Chapter - Oscillations

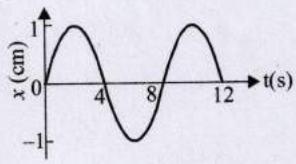
Topic-1: Displacement, Phase, Velocity and Acceleration in S.H.M.

1 MCQs with One Correct Answer

1. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A \sin \omega t$ and $x_2(t) = A\sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B \sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are **[2011]**

   (a) $\sqrt{2}A, \frac{3\pi}{4}$ (b) $A, \frac{4\pi}{3}$ (c) $\sqrt{3}A, \frac{5\pi}{6}$ (d) $A, \frac{\pi}{3}$

2. The x-t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at $t = 4/3$ s is **[2009]**



   (a) $\frac{\sqrt{3}}{32}\pi^2$ cm/s^2 (b) $\frac{-\pi^2}{32}$ cm/s^2

   (c) $\frac{\pi^2}{32}$ cm/s^2 (d) $-\frac{\sqrt{3}}{32}\pi^2$cm/s^2

6 MCQs with One or More than One Correct Answer

3. The function $x = A\sin^2 \omega t + B\cos^2 \omega t + C\sin \omega t \cos \omega t$ represent SHM for which of the option(s)

   (a) for all value of A, B and C $(C \neq 0)$ **(2006 - 5M, –1)**

   (b) $A = B, C = 2B$

   (c) $A = -B, C = 2B$

   (d) $A = B, C = 0$

4. Three simple harmonic motions in the same direction having the same amplitude a and same period are superposed. If each differs in phase from the next by 45°, then. **(1999S - 3marks)**

   (a) the resultant amplitude is $(1+\sqrt{2})a$

   (b) the phase of the resultant motion relative to the first is 90°

   (c) the energy associated with the resulting motion is $(3+2\sqrt{2})$ times the energy associated with any single motion

   (d) the resulting motion is not simple harmonic.

Topic-2: Energy in Simple Harmonic Motion

1 MCQs with One Correct Answer

1. A particle free to move along the x-axis has potential energy given by $U(x) = k[1-\exp(-x^2)]$ for $-\infty \leq x \leq +\infty$, where k is a positive constant of appropriate dimensions. Then **[1999S - 2marks]**

   (a) at points away from the origin, the particle is in unstable equilibrium

   (b) for any finite nonzero value of x, there is a force directed away from the origin

   (c) if its total mechanical energy is $k/2$, it has its minimum kinetic energy at the origin.

   (d) for small displacements from $x = 0$, the motion is simple harmonic

2. A particle executes simple harmonic motion with a frequency. f. The frequency with which its kinetic energy oscillates is **[1987 - 2marks]**

   (a) $f/2$ (b) f (c) $2f$ (d) $4f$

4 Fill in the Blanks

3. An object of mass 0.2 kg executes simple harmonic oscillation along the x-axis with a frequency of $(25/\pi)$ Hz. At the position $x = 0.04$, the object has Kinetic energy of 0.5 J and potential energy 0.4 J. The amplitude of oscillations is ....m. **[1994 - 2marks]**

6 MCQs with One or More than One Correct Answer

4. A linear harmonic oscillator of force constant 2×10^6 N/m and amplitude 0.01 m has a total mechanical energy of 160 J. Its **[1989 - 2 Mark]**

(a) maximum potential energy is 100 J
(b) maximum kinetic energy is 100 J
(c) maximum potential energy is 160 J
(d) maximum potential energy is zero

Topic-3: Time Period, Frequency, Simple Pendulum and Spring Pendulum

1 MCQs with One Correct Answer

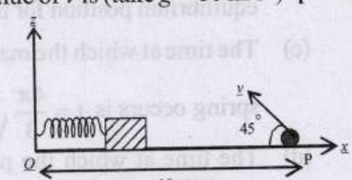
1. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency **[2013]**

(a) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$ (b) $\frac{1}{2\pi}\frac{V_0 M P_0}{A^2\gamma}$

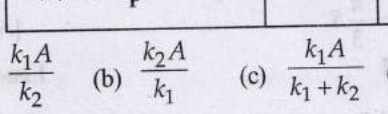
(c) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$

2. A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at $t = 0$. It then executes simple harmonic motion with angular frequency $\omega = \pi/3$ rad/s. Simultaneously at $t = 0$, a small pebble is projected with speed v form point P at an angle of 45° as shown in the figure. Point P is at a horizontal distance of 10 m from O. If the pebble hits the block at $t = 1$ s, the value of v is (take $g = 10$ m/s^2) **[2012]**

(a) $\sqrt{50}$ m/s
(b) $\sqrt{51}$ m/s
(c) $\sqrt{52}$ m/s
(d) $\sqrt{53}$ m/s



3. The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is **[2009]**



(a) $\frac{k_1 A}{k_2}$ (b) $\frac{k_2 A}{k_1}$ (c) $\frac{k_1 A}{k_1 + k_2}$ (d) $\frac{k_2 A}{k_1 + k_2}$

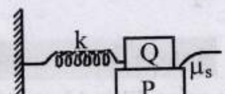
4. A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation $y = Kt^2$, $(K = 1$ m/s$^2)$ where y is the vertical displacement. The time period now becomes T_2. The ratio of $\frac{T_1^2}{T_2^2}$ is

$(g = 10$ m/s$^2)$ **[2005S]**

(a) 5/6 (b) 6/5 (c) 1 (d) 4/5

5. A block P of mass m is placed on a horizontal frictionless plane. A second block of same mass m is placed on it and is connected to a spring of spring constant k, the two blocks are pulled by distance A. Block Q oscillates without slipping. What is the maximum value of frictional force between the two blocks. **[2004S]**

(a) $kA/2$
(b) kA
(c) $\mu_s mg$
(d) zero



6. A particle of mass m is executing oscillations about the origin on the x axis. Its potential energy is $V(x) = k|x|^3$ where k is a positive constant. If the amplitude of oscillation is a, then its time period T is **[1998S - 2marks]**

(a) proportional to $1/\sqrt{a}$ (b) independent of a

(c) proportional to $\sqrt{a}$ (d) proportional to $a^{3/2}$

7. One end of a long metallic wire of length L is tied to the ceiling. The other end is tied to a massless spring of spring constant K. A mass m hangs freely from the free end of the spring. The area of cross-section and the Young's modulus of the wire are A and Y respectively. If the mass is slightly pulled down and released, it will oscillate with a time period T equal to: **[1993-2marks]**

(a) $2\pi(m/K)^{1/2}$ (b) $2\pi\sqrt{\frac{m(YA+KL)}{YAK}}$

(c) $2\pi[(mYA/KL)]^{1/2}$ (d) $2\pi[(mL/YA)]^{1/2}$

8. A highly rigid cubical block A of small mass M and side L is fixed rigidly on to another cubical block B of the same dimensions and of low modulus of rigidity η such that the lower face of A completely covers the upper face of B. The lower face of B is rigidly held on a horizontal surface. A small force F is applied perpendicular to one of the sides faces of A. After the force is withdrawn, block A executes small oscillations the time period of which is given by **[1992 - 2mark]**

(a) $2\pi\sqrt{M\eta L}$ (b) $2\pi\sqrt{\frac{M\eta}{L}}$

(c) $2\pi\sqrt{\frac{ML}{\eta}}$ (d) $2\pi\sqrt{\frac{M}{\eta L}}$

9. A uniform cylinder of length L and mass M having cross sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half- submerged in a liquid of density ρ at equilibrium position. When the cylinder is given a small downward push and released it starts oscillating vertically with small amplitude. If the force constant of the spring is k, the frequency of oscillation of the cylinder is **[1990 - 2mark]**

(a) $\frac{1}{2\pi}\left(\frac{k - A\rho g}{M}\right)^{1/2}$ (b) $\frac{1}{2\pi}\left(\frac{k + A\rho g}{M}\right)^{1/2}$

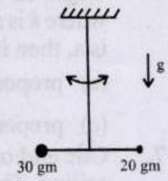
(c) $\frac{1}{2\pi}\left(\frac{k + \rho gL}{M}\right)^{1/2}$ (d) $\frac{1}{2\pi}\left(\frac{k + A\rho g}{A\rho g}\right)^{1/2}$

10. Two bodies M and N of equal masses are suspended from two separate massless springs of spring constants k_1 and k_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of vibration of M to that of N is **[1988 - 1mark]**

(a) $\frac{k_1}{k_2}$ (b) $\sqrt{k_1 / k_2}$ (c) $\frac{k_2}{k_1}$ (d) $\sqrt{k_2 / k_1}$

2 Integer Value Answer

11. Two point–like objects of masses 20 gm and 30 gm are fixed at the two ends of a rigid massless rod of length 10 cm. This system is suspended vertically from a rigid ceiling using a thin wire attached to its centre of mass, as shown in the figure. The resulting torsional pendulum undergoes small oscillations. The torsional constant of the wire is 1.2×10^{-8} N m rad^{-1}. The angular frequency of the oscillations in $n \times 10^{-3}$ rad s^{-1}. The value of n is ______.

