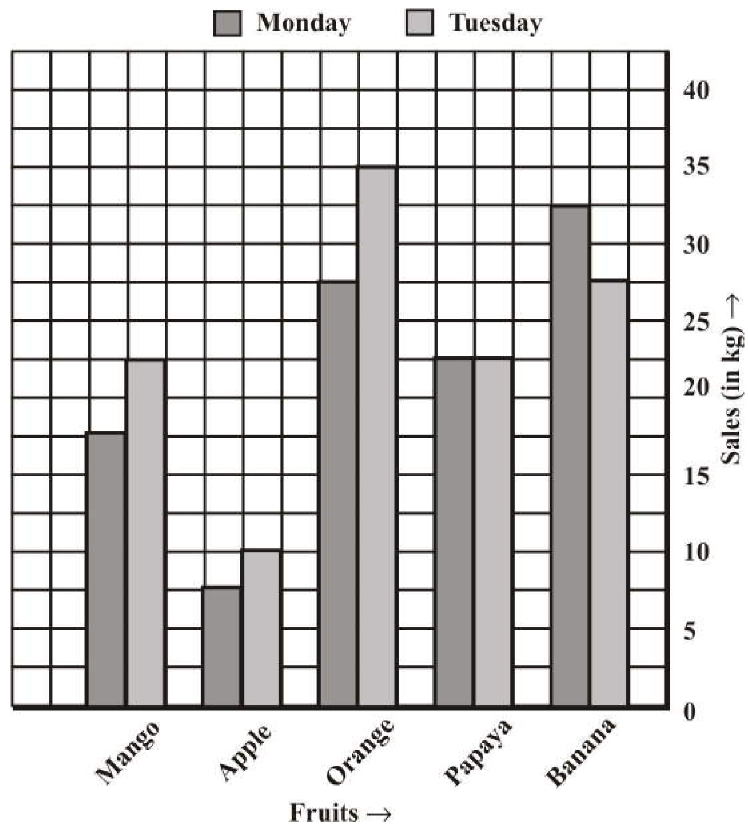


Fig 15.2

15.1.2 A Pie graph (or a circle-graph)

A pie-graph is used to compare parts of a whole. The circle represents the whole. Fig 15.3 is a pie-graph. It shows the percentage of viewers watching different types of TV channels.

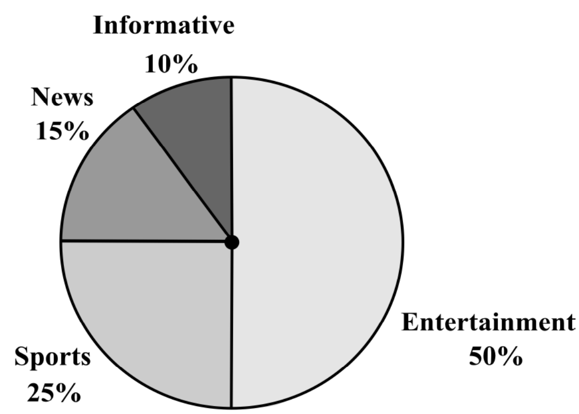


Fig 15.3

15.1.3 A histogram

A Histogram is a bar graph that shows data in intervals. It has adjacent bars over the intervals.

The histogram in Fig 15.4 illustrates the distribution of weights (in kg) of 40 persons of a locality.

Weights (kg)	40-45	45-50	50-55	55-60	60-65
No. of persons	4	12	13	6	5

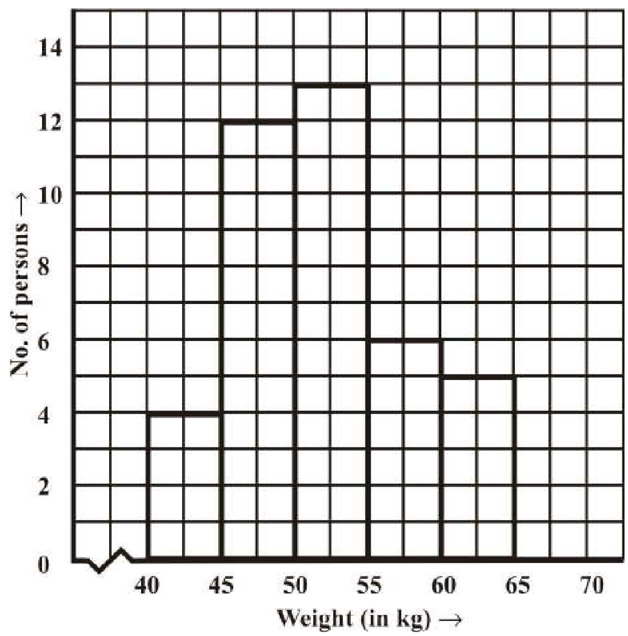


Fig 15.4

In Fig 15.4 a jagged line (⁓) has been used along horizontal line to indicate that we are not showing numbers between 0 and 40.

There are no gaps between bars, because there are no gaps between the intervals. What is the information that you gather from this histogram? Try to list them out.

15.1.4 A line graph

A **line graph** displays data that changes continuously over periods of time.

When Renu fell sick, her doctor maintained a record of her body temperature, taken every four hours. It was in the form of a graph (shown in Fig 15.5 and Fig 15.6).

We may call this a "time-temperature graph".

It is a pictorial representation of the following data, given in tabular form.

Time	6 a.m.	10 a.m.	2 p.m.	6 p.m.
Temperature(°C)	37	40	38	35

The horizontal line (usually called the x-axis) shows the timings at which the temperatures were recorded. What are labelled on the vertical line (usually called the y-axis)?

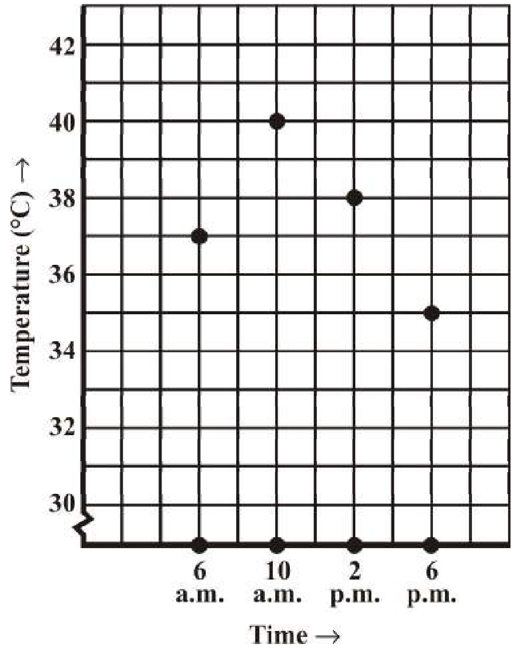


Fig 15.5

Each piece of data is shown by a point on the square grid.

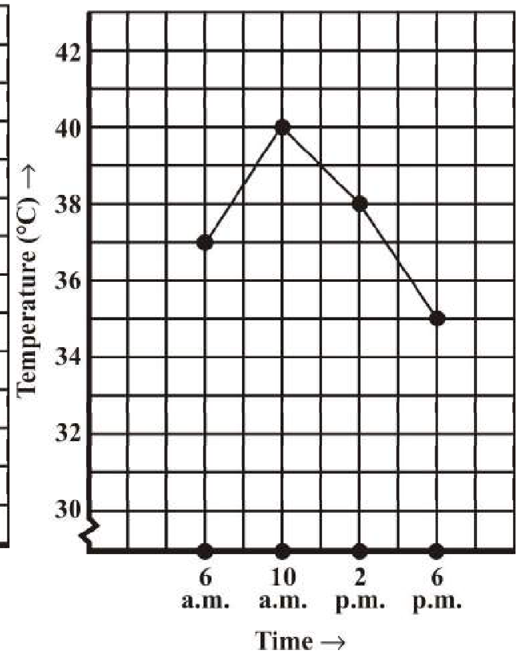


Fig 15.6

The points are then connected by line segments. The result is the **line graph**.

What all does this graph tell you? For example you can see the pattern of temperature; more at 10 a.m. (see Fig 15.5) and then decreasing till 6 p.m. Notice that the temperature increased by 3° C(= 40° C – 37° C) during the period 6 a.m. to 10 a.m.

There was no recording of temperature at 8 a.m., however the graph *suggests* that it was more than 37 °C (How?).

Example 1: (A graph on "performance")

The given graph (Fig 15.7) represents the total runs scored by two batsmen A and B, during each of the ten different matches in the year 2007. Study the graph and answer the following questions.

(i) What information is given on the two axes?

(ii) Which line shows the runs scored by batsman A?

(iii) Were the run scored by them same in any match in 2007? If so, in which match?

(iii) Among the two batsmen, who is steadier? How do you judge it?

Solution:

(i) The horizontal axis (or the x-axis) indicates the matches played during the year 2007. The vertical axis (or the y-axis) shows the total runs scored in each match.

(ii) The dotted line shows the runs scored by Batsman A. (This is already indicated at the top of the graph).

(iii) During the 4th match, both have scored the same number of 60 runs. (This is indicated by the point at which both graphs meet).

(iv) Batsman A has one great "peak" but many deep "valleys". He does not appear to be consistent. B, on the other hand has never scored below a total of 40 runs, even though his highest score is only 100 in comparison to 115 of A. Also A has scored a zero in two matches and in a total of 5 matches he has scored less than 40 runs. Since A has a lot of ups and downs, B is a more consistent and reliable batsman.

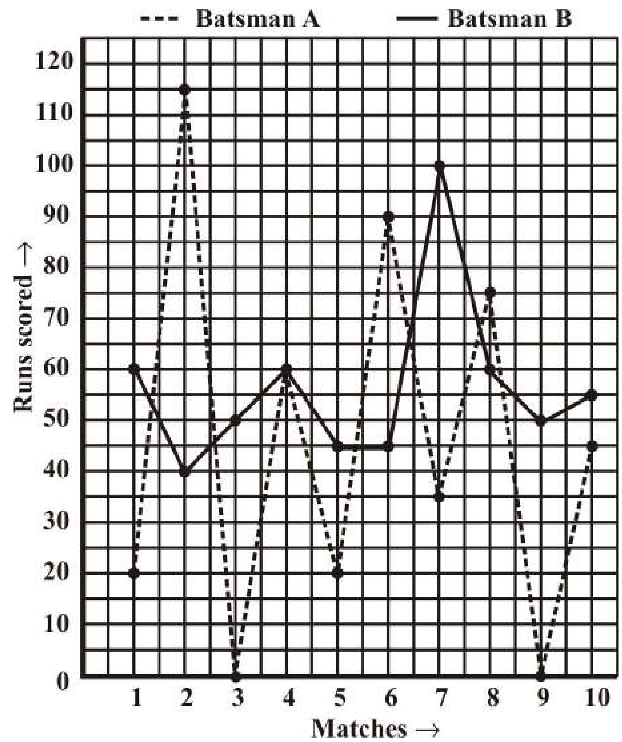


Fig 15.7

Example 2: The given graph (Fig 15.8) describes the distances of a car from a city P at different times when it is travelling from City P to City Q, which are 350 km apart. Study the graph and answer the following:

(i) What information is given on the two axes?

(ii) From where and when did the car begin its journey?

(iii) How far did the car go in the first hour?

(iv) How far did the car go during (i) the 2nd hour? (ii) the 3rd hour?

(v) Was the speed same during the first three hours? How do you know it?

(vi) Did the car stop for some duration at any place? Justify your answer.

(vii) When did the car reach City Q?

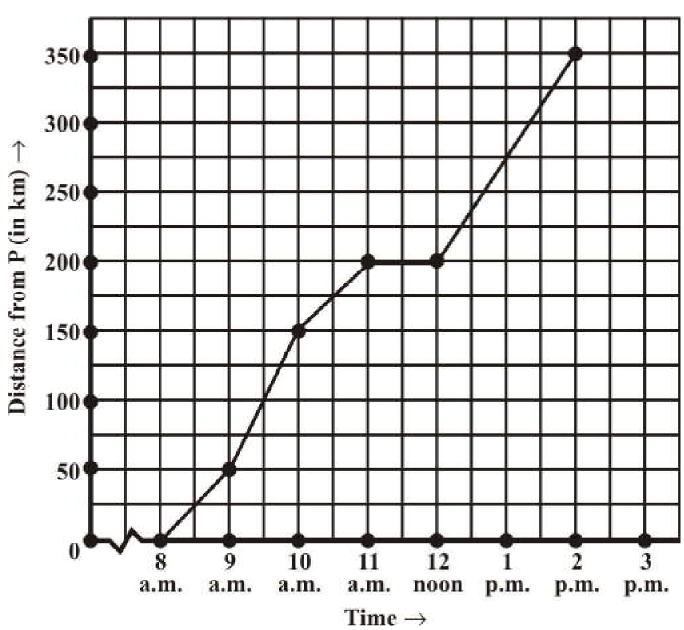


Fig 15.8

Solution:

(i) The horizontal (x) axis shows the time. The vertical (y) axis shows the distance of the car from City P.

(ii) The car started from City P at 8 a.m.

(iii) The car travelled 50 km during the first hour. [This can be seen as follows. At 8 a.m. it just started from City P. At 9 a.m. it was at the 50th km (seen from graph). Hence during the one-hour time between 8 a.m. and 9 a.m. the car travelled 50 km].

(iv) The distance covered by the car during

(a) the 2nd hour (i.e., from 9 am to 10 am) is 100 km, (150 – 50).

(b) the 3rd hour (i.e., from 10 am to 11 am) is 50 km (200 – 150).

(v) From the answers to questions (iii) and (iv), we find that the speed of the car was not the same all the time. (In fact the graph illustrates how the speed varied).

(vi) We find that the car was 200 km away from city P when the time was 11 a.m. and also at 12 noon. This shows that the car did not travel during the interval 11 a.m. to 12 noon. The horizontal line segment representing "travel" during this period is illustrative of this fact.

