CBSE Class 11 Mathematics
Sample Papers 07 (2020-21)

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks
ii. Part-A has Objective Type Questions and Part -B has Descriptive Type Questions
iii. Both Part A and Part B have choices.

Part – A:

i. It consists of two sections- I and II.
ii. Section I comprises of 16 very short answer type questions.
iii. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part – B:

i. It consists of three sections- III, IV and V.
ii. Section III comprises of 10 questions of 2 marks each.
iii. Section IV comprises of 7 questions of 3 marks each.
iv. Section V comprises of 3 questions of 5 marks each.
v. Internal choice is provided in 3 questions of Section –III, 2 questions of SectionIV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

Part - A Section - I

1. If X = {1, 2, 3}, if n represents any member of X, write the set contain all numbers represented by $\frac{n}{2}$.

OR

Write down all possible subsets of set {a, b, c}.

2. In which plane does the point (0, 5, -4) lie.
3. Find the degree measure to the radian measure $\left(use\ \pi = \frac{22}{7}\right) \frac{11}{16}$

OR

In a right-angled triangle, the difference between the two acute angles is $(\pi_1)^o$. Find the angles in degrees.

4. Solve $x^2 + 2 = 0$
5. In how many ways can the letters of the word PENCIL be arranged so that N is always next to E?

OR

How many different words can be formed with the letters of the word EQUATION so that the words begin with E and end with N?

6. The seventh term of a G.P. is 8 times the fourth term and 5th term is 48. Find the G.P.
7. Let A = {x, y, z} and B = {1, 2}. Find the number of relations from A to B.

OR

If A = {x : x ∈ W, x < 2}, B = {x : x N, 1 < x < 5} and C = {3, 5}. Find $\mathbf{A} \times (\mathbf{B} \cup \mathbf{C})$.

8. Find the equation of a circle with centre (h, k) and touching both the axes in first quadrant.
9. The probability that a student will receive A, B, C, or D grade is 0.40, 0.35, 0.15, and 0.10 respectively. Find the probability that a student will receive a B or C grade.

OR

A card is drawn from a well-shuffled deck of 52 cards. Find the odds against getting a spade.

10. Two coins are tossed together. List the sample space.
11. If a point lies on yz-plane then what is its x-coordinate?

12. Express the product in factorial: $2 \times 4 \times 6 \times 8 \times 10$

13. Find the value of $\operatorname{cosec}\left(\frac{-41\pi}{4}\right)$

14. Find sin x and tan x, if cos x = $-\frac{12}{13}$ and x lies in the third quadrant.

15. Find the value of cos (-2220°)

16. Let X = {1, 2, 3, 4} and Y = {1, 5, 9, 11, 15, 16} Determine which of the following sets are functions from X to Y. f_3 = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}.

Section - II

17. **Read the Case study given below and attempt any 4 subparts:**

Plumbers are responsible for installing and maintaining water systems within buildings, including drinking water, drainage, heating, sanitation, and sewage systems. Plumbers are not only involved with the installation and development of new houses and plumbing systems, but also with assessing and fixing problems in existing and older systems.

A plumber can be paid under two schemes as given below:

I: Rs 600 and Rs 50 per hour.

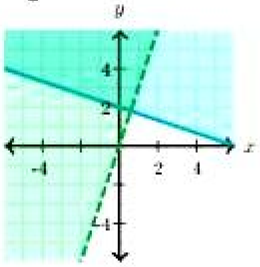
II: Rs 170 per hour.

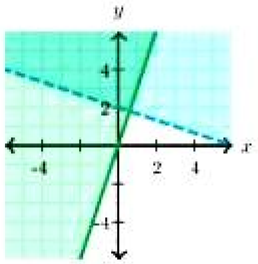
i. If the job takes n hours, then the values of n for which the scheme I will give the plumber better wages are
   a. less than 4 hours
   b. less than 5 hours
   c. more than 5 hours
   d. 4 hours

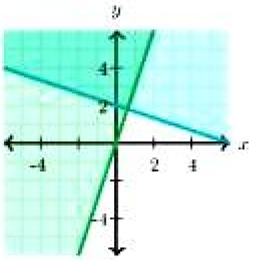
ii. Solve for x: 3x-91> -87 and 17x-16>18
   a. x > 2
   b. x > $\frac{4}{3}$
   c. $\frac{4}{3}$ < x < 2

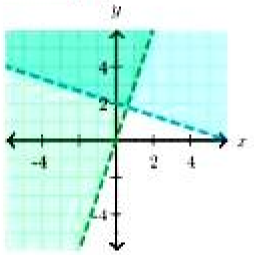
d. all value of x is the solutions

iii. $y \geq \frac{-1}{3}x+2$ and $y \geq 3x$, Which graph represents the system of inequalities

a. 

b. 

c. 

d. 

iv. Solve: f(x) = {(x – 1)×(2 – x)}/(x – 3) ≥ 0

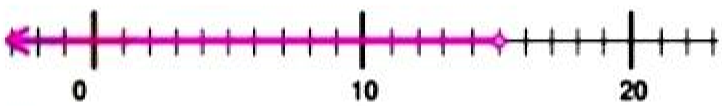
a. $(-\infty, 1] \cup (2, \infty)$

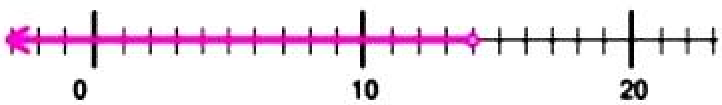
b. $(-\infty, 1] \cup (2, 3)$

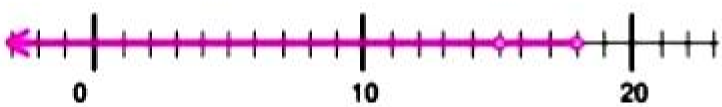
c. $(-\infty, 1] \cup (3, \infty)$

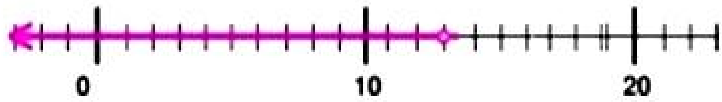
d. None of these

v. Graph the following inequality on the number line: $x < 14$

a. 

