14

Constructions

14.01 Introduction

In previous chapter we have understood and learnt the properties related to circle, secant line, tangent and alternate segment. Here we will learn how to construct the constructions related to theorems studied in previous chapters. We have discussed about congruent lines and points in the chapter of Locus, in whcih we have studied about incentre and circumcentre.

Now, we will try to understand these principles through construction by using basic principles and theorems.

Construction : In geometry any geometrical construction related to the puzzle is called 'construction'.

14.02. Internal Division of a Line Segment in a Given Ratio

Construction 1 : Draw a line segment of length 7.4 cm and divide it into 3 : 5 internally.

Steps of Construction

1. Draw a line segment AB = 7.4 cm.
2. Make an actue angle BAX.
3. Taking any suitable radius in the compass, mark eight (3+5) arcs $P_1, P_2, P_3, ..., P_8$ such that $AP_1 = P_2P_3 = ... = P_7P_8$
4. Join B to P_8.
5. Draw $P_3C \parallel P_8B$ from P_3 (for it draw $\angle AP_3C = \angle AP_8B$ which meets AB at C)

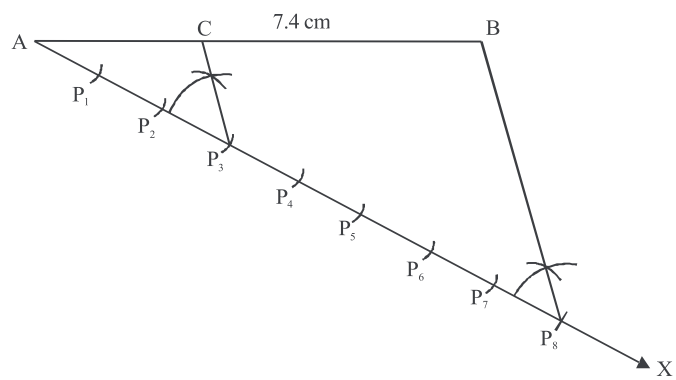


Fig. 14.01

Hence, the line segment AB is divided at C in the ratio of 3 : 5.

Construction 2 : Draw a line segment ML of length 9.7 cm on which find a point N such that

$$MN = \frac{4}{5} ML$$

Steps of Construction

1. Draw a line segment ML = 9.7 cm.
2. Construct on actue angle LMX.
3. Taking any suitable radius in the compass mark five arcs A_1, A_2, A_3, A_4 and A_5 such that

$MA_1 = A_1A_2 = ... = A_4A_5$

4. Join A_5 to L.

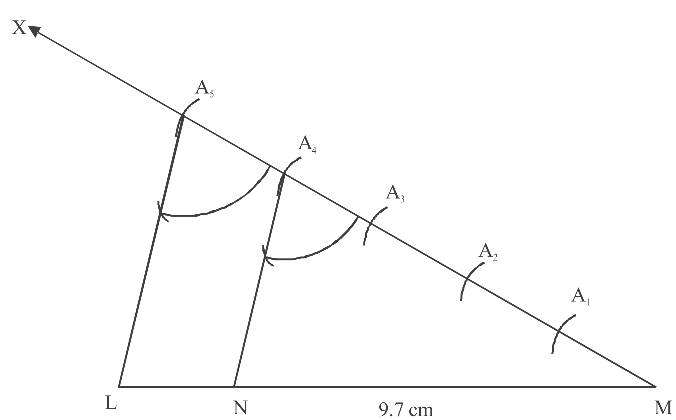


Fig. 14.02

5. Draw $A_4N \parallel A_5L$ from A_4 (for it construct $\angle MA_4N = \angle MA_5L$)

6. Point N divides the line segment ML such that $MN = \frac{4}{5}M$

Verification : In the triangle $MLA_5, NA_4 \parallel LA_5$

$\therefore \quad \frac{LN}{NM} = \frac{A_5A_4}{MA_4}$ (fundamental proportional theorem)

$\Rightarrow \quad \frac{LN}{NM} + 1 = \frac{A_5A_4}{MA_4} + 1$

$\Rightarrow \quad \frac{LN + NM}{NM} = \frac{A_5A_4 + MA_4}{MA_4}$

$\Rightarrow \quad \frac{ML}{NM} = \frac{MA_5}{MA_4} = \frac{5}{4}$

$\Rightarrow \quad \frac{ML}{NM} = \frac{4}{5}$

Hence, N is the point on the line segment ML such that $MN = \frac{4}{5}ML$

14.03 Construction of a Tangent to a Point on the Circle

Construction : Draw a tangent on a point at the circle whose radius is given.

Steps of construction:

(i) Take O as centre and draw a circle of radius r.

(ii) Mark a point P at the circumference of the circle and join O to P.

(iii) Draw a perpendicular AB at the point B.

Here AB is the required tangent.