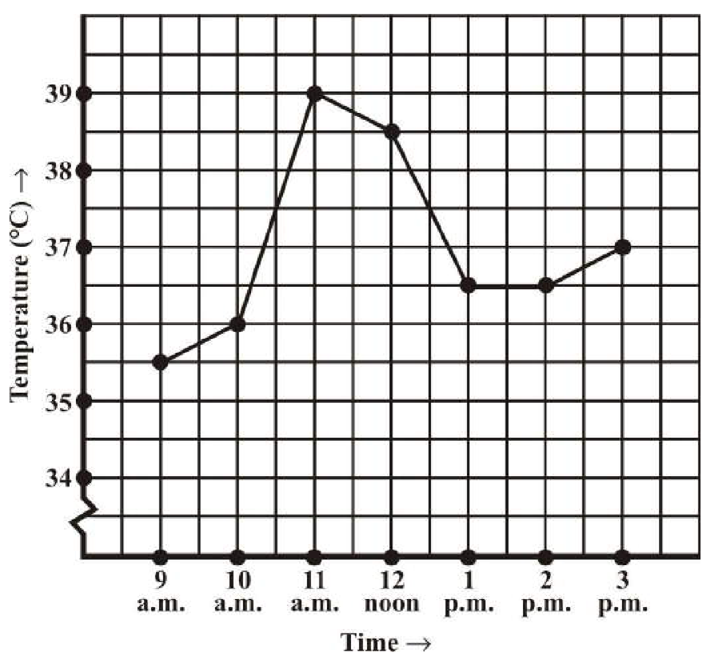
(vii) The car reached City Q at 2 p.m.

EXERCISE 15.1

1. The following graph shows the temperature of a patient in a hospital, recorded every hour.

(a) What was the patient's temperature at 1 p.m. ?

(b) When was the patient's temperature 38.5° C?



(c) The patient's temperature was the same two times during the period given. What were these two times?

(d) What was the temperature at 1.30 p.m.? How did you arrive at your answer?

(e) During which periods did the patients' temperature showed an upward trend?

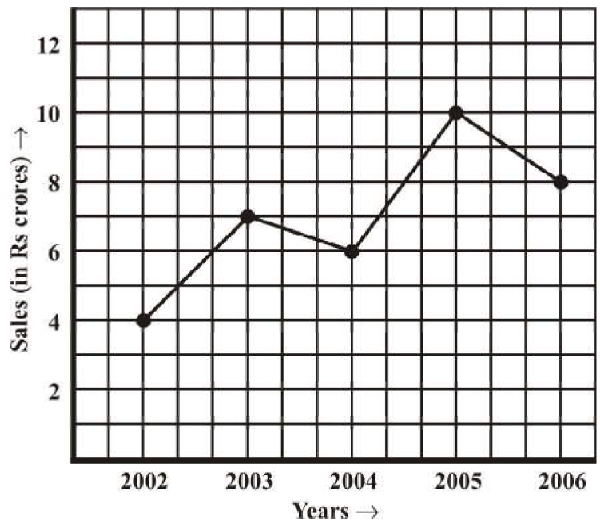
2. The following line graph shows the yearly sales figures for a manufacturing company.

(a) What were the sales in (i) 2002 (ii) 2006?

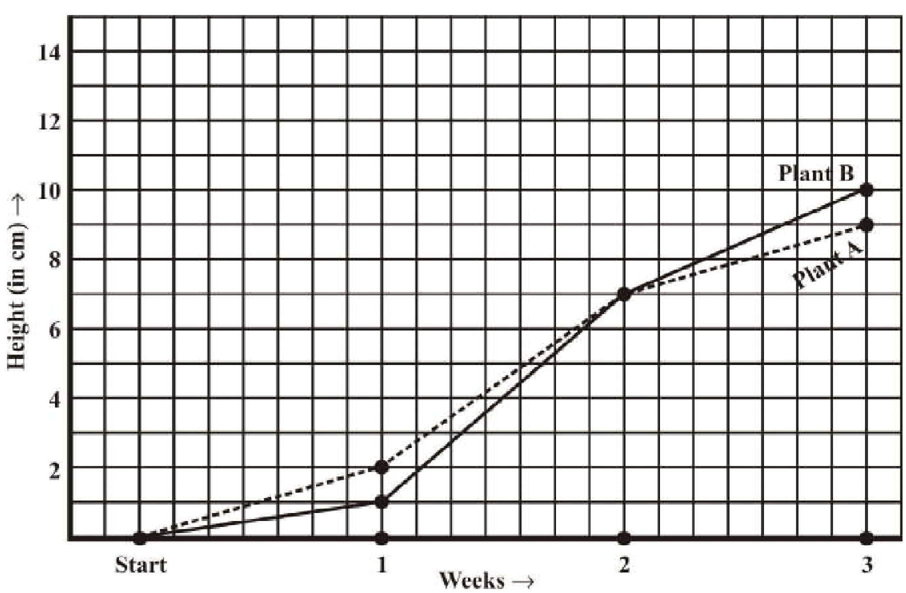
(b) What were the sales in (i) 2003 (ii) 2005?

(c) Compute the difference between the sales in 2002 and 2006.

(d) In which year was there the greatest difference between the sales as compared to its previous year?

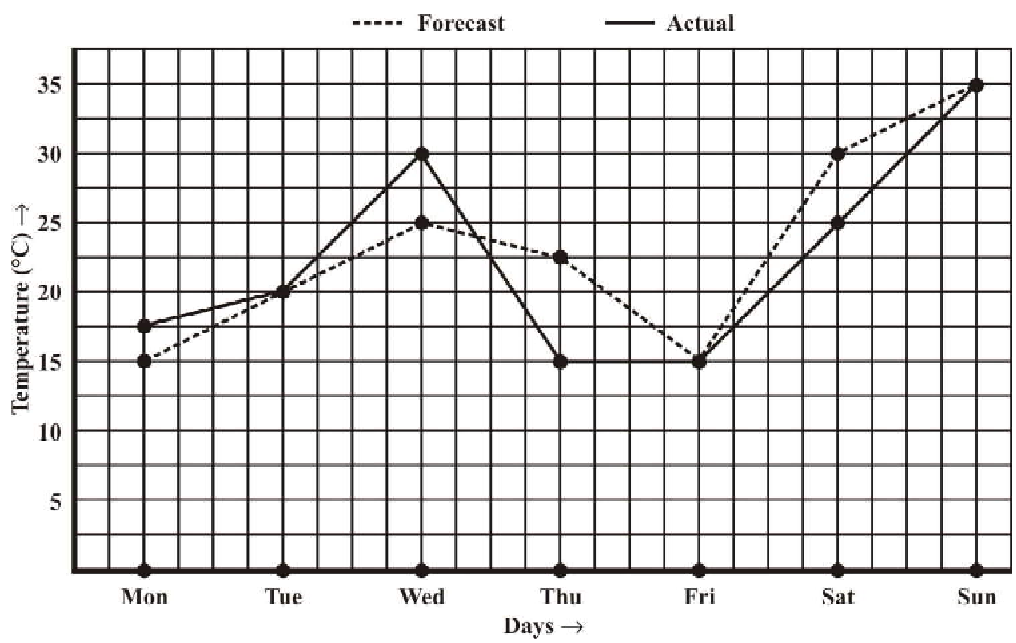


3. For an experiment in Botany, two different plants, plant A and plant B were grown under similar laboratory conditions. Their heights were measured at the end of each week for 3 weeks. The results are shown by the following graph.



(a) How high was Plant A after (i) 2 weeks (ii) 3 weeks?
(b) How high was Plant B after (i) 2 weeks (ii) 3 weeks?
(c) How much did Plant A grow during the 3rd week?
(d) How much did Plant B grow from the end of the 2nd week to the end of the 3rd week?
(e) During which week did Plant A grow most?
(f) During which week did Plant B grow least?
(g) Were the two plants of the same height during any week shown here? Specify.

4. The following graph shows the temperature forecast and the actual temperature for each day of a week.
(a) On which days was the forecast temperature the same as the actual temperature?
(b) What was the maximum forecast temperature during the week?
(c) What was the minimum actual temperature during the week?
(d) On which day did the actual temperature differ the most from the forecast temperature?



5. Use the tables below to draw linear graphs.
(a) The number of days a hill side city received snow in different years.

Year	2003	2004	2005	2006
Days	8	10	5	12

(b) Population (in thousands) of men and women in a village in different years.

Year	2003	2004	2005	2006	2007
Number of Men	12	12.5	13	13.2	13.5
Number of Women	11.3	11.9	13	13.6	12.8

6. A courier-person cycles from a town to a neighbouring suburban area to deliver a parcel to a merchant. His distance from the town at different times is shown by the following graph.

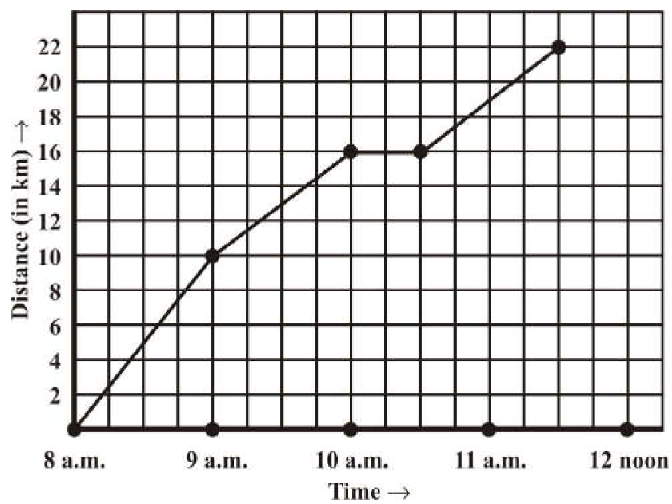
(a) What is the scale taken for the time axis?

(b) How much time did the person take for the travel?

(c) How far is the place of the merchant from the town?

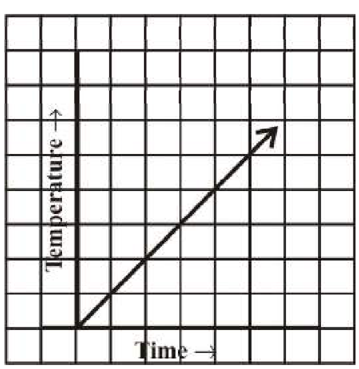
(d) Did the person stop on his way? Explain.

(e) During which period did he ride fastest?

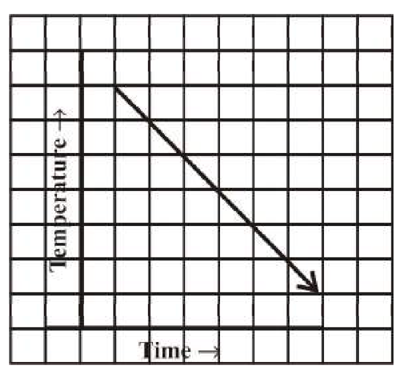


7. Can there be a time-temperature graph as follows? Justify your answer.

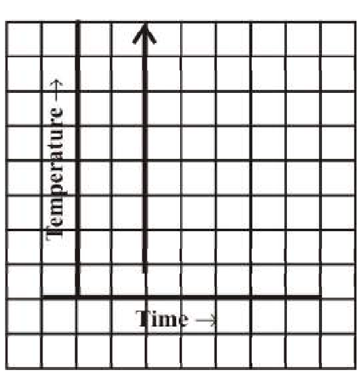
(i)



(ii)



(iii)



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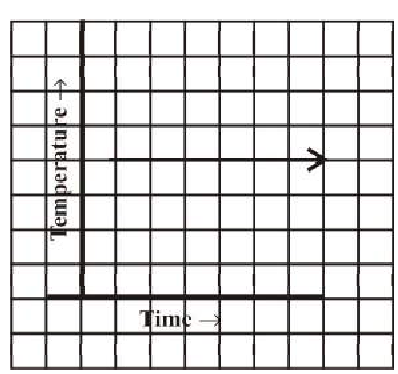
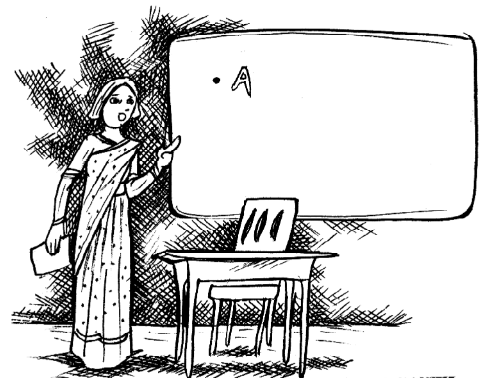


15.2 Linear Graphs



A line graph consists of bits of line segments joined consecutively. Sometimes the graph may be a whole unbroken line. Such a graph is called a **linear graph**. To draw such a line we need to locate some points on the graph sheet. We will now learn how to locate points conveniently on a graph sheet.

15.2.1 Location of a point

The teacher put a dot on the black-board. She asked the students how they would describe its location. There were several responses (Fig 15. 9).

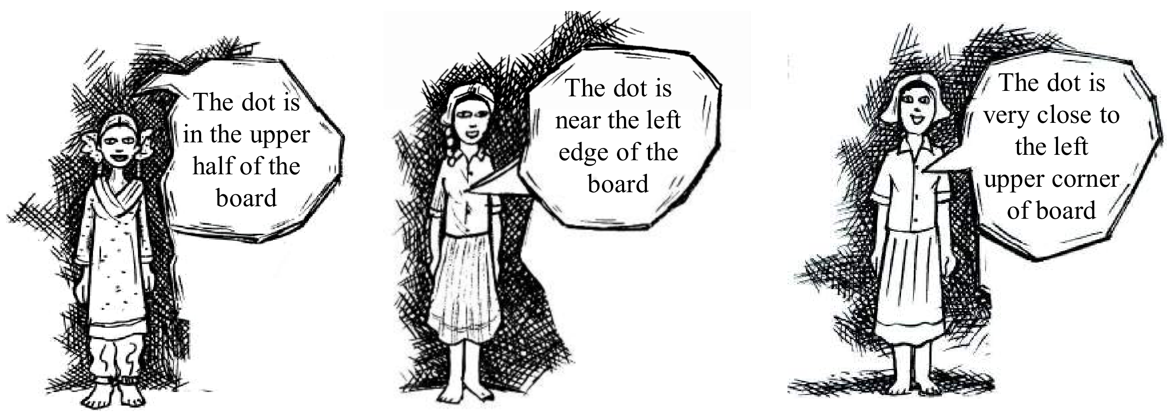


Fig 15.9

Can any one of these statements help fix the position of the dot? No! Why not? Think about it.

John then gave a suggestion. He measured the distance of the dot from the left edge of the board and said, "The dot is 90 cm from the left edge of the board". Do you think John's suggestion is really helpful? (Fig 15.10)

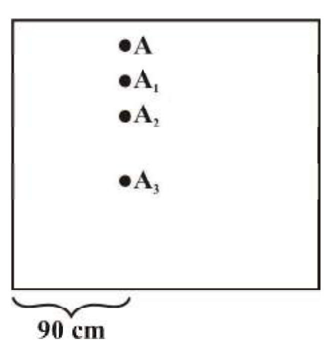


Fig 15.10

A, A_1, A_2, A_3 are all 90 cm away from the left edge.