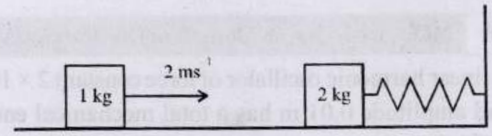


[Adv. 2023]

3 Numeric Answer

12. On a frictionless horizontal plane, a bob of mass $m = 0.1\ kg$ is attached to a spring with natural length $l_0 = 0.1\ m$. The spring constant is $k_1 = 0.009\ Nm^{-1}$ when the length of the spring $l > l_0$ and is $k_2 = 0.016\ Nm^{-1}$ when $l < l_0$. Initially the bob is released from $l = 0.15\ m$. Assume that Hooke's law remains valid throughout the motion. If the time period of the full oscillation is $T = (n\pi)\ s$, then the integer closest to n is **[Adv. 2022]**

13. A spring-block system is resting on a frictionless floor as shown in the figure. The spring constant is 2.0 Nm^{-1} and the mass of the block is 2.0 kg. Ignore the mass of the spring. Initially the spring is in an unstretched condition. Another block of mass 1.0 kg moving with a speed of 2.0 ms^{-1} collides elastically with the first block. The collision is such that the 2.0 kg block does not hit the wall. The distance, in metres, between the two blocks when the spring returns to its unstretched position for the first time after the collision is ______. **[Adv. 2018]**



6 MCQs with One or More than One Correct Answer

14. A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0. Consider two cases: (i) when the block is at x_0; and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle with mass m(<M) is softly placed on the block after which they stick to each other. Which of the following statement(s) is (are) true about the motion after the mass m is placed on the mass M? **[Adv. 2016]**

(a) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged.

(b) The final time period of oscillation in both the cases is same.

(c) The total energy decreases in both the cases.

(d) The instantaneous speed at x_0 of the combined masses decreases in both the cases

15. A particle of mass m is attached to one end of a mass-less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time t = 0 with an initial velocity u_0. When the speed of the particle is 0.5 u_0, it collides elastically with a rigid wall. After this collision **[Adv. 2013]**

(a) The speed of the particle when it returns to its equilibrium position is u_0

(b) The time at which the particle passes through the equilibrium position for the first time is $t = \pi\sqrt{\frac{m}{k}}$

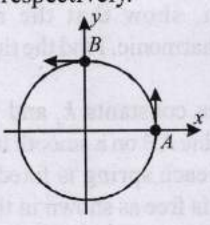
(c) The time at which the maximum compression of the spring occurs is $t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$

(d) The time at which the particle passes through the equilibrium position for the second time is $t = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$

16. List I describes four systems, each with two particles A and B in relative motion as shown in figures. List II gives possible magnitudes of their relative velocities (in m s^{-1}) at time $t = \frac{\pi}{3}S$. **[Adv. 2022]**

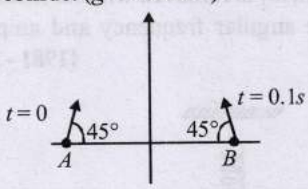
List-I	List-II
(I) A and B are moving on a horizontal circle of radius 1 m with uniform angular speed $\omega = 1\ rad\,s^{-1}$. The initial	(P) $\frac{\sqrt{3}+1}{2}$

angular positions of A and B at time $t = 0$ are $\theta = 0$ and $\theta = \frac{\pi}{2}$, respectively.



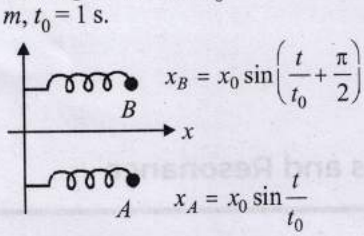
(II) Projectiles A and B are fired (in the same vertical plane) at $t = 0$ and $t = 0.1\ s$ respectively, with the same speed $v = \frac{5\pi}{\sqrt{2}}\ m\ s^{-1}$ and at 45° from the horizontal plane. The initial separation between A and B is large enough so that they do not collide. ($g = 10\ m\ s^{-2}$).

(Q) $\frac{(\sqrt{3}-1)}{\sqrt{2}}$



(III) Two harmonic oscillators A and B moving in the x direction according to $x_A = x_0 \sin\frac{t}{t_0}$ and $x_B = x_0 \sin\left(\frac{t}{t_0} + \frac{\pi}{2}\right)$ respectively, starting $t = 0$. Take $x_0 = 1\ m$, $t_0 = 1$ s.

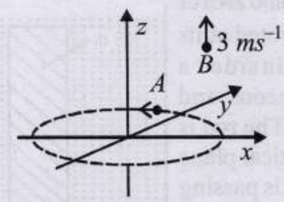
(R) $\sqrt{10}$



(IV) Particle A is rotating in a horizontal circular path of radius 1 m on the xy plane, with constant angular speed $\omega = 1\ rad\ s^{-1}$. Particle B is moving up at a constant speed 3 $m\ s^{-1}$ in the vertical direction as shown in the figure. (Ignore gravity.)

(S) $\sqrt{2}$

(T) $\sqrt{25\pi^2 + 1}$



Which one of the following options is correct?

(a) (I) → (R); (II) → (T); (III) → (P); (IV) → (S)

(b) (I) → (S); (II) → (P); (III) → (Q); (IV) → (R)

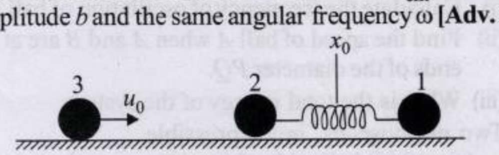
(c) (I) → (S); (II) → (T); (III) → (P); (IV) → (R)

(d) (I) → (T); (II) → (P); (III) → (R); (IV) → (S)

8 Comprehension/Passage Based Questions

Passage

Two particles, 1 and 2, each of mass m, are connected by a massless spring, and are on a horizontal frictionless plane, as shown in the figure. Initially, the two particles, with their center of mass at x_0, are oscillating with amplitude a and angular frequency ω. Thus, their positions at time t are given by $x_1(t) = (x_0 + d) + a \sin \omega t$ and $x_2(t) = (x_0 - d) - a \sin \omega t$, respectively, where $d > 2_a$. Particle 3 of mass m moves towards this system with speed $u_0 = \frac{a\omega}{2}$, and undergoes instantaneous elastic collision with particle 2, at time t_0. Finally, particles 1 and 2 acquire a center of mass speed v_{cm} and oscillate with amplitude b and the same angular frequency ω **[Adv. 2024]**

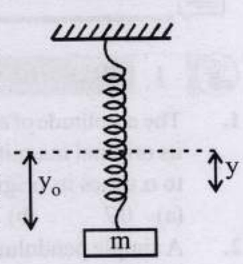


17. If the collision occurs at time $t_0 = 0$, the value of $\frac{v_{cm}}{(a\omega)}$ will be ________.

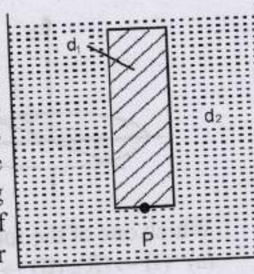
18. If the collision occurs at time $t_0 = \frac{\pi}{(2\omega)}$, then the value of $\frac{4b^2}{a^2}$ will be ________.

10 Subjective Problems

19. A small body attached to one end of a vertically hanging spring is performing SHM about its mean position with angular frequency ω and amplitude a. If at a height y^* from the mean position, the body gets detached from the spring, calculate the value of y^* so that the height H attained by the mass is maximum. The body does not interact with the spring during its subsequent motion after detachment. ($a\omega^2 > g$) **[2005 - 4 Marks]**



20. A thin rod of length L and area of cross-section S is pivoted at its lowest point P inside a stationary, homogeneous and non-viscous liquid. The rod is free to rotate in a vertical plane about a horizontal axis passing through P. The density d_1 of the material of the rod is smaller than the density d_2 of the liquid. The rod is displaced by a small angle θ from its equilibrium position and then released. Show that the motion of the rod is simple harmonic and determine its angular frequency in terms of the given parameters. **[1996 - 5 Marks]**



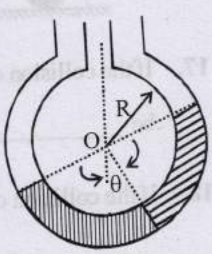
21. Two identical balls A and B each of mass 0.1 kg, are attached to two identical massless springs. The spring-mass system is constrained to move inside a rigid smooth pipe bent in the form of a circle as shown in Fig. The pipe is fixed in a horizontal plane. The centres of the balls can move in a circle of radius $0.06\,\pi$ meter. Each spring has a natural length z of 0.06π meter and spring constant 0.1 N/m. Initially, both the balls are displaced by an angle $\theta = \pi/6$ radian with respect to the diameter PQ of the circle (as shown in Fig.) and released from rest. **[1993 - 6 marks]**

(i) Calculate the frequency of oscillation of ball B.

(ii) Find the speed of ball A when A and B are at the two ends of the diameter PQ.

(iii) What is the total energy of the system

22. Two non-viscous, incompressible and immiscible liquids of densities ρ and $1.5\,\rho$ are poured into the two limbs of a circular tube of radius R and small cross section kept fixed in a vertical plane as shown in fig. Each liquid occupies one fourth the circumference of the tube.

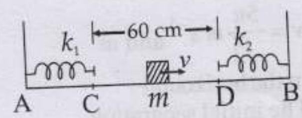


[1991 - 4 + 4 marks]

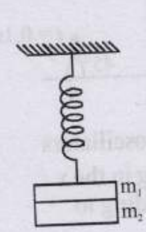
(a) Find the angle θ that the radius to the interface makes with the vertical in equilibrium position.

(b) If the whole is given a small displacement from its equilibrium position, show that the resulting oscillations are simple harmonic. Find the time period of these oscillations.