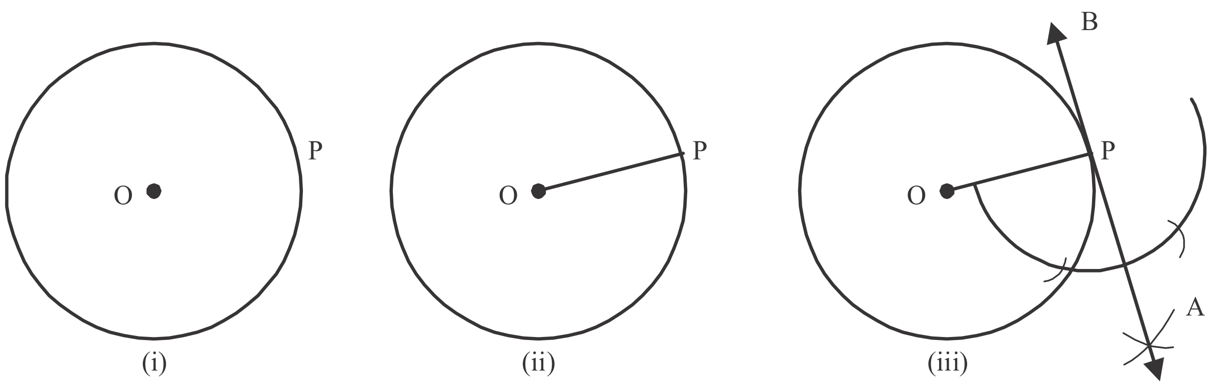


Fig. 14.03

Wrong method :

If we draw a circle and choose any point on it and then draw a line passing through this point, then this will not be a perfect method.

14.04 Construction of a Tangent from a Point Outside the Circle

Construction 4. To construct a tangent to a point outside the circle when centre is given.

Steps of construction

(i) Take a point *P* outside the circle whose centre is O.

(ii) Meet *P* to *O*.

(iii) Draw a perpendicular bisector of *OP* that intersects *OP* at *M*.

(iv) Taking *M* as the centre and radius *MO* draw a circle which intersects the first circle at A and B.

(v) Join A to P and B to P.

Hence AP and BP are the required tangents.

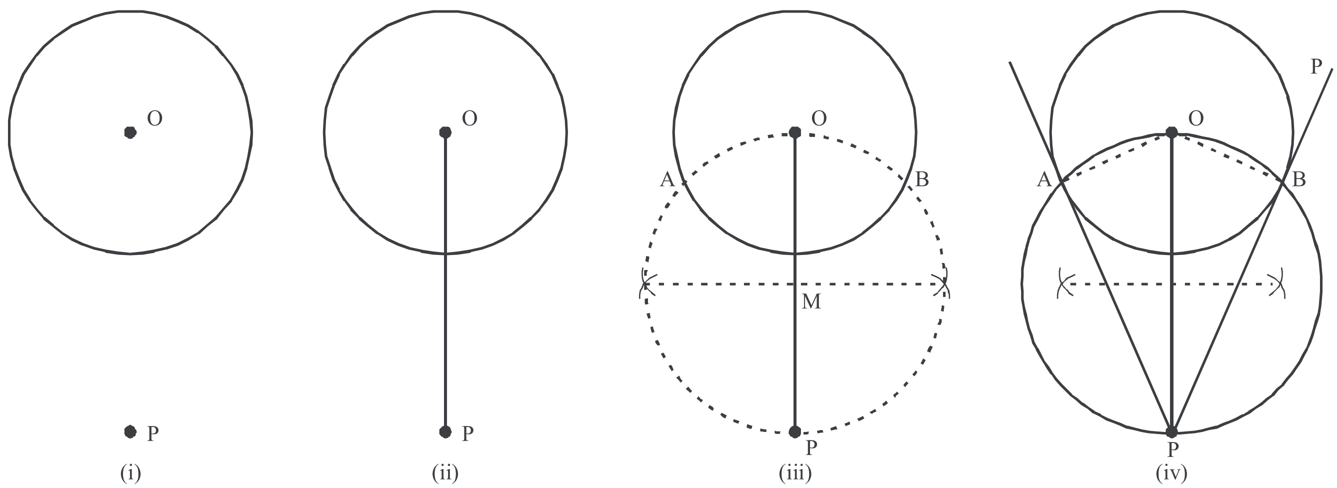
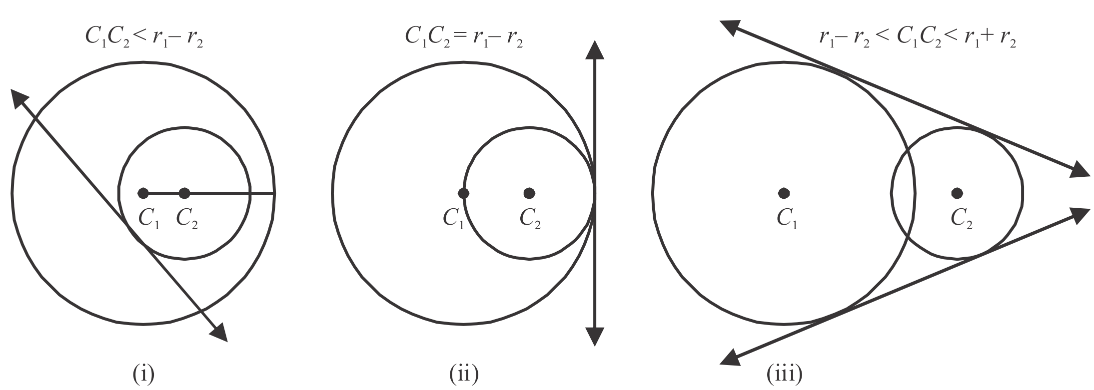


Fig. 14.04

14.05. Common Tangents

To find the maximum number of tangents to a circle or more than one circle we have to consider the different conditions. See the figures given below:



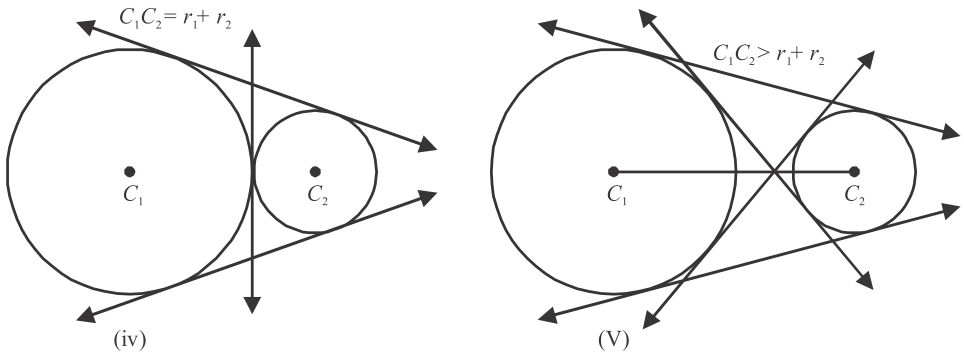


Fig. 14.05

I. In figure 14.05(i), if a tangent is drawn to the smaller ercircle, it intersects the larger circle. So, if $C_1C_2 < r_1 - r_2$ then the number of common tangents is zero.

II. In figure 14.05(ii), there is only one common tangent to both the circles and these circles are in the same side of the tangent.This is called a direct common tangent. Now when $C_1C_2 = r_1 - r_2$, then the number of common tangent is only one.

III. In the figure 14.05(iii), both the circles intersect each other so there are two tangents to both the circles in two opposite sides.Since two circles are in the middle of the tangents i.e., in the same side of each tangent. So both the tangents are considered as direct common tangents. When $r_1 - r_2 < C_1C_2 < r_1 + r_2$, then the number of direct common tangents will be 2.

IV. In the figure 14.05(iv), both the circles touch each other externally, so the number of common tangents is three. Here one common tangent is situated at the common touch point of the two circles, which is called the transversal common tangent, as two circle are situated at the two different sides of this tangent.

Thus if $C_1C_2 = r_1 + r_2$, then the total number of common tangents will be 3 (2 direct common tangents and one transversal common tangent).

V. In the figure 14.05(v), two circles do not touch each other, so the number of common tangents to the two circles is 4. (2 direct common tangents and two transversal common tangents).

Thus, when $C_1C_2 > r_1 + r_2$, then the total number of common tangents is 4 in which there 'two' direct and two transversal tangents.

Construction 5 : To construct a common tangents of both circles where distance between centres of both the circles having different radius is known.

Steps of construction

(i) Draw a line segment $C_1C_2 = 5.5$ cm, with centres C_1 and C_2 and radii $r_1 = 3.5$ cm and $r_2 = 2.5$ cm, draw two circles respectively.

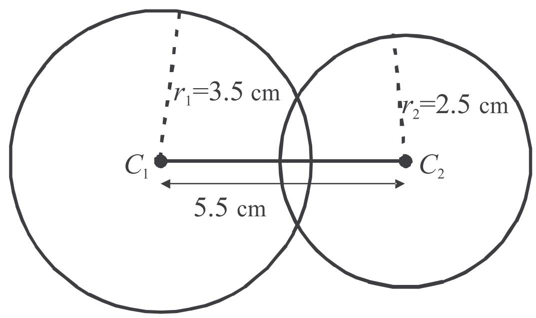


Fig. 14.06

(ii) With the centre of a larger circle C_1 and radius $r_1 - r_2 = 3.5 - 2.5 = 1$ cm, draw another circle. Taking C_1C_2 as diameter draw a dotted circle which intersects the smallest circle at P.

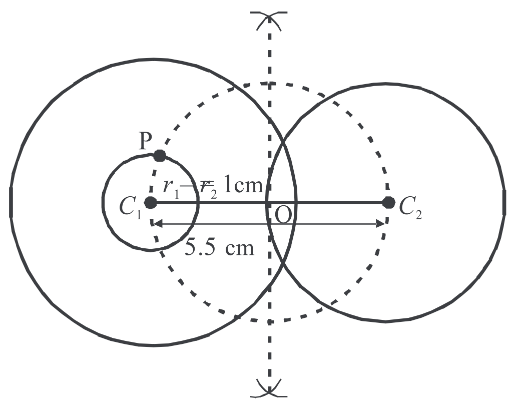


Fig. 14.07

(iii) Join P to C_2 and from C_2 draw a tangent to the circle of radius $r_1 - r_2$. Join C_1 to P and extend it to meet the largest circle at Q of radius r_1. Taking Q as center and radius PC_2 draw an arc which intersects the circle of radius r_2 at R. Join Q to R.

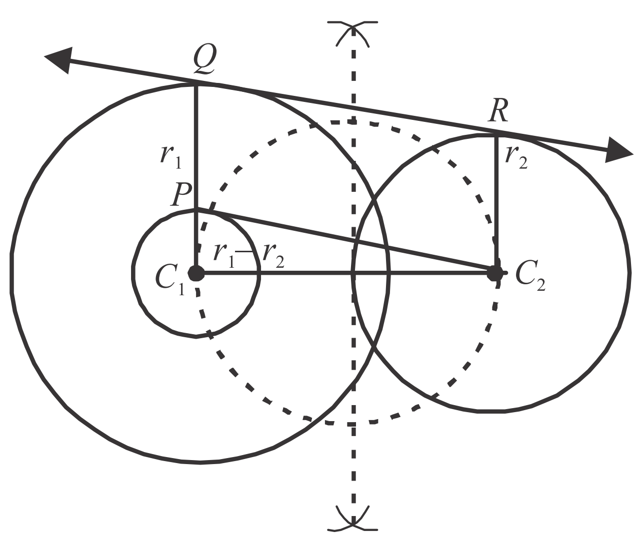


Fig. 14.08

Note: In this way we get a rectangle $PQRC_2$. Therefore C_1Q and C_2R are the perpendiculars at the line QR. So QR is called the direct common tangent to both the circles. Similarly, another tangent line can also be drawn in opposite side.

