DAY SIXTEEN

Transfer of Heat

Learning & Revision for the Day

- Modes of Heat Transfer
- Some Common Terms and Points
- Perfectly Black Body
- Kirchhoff's Law of Radiation
- Stefan's Law
- Newtons Law of Cooling
- Wien's Displacement Law

Heat is a form of energy which characterises the thermal state of matter. It is transferred from one body to the other due to temperature difference between them.

Heat is a scalar quantity with dimensions $[ML^2T^{-2}]$ and its SI unit is joule (J) while practical unit is calorie (cal); 1 cal = 4.18 J.

The heat can be transferred from one body to the another body, through the following modes

(i) Conduction (ii) Convection (iii) Radiation

Conduction

The process of heat-transmission in which the particles of the body do not leave their position is called conduction.

Thermal Conductivity

The amount of heat transmitted through a conductor is given by $Q = \frac{KA\Delta T\, t}{l}$

where, A = area of cross-section,

ΔT = temperature difference $= T_2 - T_1$,

t = time elapsed,

K = thermal conductivity

and l = length of conductor

The rate of transmission of heat by conduction is given by

$$H = \frac{\Delta Q}{\Delta t} = \frac{KA\Delta T}{l}$$

The unit of thermal conductivity is $Wm^{-1}K^{-1}$.

Thermal Resistance

$$|H| = \left|\frac{\Delta Q}{\Delta t}\right| = \frac{KA}{l} \cdot \Delta T = \frac{\Delta T}{l / KA}$$

The term $\frac{l}{KA}$ is generally called the **thermal resistance** (R).

- Equation for rate of heat conduction can be written as

$$H = \frac{Q}{t} = \frac{\Delta T}{R_{\text{thermal}}}$$

It is equivalent/analysis to ohm's law which states that

$$I = \frac{V}{R_{(\text{electrical})}}$$

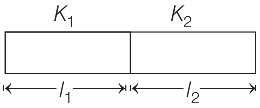
where, $H = \frac{Q}{t}$ is equivalent of electric current and called as heat, ΔT is equivalent of voltage (PD) and R_{thermal} is equivalent of $R_{\text{electrical}}$.

Combination of Metallic Rods

1. **Series Combination** In a series combination of two metal rods, equivalent thermal conductivity is given by

$$K_s = \frac{l_1 + l_2}{\frac{l_1}{K_1} + \frac{l_2}{K_2}}$$

or $$K_s = \frac{2K_1K_2}{K_1 + K_2} \quad [\text{if } l_1 = l_2]$$



Series combination of rods

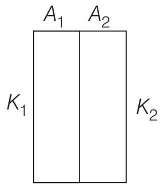
If temperature of the interface of the series combination be T, then

$$T = \frac{K_1T_1 + K_2T_2}{K_1 + K_2}$$

2. **Parallel Combinations** In a parallel combination of two metal rods, thermal conductivity is given by

$$K_p = \frac{K_1A_1 + K_2A_2}{A_1 + A_2}$$

or $$K_p = \frac{K_1 + K_2}{2} \quad [\text{if } A_1 = A_2]$$



Parallel combination of rods

Formation and Growth of Ice on a Lake

Time required for the thickness of the layer of ice to increase from d_1 to d_2 will be

$$t = \frac{\rho L_f}{2KT}(d_2^2 - d_1^2)$$

where, ρ = density of ice,
L_f = latent heat of fusion of ice
and K = thermal conductivity of ice

Widemann-Franz Law

According to the Widemann-Franz law, the ratio of thermal and electrical conductivities is same for the metals at a particular temperature and is proportional to the absolute temperature of the metal.

i.e. $$\frac{K}{\sigma} \propto T$$

or $$\frac{K}{\sigma T} = \text{constant}$$

Convection

The process of heat-transmission in which the particles of the fluid move is called convenction.

Natural Convection

In natural convection gravity plays an important role. When a fluid is heated, the hot part expands and becomes less dense. Consequently it rises and the upper colder part is replaced. This again gets hot, rises up and is replaced by the colder part of the fluid.

Forced Convection

In a forced convection the material is forced to move up by a pump or by some other physical means. Common examples of forced convection are human circulatory system, cooling system of an automobile engine and forced air heating system in offices, etc.

Radiation

The process of the transfer of heat from one place to another place without heating the intervening medium is called radiation.

Interaction of Radiation with Matter

When radiant energy Q is incident on a body, a part of it Q_a is absorbed, another part Q_r is reflected back and yet another part Q_t is transmitted such that

$$Q = Q_a + Q_r + Q_t$$

or $$\frac{Q_a}{Q} + \frac{Q_r}{Q} + \frac{Q_t}{Q} = 1$$

or $$a + r + t = 1$$

where, $a = \frac{Q_a}{Q}$ = absorbing power or absorptance,

$r = \frac{Q_r}{Q}$ = reflecting power or reflectance

and $t = \frac{Q_t}{Q}$ = transmitting power or transmittance

Some Common Terms and Points

- **Absorptive power** (α) It is defined as the ratio of the radiant energy absorbed by it in a given time to the total radiant energy incident on it in the same interval of time.

$$\alpha = \frac{\text{Energy absorbed}}{\text{Energy incident}}$$

As a perfectly black body absorbs all radiations incident on it, the absorptive power of a perfectly black body is maximum and unity.

- **Spectral absorptive power** (a_λ) It is the ratio of radiant energy absorbed by a surface to the radiant energy incident on it for a particular wavelength λ. The spectral absorptive power a_λ is related to absorptive power a through the relation

$$a = \int_0^\infty a_\lambda d\lambda$$

- **Emissive power** (e) It is the total amount of energy radiated by a body per second per unit area of surface

$$e = \frac{1}{A}\frac{\Delta Q}{\Delta t}$$

- **Spectral emissive power** (e_λ) It is emissive power for a particular wavelength λ. Thus,

$$e = \int_0^\infty e_\lambda d\lambda$$

- **Emissivity** (ε) Emissivity of a body at a given temperature is defined as the ratio of the total emissive power of the body (e) to the total emissive power of a perfect black body (E) at that temperature,

i.e. $$\varepsilon = \frac{e}{E}$$

Perfectly Black Body

A perfectly black body is the one which completely absorbs the radiations of all the wavelengths that are incident on it. Thus, absorbing power of a perfectly black body is 1 (i.e $a = 1$).