23. Two light springs of force constants k_1 and k_2 and a block of mass m are in one line AB on a smooth horizontal table such that one end of each spring is fixed on rigid supports and the other end is free as shown in the figure. The distance CD between the free ends of the springs is 60 cms. If the block moves along AB with a velocity 120 cm/sec in between the springs, calculate the period of oscillation of the block ($k_1 = 1.8$ N/m, $k_2 = 3.2$ N/m, $m =$ 200 gm) **[1985 - 6 Marks]**



24. Two masses m_1 and m_2 are suspended together by a massless spring of spring constant k. When the masses are in equilibrium, m_1 is removed without disturbing the system. Find the angular frequency and amplitude of oscillation of m_2. **[1981 - 3 marks]**



25. A mass M attached to a spring, oscillates with a period of 2sec. If the mass is increased by 2 kg the period increases by one sec. Find the initial mass M assuming that Hook's Law is obeyed. **[1979]**

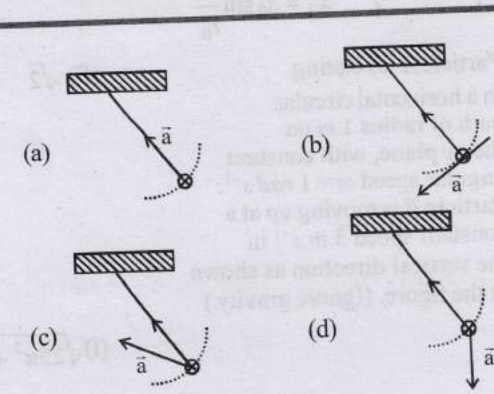
Topic-4: Damped, Forced Oscillations and Resonance

1 MCQs with One Correct Answer

1. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5s. In another 10s it will decrease to α times its original magnitude, where α equals **[2013]**

(a) 0.7 (b) 0.81 (c) 0.729 (d) 0.6

2. A simple pendulum is oscillating without damping. When the displacement of the bob is less than maximum, its acceleration vector $\vec{a}$ is correctly shown in : **[2002S]**



3. A particle executes simple harmonic motion between $x = -A$ and $x = +A$. The time taken for it to go from 0 to $A/2$ is T_1 and to go from $A/2$ to A is T_2. Then **[2001S]**

(a) $T_1 < T_2$ (b) $T_1 > T_2$

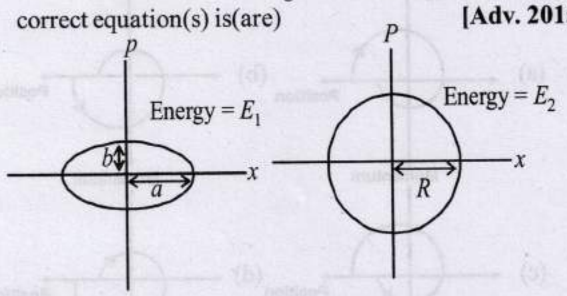
(c) $T_1 = T_2$ (d) $T_1 = 2T_2$

2 Integer Value Answer

4. A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1m and its crosssectional area is 4.9×10^{-7} m^2. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad s^{-1}. If the Young's modulus of the material of the wire is $n \times 10^9$ Nm^{-2}, the value of n is **[2010]**

6 MCQs with One or More than One Correct Answer

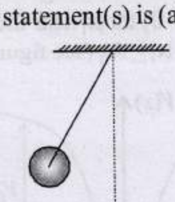
5. Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in the figures. If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n$, then the correct equation(s) is(are) **[Adv. 2015]**



(a) $E_1\omega_1 = E_2\omega_2$ (b) $\frac{\omega_2}{\omega_1} = n^2$

(c) $\omega_1\omega_2 = n^2$ (d) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

6. A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disc of mass 'M' and radius 'R' (<L) is attached at its center to the free end of the rod. Consider two ways the disc is attached: (case A). The disc is not free to rotate about its centre and (case B) the disc is free to rotate about its centre.
The rod disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is (are) true? **[2011]**



(a) Restoring torque in case A = Restoring torque in case B

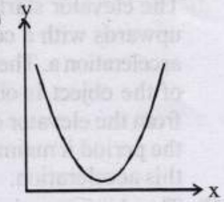
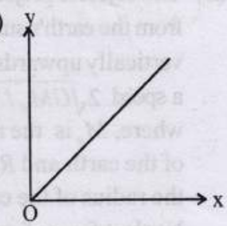
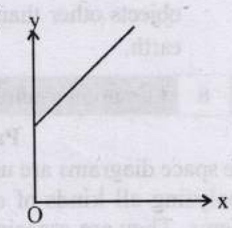
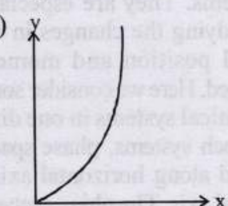
(b) restoring torque in case A < Restoring torque in case B

(c) Angular frequency for case A > angular frequency for case B.

(d) Angular frequency for case A < Angular frequency for case B.

7 Match the Following

7. **[2008]**

Column I	Column II
(A) Potential energy of a simple pendulum (y axis) as a function of displacement (x axis)	(p) 
(B) Displacement (y axis) as a function of time (x axis) for a one dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction.	(q) 
(C) Range of a projectile (y axis) as a function of its velocity (x axis) when projected at a fixed angle.	(r) 
(D) The square of the time period (y axis) of a simple pendulum as a function of its length (x axis)	(s) 

8. **[2007]**

Column I	Column II
(A) The object moves on the x -axis under conservative force in such a way that its "speed" and position" satisfy $v = c_1\sqrt{c_2 - x^2}$ where c_1 and c_2 are positive constants.	(p) The object executes a simple harmonic motion.
(B) The object moves on the x- axis in such a way that its velocity and its displacement from the origin satisfy $v = -kx$, where k is a positive constant.	(q) The object does not change its direction.

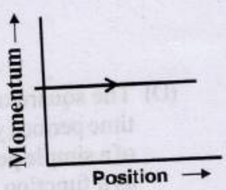
(C) The object is attached to one end of a mass-less spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration a. The motion of the object is observed from the elevator during the period it maintains this acceleration. — (r) The kinetic energy of the object keeps on decreasing

(D) The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{GM_e / R_e}$, where, M_e is the mass of the earth and R_e is the radius of the earth, Neglect forces from objects other than the earth. — (s) The object can change its direction only once.

8 Comprehension/Passage Based Questions

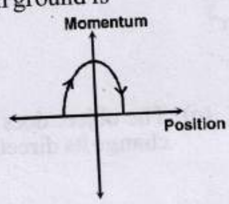
Passage- 1

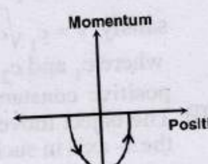
Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momenum are changed. Here we consider some simple dynamical systems in one dimension.

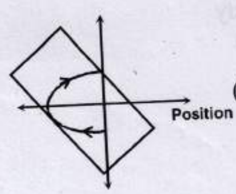


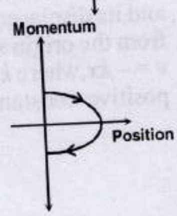
For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is x(t) vs. p(t) curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which positon or momentum upwards (or to right) is positive and downwards (or to left) is negative. **[2011]**

9. The phase space diagram for a ball thrown vertically up from ground is

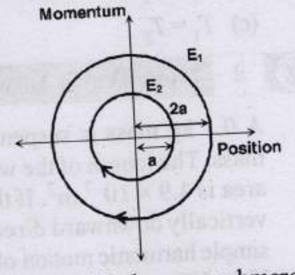
(a)


(b)


(c)


(d)


10. The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then



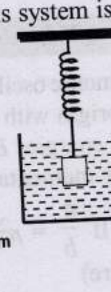
(a) $E_1 = \sqrt{2}E_2$

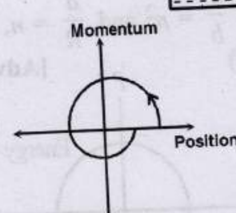
(b) $E_1 = 2E_2$

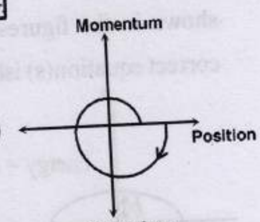
(c) $E_1 = 4E_2$

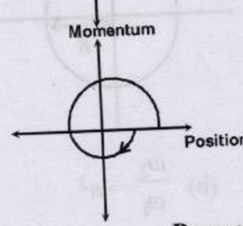
(d) $E_1 = 16E_2$

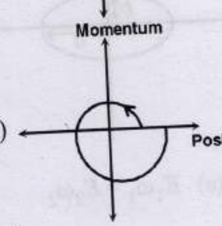
11. Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is



(a)


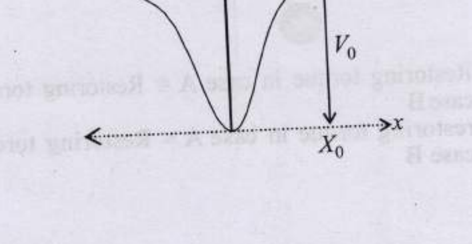
(b)


(c)


(d)


Passage - 2

When a particle of mass m moves on the x-axis in a potential of the form $V(x) = kx^2$ it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$,as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of $x = 0$ in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x-axis. Its potential energy is $V(x) = \alpha x^4$ $(\alpha > 0)$ for $|x|$ near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see figure). **[2010]**



12. If the total energy of the particle is E, it will perform periodic motion only if
(a) $E < 0$ (b) $E > 0$
(c) $V_0 > E > 0$ (d) $E > V_0$

13. For periodic motion of small amplitude A, the time period T of this particle is proportional to
(a) $A\sqrt{\frac{m}{\alpha}}$ (b) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$
(c) $A\sqrt{\frac{\alpha}{m}}$ (d) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

14. The acceleration of this particle for $|x| > X_0$ is
(a) proportional to V_0 (b) proportional to $\frac{V_0}{mX_0}$
(c) proportional to $\sqrt{\frac{V_0}{mX_0}}$ (d) zero

10 Subjective Problems

15. A point mass m is suspended at the end of a massless wire of length l and cross section A. If Y is the Young's modulus for the wire, obtain the frequency of oscillation for the simple harmonic motion along the vertical line. **[1978]**

Answer Key

Topic-1 : Displacement, Phase, Velocity and Acceleration in S.H.M.