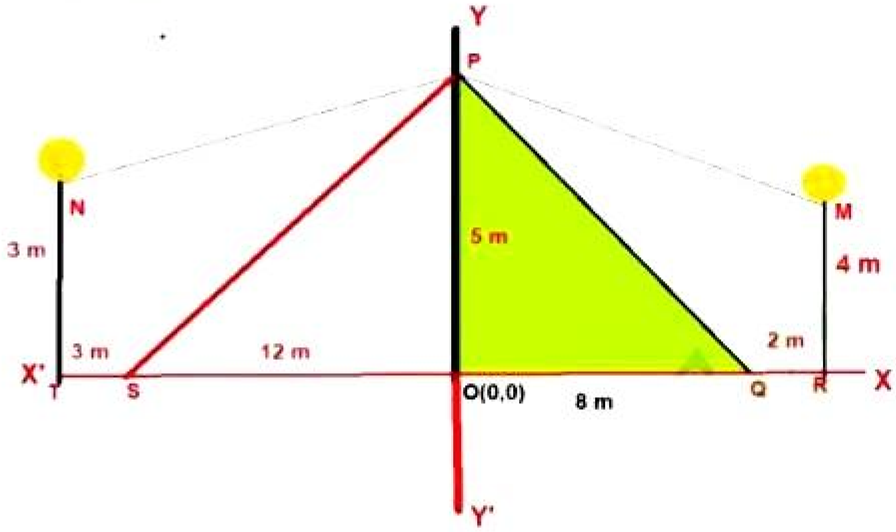
b. 

c. 

d. 

18. **Read the Case study given below and attempt any 4 subparts:**

In a colony, as shown in the following picture, an electric pole has been installed. The pole has been tied by strong wire PQ to support this pole, some electricians were working on the staircase PS.



In the left and right side of the pole, two street lights are fixed at a height of 3m and 4m respectively. These lights are given supply by wires PM and PN. The height OP=5 m and O is the origin.

Now answer the following questions:

i. What are the coordinates of point P?
   a. (5, 0)
   b. (0, 5)
   c. (0, -5)
   d. (0, 10)

ii. What is the length of the staircase?
   a. 12 m
   b. 15 m
   c. 13m
   d. 20 m

iii. What is the area of the ΔOPQ?
   a. 12 m^2
   b. 15 m^2
   c. 13m^2

d. 20 m^2

iv. What is the equation of line PN?

a. x + 15y = -75

b. 15x - 2y + 6 = 0

c. x + 5y = 50

d. x - 5y = 20

v. What is the length of wire PM?

a. √101

b. √26 m

c. 26 m

d. 25 m

Part - B Section - III

19. Let A and B be two sets. Then, prove that A = B $\Leftrightarrow A \subseteq B$ and $B \subseteq A$.

20. If A = {a, b}, B = {c, d} and C = {d, e}, then find A × (B ∪ C).

OR

Let A be the set of first five natural numbers and let R be a relation on A, defined by (x, y) $\in R \Leftrightarrow x \leq y$.

Express R and R^{-1} as sets of ordered pairs. Find: dom (R^{-1}) and range (R).

21. A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

22. Express $\frac{5+\sqrt{2}i}{1-\sqrt{2}i}$ in the form of a + ib.

23. Solve the equation $9x^2 + 4 = 0$ quadratic equations by factorization method only.

OR

Solve the equation $x^2 - \sqrt{2}ix + 12 = 0$ by factorization method.

24. Find the domain and the range of the real function, f(x) = $\frac{x-3}{x-5}$

25. Evaluate the following limits: $\lim\limits_{x \to 0} \frac{\sqrt{a+x}-\sqrt{a}}{x\sqrt{a^2+ax}}$.

26. Evaluate $\lim\limits_{x \to 0} \frac{(1+x)^6-1}{(1+x)^2-1}$.

27. In a class of a certain school, 50 students offered mathematics, 42 offered biology and 24

offered both the subjects. Find the number of students offering biology only.

28. Express $(1 + 2i)^{-3}$ the complex numbers in the standard form a + ib.

OR

Find real θ such that $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real.

Section - IV

29. Find the value of $\lim\limits_{x\to\pi/2}\frac{2x-\pi}{\cos x}$.

30. From a well-shuffled pack of 52 cards, a card is drawn at random. Find the probability of its being a king or a queen.

31. The first term of an A.P. is 2 and the last term is 50. The sum of all these terms is 442. Find the common difference.

OR

Find the 10th and nth terms of the GP $\frac{-3}{4},\frac{1}{2},\frac{-1}{3},\frac{2}{9},....$

32. If the vertex of the parabola is the point (- 3, 0) and the directrix is the line x + 5 = 0, then find its equation.

33. Find r if ${}^5P_r={}^6P_{r-1}$

34. In a survey of 200 students of a school, it was found that 120 study Mathematics, 90 study Physics and 70 study Chemistry, 40 study Mathematics and Physics, 30 study Physics and Chemistry, 50 study Chemistry and Mathematics and 20 none of these subjects. Find the number of students who study all the three subjects.