When perfectly black body is heated to a suitable high temperature, it emits radiation of all possible wavelengths.

e.g. temperature of the sun is very high (6000 k approx.) it emits all possible radiations. So, it is an example of black body.

- For perfectly black body, $a = 1$, $r = t = 0$
- For a perfect reflector, $a = t = 0$, $r = 1$
- For a perfect transmitter, $a = r = 0$, $t = 1$.

Kirchhoff's Law of Radiation

Kirchhoff's law of radiation states that the ratio of emissive power to absorptive power of a body, is same for all surfaces at the same temperature and is equal to the emissive power of a perfectly black body at that temperature.

Mathematically, $$\frac{e_1}{a_1} = \frac{e_2}{a_2} = \ldots = E \quad \text{(Black body)}$$

- Kirchhoff's law implies that 'a good absorber is a good emitter (or radiator) too'.
- Fraunhoffer's lines (dark lines observed in solar spectrum) can be easily explained on the basis of Kirchhoff's laws.

Stefan's Law

According to the Stefan's law, the emissive power of a perfectly black body (energy emitted by black body per unit surface area per unit time) is directly proportional to the fourth power of its absolute temperature.

Mathematically, $E \propto T^4$

or $E = \sigma T^4$

or $E = \sigma (T^4 - T_0^4)$

where, σ is a constant known as the **Stefan's constant** and its value is 5.67×10^{-8} Wm^{-2} K^{-4} and T_0 is the temperature of surrounding of black body.

- For a body, whose emissivity is ε, Stefan's law is modified as, $e = \varepsilon\sigma T^4$
- The total radiant energy Q emitted by a body of surface area A in time t, is given by

$$Q = Ate = At\varepsilon\sigma T^4$$

- The radiant power (P), i.e. energy radiated by a body per unit time is given by

$$P = \frac{Q}{t} = A\varepsilon\sigma T^4$$

- If a body at temperature T is surrounded by another body at temperature T_0 (where, $T_0 < T$), then according to Stefan's law of power

$$P = \varepsilon\sigma A(T^4 - T_o^4)$$

- If a body at temperature T is surrounded by another body at temperature T_0 (where, $T_0 < T$), then Stefan's law is modified as,

$$E = \sigma (T^4 - T_0^4) \quad \text{[black body]}$$

and $$e = \varepsilon\sigma (T^4 - T_0^4) \quad \text{[any body]}$$

Newton's Law of Cooling

According to the Newton's law of cooling, rate of cooling of a body is directly proportional to the temperature difference between the body and the surroundings, provided the temperature difference is small.

Mathematically, $\frac{dT}{dt} \propto (T - T_0)$ or $-\frac{dT}{dt} = k(T - T_0)$

where, k is a constant.

If a body cools by radiation through a small temperature difference from T_1 to T_2 in a short time t when the surrounding temperature is T_0, then

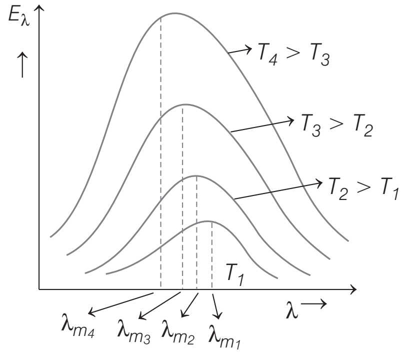
$$\frac{dT}{dt} \simeq \frac{T_1 - T_2}{t} \quad \text{and } T = \frac{T_1 + T_2}{2}$$

The Newton's law of cooling becomes

$$\left[\frac{T_1 - T_2}{t}\right] = k\left[\frac{T_1 + T_2}{2} - T_0\right].$$

Black Body Spectrum

The black body spectrum is a continuous spectrum as shown in the figure. At a given temperature, initially the intensity of thermal radiation increases with an increase in wavelength and reaches a maximum value at a particular wavelength λ_m. On increasing the wavelength beyond λ_m, the intensity of radiation E_λ starts decreasing.



Graph between intensity E_λ and λ

Variation of intensity of thermal radiation with wavelength is shown in fig. The total area under E_λ-λ curve gives the total intensity of radiation at that temperature. The area, in accordance with the Stefan's law of radiation, is directly proportional to the fourth power of the temperature.

Wien's Displacement Law

According to Wien's law, the product of wavelength corresponding to maximum intensity of radiation and temperature of body is constant i.e. $\lambda_m T$ = constant = b, where b is known as the Wien's constant and its value is 2.89×10^{-3} mK.

Solar Constant

The amount of heat received from the sun by one square centimeter area of a surface placed normally to the sun rays at mean distance of the earth from the sun is known as solar constant. It is denoted by S.

$$S = \left(\frac{r}{R}\right)^2 \sigma T^4$$

where, r is the radius of sun and R is the mean earth's distance from sun value of solar constant $S = 1.937$ cal/cm^2/min.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

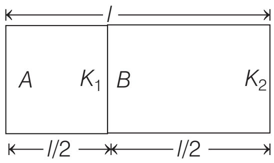
1 A cylindrical rod is having temperatures T_1 and T_2 at its ends. The rate of flow of heat is Q_1. If all the linear dimensions are doubled keeping the temperature constant, then rate of flow of heat Q_2 will be

(a) $4Q_1$ (b) $2Q_1$ (c) $\frac{Q_1}{4}$ (d) $\frac{Q_1}{2}$

2 A uniform metallic rod rotates about its perpendicular bisector with constant angular speed. If it is heated uniformly to raise its temperature slightly

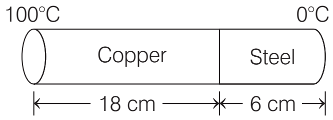
(a) Its speed of rotation increases
(b) Its speed of rotation decreases
(c) Its speed of rotation remains same
(d) Its speed increases because its moment of inertia increases

3 Two slabs A and B of different materials but with the same thickness are joined as shown in the figure. The thermal conductivities of A and B are K_1 and K_2, respectively. The thermal conductivity of the composite slab will be



(a) $\frac{1}{2}(K_1 + K_2)$ (b) $\sqrt{K_1K_2}$ (c) $(K_1 + K_2)$ (d) $\frac{2K_1K_2}{(K_1 + K_2)}$

4 The coefficient of thermal conductivity of copper is 9 times that of steel. In the composite cylindrical bar shown in the figure, what will be the temperature at the junction of copper and steel?



