Probability

Case Study Based Questions

Case Study 1

The reliability of a COVID PCR test is specified as follows:

Of people having COVID, 90% of the test detects the disease but 10 % goes undetected. Of people free of COVID, 99% of the test is judged COVID negative but 1% are diagnosed as showing COVID positive. From a large population of which only 0.1% have COVID, one person is selected at random, given the COVID PCR test, and the pathologist reports him/her as COVID positive.

Based on the given information, solve the following questions:

Q1. What is the probability of the 'person to be tested as COVID positive' given that 'he is actually having COVID' ?

a. 0.001 b. 0.1

c. 0.8 d. 0.9

Q2. What is the probability of the 'person to be tested as COVID positive' given that 'he is actually not having COVID' ?

a. 0.01 b. 0.99

c. 0.1 d. 0.001

Q3. What is the probability that the 'person is actually not having COVID' ?

a. 0.998 b. 0.999

c. 0.001 d. 0.111

Q4. What is the probability that the 'person is actually having COVID' given that 'he is tested as COVID positive' ?

a. 0.83 b. 0.0803

c. 0.083 d. 0.089

Q5. What is the probability that the 'person selected will be diagnosed as COVID positive' ?

a. 0.1089 b. 0.01089

c. 0.0189 d. 0.189

Solutions

Consider the events

E_1 = People having COVID,

E_2 = People free of COVID

A = Person to be tested as COVID positive

1. Probability of the 'person to be tested as COVID positive' given that he is actually having COVID

$$= P\left(\frac{A}{E_1}\right) = 90\% = \frac{90}{100} = 0.9$$

So, option (d) is correct.

2. Probability of the 'person to be tested as COVID positive' given that he is actually not having COVID

$$= P\left(\frac{A}{E_2}\right) = 1\% = \frac{1}{100} = 0.01$$

So, option (a) is correct.

3. Probability that the person is actually not having COVID = $P(E_2)$ = 99.9%

$$= \frac{999}{1000} = 0.999$$

So, option (b) is correct.

4. Probability that the person is actually having COVID given that he is tested as COVID positive

$$= P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \times P\left(\frac{A}{E_1}\right)}{P(E_1) \times P\left(\frac{A}{E_1}\right) + P(E_2) \times P\left(\frac{A}{E_2}\right)}$$

$$= \frac{0.1\% \times 90\%}{0.1\% \times 90\% + 99.9\% \times 1\%}$$

$$= \frac{0.1 \times 90}{0.1 \times 90 + 99.9 \times 1}$$

$$= \frac{9}{9 + 99.9} = \frac{9}{108.9} = 0.0826$$

$\therefore$ Required probability $= 0.083$

So, option (c) is correct.

5. Probability that the person selected will be diagnosed as COVID positive

$$P(A) = P(E_1) \times P\left(\frac{A}{E_1}\right) + P(E_2) \times P\left(\frac{A}{E_2}\right)$$

$$= 0.1\% \times 90\% + 99.9\% \times 1\%$$

$$= \frac{90}{100000} + \frac{999}{100000} = \frac{1089}{100000} = 0.01089$$

So, option (b) is correct.

Case Study 2

An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The company's statistics show that an accident-prone person will have an accident at sometime within a fixed one-year period with probability 0.6, whereas this probability is 0.2 for a person who is not accident prone. The company knows that 20 per cent of the population is accident prone.

Based on the above information, solve the following questions:

(CBSE SQP 2022 Term-2)

Q1. What is the probability that a new policyholder will have an accident within a year of purchasing a policy?

Q2. Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone?

Solutions

1. Let $E_1 =$ The policy holder is accident prone.

$E_2 =$ The policy holder is not accident prone.

$E =$ The new policy holder has an accident within a year of purchasing a policy.

$$\therefore \quad P(E) = P(E_1) \times P(E/E_1) + P(E_2) \times P(E/E_2)$$

$$= \frac{6}{10} \times \frac{20}{100} + \frac{2}{10} \times \frac{80}{100} = \frac{280}{1000} = \frac{7}{25}$$

2. By Bayes' theorem,

$$P(E_1/E) = \frac{P(E_1) \times P(E/E_1)}{P(E)}$$

$$= \frac{\frac{6}{10} \times \frac{20}{100}}{\frac{280}{1000}} = \frac{3}{7}$$

Case Study 3

A shopkeeper sells three types of flower seeds A_1, A_2, A_3. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2, respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

Based on the above information, solve the following questions:

(CBSE SQP 2022 Term-2)

Q1. Calculate the probability that a randomly chosen seed will germinate.

Q2. Calculate the probability that the seed is of type A_2, given that a randomly chosen seed germinates.

Solutions

1. We have, $A_1 : A_2 : A_3 = 4 : 4 : 2$

$$P(A_1) = \frac{4}{10}, P(A_2) = \frac{4}{10} \text{ and } P(A_3) = \frac{2}{10}$$

where A_1, A_2 and A_3 denote the three types of flower seeds.

Let E be the event that a seed germinates.

$$\therefore \quad P(E / A_1) = \frac{45}{100}, P(E / A_2) = \frac{60}{100}$$

and $$P(E / A_3) = \frac{35}{100}$$

$$\therefore \quad P(E) = P(A_1) \cdot P(A / E_1)$$

$$P(A_2) \cdot P(A / E_2) + P(A_3) \cdot P(A / E_3)$$

$$= \frac{4}{10} \times \frac{45}{100} + \frac{4}{10} \times \frac{60}{100} + \frac{2}{10} \times \frac{35}{100}$$

$$= \frac{490}{1000} = 0.49$$

2. $$P\left(\frac{A_2}{E}\right) = \frac{P(A_2) \cdot P(E / A_2)}{P(A_1) P(E / A_1) + P(A_2) P(E / A_2) + P(A_3) P(E / A_3)}$$

[By Baye's theorem]

$$= \frac{\frac{4}{10} \times \frac{60}{100}}{\frac{4}{10} \times \frac{45}{100} + \frac{4}{10} \times \frac{60}{100} + \frac{2}{10} \times \frac{35}{100}}$$

$$= \frac{240 / 1000}{490 / 1000}$$

$$= \frac{240}{490} = \frac{24}{49}.$$

Case Study 4

There are two antiaircraft guns, name as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

Based on the above information, solve the following questions: (CBSE SQP 2022-23)

Q1. What is the probability that the shell fired from exactly one of them hit the plane?

Q2. If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B?

Solutions

1. P (Shell fired from exactly one of them hits the plane) = P [(Shell from A hits the plane and shell from B does not hit the plane) or (shell from A does not hit the plane and shell from B hits the plane)] = 0.3 × 0.8 + 0.7 × 0.2 = 0.24 + 0.14 = 0.38.

2. P (Shell fired from B hit the plane/Exactly one of them hit the plane)

$$= \frac{P\text{ (Shell fired from } B \text{ hit the plane } \cap \text{Exactly one of them hit the plane)}}{P\text{ (Exactly one of them hit the plane)}}$$

$$= \frac{P\text{ (Shell from only } B \text{ hit the plane)}}{P\text{ (Exactly one of them hit the plane)}}$$

$$= \frac{0.14}{0.38} = \frac{7}{19}.$$

$$[\because P(\bar{A} \cap B) = P(\bar{A}) \times P(B) = (1 - 0.3) \times 0.2 = 0.14]$$

Case Study 5

A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0.65. The probability that many are not present and still the work gets completed on time is 0.35. The probability that work will be completed on time when all workers are present is 0.80.

Let E_1 : represent the event when many workers were not present for the job

E_2 : represent the event when all workers were present and

E : represent completing the construction work on time. (CBSE 2023)

Based on the above information, solve the following questions:

Q1. What is the probability that all the workers are present for the job?

Q2. What is the probability that construction will be completed on time?

Q3. What is the probability that many workers are not present given that the construction work is completed on time?

Or

What is the probability that all workers were present given that the construction job was completed on time?

Solutions

1. Given $P(E_1) = 0.65, P\left(\frac{E}{E_1}\right) = 0.35, P\left(\frac{E}{E_2}\right) = 0.80$

$$\therefore \quad P(E_2) = 1 - P(E_1) = 1 - 0.65 = 0.35$$

2. $P(E) = P(E_1) \times P\left(\frac{E}{E_1}\right) + P(E_2) \times P\left(\frac{E}{E_2}\right)$

$$= 0.65 \times 0.35 + 0.35 \times 0.80$$
$$= 0.2275 + 0.28 = 0.5075$$

3. $P\left(\frac{E}{E_1}\right) = 0.35$

Or

$P\left(\frac{E}{E_2}\right) = 0.80$

Case Study 6

In an office three employees James, Sophia and Oliver process incoming copies of a certain form. James processes 50% of the forms, Sophia processes 20% and Oliver the remaining 30% of the forms. James has an error rate of 0.06, Sophia has an error rate of 0.04 and Oliver has an error rate of 0.03.

Based on the above information, solve the following questions: (CBSE SQP 2023-24)

Q1. Find the probability that Sophia processed the form and committed an error.

Q2. Find the total probability of committing an error in processing the form.

Q3. The manager of the company wants to do a quality check. During inspection, he selects a form at random from the days output of processed form. If the form selected at random has an error, find the probability that the form is not processed by James.