OR

In a town of 10,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C. 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three news papers, find

a. The number of families which buy newspaper A only
b. The number of families which buy none of A, B and C.

35. i. Let A = {8,11,12,15,18,23} and f is a function from A $\rightarrow$ N such that f(x) = highest prime factor of x, find f and its range.

ii. Find the domain of the function $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$.

Section - V

36. A farmer buys a used tractor for ₹12000. He pays ₹ 6000 cash and agrees to pay the balance in annual installments of ₹ 500 plus 12% interest on the unpaid amount. How much will the tractor cost him?

OR

If in an A.P. the sum of m terms is equal to n and the sum of n terms is equal to m, then prove that the sum of(m + n) terms is - (m + n). Also, find the sum of the first (m - n) terms (m > n).

37. The measurements of the diameters (in mm) of the heads of 107 screws are given below:

Diameter (in mm)	33-35	36-38	39-41	42-44	45-47
No. of screws	17	19	23	21	27

Calculate the standard deviation.

OR

Calculate the mean and standard deviation for the following table, given the age distribution of a group of people:

Age:	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of persons:	3	51	122	141	130	51	2

38. Solve graphically 4x + 3y $\geq$ 12 and 4x - 5y $\geq$ - 20.

OR

Solve the system of inequation graphically: $x + 2y \leq 100, 2x + y \leq 120,$
$x + y \leq 70, x \geq 0, y \geq 0$

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Solution

Part - A Section - I

1. If X = {1, 2, 3}

 Then,
 $\left\{\frac{n}{2}|n \in X\right\}$
 = $\left\{\frac{1}{2}, \frac{2}{2}, \frac{3}{2}\right\}$

OR

 The set has three elements , so power set has 2^3 = 8 elements
 namely, ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}, {a, b, c}.

2. The point (0, 5, -4) lies in the yz-plane.
3. To convert radian into degree measures, we multiply by $\frac{180}{\pi}$
 $\therefore \left(\frac{11}{16}\right)^C = \left(\frac{11}{16} \times \frac{180}{\pi}\right)^o = \left(\frac{11}{16} \times \frac{180 \times 7}{22}\right)^o = \left(\frac{315}{8}\right)^o = \left(39\frac{3}{8}\right)^o$
 $= 39^\circ\left(\frac{3}{8} \times 60\right)' = 39^\circ\left(\frac{45}{2}\right)' = 39^\circ\left(22\frac{1}{2}\right)'$
 $= 39^\circ 22'\left(\frac{1}{2} \times 60\right)''$ = 39°22'30"

OR

 Clearly, the sum of the two acute angles of a right triangle is 90°.
 Difference between the acute angles $= \left(\frac{\pi}{15}\right)^c = \left(\frac{\pi}{15} \times \frac{180}{\pi}\right)^\circ = 12^\circ$
 Let the two acute angles be x° and y°. Then,
 x + y = 90
 X - y =12
 we get x = 51 and y = 39.
 Hence, the angles of the triangle are 51°, 39° and 90°.

4. We have $x^2 + 2 = 0$

or x^2 = -2

taking squareroot both sides

i.e., $x = \pm\sqrt{-2} = \pm\sqrt{2}i$

5. To Find: Number of ways to arrange letters P,E,N,C,I,L

   Condition: N is always next to E

   Here we need EN together in all arrangements.

   So, we will consider EN as a single letter.

   Now, we have 5 letters, i.e. P,C,I,L and 'EN'.

   5 letters can be arranged in ${}^5p_5$ ways

   $\Rightarrow {}^5p_5$

   $\Rightarrow \frac{5!}{(5-5)!}$

   $\Rightarrow \frac{5!}{0!}$

   $\Rightarrow 120$

   In 120 ways we can arrange the letters of the word 'PENCIL' so that N is always next to E

OR

Since all words must begin with E and end with N.

So, we fix E in the first place and N at the last place. Now, the remaining 6 letters can be arranged in ${}^6P_6$ = 6! ways.

Hence, the required number of words = ${}^6P_6$ = 6!

6. Let a be the first term and r be the common ratio

   $\therefore a_7 = 8a_4$ and $a_5 = 48$

   $\Rightarrow ar^6 = 8ar^3$ and $ar^4 = 48$

   $\Rightarrow r^3 = 8$

   $\Rightarrow r^3 = 2^3$

   $\Rightarrow r = 2$

   Put r = 2 in $r^4 = 48$

   $a(2)^4 = 48$

   $\Rightarrow a = 3$

   using condition of G.P we can write ,

   Thus, the given G.P. is 3, 6, 12,.....

7. Here A ={x, y, z} and B = {1, 2}

Number of elements in set A = 3

Number of elements in set B = 2

Number of subsets of $A \times B = 3 \times 2 = 6$

Number of relations from A to B = 2^6

OR

Here we have:

A = {x: x∈W, x < 2} = {0, 1};

B = {x: x ∈ N, 1 < x < 5} = {2, 3, 4}; and C = {3, 5}

$(B \cup C)$ = {2,3,4,5}

Thus, $A \times (B \cup C) = \{0,1\} \times \{2,3,4,5\}$

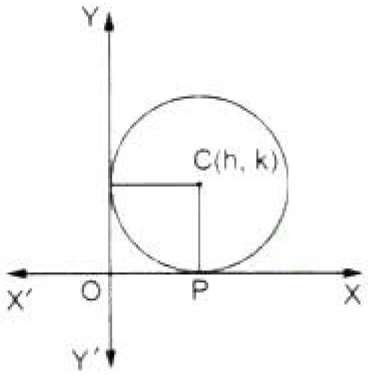
= {(0,2), (0,3), (0,4), (0,5), (1,2), (1,3), (1,4), (1,5)}

8. Circle is touching both the axes in first quadrant and centre is (h, k)

radius = h = k = c (say).