Important note : To construct a direct common tangent line.

(i) Find out $r_1 - r_2$.

(ii) Draw circle with radius $r_1 - r_2$ must be co-centred of the larger circle.

(iii) If twocircles have equal radii, then join C_1 to C_2 and draw perpendiculars at both ends C_1C_2, and join these intersecting points what is obtained is the required tangent.

Construction 6 : To construct a common transversal tangent to two circles (radii r_1 and r_2 and centres C_1 and C_2 are given).

Example : The centres of two circles with radii 2.5 cm and 1.0 cm are at a distance of 7 cm to each other. Draw a common transversal tangent line.

Steps of Constructions :

(i) Draw a line segment C_1C_2 = 7 cm with the centres C_1 and C_2 and radii 2.5 cm and 1.0 cm, draw two circles at both end points.

(ii) With the radius equal to the sum of the radii of both the circles *i.e.*, $r_1 + r_2 = 2.5 + 1.00 = 3.5$ cm and from centre C_2 of smaller circle, draw a dotted circle of radius 3.5 cm. Find the bisector of the diameter C_1C_2 and taking the bisecting point O as centre draw a dotted circle taking radius as $OC_1 = OC_2$. Which intersects circle with radius 3.5 cm at R. Join C_1 to R, and draw the tangent to the circle of radius $r_1 + r_2$ from C_1.

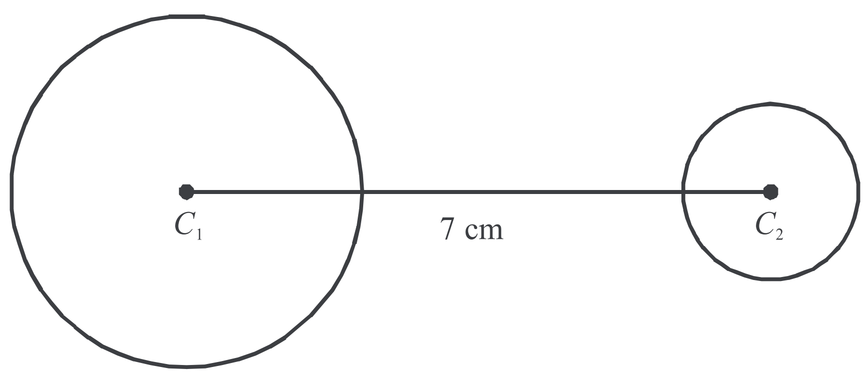


Fig. 14.09

(iii) Join R to C_2 which touches the smaller circle at Q. Taking Q as centre and RC_1 as radius draw an arc whcih intersect the larger circle at P.

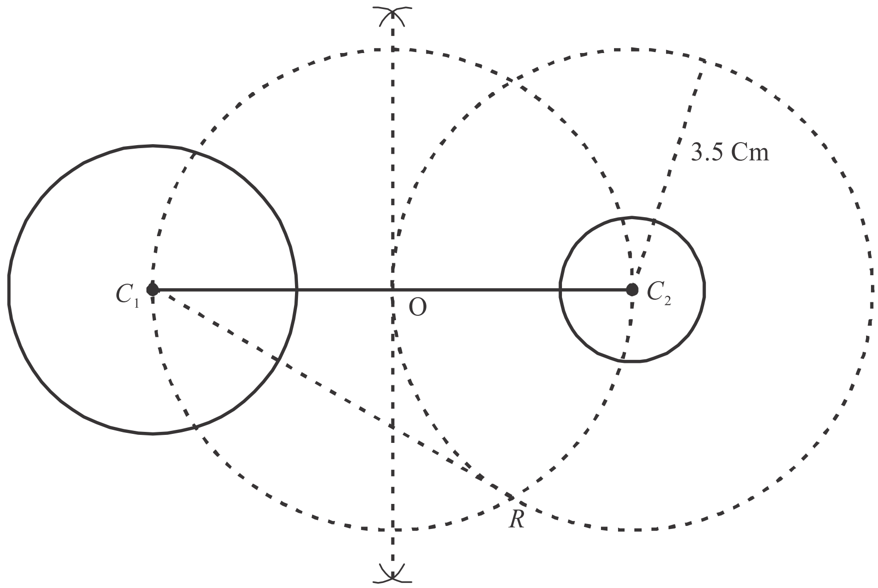


Fig. 14.10

Joint P to Q. Hence PQ, is the required tangent.

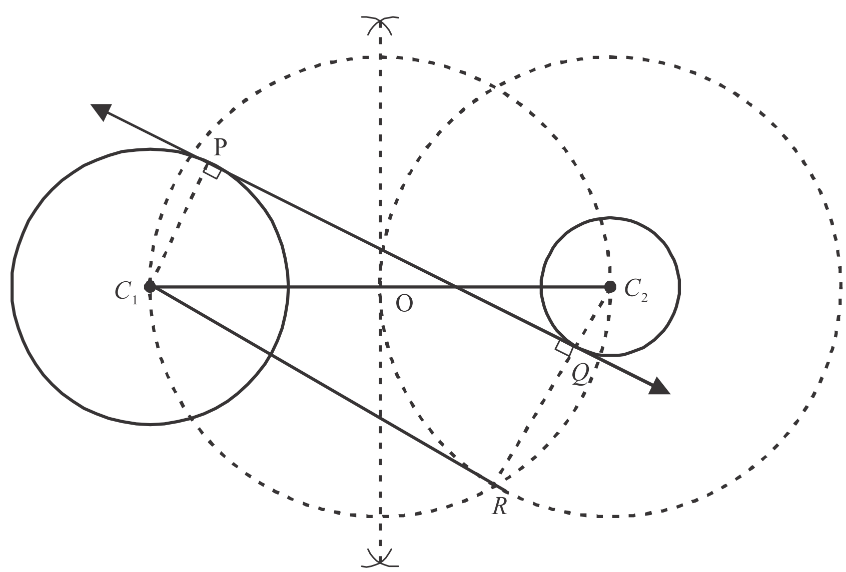


Fig. 14.11

Illustrative Examples

Example 1. Draw a tangent to a circle of radius 2.5 cm.