Or

Let E be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that James, Sophia and Oliver processed the form. Find the value of

$$\sum_{i=1}^{3} P(E_i / E).$$

Solutions

$$\text{Given } P(J) = 50\% = \frac{50}{100} = \frac{1}{2}$$

$$P(S) = 20\% = \frac{20}{100} = \frac{1}{5}$$

$$P(O) = 30\% = \frac{30}{100} = \frac{3}{10}$$

$$P\left(\frac{E}{J}\right) = 0.06,\ P\left(\frac{E}{S}\right) = 0.04 \text{ and } P\left(\frac{E}{O}\right) = 0.03$$

1. Probability that Sophia processed the form and committed an error $= P(S)P\left(\frac{E}{S}\right)$

$$= \frac{1}{5} \times 0.04 = 0.008$$

2. The total probability of committing an error in processing the form

$$= P(J)P\left(\frac{E}{J}\right) + P(S)P\left(\frac{E}{S}\right) + P(O)P\left(\frac{E}{O}\right)$$

$$= \frac{1}{2}\times 0.06 + \frac{1}{5}\times 0.04 + \frac{3}{10}\times 0.03$$

$$= 0.03 + 0.008 + 0.009$$

$$= 0.047$$

3. Probability that an error is committed by james in processing,

$$P\left(\frac{J}{E}\right) = \frac{P(J)\times P\left(\frac{E}{J}\right)}{P(J)P\left(\frac{E}{J}\right) + P(S)P\left(\frac{E}{S}\right) + P(O)P\left(\frac{E}{O}\right)}$$

$$= \frac{\frac{1}{2}\times 0.06}{0.047} = \frac{3\times 10}{47} = \frac{30}{47}$$

∴ Required probability that error was not processed by James $= 1 - P\left(\frac{J}{E}\right) = 1 - \frac{30}{47} = \frac{17}{47}$

Or

$$\sum_{i=1}^{3} P\left(\frac{E_i}{E}\right) = P\left(\frac{E_1}{E}\right) + P\left(\frac{E_2}{E}\right) + P\left(\frac{E_3}{E}\right)$$

$$= \frac{P(E_1)\times P\left(\frac{E}{E_1}\right) + P(E_2)\times P\left(\frac{E}{E_2}\right) + P(E_3)P\left(\frac{E}{E_3}\right)}{P(E_1)P\left(\frac{E}{E_1}\right) + P(E_2)P\left(\frac{E}{E_2}\right) + P(E_3)\left(\frac{E}{E_3}\right)}$$

$$= 1$$

Solutions for Questions 7 to 27 are Given Below

Case Study 7

Three friends A, B and C are playing a dice game. The numbers rolled up by them in their first three chances were noted and given by $A = \{1, 5\}$, $B = \{2, 4, 5\}$ and $C = \{1, 2, 5\}$ as A reaches the cell 'SKIP YOUR NEXT TURN' in second throw.

Based on the above information, answer the following questions.

(i) $P(A \mid B) =$

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

(ii) $P(B \mid C) =$

(a) $\frac{2}{3}$ (b) $\frac{1}{12}$ (c) $\frac{1}{9}$ (d) 0

(iii) $P(A \cap B \mid C) =$

(a) $\frac{1}{6}$ (b) $\frac{1}{2}$ (c) $\frac{1}{12}$ (d) $\frac{1}{3}$

(iv) $P(A \mid C) =$

(a) $\frac{1}{4}$ (b) 1 (c) $\frac{2}{3}$ (d) None of these

(v) $P(A \cup B \mid C) =$

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) 1

Case Study 8

In a play zone, Aastha is playing crane game. It has 12 blue balls, 8 red balls, 10 yellow balls and 5 green balls. If Aastha draws two balls one after the other without replacement, then answer the following questions.

(i) What is the probability that the first ball is blue and the second ball is green?

(a) $\frac{5}{119}$ (b) $\frac{12}{119}$ (c) $\frac{6}{119}$ (d) $\frac{15}{119}$

(ii) What is the probability that the first ball is yellow and the second ball is red?

(a) $\frac{16}{119}$ (b) $\frac{8}{119}$ (c) $\frac{24}{119}$ (d) None of these

(iii) What is the probability that both the balls are red?

(a) $\frac{4}{85}$ (b) $\frac{24}{595}$ (c) $\frac{12}{119}$ (d) $\frac{64}{119}$

(iv) What is the probability that the first ball is green and the second ball is not yellow?

(a) $\frac{10}{119}$ (b) $\frac{6}{85}$ (c) $\frac{12}{119}$ (d) None of these

(v) What is the probability that both the balls are not blue?

(a) $\frac{6}{595}$ (b) $\frac{12}{85}$ (c) $\frac{15}{17}$ (d) $\frac{253}{595}$

Case Study 9

Ajay enrolled himself in an online practice test portal provided by his school for better practice. Out of 5 questions in a set-I, he was able to solve 4 of them and got stuck in the one which is as shown below.

If A and B are independent events, $P(A) = 0.6$ and $P(B) = 0.8$, then answer the following questions.

(i) $P(A \cap B) =$

(a) 0.2 (b) 0.9 (c) 0.48 (d) 0.6

(ii) $P(A \cup B) =$

(a) 0.92 (b) 0.08 (c) 0.48 (d) 0.64

(iii) $P(B \mid A) =$

(a) 0.14 (b) 0.2 (c) 0.6 (d) 0.8

(iv) $P(A \mid B) =$

(a) 0.6 (b) 0.9 (c) 0.19 (d) 0.11

(v) P (not A and not B) =

(a) 0.01 (b) 0.48 (c) 0.08 (d) 0.91

Case Study 10

A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by cab, metro, bike or by other means of transport are respectively 0.3, 0.2, 0.1 and 0.4. The probabilities that he will be late are 0.25, 0.3, 0.35 and 0.1 if he comes by cab, metro, bike and other means of transport respectively.

Based on the above information, answer the following questions.

(i) When the doctor arrives late, what is the probability that he comes by metro?

(a) $\frac{5}{14}$ (b) $\frac{2}{7}$ (c) $\frac{5}{21}$ (d) $\frac{1}{6}$

(ii) When the doctor arrives late, what is the probability that he comes by cab?

(a) $\frac{4}{21}$ (b) $\frac{1}{7}$ (c) $\frac{5}{14}$ (d) $\frac{2}{21}$

(iii) When the doctor arrives late, what is the probability that he comes by bike?

(a) $\frac{5}{21}$ (b) $\frac{4}{7}$ (c) $\frac{5}{6}$ (d) $\frac{1}{6}$

(iv) When the doctor arrives late, what is the probability that he comes by other means of transport?

(a) $\frac{6}{7}$ (b) $\frac{5}{14}$ (c) $\frac{4}{21}$ (d) $\frac{2}{7}$

(v) What is the probability that the doctor is late by any means?

(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

Case Study 11

Suman was doing a project on a school survey, on the average number of hours spent on study by students selected at random. At the end of survey, Suman prepared the following report related to the data.

Let X denotes the average number of hours spent on study by students. The probability that X can take the values x, has the following form, where k is some unknown constant.

$$P(X=x)=\begin{cases} 0.2, \text{ if } x=0 \\ kx, \text{ if } x=1 \text{ or } 2 \\ k(6-x), \text{ if } x=3 \text{ or } 4 \\ 0, \text{ otherwise} \end{cases}$$

Based on the above information, answer the following questions.

(i) Find the value of k.

(a) 0.1 (b) 0.2 (c) 0.3 (d) 0.05

(ii) What is the probability that the average study time of students is not more than 1 hour?

(a) 0.4 (b) 0.3 (c) 0.5 (d) 0.1

(iii) What is the probability that the average study time of students is at least 3 hours?

(a) 0.5 (b) 0.9 (c) 0.8 (d) 0.1

(iv) What is the probability that the average study time of students is exactly 2 hours?

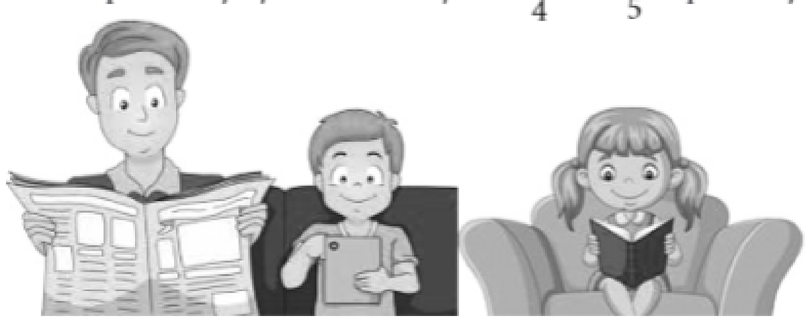
(a) 0.4 (b) 0.5 (c) 0.7 (d) 0.2

(v) What is the probability that the average study time of students is at least 1 hour?

(a) 0.2 (b) 0.4 (c) 0.8 (d) 0.6

Case Study 12

On a holiday, a father gave a puzzle from a newspaper to his son Ravi and his daughter Priya. The probability of solving this specific puzzle independently by Ravi and Priya are $\frac{1}{4}$ and $\frac{1}{5}$ respectively.



Based on the above information, answer the following questions.

(i) The chance that both Ravi and Priya solved the puzzle, is

(a) 10% (b) 5% (c) 25% (d) 20%

(ii) Probability that puzzle is solved by Ravi but not by Priya, is

(a) $\frac{1}{2}$ (b) $\frac{1}{5}$ (c) $\frac{3}{5}$ (d) $\frac{1}{3}$

(iii) Find the probability that puzzle is solved.