∴ the equation of the circle is $(x - c)^2 + (y - c)^2 = c^2$

$\Rightarrow x^2 + y^2 - 2c(x + y) + c^2 = 0$, where c = h = k.



9. Let E_1, E_2, E_3, and E_4 denote respectively the events that a student will receive, A, B ,C and D, grades. Then, we have,

Required probability = $P(E_2 \cup E_3) = P(E_2) + P(E_3)$ [Since, E_2 and E_3 are mutually exclusive events]

= 0.35 + 0.15 = 050

OR

let S be the sample space. Then, n(S) = 52

Let E_2 be the event of getting a spade. Then, $n(E_2)$ = 13

$\therefore P(E_2) = \frac{n(E_2)}{n(S)} = \frac{13}{52} = \frac{1}{4}$

$\Rightarrow$ odds against getting a spade = $\frac{\{1-P(E_2)\}}{P(E_2)}$

$= \frac{\left(1-\frac{1}{4}\right)}{(1/4)} = \frac{3}{1}$

10. When two coins are tossed together, the sample space is

    S = {HT, TH, HH, TT}.

    Here, HT shows a head on the first coin and a tail on the second. Similarly, TH means a tail on the first and a head on the second and so on.

11. If a point lies on yz-plane its x-coordinate is 0.

12. We have,

    $2 \times 4 \times 6 \times 8 \times 10 = (2 \times 1) \times (2 \times 2) \times (2 \times 3) \times (2 \times 4) \times (2 \times 5)$

    $= 2^5 \times (1 \times 2 \times 3 \times 4 \times 5)$

    $= 2^5 \times (5!)$

13. Let y = cosec$\left(-\frac{41\pi}{4}\right)$, then

    $y = \text{cosec}\left(-\frac{41\pi}{4}\right) = -\text{cosec}\left(\frac{41\pi}{4}\right)$

    Putting $\pi = 180^\circ$

    $y = -\text{cosec}\left(\frac{41\times180^\circ}{4}\right) = -\text{cosec}(41 \times 45^\circ)$ = -cosec (1845°)

    $= -\text{cosec}(90^\circ \times 20 + 45^\circ) = -\text{cosec}\,45^\circ = -\sqrt{2}$

14. We know that

    $cos^2x + sin^2x = 1$

    $\Rightarrow sinx = \pm\sqrt{1-\cos^2 x}$

    In third quadrant sin x is negative.

    $\therefore sinx = -\sqrt{1-\cos^2 x} \Rightarrow sinx = -\sqrt{1-\left(-\frac{12}{13}\right)^2} = -\frac{5}{13}$

    and, $tanx = \frac{sin\,x}{cos\,x} \Rightarrow \tan x = -\frac{5}{13} \times \frac{13}{-12} = \frac{5}{12}$

15. Let y = cos 2220°, then

    y = cos (-2220°) = cos 2220°

    = cos (2160 + 30°) $= \cos(360^\circ \times 6 + 60^\circ) = \cos 60^\circ = \frac{1}{2}$

16. f_3 = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}

Every element of set X has an ordered pair in the relation f_3.

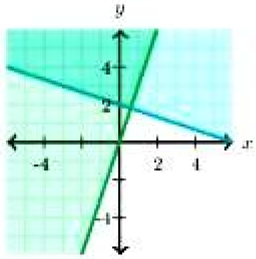
However, two ordered pairs (2, 9) and (2, 11) have the same first component but different second components.

Hence, the given relation f_3 is not a function.

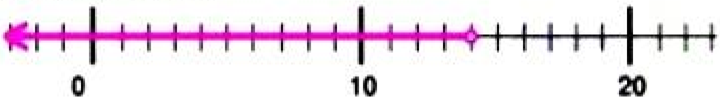
Section - II

17. i. (b) less than 5 hours

ii. (a) x>2

iii. c) 

iv. (b) $(-\infty, 1] \cup (2, 3)$

v. (b) 

18. i. (b) (0, 5)

ii. (c) 13 m

iii. (d) 20 m^2

iv. (b) 15x - 2y + 6 = 0

v. (a) $\sqrt{101}$

Part - B Section - III

19. **Given:** $A = B$

To Prove: $A \subseteq B$ and $B \subseteq A$

Proof: As we know that every element of A is in B and every element of B is in A in equal sets

$\therefore A \subseteq B$ and $B \subseteq A$

$\therefore A = B \Rightarrow A \subseteq B$ and $B \subseteq A$

Now, Suppose $A \subseteq B$ and $B \subseteq A$

By the definition of a subset, if $A \subseteq B$ then it follows that every element of A is in B and if $B \subseteq A$ then it follows that every element of B is in A.

$\therefore A = B$

$\therefore A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$

Hence Proved.

20. Given, A = {a, b}, B = {c, d}, C = {d, e}

Now, B ∪ C = {c, d} ∪ {d, e} = {c, d, e}

∴ A × (B ∪ C) = {a, b} × {c, d, e}

= $\{(a,c),(a,d),(a,e),(b,c),(b,d),(b,e)\}$

OR

Here it is given that A is the set of first five natural numbers and R is a relation on A, defined by (x, y) ∈ $R \leftrightarrow x \leq y$

Now, A = {1, 2, 3, 4, 5}

Since, $x \leq y$

R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 2), (2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5), (4, 4), (4, 5), (5, 5)}

The domain of R is the set of first co-ordinates of R.