(a) 75°C (b) 67°C (c) 25°C (d) 33°C

5 Three objects coloured black, grey and white can withstand hostile conditions at 2800°C. These objects are thrown into furnace where each of them attains a temperature of 2000°C. Which object will have the brightest glow?

(a) The white object
(b) The black object
(c) All glow with equal brightness
(d) Grey object

6 A black body maintained at a certain temperature radiates heat energy at the rate Q Watt. If its surface is smoothened, so as to lower its emissivity by 10%, what will be the increase in its rate of radiation at double the initial temperature?

(a) $(0.9 \times 2^4 - 1)\,Q$ W (b) $0.9 \times 2^4\,Q$ W
(c) $(0.9 \times 2)^4\,Q$ W (d) $(0.9)^4 \times 2Q$ W

7 We consider the radiation emitted by the human body. Which of the following statement is true?

(a) The radiation is emitted during the summers and absorbed during the winters.
(b) The radiation emitted lies in the ultraviolet region and hence is not visible.
(c) The radiation emitted is in the infrared region.
(d) The radiation is emitted only during the day.

8 Parallel rays of light of intensity $I = 912\ \text{Wm}^{-2}$ are incident on a spherical black body kept in surroundings of temperature 300 K. Take, Stefan constant $\sigma = 5.7 \times 10^{-8}\ \text{Wm}^{-2}\ \text{K}^{-4}$ and assume that the energy exchange with the surroundings is only through radiation. The final steady state temperature of the black body is close to **→ 2014 Main**

(a) 330 K (b) 660 K (c) 990 K (d) 1550

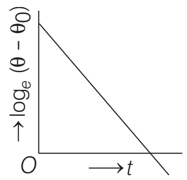
9 The spectral energy distribution of a star is maximum at twice temperature as that of the sun. The total energy radiated by the star is

(a) twice as that of the sun
(b) same as that of the sun
(c) sixteen times as that of the sun
(d) one-sixteenth of the sun

10 Newton's law of cooling holds good only, if the temperature difference between the body and the surroundings is

(a) less than 10°C (b) more than 10°C
(c) less than 100°C (d) more than 100°C

11 A liquid in a beaker has temperature $\theta(t)$ at time t and θ_0 is temperature of surroundings, then according to Newton's law of cooling, the correct graph between $\log_e(\theta - \theta_0)$ and t is

(a)


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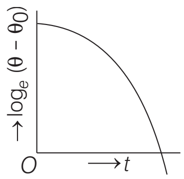

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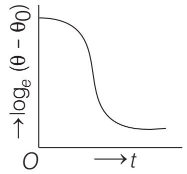

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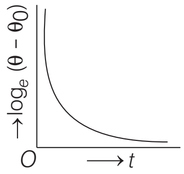
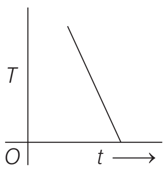
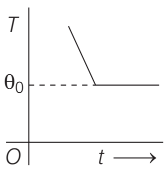
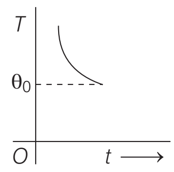
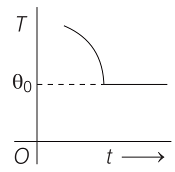


12 If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0, the graph between the temperature T of the metal and time will be closed to **→ JEE Main 2013**

(a)


(b)


(c)


(d)


13 A sphere, a cube and a thin circular plate, all of same material and same mass are initially heated to same high temperature. Then

(a) plate will cool fastest and cube the slowest
(b) sphere will cool fastest and cube the slowest
(c) plate will cool fastest and sphere the slowest
(d) cube will cool fastest and plate the slowest

14 Temperatures of two stars are in the ratio 3 : 2. If wavelength for the maximum intensity of the first body is 4000 Å, what is the corresponding wavelength of the second body?

(a) 9000 Å (b) 6000 Å
(c) 2000 Å (d) 8000 Å

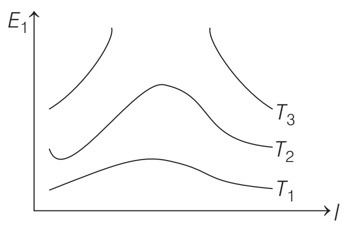
15 The energy spectrum of a black body exhibits a maximum around a wavelength λ_0. The temperature of the black body is now changed such that the energy is maximum around a wavelength $\frac{3\lambda_0}{4}$. The power radiated by the two black bodies will now increase by a factor of

(a) 64/27 (b) 256/81 (c) 4/3 (d) 16/9

16 Three discs, A, B and C having radii 2 m, 4 m and 6 m respectively, are coated with carbon black on their outer surfaces. The wavelengths corresponding to maximum intensity are 300 nm, 400 nm and 500 nm, respectively. The power radiated by them are Q_A, Q_B and Q_C, respectively

(a) Q_A is maximum
(b) Q_B is maximum
(c) Q_C is maximum
(d) $Q_A = Q_B = Q_C$

17 Variation of radiant energy emitted by sun, filament of tungsten lamp and welding arc as a function of its wavelength is shown in figure. Which of the following options is the correct match?