(a) $\frac{1}{2}$ (b) $\frac{1}{5}$ (c) $\frac{2}{5}$ (d) $\frac{5}{6}$

(iv) Probability that exactly one of them solved the puzzle, is

(a) $\frac{1}{30}$ (b) $\frac{1}{20}$ (c) $\frac{7}{20}$ (d) $\frac{3}{20}$

(v) Probability that none of them solved the puzzle, is

(a) $\frac{1}{5}$ (b) $\frac{3}{5}$ (c) $\frac{2}{5}$ (d) None of these

Case Study 13

A card is lost from a pack of 52 cards. From the remaining cards two cards are drawn at random.

Based on the above information, answer the following questions.

(i) The probability of drawing two diamonds, given that a card of diamond is missing, is

(a) $\frac{21}{425}$ (b) $\frac{22}{425}$ (c) $\frac{23}{425}$ (d) $\frac{1}{425}$

(ii) The probability of drawing two diamonds, given that a card of heart is missing, is

(a) $\frac{26}{425}$ (b) $\frac{22}{425}$ (c) $\frac{19}{425}$ (d) $\frac{23}{425}$

(iii) Let A be the event of drawing two diamonds from remaining 51 cards and E_1, E_2, E_3 and E_4 be the events that lost card is of diamond, club, spade and heart respectively, then the approximate value of $\sum_{i=1}^{4} P(A \mid E_i)$ is

(a) 0.17 (b) 0.24 (c) 0.25 (d) 0.18

(iv) All of a sudden, missing card is found and, then two cards are drawn simultaneously without replacement. Probability that both drawn cards are king is

(a) $\frac{1}{52}$ (b) $\frac{1}{221}$ (c) $\frac{1}{121}$ (d) $\frac{2}{221}$

(v) If two cards are drawn from a well shuffled pack of 52 cards, one by one with replacement, then probability of getting not a king in 1^{st} and 2^{nd} draw is

(a) $\frac{144}{169}$ (b) $\frac{12}{169}$ (c) $\frac{64}{169}$ (d) none of these

Case Study 14

One day, a sangeet mahotsav is to be organised in an open area of Rajasthan. In recent years, it has rained only 6 days each year. Also, it is given that when it actually rains, the weatherman correctly forecasts rain 80% of the time. When it doesn't rain, he incorrectly forecasts rain 20% of the time.
If leap year is considered, then answer the following questions.

(i) The probability that it rains on chosen day is

(a) $\frac{1}{366}$ (b) $\frac{1}{73}$ (c) $\frac{1}{60}$ (d) $\frac{1}{61}$

(ii) The probability that it does not rain on chosen day is

(a) $\frac{1}{366}$ (b) $\frac{5}{366}$ (c) $\frac{360}{366}$ (d) none of these

(iii) The probability that the weatherman predicts correctly is

(a) $\frac{5}{6}$ (b) $\frac{7}{8}$ (c) $\frac{4}{5}$ (d) $\frac{1}{5}$

(iv) The probability that it will rain on the chosen day, if weatherman predict rain for that day, is

(a) 0.0625 (b) 0.0725 (c) 0.0825 (d) 0.0925

(v) The probability that it will not rain on the chosen day, if weatherman predict rain for that day, is

(a) 0.94 (b) 0.84 (c) 0.74 (d) 0.64

Case Study 15

In a family there are four children. All of them have to work in their family business to earn their livelihood at the age of 18.

Based on the above information, answer the following questions.

(i) Probability that all children are girls, if it is given that elder child is a boy, is
(a) 3/8 (b) 1/8 (c) 5/8 (d) none of these

(ii) Probability that all children are boys, if two elder children are boys, is
(a) 1/4 (b) 3/4 (c) 1/2 (d) none of these

(iii) Find the probability that two middle children are boys, if it is given that eldest child is a girl.
(a) 0 (b) 3/4 (c) 1/4 (d) none of these

(iv) Find the probability that all children are boys, if it is given that at most one of the children is a girl.
(a) 0 (b) 1/5 (c) 2/5 (d) 4/5

(v) Find the probability that all children are boys, if it is given that at least three of the children are boys.
(a) 1/5 (b) 2/5 (c) 3/5 (d) 4/5

Case Study 16

To teach the application of probability a maths teacher arranged a surprise game for 5 of his students namely Archit, Aadya, Mivaan, Deepak and Vrinda. He took a bowl containing tickets numbered 1 to 50 and told the students go one by one and draw two tickets simultaneously from the bowl and replace it after noting the numbers.

Based on the above information, answer the following questions.

(i) Teacher ask Vrinda, what is the probability that both tickets drawn by Archit shows even number?

(a) 1/50 (b) 12/49 (c) 13/49 (d) 15/49

(ii) Teacher ask Mivaan, what is the probability that both tickets drawn by Aadya shows odd number?

(a) 1/50 (b) 2/49 (c) 12/49 (d) 5/49

(iii) Teacher ask Deepak, what is the probability that tickets drawn by Mivaan, shows a multiple of 4 on one ticket and a multiple 5 on other ticket?

(a) 14/245 (b) 16/245 (c) 24/245 (d) None of these

(iv) Teacher ask Archit. what is the probability that tickets are drawn by Deepak, shows a prime number on one ticket and a multiple of 4 on other ticket?

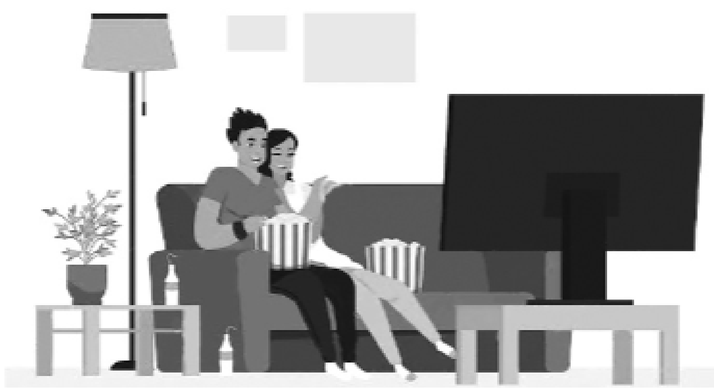
(a) 3/245 (b) 17/245 (c) 18/245 (d) 36/245

(v) Teacher ask Aadya, what is the probability that tickets drawn by Vrinda, shows an even number on first ticket and an odd number on second ticket?

(a) 15/98 (b) 25/98 (c) 35/98 (d) none of these

Case Study 17

A pharmaceutical company wants to advertise a new product on T.V., where the product is specially designed for women. For that an advertising executive is hired to study television-viewing habits of married couples during prime time hours. Based on past viewing records he has determined that during prime time husbands are watching television 70% of the time. It has also been determined that when the husband is watching television, 30% of the time the wife is also watching. When the husband is not watching television, 40% of the time the wife is watching television.



Based on the above information, answer the following questions.

(i) The probability that the husband is not watching television during prime time, is

(a) 0.6 (b) 0.3 (c) 0.4 (d) 0.5

(ii) If the wife is watching television, the probability that husband is also watching television, is

(a) 2/11 (b) 7/11 (c) 5/11 (d) 8/11

(iii) The probability that both husband and wife are watching television during prime time, is

(a) 0.21 (b) 0.5 (c) 0.3 (d) 0.4

(iv) The probability that the wife is watching television during prime time, is

(a) 0.24 (b) 0.33 (c) 0.3 (d) 0.4

(v) If the wife is watching television, then the probability that husband is not watching television, is

(a) 2/11 (b) 4/11 (c) 1/11 (d) 5/11

Case Study 18

In a bilateral cricket series between India and South Africa, the probability that India wins the first match is 0.6. If India wins any match, then the probability that it wins the next match is 0.4, otherwise the probability is 0.3. Also, it is given that there is no tie in any match.

Based on the above information answer the following questions.

(i) The probability that India won the second match, if India has already loose the first match is
(a) 0.5 (b) 0.4 (c) 0.3 (d) 0.6

(ii) The probability that India losing the third match, if India has already loose the first two matches is
(a) 0.2 (b) 0.3 (c) 0.4 (d) 0.7

(iii) The probability that India losing the first two matches is
(a) 0.12 (b) 0.28 (c) 0.42 (d) 0.01

(iv) The probability that India winning the first three matches is
(a) 0.92 (b) 0.96 (c) 0.94 (d) 0.096

(v) The probability that India winning exactly one of the first three matches is
(a) 0.205 (b) 0.21 (c) 0.408 (d) 0.312

Case Study 19

A student is preparing for the competitive examinations LIC AAO, SSC CGL and Bank P.O. The probabilities that the student is selected independently in competitive examination of LIC AAO, SSC CGL and Bank P.O. are a, b and c respectively. Of these examinations, students has 50% chance of selection in at least one, 40% chance of selection in at least two and 30% chance of selection in exactly two examinations.

Based on the above information, answer the following questions.