1. (b)	2. (d)	3. (a,b,c)	4. (a,c)

Topic-2 : Energy in Simple Harmonic Motion

1. (d)	2. (c)	4. (b, c)

Topic-3 : Time Period, Frequency, Simple Pendulum and Spring Pendulum

1. (c)	2. (a)	3. (d)	4. (b)	5. (a)	6. (a)	7. (b)	8. (d)	9. (b)	10. (d)
11. (10)	12. (6)	13. (2.09)	14. (a, b, d)	15. (a, d)	16. (c)	17. (0.75)	18. (4.25)		

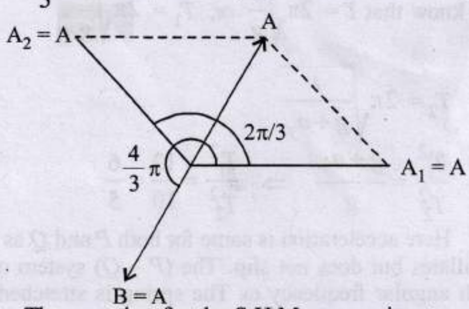
Topic-4 : Damped, Forced Oscillations and Resonance

1. (c)	2. (c)	3. (a)	4. (4)	5. (b, d)	6. (a, d)	7. A → p; B → q, r, s; C → s; D → q
8. A → p; B → q, r; C → p; D → q, r	9. (d)	10. (c)	11. (b)	12. (c)	13. (b)	14. (d)

Hints & Solutions

Topic-1: Displacement, Phase, Velocity and Acceleration in S.H.M.

1. **(b)** Two sinusoidal displacements $x_1(t) = A \sin \omega t$ and $x_2(t) = A \sin\left(\omega t + \frac{2}{3}\pi\right)$ have amplitude A each, with a phase difference of $2\frac{\pi}{3}$. It is given that sinusoidal displacement $x_3(t) = B(\sin \omega t + \phi)$ brings the mass to a complete rest. This is possible when the amplitude of third $B = A$ and is having a phase difference of $\phi = 4\frac{\pi}{3}$ with respect to $x_1(t)$ as shown in the figure.



2. **(d)** The equation for the S.H.M. $x = a \sin \omega t$

$$\Rightarrow \quad x = a \sin\left(\frac{2\pi}{T}\right) \times t = 1 \sin\left(\frac{2\pi}{8}\right) t = \sin\frac{\pi}{4} t$$

($\because$ $a = 1$ cm, $T = 8s$ from graph)

or, velocity, $v = \frac{dx}{dt} = \frac{d}{dt}\left[\sin\left(\frac{\pi}{4}\right)t\right] = \frac{\pi}{4}\cos\left(\frac{\pi}{4}\right)t$

$\therefore$ Acceleration $a = \frac{d^2x}{dt^2} = -\left(\frac{\pi}{4}\right)^2 \sin\left(\frac{\pi}{4}\right)t$

At $t = \frac{4}{3}s$ acceleration

$$a = \frac{d^2x}{dt^2} = -\left(\frac{\pi}{4}\right)^2 \sin\frac{\pi}{4} \times \frac{4}{3} = \frac{-\pi^2}{16}\sin\frac{\pi}{3}$$

$$= \frac{-\sqrt{3}\pi^2}{32} \text{ cm}/s^2$$

3. **(a, b, c)** The given equation

$x = A \sin^2 \omega t + B \cos^2 \omega t + C \sin \omega t \cos \omega t$

Rearranging the equation for SHM the sine and cosine functions should have linear power.

$$\therefore \ x = \frac{A}{2}(2\sin^2 \omega t) + \frac{B}{2}(2\cos^2 \omega t) + \frac{C}{2}(2\sin \omega t \cos \omega t)$$

$$= \frac{A}{2}[1 - \cos 2\omega t] + \frac{B}{2}[1 + \cos 2\omega t] + \frac{C}{2}[\sin 2\omega t]$$

(a) For $A = 0$ and $B = 0$, $x = \frac{C}{2}\sin(2\omega t)$

The above equation represents SHM.

(b) If $A = B$ and $C = 2B$ then $x = B + B \sin 2\omega t$

This is an equation of SHM.

(c) $A = -B, C = 2B;$

$\therefore \quad x = B \cos 2\omega t + B \sin 2\omega t$

Two SHMs are superposed to give another SHM equation.

(d) $A = B, C = 0 \therefore \ x = A.$

This equation does not represents SHM.

4. **(a, c)** Applying superposition principle

$$y = y_1 + y_2 + y_3$$
$$= a \sin \omega t + a \sin(\omega t + 45°) + a \sin(\omega t + 90°)$$
$$= a[\sin \omega t + \sin(\omega t + 90°] + a \sin(\omega t + 45°)$$
$$= 2a \sin(\omega t + 45°) \cos 45° + a \sin(\omega t + 45°)$$
$$= (\sqrt{2} + 1)\, a \sin(\omega t + 45°)$$
$$= A \sin(\omega t + 45°)$$

Clearly, resultant motion is SHM of amplitude $A = (\sqrt{2} + 1)\, a$ and differ in phase by 45° not by 90° relative to the first.

Energy E in SHM $\propto$ (amplitude)^2 $\left[\because E = \frac{1}{2}mA^2\omega^2\right]$

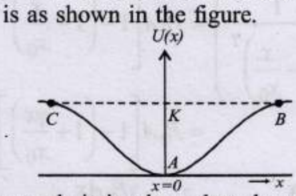
$$\frac{E_{\text{resultant}}}{E_{\text{single}}} = \left(\frac{A}{a}\right)^2 = (\sqrt{2}+1)^2 = (3 + 2\sqrt{2})$$

$$\therefore \quad E_{\text{resultant}} = (3 + 2\sqrt{2})\, E_{\text{single}}$$

Topic-2: Energy in Simple Harmonic Motion

1. **(d)** Given: $U(x) = K\left[1 - e^{-x^2}\right]$, $\therefore F = -\frac{dU}{dx} = 2kxe^{-x^2}$

The corresponding $u - x$ graph of the given $u(x) = K[1 - e^{-x^2}]$ which is an exponentially increasing graph of u with x^2 is as shown in the figure.



From the graph it is clear that the potential energy is minimum at $x = 0$. Hence, $x = 0$ is the state of stable equilibrium. Now if we displace the particle from $x = 0$ then for displacements the particle tends to regain the position $x = 0$ with a force $F = \frac{2kx}{e^{x^2}}$. Hence, for small values of x we have $F \propto x$.

Hence for small displacement from $x = 0$ the motion is simple harmonic.

2. (c) During one complete oscillation, the kinetic energy of particle executing SHM will become maximum twice. Therefore the frequency with which its kinetic energy oscillates will be $2f$.

3. Given: $m = 0.2$ kg, $\omega = \frac{25}{\pi}$, K.E. = 0.5 J, P.E. = 0.4 J

and $a = ?$

T.E. = K.E. + P.E. = (0.5 + 0.4) J = 0.9 J

Also, T.E. = $\frac{1}{2}m\omega^2a^2 = \frac{1}{2}m \times 4\pi^2\nu^2a^2$

$\Rightarrow \ 0.9 = \frac{1}{2} \times 0.2 \times 4\pi^2 \times \frac{25}{\pi} \times \frac{25}{\pi} \times a^2 \ \therefore \ a = 0.06\text{m}$

4. (b, c) Maximum kinetic energy

$= \frac{1}{2}kA^2 = \frac{1}{2} \times 2 \times 10^6 \times (0.01)^2 = 100\text{ J}$

Total mechanical energy E = 160 J

Extreme position	Mean position
$x = A$	$x = 0$
K.E. = 0	K.E. = 100J
E = 160 J	E = 160 J
P. E. = 160J	P. E. = 60J

Topic-3: Time Period, Frequency, Simple Pendulum and Spring Pendulum

1. (c) $\frac{Mg}{A} = P_0$

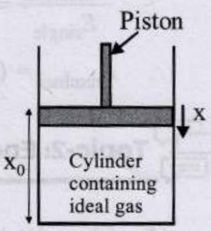
$P_0V_0^\gamma = PV^\gamma$

Mg = P_0A ...(1)

Let piston is displaced by distance x

$P_0Ax_0^\gamma = PA(x_0 - x)^\gamma$

$P = \frac{P_0x_0^\gamma}{(x_0 - x)^\gamma}$



$Mg - \left(\frac{P_0x_0^\gamma}{(x_0 - x)^\gamma}\right)A = F_{restoring}$

$P_0A\left(1 - \frac{x_0^\gamma}{(x_0 - x)^\gamma}\right) = F_{restoring} \quad [x_0 - x \approx x_0]$

$P_0A\left(1 - \frac{1}{\left(1 - \frac{x}{x_0}\right)^\gamma}\right) = P_0A\left[1 - \left(1 - \frac{x}{x_0}\right)^\gamma\right]$

$= P_0A\left[1 - \left(1 + \frac{\gamma x}{x_0}\right)\right]$

$F = -\frac{\gamma P_0Ax}{x_0} \Rightarrow M\omega^2x = \frac{\gamma P_0Ax}{x_0} \Rightarrow \omega = \sqrt{\frac{P_0A\gamma}{x_0M}}$

∴ Frequency with which piston executes SHM.

$f = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0A}{x_0M}} = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0A^2}{MV_0}}$

2. (a) Here, we don't need to consider the SHM part rather we will focus on projectile part only. Since the pebble hits the block after 1 sec. We can easily calculate the speed of projection (V) from this time of flight.