(a) Sun-T_1, tungsten filament-T_2, welding arc-T_3
(b) Sun-T_2, tungsten filament-T_1, welding arc-T_3
(c) Sun-T_3, tungsten filament-T_2, welding arc-T_1
(d) Sun-T_1, tungsten filament-T_3, welding arc-T_2

Direction (Q. Nos. 18-20) *Each of these questions contains two statements : Statement* I *and Statement* II. *Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c), (d) given below.*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

18 **Statement I** A solid sphere of copper of radius R and a hollow sphere of the same material of inner radius r and outer radius R are heated to the same temperature and allowed to cool down in the same environment. The hollow sphere cools faster.

Statement II Rate of cooling follows the Stefan's law which is $E \propto T^4$.

19 **Statement I** A body that is a good radiator is also a good absorber of radiation at a given wavelength.

Statement II According to Kirchhoff's law, the absorptivity of a body is equal to its emissivity at a given wavelength.

20 **Statement I** For higher temperatures, the peak emission wavelength of a black body shifts towards the lower wavelength side.

Statement II Peak emission wavelength of a black body is proportional to the fourth-power of the temperature.

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 A metallic sphere cools from 50°C to 40°C in 300 s. If the room temperature is 20°C, then its temperature in the next 5 min will be

(a) 38°C (b) 33.3°C (c) 30°C (d) 36°C

2 A pan filled with hot food cools from 94° C to 86 °C in 2 min, when the room temperature is at 20 °C, how long will it take to cool from 71 °C to 69 °C?

(a) 14 s (b) 3 s (c) 42 s (d) 13 s

3 Two slabs A and B of equal surface area are placed one over the other such that their surfaces are completely in contact. The thickness of slab A is twice that of B. The coefficient of thermal conductivity of slab A is twice that of B. The first surface of slab A is maintained at 100°C, while the second surface of slab B is maintained at 25°C. The temperature at the contact of their surfaces is

(a) 62.5°C (b) 45°C (c) 55°C (d) 85°C

4 Assuming the sun to be a spherical body of radius R at a temperature of T K, evaluate the total radiant power, incident on the earth, at a distance r from the sun.

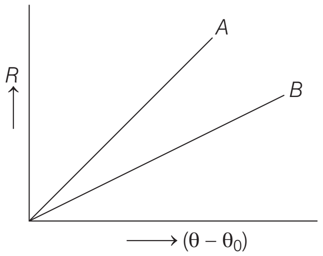
(a) $4\pi r_0^2 R^2 \sigma T^4 / r^2$ (b) $\pi r_0^2 R^2 \sigma T^4 / r^2$
(c) $r_0^2 R^2 \sigma T^4 / 4\pi r^2$ (d) $R^2 \sigma T^4 / r^2$

where, r_0 is the radius of the earth and σ is the Stefan's constant.

5 Two identical conducting rods are first connected independently to two vessels, one containing water at 100°C and the other containing ice at 0°C. In the second case, the rods are joined end to end and connected to the same vessels. Let q_1 and q_2 g/s be the rate of melting of ice in two cases respectively. The ratio of q_1 / q_2 is

(a) $\frac{1}{2}$ (b) $\frac{2}{1}$ (c) $\frac{4}{1}$ (d) $\frac{1}{4}$

6 Two circular discs A and B with equal radii are blackened. They are heated to same temperature and are cooled under identical conditions. What inference do you draw from their cooling curves?



(a) A and B have same specific heats
(b) Specific heat of A is less
(c) Specific heat of B is less
(d) None of the above

7 Three rods of copper, brass and steel are welded together to form a Y-shaped structure. Area of cross-section of each rod is 4 cm^2. End of copper rod is maintained at 100°C whereas ends of brass and steel are kept at 0°C. Lengths of the copper, brass and steel rods are 46, 13 and 12 cm respectively.
The rods are thermally insulated from surroundings except at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 in CGS units, respectively. Rate of heat flow through copper rod is

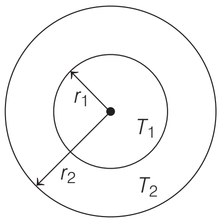
→ **JEE Main 2014**

(a) 1.2 cals^{-1} (b) 2.4 cals^{-1}
(c) 4.8 cals^{-1} (d) 6.0 cals^{-1}

8 A mass of 50 g of water in a closed vessel, with surroundings at a constant temperature takes 2 min to cool from 30°C to 25°C. A mass of 100g of another liquid in an identical vessel with identical surroundings takes the same time to cool from 30°C to 25°C. The specific heat of the liquid is (The water equivalent of the vessel is 30 g.) **→ JEE Main (Online) 2013**

(a) 2.0 kcal/kg (b) 7 kcal/kg
(c) 3 kcal/kg (d) 0.5 kcal/kg

9 The figure shows a system of two concentric spheres of radii r_1 and r_2 and kept at temperatures T_1 and T_2 respectively. The radial rate of flow of heat in a substance between the two concentric spheres, is proportional to



(a) $\frac{(r_2 - r_1)}{(r_1 r_2)}$ (b) $\ln\left(\frac{r_2}{r_1}\right)$ (c) $\frac{r_1 r_2}{(r_2 - r_1)}$ (d) $(r_2 - r_1)$

10 A slab of stone of area 3600 cm^2 and thickness 10 cm is exposed on the lower surface to steam at 100°C. A block of ice at 0°C rests on the upper surface of the slab. If in 1 h 4.8 kg of ice melted the thermal conductivity of the stone is

(a) 1.24 W/m/k (b) 2.24 W/m/k
(c) 0.24 W/m/k (d) 1.54 W/m/k

11 Two spherical stars *A* and *B* emit black body radiation. The radius of *A* is 400 times that of *B* and *A* emits 10^4 times the power emitted from *B*. The ratio $\left(\frac{\lambda_A}{\lambda_B}\right)$ of their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is

(a) 1 (b) 2
(c) 3 (d) 5

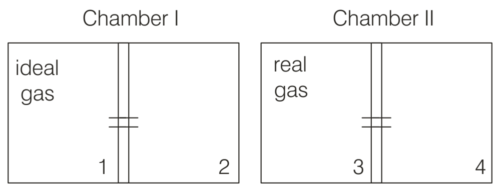
12 A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2(P/P_0)$, where P_0 is a constant. When the metal surface is at a temperature of 487°C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767°C?