Solution :

Steps of Construction

(i) Draw a circle with the centre O and radius 2.5cm.

(ii) Take a point P on the circle and join O to P.

(iii) Draw a perpendicular at P i.e., draw $\angle OPA = 90°$ Extend the perpendicular to both sides upto A and B.

Hence, APB is the required tangent.

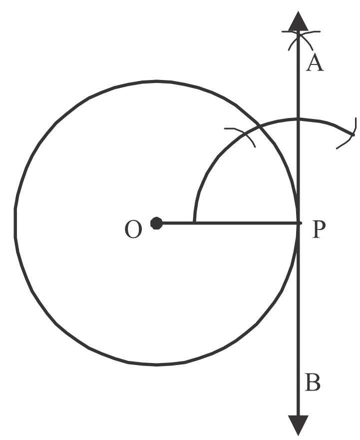


Fig. 14.12

Example 2. Construct a circle with the centre O and radius 2.8 cm. Take a point P at a distance of 4.3 cm from the centre and draw a pair of tangents to the circle from it. Also measure the tangents and verify that they are equal.

Steps of Construction:

(i) With the centre O and radius 2.8 cm draw a circle.

(ii) Take a point P outside the circle at a distance of 4.3cm from its centre and join O to P.

(iii) Draw the perpendicular bisector of OP, find its mid point M.

(iv) Take M as centre and radius $OM = PM$ and draw a circle which intersects the given circle at the points A and B.

(v) Join A to P and B to P.

Now PA and PB are the required tangents.

By measuring PA and PB, we find that PA = PB = 3.2 cm.

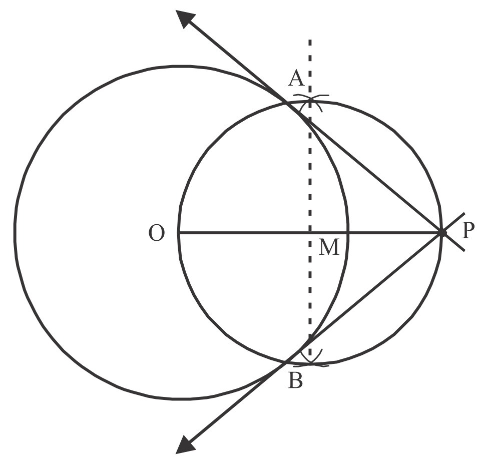


Fig. 14.13

Example 3. Draw a circle with center O and radius 3 cm. Draw its two radii OA and OB such that the angle between them is 180^{o} and draw a pair of tangents at A and B.

Steps of Construction :

(i) Draw a circle with centre O and radius 3 cm.

(ii) Draw two radii OA and OB such that angle between them be 120^{o}.

(iii) Draw the perpendicular at A and B on two radii which intersect each other at P.

(iv) Join P to A and P to B.

Here PA and PB are the required pair of tangents.

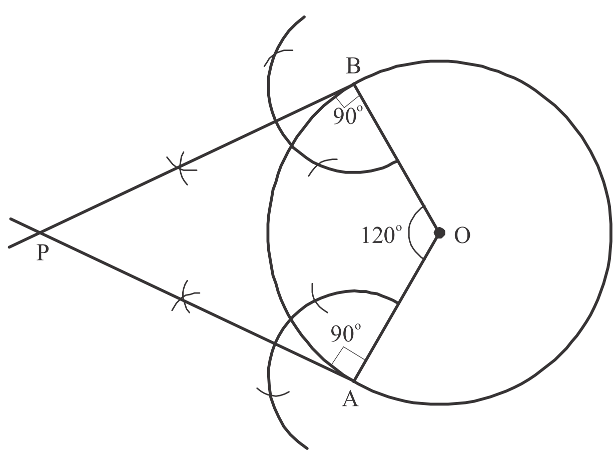


Fig. 14.14

Example 4. Draw two tangents to the circle of radius 4 cm such that the measure of the angle between them is 80^{o}.

Solution : We have $\angle APB = 80°$

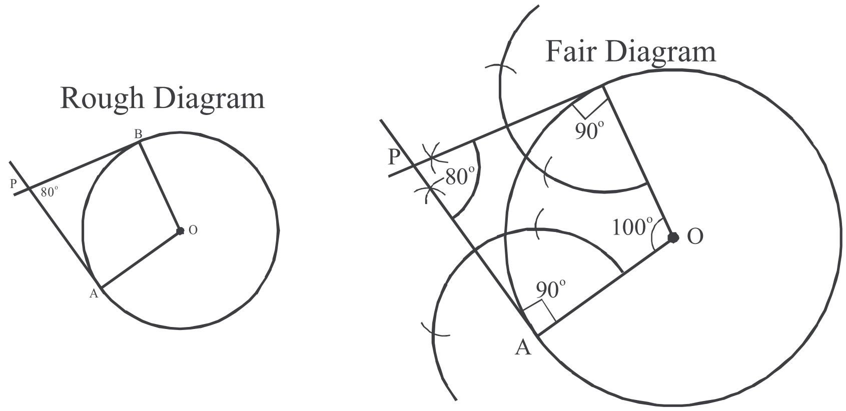
and we know that $\angle A = \angle B = 90°$

$\therefore$ In a quadrilateral $AOBP$

fourth angle $\angle AOB = 360 - (80 + 90 + 90) = 360 - 260 = 100°$

$\therefore$ the angle between two radii OA and OB

$= \angle AOB = 100°$



(i) (ii)

Fig. 14.15

Steps of Construction :

(i) Draw a circle of radius 4 cm with centre O.