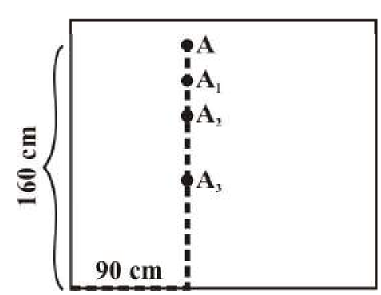


Fig 15.11

A is 90 cm from left edge and 160 cm from the bottom edge.

Rekha then came up with a modified statement : "The dot is 90 cm from the left edge and 160 cm from the bottom edge". That solved the problem completely! (Fig 15.11) The teacher said, "We describe the position of this dot by writing it as (90, 160)". Will the point (160, 90) be different from (90, 160)? Think about it.

*The 17th century mathematician **Rene Descartes**, it is said, noticed the movement of an insect near a corner of the ceiling and began to think of determining the position of a given point in a plane. His system of fixing a point with the help of two measurements, vertical and horizontal, came to be known as Cartesian system, in his honour.*

Rene Descartes (1596-1650)

15.2.2 Coordinates

Suppose you go to an auditorium and search for your reserved seat. You need to know two numbers, the row number and the seat number. This is the basic method for fixing a point in a plane.

Observe in Fig 15.12 how the point (3, 4) which is 3 units from left edge and 4 units from bottom edge is plotted on a graph sheet. The graph sheet itself is a square grid. We draw the x and y axes conveniently and then fix the required point. 3 is called the ***x*-coordinate** of the point; 4 is the ***y*-coordinate** of the point. We say that the **coordinates** of the point are (3, 4).

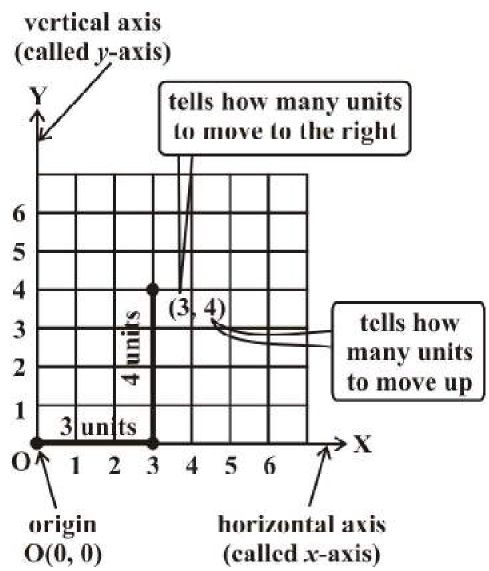


Fig 15.12

Example 3: Plot the point (4, 3) on a graph sheet. Is it the same as the point (3, 4)?

Solution: Locate the x, y axes, (they are actually number lines!). Start at O (0, 0). Move 4 units to the right; then move 3 units up, you reach the point (4, 3). From Fig 15.13, you can see that the points (3, 4) and (4, 3) are two different points.

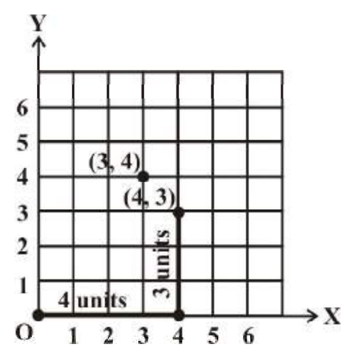


Fig 15.13

Example 4: From Fig 15.14, choose the letter(s) that indicate the location of the points given below:

(i) (2, 1)

(ii) (0, 5)

(iii) (2, 0)

Also write

(iv) The coordinates of A.

(v) The coordinates of F.

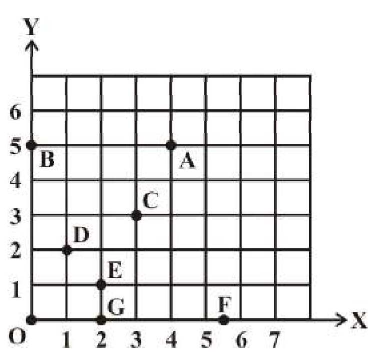


Fig 15.14

Solution:

(i) (2, 1) is the point E (It is not D!).

(ii) (0, 5) is the point B (why? Discuss with your friends!). (iii) (2, 0) is the point G.

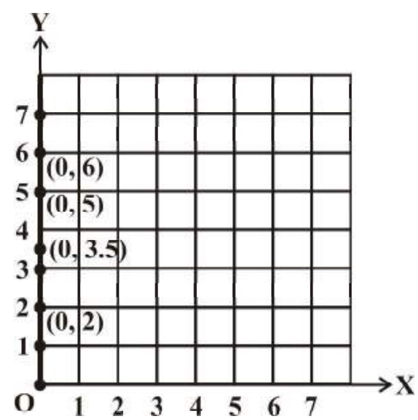
(iv) Point A is (4, 5) (v) F is (5.5, 0)

Example 5: Plot the following points and verify if they lie on a line. If they lie on a line, name it.

(i) (0, 2), (0, 5), (0, 6), (0, 3.5) (ii) A (1, 1), B (1, 2), C (1, 3), D (1, 4)

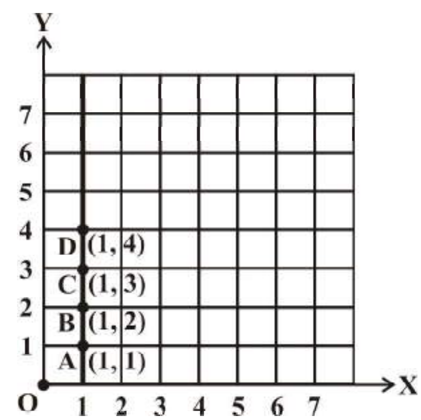
(iii) K (1, 3), L (2, 3), M (3, 3), N (4, 3) (iv) W (2, 6), X (3, 5), Y (5, 3), Z (6, 2)

Solution:



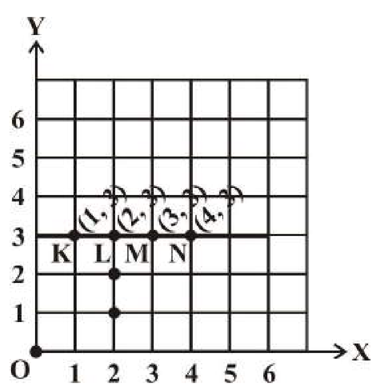
(i)

These lie on a line.
The line is y-axis.

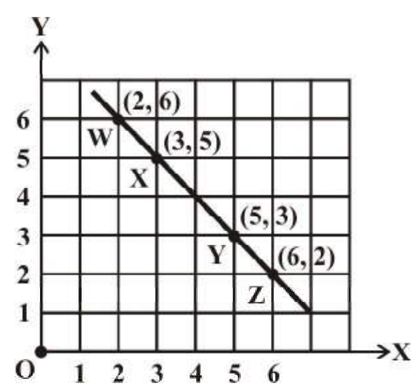


(ii)

These lie on a line. The line is AD.
(You may also use other ways
of naming it). It is parallel to the y-axis



(iii)



(iv)

Fig 15.15

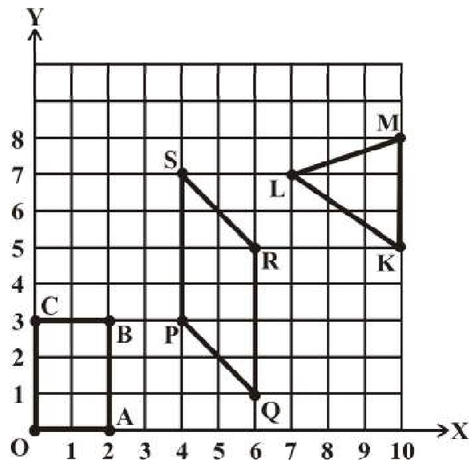
These lie on a line. We can name it as KL or KM or MN etc. It is parallel to x-axis

These lie on a line. We can name it as XY or WY or YZ etc.

Note that in each of the above cases, graph obtained by joining the plotted points is a line. Such graphs are called **linear graphs**.

EXERCISE 15.2

1. Plot the following points on a graph sheet. Verify if they lie on a line
   (a) A(4, 0), B(4, 2), C(4, 6), D(4, 2.5)
   (b) P(1, 1), Q(2, 2), R(3, 3), S(4, 4)
   (c) K(2, 3), L(5, 3), M(5, 5), N(2, 5)
2. Draw the line passing through (2, 3) and (3, 2). Find the coordinates of the points at which this line meets the *x*-axis and *y*-axis.



3. Write the coordinates of the vertices of each of these adjoining figures.
4. State whether True or False. Correct that are false.
   (i) A point whose *x* coordinate is zero and *y*-coordinate is non-zero will lie on the *y*-axis.
   (ii) A point whose *y* coordinate is zero and *x*-coordinate is 5 will lie on *y*-axis.
   (iii) The coordinates of the origin are (0, 0).

15.3 Some Applications

In everyday life, you might have observed that the more you use a facility, the more you pay for it. If more electricity is consumed, the bill is bound to be high. If less electricity is used, then the bill will be easily manageable. This is an instance where one quantity affects another. Amount of electric bill depends on the quantity of electricity used. We say that the quantity of electricity is an **independent variable** (or sometimes **control variable**) and the amount of electric bill is **the dependent variable**. The relation between such variables can be shown through a graph.

THINK, DISCUSS AND WRITE

The number of litres of petrol you buy to fill a car's petrol tank will decide the amount you have to pay. Which is the independent variable here? Think about it.

Example 6: **(Quantity and Cost)**

The following table gives the quantity of petrol and its cost.

No. of Litres of petrol	10	15	20	25
Cost of petrol in ₹	500	750	1000	1250

Plot a graph to show the data.

Solution: (i) Let us take a suitable scale on both the axes (Fig 15.16).

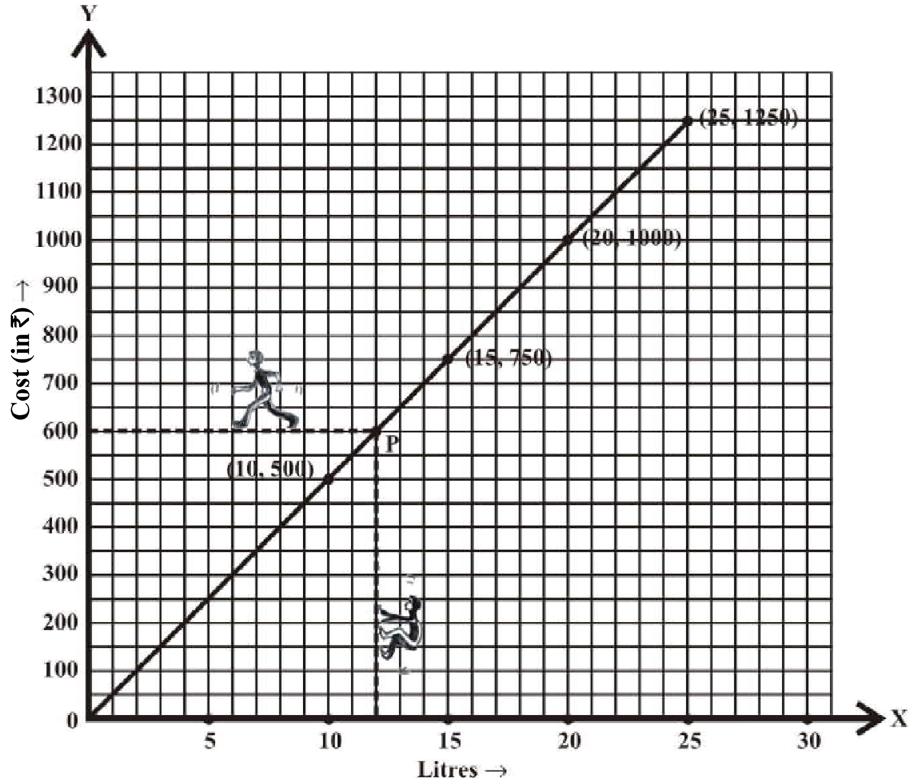


Fig 15.16

(ii) Mark number of litres along the horizontal axis.

(iii) Mark cost of petrol along the vertical axis.

(iv) Plot the points: (10,500), (15,750), (20,1000), (25,1250).

(v) Join the points.

We find that the graph is a line. (It is a linear graph). Why does this graph pass through the origin? Think about it.

This graph can help us to estimate a few things. Suppose we want to find the amount needed to buy 12 litres of petrol. Locate 12 on the horizontal axis.

Follow the vertical line through 12 till you meet the graph at P (say).

From P you take a horizontal line to meet the vertical axis. This meeting point provides the answer.

This is the graph of a situation in which two quantities, are in direct variation. (How ?). In such situations, the graphs will always be linear.

TRY THESE

In the above example, use the graph to find how much petrol can be purchased for ₹ 800.

Example 7: (Principal and Simple Interest)

A bank gives 10% Simple Interest (S.I.) on deposits by senior citizens. Draw a graph to illustrate the relation between the sum deposited and simple interest earned. Find from your graph

(a) the annual interest obtainable for an investment of ₹ 250.

(b) the investment one has to make to get an annual simple interest of ₹ 70.

Solution:

Sum deposited	Simple interest for a year
₹ 100	₹ $\frac{100 \times 1 \times 10}{100}$ = ₹ 10
₹ 200	₹ $\frac{200 \times 1 \times 10}{100}$ = ₹ 20
₹ 300	₹ $\frac{300 \times 1 \times 10}{100}$ = ₹ 30
₹ 500	₹ $\frac{500 \times 1 \times 10}{100}$ = ₹ 50
₹ 1000	₹ 100

Steps to follow:

1. Find the quantities to be plotted as Deposit and SI.
2. Decide the quantities to be taken on x-axis and on y-axis.
3. Choose a scale.
4. Plot points.
5. Join the points.

We get a table of values.

Deposit (in ₹)	100	200	300	500	1000
Annual S.I. (in ₹)	10	20	30	50	100

(i) Scale : 1 unit = ₹ 100 on horizontal axis; 1 unit = ₹ 10 on vertical axis.

(ii) Mark Deposits along horizontal axis.

