Verify that the Angle Subtended By An Arc Of a Circle

OBJECTIVE

To verify that the angle subtended by an arc of a circle at the centre is double the angle subtended by it at an point on the remaining part of the circle.

Materials Required

1. Coloured drawing sheets
2. Cardboard
3. Geometry box
4. White paper
5. Adhesive
6. Transparent sheet
7. Cutter/Scissors

Prerequisite Knowledge

1. All the basic knowledge related to the circle.
2. Angle subtended by an arc.

Theory

1. The collection of all the points in a plane, which are at a fixed distance from a fixed point in the plane, is called a circle. The fixed point is called the centre of the circle, the line segment joining the centre and any point on the circle is called radius of circle.
2. A line segment joining two points on the circle is called a chord of the circle.
3. A chord which passes through the centre of the circle is called a diameter of the circle.
4. The length of the complete circle is called its circumference.

5. A piece of a circumference of circle between two points is called an arc.

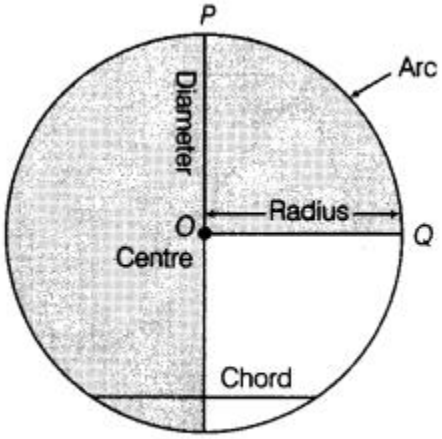

Fig. 23.1

6. Angle Subtended by an Arc of a Circle.
   Let we have a circle with centre at O and AB be its arc. Here, ∠AOB is the angle subtended by arc AB ($\bar{AB}$) at the centre of the circle.

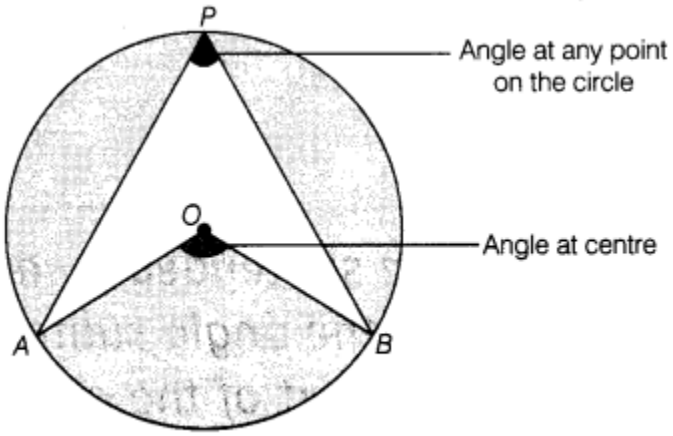

Fig. 23.2

   Also, ∠APS is the angle subtended by arc AB ($\bar{AB}$) at a point P on the remaining part of the circle.

7. Important Points about Angle Subtended by an Arc
   1. The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
   2. Angles in the same segment of a circle are equal.
   3. Angle in a semi-circle is a right angle.
   4. If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle (i.e. they are concyclic).

Procedure

1. Take a rectangular piece of cardboard of suitable size and by using adhesive, paste a white paper on it.
2. Cut out a circle of suitable radius with centre O from drawing sheet and paste it on the cardboard.
3. Take a pair of points O and R on the circle to obtain the arc QR. (see Fig. 23.3)