Time of flight of projectile,

$T = \frac{2V\sin\theta}{g}$

$\therefore \quad 1 = \frac{2V\sin 45^\circ}{g} \quad \therefore \ V = \sqrt{50}\text{ ms}^{-1}$

Hence pebble is projected with a speed $V = \sqrt{50}$ ms^{-1}

3. (d) If the spring of spring constant k_1 is compressed by x_1 and that of spring constant k_2 is compressed by x_2 then

$x_1 + x_2 = A$ ...(i)

and $k_1x_1 = k_2x_2 \Rightarrow x_2 = \frac{k_1x_1}{k_2}$ ...(ii)

Solving eqs. (i) & (ii) we get

$x_1 = \frac{k_2A}{k_2 + k_1}$

4. (b) Given : $y = kt^2$

$\therefore \ \frac{dy}{dt} = v = 2kt$ and, $\frac{d^2y}{dt^2} = a = 2k$

or $a_y = 2\text{m/s}^2$ ($\because k = 1$ m/s^2 given)

We know that $T = 2\pi\sqrt{\frac{\ell}{g}}$ or, $T_1 = 2\pi\sqrt{\frac{l}{g}}$

$T_2 = 2\pi\sqrt{\frac{l}{g + a_y}}$

$\therefore \ \frac{T_1^2}{T_2^2} = \frac{g + a_y}{g} \Rightarrow \frac{T_1^2}{T_2^2} = \frac{12}{10} = \frac{6}{5}$

5. (a) Here acceleration is same for both P and Q as block Q oscillates but does not slip. The $(P - Q)$ system oscillates with angular frequency ω. The spring is stretched by A.

Angular frequency of the system,

$\omega = \sqrt{\frac{k}{m + m}} = \sqrt{\frac{k}{2m}}$

Maximum acceleration of the system in SHM

$a_{max} = A\omega^2 = A\sqrt{\left(\frac{k}{2m}\right)^2} = \frac{KA}{2m}$

This acceleration to the lower block is provided by friction

∴ Maximum force of friction

$f_{max} = ma_{max} = m\left(\frac{kA}{2m}\right) = \frac{kA}{2}$

6. (a) Given : $U(x) = k|x|^3$

$\therefore \ [k] = \frac{[U]}{[x^3]} = \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}$

Now time period may depend on $T \propto$ (mass)^x (amplitude)^y $(k)^z$

$\therefore \ [M^0L^0T] = [M]^x[L]^y[ML^{-1}T^{-2}]^z$
$= [M^{x+z}L^{y-z}T^{-2z}]$

Equating the powers, we get

$-2z = 1$ or $z = -1/2$

$y - z = 0$ or $y = z = -1/2$

Hence $T \propto$ (amplitude)$^{-1/2} \propto a^{-1/2}$

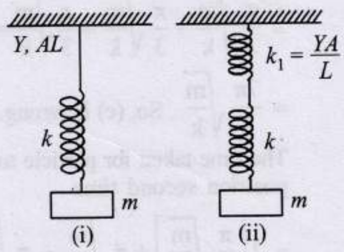
Therefore, $T \propto \frac{1}{\sqrt{a}}$

7. **(b)** Let us consider the wire also as a spring of spring constant k'. Then the case becomes that of two spring attached in series. The equivalent spring constant

$$\frac{1}{k_{eq}}=\frac{1}{k}+\frac{1}{k'} \Rightarrow k_{eq}=\frac{kk'}{k+k'}$$

Force constant of spring = k

$$\therefore \frac{\text{force}}{\text{length}}=k \quad ...(i)$$



For wire,

$$Y=\frac{F}{A}\frac{L}{l}$$

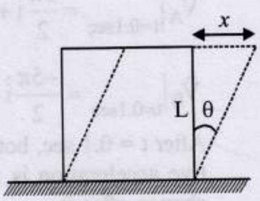
$$\text{or } \frac{F}{l}=\frac{YA}{L}$$

$$\therefore \frac{\text{force}}{\text{length}}=\frac{YA}{L} \text{ or } k_1=\frac{YA}{L} \quad ...(ii)$$

$$\therefore T=2\pi\sqrt{\frac{m}{k_{eq}}}=2\pi\sqrt{\frac{m(k+k_1)}{kk_1}}$$

$$=2\pi\sqrt{\frac{m\left(k+\frac{YA}{L}\right)}{k\times\frac{YA}{L}}}=2\pi\sqrt{\frac{m(YA+kL)}{YAk}}$$

8. **(d)** Here, $A=L^2$ and $\theta=\frac{x}{L}$



Modulus of rigidity, $\eta=F/A\,\theta$

$\therefore$ Restoring force

$$F=-\eta A\theta=-\eta Lx$$

or, acceleration,

$$a=\frac{F}{M}=-\frac{\eta L}{M}x$$

Since, $a\propto -x$ So, oscillations are SHM. Therefore, time period of which is given by

$$T=2\pi\sqrt{\left|\frac{\text{displacement}}{\text{acceleration}}\right|}=2\pi\sqrt{\left|\frac{x}{a}\right|}$$

$$\text{or, } T=2\pi\sqrt{\frac{M}{\eta L}}$$

9. **(b)** If x is the displacement then,

Net restoring force = extra upthrust + extra spring force

$$\therefore M\omega^2x=[\rho Ag+k]x$$

$$\Rightarrow \omega=\left[\frac{\rho Ag+k}{M}\right]^{1/2} \Rightarrow \nu=\frac{1}{2\pi}\left[\frac{\rho Ag+k}{M}\right]^{1/2}$$

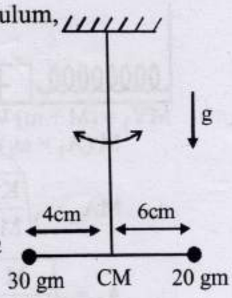
10. **(d)** Given maximum velocity, $v_1=v_2$ or, $a_1\omega_1=a_2\omega_2$

$\because V_{\text{maximum}}=a\omega$

$$\Rightarrow a_1\times\frac{2\pi}{T_1}=a_2\times\frac{2\pi}{T_2}$$

$$\therefore \frac{a_1}{a_2}=\frac{T_1}{T_2}=\frac{2\pi\sqrt{\frac{m}{k_1}}}{2\pi\sqrt{\frac{m}{k_2}}}=\sqrt{\frac{k_2}{k_1}}$$

11. **(10)** Time period of torsional pendulum,



$$T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{I}{C}}$$

$$\therefore \omega=\sqrt{\frac{C}{I}}$$

Where I = moment of inertia

$I=(30)(4)^2+(20)(6)^2=1200\text{ gcm}^2$

$$\text{or, } I=1.2\times10^{-4}\text{ kg–m}^2=\sqrt{\frac{C}{I}}\omega=\sqrt{\frac{1.2\times10^{-8}}{1.2\times10^{-4}}}$$

[$\because$ Torsional constant of the wire $C=1.2\times10^{-8}$ N m rad^{-1} given]

$$\therefore \omega=\sqrt{10^{-4}}=10^{-2}$$

$$\therefore n\times10^{-3}=10^{-2} \therefore n=10$$

12. **(6)** We have

$\ell>\ell_0\Rightarrow k=k_1$; $\ell<\ell_0\Rightarrow k=k_2$

Time period of oscillation,

$$T=\pi\sqrt{\frac{m}{k_1}}+\pi\sqrt{\frac{m}{k_2}}=\pi\sqrt{\frac{0.1}{0.009}}+\pi\sqrt{\frac{0.1}{0.016}}$$

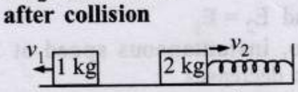
$$=\frac{\pi}{0.3}+\frac{\pi}{0.4}=\pi\times\frac{0.70}{0.12}=5.83\pi\simeq6\pi$$

So, n = 6

13. **(2.09)**

Let velocities of 1 kg and 2 kg blocks just after collision be v_1 and v_2 respectively.

Just after collision



From momentum conservation principle,

$$m_1u_1+m_2u_2=m_1v_1+m_2v_2$$

$$0+1\times2=-1v_1+2v_2 \quad .... (i)$$

Collision is elastic. Hence $e=1$

$$e=1=\frac{2-0}{v_1+v_2}$$

$$\Rightarrow v_2+v_1=2 \quad .... (ii)$$

From eqs. (i) and (ii),

$$v_2=\frac{4}{3}\text{ m/s}, v_1=\frac{2}{3}\text{ m/s}$$

2 kg block will perform SHM after collision, so spring returns to its unstretched position for the first time after.

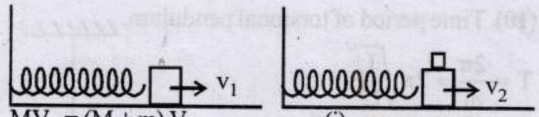
$$t=\frac{T}{2}=\pi\sqrt{\frac{m}{k}}=\pi\sqrt{\frac{2}{2}}=\pi=3.14\text{ s}$$

Distance or required separation between the blocks

$$=|v_1|t=\frac{2}{3}\times3.14=2.093=2.09\text{ m}$$

14. **(a, b, d)**

Case (i) : Applying principle of conservation of linear momentum.