(a) 1 (b) 4 (c) 7 (d) 9

13 Two spherical bodies *A* (radius 6 cm) and *B* (radius 18 cm) are at temperatures T_1 and T_2, respectively. The maximum intensity in the emission spectrum of *A* is at 500 nm and in that of *B* is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by *A* to that of *B*?

(a) 9 (b) 7 (c) 5 (d) 1

14



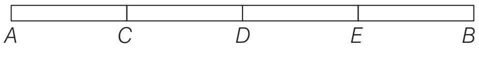
There are two identical chambers, completely thermally insulated from surrounding. Both chambers have a partition wall dividing the chambers in two compartments. Compartment 1 is filled with an ideal gas and compartment 3 is filled with a real gas. Compartments 2 and 4 are vacuum. A small hole (orifice) is made in the partition walls and the gases are allowed to expand in vacuum.

Statement I No change in the temperature of the gas takes place when ideal gas expands in vacuum. However, the temperature of real gas goes down (cooling) when it expands in vacuum.

Statement II The internal energy of an ideal gas is only kinetic. The internal energy of a real gas is kinetic as well as potential. **→ JEE Main (Online) 2013**

(a) Statement I is false and Statement II is true
(b) Statement I and Statement II both are true. Statement II is the correct explanation of Statement I
(c) Statement I is true and Statement II is false
(d) Statement I and Statement II both are true, but Statement II is not the correct explanation of Statement I

15 A rod *AB* of uniform cross-section consists of four section *AC, CD, DE* and *EB* of different metals with thermal conductivities *K*, (0.8) *K*, (1.2) *K* and (1.50) *K*, respectively. Their lengths are respectively *L*, (1.2) *L*, (1.5) *L* and (0.6) *L*. They are joined rigidly in succession at *C*, *D* and *E* to form the rod *AB*. The end *A* is maintained at 100 °C and the end *B* is maintained at 0°C. The steady state temperatures of the joints *C*, *D* and *E* are respectively T_C, T_D and T_E. Column I lists the temperature differences $(T_A - T_C), (T_C - T_D), (T_D - T_E)$ and $(T_E - T_B)$ in the four sections and column II their values jumbled up. Match each item in column I with its correct value in column II.



	Column I		Column II
A.	$(T_A - T_C)$	1.	9.6
B.	$(T_C - T_D)$	2.	30.1
C.	$(T_D - T_E)$	3.	24.1
D.	$(T_E - T_B)$	4.	36.2

	A	B	C	D		A	B	C	D
(a)	3	4	2	1	(b)	1	2	4	3
(c)	3	4	1	2	(d)	3	2	1	4

ANSWERS

SESSION 1	**1** (b)	**2** (b)	**3** (d)	**4** (a)	**5** (b)	**6** (a)	**7** (c)	**8** (a)	**9** (c)	**10** (a)
	11 (a)	**12** (c)	**13** (c)	**14** (b)	**15** (b)	**16** (b)	**17** (c)	**18** (c)	**19** (a)	**20** (c)
SESSION 2	**1** (b)	**2** (c)	**3** (a)	**4** (b)	**5** (c)	**6** (b)	**7** (c)	**8** (d)	**9** (c)	**10** (a)
	11 (b)	**12** (d)	**13** (a)	**14** (c)	**15** (a)					

Hints and Explanations

SESSION 1

1 Initially, $Q_1 = \dfrac{KA_1(T_1 - T_2)}{l_1}$ but on doubling all dimensions $l_2 = 2l_1$ and $A_2 = 4A_1$.

Hence, $Q_2 = \dfrac{K A_2(T_1 - T_2)}{l_2}$

$= \dfrac{K 4 A_1(T_1 - T_2)}{2l_1} = 2\dfrac{K A_1(T_1 - T_2)}{l_1}$

$= 2Q_1$

2 When a metallic rod is heated it expands. Its moment of inertia (I) about a perpendicular bisector increases. According to law of conservation of angular momentum, its angular speed (ω) decreases, since $\omega \propto 1/I$. (According to law of conservation of angular momentum).

3 The thermal resistance of a slab of length l_1, area of cross-section A and thermal conductivity K is given by

$$R = \frac{l}{KA}$$

Since, the slabs are joined in series, the thermal resistance of the composite slab is

$\therefore \quad R_C = R_1 + R_2$

or $\quad \dfrac{l}{K_C A} = \dfrac{l/2}{K_1 A} + \dfrac{l/2}{K_2 A}$

or $\quad K_C = \dfrac{2K_1 K_2}{(K_1 + K_2)}$

4 From temperature of interface,

$$\theta = \frac{K_1\theta_1 l_2 + K_2\theta_2 l_1}{K_1 l_2 + K_2 l_1}$$

It is given that $K_{Cu} = 9K_s$.
So, if $K_s = K_1 = K$, then

$$K_{Cu} = K_2 = 9K$$

$\Rightarrow \quad \theta = \dfrac{9K \times 100 \times 6 + K \times 0 \times 18}{9K \times 6 + K \times 18}$

$= \dfrac{5400\,K}{72\,K} = 75^\circ\text{C}$

5 An ideal black body absorbs all the radiations incident upon it and has an emissivity equal to 1. If a black body and an identical body are kept at the same temperature, then the black body will radiate the maximum power.

Hence, the black object at a temperature of 2000°C will have the brightest glow.