(i) The value of $a + b + c - ab - bc - ca + abc$ is

(a) 0.3 (b) 0.5 (c) 0.7 (d) 0.6

(ii) The value of $ab + bc + ac - 2abc$ is

(a) 0.5 (b) 0.3 (c) 0.4 (d) 0.6

(iii) The value of abc is

(a) 0.1 (b) 0.5 (c) 0.7 (d) 0.3

(iv) The value of $ab + bc + ac$ is

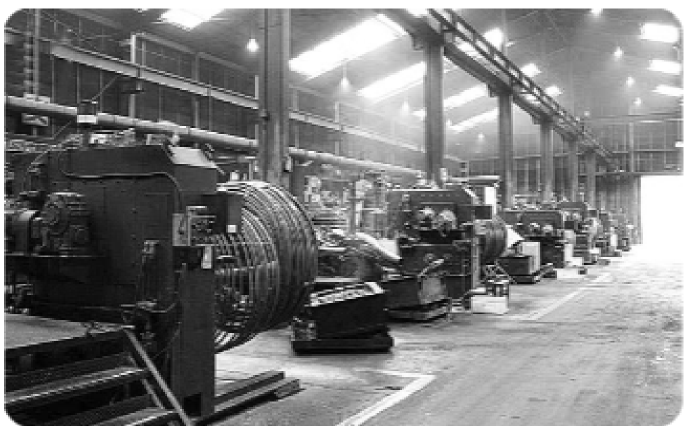
(a) 0.1 (b) 0.6 (c) 0.5 (d) 0.3

(v) The value of $a + b + c$ is

(a) 1 (b) 1.5 (c) 1.6 (d) 1.4

Case Study 20

A factory has three machines A, B and C to manufacture bolts. Machine A manufacture 30%, machine B manufacture 20% and machine C manufacture 50% of the bolts respectively. Out of their respective outputs 5%, 2% and 4% are defective. A bolt is drawn at random from total production and it is found to be defective.



Based on the above information, answer the following questions.

(i) Probability that defective bolt drawn is manufactured by machine A, is

(a) $\frac{4}{13}$ (b) $\frac{5}{13}$ (c) $\frac{6}{13}$ (d) $\frac{9}{13}$

(ii) Probability that defective bolt drawn is manufactured by machine B, is

(a) 0.3 (b) 0.1 (c) 0.2 (d) 0.4

(iii) Probability that defective bolt drawn is manufactured by machine C, is

(a) $\frac{16}{39}$ (b) $\frac{17}{39}$ (c) $\frac{20}{39}$ (d) $\frac{15}{39}$

(iv) Probability that defective bolt is not manufactured by machine B, is

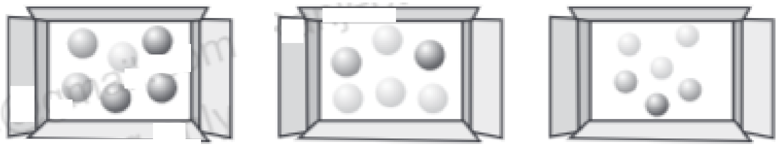
(a) $\frac{35}{39}$ (b) $\frac{61}{39}$ (c) $\frac{41}{39}$ (d) none of these

(v) Probability that defective bolt is not manufactured by machine C, is

(a) 0.03 (b) 0.09 (c) 0.5 (d) 0.9

Case Study 21

Box I contains 1 white, 3 black and 2 red balls. Box II contains 2 white, 1 black and 3 red balls. Box III contains 3 white, 2 black and 1 red balls. One box is chosen at random and two balls are drawn with replacement.



If E_1, E_2, E_3 be the events that the balls drawn from box I, box II and box III respectively and E be the event that balls drawn are one white and one red, then answer the following questions.

(i) Probability of occurrence of event E given that the balls drawn are from box I, is

(a) $\frac{1}{9}$ (b) $\frac{2}{6}$ (c) $\frac{3}{5}$ (d) $\frac{1}{7}$

(ii) Probability of occurrence of event E, given that the balls drawn are from box II, is

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) $\frac{3}{5}$

(iii) Probability of occurrence of event E, given that balls drawn are from box III, is

(a) $\frac{1}{12}$ (b) $\frac{3}{11}$ (c) $\frac{1}{6}$ (d) $\frac{4}{11}$

(iv) The value of $\sum_{i=1}^{3} P(E|E_i)$ is equal to

(a) $\frac{5}{18}$ (b) $\frac{1}{2}$ (c) $\frac{1}{18}$ (d) $\frac{11}{18}$

(v) The probability that the balls drawn are from box II, given that event E has already occurred, is

(a) $\frac{1}{11}$ (b) $\frac{6}{11}$ (c) $\frac{5}{11}$ (d) none of these

Case Study 22

Nisha and Ayushi appeared for first round of an interview for two vacancies. The probability of Nisha's selection is 1/3 and that of Ayushi's selection is 1/2.

Based on the above information, answer the following questions.

(i) The probability that both of them are selected, is

(a) $\frac{1}{12}$ (b) $\frac{1}{24}$ (c) $\frac{1}{6}$ (d) $\frac{1}{2}$

(ii) The probability that none of them is selected, is

(a) $\frac{2}{7}$ (b) $\frac{3}{8}$ (c) $\frac{5}{8}$ (d) $\frac{1}{3}$

(iii) The probability that only one of them is selected, is

(a) $\frac{5}{8}$ (b) $\frac{2}{3}$ (c) $\frac{2}{5}$ (d) $\frac{1}{2}$

(iv) The probability that atleast one of them is selected, is

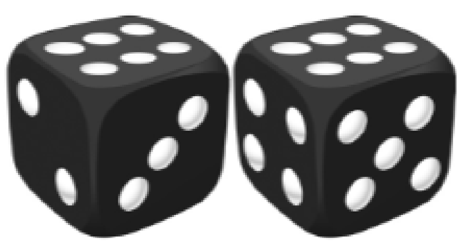
(a) $\frac{2}{3}$ (b) $\frac{1}{8}$ (c) $\frac{3}{5}$ (d) $\frac{2}{5}$

(v) Suppose Nisha is selected by the manager and told her about two posts I and II for which selection is independent. If the probability of selection for post I is $\frac{1}{6}$ and for post II is $\frac{1}{5}$, then the probability that Nisha is selected for at least one post, is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{8}$ (d) $\frac{1}{2}$

Case Study 23

Varun and Isha decided to play with dice to keep themselves busy at home as their schools are closed due to coronavirus pandemic. Varun throw a dice repeatedly until a six is obtained. He denote the number of throws required by X.



Based on the above information, answer the following questions.

(i) The probability that $X = 2$ equals

(a) $\frac{1}{6}$ (b) $\frac{5}{6^2}$ (c) $\frac{5}{3^6}$ (d) $\frac{1}{6^3}$

(ii) The probability that $X = 4$ equals

(a) $\frac{1}{6^4}$ (b) $\frac{1}{6^6}$ (c) $\frac{5^3}{6^4}$ (d) $\frac{5}{6^4}$

(iii) The probability that $X \geq 2$ equals

(a) $\frac{25}{216}$ (b) $\frac{1}{36}$ (c) $\frac{5}{6}$ (d) $\frac{25}{36}$

(iv) The value of $P(X \geq 6)$ is

(a) $\frac{5^5}{6^5}$ (b) $1 - \frac{5^3}{6^5}$ (c) $\frac{5^3 \times 61}{6^5}$ (d) $\frac{5^3}{6^4}$

(v) The probability that $X > 3$ equals

(a) $\frac{36}{25}$ (b) $\frac{5^2}{6^2}$ (c) $\frac{5}{6}$ (d) $\frac{5^3}{6^3}$

Case Study 24

In pre-board examination of class XII, commerce stream with Economics and Mathematics of a particular school, 50% of the students failed in Economics, 35% failed in Mathematics and 25% failed in both Economics and Mathematics. A student is selected at random from the class.

Based on the above information, answer the following questions.

(i) The probability that the selected student has failed in Economics, if it is known that he has failed in Mathematics, is

(a) $\frac{3}{10}$ (b) $\frac{12}{25}$ (c) $\frac{1}{4}$ (d) $\frac{5}{7}$

(ii) The probability that the selected student has failed in Mathematics, if it is known that he has failed in Economics, is

(a) $\frac{22}{25}$ (b) $\frac{12}{25}$ (c) $\frac{1}{2}$ (d) $\frac{3}{25}$

(iii) The probability that the selected student has passed in at least one of the two subjects, is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) None of these

(iv) The probability that the selected student has failed in at least one of the two subjects, is

(a) $\frac{3}{5}$ (b) $\frac{22}{25}$ (c) $\frac{2}{5}$ (d) $\frac{43}{100}$

(v) The probability that the selected student has passed in Mathematics, if it is known that he has failed in Economics, is

(a) $\frac{2}{5}$ (b) $\frac{3}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

Case Study 25

In a wedding ceremony, consists of father, mother, daughter and son line up at random for a family photograph, as shown in figure.

Based on the above information, answer the following questions.

(i) Find the probability that daughter is at one end, given that father and mother are in the middle.

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

(ii) Find the probability that mother is at right end, given that son and daughter are together.

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 0

(iii) Find the probability that father and mother are in the middle, given that son is at right end.

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

(iv) Find the probability that father and son are standing together, given that mother and daughter are standing together.

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

(v) Find the probability that father and mother are on either of the ends, given that son is at second position from the right end.

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{4}$ (d) $\frac{2}{5}$

Case Study 26

Between students of class XII of two schools A and B basketball match is organised. For which, a team from each school is chosen, say T_1 be the team of school A and T_2 be the team of school B. These teams have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probability of T_1 winning, drawing and losing a game against T_2 are $\frac{1}{2}, \frac{3}{10}$ and $\frac{1}{5}$ respectively.

Each team gets 2 points for a win, 1 point for a draw and 0 point for a loss in a game.