$MV_1 = (M+m)\,V_2$ ... (i)

$M\,(A_1 \times \omega_1) = (M+m)\,(A_2 \times \omega_2)$

$\therefore\ MA_1 \times \sqrt{\frac{K}{M}} = (M+m)\,A_2 \times \sqrt{\frac{K}{M+m}}$

$\therefore\ A_2 = \sqrt{\frac{M}{M+m}}\,A_1 \Rightarrow \frac{A_2}{A_1} = \sqrt{\frac{M}{M+m}}$

Also $E_1 = \frac{1}{2}MV_1^2$

and $E_2 = \frac{1}{2}(M+m)V_2^2 = \frac{1}{2}(M+m)$

$\left(\because\ V_2 = \left(\frac{M}{M+m}\right)V_1 \text{ from eq (i)}\right)$

$\times \frac{M^2V_1^2}{(M+m)^2} = \frac{1}{2}\left(\frac{M}{M+m}\right)^2 V_1^2$

Clearly $E_1 > E_2$

The new time Period $T_2 = 2\sqrt{\frac{m+M}{K}}$

Instantaneous speed at X_0 of the combined masses

$V_2 = \frac{MV_1}{M+m} < V_1$

Case (ii) : The new time Period $T_2 = 2\sqrt{\frac{m+M}{K}}$

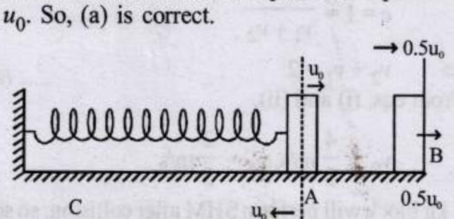
Also $A_2 = A_1$ and $E_2 = E_1$

In this case also, instantaneous speed at X_o of the combined masses decreases.

15. (a, d) The particle collides elastically with rigid wall.

$\therefore\ e = \frac{V}{0.5u_0} = 1 \Rightarrow V = 0.5u_0$

i.e., the particle rebounds with the same speed. Therefore the particle will return to its equilibrium position with speed u_0. So, (a) is correct.



Equilibrium position

The velocity of the particle becomes $0.5u_0$ after time t. Using, equation $V = V_{max}\cos\omega t$

$0.5u_0 = u_0\cos\omega t$

$\therefore\ \frac{\pi}{3} = \frac{2\pi}{t} \times T \Rightarrow t = \frac{T}{6}$

The time period $T = 2\pi\sqrt{\frac{m}{k}}\ \therefore\ t = \frac{\pi}{3}\sqrt{\frac{m}{k}}$

The time taken by the particle to pass through the equilibrium for the first time $= 2t = \frac{2\pi}{3}\sqrt{\frac{m}{k}}$. So, (b) is wrong.

The time taken for the maximum compression

$= t_{AB} + t_{BA} + t_{AC}$

$= \frac{\pi}{3}\sqrt{\frac{m}{k}} + \frac{\pi}{3}\sqrt{\frac{m}{k}} + \frac{\pi}{3}\sqrt{\frac{m}{k}} = \pi\sqrt{\frac{m}{k}}\left[\frac{1}{3} + \frac{1}{3} + \frac{1}{2}\right]$

$= \frac{7\pi}{6}\sqrt{\frac{m}{k}}$. So, (c) is wrong.

The time taken for particle to pass through the equilibrium position second time

$= 2\left[\frac{\pi}{3}\sqrt{\frac{m}{k}}\right] + \pi\sqrt{\frac{m}{k}} = \pi\sqrt{\frac{m}{k}}\left(\frac{2}{3} + 1\right) = \frac{5}{3}\pi\sqrt{\frac{m}{k}}$. So (d) is correct.

16. (c) (I) $V_{BA}^2 = V_A^2 + V_B^2 - 2V_BV_A\cos\theta$

As $\omega_A = \omega_B$, $\theta = 90°$ remains constant

Also, $V_A = V_B = 1$ m/s $\quad [\because\ V = \omega R]$

$V_{BA} = \sqrt{2}$ m/s. So I → S.

(II) $\vec{u}_A = \frac{5\pi}{2}\hat{i} + \frac{5\pi}{2}\hat{j}$

$\vec{V}_A\Big|_{t=0.1\,sec} = \frac{5\pi}{2}\hat{i} + \left(\frac{5\pi}{2} - 10 \times 0.1\right)\hat{j} = \frac{5\pi}{2}\hat{i} + \left(\frac{5\pi}{2} - 1\right)\hat{j}$

$\vec{V}_B\Big|_{t=0.1\,sec} = \frac{-5\pi}{2}\hat{i} + \frac{5\pi}{2}\hat{j}$

After t = 0.1 sec, both projectile came in air. So there relative acceleration is zero. So relative velocity should not change after it.

$V_{rel} = V_{rel}(t = 0.1\,sec) = \left|5\pi\hat{i} - \hat{j}\right| = \sqrt{25\pi^2 + 1}$. So II → T

(III) $x = x_A - x_B$

$= x_0\sin t - x_0\sin\left(t + \frac{\pi}{2}\right) \quad [\because\ t_0 = 1]$

$= \sin t - \cos t = \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin t - \frac{1}{\sqrt{2}}\cos t\right)$

$= \sqrt{2}\sin\left(t - \frac{\pi}{4}\right)$

$V_{rel} = \frac{dx}{dt} = \sqrt{2}\cos\left(t - \frac{\pi}{4}\right) = \sqrt{2}\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= \sqrt{2} \times \frac{\sqrt{3}+1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2}$. So, III → P

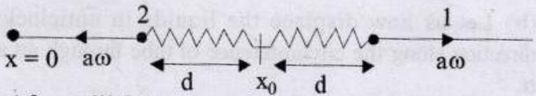
(IV) $\vec{V}_A$ and $\vec{V}_B$ are always perpendicular

So, $\left|\vec{V}_{BA}\right| = \sqrt{V_A^2 + V_B^2} = \sqrt{3^2 + 1^2} = \sqrt{10}$ m/s.

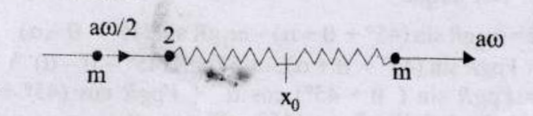
So IV → R

17. **(0.75)** At time $t_0 = 0$ collision occurs

Before collision



After collision

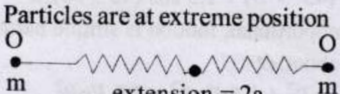


$$V_{CM} = \frac{m.\frac{a\omega}{2} + m.a\theta}{m+m} = \frac{3a\omega}{4}$$

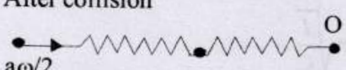
$$\therefore \quad \frac{V_{CM}}{a\omega} = \frac{3}{4} = 0.75$$

18. **(4.25)** If the collision occurs at time $t_0 = \frac{\pi}{2g\omega} = \frac{T}{4}$

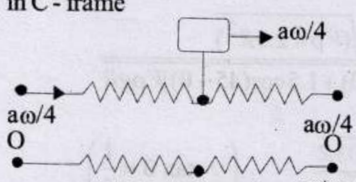
Particles are at extreme position



After collision



in C - frame



using work - energy theorem $W_{spring} = \Delta K$

$$\frac{1}{2}k(2b)^2 - \frac{1}{2}k(2a)^2 = 2 \times \frac{1}{2} m \times \left(\frac{a\omega}{4}\right)^2$$

$$4kb^2 - 4ka^2 = 2 \times m \times \frac{a^2}{16} \times \frac{2k}{m}$$

$$\Rightarrow \quad 4b^2 = \frac{17}{4}a^2 \quad \therefore \quad \frac{4b^2}{a^2} = 4.25$$

19. Under SHM, at a distance y above the mean position velocity of block, $v = \omega\sqrt{a^2 - y^2}$

After detaching from spring, net downward acceleration of the block = g.

$\therefore$ Height attained by the block = h

$$\therefore \quad h = y + \frac{v^2}{2g} \text{ or } h = y + \frac{\omega^2\left(a^2 - y^2\right)}{2g}$$

For h to be maximum, $\frac{dh}{dy} = 0, y = y^*$.

$$\therefore \frac{dh}{dy} = 1 + \frac{\omega^2}{2g}(-2y^*) \text{ or } 0 = 1 - \frac{2\omega^2 y^*}{2g}$$

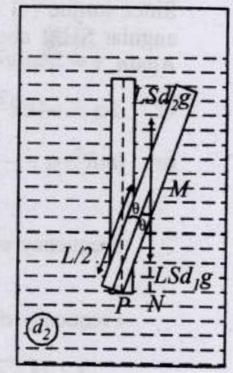
$$\text{or} \quad \frac{\omega^2 y^*}{g} = 1 \text{ or } y^* = \frac{g}{\omega^2}$$

Since $a\omega^2 > g$ (given)

$\therefore \; a > \frac{g}{\omega^2} \therefore a > y^*. y^*$ from mean position $< a$.

Hence $y^* = \frac{g}{\omega^2}$

20. Volume of the rod = LS



Weight of the rod = $LS\, d_1 g$

Upthrust acting on rod = $LS\, d_2 g$

Since, $d_2 > d_1$ (given).

$\therefore$ Net force acting at the centre of mass of the rod at tilted position $(LS\, d_2 g - LSd_1 g)$

Torque about this force about P

$\tau = F \times r_1 = (LSd_2 g - LSd_1 g) \times PN$

$$\therefore \quad \tau = LSg\,(d_2 - d_1) \times \frac{L}{2}\sin\theta$$

when θ is small, $\sin\theta \approx \theta$

$$\therefore \quad \tau = \frac{L^2 Sg}{2}(d_2 - d_1)\theta. \quad \text{....(i)}$$

Since, $\tau \propto \theta$, hence motion is simple harmonic.