6 For black body,
Rate of radiation $Q = \sigma T^4$

After smoothing and doubling the temperature = Rate Q

$= 0.9\,\sigma\,(2T)^4$

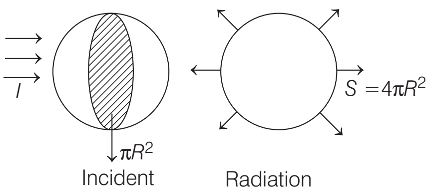
$= 0.9 \times 2^4\,Q$

Change $= (0.9 \times 2^4 - 1)\,Q$ W

7 The heat radiation emitted by the human body have wavelength of the order of 7.9×10^{-7} m to 10^{-3} m, which is ofcourse the range of infrared region.

Hence, human body emits radiation in infrared region.

8 In steady state



Energy incident per second = Energy radiated per second

$\therefore \quad I\pi R^2 = \sigma(T^4 - T_0^4)\,4\pi R^2$

$\Rightarrow \quad I = \sigma(T^4 - T_0^4)\,4$

$\Rightarrow \quad T^4 - T_0^4 = 40 \times 10^8$

$\Rightarrow \quad T^4 - 81 \times 10^8 = 40 \times 10^8$

$\Rightarrow \quad T^4 = 121 \times 10^8$

$\Rightarrow \quad T \approx 330$ K

9 From Stefan's law of radiation, $E = \sigma T^4$
where, σ is Stefan's constant.
Given, $T = 2T_S$

$\therefore \quad E' = \sigma(2T_S)^4 = 16\,\sigma T_S^4 = 16E_S$

Hence, total energy radiated by star is sixteen times as that of the sun.

10 Newton's law of cooling states, that," the rate of cooling of a body is directly proportional to temperature difference between the body and the surroundings, provided the temperature difference is small, (less than 10°C)" and Newton's law of cooling is given by

$$\frac{dT}{dt} \propto (\theta - \theta_0)$$

11 According to Newton's law of cooling, rate of fall in temperature is proportional to the difference in temperature of the body with surrounding, i.e.

$$-\frac{d\theta}{dt} = k(\theta - \theta_0)$$

$\Rightarrow \quad \displaystyle\int \frac{d\theta}{\theta - \theta_0} = \int -k\,dt$

$\Rightarrow \quad \ln(\theta - \theta_0) = kt + C$

which is a straight line with negative slope.

12 According to Newton's cooling law,

$$T_2 = T_1 + Ce^{-Kt}$$

where, $\quad C = T_i - T_1$

(difference in temperature of body and surrounding)

$\Rightarrow \quad T_2 \propto e^{-Kt}$

Thus, the graph decays exponentially. This is shown in fig. (c).

13 We know that, the rate of loss of heat from a body is directly proportional to the surface area of the body. For a given mass of a material, the surface area of a circular plate is maximum and of sphere is least. Hence, plate will cool fastest and sphere the slowest.

14 According to Wien's displacement law,

$$\lambda_m T = \text{constant}$$

$$\therefore \quad \frac{(\lambda_m)_1}{(\lambda_m)_2} = \frac{T_2}{T_1}$$

Here, $\frac{T_1}{T_2} = \frac{3}{2}$

$$(\lambda_m)_1 = 4000\,\text{Å}$$

$$\therefore \quad (\lambda_m)_2 = \frac{4000 \times 3}{2} = 6000\,\text{Å}$$

15 We know that, $\lambda_m T$ = constant and the power radiated by a black body is proportional to T^4 i.e. $P \propto T^4$, Hence, $P \propto (\lambda_m)^{-4}$

$$\Rightarrow \quad \frac{P_2}{P_1} = \left(\frac{\lambda_{m_1}}{\lambda_{m_2}}\right)^4 = \left(\frac{\lambda_0}{3\lambda_0 4}\right)^4 = \left(\frac{4}{3}\right)^4 = \frac{256}{81}$$

16 $Q \propto AT^4$ and $\lambda_m T$ = constant.

Hence, $Q \propto \frac{A}{(\lambda_m)^4}$ or $Q \propto \frac{r^2}{(\lambda_m)^4}$

$$Q_A : Q_B : Q_C = \frac{(2)^2}{(3)^4} : \frac{(4)^2}{(4)^4} : \frac{(6)^2}{(5)^4} = \frac{4}{81} : \frac{1}{16} : \frac{36}{625} = 0.05 : 0.0625 : 0.0576$$

i.e. Q_B is maximum.

17 $\lambda_m T$ = constant

From the graph $T_3 > T_2 > T_1$

Temperature of sun will be maximum. Therefore, (c) is the correct option.

18 As external radii of both the spheres are equal, the surface areas of the two are also equal. Therefore, when the two spheres are heated to the same temperature, both radiate heat at the same rate.

Now, rate of loss of heat from a sphere $= Mc \frac{d\theta}{dt}$

Therefore, rate of cooling

$$\frac{d\theta}{dt} = \frac{\text{rate of loss of heat}}{Mc}$$

or $\frac{d\theta}{dt} \propto \frac{1}{M}$

Since, mass of a hollow sphere is less, its rate of cooling will be fast.

19 According to Kirchhoff's law of radiation, the ratio of emissive power to absorptive power of a body, is same for all surfaces at the same temperature and at a particular wavelength.

Thus, Kirchhoff's law implies that a good absorber is a good emitter (or radiator) too or *vice-versa*.

20 As the temperature of the black body increases, two distinct behaviours are observed. The first effect is that the peak of the distribution shifts towards the shorter wavelength side. This shift is found to obey the following relationship called the Wien's displacement law, which is given by $\lambda_m T$ = constant.

The second effect is that the total amount of energy, the black body emits per unit area per unit time increases with fourth power of the absolute temperature T.