Let X and Y denote the total points scored by team A and B respectively, after two games.

Based on the above information, answer the following questions.

(i) $P(T_2$ winning a match against $T_1)$ is equal to

(a) 1/5 (b) 1/6 (c) 1/3 (d) none of these

(ii) $P(T_2$ drawing a match against $T_1)$ is equal to

(a) 1/2 (b) 1/3 (c) 1/6 (d) 3/10

(iii) $P(X > Y)$ is equal to

(a) 1/4 (b) 5/12 (c) 1/20 (d) 11/20

(iv) $P(X = Y)$ is equal to

(a) 11/100 (b) 1/3 (c) 29/100 (d) 1/2

(v) $P(X + Y = 8)$ is equal to

(a) 0 (b) 5/12 (c) 13/36 (d) 7/12

Case Study 27

In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms, Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03.

Based on the above information, answer the following questions.

(i) The conditional probability that an error is committed in processing given that Sonia processed the form is

(a) 0.0210 (b) 0.04 (c) 0.47 (d) 0.06

(ii) The probability that Sonia processed the form and committed an error is

(a) 0.005 (b) 0.006 (c) 0.008 (d) 0.68

(iii) The total probability of committing an error in processing the form is

(a) 0 (b) 0.047 (c) 0.234 (d) 1

(iv) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is NOT processed by Vinay is

(a) 1 (b) $\frac{30}{47}$ (c) $\frac{20}{47}$ (d) $\frac{17}{47}$

(v) Let A be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that Vinay, Sonia and Iqbal processed the form. The value of $\sum_{i=1}^{3} P(E_i \mid A)$ is

(a) 0 (b) 0.03 (c) 0.06 (d) 1

HINTS & EXPLANATIONS

7. Here, sample space = {1, 2, 3, 4, 5, 6}, $A \cap B = \{5\}$, $B \cap C = \{2, 5\}$, $A \cap C = \{1, 5\}$, $A \cap B \cap C = \{5\}$ and $\{A \cup B\} \cap C = \{1, 2, 5\}$

Also, $P(A) = \frac{2}{6}$, $P(B) = \frac{3}{6}$, $P(C) = \frac{3}{6}$

$P(A \cap B) = \frac{1}{6}$, $P(B \cap C) = \frac{2}{6}$, $P(A \cap C) = \frac{2}{6}$,

$P(A \cap B \cap C) = \frac{1}{6}$ and $P((A \cup B) \cap C) = \frac{3}{6}$

(i) (b): $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/6}{3/6} = \frac{1}{3}$

(ii) (a): $P(B|C) = \frac{P(B \cap C)}{P(C)} = \frac{2/6}{3/6} = \frac{2}{3}$

(iii) (d): $P(A \cap B|C) = \frac{P(A \cap B \cap C)}{P(C)} = \frac{1/6}{3/6} = \frac{1}{3}$

(iv) (c): $P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{2/6}{3/6} = \frac{2}{3}$

(v) (d): $P(A \cup B|C) = \frac{P((A \cup B) \cap C)}{P(C)} = \frac{3/6}{3/6} = 1$

8. Let B, R, Y and G denote the events that ball drawn is blue, red, yellow and green respectively.

$\therefore$ $P(B) = \frac{12}{35}$, $P(R) = \frac{8}{35}$, $P(Y) = \frac{10}{35}$ and $P(G) = \frac{5}{35}$

(i) (c): $P(G \cap B) = P(B) \cdot P(G \mid B) = \frac{12}{35} \cdot \frac{5}{34} = \frac{6}{119}$

(ii) (b): $P(R \cap Y) = P(Y) \cdot P(R \mid Y) = \frac{10}{35} \cdot \frac{8}{34} = \frac{8}{119}$

(iii) (a): Let E = event of drawing a first red ball and F = event of drawing a second red ball

Here, $P(E) = \frac{8}{35}$ and $P(E) = \frac{7}{34}$

$\therefore$ $P(F \cap E) = P(E) \cdot P(F \mid E) = \frac{8}{35} \cdot \frac{7}{34} = \frac{4}{85}$

(iv) (c): $P(Y' \cap G) = P(G) \cdot (Y' \mid G) = \frac{5}{35} \cdot \frac{24}{34} = \frac{12}{119}$

(v) (d): Let E = event of drawing a first non-blue ball and F = event of drawing a second non-blue ball

Here, $P(E) = \frac{23}{35}$ and $P(F) = \frac{22}{34}$

$\therefore$ $P(F \cap E) = P(E) \cdot P(F \mid E) = \frac{23}{35} \cdot \frac{22}{34} = \frac{253}{595}$

9. Here, $P(A) = 0.6$ and $P(B) = 0.8$

(i) (c): $P(A \cap B) = P(A) \cdot P(B) = (0.6)(0.8) = 0.48$

(ii) (a): $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.6 + 0.8 - 0.48 = 0.92$

(iii) (d): $P(B \mid A) = P(B)$ ($\because$ A and B are independent)
$= 0.8$

(iv) (a): $P(A \mid B) = P(A)$ ($\because$ A and B are independent)
$= 0.6$

(v) (c): $P(\text{not } A \text{ and not } B) = P(A' \cap B') = P(A \cup B)'$
$= 1 - P(A \cup B) = 1 - 0.92 = 0.08$

10. Let E be the event that the doctor visit the patient late and let A_1, A_2, A_3, A_4 be the events that the doctor comes by cab, metro, bike and other means of transport respectively.

$P(A_1) = 0.3$, $P(A_2) = 0.2$, $P(A_3) = 0.1$, $P(A_4) = 0.4$

$P(E|A_1)$ = Probability that the doctor arriving late when he comes by cab = 0.25

Similarly, $P(E \mid A_2) = 0.3$, $P(E \mid A_3) = 0.35$ and $P(E \mid A_4) = 0.1$

(i) (b): $P(A_2|E)$ = Probability that the doctor arriving late and he comes by metro

$$=\frac{P(A_2)P(E|A_2)}{\Sigma P(A_i)P(E|A_i)}$$

$$=\frac{(0.2)(0.3)}{(0.3)(0.25)+(0.2)(0.3)+(0.1)(0.35)+(0.4)(0.1)}$$

$$=\frac{0.06}{0.21}=\frac{2}{7}$$

(ii) (c) : $P(A_1|E)$ = Probability that the doctor arriving late and he comes by cab

$$=\frac{P(A_1)P(E|A_1)}{\Sigma P(A_i)P(E|A_i)}$$

$$=\frac{(0.3)(0.25)}{(0.3)(0.25)+(0.2)(0.3)+(0.1)(0.35)+(0.4)(0.1)}$$

$$=\frac{0.075}{0.21}=\frac{5}{14}$$

(iii) (d): $P(A_3|E)$ = Probability that the doctor arriving late and he comes by bike

$$=\frac{P(A_3)P(E|A_3)}{\Sigma P(A_i)P(E|A_i)}$$

$$=\frac{(0.1)(0.35)}{(0.3)(0.25)+(0.2)(0.3)+(0.1)(0.35)+(0.4)(0.1)}$$

$$=\frac{0.035}{0.21}=\frac{1}{6}$$

(iv) (c) : $P(A_4|E)$ = Probability that the doctor arriving late and he comes by other means of transport

$$=\frac{P(A_4)P(E|A_4)}{\Sigma P(A_i)P(E|A_i)}$$

$$=\frac{(0.4)(0.1)}{(0.3)(0.25)+(0.2)(0.3)+(0.1)(0.35)+(0.4)(0.1)}$$

$$=\frac{0.04}{0.21}=\frac{4}{21}$$

(v) (a) : Probability that the doctor is late by any means

$$=\frac{2}{7}+\frac{5}{14}+\frac{1}{6}+\frac{4}{21}=1$$

11. (i) (a) : We know that $\Sigma P_i = 1$

Then $0.2 + k + 2k + 3k + 2k + 0 = 1$

$\Rightarrow 8k = 1 - 0.2 = 0.8 \Rightarrow k = 0.1$

(ii) (b): P(Average study time is not more than 1 hour)

$= P(X \le 1) = P(X = 0) + P(X = 1) = 0.2 + 0.1 = 0.3$

(iii) (a) : P(Average study time is at least 3 hours)

$= P(X \ge 3) = P(X = 3) + P(X = 4) = 0.3 + 0.2 = 0.5$

(iv) (d): P(Average study time is exactly 2 hours)

$= P(X = 2) = 0.2$

(v) (c) : P(Average study time is at least 1 hour)

$= 1 - P(X = 0) = 1 - 0.2 = 0.8$

12. Let E_1 be the event that Ravi solved the puzzle and E_2 be the event that Priya solved the puzzle.

Then, $P(E_1) = 1/4$ and $P(E_2) = 1/5$

(i) (b): Since, E_1 and E_2 are independent events.