(ii) Make an angle AOB of 100^o at the center O.

(iii) Draw the perpendiculars at A and B respectively, which intersects each other at P.

(iv) Join A to P and B to P.

Here AP and AB are the required tangents including the angle APB = 80^o.

Example 5. The distance between the centres of two circles whose radii are 4 cm and 3 cm respectively is 6.5 cm, draw any one direct common tangent to these circles.

Steps of Construction :

Let C_1 and C_2 are the centre of both the circles respectively where C_1C_2 = 6.5 cm.

(i) Draw a line segment C_1C_2 = 6.5 cm and aslo draw two circles with C_1 as centre and radius 4 cm and C_2 as centre and radius 3 cm.

(ii) Taking the centre C_1 of the larger circle and radius $r_1 - r_2 = 4 - 3 = 1$cm, draw a circle.

(iii) Bisect C_1C_2 at M.

(iv) Take M as centre and $C_1M = C_2M$ as radius, draw another circle, which intersects the circle of radius 1 cm at P.

(v) Join C_2 to P and join C_1 to P and produce it to Q.

, (vi) Join Q to R.

Here QR is the required tangent.

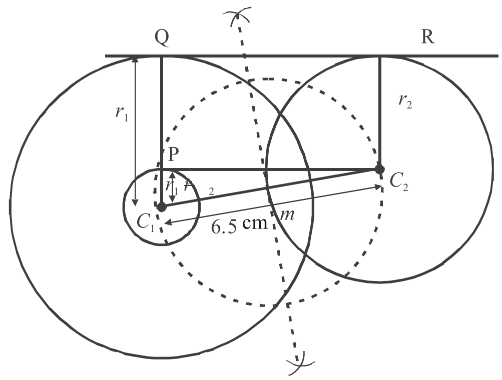


Fig. 14.16

Example 6. The distance between the centres of the two circles of 8 cm. Radius of one circle is 2.5 cm and that of other is 1.5 cm. Draw transversal common tangent line to the circles.

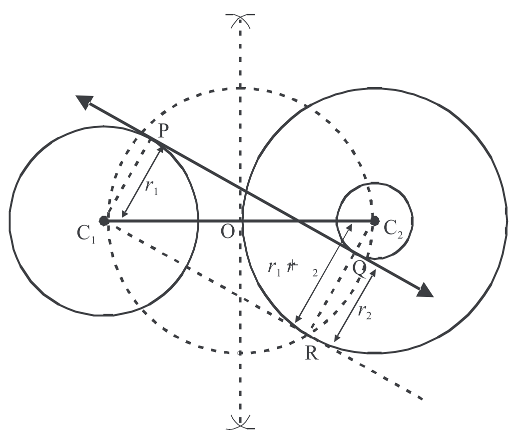


Fig. 14.17

Steps of Construction :

Let C_1, C_2 and r_1, r_2 are centre and radii of both the circles respectively where r_1 = 2.5 cm, r_2 = 1.5 cm and C_1C_2 = 8 cm.

(i) Draw a line segment C_1C_2 = 8 cm.

(ii) Draw two circles with centre C_1 and C_2 radius 2.5 cm and 1.5 cm respectively at both the ends.

(iii) Draw a circle with C_2 as centre and radius 2.5 + 1.5 = 4.0 cm.

(iv) Take C_1C_2 as diameter. Draw another circle which intersects the circle of radius 4 cm at *R*. Join C_1 at *R*. Join C_2 to *R* which intersects the circle of radius 1.5 cm at *Q*.

(v) Take *Q* as centre and radius equal to RC_1. Draw an arc which intersect the circle with centre C_1 at *P*. Join *Q* to *P*.

Here *PQ* is the required transversal tangent.

Exercise 14.1

1. Draw a line segment of length 6.7 cm and divide it into the ratio of 2 : 3 internally.
2. Draw a line segment AB of length 8.3 cm. Find a point C at AB such that $AC = \frac{1}{3}AB$ and also verify it.
3. Take a point P at the circle of radius 2.8 cm and draw a tangent to the circle at P.
4. Draw the tangents to both the ends of the diameter of circle of radius 3 cm. Will the tangents intersects each other? If yes, justify your answer.
5. Draw a chord of lengh 2.3 cm in a circle of radius 13.1 cm and draw the tangent to its both ends.
6. Draw a tangent to a circle of radius 2.7 cm.
7. Draw a circle with centre O and radius 2.4 cm. Draw two radii OA and OB such that the angle between them is 60^{o}. Draw the tangents to A and B to intersect each other at T. Measure the angle ATB.
8. Draw two tangents to the circle of radius 13.2 cm, such that the angle between them is 70^{o}.
9. Draw a circle of radius 3cm. Construct a pair of tangents from an exterior point 5 cm away from its centre.
10. The distance between the centres of two circles is 8 cm. Radius of one circle is 3 cm and that of other is 4 cm. How many common tangents can be drawn on these circles and construct two direct common tangents.
11. Construct a common transversal tangents of the circles whose radii are 1.7 cm and 2.8 cm respectively and centres are 6cm apart.

