
Sample Paper-02 (solved)
Mathematics
Class – XI

Time allowed: 3 hours

Maximum Marks: 100

General Instructions:

- a) All questions are compulsory.
- b) The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
- c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- d) Use of calculators is not permitted.

Section A

1. Check whether the given lines are parallel or perpendicular.
 $ax - by + c = 0$ and $\frac{ax}{2} - \frac{by}{2} + d = 0$
2. Find the area of the triangle whose vertices are $(2,0), (5,3), (2,6)$
3. Write the equation of a circle with center $(0,0)$ and radius 5
4. Identify a function $f(x)$ so that $f(x).f(y) = f(x+y)$
5. If $A = \{(x, y) : y = a^x, x \in R\}$ and $B = \{(x, y) : y = a^{-x}, x \in R\}$ then what is $(A \cap B)$
6. If R is a relation from a set A containing p elements to a set B containing q elements then find the number of subsets of $A \times B$

Section B

7. If $f(x)$ is a function that contains 3 in its domain and range and satisfy the relation $f(f(x)).(1 + f(x)) = -f(x)$ find $f(3)$
 8. If $\tan A = \frac{1}{3}$ and $\tan B = \frac{1}{2}$ prove that $\sin 2(A + B) = 1$
 9. Find two numbers such that their arithmetic mean is 15 and Geometric mean is 9 without using the identity $(a + b)^2 = (a - b)^2 + 4ab$
 10. Let $f : R \rightarrow R$ be a function given by $f(x) = x^2 + 2$ find $f^{-1}(27)$
 11. Find the domain and range of the function $f(x) = \frac{x - a}{a + 1 - x}$ where a is a positive integer.
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12. Find the limit of $\lim_{x \rightarrow 0} \frac{\sqrt{a+x} - \sqrt{a}}{x}$
 13. Find the sign and value of the expression $\sin 75^\circ + \cos 75^\circ$
 14. Solve $\cos 3x = -\frac{1}{2}$
 15. Prove by mathematical induction that $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$
 16. Find the square root of $\sqrt{-8i}$
 17. Solve the inequality $\frac{2x+5}{x-2} \geq 3$
 18. Find the value of x if ${}^{12}C_x = {}^{12}C_{x+4}$
 19. Three cars are there in a race. Car A is 3 times as likely to win as car B. Car B is twice as likely to win as car C. What is the probability of winning each car.

Section C

20. If the ratio of the roots of the equation $x^2 + px + q = 0$ is the same as $x^2 + p_1x + q_1 = 0$ then prove that $p^2q_1 = p_1^2q$
 21. Prove that $a \cdot a^{\frac{1}{2}} \cdot a^{\frac{1}{4}} \cdot a^{\frac{1}{8}} \dots \infty = a^2$
 22. In a survey of 700 students in a medical college 200 went for regular entrance coaching, 295 attended only correspondence coaching, 115 attended both regular and correspondence coaching. Find how many got admission without any entrance coaching.
 23. In how many ways can 3 students from Class 12, 4 from class 11, 4 from class 10 and 2 from class 9 be seated in a row so that those of the same classes sit together. Also find the number of ways they can be arranged in at a round table
 24. A circle represented by the equation $(x-a)^2 + (y-b)^2 = r^2$
This makes two complete revolutions along the positive direction of the x axis. Find the equation of the circle in the new position
 25. Show that the equation $x^2 + 4y^2 + 4x + 16y + 16 = 0$ represents an ellipse.
 26. Calculate the mean deviation about the mean from the following data

x_i	2	15	17	23	27
f_i	12	6	12	9	5
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ANSWERS

Section A

1. Solution

They are parallel since

$$\begin{vmatrix} a & -b \\ a & -b \\ 2 & 2 \end{vmatrix} = 0$$

2. Solution

Area of a triangle

$$\frac{1}{2} \begin{vmatrix} 2-2 & 0-6 \\ 5-2 & 3-6 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} 0 & -6 \\ 3 & -3 \end{vmatrix} = 9$$