$\therefore$ P(both solved the puzzle) $= P(E_1 \cap E_2)$

$$= P(E_1)\cdot P(E_2) = \frac{1}{4}\times\frac{1}{5}=\frac{1}{20}=\frac{1}{20}\times 100\% = 5\%$$

(ii) (b): P(puzzle is solved by Ravi but not by Priya)

$$= P(\bar{E}_2)P(E_1) = \left(1-\frac{1}{5}\right)\cdot\frac{1}{4}=\frac{4}{5}\cdot\frac{1}{4}=\frac{1}{5}$$

(iii) (c) : P(puzzle is solved) $= P(E_1 \text{ or } E_2)$

$= P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$$=\frac{1}{4}+\frac{1}{5}-\frac{1}{20}=\frac{8}{20}=\frac{2}{5}$$

(iv) (c) : P(Exactly one of them solved the puzzle)

$= P[(E_1 \text{ and } \bar{E}_2) \text{ or } (E_2 \text{ and } \bar{E}_1)]$

$= P(E_1 \cap \bar{E}_2) + P(E_2 \cap \bar{E}_1)$

$= P(E_1) \times P(\bar{E}_2) + P(E_2) \times P(\bar{E}_1)$

$$=\frac{1}{4}\times\frac{4}{5}+\frac{1}{5}\times\frac{3}{4} \qquad [\because P(\bar{E}_1) = 1 - P(E_1)]$$

$$=\frac{4}{20}+\frac{3}{20}=\frac{7}{20}$$

(v) (b): P(none of them solved the puzzle)

$$= P(\bar{E}_1 \cap \bar{E}_2) = P(\bar{E}_1)\cdot P(\bar{E}_2) = \frac{3}{4}\times\frac{4}{5}=\frac{3}{5}$$

13. (i) (b): Required probability $=\dfrac{{}^{12}C_2}{{}^{51}C_2}$

$$=\frac{12\times 11}{51\times 50}=\frac{22}{425}$$

(ii) (a) : Required probability $=\dfrac{{}^{13}C_2}{{}^{51}C_2}=\dfrac{13\times 12}{51\times 50}=\dfrac{26}{425}$

(iii) (b): Clearly, $P(A|E_1)=\dfrac{{}^{12}C_2}{{}^{51}C_2}=\dfrac{22}{425}$

$$P(A|E_2)=\frac{{}^{13}C_2}{{}^{51}C_2}=\frac{26}{425}$$

$$P(A|E_3)=P(A|E_4)=\frac{26}{425}$$

$$\therefore \sum_{i=1}^{4}P(A|E_i)=\frac{22}{425}+\frac{26}{425}+\frac{26}{425}+\frac{26}{425}=\frac{100}{425}=0.24$$

(iv) (b): P(getting both king) $=\dfrac{{}^{4}C_2}{{}^{52}C_2}=\dfrac{4\times 3}{52\times 51}=\dfrac{1}{221}$

(v) (a): $P(\text{drawing a king}) = \frac{4}{52} = \frac{1}{13}$

$\therefore$ $P(\text{not drawing a king}) = 1 - \frac{1}{13} = \frac{12}{13}$

$\therefore$ Required probability $= \frac{12}{13} \times \frac{12}{13} = \frac{144}{169}$

14. (i) (d): Since, it rained only 6 days each year, therefore, probability that it rains on chosen day is $\frac{6}{366} = \frac{1}{61}$

(ii) (c): The probability that it does not rain on chosen day $= 1 - \frac{1}{61} = \frac{60}{61} = \frac{360}{366}$

(iii) (c): It is given that, when it actually rains, the weatherman correctly forecasts rain 80% of the time.

$\therefore$ Required probability $= \frac{80}{100} = \frac{8}{10} = \frac{4}{5}$

(iv) (a): Let A_1 be the event that it rains on chosen day, A_2 be the event that it does not rain on chosen day and E be the event the weatherman predict rain.

Then we have, $P(A_1) = \frac{6}{366}$, $P(A_2) = \frac{360}{366}$,

$P(E \mid A_1) = \frac{8}{10}$ and $P(E \mid A_2) = \frac{2}{10}$

Required probability $= P(A_1 \mid E)$

$$= \frac{P(A_1) \cdot P(E \mid A_1)}{P(A_1) \cdot P(E \mid A_1) + P(A_2) \cdot P(E \mid A_2)}$$

$$= \frac{\frac{6}{366} \times \frac{8}{10}}{\frac{6}{366} \times \frac{8}{10} + \frac{360}{366} \times \frac{2}{10}} = \frac{48}{768} = 0.0625$$

(v) (a): Required probability $= 1 - P(A_1 \mid E)$
$= 1 - 0.0625 = 0.9375 \approx 0.94$

15. Let B and G denote the boy and girl respectively. If a family has 4 children then each of four children can either boy or girl.

Sample space is given by

$S = \{BBBB, BBBG, BBGB, BGBB, BBGG, BGBG, BGGB, BGGG, GBBB, GBBG, GBGB, GBGG, GGBB, GGBG, GGGB, GGGG\}$

(i) (d): Let E = All children are girls.

$\therefore E = \{GGGG\}$ i.e., $n(E) = 1$

F = Elder child is a boy

$\therefore F = \{BBBB, BBBG, BBGB, BGBB, BBGG, BGBG, BGGB, BGGG\}$ i.e., $n(F) = 8$

Now, $n(E \cap F) = 0$

$\therefore P(E|F) = \frac{(E \cap F)}{n(F)} = 0$

(ii) (a): Let E = All are boys.

$\therefore$ $E = \{BBBB\}$ i.e., $n(E) = 1$

F = Two elder children are boys

$\therefore$ $F = \{BBBB, BBBG, BBGB, BBGG\}$ i.e., $n(F) = 4$

Now, $n(E \cap F) = 1$

$\therefore P(E|F) = \frac{n(E \cap F)}{n(F)} = \frac{1}{4}$

(iii) (c): Let E = Two middle children are boys.

$\therefore$ $E = \{BBBB, BBBG, GBBB, GBBG\}$ i.e., $n(E) = 4$

F = Eldest child is a girl

$\therefore$ $F = \{GBBB, GBBG, GBGB, GBGG, GGBB, GGBG, GGGB, GGGG\}$ i.e., $n(F) = 8$

Now, $n(E \cap F) = 2$

$\therefore P(E \mid F) = \frac{2}{8} = \frac{1}{4}$

(iv) (b): Let E = All are boys.

$\therefore$ $E = \{BBBB\}$ i.e., $n(E) = 1$

F = At most one child is girl.

$\therefore$ $F = \{BBBB, BBBG, BBGB, BGBB, GBBB\}$ i.e., $n(F) = 5$

Now, $n(E \cap F) = 1$

$\therefore$ $P(E \mid F) = \frac{1}{5}$

(v) (a): Let E = All are boys.

$\therefore$ $E = \{BBBB\}$ i.e., $n(E) = 1$

F = At least three of the children are boys.

$\therefore$ $F = \{BBBB, BBBG, BBGB, BGBB, GBBB\}$ i.e., $n(F) = 5$

Now, $n(E \cap F) = 1$

$\therefore$ $P(E \mid F) = \frac{1}{5}$

16. (i) (b): Total number of tickets = 50

Let event A = First ticket shows even number

and B = Second ticket shows even number

Now, $P(\text{Both tickets show even number}) = P(A) \cdot P(B|A)$

$= \frac{25}{50} \cdot \frac{24}{49} = \frac{12}{49}$

(ii) (c): Let the event A = First ticket shows odd number and B = Second ticket shows odd number

$P(\text{Both tickets show odd number})$

$= \frac{25}{50} \times \frac{24}{49} = \frac{12}{49}$

(iii) (c): Required probability = P(one number is a multiple of 4 and other is a multiple of 5)

= P(multiple of 5 on first ticket and multiple of 4 on second ticket) + P(multiple of 4 on first ticket and multiple of 5 on second ticket)

$$=\frac{10}{50}\cdot\frac{12}{49}+\frac{12}{50}\times\frac{10}{49}=\frac{12}{245}+\frac{12}{245}=\frac{24}{245}$$

(iv) (d): Required probability = P(one ticket with prime number and other ticket with a multiple of 4)

$$=2\left(\frac{15}{50}\times\frac{12}{49}\right)=\frac{36}{245}$$

(v) (b): Let the event A = First ticket shows even number and B = Second ticket shows odd number

Now, P(First ticket shows an even number and second ticket shows an odd number) = $P(A)\cdot P(B|A)$

$$=\frac{25}{50}\times\frac{25}{49}=\frac{25}{98}$$

17. **(i) (b):** Since, it is given that during prime time husband is watching T.V. 70% of the time

∴ Required probability

= 1 – P(husband is watching television during prime time)

= 1 – 0.7 = 0.3

(ii) (b): Let H be the event that husband is watching T.V., W be the event that wife is watching T.V.

Then, $P(\text{H}) = 0.7$, $P(\overline{\text{H}}) = 0.3$

$P(\text{W}\mid\text{H}) = 0.3$ and $P(\text{W}\mid\overline{\text{H}}) = 0.4$

∴ Required probability = $P(\text{H}\mid\text{W})$

$$=\frac{P(\text{H})\cdot P(\text{W}|\text{H})}{P(\text{H})\cdot P(\text{W}\mid\text{H})+P(\overline{\text{H}})P(\text{W}\mid\overline{\text{H}})}$$

$$=\frac{0.7\times0.3}{0.7\times0.3+0.4\times0.3}=\frac{0.21}{0.33}=\frac{7}{11}$$

(iii) (a): Required probability = $P(\text{H}\cap\text{W})$

$= P(\text{H})P(\text{W}\mid\text{H}) = 0.7\times0.3 = 0.21$

(iv) (b): Required probability = $P(W)=\dfrac{P(\text{H}\cap\text{W})}{P(\text{H}|\text{W})}$

$$=\frac{0.21}{7/11}=\frac{21}{100}\times\frac{11}{7}=\frac{33}{100}=0.33$$

(v) (b): Required probability = $P(\overline{\text{H}}\mid\text{W})$

$$=1-P(\text{H}\mid\text{W})=1-\frac{7}{11}=\frac{4}{11}$$

18. **(i) (c):** It is given that if India loose any match, then the probability that it wins the next match is 0.3.