Dom (R) = {1, 2, 3, 4, 5}

The range of R is the set of second co-ordinates of R.

Therefore, Range (R) = {1, 2, 3, 4, 5}

21. Number of revolutions in one minute = 360 revolutions

Number of revolutions in 60 seconds = 360 revolutions

Number of revolutions in 1 second $= \frac{360}{60}$ = 6 revolutions.

Angle made by wheel in 6 revolutions $= 360 \times 6 = 2160^\circ$

Now $2160^\circ = \left(2160 \times \frac{\pi}{180}\right)^C = (12\pi)^C$

22. Let z = $\frac{5+\sqrt{2}i}{1-\sqrt{2}i} = \frac{5+\sqrt{2}i}{1-\sqrt{2}i} \times \frac{1+\sqrt{2}i}{1+\sqrt{2}i}$

[multiplying numerator and denominator by 1 + $\sqrt{2}i$]

= $\frac{5+5\sqrt{2}i+\sqrt{2}i-2}{1-(\sqrt{2}i)^2}$

= $\frac{3+6\sqrt{2}i}{1+2}$

= $\frac{3(1+2\sqrt{2}i)}{3}$

= 1 + $2\sqrt{2}$ i

23. Given: $9x^2 + 4 = 0$

$\Rightarrow (3x)^2 + 2^2 = 0$

$\Rightarrow (3x)^2 - (2i)^2 = 0$ [$i^2 = -1$]

$\Rightarrow (3x + 2i)(3x - 2i) = 0\ [(a^2 - b^2) = (a + b)(a - b)]$

$\Rightarrow (3x + 2i) = 0$ or, $(3x - 2i) = 0$

$\Rightarrow 3x = -2i$ or $3x = 2i$

$\Rightarrow x = -\frac{2i}{3}$ or $x = \frac{2i}{3}$

Hence, the roots of the equation are $\frac{2i}{3}$ and $-\frac{2i}{3}$

OR

We have,

$x^2 - \sqrt{2}ix + 12 = 0$

$\Rightarrow x^2 - \sqrt{2}ix - i^2(12) = 0$

$\Rightarrow x^2 - 3\sqrt{2}i\,x + 2\sqrt{2}ix - 12i^2 = 0$

$\Rightarrow x(x - 3\sqrt{2}i) + 2\sqrt{2}i(x - 3\sqrt{2}i) = 0$

$\Rightarrow (x - 3\sqrt{2}i)(x + 2\sqrt{2}i) = 0$

$\Rightarrow x - 3\sqrt{2}i = 0$ or, $x + 2\sqrt{2}i = 0$

$\Rightarrow x = 3\sqrt{2}i$ or, $x = -2\sqrt{2}\,i$

Hence, the roots of the given equation are $-2\sqrt{2}\,i$ and $3\sqrt{2}\,i$.

24. Here we are given that, $f(x) = \frac{x-3}{x-5}$

Clearly, f(x) is defined for all real values of x except that at which $x - 5 = 0$, i.e., $x = 5$

$\therefore$ dom(c) = R - {5}

Let y = f(x). Then,

$y = \frac{x-3}{x-5} \Rightarrow xy - 5y = x - 3$

$\Rightarrow x(y - 1) = 5y - 3 \Rightarrow x = \frac{5y-3}{y-1}$ ....(i)

It is clear from (i) that x is not defined when $y - 1 = 0$. i.e., when $y = 1$

$\therefore$ range (f) = R - {1}.

Hence, dom (f) = R - {5} and range (f) = R - {1}.

25. Given: $\lim\limits_{x \to 0} \frac{\sqrt{a+x}-\sqrt{a}}{x\sqrt{a^2+ax}}$

Rationalizing the given equation,

$= \lim\limits_{x \to 0} \frac{(\sqrt{a+x}-\sqrt{a})}{(x\sqrt{a^2+ax})} \frac{(\sqrt{a+x}+\sqrt{a})}{(\sqrt{a+x}+\sqrt{a})}$

Formula: $(a + b)(a - b) = a^2 - b^2$

$= \lim\limits_{x \to 0} \frac{(a+x-a)}{(x\sqrt{a^2+ax})} \frac{1}{(\sqrt{a+x}+\sqrt{a})}$

$= \lim\limits_{x \to 0} \frac{(x)}{(x\sqrt{a^2+ax})} \frac{1}{(\sqrt{a+x}+\sqrt{a})}$

$= \lim\limits_{x \to 0} \frac{1}{(\sqrt{a^2+ax})(\sqrt{a+x}+\sqrt{a})}$

$= \lim\limits_{x \to 0} \frac{\sqrt{a+x}-\sqrt{a}}{x\sqrt{a^2+ax}} = \frac{1}{a(2\sqrt{a})} = \frac{1}{2a\sqrt{a}}$

26. Put 1 + x = y and when x $\to$ 0, then y $\to$ 1

$\therefore \lim\limits_{x \to 0} \frac{(1+x)^6-1}{(1+x)^2-1} = \lim\limits_{y \to 1} \frac{y^6-1}{y^2-1}$

$= \lim\limits_{y \to 1} \left(\frac{\frac{y^6-1}{y-1}}{\frac{y^2-1}{y-1}} \right)$ [dividing numerator and denominator by y - 1]

$= \lim\limits_{y \to 1} \frac{y^6-1}{y-1} \div \lim\limits_{y \to 1} \frac{y^2-1}{y-1}$

$= \frac{6(1)^{6-1}}{2(1)^2-1} \left[\because \lim_{x \to a} \frac{x^n-a^n}{x-a} = na^{n-1}\right]$