(iii) Mark Simple Interest along vertical axis.

(iv) Plot the points : (100,10), (200, 20), (300, 30), (500,50) etc.

(v) Join the points. We get a graph that is a line (Fig 15.17).

(a) Corresponding to ₹ 250 on horizontal axis, we get the interest to be ₹ 25 on vertical axis.

(b) Corresponding to ₹ 70 on the vertical axis, we get the sum to be ₹ 700 on the horizontal axis.

TRY THESE

Is Example 7, a case of direct variation?

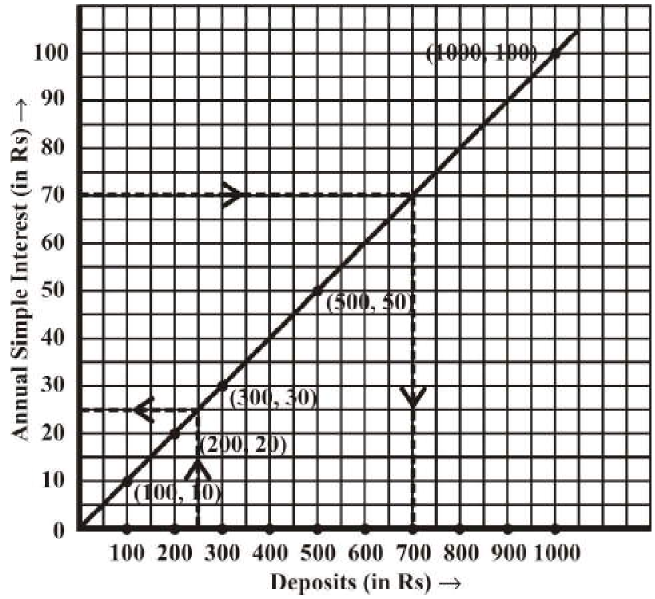


Fig 15.17

Example 8: **(Time and Distance)**

Ajit can ride a scooter constantly at a speed of 30 kms/hour. Draw a time-distance graph for this situation. Use it to find

(i) the time taken by Ajit to ride 75 km. (ii) the distance covered by Ajit in $3\frac{1}{2}$ hours.

Solution:

Hours of ride	Distance covered
1 hour	30 km
2 hours	2 × 30 km = 60 km
3 hours	3 × 30 km = 90 km
4 hours	4 × 30 km = 120 km and so on.

We get a table of values.

Time (in hours)	1	2	3	4
Distance covered (in km)	30	60	90	120

(i) Scale: (Fig 15.18)

Horizontal: 2 units = 1 hour

Vertical: 1 unit = 10 km

(ii) Mark time on horizontal axis.

(iii) Mark distance on vertical axis.

(iv) Plot the points: (1, 30), (2, 60), (3, 90), (4, 120).

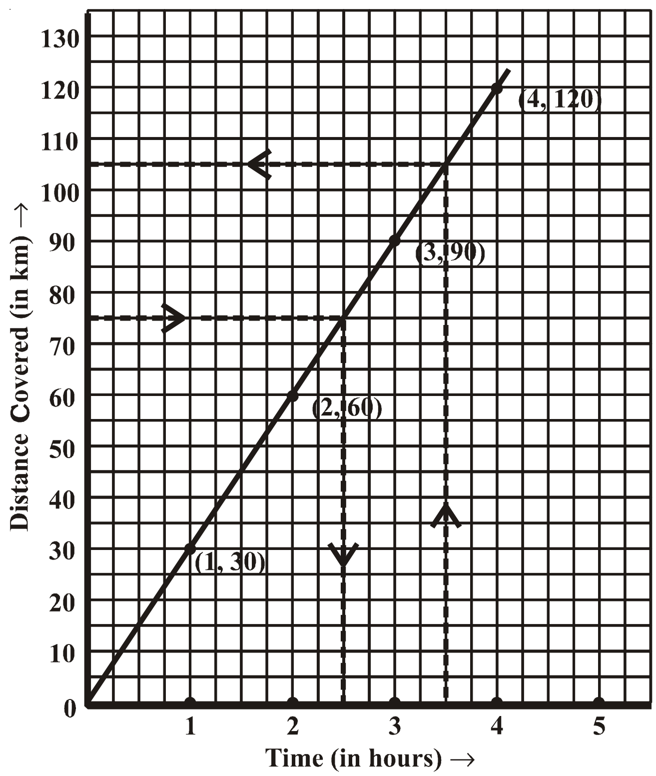


Fig 15.18

(v) Join the points. We get a linear graph.

(a) Corresponding to 75 km on the vertical axis, we get the time to be 2.5 hours on the horizontal axis. Thus 2.5 hours are needed to cover 75 km.

(b) Corresponding to $3\frac{1}{2}$ hours on the horizontal axis, the distance covered is 105 km on the vertical axis.

EXERCISE 15.3

1. Draw the graphs for the following tables of values, with suitable scales on the axes.

(a) Cost of apples

Number of apples	1	2	3	4	5
Cost (in ₹)	5	10	15	20	25

(b) Distance travelled by a car

Time (in hours)	6 a.m.	7 a.m.	8 a.m.	9 a.m.
Distances (in km)	40	80	120	160

(i) How much distance did the car cover during the period 7.30 a.m. to 8 a.m?

(ii) What was the time when the car had covered a distance of 100 km since it's start?

(c) Interest on deposits for a year.

Deposit (in ₹)	1000	2000	3000	4000	5000
Simple Interest (in ₹)	80	160	240	320	400

(i) Does the graph pass through the origin?

(ii) Use the graph to find the interest on ₹ 2500 for a year.

(iii) To get an interest of ₹ 280 per year, how much money should be deposited?

2. Draw a graph for the following.

(i)

Side of square (in cm)	2	3	3.5	5	6
Perimeter (in cm)	8	12	14	20	24

Is it a linear graph?

(ii)

Side of square (in cm)	2	3	4	5	6
Area (in cm^2)	4	9	16	25	36

Is it a linear graph?

WHAT HAVE WE DISCUSSED?

1. Graphical presentation of data is easier to understand.

2. (i) A **bar graph** is used to show comparison among categories.

(ii) A **pie graph** is used to compare parts of a whole.

(iii) A **Histogram** is a bar graph that shows data in intervals.

3. A **line graph** displays data that changes continuously over periods of time.

4. A line graph which is a whole unbroken line is called a **linear graph**.

5. For fixing a point on the graph sheet we need, ***x*-coordinate** and ***y*-coordinate**.

6. The relation between **dependent variable** and **independent variable** is shown through a graph.

CHAPTER 16

Playing with Numbers

16.1 Introduction

You have studied various types of numbers such as natural numbers, whole numbers, integers and rational numbers. You have also studied a number of interesting properties about them. In Class VI, we explored finding factors and multiples and the relationships among them.

In this chapter, we will explore numbers in more detail. These ideas help in justifying tests of divisibility.

16.2 Numbers in General Form

Let us take the number 52 and write it as

$$52 = 50 + 2 = 10 \times 5 + 2$$

Similarly, the number 37 can be written as

$$37 = 10 \times 3 + 7$$

In general, any two digit number *ab* made of digits *a* and *b* can be written as

$$ab = 10 \times a + b = 10a + b$$

What about *ba*? $ba = 10 \times b + a = 10b + a$

Here *ab* does not mean $a \times b$!

Let us now take number 351. This is a three digit number. It can also be written as

$$351 = 300 + 50 + 1 = 100 \times 3 + 10 \times 5 + 1 \times 1$$

Similarly $497 = 100 \times 4 + 10 \times 9 + 1 \times 7$

In general, a 3-digit number *abc* made up of digits *a*, *b* and *c* is written as

$$abc = 100 \times a + 10 \times b + 1 \times c$$
$$= 100a + 10b + c$$

In the same way,

$$cab = 100c + 10a + b$$
$$bca = 100b + 10c + a$$

and so on.

TRY THESE

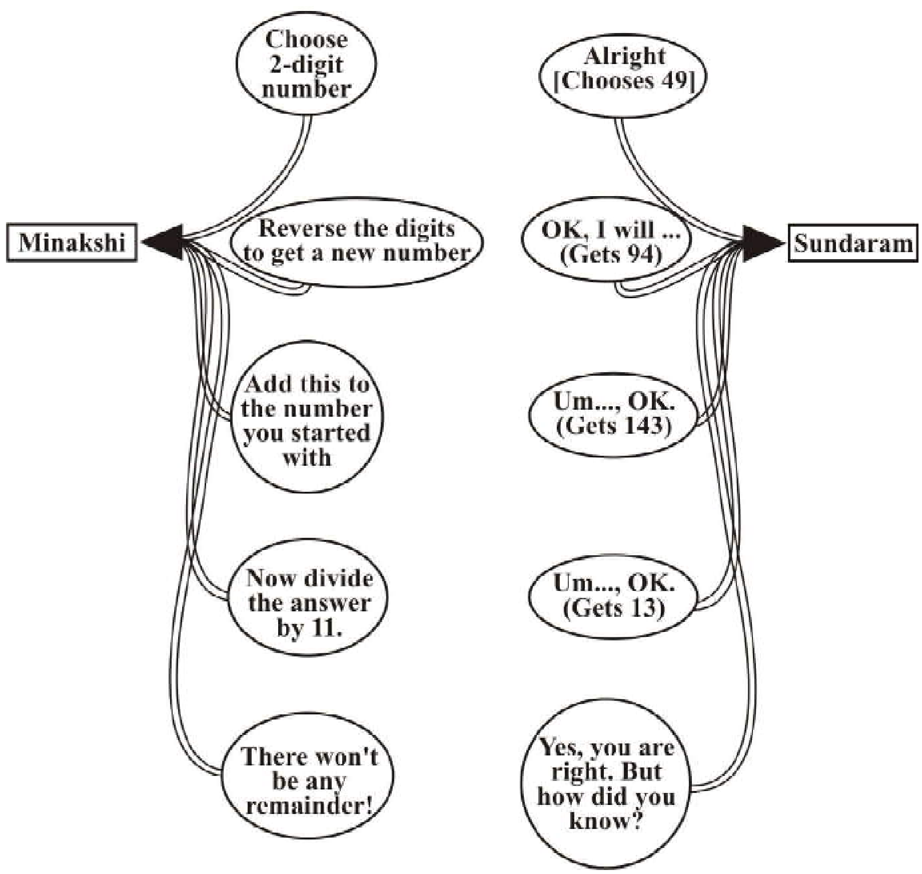
1. Write the following numbers in generalised form.
   (i) 25 (ii) 73 (iii) 129 (iv) 302
2. Write the following in the usual form.
   (i) $10 \times 5 + 6$ (ii) $100 \times 7 + 10 \times 1 + 8$ (iii) $100 \times a + 10 \times c + b$

16.3 Games with Numbers

(i) **Reversing the digits – two digit number**

Minakshi asks Sundaram to think of a 2-digit number, and then to do whatever she asks him to do, to that number. Their conversation is shown in the following figure. **Study the figure carefully before reading on**.

Conversations between Minakshi and Sundaram: First Round ...



It so happens that Sundaram chose the number 49. So, he got the reversed number 94; then he added these two numbers and got $49 + 94 = 143$. Finally he divided this number by 11 and got $143 \div 11 = 13$, with no remainder. This is just what Minakshi had predicted.

TRY THESE

Check what the result would have been if Sundaram had chosen the numbers shown below.

1. 27 2. 39 3. 64 4. 17

Now, let us see if we can **explain** Minakshi's "trick".

Suppose Sundaram chooses the number *ab*, which is a short form for the 2-digit number $10a + b$. On reversing the digits, he gets the number $ba = 10b + a$. When he adds the two numbers he gets:

$$(10a + b) + (10b + a) = 11a + 11b$$
$$= 11\ (a + b).$$

So, the sum is always a multiple of 11, just as Minakshi had claimed.

Observe here that if we divide the sum by 11, the quotient is $a + b$, which is exactly the sum of the digits of chosen number *ab*.

You may check the same by taking any other two digit number.

The game between Minakshi and Sundaram continues!

Minakshi: Think of another 2-digit number, but don't tell me what it is.

Sundaram: Alright.

Minakshi: Now reverse the digits of the number, and *subtract* the smaller number from the larger one.

Sundaram: I have done the subtraction. What next?

Minakshi: Now divide your answer by 9. I claim that there will be no remainder!

Sundaram: Yes, you are right. There is indeed no remainder! But this time I think I know how you are so sure of this!

In fact, Sundaram had thought of 29. So his calculations were: first he got the number 92; then he got $92 – 29 = 63$; and finally he did $(63 \div 9)$ and got 7 as quotient, with no remainder.

TRY THESE

Check what the result would have been if Sundaram had chosen the numbers shown below.

1. 17 2. 21 3. 96 4. 37

Let us see how Sundaram explains Minakshi's second "trick". (Now he feels confident of doing so!)

Suppose he chooses the 2-digit number $ab = 10a + b$. After reversing the digits, he gets the number $ba = 10b + a$. Now Minakshi tells him to do a subtraction, the smaller number from the larger one.

- If the tens digit is larger than the ones digit (that is, $a > b$), he does:

$$(10a + b) – (10b + a) = 10a + b – 10b – a$$
$$= 9a – 9b = 9(a – b).$$

- If the ones digit is larger than the tens digit (that is, $b > a$), he does:

$$(10b + a) - (10a + b) = 9(b - a).$$

- And, of course, if $a = b$, he gets 0.

In each case, the resulting number is divisible by 9. So, the remainder is 0. Observe here that if we divide the resulting number (obtained by subtraction), the quotient is $a - b$ or $b - a$ according as $a > b$ or $a < b$. You may check the same by taking any other two digit numbers.

(ii) **Reversing the digits – three digit number**.

Now it is Sundaram's turn to play some tricks!

Sundaram: Think of a 3-digit number, but don't tell me what it is.

Minakshi: Alright.

Sundaram: Now make a new number by putting the digits in reverse order, and subtract the smaller number from the larger one.

Minakshi: Alright, I have done the subtraction. What next?

Sundaram: Divide your answer by 99. I am sure that there will be no remainder!

In fact, Minakshi chose the 3-digit number 349. So she got:

- Reversed number: 943;
- Difference: $943 - 349 = 594$;
- Division: $594 \div 99 = 6$, with no remainder.

TRY THESE

Check what the result would have been if Minakshi had chosen the numbers shown below. In each case keep a record of the quotient obtained at the end.