SESSION 2

1 According to the Newton's law of cooling,

$$\frac{50 - 40}{300} = K\left[\frac{50 + 40}{2} - 20\right]$$

$$\Rightarrow \quad \frac{10}{300} = K\left[\frac{90}{2} - 20\right] = K \times 25$$

$$\Rightarrow \quad K = \frac{10}{300 \times 25} = \frac{1}{30 \times 25}$$

Similarly,

$$\frac{40 - \theta}{300} = K\left[\frac{40 + \theta}{2} - 20\right] = K\left[20 + \frac{\theta}{2} - 20\right] = \frac{K\theta}{2} = \frac{\theta}{2 \times 30 \times 25} = \frac{\theta}{1500}$$

$$\Rightarrow \quad 300\theta = 1500(40 - \theta) = 60000 - 1500\,\theta$$

$$\Rightarrow \quad 1800\,\theta = 60000$$

$$\Rightarrow \quad \theta = \frac{60000}{1800} = 33.3\,°\text{C}$$

2 The average temperature of 94 °C and 86 °C is 90 °C, which is 70 °C above the room temperature, under these conditions the pan cools 8°C in 2 min, we have

$$\frac{\text{Change in temperature}}{\text{Time}} = k\Delta T$$

$$\frac{8°\text{C}}{2\,\text{min}} = k(70°\text{C}) \quad \ldots\text{(i)}$$

The average of 69 °C and 71°C is 70 °C, which is 50 °C above room temperature. K is same for this situation as for the original

$$\frac{2°\text{C}}{\text{Time}} = k(50°\text{C}) \quad \ldots\text{(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

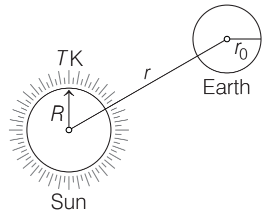
$$\frac{8°\text{C}/2\,\text{min}}{2°\text{C}/\,\text{time}} = \frac{k(70°\text{C})}{k(50°\text{C})},$$

$$T = 0.7\,\text{min} = 42\text{s}$$

3 According to question, the temperature at the contact of the surface is given by

$$= \frac{K_1 d_2 \theta_1 + K_2 d_1 \theta_2}{K_1 d_2 + K_2 d_1} = \frac{2K_2 d_2 \times 100 + 2d_2 \times K_2 \times 25}{2K_2 d_2 + K_2 2d_2} = \frac{200 + 50}{4} = 62.5°\text{C}$$

4 From Stefan's law, the rate at which energy is radiated by sun from its surface is



$$P = \sigma \times 4\pi R^2 \times T^4$$

[Sun is a perfect black body as it emits radiations of all wavelengths and so for it, $e = 1$]

The intensity of this power at the surface of the earth

[under the assumption $r >> r_0$] is

$$I = \frac{P}{4\pi r^2} = \frac{\sigma \times 4\pi R^2 T^4}{4\pi r^2} = \frac{\sigma R^2 T^4}{r^2}$$

The area of the earth which receives this energy is only one-half of the total surface area of earth, whose projection would be πr_0^2.

$\therefore$ Total radiant power as received by the earth $= \pi r_0^2 \times I = \frac{\pi r_0^2 \, R^2 \sigma T^4}{r^2}$

5 When the rods are placed in vessels

$$\frac{\theta}{t} = \frac{(T_1 - T_2)}{R}$$

$$= \left(\frac{\theta}{t}\right)_1 = \frac{mL}{t} = q_1 L = \frac{(100 - 0)}{R/2} \quad \ldots\text{(i)}$$

When the rods are joined end to end

$$\left(\frac{\theta}{t}\right)_2 = \frac{mL}{t} = q_2 L = \frac{(100 - 0)}{2R} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{q_1}{q_2} = \frac{4}{1}.$$

6 According to Newton's law of cooling, rate of cooling is given by,

$$\left(\frac{-dT}{dt}\right) = \frac{eA\sigma}{mc}(T^4 - T_0^4)$$

where, c is specific heat of material.

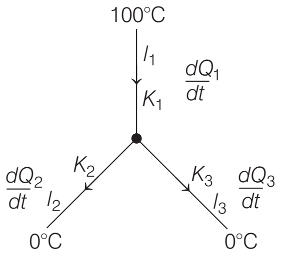
or $\left(\frac{-dT}{dt}\right) \propto \frac{1}{c}$

i.e. rate of cooling varies inversely as specific heat. From the graph, for A, rate of cooling is larger. Therefore, specific heat of A is smaller.

7 In thermal conduction, it is found that in steady state the heat current is directly proportional to the area of cross-section A, which is proportional to the change in temperature $(T_1 - T_2)$.

Then, $\dfrac{\Delta Q}{\Delta t} = \dfrac{KA(T_1 - T_2)}{x}$

According to thermal conductivity, we get



i.e. $$\frac{dQ_1}{dt} = \frac{dQ_2}{dt} + \frac{dQ_3}{dt}$$

$$\frac{0.92(100 - T)}{46} = \frac{0.26(T - 0)}{13} + \frac{0.12(T - 0)}{12}$$

$\Rightarrow \quad T = 40°\text{ C}$

$\therefore \quad \dfrac{dQ_1}{dt} = \dfrac{0.92 \times 4(100 - 40)}{46}$

$= 4.8 \text{ cals}^{-1}$

8 As, $\Delta Q = ms\Delta\theta$ (for water)

$= 50 \times s \times 5$

$\Rightarrow \left(\dfrac{dQ}{dt}\right)_s$ = rate of cooling

$= \dfrac{250}{2 \times 60} = \dfrac{25}{2 \times 6}$

$(\because S_w = 1 \text{ cal/g})$

Now, other liquid $\left(\dfrac{dQ}{dt}\right)_l$

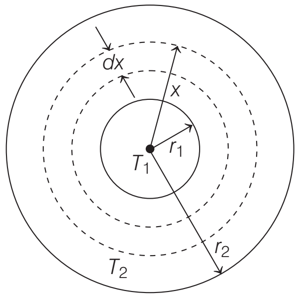
= rate of cooling

$= \dfrac{100 \times s \times 5}{2 \times 60} = \dfrac{50}{2 \times 6} s$

Now, $\left(\dfrac{dQ}{dt}\right)_l = \left(\dfrac{dQ}{dt}\right)_s$

$\Rightarrow \quad s = 0.5 \text{ cal/g} = 0.5 \text{ kcal/kg}$

9 To measure the radial rate of heat flow, we have to go for integration technique as here the area of the surface through which heat will flow is not constant.