3. Solution

$$x^2 + y^2 = 25$$

4. Solution :

$$f(x) = a^x$$

$$f(y) = a^y$$

$$f(x).f(y) = a^x.a^y = a^{x+y} = f(x).f(y)$$

5. Solution :

When $x = 0, y = 1$ in both cases. Hence

$$(A \cap B) = \{0, 1\}$$

6. Solution : 2^{pq}

Section B

7. Solution :

Let a satisfy the relation $f(a) = 3$

$$f(f(a)).(1 + f(a)) = -f(a)$$

$$f(3).(4) = -3$$

$$f(3) = -\frac{3}{4}$$

8. Solution:

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\frac{1}{3} + \frac{1}{2}}{1 - \frac{1}{3} \cdot \frac{1}{2}}$$

$$= 1$$

$$A + B = 45$$

$$2(A+B) = 90$$

$$\sin 90 = 1$$

9. Solution:

Form a quadratic equation sum of whose roots are 30 and product of the roots is 81

$$x^2 - x(30) + 81 = 0$$

$$x^2 - 3x - 27x + 81 = 0$$

$$x(x-3) - 27(x-3)$$

$$(x-3)(x-27) = 0$$

Hence the numbers are 3 and 27

10. Solution:

Let $f: R \rightarrow R$ be a function given by $f(x) = x^2 + 2$ find $f^{-1}(27)$

$$f(x) = x^2 + 2$$

$$x^2 + 2 = 27$$

$$x^2 = 25$$

$$x = \pm 5$$

$$f^{-1}(27) = \{-5, 5\}$$

11. Solution:

The function is defined for all values of x where the denominator is not equal to zero

$$a+1-x \neq 0$$

Hence domain =

$$R - \{(a+1)\}$$

Range of f

Let $y = f(x)$

$$y = \frac{x-a}{a+1-x}$$

$$(a+1)y - xy = x - a$$

$$x(y+1) = (a+1)y + a$$

$$x = \frac{(a+1)y + a}{y+1}$$

Range of $f = R - \{-1\}$

12. Solution

Rationalize the numerator

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sqrt{a+x} - \sqrt{a}}{x} \\ = \lim_{x \rightarrow 0} \frac{(\sqrt{a+x} - \sqrt{a})(\sqrt{a+x} + \sqrt{a})}{x(\sqrt{a+x} + \sqrt{a})} \end{aligned}$$

$$= \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{a+x} + \sqrt{a})}$$

$$= \frac{1}{2\sqrt{a}}$$

13. Solution:

$$\sin 75^\circ + \cos 75^\circ$$

$$= \sqrt{2} \left(\frac{1}{\sqrt{2}} \sin 75^\circ + \frac{1}{\sqrt{2}} \cos 75^\circ \right)$$

$$= \sqrt{2} (\cos 45^\circ \sin 75^\circ + \sin 45^\circ \cos 75^\circ)$$

$$= \sqrt{2} \sin(75^\circ + 45^\circ)$$

$$= \sqrt{2} \sin 120^\circ$$

Hence sign is positive and value is $\frac{\sqrt{2} \cdot \sqrt{3}}{2} = \frac{\sqrt{6}}{2}$

14. Solution:

$$\cos 3x = \cos \frac{2\pi}{3}$$

$$3x = 2n\pi \pm \frac{2\pi}{3}$$

$$x = \frac{2n\pi}{3} \pm \frac{2\pi}{9}, n \in \mathbb{Z}$$

15. Solution:

Let $P(n)$ be the statement given by $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$

$$P(1) = \frac{1(1+1)}{2}$$

$= 1, \text{True}$

Let it be true for $n = m$

$$1 + 2 + 3 + \dots + m = \frac{m(m+1)}{2}$$

$$1 + 2 + 3 + \dots + m + (m+1) = \frac{m(m+1)}{2} + (m+1)$$

$$P(m+1) = \frac{m(m+1)}{2} + (m+1)$$

$$P(m+1) = \frac{m^2 + 3m + 2}{2}$$

$$P(m+1) = \frac{(m+1)(m+2)}{2}$$

Thus $P(m)$ is true $\Rightarrow P(m+1)$ is True

16. Solution:

$$\text{Let } \sqrt{z} = \sqrt{-8i}$$

$$\sqrt{z} = \pm \left\{ \frac{\sqrt{|z| - \operatorname{Re}(z)}}{\sqrt{2}} \right\} - i \left\{ \frac{\sqrt{|z| - \operatorname{Re}(z)}}{\sqrt{2}} \right\}, \operatorname{Im}(z) < 0$$

$$\sqrt{-8i} = \pm \left\{ \frac{\sqrt{8+0}}{\sqrt{2}} - i \frac{\sqrt{8-0}}{\sqrt{2}} \right\}, \operatorname{Im}(z) < 0$$