1. 132
2. 469
3. 737
4. 901

Let us see how this trick works.

Let the 3-digit number chosen by Minakshi be $abc = 100a + 10b + c$.

After reversing the order of the digits, she gets the number $cba = 100c + 10b + a$. On subtraction:

- If $a > c$, then the difference between the numbers is

$$(100a + 10b + c) - (100c + 10b + a) = 100a + 10b + c - 100c - 10b - a$$
$$= 99a - 99c = 99(a - c).$$

- If $c > a$, then the difference between the numbers is

$$(100c + 10b + a) - (100a + 10b + c) = 99c - 99a = 99(c - a).$$

- And, of course, if $a = c$, the difference is 0.

In each case, the resulting number is divisible by 99. So the remainder is 0. Observe that quotient is $a - c$ or $c - a$. You may check the same by taking other 3-digit numbers.

(iii) **Forming three-digit numbers with given three-digits**.

Now it is Minakshi's turn once more.

Minakshi: Think of any 3-digit number.

Sundaram: Alright, I have done so.

Minakshi: Now use this number to form two more 3-digit numbers, like this: if the number you chose is *abc*, then

- 'the first number is *cab* (i.e., with the ones digit shifted to the "left end" of the number);
- the other number is *bca* (i.e., with the hundreds digit shifted to the "right end" of the number).

Now add them up. Divide the resulting number by 37. I claim that there will be no remainder.

Sundaram: Yes. You are right!

In fact, Sundaram had thought of the 3-digit number 237. After doing what Minakshi had asked, he got the numbers 723 and 372. So he did:

$$\begin{array}{r} 2\ 3\ 7 \\ +\ \ 7\ 2\ 3 \\ +\ \ 3\ 7\ 2 \\ \hline 1\ 3\ 3\ 2 \\ \hline \end{array}$$

Form all possible 3-digit numbers using all the digits 2, 3 and 7 and find their sum. Check whether the sum is divisible by 37! Is it true for the sum of all the numbers formed by the digits *a*, *b* and *c* of the number *abc*?

Then he divided the resulting number 1332 by 37:

$$1332 \div 37 = 36, \quad \text{with no remainder.}$$

TRY THESE

Check what the result would have been if Sundaram had chosen the numbers shown below.

1. 417   2. 632   3. 117   4. 937

Will this trick always work?

Let us see.

$$abc = 100a + 10b + c$$

$$cab = 100c + 10a + b$$

$$bca = 100b + 10c + a$$

$$abc + cab + bca = 111(a + b + c)$$

$$= 37 \times 3(a + b + c), \text{ which is divisible by 37}$$

16.4 Letters for Digits

Here we have puzzles in which letters take the place of digits in an arithmetic 'sum', and the problem is to find out which letter represents which digit; so it is like cracking a code. Here we stick to problems of addition and multiplication.

Here are two rules we follow while doing such puzzles.

1. *Each letter in the puzzle must stand for just one digit. Each digit must be represented by just one letter.*
2. *The first digit of a number cannot be zero.* Thus, we write the number "sixty three" as 63, and not as 063, or 0063.

A rule that we would *like* to follow is that the puzzle must have just one answer.

Example 1: Find Q in the addition.

$$\begin{array}{r} 3\ 1\ Q \\ +\ 1\ Q\ 3 \\ \hline 5\ 0\ 1 \\ \hline \end{array}$$

Solution:

There is just one letter Q whose value we have to find.

Study the addition in the ones column: from Q + 3, we get '1', that is, a number whose ones digit is 1.

For this to happen, the digit Q should be 8. So the puzzle can be solved as shown below.

$$\begin{array}{r} 3\ 1\ 8 \\ +\ 1\ 8\ 3 \\ \hline 5\ 0\ 1 \\ \hline \end{array}$$

That is, Q = 8

Example 2: Find A and B in the addition.

$$\begin{array}{rr} & A \\ + & A \\ + & A \\ \hline B & A \\ \hline \end{array}$$

Solution: This has *two* letters A and B whose values are to be found.

Study the addition in the ones column: the sum of *three* A's is a number whose ones digit is A. Therefore, the sum of *two* A's must be a number whose ones digit is 0.
This happens only for A = 0 and A = 5.

If A = 0, then the sum is 0 + 0 + 0 = 0, which makes B = 0 too. We do not want this (as it makes A = B, and then the tens digit of BA too becomes 0), so we reject this possibility. So, A = 5.
Therefore, the puzzle is solved as shown below.

$$\begin{array}{rr} & 5 \\ + & 5 \\ + & 5 \\ \hline 1 & 5 \\ \hline \end{array}$$

That is, A = 5 and B = 1.

Example 3: Find the digits A and B.

$$\begin{array}{r} \text{B A} \\ \times \quad \text{B 3} \\ \hline \text{5 7 A} \\ \hline \end{array}$$

Solution:

This also has two letters A and B whose values are to be found.
Since the ones digit of 3 × A is A, it must be that A = 0 or A = 5.

Now look at B. If B = 1, then BA × B3 would *at most* be equal to 19 × 19; that is, it would at most be equal to 361. But the product here is 57A, which is more than 500. So we cannot have B = 1.

If B = 3, then BA × B3 would be more than 30 × 30; that is, more than 900. But 57A is less than 600. So, B can not be equal to 3.

Putting these two facts together, we see that B = 2 only. So the multiplication is either 20 × 23, or 25 × 23.

The first possibility fails, since 20 × 23 = 460. But, the second one works out correctly, since 25 × 23 = 575.

$$\begin{array}{r} 2\;\;5 \\ \times \quad 2\;\;3 \\ \hline 5\;\;7\;\;5 \\ \hline \end{array}$$

So the answer is A = 5, B = 2.

DO THIS

Write a 2-digit number *ab* and the number obtained by reversing its digits i.e., *ba*. Find their sum. Let the sum be a 3-digit number *dad*

$$\text{i.e.,} \qquad ab + ba = dad$$
$$(10a + b) + (10b + a) = dad$$
$$11(a + b) = dad$$

The sum $a + b$ can not exceed 18 (Why?).

Is *dad* a multiple of 11?

Is *dad* less than 198?

Write all the 3-digit numbers which are multiples of 11 upto 198.

Find the values of *a* and *d*.

EXERCISE 16.1

Find the values of the letters in each of the following and give reasons for the steps involved.

1.
$$\begin{array}{r} 3\;\;\text{A} \\ +\;\;2\;\;5 \\ \hline \text{B}\;\;2 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 4\;\;\text{A} \\ +\;\;9\;\;8 \\ \hline \text{C}\;\;\text{B}\;\;3 \\ \hline \end{array}$$

3.
$$\begin{array}{r} 1\;\;\text{A} \\ \times\;\;\text{A} \\ \hline 9\;\;\text{A} \\ \hline \end{array}$$

4.
$$\begin{array}{r} A\ B \\ +\ 3\ 7 \\ \hline 6\ A \\ \hline \end{array}$$

5.
$$\begin{array}{r} A\ B \\ \times\ \ 3 \\ \hline C\ A\ B \\ \hline \end{array}$$

6.
$$\begin{array}{r} A\ B \\ \times\ \ 5 \\ \hline C\ A\ B \\ \hline \end{array}$$

7.
$$\begin{array}{r} A\ B \\ \times\ \ 6 \\ \hline B\ B\ B \\ \hline \end{array}$$

8.
$$\begin{array}{r} A\ 1 \\ +\ 1\ B \\ \hline B\ 0 \\ \hline \end{array}$$

9.
$$\begin{array}{r} 2\ A\ B \\ +\ A\ B\ 1 \\ \hline B\ 1\ 8 \\ \hline \end{array}$$

10.
$$\begin{array}{r} 1\ 2\ A \\ +\ 6\ A\ B \\ \hline A\ 0\ 9 \\ \hline \end{array}$$

16.5 Tests of Divisibility

In Class VI, you learnt how to check divisibility by the following divisors.

10, 5, 2, 3, 6, 4, 8, 9, 11.

You would have found the tests easy to do, but you may have wondered at the same time *why* they work. Now, in this chapter, we shall go into the "why" aspect of the above.

16.5.1 Divisibility by 10

This is certainly the easiest test of all! We first look at some multiples of 10.

10, 20, 30, 40, 50, 60, ... ,

and then at some non-multiples of 10.

13, 27, 32, 48, 55, 69,

From these lists we see that if the ones digit of a number is 0, then the number is a multiple of 10; and if the ones digit is *not* 0, then the number is *not* a multiple of 10. So, we get a test of divisibility by 10.

Of course, we must not stop with just stating the test; we must also explain *why* it "works". That is not hard to do; we only need to remember the rules of place value.

Take the number. ... cba; this is a short form for

$$\ldots + 100c + 10b + a$$

Here a is the one's digit, b is the ten's digit, c is the hundred's digit, and so on. The dots are there to say that there may be more digits to the left of c.

Since 10, 100, ... are divisible by 10, so are $10b$, $100c$, ... And as for the number a is concerned, it must be a divisible by 10 if the given number is divisible by 10. This is possible only when $a = 0$.

Hence, a number is divisible by 10 when its one's digit is 0.

16.5.2 Divisibility by 5

Look at the multiples of 5.

5, 10, 15, 20, 25, 30, 35, 40, 45, 50,

We see that *the one's digits are alternately* 5 *and* 0, *and no other digit ever appears in this list.*

So, we get our test of divisibility by 5.

If the ones digit of a number is 0 *or* 5, *then it* is *divisible by* 5.

Let us explain this rule. Any number ... *cba* can be written as:

... + 100c + 10b + a

Since 10, 100 are divisible by 10 so are 10b, 100c, ... which in turn, are divisible by 5 because 10 = 2 × 5. As far as number a is concerned it must be divisible by 5 if the number is divisible by 5. So a has to be either 0 or 5.

TRY THESE

(The first one has been done for you.)

1. If the division N ÷ 5 leaves a remainder of 3, what might be the ones digit of N? (The one's digit, when divided by 5, must leave a remainder of 3. So the one's digit must be either 3 or 8.)
2. If the division N ÷ 5 leaves a remainder of 1, what might be the one's digit of N?
3. If the division N ÷ 5 leaves a remainder of 4, what might be the one's digit of N?

16.5.3 Divisibility by 2

Here are the even numbers.

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, ... ,

and here are the odd numbers.

1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, ... ,

We see that a natural number is even if its one's digit is

2, 4, 6, 8 or 0

A number is odd if its one's digit is

1, 3, 5, 7 or 9

Recall the test of divisibility by 2 learnt in Class VI, which is as follows.

If the one's digit of a number is 0, 2, 4, 6 or 8 then the number is divisible by 2.

The explanation for this is as follows.

Any number *cba* can be written as 100c + 10b + a

First two terms namely 100c, 10b are divisible by 2 because 100 and 10 are divisible by 2. So far as a is concerned, it must be divisible by 2 if the given number is divisible by 2. This is possible only when a = 0, 2, 4, 6 or 8.

TRY THESE

(The first one has been done for you.)

1. If the division N ÷ 2 leaves a remainder of 1, what might be the one's digit of N? (N is odd; so its one's digit is odd. Therefore, the one's digit must be 1, 3, 5, 7 or 9.)

2. If the division N ÷ 2 leaves no remainder (i.e., zero remainder), what might be the one's digit of N?
3. Suppose that the division N ÷ 5 leaves a remainder of 4, and the division N ÷ 2 leaves a remainder of 1. What must be the one's digit of N?

16.5.4 Divisibility by 9 and 3

Look carefully at the three tests of divisibility found till now, for checking division by 10, 5 and 2. We see something common to them: *they use only the one's digit of the given number; they do not bother about the 'rest' of the digits*. Thus, *divisibility is decided just by the one's digit.* 10, 5, 2 are divisors of 10, which is the key number in our place value.

But for checking divisibility by 9, this will not work. Let us take some number say 3573.

Its expanded form is: $3 \times 1000 + 5 \times 100 + 7 \times 10 + 3$

This is equal to $3 \times (999 + 1) + 5 \times (99 + 1) + 7 \times (9 + 1) + 3$

$$= 3 \times 999 + 5 \times 99 + 7 \times 9 + (3 + 5 + 7 + 3) \quad \ldots (1)$$

We see that the number 3573 will be divisible by 9 or 3 if $(3 + 5 + 7 + 3)$ is divisible by 9 or 3.

We see that $3 + 5 + 7 + 3 = 18$ is divisible by 9 and also by 3. Therefore, the number 3573 is divisible by both 9 and 3.

Now, let us consider the number 3576. As above, we get

$$3576 = 3 \times 999 + 5 \times 99 + 7 \times 9 + (3 + 5 + 7 + 6) \quad \ldots (2)$$

Since $(3 + 5 + 7 + 6)$ i.e., 21 is not divisible by 9 but is divisible by 3,

therefore 3576 is not divisible by 9. However 3576 is divisible by 3. Hence,

(i) A number N is divisible by 9 if the sum of its digits is divisible by 9. Otherwise it is not divisible by 9.

(ii) A number N is divisible by 3 if the sum of its digits is divisible by 3. Otherwise it is not divisible by 3.

If the number is '*cba*', then, $100c + 10b + a = 99c + 9b + (a + b + c)$

$$= \underbrace{9(11c + b)}_{\text{divisible by 3 and 9}} + (a + b + c)$$

Hence, divisibility by 9 (or 3) is possible if $a + b + c$ is divisible by 9 (or 3).

Example 4: Check the divisibility of 21436587 by 9.

Solution: The sum of the digits of 21436587 is $2 + 1 + 4 + 3 + 6 + 5 + 8 + 7 = 36$. This number is divisible by 9 (for $36 \div 9 = 4$). We conclude that 21436587 is divisible by 9.

We can double-check:

$$\frac{21436587}{9} = 2381843 \qquad \text{(the division is exact).}$$

Example 5: Check the divisibility of 152875 by 9.

Solution: The sum of the digits of 152875 is 1 + 5 + 2 + 8 + 7 + 5 = 28. This number is **not** divisible by 9. We conclude that 152875 is not divisible by 9.

TRY THESE

Check the divisibility of the following numbers by 9.

1. 108 2. 616 3. 294 4. 432 5. 927

Example 6: If the three digit number 24x is divisible by 9, what is the value of x?

Solution: Since 24x is divisible by 9, sum of it's digits, i.e., 2 + 4 + x should be divisible by 9, i.e., 6 + x should be divisible by 9.

This is possible when 6 + x = 9 or 18, ...
But, since x is a digit, therefore, 6 + x = 9, i.e., x = 3.