$= \frac{6}{2} = 3$

27. Let us consider,

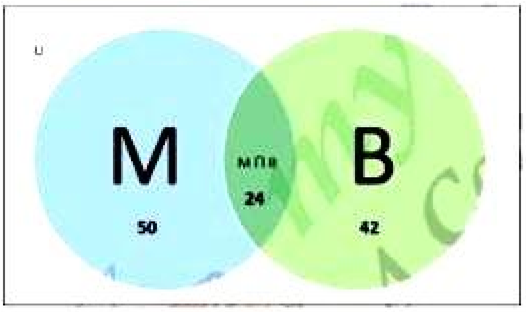
Number of students offered Mathematics = n(M) = 50

Number of students offered Biology = n(B) = 42

Number of students offered Mathematics & Biology both = n(M ∩ B) = 24

Number of students offered Biology only= n(B - M)

Venn diagram:



Now, we know that

n(B - M) = n(B) - n(M ∩ B)

= 42 - 24 = 18

Thus, Number of students offered Biology only = 18

28. $(1 + 2i)^{-3}$

$= \frac{1}{(1+2i)^3}$

$= \frac{1}{1+8i^3+6i+12i^2}$

$= \frac{1}{1-8i+6i-12}$ ($\because$ $i^2 = -1$ and $i^3 = -i$)
$= \frac{1}{-2i-11}$
$= \frac{1}{-2i-11} \times \frac{-2i+11}{-2i+11}$ [multiply and divide by -2i + 11]
$= \frac{-2i+11}{4i^2-121}$
$= \frac{-2i+11}{-4-121}$
$= \frac{-2i+11}{-125} = -\frac{11}{125} + \frac{2i}{125}$

OR

We have, $\frac{3+2i\sin\theta}{1-2i\sin\theta} = \frac{3+2i\sin\theta}{1-2i\sin\theta} \times \frac{1+2i\sin\theta}{1+2i\sin\theta}$
[multiplying numerator and denominator by $1 + 2i\sin\theta$]
$= \frac{3+6i\sin\theta+2i\sin\theta-4\sin^2\theta}{1-4(i)^2\sin^2\theta}$ [$\because$ $(a - b)(a + b) = a^2 - b^2$]
$= \frac{3-4\sin^2\theta+8i\sin\theta}{1+4\sin^2\theta}$
$= \frac{3-4\sin^2\theta}{1+4\sin^2\theta} + \frac{8i\sin\theta}{1+4\sin^2\theta}$
We are given the complex number to be real.
$\therefore \frac{8\sin\theta}{1+4\sin^2\theta} = 0$
i.e., $\sin\theta = 0$
Thus, $\theta = n\pi, n \in Z$

Section - IV

29. We have to find $\lim\limits_{x\to\pi/2} \frac{2x-\pi}{\cos x}$

LHL = $\lim\limits_{x\to\frac{\pi}{2}^-} \left[\frac{2x-\pi}{\cos x}\right]$
Let $x = \frac{\pi}{2} - h$
If $x \to \frac{\pi}{2}$, then we have,
$h \to 0$
$LHL = \lim\limits_{h\to 0}\left[\frac{2\left(\frac{\pi}{2}-h\right)-\pi}{\cos\left(\frac{\pi}{2}-h\right)}\right]$
$= \lim\limits_{h\to 0}\left[\frac{\pi-2h-\pi}{\sin h}\right]$
$= -2\lim\limits_{h\to 0}\frac{h}{\sin h}$
= - 2
RHL=

$\lim_{x \to \frac{\pi}{2}^+} \left[\frac{2x-\pi}{\cos x}\right]$

Let $x = \frac{\pi}{2} + h$

If $x \to \frac{\pi}{2}$, then we have,

$h \to 0$

$RHL = \lim_{h \to 0} \left[\frac{2\left(\frac{\pi}{2}+h\right)-\pi}{\cos\left(\frac{\pi}{2}+h\right)}\right]$

$= -2 \lim_{h \to 0} \frac{h}{\sin h}$ =-2

$\Rightarrow \lim_{x \to \frac{\pi}{2}} \left(\frac{2x-\pi}{\cos x}\right) = -2$

30. We have to find the probability of its being a king or a queen.

Let A denote the event that the card drawn is king and B denote the event that card drawn is queen.

In a pack of 52 cards, there are 4 king cards and 4 queen cards

Given : P(A) = $\frac{4}{52}$, P(B) = $\frac{4}{52}$

To find : Probability that card drawn is king or queen = P(A or B)

The formula used : Probability of an event = $\frac{\text{favourable number of outcomes}}{\text{total number of outcomes}}$

P(A or B) = P(A) + P(B) - P(A and B)

P(A) = $\frac{4}{52}$ (as favourable number of outcomes = 4 and total number of outcomes = 52)

P(B) = $\frac{4}{52}$ (as favourable number of outcomes = 4 and total number of outcomes = 52)

Probability that card drawn is king or queen = P(A and B) = 0

(as a card cannot be both king and queen in the same time)

P(A or B) = $\frac{4}{52} + \frac{4}{52}$ - 0

P(A or B) = $\frac{4+4}{52} = \frac{8}{52} = \frac{2}{13}$

P(A or B) = $\frac{2}{13}$

Probability of a card drawn is king or queen = P(A or B) = $\frac{2}{13}$

31. Given, as per the given question,

$a^2 = 2$

l = 50

l = a + (n - 1) d

50 = 2 + (n - 1) d

(n - 1)d = 48 ...(i)

S_n of all n terms is given 442

$\therefore S_n = \frac{n}{2}[a+1]$

$442 = \frac{n}{2}[2 + 50]$

Qr n = 17 ...(ii)

From (i) and (ii) $d = \frac{48}{n-1} = \frac{48}{16} = 3$

The common difference is 3.