On comparing it with $\tau = C\,\theta$, we get

$$C = \frac{L^2 Sg}{2}(d_2 - d_1)$$

$$\Rightarrow \; I\omega^2 = \frac{L^2 Sg}{2}(d_2 - d_1) \quad \text{...(ii)}$$

The moment of inertia I of the rod about P,

$$I = \frac{1}{3}ML^2 = \frac{1}{3}LSd_1 L^2$$

Putting this value of I in eq. (ii)

$$\omega^2 \times \frac{L^3}{3}Sd_1 = \frac{L^2 Sg}{2}(d_2 - d_1)$$

$$\Rightarrow \quad \omega = \sqrt{\frac{3Sg(d_2 - d_1)}{2LSd_1}} \Rightarrow \omega = \sqrt{\frac{3(d_2 - d_1)g}{2d_1 L}}$$

Hence angular frequency of oscillation,

$$\omega = \sqrt{\frac{3g(d_2 - d_1)}{2d_1 L}}$$

21. (i) Given : Natural length of spring, $l_0 = 0.06\,\pi = \pi R$

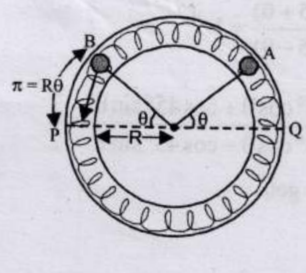
Spring constant, $k = 0.1$ N/m

Mass of each block A and B, $m = 0.1$ kg

Radius of circle, R= 0.06 m

As both the balls are displaced by an angle $\theta = \pi/6$ radian with respect to the diameter PQ of the circle and released from rest.

It results into compression of spring in upper segment and an equal elongation of spring in lower segment. Let it be x. PB and QA denote x in the figure.



Compression = $R\theta$ = elongation = x

$\therefore$ Force exerted by each spring on each ball = 2 kx

$\therefore$ Total force on each ball due to two springs = 4 kx

$\therefore$ Restoring torque about origin $O = -(4\,kx)R$

$\therefore$ $\tau = -4k(R\theta)R$, where θ = Angular displacement

or $\tau = -4kR^2\theta$

Since torque (τ) is proportional to θ, each ball executes angular SHM about the centre O.

Again, $\tau = -4kR^2\theta$

or $I\alpha = -4kR^2\theta$ where α = angular acceleration

or $(mR^2)\,\alpha = -4kR^2\theta$ or $\alpha = -\left(\frac{4k}{m}\right)\theta$

$\therefore$ Frequency of oscillation $f = \frac{1}{2\pi}\sqrt{\frac{\alpha}{\theta}}$

$\therefore$ Frequency of each ball $= \frac{1}{2\pi}\sqrt{\frac{4k}{m}}$

$= \frac{1}{2\pi}\sqrt{\frac{4\times 0.1}{0.1}} = \frac{1}{\pi}\text{ sec}^{-1}$ ..(ii)

(ii) v_{max} be the velocity at the mean position

$\therefore$ Loss in elastic potential energy = gain in kinetic energy

$$2\left[\frac{1}{2}K\left(2R\frac{\pi}{6}\right)^2\right] = 2\times\left[\frac{1}{2}mv_{max}^2\right]$$

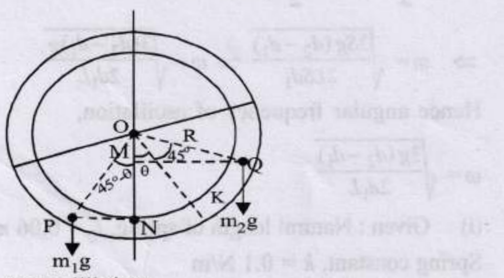
$\therefore$ $v_{max} = \sqrt{\frac{K}{m}}\times\frac{R\pi}{3} = 0.02\,\pi$ m/s $= 0.628$ m/s

(iii) Total energy = $2\left[\frac{1}{2}mv_{max}^2\right]$ (= K.E. at mean position)

$= 2\left[\frac{1}{2}\times 0.1(0.02\pi)^2\right]$

$= 3.95\times 10^{-4}$ J

22.



At equilibrium,

$\left(\begin{array}{c}\text{Torque at q due to}\\ \text{liquid of density } \rho\end{array}\right) = \left(\begin{array}{c}\text{Torque at p due to}\\ \text{liquid of density } 1.5\rho\end{array}\right)$

$m_2g\times QM = m_1g\times PN$

$\therefore$ $m_2g\,R\sin(45°+\theta) = m_1gR\sin(45°-\theta)$

$V\rho gR\sin(45°+\theta) = 1.5V\rho gR\sin(45°-\theta)$... (i)

$\Rightarrow \frac{\sin(45+\theta)}{\sin(45-\theta)} = 1.5$

$\Rightarrow \frac{\sin 45°\cos\theta + \cos 45°\sin\theta}{\sin 45°\cos\theta - \cos 45°\sin\theta} = \frac{3}{2}$

Solving we get,

$\tan\theta = \frac{1}{5}$

(b) Let us now displace the liquids in anticlockwise direction along the circumference of tube through an angle α.

$\therefore$ Net torque

$\tau = m_2gR\sin(45°+\theta+\alpha) - m_1gR\sin(45°-\theta-\alpha)$

$= V\rho gR\sin(45°+\theta+\alpha) - 1.5V\rho gR(45°-\theta-\alpha)$

$= V\rho gR\sin(\theta+45°)\cos\alpha + V\rho gR\cos(45°+\theta)\sin\alpha - 1.5\,V\rho gR\sin(45°-\theta)\cos\alpha + 1.5\,V\rho gR\cos(45°-\theta)\sin\alpha$

Using eq. (i) we get

$\tau = V\rho gR\left[\cos(45°+\theta)\sin\alpha + 1.5\cos(45°-\theta)\sin\alpha\right]$

$\tau = V\rho gR\,[\cos(45°+\theta) + 1.5\cos(45°-\theta)]\sin\alpha$

when α is small (given) $\therefore$ $\sin\alpha \approx \alpha$

$\therefore$ $\tau = V\rho gR\,[\cos(45°+\theta) + 1.5\cos(45°-\theta)]\alpha$

Since, τ and α are proportional, motion is simple harmonic.

Moment of inertia about O

$I = m_1R^2 + m_2R^2 = V\rho R^2 + 1.5\,V\rho R^2 = 2.5\,V\rho R^2$

Now from $T = 2\pi\sqrt{\frac{I}{C}}$

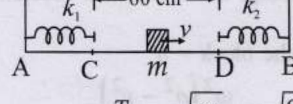
$= 2\pi\frac{\sqrt{(V\rho\times 2.5R^2)}}{\sqrt{[\cos(45+\theta)+1.5\cos(45-\theta)]V\rho gR}}$

or, $T = 2\pi\frac{\sqrt{1.803R}}{\sqrt{g}}$ $\left(\because \tan\theta = \frac{1}{5}\right)$

23. Since AB is a smooth table. So no acceleration or retardation in this region so motion from C to D to C is uniform with speed 120 m/s.

The mass will strike the right spring, compress it. The K.E. of the mass will convert into P.E. of the spring. Again the spring will return to its normal size thereby converting its P.E. to K.E. of the block.

For this process, time taken $= \frac{T}{2}$, where $T = 2\pi\sqrt{\frac{m}{k}}$.



$\therefore$ $t_1 = \frac{T}{2} = \pi\sqrt{\frac{m}{k_2}} = \pi\sqrt{\frac{0.2}{3.2}} = 0.785$ sec

Now, time taken to travel from D to C

$t_2 = \frac{\text{distance}}{\text{velocity}} = \frac{60}{120} = 0.5$

Now the block will compress the left spring and then the spring again attains its normal length. The time taken C to A

$t_3 = \pi\sqrt{\frac{m}{k_1}} = \pi\sqrt{\frac{0.2}{1.8}} = 1.05$ sec.

Again time taken to travel from C to D

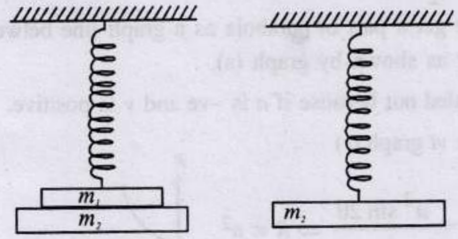
$t_4 = \frac{60}{120} = 0.5$

$\therefore$ Total time of oscillation

$T = t_1 + t_2 + t_3 + t_4$

$= 0.785 + 0.5 + 1.05 + 0.5 = 2.83$ (Approximately)

24.



Let x_1 be the extension in equilibrium when both m_1 and m_2 are suspended.

$\therefore \quad (m_1 + m_2)g = kx_1 \Rightarrow x_1 = \frac{(m_1+m_2)g}{k}$ ...(i)

Let x_2 be the extension when only m_2 is left.

$\therefore \quad kx_2 = m_2 g \quad$ or $\quad x_1 = \frac{m_2 g}{k}$ ...(ii)

From eqs. (i) and (ii), amplitude of oscillation

$A = x_1 - x_2 = \frac{m_1 g}{k}$

Angular frequency when m_1 is removed only m_2 is left,

$\omega = \sqrt{\frac{k}{m_2}}$

25. According to question, if the mass is increased by 2 kg the period increases by 1 s. Time period the spring

$T = 2\pi\sqrt{\frac{M}{k}}$

or, $2 = 2\pi\sqrt{\frac{M}{k}}$ .... (i)

When mass is increased by 2 kg

$3 = 2\pi\sqrt{\frac{M+2}{k}}$ ... (ii)

Dividing eq. (i) by (ii)

$\frac{2}{3} = \sqrt{\frac{M}{M+2}} \Rightarrow \frac{4}{9} = \frac{M}{M+2} \Rightarrow M = 1.6$ kg.

Topic-4: Damped, Forced Oscillations and Resonance

1. **(c)** $\because A = A_0 e^{\frac{bt}{2m}}$

(where, A_0 = maximum amplitude)

According to the questions, after 5 second,

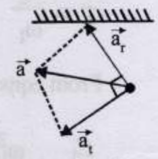
$0.9A_0 = A_0 e^{-\frac{b(5)}{2m}}$ ...(i)

After 10 more second,

$\alpha A_0 = A_0 e^{-\frac{b(15)}{2m}}$ ...(ii)

From eq^{n}s (i) and (ii) $\alpha = 0.729$

2. **(c)** The resultant of transverse $\vec{a}_t$ and radial $\vec{a}_r$ component of the acceleration is represented by $\vec{a}$



3. **(a)** In SHM, velocity of particles goes on decreasing from maximum value to zero as the particles travel from mean position to extreme position.