THINK, DISCUSS AND WRITE

1. You have seen that a number 450 is divisible by 10. It is also divisible by 2 and 5 which are factors of 10. Similarly, a number 135 is divisible 9. It is also divisible by 3 which is a factor of 9.
   Can you say that if a number is divisible by any number m, then it will also be divisible by each of the factors of m?
2. (i) Write a 3-digit number abc as $100a + 10b + c$

$$= 99a + 11b + (a - b + c)$$
$$= 11(9a + b) + (a - b + c)$$

   If the number abc is divisible by 11, then what can you say about $(a - b + c)$?
   Is it necessary that $(a + c - b)$ should be divisible by 11?

   (ii) Write a 4-digit number $abcd$ as $1000a + 100b + 10c + d$

$$= (1001a + 99b + 11c) - (a - b + c - d)$$
$$= 11(91a + 9b + c) + [(b + d) - (a + c)]$$

   If the number $abcd$ is divisible by 11, then what can you say about $[(b + d) - (a + c)]$?

   (iii) From (i) and (ii) above, can you say that a number will be divisible by 11 if the difference between the sum of digits at its odd places and that of digits at the even places is divisible by 11?

Example 7: Check the divisibility of 2146587 by 3.

Solution: The sum of the digits of 2146587 is 2 + 1 + 4 + 6 + 5 + 8 + 7 = 33. This number is divisible by 3 (for 33 ÷ 3 = 11). We conclude that 2146587 is divisible by 3.

Example 8: Check the divisibility of 15287 by 3.

Solution: The sum of the digits of 15287 is $1 + 5 + 2 + 8 + 7 = 23$. This number is not divisible by 3. We conclude that 15287 too is not divisible by 3.

TRY THESE

Check the divisibility of the following numbers by 3.

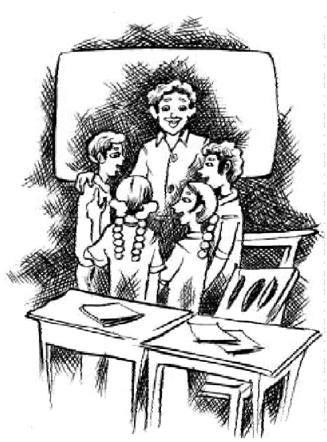
1. 108
2. 616
3. 294
4. 432
5. 927

EXERCISE 16.2

1. If $21y5$ is a multiple of 9, where y is a digit, what is the value of y?
2. If $31z5$ is a multiple of 9, where z is a digit, what is the value of z?
   You will find that there are *two* answers for the last problem. Why is this so?
3. If $24x$ is a multiple of 3, where x is a digit, what is the value of x?
   (Since $24x$ is a multiple of 3, its sum of digits $6 + x$ is a multiple of 3; so $6 + x$ is one of these numbers: 0, 3, 6, 9, 12, 15, 18, ... But since x is a digit, it can only be that $6 + x = 6$ or 9 or 12 or 15. Therefore, $x = 0$ or 3 or 6 or 9. Thus, x can have any of four different values.)
4. If $31z5$ is a multiple of 3, where z is a digit, what might be the values of z?

WHAT HAVE WE DISCUSSED?

1. Numbers can be written in general form. Thus, a two digit number ab will be written as $ab = 10a + b$.
2. The general form of numbers are helpful in solving puzzles or number games.
3. The reasons for the divisibility of numbers by 10, 5, 2, 9 or 3 can be given when numbers are written in general form.



ANSWERS

EXERCISE 1.1

1. (i) 2 (ii) $\frac{-11}{28}$

2. (i) $\frac{-2}{8}$ (ii) $\frac{5}{9}$ (iii) $\frac{-6}{5}$ (iv) $\frac{2}{9}$ (v) $\frac{19}{6}$

4. (i) $\frac{-1}{13}$ (ii) $\frac{-19}{13}$ (iii) 5 (iv) $\frac{56}{15}$ (v) $\frac{5}{2}$ (vi) -1

5. (i) 1 is the multiplicative identity (ii) Commutativity
(iii) Multiplicative inverse

6. $\frac{-96}{91}$ **7.** Associativity **8.** No, because the product is not 1.

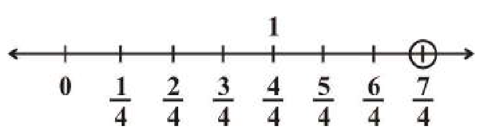
9. Yes, because $0.3 \times 3\frac{1}{3} = \frac{3}{10} \times \frac{10}{3} = 1$

10. (i) 0 (ii) 1 and (-1) (iii) 0

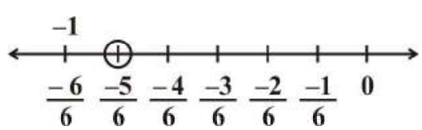
11. (i) No (ii) $1, -1$ (iii) $\frac{-1}{5}$ (iv) x (v) Rational number
(vi) positive

EXERCISE 1.2

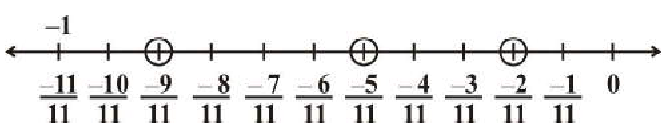
1. (i)



(ii)



2.



3. Some of these are $1, \frac{1}{2}, 0, -1, \frac{-1}{2}$

4. $\frac{-7}{20}, \frac{-6}{20}, \frac{-5}{20}, \frac{-4}{20}, \frac{-3}{20}, \frac{-2}{20}, \frac{-1}{20}, 0, \ldots, \frac{1}{20}, \frac{2}{20}$ (There can be many more such rational numbers)

5. (i) $\frac{41}{60}, \frac{42}{60}, \frac{43}{60}, \frac{44}{60}, \frac{45}{60}$ (ii) $\frac{-8}{6}, \frac{-7}{6}, 0, \frac{1}{6}, \frac{2}{6}$ (iii) $\frac{9}{32}, \frac{10}{32}, \frac{11}{32}, \frac{12}{32}, \frac{13}{32}$
(There can be many more such rational numbers)

6. $-\frac{3}{2}, -1, \frac{-1}{2}, 0, \frac{1}{2}$ (There can be many more such rational numbers)

7. $\frac{97}{160}, \frac{98}{160}, \frac{99}{160}, \frac{100}{160}, \frac{101}{160}, \frac{102}{160}, \frac{103}{160}, \frac{104}{160}, \frac{105}{160}, \frac{106}{160}$

(There can be many more such rational numbers)

EXERCISE 2.1

1. $x = 9$ **2.** $y = 7$ **3.** $z = 4$ **4.** $x = 2$ **5.** $x = 2$ **6.** $t = 50$

7. $x = 27$ **8.** $y = 2.4$ **9.** $x = \frac{25}{7}$ **10.** $y = \frac{3}{2}$ **11.** $p = -\frac{4}{3}$ **12.** $x = -\frac{8}{5}$

EXERCISE 2.2

1. $\frac{3}{4}$ **2.** length = 52 m, breadth = 25 m **3.** $1\frac{2}{5}$ cm **4.** 40 and 55

5. 45, 27 **6.** 16, 17, 18 **7.** 288, 296 and 304 **8.** 7, 8, 9

9. Rahul's age: 20 years; Haroon's age: 28 years **10.** 48 students

11. Baichung's age: 17 years; Baichung's father's age: 46 years;

Baichung's grandfather's age = 72 years **12.** 5 years **13** $-\frac{1}{2}$

14. ₹ 100 → 2000 notes; ₹ 50 → 3000 notes; ₹ 10 → 5000 notes

15. Number of ₹ 1 coins = 80; Number of ₹ 2 coins = 60; Number of ₹ 5 coins = 20

16. 19

EXERCISE 2.3

1. $x = 18$ **2.** $t = -1$ **3.** $x = -2$ **4.** $z = \frac{3}{2}$ **5.** $x = 5$ **6.** $x = 0$

7. $x = 40$ **8.** $x = 10$ **9.** $y = \frac{7}{3}$ **10.** $m = \frac{4}{5}$

EXERCISE 2.4

1. 4 **2.** 7, 35 **3.** 36 **4.** 26 (or 62)

5. Shobo's age: 5 years; Shobo's mother's age: 30 years

6. Length = 275 m; breadth = 100 m **7.** 200 m **8.** 72

9. Grand daughter's age: 6 years; Grandfather's age: 60 years

10. Aman's age: 60 years; Aman's son's age: 20 years

EXERCISE 2.5

1. $x = \frac{27}{10}$ **2.** $n = 36$ **3.** $x = -5$ **4.** $x = 8$ **5.** $t = 2$

6. $m = \frac{7}{5}$ **7.** $t = -2$ **8.** $y = \frac{2}{3}$ **9.** $z = 2$ **10.** $f = 0.6$

EXERCISE 2.6

1. $x = \frac{3}{2}$ **2.** $x = \frac{35}{33}$ **3.** $z = 12$ **4.** $y = -8$ **5.** $y = -\frac{4}{5}$

6. Hari's age = 20 years; Harry's age = 28 years **7.** $\frac{13}{21}$

EXERCISE 3.1

1. (a) 1, 2, 5, 6, 7 (b) 1, 2, 5, 6, 7 (c) 1, 2
(d) 2 (e) 1

2. (a) 2 (b) 9 (c) 0 **3.** 360°; yes.

4. (a) 900° (b) 1080° (c) 1440° (d) $(n-2)180°$

5. A polygon with equal sides and equal angles.
(i) Equilateral triangle (ii) Square (iii) Regular hexagon

6. (a) 60° (b) 140° (c) 140° (d) 108°

7. (a) $x + y + z = 360°$ (b) $x + y + z + w = 360°$

EXERCISE 3.2

1. (a) $360° - 250° = 110°$ (b) $360° - 310° = 50°$

2. (i) $\frac{360°}{9} = 40°$ (ii) $\frac{360°}{15} = 24°$

3. $\frac{360}{24} = 15$ (sides) **4.** Number of sides = 24

5. (i) No; (Since 22 is not a divisor of 360)
(ii) No; (because each exterior angle is $180° - 22° = 158°$, which is not a divisor of 360°).

6. (a) The equilateral triangle being a regular polygon of 3 sides has the least measure of an interior angle = 60°.
(b) By (a), we can see that the greatest exterior angle is 120°.

EXERCISE 3.3

1. (i) BC(Opposite sides are equal) (ii) ∠DAB (Opposite angles are equal)

(iii) OA (Diagonals bisect each other)

(iv) 180° (Interior opposite angles, since $\overline{AB} \parallel \overline{DC}$)

2. (i) $x = 80°; y = 100°; z = 80°$ (ii) $x = 130°; y = 130°; z = 130°$

(iii) $x = 90°;\ y = 60°;\ z = 60°$ (iv) $x = 100°;\ y = 80°;\ z = 80°$

(v) $y = 112°; x = 28°; z = 28°$

3. (i) Can be, but need not be.

(ii) No; (in a parallelogram, opposite sides are equal; but here, AD ≠ BC).

(iii) No; (in a parallelogram, opposite angles are equal; but here, ∠A ≠ ∠C).

4. A kite, for example **5.** 108°; 72°; **6.** Each is a right angle.

7. $x = 110°;\ y = 40°;\ z = 30°$

8. (i) $x = 6;\ y = 9$ (ii) $x = 3;\ y = 13$; **9.** $x = 50°$

10. $\overline{NM} \parallel \overline{KL}$ (sum of interior opposite angles is 180°). So, KLMN is a trapezium.

11. 60° **12.** ∠P = 50°; ∠S = 90°

EXERCISE 3.4

1. (b), (c), (f), (g), (h) are true; others are false.

2. (a) Rhombus; square. (b) Square; rectangle

3. (i) A square is 4 – sided; so it is a quadrilateral.

(ii) A square has its opposite sides parallel; so it is a parallelogram.

(iii) A square is a parallelogram with all the 4 sides equal; so it is a rhombus.

(iv) A square is a parallelogram with each angle a right angle; so it is a rectangle.

4. (i) Parallelogram; rhombus; square; rectangle.

(ii) Rhombus; square (iii) Square; rectangle

5. Both of its diagonals lie in its interior.

6. $\overline{AD} \parallel \overline{BC}$; $\overline{AB} \parallel \overline{DC}$. So, in parallelogram ABCD, the mid-point of diagonal $\overline{AC}$ is O.

EXERCISE 5.1

1. (b), (d). In all these cases data can be divided into class intervals.

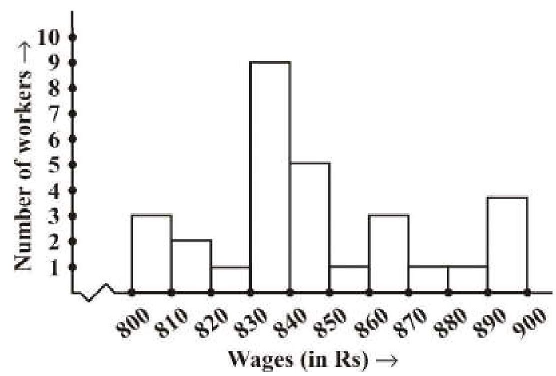
2.

Shopper	Tally marks	Number
W	𝍸 𝍸 𝍸 𝍸 𝍸 |||	28
M	𝍸 𝍸 𝍸	15
B	𝍸	5
G	𝍸 𝍸 ||	12

3.

Interval	Tally marks	Frequency
800 - 810	|||	3
810 - 820	||	2
820 - 830	|	1
830 - 840	𝍸 ||||	9
840 - 850	𝍸	5
850 - 860	|	1
860 - 870	|||	3
870 - 880	|	1
880 - 890	|	1
890 - 900	||||	4
	Total	**30**

4. (i) 830 - 840 (ii) 10
(iii) 20

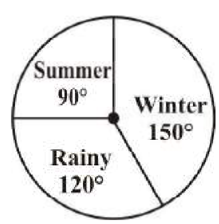


5. (i) 4 - 5 hours (ii) 34
(iii) 14

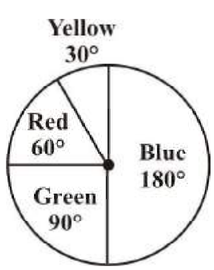
EXERCISE 5.2

1. (i) 200 (ii) Light music (iii) Classical - 100, Semi classical - 200, Light - 400, Folk - 300

2. (i) Winter (ii) Winter - 150°, Rainy - 120°, Summer - 90° (iii)

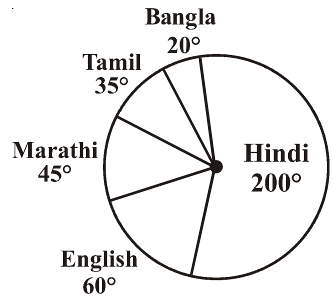


3.