14.06. Constuction of Circumcircle and Incircle of a Triangle

(A) Construction of circumcircle of a triangle

The *circumcircle* of a triangle is a circle passing through the vertices of the triangle. The centre of the circle is called the circumcentre of the triangle. The circumcentre of a triangle is a point where the perpendicular bisectors of three sides of triangle intersect each other.

To construct a circumcircle of triangle, the following steps are to be taken :

(i) To find the circumcircle of a triangle, perpendicular bisectors of any of the two sides are drawn. The intersection points of these bisectors is the circumcentre of the triangle.

(ii) Take circumcentre as centre and radius between the centre and any of the vertex.

(iii) Draw a circle which passes through three vertices of the triangle. It is the required circumcircle.

Example 7. Construct a ΔABC in which BC = 3.8 cm, $\angle B = 60^\circ$ and $\angle C = 55^\circ$. Draw a circumcircle of this triangle and check the position of circumcentre.

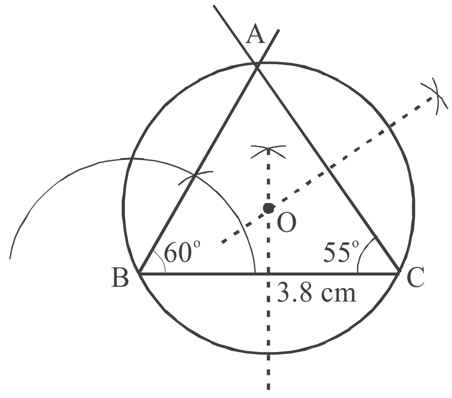


Fig. 14.18

Steps of Construction :

(i) Construct ΔABC according the given measurements.

(ii) Draw the perpendicular bisectors of the sides BC and AC that intersect each other at the point O.

(iii) Take O as centre and radii $OA = OB = OC$, draw a circle.

This is the required circumcircle

Note : *The circumcentre of an acute angled triangle always lies inside it.*

Example 8. Construct ΔABC in which side BC = 4 cm, $\angle B = 40^\circ$ and $\angle A = 90^\circ$. Construct the circumcircle of the triangle. Also find the position of its circumcentre.

Steps of Construction :

Since, in a triangle ABC

$\angle A + \angle B + \angle C = 180^\circ$

$\therefore$ $90° + 40° + \angle C = 180°$

$\Rightarrow$ $\angle C = 180° - 135° = 50°$

(i) Draw a line segment BC = 4 cm, $\angle B = 40^\circ$ and $\angle C = 50^\circ$, in this way we get $\angle A = 90^\circ$.

(ii) Draw the perpendicular bisectors of the sides AB and AC to intersect each other at O.

(iii) Take O as centre and radius $OA = OB = OC$ draw a circle, which passes through the vertices of ΔABC.

This is the required circumcircle of ΔABC.

Note : The circumcentre of a right angled triangle always falls at the hypotenuse.

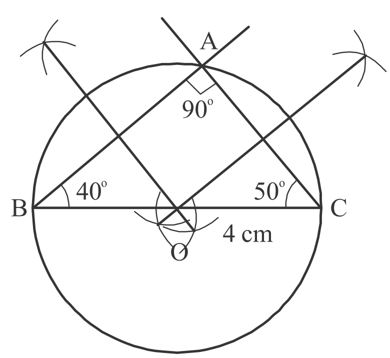


Fig. 14.19

Example 9. Construct a circumcircle of a triangle with base BC= 6 cm, $\angle B = 30^\circ$ and $\angle C = 25^\circ$. Also find the position of the circumcentre of the triangle.

Steps of Construction :

(i) Construct the ΔABC according to the given measures.

(ii) Draw the perpendicular bisector of sides AB and AC which intersect each other at the point O. It is the circumcentre of the triangle.

(iii) Take O as centre and radii = OA = OB = OC, draw a circle, which passes through the three vertices A, B and C of given triangle.

Now the constructed circle is the required circle.

Note : The circumcentre of an obtuse angled triangle always falls in somewhere outside the triangle. According to the nature of the triangles the position of the circumcentre is always different.

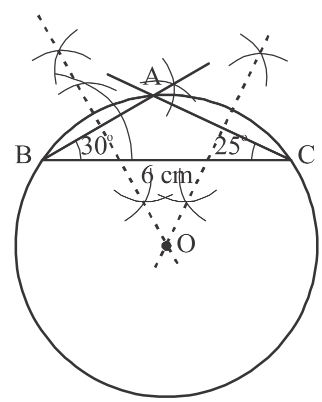


Fig. 14.20

(a) In an acute angled triangle, the position of its circumcentre is inside the triangle.

(b) In an right angled triangle, the position of its circumcentre is always at its hypotenuse.

(c) In an obtuse angled triangle the position of its circumcentre is always at an external point of the triangle.

(B) Construction of an incircle of triangle

The incircle of a triangle is a circle drawn inside a triangle touching all the sides of the triangle. The centre of this circle is called the incentre of the triangle. The intersecting point of the bisectors of angles of a triangle is called the incentre of the triangle. We use the following steps to understand the construction of incircle.

(i) Draw bisector of any two angles of the given triangle.

(ii) Draw a perpendicular to any one side of the triangle form the intersecting point of bisetors (bisector of any angle is at the same distance from the two sides of the angle).