Therefore if the time taken for the body to go from O to $A/2$ is T_1 and to go from $A/2$ to A is T_2 then obviously $T_1 < T_2$

4. **(4)** As we know, $Y = \frac{FL}{Al} \Rightarrow F = \left(\frac{YA}{L}\right) l$

Also, $F = Kl$

or, $Kl = \left(\frac{YA}{L}\right) l \quad \Rightarrow \quad K = \frac{YA}{L}$

Angular frequency $\omega = \sqrt{\frac{K}{m}}$ or, $\omega = \sqrt{\frac{YA}{ml}}$

or, $140 = \sqrt{\frac{n \times 10^9 \times 4.9 \times 10^{-7}}{0.1 \times 1}}$ $(\because Y = 9 \times 10^9$ given)

$\therefore \quad n = 4$

5. **(b, d)** For first harmonic oscillator,

Mass = m

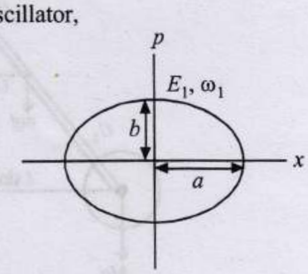
Angular frequency = ω_1

Amplitude = a

Total energy = E_1

Maximum momentum,

$p_{max} = b$



$E_1 = \frac{1}{2} m\omega_1^2 a^2$ ...(i)

$p_{max} = mv_{max} = m\, a\, \omega_1 \Rightarrow b = m\, a\, \omega_1$

$\frac{a}{b} = \frac{1}{m\omega_1}$ ...(ii)

For second harmonic oscillator,

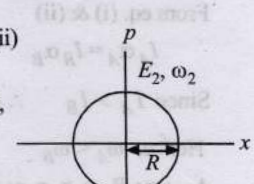
Mass = m

Angular frequency = ω_2

Amplitude = R

Maximum momentum, $p_{max} = R$

Total energy = E_2



$$E_2 = \frac{1}{2}m\omega_2^2 R^2 \quad ...(iii)$$

$$p_{max} = mv_{max} = m\omega_2 R$$

$$R = m\omega_2 R \Rightarrow m\omega_2 = 1 \quad ...(iv)$$

From eqns. (ii) and (iv).

$$\frac{a}{b} = \frac{\omega_2}{\omega_1} \quad ...(v)$$

From eqns. (i) and (iii),

$$\frac{E_1}{E_2} = \frac{\omega_1^2 a^2}{\omega_2^2 R^2}$$

If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n$ then from eqn. (v)

$$\frac{\omega_2}{\omega_1} = n^2$$

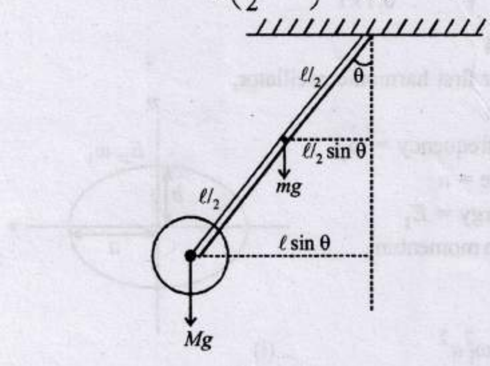
and from eqn. (vi)

$$\frac{E_1}{E_2} = \frac{\omega_1^2}{\omega_2^2} \times n^2 = \frac{\omega_1}{\omega_2} \quad \therefore \quad \frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$$

6. **(a, d)**

Applying, torque $\tau = I\alpha$

For case A : $mg\left(\frac{\ell}{2}\sin\theta\right) + Mg(\ell \sin\theta) = I_A \alpha_A$ ...(i)



For case B : $mg\left(\frac{\ell}{2}\sin\theta\right) + Mg(\ell \sin\theta) = I_B \alpha_B$ ...(ii)

From eq. (i) & (ii)

$$I_A \alpha_A = I_B \alpha_B$$

Since $I_A > I_B \quad \therefore \alpha_A < \alpha_B$

Hence, $\omega_A < \omega_B$

7. **A → p; B → q, r, s; C → s; D → q**

(A) : Potential energy k minimum at mean position and maximum at extreme position. In case of a S.H.M. we get a parabola for potential energy versus displacement graph.

(B) : S = ut for a = 0. Therefore we get a straight line passing through the origin, as shown in graph (a).

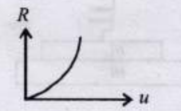
If at t = 0, $Y \neq 0$ and $Y = Y_0$. Then for constant acceleration, we have graph as shown in (r).

$S = ut + \frac{1}{2}at^2$ for constant positive acceleration. In this case we get a part of parabola as a graph line between *s* versus *t* as shown by graph (s).

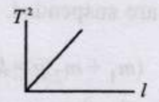
(p) is ruled out because if *a* is –ve and *v* is positive.

$S = S_0 + vt$ graph (r)

(C) : $R = \frac{u^2 \sin 2\theta}{g} \Rightarrow R \propto u^2$



(D) : $T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow T^2 \propto \ell$



8. **A → p; B → q, r; C → p; D → q, r**

(A) : For a simple harmonic motion $v = \omega\sqrt{a^2 - x^2}$. On comparing it with $v = c_1\sqrt{c_2 - x^2}$ this equation is SHM with $w = c_1$ and $a^2 = c_2$

(B) : $v = -k\,x$

when *x* is positive; *v* is – ve, and as *x* decreases, *v* decreases. Therefore kinetic energy will decrease. When *x* = 0, *v* = 0. Therefore the object does not change its direction.

When *x* is negative, *v* is positive. But as *x* decreases in magnitude, *v* also decreases. Therefore kinetic energy decreases. When *x* = 0, *v* = 0. Therefore the object does not change its direction.

(C) : When *a* = 0, let the spring have an extension *x*. Then $kx = mg$.

When the elevator starts going upwards with a constant acceleration, as seen by the observer in the elevator, the object is at rest.

$\therefore \quad ma + mg = k\,x'$

$\Rightarrow \quad ma = k\,(x' - x)$ (Since *a* is constant)

(D) : The object is projected with a speed is $\sqrt{2}$ times the escape speed $V_e = \sqrt{\frac{2GM_e}{R_e}}$. Therefore the object will leave the earth. It will therefore not change the direction keeps on moving with decreasing speed.

9. **(d)** When the ball is thrown upwards, at the point of throw (O) the linear momentum is in upwards direction (and has a maximum value) and the position is zero. As the time passes, the ball moves upwards and its momentum goes on decreasing and the position becomes positive. The momentum becomes zero at the topmost point.

As the time increases, the ball starts moving down with an increasing linear momentum in the downward direction (negative) and reaches back to its original position.

10. **(c)** In SHM mechanical energy, $E \propto (\text{amplitude})^2$

$\therefore \quad E_1 \propto (2a)^2 \quad \& \quad E_2 \propto a^2$

$\therefore \quad \frac{E_1}{E_2} = 4$

11. **(b)** When the position of the mass is at one extreme end in the positive side (the topmost point), the momentum is zero. As the mass moves towards the mean position the momentum increases in the negative direction.

As the mass is osicillating in water its amplitude will go on decreasing and the amplitude will decrease with time.

12. **(c)** The particle will not perform oscillations if energy ≤ 0. Therefore E > 0. If $E = V_0$, the potential energy will become constant as depicted in the graph given. In this case also the particle will not oscillate.

$\therefore \quad E > V_0$

$\therefore \quad V_0 > E > 0$

13. **(b)** Potential energy, $V = \propto x^4$ given

$$\therefore \quad \alpha = \frac{\text{Potential energy}}{x^4} = \frac{ML^2T^{-2}}{L^4} = [ML^{-2}T^{-2}]$$

$$\text{Now } \frac{1}{A}\sqrt{\frac{m}{\alpha}} = \frac{1}{L}\sqrt{\frac{M}{ML^{-2}T^{-2}}} = T$$

14. **(d)** $F = \frac{-dV(x)}{dx}$

$\because$ As $V(x)$ = constant for $x > X_0$

$\therefore$ $F = 0$ and hence $a = 0$ for $x > X_0$

15. A point mass m is suspended at the end of a massless wire of length L fig. (a).

From fig. (b), due to equilibrium

$T = mg \quad$ ... (i)

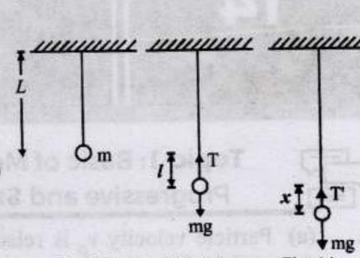


Fig. (a) Fig. (b) Fig. (c)

From $\quad Y = \frac{T/A}{\ell/L}$

$\Rightarrow T = \frac{YA\ell}{L} \quad$ ... (ii)

From eq. (i) and (ii)

$$mg = \frac{YA\ell}{L} \quad \text{... (iii)}$$

From fig. (c)

Restoring force

$$= -[T' - mg] = -\left[\frac{YA(\ell + x)}{L} - \frac{YA\ell}{L}\right] \quad \text{[from (iii)]}$$

$$= \frac{-YAx}{L}$$

On comparing this equation with $F = -m\omega^2 x$

$$m\omega^2 = \frac{YA}{L} \Rightarrow \omega = \sqrt{\frac{YA}{mL}} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{YA}{mL}}$$

$$\text{or, frequency } f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$$