4. (i) Hindi (ii) 30 marks (iii) Yes

5.



EXERCISE 5.3

1. (a) Outcomes → A, B, C, D

(b) HT, HH, TH, TT (Here HT means Head on first coin and Tail on the second coin and so on).

2. Outcomes of an event of getting

(i) (a) 2, 3, 5 (b) 1, 4, 6

(ii) (a) 6 (b) 1, 2, 3, 4, 5

3. (a) $\frac{1}{5}$ (b) $\frac{1}{13}$ (c) $\frac{4}{7}$

4. (i) $\frac{1}{10}$ (ii) $\frac{1}{2}$ (iii) $\frac{2}{5}$ (iv) $\frac{9}{10}$

5. Probability of getting a green sector $= \frac{3}{5}$; probability of getting a non-blue sector $= \frac{4}{5}$

6. Probability of getting a prime number $= \frac{1}{2}$; probability of getting a number which is not prime $= \frac{1}{2}$

Probability of getting a number greater than 5 $= \frac{1}{6}$

Probability of getting a number not greater than 5 $= \frac{5}{6}$

EXERCISE 6.1

1. (i) 1 (ii) 4 (iii) 1 (iv) 9 (v) 6 (vi) 9

(vii) 4 (viii) 0 (ix) 6 (x) 5

2. These numbers end with

(i) 7 (ii) 3 (iii) 8 (iv) 2 (v) 0 (vi) 2

(vii) 0 (viii) 0

3. (i), (iii)

4. 10000200001, 100000020000001

5. 1020304030201, 101010101^2

6. 20, 6, 42, 43

7. (i) 25 (ii) 100 (iii) 144

8. (i) 1 + 3 + 5 + 7 + 9 + 11 + 13

(ii) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21

9. (i) 24 (ii) 50 (iii) 198

EXERCISE 6.2

1.	(i)	1024	(ii)	1225	(iii)	7396	(iv)	8649	(v)	5041	(vi)	2116
2.	(i)	6,8,10	(ii)	14,48,50	(iii)	16,63,65	(iv)	18,80,82				

EXERCISE 6.3

1. (i) 1, 9 (ii) 4, 6 (iii) 1, 9 (iv) 5

2. (i), (ii), (iii) **3.** 10, 13

4.	(i)	27	(ii)	20	(iii)	42	(iv)	64	(v)	88	(vi)	98
	(vii)	77	(viii)	96	(ix)	23	(x)	90				
5.	(i)	7; 42	(ii)	5; 30	(iii)	7, 84	(iv)	3; 78	(v)	2; 54	(vi)	3; 48
6.	(i)	7; 6	(ii)	13; 15	(iii)	11; 6	(vi)	5; 23	(v)	7; 20	(vi)	5; 18

7. 49 **8.** 45 rows; 45 plants in each row **9.** 900 **10.** 3600

EXERCISE 6.4

1.	(i)	48	(ii)	67	(iii)	59	(iv)	23	(v)	57	(vi)	37
	(vii)	76	(viii)	89	(ix)	24	(x)	32	(xi)	56	(xii)	30
2.	(i)	1	(ii)	2	(iii)	2	(iv)	3	(v)	3		
3.	(i)	1.6	(ii)	2.7	(iii)	7.2	(iv)	6.5	(v)	5.6		
4.	(i)	2; 20	(ii)	53; 44	(iii)	1; 57	(iv)	41; 28	(v)	31; 63		
5.	(i)	4; 23	(ii)	14; 42	(iii)	4; 16	(iv)	24; 43	(v)	149; 81		

6. 21 m **7.** (a) 10 cm (b) 12 cm

8. 24 plants **9.** 16 children

EXERCISE 7.1

1. (ii) and (iv)

2.	(i)	3	(ii)	2	(iii)	3	(iv)	5	(v)	10
3.	(i)	3	(ii)	2	(iii)	5	(iv)	3	(v)	11

4. 20 cuboids

EXERCISE 7.2

1.	(i)	4	(ii)	8	(iii)	22	(iv)	30	(v)	25	(vi)	24
	(vii)	48	(viii)	36	(ix)	56						
2.	(i)	False	(ii)	True	(iii)	False	(iv)	False	(v)	False	(vi)	False
	(vii)	True										

3. 11, 17, 23, 32

EXERCISE 8.1

1. (a) 1 : 2 (b) 1 : 2000 (c) 1 : 10
2. (a) 75% (b) $66\frac{2}{3}$% 3. 28% students 4. 25 matches 5. ₹ 2400
6. 10%, cricket → 30 lakh; football → 15 lakh; other games → 5 lakh

EXERCISE 8.2

1. ₹ 1,40,000 2. 80% 3. ₹ 34.80 4. ₹ 18,342.50
5. Gain of 2% 6. ₹ 2,835 7. Loss of ₹ 1,269.84
8. ₹ 14,560 9. ₹ 2,000 10. ₹ 5,000

EXERCISE 8.3

1. (a) Amount = ₹ 15,377.34; Compound interest = ₹ 4,577.34
   (b) Amount = ₹ 22,869; Interest = ₹ 4869 (c) Amount = ₹ 70,304, Interest = ₹ 7,804
   (d) Amount = ₹ 8,736.20, Interest = ₹ 736.20
   (e) Amount = ₹ 10,816, Interest = ₹ 816
2. ₹ 36,659.70 3. Fabina pays ₹ 362.50 more 4. ₹ 43.20
5. (ii) ₹ 63,600 (ii) ₹ 67,416 6. (ii) ₹ 92,400 (ii) ₹ 92,610
7. (i) ₹ 8,820 (ii) ₹ 441
8. Amount = ₹ 11,576.25, Interest = ₹ 1,576.25 Yes.
9. ₹ 4,913 10. (i) About 48,980 (ii) 59,535 11. 5,31,616 (approx)
12. ₹ 38,640

EXERCISE 9.1

1.

	Term	Coefficient
(i)	$5xyz^2$ $-3zy$	5 –3
(ii)	1 x x^2	1 1 1
(iii)	$4x^2y^2$ $-4x^2y^2z^2$ z^2	4 – 4 1
(iv)	3 $-pq$ qr $-rp$	3 –1 1 –1
(v)	$\frac{x}{2}$ $\frac{y}{2}$ $-xy$	$\frac{1}{2}$ $\frac{1}{2}$ –1
(vi)	$0.3a$ $-0.6ab$ $0.5b$	0.3 – 0.6 0.5

2. Monomials: 1000, pqr

Binomials: $x + y$, $2y - 3y^2$, $4z - 15z^2$, $p^2q + pq^2$, $2p + 2q$

Trinomials : $7 + y + 5x$, $2y - 3y^2 + 4y^3$, $5x - 4y + 3xy$

Polynomials that do not fit in these categories: $x + x^2 + x^3 + x^4$, $ab + bc + cd + da$

3. (i) 0 (ii) $ab + bc + ac$ (iii) $-p^2q^2 + 4pq + 9$

(iv) $2(l^2 + m^2 + n^2 + lm + mn + nl)$

4. (a) $8a - 2ab + 2b - 15$ (b) $2xy - 7yz + 5zx + 10xyz$

(c) $p^2q - 7pq^2 + 8pq - 18q + 5p + 28$

EXERCISE 9.2

1. (i) $28p$ (ii) $-28p^2$ (iii) $-28p^2q$ (iv) $-12p^4$ (v) 0

2. pq; $50\ mn$; $100\ x^2y^2$; $12x^3$; $12mn^2p$

3.

First monomial → / Second monomial ↓	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	$-10xy$	$6x^3$	$-8x^2y$	$14x^3y$	$-18x^3y^2$
$-5y$	$-10xy$	$25y^2$	$-15x^2y$	$20xy^2$	$-35x^2y^2$	$45x^2y^3$
$3x^2$	$6x^3$	$-15x^2y$	$9x^4$	$-12x^3y$	$21x^4y$	$-27x^4y^2$
$-4xy$	$-8x^2y$	$20xy^2$	$-12x^3y$	$16x^2y^2$	$-28x^3y^2$	$36x^3y^3$
$7x^2y$	$14x^3y$	$-35x^2y^2$	$21x^4y$	$-28x^3y$	$49x^4y^2$	$-63x^4y^3$
$-9x^2y^2$	$-18x^3y^2$	$45x^2y^3$	$-27x^4y^2$	$36x^3y^2$	$-63x^4y^3$	$81x^4y^4$

4. (i) $105a^7$ (ii) $64pqr$ (iii) $4x^4y^4$ (iv) $6abc$

5. (i) $x^2y^2z^2$ (ii) $-a^6$ (iii) $1024y^6$ (iv) $36a^2b^2c^2$ (v) $-m^3n^2p$

EXERCISE 9.3

1. (i) $4pq + 4pr$ (ii) $a^2b - ab^2$ (iii) $7a^3b^2 + 7a^2b^3$

(iv) $4a^3 - 36a$ (v) 0

2. (i) $ab + ac + ad$ (ii) $5x^2y + 5xy^2 - 25xy$

(iii) $6p^3 - 7p^2 + 5p$ (iv) $4p^4q^2 - 4p^2q^4$

(v) $a^2bc + ab^2c + abc^2$

3. (i) $8a^{50}$ (ii) $-\frac{3}{5}x^3y^3$ (iii) $-4p^4q^4$ (iv) x^{10}

4. (a) $12x^2 - 15x + 3$; (i) 66 (ii) $\frac{-3}{2}$

(b) $a^3 + a^2 + a + 5$; (i) 5 (ii) 8 (iii) 4

5. (a) $p^2 + q^2 + r^2 - pq - qr - pr$ (b) $-2x^2 - 2y^2 - 4xy + 2yz + 2zx$

(c) $5l^2 + 25ln$ (d) $-3a^2 - 2b^2 + 4c^2 - ab + 6bc - 7ac$

EXERCISE 9.4

1. (i) $8x^2 + 14x - 15$ (ii) $3y^2 - 28y + 32$ (iii) $6.25l^2 - 0.25m^2$
(iv) $ax + 5a + 3bx + 15b$ (v) $6p^2q^2 + 5pq^3 - 6q^4$ (vi) $3a^4 + 10a^2b^2 - 8b^4$

2. (i) $15 - x - 2x^2$ (ii) $7x^2 + 48xy - 7y^2$ (iii) $a^3 + a^2b^2 + ab + b^3$
(iv) $2p^3 + p^2q - 2pq^2 - q^3$

3. (i) $x^3 + 5x^2 - 5x$ (ii) $a^2b^3 + 3a^2 + 5b^3 + 20$ (iii) $t^3 - st + s^2t^2 - s^3$
(iv) $4ac$ (v) $3x^2 + 4xy - y^2$ (vi) $x^3 + y^3$
(vii) $2.25x^2 - 16y^2$ (viii) $a^2 + b^2 - c^2 + 2ab$

EXERCISE 9.5

1. (i) $x^2 + 6x + 9$ (ii) $4y^2 + 20y + 25$ (iii) $4a^2 - 28a + 49$
(iv) $9a^2 - 3a + \frac{1}{4}$ (v) $1.21m^2 - 0.16$ (vi) $b^4 - a^4$
(vii) $36x^2 - 49$ (viii) $a^2 - 2ac + c^2$ (ix) $\frac{x^2}{4} + \frac{3xy}{4} + \frac{9y^2}{16}$
(x) $49a^2 - 126ab + 81b^2$

2. (i) $x^2 + 10x + 21$ (ii) $16x^2 + 24x + 5$ (iii) $16x^2 - 24x + 5$
(iv) $16x^2 + 16x - 5$ (v) $4x^2 + 16xy + 15y^2$ (vi) $4a^4 + 28a^2 + 45$
(vii) $x^2y^2z^2 - 6xyz + 8$

3. (i) $b^2 - 14b + 49$ (ii) $x^2y^2 + 6xyz + 9z^2$ (iii) $36x^4 - 60x^2y + 25y^2$
(iv) $\frac{4}{9}m^2 + 2mn + \frac{9}{4}n^2$ (v) $0.16p^2 + 0.04pq + 0.25q^2$ (vi) $4x^2y^2 + 20xy^2 + 25y^2$

4. (i) $a^4 - 2a^2b^2 + b^4$ (ii) $40x$ (iii) $98m^2 + 128n^2$
(iv) $41m^2 + 80mn + 41n^2$ (v) $4p^2 - 4q^2$ (vi) $a^2b^2 + b^2c^2$ (vii) $m^4 + n^4m^2$

6. (i) 5041 (ii) 9801 (iii) 10404 (iv) 996004
(v) 27.04 (vi) 89991 (vii) 6396 (viii) 79.21
(ix) 9.975

7. (i) 200 (ii) 0.08 (iii) 1800 (iv) 84

8. (i) 10712 (ii) 26.52 (iii) 10094 (iv) 95.06

EXERCISE 10.1

1. (a) → (iii) → (iv) (b) → (i) → (v) (c) → (iv) → (ii)
(d) → (v) → (iii) (e) → (ii) → (i)

2. (a) (i) → Front, (ii) → Side, (iii) → Top (b) (i) → Side, (ii) → Front, (iii) → Top
(c) (i) → Front, (ii) → Side, (iii) → Top (d) (i) → Front, (ii) → Side, (iii) → Top

3. (a) (i) → Top, (ii) → Front, (iii) → Side (b) (i) → Side, (ii) → Front, (iii) → Top
(c) (i) → Top, (ii) → Side, (iii) → Front (d) (i) → Side, (ii) → Front, (iii) → Top
(e) (i) → Front, (ii) → Top, (iii) → Side

EXERCISE 10.3

1. (i) No (ii) Yes (iii) Yes
2. Possible, only if the number of faces are greater than or equal to 4
3. only (ii) and (iv)
4. (i) A prism becomes a cylinder as the number of sides of its base becomes larger and larger.
   (ii) A pyramid becomes a cone as the number of sides of its base becomes larger and larger.
5. No. It can be a cuboid also
7. Faces $\to$ 8, Vertices $\to$ 6, Edges $\to$ 30
8. No