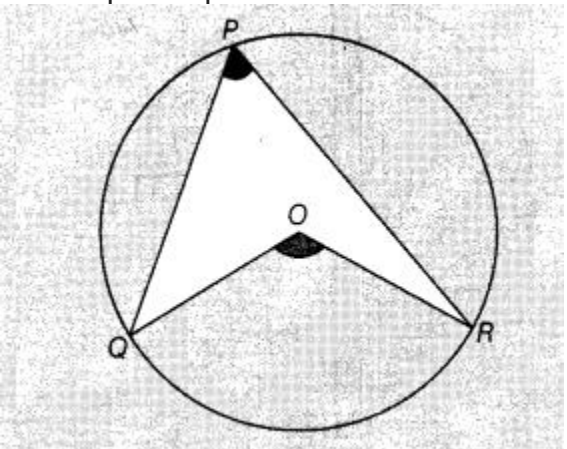

Fig. 23.3

4. To obtain the angle subtended by arc QR at centre O, join the points O and R to the centre O. (see Fig. 23.3)
5. Taking a point P on the remaining part of circle, join it to Q and R to get ∠QPR subtended by arc QR on point P on the remaining part of circle, (see Fig. 23.3)
6. Mark ∠QPR and ∠QOR.
7. Make a cut out of ∠QOR and a pair of cut outs of ∠QPR using transparent sheet, (see Fig. 23.4)

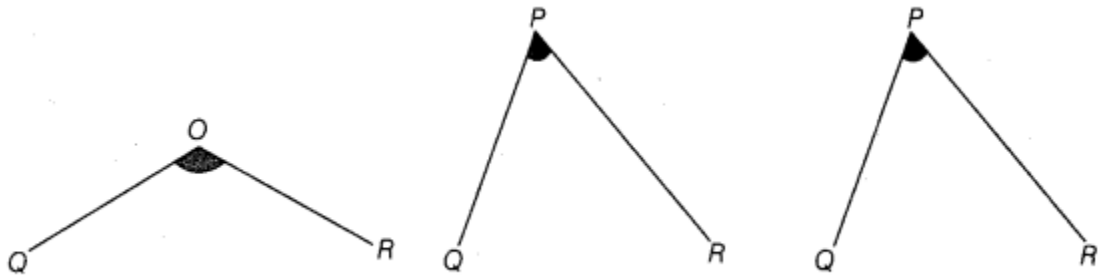

Fig. 23.4

8. Now, place the pair of cut outs of ∠QPR on the cut out of ∠QOR, adjacent to each other, (see Fig. 23.5)

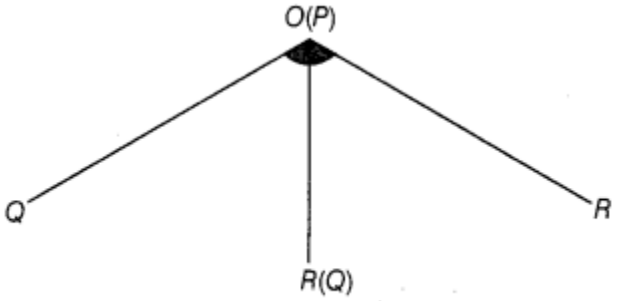

Fig. 23.5

Demonstration

Flere, ∠QOR = 2 ∠QPR
We find that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of circle.

Observation

By actual measurement, ∠QOR = …… ,
∠QPR = …… ,
Therefore, ∠QOR = 2 …… .

Result

We find that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Application

The property is used in proving many other important results such as opposite angles of a cyclic quadrilateral are always supplementary angles and angles in the same segment of a circle are always equal.

Viva Voce

Question 1:
Define a circle.
Answer:
A circle is the collection of all points in a plane which are equidistant from a fixed point within the same plane.

Question 2:
What is chord?
Answer:
A line segment which is formed by joining any two points on the circumference of circle.

Question 3:
How many circles can pass through three non-collinear points?
Answer:
There is only one such circle.

Question 4:
What is the diameter?
Answer:
The chord which passes through the centre of circle is known as the diameter of circle.

Question 5:
Which is the longest chord of circle?
Answer:
Diameter

Question 6:
If the angle subtended by an arc at centre is 110°, then what will be the angle on the remaining part of circle subtended by same arc?
Answer:
55°

Question 7:
If two chords are equal, then what will be the lengths of their corresponding arcs?
Answer:
For equal chords, the lengths of their corresponding arcs are always equal.

Question 8:
If the angles subtended by the chords of a circle at centre are equal, then what will be the length of chords?
Answer:
Both chords will be of equal length.

Question 9:
What will be the distance of two equal chords from the centre?
Answer:
Both chords are at equal distance from the centre.

Suggested Activity

To verify the converse of this theorem experimentally.