OR

Here,it is given the series, $-\frac{3}{4}, \frac{1}{2}, -\frac{1}{3}, \frac{2}{9}$.....

The standard form of GP is, a, ar, ar^2, ar^3, ...

The common ratio is r.

First term in the given GP, $a_1 = a = -\frac{3}{4}$

Second term in GP, $a_2 = \frac{1}{2}$

The common ratio, $r = \frac{a_2}{a_1}$, $r = -\frac{\frac{1}{2}}{\frac{3}{4}} = -\frac{2}{3}$

Now, nth term of GP is, $a^n = ar^{n-1}$

Therefore, the 10th term, $a^{10} = ar^9$

$a_{10} = ar^9 = \left(-\frac{3}{4}\right)\left(-\frac{2}{3}\right)^9 = \frac{128}{6561}$

Now, the required nth term, $a^n = ar^{n-1}$

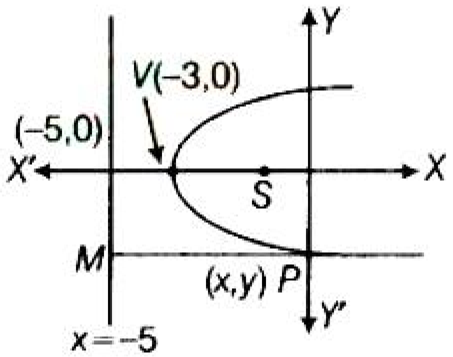
$a_n = ar^{n-1} = \left(-\frac{3}{4}\right)\left(-\frac{2}{3}\right)^{n-1} = \left(\frac{9}{8}\right)\left(-\frac{2}{3}\right)^n$

Hence, the 10th term $a_{10} = \frac{128}{6561}$ and the nth term $a_n = \left(\frac{9}{8}\right)\left(-\frac{2}{3}\right)^n$ of the given GP.

32. Given, vertex (- 3, 0) and directrix x + 5 = 0.

Let the coordinate of focus be S (a, 0).

We know that vertex is the mid-point of directrix and focus.



$\therefore$ (- 3, 0) = $\left(\frac{-5+a}{2}, 0\right)$

Then, - 3 = $\frac{-5+a}{2}$ $\Rightarrow$ a = - 1

So, coordinate of focus is (- 1, 0).

By definition of parabola, $PM^2 = PS^2$

$\therefore (x + 5)^2 = (x + 1)^2 + y^2$

$\Rightarrow x^2 + 25 + 10x = x^2 + 1 + 2x + y^2$

$\Rightarrow y^2 = 24 + 8x \Rightarrow y^2 = 8(3 + x)$

33. ${}^5P_r = {}^6P_{r-1}$

$\therefore \frac{5!}{(5-r)!} = \frac{6!}{(6-r+1)!}$

$\Rightarrow \frac{5!}{(5-r)!} = \frac{6\times5!}{(7-r)(6-r)(5-r)!}$

$\Rightarrow 1 = \frac{6}{(7-r)(6-r)} \Rightarrow r^2 - 13r + 42 = 6$

$\Rightarrow r^2 - 13r + 36 = 0 \Rightarrow r^2 - 9r - 4r + 36 = 0$

$\Rightarrow r(r-9) - 4(r-9) = 0 \Rightarrow (r-9)(r-4) = 0$

$\Rightarrow$ r = 9 or r = 4

Now r = 9 is not possible because r > n.

Thus r = 4

34. Let M, P and C denote the students studying Mathematics, Physics and Chemistry, respectively.

Then, we have,

n (U) = 200, n (M) =120, n (P) =90, n (C) = 70,

$n(M\cap P) = 40$, $n(P\cap C) = 30$, $n(M \cap C) = 50$ and

n $(M \cup P \cup C)'$ = 20

Now, $n(M \cup P \cup C)' = n(U) - n(M \cup P \cup C)$

20 = 200 - $n(M \cup P \cup C)$

$n(M \cup P \cup C) = 200 - 20 = 180$

We know that, $n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P) - n(P \cap C) - n(C \cap M) + n(M \cap C \cap P)$

$\therefore 180 = 120 + 90 + 70 - 40 - 30 - 50 + n(M \cap C \cap P)$

$\Rightarrow 180 = 280 - 120 + n(M \cap C \cap P)$

$\Rightarrow 180 + 120 - 280 = n(M \cap C \cap P)$

$\therefore n(M \cap C \cap P) = 300 - 280 = 20$

Hence, 20 students study all the three subjects.

OR

We have,total number of families = 10,000

Number of families buy newspaper A = 40%

Number of families buy newspaper B = 20%

Number of families buy newspaper C = 10%

Number of families buy newspaper A and B = 5%

Number of families buy newspaper B and C = 3%

Number of families buy newspaper A and C = 4%

Number of families buy all three newspapers = 2%

Suppose U be the total number of families, A, B and C be the number of families buy newspaper A, B and C respectively

a. Now to find: number of families which buy newspaper A only

   We have ,n(A) = 40%, n(B) = 20%, n(C) = 10%, n(A ∩ B) = 5%

   n(B ∩ C) = 3%, n(A ∩ C) = 4%, n(A ∩ B ∩ C) = 2%

   Percentage of families which buy newspaper A only,so

   = n(A) – n(A ∩ B) – n(A ∩ C) + n(A ∩ B ∩ C)

   = 40 – 5 – 4 + 2 = 33%

   Number of families which buy newspaper A only therefore,$= \frac{33}{100} \times 10000$= 3300