EXERCISE 11.1

1. (a)
2. ₹ 17,875
3. Area = 129.5 m^2; Perimeter = 48 m
4. 45000 tiles
5. (b)

EXERCISE 11.2

1. 0.88 m^2
2. 7 cm
3. 660 m^2
4. 252 m^2
5. 45 cm^2
6. 24 cm^2, 6 cm
7. ₹ 810
8. 140 m
9. 119 m^2
10. Area using Jyoti's way $= 2 \times \frac{1}{2} \times \frac{15}{2} \times (30 + 15)\ m^2 = 337.5\ m^2$,

    Area using Kavita's way $= \frac{1}{2} \times 15 \times 15 + 15 \times 15 = 337.5\ m^2$
11. 80 cm^2, 96 cm^2, 80 cm^2, 96 cm^2

EXERCISE 11.3

1. (a)
2. 144 m
3. 10 cm
4. 11 m^2
5. 5 cans
6. Similarity $\to$ Both have same heights. Difference $\to$ one is a cylinder, the other is a cube. The cube has larger lateral surface area
7. 440 m^2
8. 322 cm
9. 1980 m^2
10. 704 cm^2

EXERCISE 11.4

1. (a) Volume (b) Surface area (c) Volume
2. Volume of cylinder B is greater; Surface area of cylinder B is greater.
3. 5 cm
4. 450
5. 1 m
6. 49500 L
7. (i) 4 times (ii) 8 times
8. 30 hours

EXERCISE 12.1

1. (i) $\frac{1}{9}$ (ii) $\frac{1}{16}$ (iii) 32

2. (i) $\frac{1}{(-4)^3}$ (ii) $\frac{1}{2^6}$ (iii) $(5)^4$ (iv) $\frac{1}{(3)^2}$ (v) $\frac{1}{(-14)^3}$

3. (i) 5 (ii) $\frac{1}{2}$ (iii) 29 (iv) 1 (v) $\frac{81}{16}$

4. (i) 250 (ii) $\frac{1}{60}$ **5.** $m = 2$ **6.** (i) –1 (ii) $\frac{512}{125}$

7. (i) $\frac{625t^4}{2}$ (ii) 5^5

EXERCISE 12.2

1. (i) 8.5×10^{-12} (ii) 9.42×10^{-12} (iii) 6.02×10^{15}
(iv) 8.37×10^{-9} (v) 3.186×10^{10}

2. (i) 0.00000302 (ii) 45000 (iii) 0.00000003
(iv) 1000100000 (v) 5800000000000 (vi) 3614920

3. (i) 1×10^{-6} (ii) 1.6×10^{-19} (iii) 5×10^{-7}
(iv) 1.275×10^{-5} (v) 7×10^{-2}

4. 1.0008×10^2

EXERCISE 13.1

1. No

2.

Parts of red pigment	1	4	7	12	20
Parts of base	8	32	56	96	160

3. 24 parts **4.** 700 bottles **5.** 10^{-4} cm; 2 cm **6.** 21 m

7. (i) 2.25×10^7 crystals (ii) 5.4×10^6 crystals **8.** 4 cm

9. (i) 6 m (ii) 8 m 75 cm **10.** 168 km

EXERCISE 13.2

1. (i), (iv), (v) **2.** $4 \rightarrow 25{,}000$; $5 \rightarrow 20{,}000$; $8 \rightarrow 12{,}500$; $10 \rightarrow 10{,}000$; $20 \rightarrow 5{,}000$
Amount given to a winner is inversely proportional to the number of winners.

3. $8 \rightarrow 45°$, $10 \rightarrow 36°$, $12 \rightarrow 30°$ (i) Yes (ii) 24° (iii) 9

4. 6 **5.** 4 **6.** 3 days **7.** 15 boxes

8. 49 machines **9.** $1\frac{1}{2}$ hours **10.** (i) 6 days (ii) 6 persons **11.** 40 minutes

EXERCISE 14.1

1. (i) 12 (ii) $2y$ (iii) $14pq$ (iv) 1 (v) $6ab$ (vi) $4x$
(vii) 10 (viii) x^2y^2

2. (i) $7(x-6)$ (ii) $6(p-2q)$ (iii) $7a(a+2)$ (iv) $4z(-4+5z^2)$
(v) $10\,lm(2l+3a)$ (vi) $5xy(x-3y)$ (vii) $5(2a^2-3b^2+4c^2)$
(viii) $4a(-a+b-c)$ (ix) $xyz(x+y+z)$ (x) $xy(ax+by+cz)$

3. (i) $(x+8)(x+y)$ (ii) $(3x+1)(5y-2)$ (iii) $(a+b)(x-y)$
(iv) $(5p+3)(3q+5)$ (v) $(z-7)(1-xy)$

EXERCISE 14.2

1. (i) $(a+4)^2$ (ii) $(p-5)^2$ (iii) $(5m+3)^2$ (iv) $(7y+6z)^2$
(v) $4(x-1)^2$ (vi) $(11b-4c)^2$ (vii) $(l-m)^2$ (viii) $(a^2+b^2)^2$

2. (i) $(2p-3q)(2p+3q)$ (ii) $7(3a-4b)(3a+4b)$ (iii) $(7x-6)(7x+6)$
(iv) $16x^3(x-3)(x+3)$ (v) $4lm$ (vi) $(3xy-4)(3xy+4)$
(vii) $(x-y-z)(x-y+z)$ (viii) $(5a-2b+7c)(5a+2b-7c)$

3. (i) $x(ax+b)$ (ii) $7(p^2+3q^2)$ (iii) $2x(x^2+y^2+z^2)$
(iv) $(m^2+n^2)(a+b)$ (v) $(l+1)(m+1)$ (vi) $(y+9)(y+z)$
(vii) $(5y+2z)(y-4)$ (viii) $(2a+1)(5b+2)$ (ix) $(3x-2)(2y-3)$

4. (i) $(a-b)(a+b)(a^2+b^2)$ (ii) $(p-3)(p+3)(p^2+9)$
(iii) $(x-y-z)(x+y+z)[x^2+(y+z)^2]$ (iv) $z(2x-z)(2x^2-2xz+z^2)$
(v) $(a-b)^2(a+b)^2$

5. (i) $(p+2)(p+4)$ (ii) $(q-3)(q-7)$ (iii) $(p+8)(p-2)$

EXERCISE 14.3

1. (i) $\frac{x^3}{2}$ (ii) $-4y$ (iii) $6pqr$ (iv) $\frac{2}{3}x^2y$ (v) $-2a^2b^4$

2. (i) $\frac{1}{3}(5x-6)$ (ii) $3y^4-4y^2+5$ (iii) $2(x+y+z)$
(iv) $\frac{1}{2}(x^2+2x+3)$ (v) q^3-p^3

3. (i) $2x-5$ (ii) 5 (iii) $6y$ (iv) xy (v) $10abc$

4. (i) $5(3x+5)$ (ii) $2y(x+5)$ (iii) $\frac{1}{2}r(p+q)$ (iv) $4(y^2+5y+3)$
(v) $(x+2)(x+3)$

5. (i) $y+2$ (ii) $m-16$ (iii) $5(p-4)$ (iv) $2z(z-2)$ (v) $\frac{5}{2}q(p-q)$
(vi) $3(3x-4y)$ (vii) $3y(5y-7)$

EXERCISE 14.4

1. $4(x-5)=4x-20$ **2.** $x(3x+2)=3x^2+2x$ **3.** $2x+3y=2x+3y$
4. $x+2x+3x=6x$ **5.** $5y+2y+y-7y=y$ **6.** $3x+2x=5x$

7. $(2x)^2 + 4(2x) + 7 = 4x^2 + 8x + 7$

8. $(2x)^2 + 5x = 4x^2 + 5x$

9. $(3x + 2)^2 = 9x^2 + 12x + 4$

10. (a) $(-3)^2 + 5(-3) + 4 = 9 - 15 + 4 = -2$ (b) $(-3)^2 - 5(-3) + 4 = 9 + 15 + 4 = 28$

(c) $(-3)^2 + 5(-3) = 9 - 15 = -6$

11. $(y - 3)^2 = y^2 - 6y + 9$

12. $(z + 5)^2 = z^2 + 10z + 25$

13. $(2a + 3b)(a - b) = 2a^2 + ab - 3b^2$

14. $(a + 4)(a + 2) = a^2 + 6a + 8$

15. $(a - 4)(a - 2) = a^2 - 6a + 8$

16. $\frac{3x^2}{3x^2} = 1$

17. $\frac{3x^2+1}{3x^2} = \frac{3x^2}{3x^2} + \frac{1}{3x^2} = 1 + \frac{1}{3x^2}$

18. $\frac{3x}{3x+2} = \frac{3x}{3x+2}$

19. $\frac{3}{4x+3} = \frac{3}{4x+3}$

20. $\frac{4x+5}{4x} = \frac{4x}{4x} + \frac{5}{4x} = 1 + \frac{5}{4x}$

21. $\frac{7x+5}{5} = \frac{7x}{5} + \frac{5}{5} = \frac{7x}{5} + 1$

EXERCISE 15.1

1. (a) 36.5° C (b) 12 noon (c) 1 p.m, 2 p.m.

(d) 36.5° C; The point between 1 p.m. and 2 p.m. on the x-axis is equidistant from the two points showing 1 p.m. and 2 p.m., so it will represent 1.30 p.m. Similarly, the point on the y-axis, between 36° C and 37° C will represent 36.5° C.

(e) 9 a.m. to 10 a.m., 10 a.m. to 11 a.m., 2 p.m. to 3 p.m.

2. (a) (i) ₹ 4 crore (ii) ₹ 8 crore

(b) (i) ₹ 7 crore (ii) ₹ 8.5 crore (approx.)

(c) ₹ 4 crore (d) 2005

3. (a) (i) 7 cm (ii) 9 cm

(b) (i) 7 cm (ii) 10 cm

(c) 2 cm (d) 3 cm (e) Second week (f) First week

(g) At the end of the 2nd week

4. (a) Tue, Fri, Sun (b) 35° C (c) 15° C (d) Thurs

6. (a) 4 units = 1 hour (b) $3\frac{1}{2}$ hours (c) 22 km

(d) Yes; This is indicated by the horizontal part of the graph (10 a.m. - 10.30 a.m.)

(e) Between 8 a.m. and 9 a.m.

7. (iii) is not possible

EXERCISE 15.2

1. Points in (a) and (b) lie on a line; Points in (c) do not lie on a line

2. The line will cut x-axis at (5, 0) and y-axis at (0, 5)

3. O(0, 0), A(2, 0), B(2, 3), C(0, 3), P(4, 3), Q(6, 1), R(6, 5), S(4, 7), K(10, 5), L(7, 7), M(10, 8)
4. (i) True (ii) False (iii) True

EXERCISE 15.3

1. (b) (i) 20 km (ii) 7.30 a.m. (c) (i) Yes (ii) ₹ 200 (iii) ₹ 3500
2. (a) Yes (b) No

EXERCISE 16.1

1. A = 7, B = 6
2. A = 5, B = 4, C = 1
3. A = 6
4. A = 2, B = 5
5. A = 5, B = 0, C = 1
6. A = 5, B = 0, C = 2
7. A = 7, B = 4
8. A = 7, B = 9
9. A = 4, B = 7
10. A = 8, B = 1

EXERCISE 16.2

1. $y = 1$
2. $z = 0$ or 9
3. $z = 0, 3, 6$ or 9
4. 0, 3, 6 or 9

JUST FOR FUN

1. **More about Pythagorean triplets**

   We have seen one way of writing pythagorean triplets as $2m, m^2 - 1, m^2 + 1$.

   A pythagorean triplet a, b, c means $a^2 + b^2 = c^2$. If we use two natural numbers m and $n (m > n)$, and take $a = m^2 - n^2$, $b = 2mn$, $c = m^2 + n^2$, then we can see that $c^2 = a^2 + b^2$.

   Thus for different values of m and n with $m > n$ we can generate natural numbers a, b, c such that they form Pythagorean triplets.

   For example: Take, $m = 2, n = 1$.

   Then, $a = m^2 - n^2 = 3$, $b = 2mn = 4$, $c = m^2 + n^2 = 5$, is a Pythagorean triplet. (Check it!)

   For, $m = 3, n = 2$, we get,

   $a = 5, b = 12, c = 13$ which is again a Pythagorean triplet.

   Take some more values for m and n and generate more such triplets.

2. When water freezes its volume increases by 4%. What volume of water is required to make 221 cm^3 of ice?

3. If price of tea increased by 20%, by what per cent must the consumption be reduced to keep the expense the same?

4. Ceremony Awards began in 1958. There were 28 categories to win an award. In 1993, there were 81 categories.
   (i) The awards given in 1958 is what per cent of the awards given in 1993?
   (ii) The awards given in 1993 is what per cent of the awards given in 1958?
5. Out of a swarm of bees, one fifth settled on a blossom of *Kadamba*, one third on a flower of *Silindhiri,* and three times the difference between these two numbers flew to the bloom of *Kutaja*. Only ten bees were then left from the swarm. What was the number of bees in the swarm? (Note, *Kadamba, Silindhiri* and *Kutaja* are flowering trees. The problem is from the ancient Indian text on algebra.)
6. In computing the area of a square, Shekhar used the formula for area of a square, while his friend Maroof used the formula for the perimeter of a square. Interestingly their answers were numerically same. Tell me the number of units of the side of the square they worked on.
7. The area of a square is numerically less than six times its side. List some squares in which this happens.
8. Is it possible to have a right circular cylinder to have volume numerically equal to its curved surface area? If yes state when.
9. Leela invited some friends for tea on her birthday. Her mother placed some plates and some *puris* on a table to be served. If Leela places 4 *puris* in each plate 1 plate would be left empty. But if she places 3 *puris* in each plate 1 *puri* would be left. Find the number of plates and number of *puris* on the table.
10. Is there a number which is equal to its cube but not equal to its square? If yes find it.
11. Arrange the numbers from 1 to 20 in a row such that the sum of any two adjacent numbers is a perfect square.

Answers

2. $212\frac{1}{2}$ cm^3

3. $16\frac{2}{3}\%$

4. (i) 34.5% (ii) 289%
5. 150
6. 4 units
7. Sides = 1, 2, 3, 4, 5 units
8. Yes, when radius = 2 units
9. Number of *puris* = 16, number of plates = 5
10. – 1
11. One of the ways is, 1, 3, 6, 19, 17, 8 (1 + 3 = 4, 3 + 6 = 9 etc.). Try some other ways.