(iii) Take radius equal to the length of the perpendicular drawn and intersecting point as centre ad draw a circle. It will be the incircle of the given triangle.

Example 10. Construct an incircle of the ΔABC in which base BC = 5.8 cm, AB = 5 cm and $\angle B = 55^\circ$.

Steps of Construction :

(i) Draw a triangle ABC accord to the given measurements in the question.

(ii) Draw the bisector of the $\angle B$ and $\angle C$, which intersect each other at *O*. *O* is the incenntre.

(iii) Draw $OM \perp BC$ form *O*.

(iv) Take *O* as centre and *OM* as radius. Draw a circle which touches the sides of ΔABC at *P, M* and *N* respectively.

This is the required incircle of ΔABC

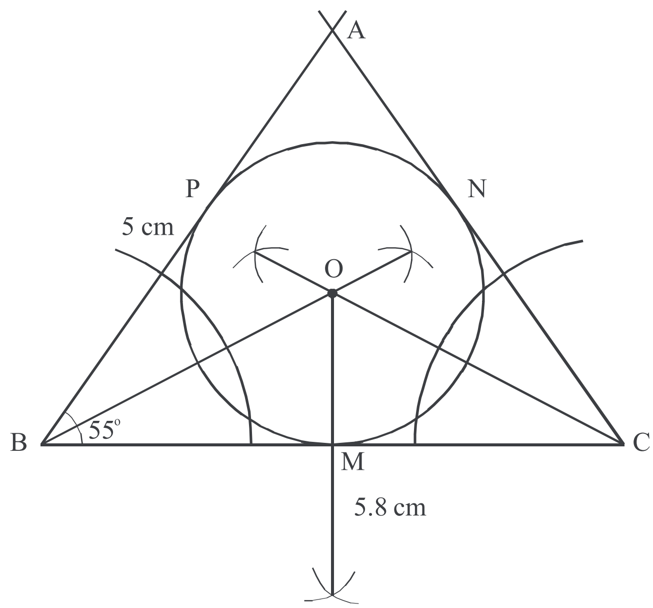


Fig. 14.21

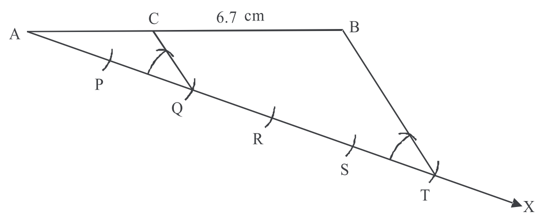
Exercise 14.2

1. Find the true or false in the following statements and justify your answer if possible.
   (i) Circumcircle and incircle of an equilateral triangle can be drawn form the same centre.
   (ii) All three sides of a triangle touch its incircle.
   (iii) If the triangle is obtuse angled, its circumcentre will fall at one of its sides.
   (iv) The circumference of the triangle lies inside, if the triangle is an acute triangle.
   (v) The construction of incircle is being done by obtaining the point of intersection of two perpendicular of sides and bisector of two angles.
2. Construct an incircle of an equilateral triangle with side 4.6 cm, Is its incentre and circumcentre are coincidence? Justify your answer.
3. Construct an incircle of a triangle with AB = 4.6 cm, AC = 4.2 cm and $\angle A = 90^{\circ}$.
4. Draw a circumcircle of a triangle with sides respectively 5, 12 and 13 cm. Why does its circumcentre falls at the side of length 13 cm?
5. Draw a circumcircle of a triangle whose sides are 5 cm, 4.5 cm and 7 cm. Where and why does its circumcentre lie?
6. Construct a ΔABC in which AB = 6 cm, BC = 4 cm and $\angle B = 120^{\circ}$ also construct incircle of this triangle.

Answers

Exercise 14.1

1\.



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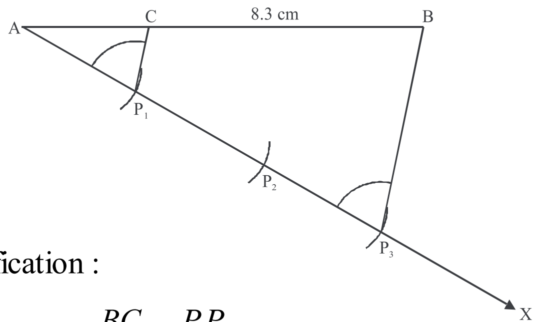

Verification :

$\therefore$ $$\frac{BC}{AC} = \frac{P_1P_3}{AP_1}$$ (Fundamental Proportional Theorem)

or $$\frac{BC}{AC} + 1 = \frac{P_1P_3}{AP_1} + 1$$

or $$\frac{BC + AC}{AC} = \frac{P_1P_3 + AP_1}{AP_1}$$

or $$\frac{AB}{AC} = \frac{AP_1 + P_1P_3}{AP_1} = \frac{3}{1}$$

or $$\frac{AC}{AB} = \frac{1}{3}$$

Hence, C is such a point on AB that $AC = \frac{1}{3}AB.$

Do other construction yourself with the help of your teacher.

Exercise 14.2

(1) (i) True, since circumcentre, incentre and orthocentre of an equilateral triangle are coincides.

(ii) True, since an incircle is drawn with the radius equal to length of perpendicular to a side from incentre.

(iii) False, a right angled triangle has its circumcentre on its hypotenuse.

(iv) True,

(v) False, to construct an incircle of a triangle by bisecting its angle not its sides.

4\. Since, the side of length 13 cm is the hypotenuse of the given triangle and circumcentre of a right angled triangle always falls on its hypotenuse.