$$= \pm(2 - 2i)$$

17. Solution

$$\frac{2x+5}{x-2} - 3 \geq 0$$

$$= \frac{2x+5-3x+6}{x-2} \geq 0$$

$$= \frac{-x+11}{x-2} \geq 0$$

$$= \frac{x-11}{x-2} \leq 0$$

$$= (x-11)(x-2) \leq 0$$

$$x \in (2, 11]$$

18. Solution

$$x + x + 4 = 12$$

$$2x = 8$$

$$x = 4$$

19. Solution

Let p be the probability of winning Car C, $P(C)$

$$P(C) = p$$

$$P(B) = 2p$$

$$P(A) = 6p$$

$$P(A) + P(B) + P(C) = 1$$

$$p + 2p + 6p = 1$$

$$9p = 1$$

$$p = \frac{1}{9}$$

$$P(C) = \frac{1}{9}$$

$$P(B) = \frac{2}{9}$$

$$P(A) = \frac{6}{9}$$

Section C**20. Solution**

Let the ratios be

$$a : b$$

$$x^2 + px + q = 0$$

$$a\alpha + b\alpha = -p$$

$$a\beta + b\beta = -p_1$$

$$a\alpha \times b\alpha = q$$

$$a\beta \times b\beta = q_1$$

$$(a + b)\alpha = -p$$

$$(a + b)\beta = -p_1$$

$$ab\alpha^2 = q$$

$$ab\beta^2 = q_1$$

$$\frac{(a + b)^2 \alpha^2}{(a + b)^2 \beta^2} = \frac{p^2}{p_1^2}$$

$$\frac{\alpha^2}{\beta^2} = \frac{p^2}{p_1^2}$$

$$\frac{\alpha^2}{\beta^2} = \frac{q}{q_1}$$

$$\frac{p^2}{p_1^2} = \frac{q}{q_1}$$

$$p^2 q_1 = p_1^2 q$$

21. Solution :

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \infty = \frac{1}{1 - \frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$$

$$a \cdot a^{\frac{1}{2}} \cdot a^{\frac{1}{4}} \cdot a^{\frac{1}{8}} \dots \infty = a^2$$

22. Solution

It is given that

$$n(U) = 700, n(A) = 200, n(B) = 295, n(A \cap B) = 115$$

We need to find out

$$n(A' \cap B')$$

$$n(A' \cap B') = n(A \cup B)'$$

$$= n(U) - n(A \cup B)$$

$$= n(U) - \{n(A) + n(B) - n(A \cap B)\}$$

$$= 700 - \{200 + 295 - 115\}$$

$$= 320$$

23. Solution:

There are 4 groups and four groups can be arranged in 4! ways. Class 12 can be arranged in 3! ways,

Class 11 can be arranged in 4! Class 10 can be arranged in 4!. Class 9 can be arranged in 2! ways

Hence Total number of ways that they can be arranged in a row $4! \times 3! \times 4! \times 4! \times 2! = 165888$

In a circular seating arrangement the four groups can be arranged only in 3! ways only. Hence the total number of ways that they can be seated at a round table = $3! \times 3! \times 4! \times 4! \times 2! = 41472$

24. Solution

The new coordinates of the centre in the new position are

$$(a + 4\pi r, b)$$

$$\{x - (a + 4\pi r)\}^2 + (y - b)^2 = r^2$$

25. Solution

$$x^2 + 4y^2 + 4x + 16y + 16 = 0$$

$$x^2 + 4x + 4 + 4y^2 + 16y + 16 = 4$$

$$(x+2)^2 + 4(y+2)^2 = 4$$

$$\frac{(x+2)^2}{2^2} + \frac{(y+2)^2}{1^2} = 1$$

This equation represents an ellipse.

26. Solution

x_i	f_i	$f_i x_i$	$ x_i - 15 $	$f_i x_i - 15 $
2	12	24	13	156
15	6	90	0	0
17	12	204	2	24
23	9	207	8	72
27	5	135	12	60
	$N = \Sigma f_i = 44$	$\Sigma f_i x_i = 660$		$f_i \Sigma x_i - 15 = 312$

$$\text{Mean} = \bar{X} = \frac{1}{N} (\Sigma f_i x_i) = \frac{660}{44} = 15$$

$$\text{MeanDeviation} = M.D = \frac{1}{N} (\Sigma f_i |x_i - 15|) = \frac{312}{44} = 7.0909$$