∴ Required probability = 0.3

(ii) (d) : It is given that, if India loose any match, then the probability that it wins the next match is 0.3.

∴ Required probability = 1 – 0.3 = 0.7

(iii) (b): Required probability = P(India losing first match) · P(India losing second match when India has already lost first match)

= 0.4 × 0.7 = 0.28

(iv) (d): Required probability = P(India winning first match) · P(India winning second match if India has already won first match) · P(India winning third match if India has already won first two matches)

= 0.6 × 0.4 × 0.4 = 0.096

(v) (c): Required probability = P(Win 1^{st} match) P(Lose 2^{nd} match) P(Lose 3^{rd} match) + P (Lose 1^{st} match) P(Win 2^{nd} match) P(Lose 3^{rd} match) + P(Lose 1^{st} match) P(Lose 2^{nd} match) P(Win 3^{rd} match)

$= 0.6\times(1-0.4)\cdot(1-0.3)+(1-0.6)\cdot(0.3)(1-0.4)+(1-0.6)(1-0.3)(0.3)$

$= 0.6\times0.6\times0.7+0.4\times0.3\times0.6+0.4\times0.7\times0.3$

$= 0.252+0.072+0.084 = 0.408$

19. Let A be the event that the student is selected for LIC AAO, B be the event that the student is selected for SSC CGL and C be the event that the student is selected for Bank P.O.

Then, $P(A)=a$; $P(B)=b$ and $P(C)=c$

(i) (b): We have, $P(A\cup B\cup C) = 0.5$

$\Rightarrow 1-P(\overline{A\cup B\cup C})=0.5$

$\Rightarrow 1-P(\bar{A}\cap\bar{B}\cap\bar{C})=0.5$

$\Rightarrow 1-P(\bar{A})\cdot P(\bar{B})\cdot P(\bar{C})=0.5$

$\Rightarrow 1-(1-a)(1-b)(1-c)=0.5$

$\Rightarrow 1-[(1-a-b+ab)(1-c)]=0.5$

$\Rightarrow 1-[1-c-a+ac-b+bc+ab-abc]=0.5$

$\Rightarrow a+b+c-ab-bc-ca+abc=0.5$ ...(i)

(ii) (c): We have, P(selection in at least two competitive exams) = 0.4

$\Rightarrow P(A\cap B\cap\bar{C})+P(\bar{A}\cap B\cap C)+P(A\cap\bar{B}\cap C)+P(A\cap B\cap C)=0.4$

$\Rightarrow ab(1-c)+(1-a)bc+a(1-b)c+abc=0.4$

$\Rightarrow ab-abc+bc-abc+ac-abc+abc=0.4$

$\Rightarrow ab+bc+ac-2abc=0.4$ ...(ii)

(iii) (a): We have, P(selection in exactly two examinations) = 0.3

$\Rightarrow P(A\cap B\cap\bar{C})+P(\bar{A}\cap B\cap C)+P(A\cap\bar{B}\cap C)=0.3$

$\Rightarrow ab(1-c)+(1-a)bc+a(1-b)c=0.3$

$\Rightarrow ab+bc+ac-3abc=0.3$ ...(iii)

On subtracting (iii) from (ii), we get

$abc = 0.1$

(iv) (b): On substituting the value of abc in (ii), we get
$ab + bc + ac = 0.6$

(v) (a): On substituting the value of $ab + bc + ac$ and abc in (i), we get
$a + b + c - 0.6 + 0.1 = 0.5$
$\Rightarrow \quad a + b + c = 1$

20. Let E_1, E_2, E_3 be the events of drawing a bolt produced by machine A, B and C, respectively.

Then $P(E_1) = \frac{30}{100} = \frac{3}{10}, P(E_2) = \frac{20}{100} = \frac{1}{5}$

and $P(E_3) = \frac{50}{100} = \frac{1}{2}$

Also, let E be the event of drawing a defective bolt.

Then $P(E|E_1) = \frac{5}{100} = \frac{1}{20}, P(E|E_2) = \frac{2}{100} = \frac{1}{50}$,

$P(E|E_3) = \frac{4}{100} = \frac{1}{25}$

(i) (b): Probability that the defective bolt is manufactured by machine $A = P(E_1|E)$

$$= \frac{P(E|E_1) \times P(E_1)}{P(E|E_1) \cdot P(E_1) + P(E|E_2) \cdot P(E_2) + P(E|E_3) \cdot P(E_3)}$$

[Using Bayes' Theorem]

$$= \frac{\frac{1}{20} \times \frac{3}{10}}{\frac{1}{20} \times \frac{3}{10} + \frac{1}{50} \times \frac{1}{5} + \frac{1}{25} \times \frac{1}{2}} = \frac{3}{200} \times \frac{1000}{39} = \frac{5}{13}$$

(ii) (b): Probability that the defective bolt is manufactured by machine $B = P(E_2|E)$

$$= \frac{P(E|E_2) \cdot P(E_2)}{P(E|E_1) \cdot P(E_1) + P(E|E_2) \cdot P(E_2) + P(E|E_3) \cdot P(E_3)}$$

$$= \frac{\frac{1}{50} \times \frac{1}{5}}{\left(\frac{1}{20} \times \frac{3}{10}\right) + \left(\frac{1}{50} \times \frac{1}{5}\right) + \left(\frac{1}{25} \times \frac{1}{2}\right)}$$

$$= \frac{1}{250} \times \frac{1000}{39} = \frac{4}{39} \approx 0.1$$

(iii) (c): Probability that the defective bolt is manufactured by machine $C = P(E_3|E)$

$$= \frac{P(E|E_3) \cdot P(E_3)}{P(E|E_1) \cdot P(E_1) + P(E|E_2) \cdot P(E_2) + P(E|E_3) \cdot P(E_3)}$$

$$= \frac{\frac{1}{25} \times \frac{1}{2}}{\left(\frac{1}{20} \times \frac{3}{10}\right) + \left(\frac{1}{50} \times \frac{1}{5}\right) + \left(\frac{1}{25} \times \frac{1}{2}\right)}$$

$$= \frac{1}{50} \times \frac{1000}{39} = \frac{20}{39}$$

(iv) (a): Probability that the defective bolt is not manufactured by machine B *i.e.*, it is manufactured by machine A or $C = \frac{15}{39} + \frac{20}{39} = \frac{35}{39}$

(v) (c): Required probability $= \frac{15}{39} + \frac{4}{39} = \frac{19}{39} \approx 0.5$

21. We have, $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

(i) (a): $P(E|E_1)$ = Probability of drawing red and white ball, if box I is selected.

$= P(\text{red}) \times P(\text{white}) + P(\text{white}) \times P(\text{red})$

$= \frac{2}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{2}{6} = \frac{4}{36} = \frac{1}{9}$

(ii) (a): $P(E|E_2)$ = Probability of drawing red and white balls, if box II is selected.

$= P(\text{red}) \times P(\text{white}) + P(\text{white}) \times P(\text{red})$

$= \frac{3}{6} \times \frac{2}{6} + \frac{2}{6} \times \frac{3}{6} = \frac{12}{36} = \frac{1}{3}$

(iii) (c): $P(E|E_3)$ = Probability of drawing red and white balls, if box III is selected.

$= P(\text{red}) \times P(\text{white}) + P(\text{white}) \times P(\text{red})$

$= \frac{1}{6} \times \frac{3}{6} + \frac{3}{6} \times \frac{1}{6} = \frac{6}{36} = \frac{1}{6}$

(iv) (d): $\sum_{i=1}^{3} P(E|E_i) = P(E|E_1) + P(E|E_2) + P(E|E_3)$

$= \frac{4}{36} + \frac{12}{36} + \frac{6}{36} = \frac{4+12+6}{36} = \frac{22}{36} = \frac{11}{18}$

(v) (b): Using Bayes' theorem,

$$P(E_2|E) = \frac{P(E|E_2) \cdot P(E_2)}{P(E|E_1) \cdot P(E_1) + P(E|E_2) \cdot P(E_2) + P(E|E_3) \cdot P(E_3)}$$

$$= \frac{\frac{1}{3} \times \frac{1}{3}}{\left(\frac{1}{9} \times \frac{1}{3}\right) + \left(\frac{1}{3} \times \frac{1}{3}\right) + \left(\frac{1}{6} \times \frac{1}{3}\right)}$$

$$= \frac{\frac{1}{3}}{\frac{1}{9} + \frac{1}{3} + \frac{1}{6}} = \frac{\frac{1}{3}}{\frac{11}{18}} = \frac{1}{3} \times \frac{18}{11} = \frac{6}{11}$$

22. Let A be the event that Nisha is selected and B be the event that Ayushi is selected. Then, we have

$P(A) = \frac{1}{3}$

$\Rightarrow \quad P(\overline{A}) = 1-\frac{1}{3}=\frac{2}{3} = P$ (Nisha is not selected)

$P(B)=\frac{1}{2}$

$\Rightarrow \quad P(\overline{B}) = 1-\frac{1}{2}=\frac{1}{2} = P$ (Ayushi is not selected)

(i) (c) : P (both are selected) $= P(A \cap B) = P(\overline{A})\cdot P(\overline{B})$

$$= \frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$$

(ii) (d): P (both are rejected) $= P(\overline{A}\cap\overline{B}) = P(\overline{A})\cdot P(\overline{B})$

$$=\frac{2}{3}\times\frac{1}{2}=\frac{1}{3}$$

(iii) (d): P (only one of them is selected)

$$=P(A\cap\overline{B})+P(\overline{A}\cap B)=P(A)\cdot P(\overline{B})+P(\overline{A})\cdot P(B)$$

$$=\frac{1}{3}\times\frac{1}{2}+\frac{2}{3}\times\frac{1}{2}=\frac{1}{6}+\frac{2}{6}=\frac{3}{6}=\frac{1}{2}$$

(iv) (a) : P (at least one of them is selected)

$= 1 - P$ (Both are rejected) $= 1-\frac{1}{3}=\frac{2}{3}$

(v) (a) : Let E_1 be the event that Nisha is selected for post I and E_2 be the event that Nisha is selected for post II.