Let us consider an element (spherical shell) of thickness dx and radius x as shown in figure. Let us first find the equivalent thermal resistance between inner and outer sphere.

Resistance of shell $= dR = \dfrac{dx}{K \times 4\pi x^2}$

$\left(\text{From } R = \dfrac{1}{KA}, \text{ where, } K = \text{thermal conductivity}\right)$

$\Rightarrow \displaystyle\int dR = R = \int_{r_1}^{r_2} \frac{dx}{4\pi K x^2}$

$= \dfrac{1}{4\pi K}\left[\dfrac{1}{r_1} - \dfrac{1}{r_2}\right] = \dfrac{r_2 - r_1}{4\pi K(r_1 r_2)}$

Rate of heat flow $= H = \dfrac{T_1 - T_2}{R}$

$= \dfrac{T_1 - T_2}{r_2 - r_1} \times 4\pi K(r_1 r_2)$

$\propto \dfrac{r_1 r_2}{r_2 - r_1}$

10 Assuming that heat loss from the sides of the slab is negligible, the amount of heat flowing through the slab is

$$Q = \frac{kA(T_1 - T_2)t}{d} \quad \text{...(i)}$$

If m is the mass of ice and L the latent heat of fusion, then

$$Q = mL \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$mL = \frac{kA(T_1 - T_2)t}{d}$$

or $$k = \frac{mLd}{A(T_1 - T_2)t} \quad \text{...(iii)}$$

Given, $m = 4.8 \text{ kg}, d = 10 \text{ cm} = 0.1 \text{ m}$,

$A = 3600 \text{ cm}^2 = 0.36 \text{ m}^2$,

$T_1 = 100°\text{C}, T_2 = 0°\text{C}$

and $t = 1 \text{ h} = (60 \times 60)\text{s}$

We know that,

$L = 80 \text{cal/g} = 80000 \text{cal/ kg}$

$= 80000 \times 4.2 \text{ J/kg} = 3.36 \times 10^5 \text{ J/kg}$

Substituting these values in Eq. (iii) and solving, we get

$k = 1.24 \text{J/s/m/°C}$ or 1.24 W/m/k

11 Power, $P = (\sigma T^4 A) = \sigma T^4 (4\pi R^2)$

or, $P \propto T^4 R^2$ ...(i)

According to Wien's law,

$$\lambda \propto \frac{1}{T}$$

(λ is the wavelength at which peak occurs)

$\therefore$ Eq. (i) will become,

$$P \propto \frac{R^2}{\lambda^4} \text{ or } \lambda \propto \left[\frac{R^2}{P}\right]^{1/4}$$

$$\Rightarrow \quad \frac{\lambda_A}{\lambda_B} = \left[\frac{R_A}{R_B}\right]^{1/2}\left[\frac{P_B}{P_A}\right]^{1/4}$$

$$= [400]^{1/2}\left[\frac{1}{10^4}\right]^{1/4} = 2$$

12 Given, $\log_2 \dfrac{P_1}{P_0} = 1$

Therefore, $\dfrac{P_1}{P_0} = 2$

According to Stefan's law,

$P \propto T^2$

$\Rightarrow \dfrac{P_2}{P_1} = \left(\dfrac{T_2}{T_1}\right)^4 = \left(\dfrac{2767 + 273}{487 + 273}\right)^4 = 4^4$

$\dfrac{P_2}{P_1} = \dfrac{P_2}{2P_0} = 4^4$

$\dfrac{P_2}{P_0} = 2 \times 4^4$

$\log_2 \dfrac{P_2}{P_0} = \log_2 [2 \times 4^4] = \log_2 2 + \log_2 4^4$

$= 1 + \log_2 2^8 = 9$

13 We know that, $\lambda_m \propto \dfrac{1}{T}$

$\therefore \quad \dfrac{\lambda_A}{\lambda_B} = \dfrac{T_B}{T_A} = \dfrac{500}{1500} = \dfrac{1}{3}$

$E \propto T^4 A$

(where, A = surface area $= 4\pi R^2$)

$\therefore \quad E \propto T^4 R^2$

$$\frac{E_A}{E_B} = \left(\frac{T_A}{T_B}\right)^4 \left(\frac{R_A}{R_B}\right)^2 = (3)^4 \left(\frac{6}{18}\right)^2 = 9$$

14 Intermolecular distance in ideal gases is assume to be large as compared to real one. Hence, the internal energy of an ideal gas and a real gas is kinetic as well as potential.

According to Newton's cooling law, option (c) is correct answer.

15 $A \to 3$; $B \to 4$; $C \to 2$; $D \to 1$

We have four sections, AB, BC, CD and DE with (dQ/dt) as the steady state thermal energy transmitted per second (A being the areas of cross-section)

$$\frac{dQ}{dt} = \frac{KA(100 - T_c)}{L}$$

$$= \frac{A(0.8)K(T_C - T_D)}{(1.2)L}$$

$$= \frac{(1.2)KA(T_D - T_E)}{(1.5)L} = \frac{(1.5)KAT_E}{(0.6)L}$$

These give

$(100 - T_C) = \left(\dfrac{0.8}{1.2}\right)(T_C - T_D)$

$= \left(\dfrac{1.2}{1.5}\right)(T_D - T_E) = \left(\dfrac{1.5}{0.6}\right)T_E$

$6(100 - T_C) = 4(T_C - T_D)$

$= (4.8)(T_D - T_E) = 15T_E$

Solving for the differences $(100 - T_C), (T_C - T_D), (T_D - T_E)$ and T_E remaining that the sum of these differences is 100, we obtain

$(T_A - T_C) = 24.1, (T_C - T_D) = 36.2$

$(T_D - T_E) = 30.1$

and $(T_E - T_B) = 9.6$