$\therefore \quad P$ (Nisha is selected for atleast one post)

$= P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$$=\frac{1}{6}+\frac{1}{5}-\frac{1}{6}\times\frac{1}{5}=\frac{10}{30}=\frac{1}{3}$$

23. (i) (b): $P(X = 2)$ = (Probability of not getting six at first throw) × (Probability of getting six at second throw)

$$=\frac{5}{6}\times\frac{1}{6}=\frac{5}{6^2}$$

(ii) (c) : $P(X = 4) = \frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}=\frac{5^3}{6^4}$

(iii) (c) : $P(X \geq 2) = 1 - P(X < 2)$

$= 1 - P(X = 1) = 1-\frac{1}{6}=\frac{5}{6}$

(iv) (a): $P(X\geq 6)=\left(\frac{5}{6}\right)^5\times\frac{1}{6}+\left(\frac{5}{6}\right)^6\times\frac{1}{6}+\ldots\infty$

$$=\frac{5^5}{6^6}\left[1+\frac{5}{6}+\left(\frac{5}{6}\right)^2+\ldots\infty\right]=\frac{5^5}{6^6}\left[\frac{1}{1-\frac{5}{6}}\right]=\left(\frac{5}{6}\right)^5$$

(v) (d): $P(X\geq 4)=\left(\frac{5}{6}\right)^3\cdot\frac{1}{6}+\left(\frac{5}{6}\right)^4\cdot\frac{1}{6}+\ldots\ldots$

$$=\frac{5^3}{6^4}\left[1+\left(\frac{5}{6}\right)+\left(\frac{5}{6}\right)^2+\ldots\ldots\right]$$

$$=\frac{5^3}{6^4}\left[\frac{1}{1-\left(\frac{5}{6}\right)}\right]=\left(\frac{5^3}{6^4}\right).6=\frac{5^3}{6^3}$$

24. Let E denote the event that student has failed in Economics and M denote the event that student has failed in Mathematics.

$\therefore \quad P(E)=\frac{50}{100}=\frac{1}{2},\ P(M)=\frac{35}{100}=\frac{7}{20}$

and $P(E\cap M)=\frac{25}{100}=\frac{1}{4}$

(i) (d): Required probability $= P(E|M)$

$$=\frac{P(E\cap M)}{P(M)}=\frac{1/4}{7/20}=\frac{1}{4}\times\frac{20}{7}=\frac{5}{7}$$

(ii) (c) : Required probability $= P(M|E)$

$$=\frac{P(M\cap E)}{P(E)}=\frac{1/4}{1/2}=\frac{1}{2}$$

(iii) (c) : Required probability $= P(E' \cup M')$

$= P(E \cap M)'$

$$=1-P(E\cap M)=1-\frac{1}{4}=\frac{3}{4}$$

(iv) (a) : Required probability $= P(E \cup M)$

$= P(E) + P(M) - P(E \cap M)$

$$=\frac{5}{10}+\frac{7}{20}-\frac{1}{4}=\frac{12}{20}=\frac{3}{5}$$

(v) (d): Required probability $= P(M'|E)$

$$=\frac{P(M'\cap E)}{P(E)}=\frac{P(E)-P(E\cap M)}{P(E)}=\frac{\frac{1}{2}-\frac{1}{4}}{\frac{1}{2}}=\frac{1}{2}$$

25. Sample space is given by

{*MFSD, MFDS, MSFD, MSDF, MDFS, MDSF, FMSD, FMDS, FSMD, FSDM, FDMS, FDSM, SFMD, SFDM, SMFD, SMDF, SDMF, SDFM DFMS, DFSM, DMSF, DMFS, DSMF, DSFM*},

where F, M, D and S represent father, mother, daughter and son respectively.

$\therefore \quad n(S) = 24$

(i) (a): Let A denotes the event that daughter is at one end.

$\therefore \quad n(A) = 12$

and B denotes the event that father and mother are in the middle.

$\therefore \quad n(B) = 4$

Also, $n(A \cap B) = 4$

$$\therefore \quad P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{4/24}{4/24} = 1$$

(ii) (b): Let A denotes the event that mother is at right end.

$\therefore \quad n(A) = 6$

and B denotes the event that son and daughter are together.

$\therefore \quad n(B) = 12$

Also, $n(A \cap B) = 4$

$$\therefore \quad P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{4/24}{12/24} = \frac{1}{3}$$

(iii) (c): Let A denotes the event that father and mother are in the middle.

$\therefore \quad n(A) = 4$

and B denotes the event that son is at right end.

$\therefore \quad n(B) = 6$

Also, $n(A \cap B) = 2$

$$\therefore \quad P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2/24}{6/24} = \frac{1}{3}$$

(iv) (d): Let A denotes the event that father and son are standing together.

$\therefore \quad n(A) = 12$

and B denotes the event that mother and daughter are standing together.

$\therefore \quad n(B) = 12$

Also, $n(A \cap B) = 8$

$$\therefore \quad P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{8/24}{12/24} = \frac{2}{3}$$

(v) (a): Let A denotes the event that father and mother are on either of the ends.

$\therefore \quad n(A) = 4$

and B denotes the event that son is at second position from the right end.

$\therefore \quad n(B) = 6$

Also, $n(A \cap B) = 2$

$$\therefore \quad P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2/24}{6/24} = \frac{1}{3}$$

26. **(i) (a):** Clearly, $P(T_2$ winning a match against $T_1)$

$= P(T_1 \text{ losing}) = \frac{1}{5}$

(ii) (d): Clearly, $P(T_2$ drawing a match against $T_1)$

$= P(T_1 \text{ drawing}) = \frac{3}{10}$

(iii) (d): According to given information, we have the following possibilities for the values of X and Y.

X	4	3	2	1	0
Y	0	1	2	3	4

Now, $P(X > Y) = P(X = 4, Y = 0) + P(X = 3, Y = 1)$

$= P(T_1 \text{ win})\, P(T_1 \text{ win}) + P(T_1 \text{ win})\, P(\text{match draw})$
$+ P(\text{match draw})\, P(T_1 \text{ win})$

$$= \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{3}{10} + \frac{3}{10}\cdot\frac{1}{2} = \frac{5+3+3}{20} = \frac{11}{20}$$

(iv) (c): $P(X = Y) = P(X = 2, Y = 2)$

$= P(T_1 \text{ win})\, P(T_2 \text{ win}) + P(T_2 \text{ win})\, P(T_1 \text{ win})$
$+ P(\text{match draw})\, P(\text{match draw})$

$$= \frac{1}{2}\cdot\frac{1}{5} + \frac{1}{5}\cdot\frac{1}{2} + \frac{3}{10}\cdot\frac{3}{10} = \frac{1}{10} + \frac{1}{10} + \frac{9}{100} = \frac{29}{100}$$

(v) (a): From the given information, it is clear that maximum sum of X and Y can be 4, therefore $P(X + Y = 8) = 0$

27. Let A be the event of commiting an error and E_1, E_2 and E_3 be the events that Vinay, Sonia and Iqbal processed the form.

(i) (b): Required probability $= P(A|E_2)$

$$= \frac{P(A \cap E_2)}{P(E_2)} = \frac{\left(0.04 \times \frac{20}{100}\right)}{\left(\frac{20}{100}\right)} = 0.04$$

(ii) (c): Required probability $= P(A \cap E_2)$

$$= 0.04 \times \frac{20}{100} = 0.008$$

(iii) (b): Total probability is given by

$P(A) = P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2) + P(E_3)\cdot P(A|E_3)$

$$= \frac{50}{100} \times 0.06 + \frac{20}{100} \times 0.04 + \frac{30}{100} \times 0.03 = 0.047$$

(iv) (d): Using Bayes' theorem, we have

$$P(E_1 | A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2) + P(E_3)\cdot P(A|E_3)}$$

$$= \frac{0.5 \times 0.06}{0.5 \times 0.06 + 0.2 \times 0.04 + 0.3 \times 0.03} = \frac{30}{47}$$

$\therefore \quad$ Required probability $= P(\bar{E}_1 | A)$

$$= 1 - P(E_1 | A) = 1 - \frac{30}{47} = \frac{17}{47}$$

(v) (d): $\sum_{i=1}^{3} P(E_i | A) = P(E_1|A) + P(E_2|A) + P(E_3|A)$

$= 1 \qquad [\because$ Sum of posterior probabilities is 1]