   Thus, there are 3300 families which buy newspaper A only

b. Now to find: number of families which buy none of A, B and C

   We have ,n(A) = 40%, n(B) = 20%, n(C) = 10%, n(A ∩ B) = 5%

   n(B ∩ C) = 3%, n(A ∩ C) = 4%, n(A ∩ B ∩ C) = 2%

   Percentage of families which buy either of A, B and C,so

   = n(A ∪ B ∪ C)

   = n(A) + n(B) + n(C) – n(A ∩ B) – n(B ∩ C) – n(A ∩ C) + n(A ∩ B ∩ C)

   = 40 + 20 + 10 – 5 – 3 – 4 + 2 = 60%

   Percentage of families which buy none of A, B and C

   = Total percentage – Number of students who play either = 100% – 60% = 40%

   Number of families which buy none of A, B and C = $\frac{40}{100} \times 10000$ = 4000

   Therefore, there are 4000 families which buy none of A, B and C

35. i. $A = \{8, 11, 12, 15, 18, 23\}$, f is function from A → N such that

   $f(x) =$ Highest prime factor of x

$\therefore f(8) =$ Highest prime factor of $8 = 2$

$f(11) =$ Highest prime factor of $11 = 11$

$f(12) =$ Highest prime factor of $12 = 3$

$f(15) =$ Highest prime factor of $15 = 5$

$f(18) =$ Highest prime factor of $18 = 3$

$f(23) =$ Highest prime factor of $23 = 23$

$\therefore$ f = {$(8,2),(11,11),(12,3),(15,5),(18,3),(23,23)$} and range of f = {2 , 3 , 5, 11, 23}

ii. Here, $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$

Since, f(x) is a rational function of x.

$\therefore$ f(x) assumes real value, except for those values of x for which $x^2 - 8x + 12 = 0$.

$\Rightarrow$ (x - 6)(x - 2) = 0

$\Rightarrow$ x = 2, 6

Hence, domain of function = R - {2 , 6}

Section - V

36. Total cost of the tractor = ₹ 12000, Cash paid = ₹ 6000

Balance to be paid = 12000 – 6000 = ₹ 6000

Annual installment = ₹ 500

$\therefore$ Number of installment $= \frac{6000}{500} = 12$

Interest of 1st installment = $\frac{6000\times12\times1}{100}$ = ₹ 720

Amount of 1st installment = 500 + 720 = ₹ 1220

Interest of 2nd installment = $\frac{5500\times12\times1}{100}$ = ₹ 660

Amount of 2nd installment = 500 + 660 = ₹ 1160

Interest of 3rd installment = $\frac{5000\times12\times1}{100}$ = ₹ 600

Amount of 3rd installment = 500 + 600 = ₹ 1100

$\therefore$ Sequence of installments is 1220, 1160, 1100, ... which is in A.P

Here, a = 1220, d = 1160 - 1220 = -60 and n = 12

$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$

= $\frac{12}{2}[2 \times 1220 + (12-1) \times (-60)]$

= 6 [2440 - 660] = ₹ 10680

Therefore, the total cost of tractor is (10680 + 6000) = ₹ 16680

OR

Let a be the first term and d be the common difference of the given A.P. Then,

$S_m = n \Rightarrow \frac{m}{2}$ {2a + (m − 1)d} = n ⇒ 2am + m (m− 1) d = 2n .....(i)

and ⇒ $S_n = m \Rightarrow \frac{m}{2}$ {2a + (m − 1) d} = m ⇒ 2an + n (n -1) d = 2m .....(ii)

Subtracting (ii) from (i), we get

2a (m - n) + {m(m -1) - n(n - 1)} d = 2n - 2m

⇒ 2a (m - n) + {($m^2 - n^2$) - (m - n)} d = - 2 (m - n)

⇒ 2a + (m + n -1) d = - 2 [On dividing both sides by (m - n)] ...(iii)

Now, $S_{m+n} = \frac{m+n}{2}$ {2a + (m + n -1) d}

⇒ $S_{m+n} = \frac{(m+n)}{2}$ (- 2) [Using (iii)]

∴ Sm + n = (m + n)

From (iii), we obtain

2a = - 2 - (m + n - 1) d ....(iv)

Substituting this value of 2a in (i), we obtain

- 2m - m (m + n - 1) d + m (m - 1) d = 2n

⇒ d = - 2 $\left(\frac{m+n}{mn}\right)$ ....(v)

Putting d = - 2 $\left(\frac{m+n}{mn}\right)$ in (iv), we obtain

2a = -2 + $\frac{2}{mn}$ (m + n -1) (m + n) ....(vi)

Now,

$S_{m-n} = \frac{m-n}{2}$ {2a + (m - n -1) d}

⇒ $S_{m-n} = \frac{m-n}{2}$ {-2 + $\frac{2}{mn}$ (m + n - 1) (m + n) - $\frac{2}{mn}$ (m - n - 1) (m + n)} [Using (v) and (vi)]

⇒ S_{m-n} = {- 2 + $\frac{4n}{mn}$ (m + n)} = $\frac{1}{m}$ (m - n) (m + 2n)

37. Here the class intervals are formed by the inclusive method. But, the mid-points of class-intervals remain the same whether they are formed by the inclusive method or exclusive method. So there is no need to convert them into an exclusive series.

Calculation of Standard Deviation

Diameter (in mm)	**Mid-values, x_i**	**No. of screws, f_i**	**$u_i = \frac{x_i - 40}{3}$**	**$f_i u_i$**	**$f_i u_i^2$**

33-35	34	17	-2	-34	68
36-38	37	19	-1	-19	19
39-41	40	23	0	0	0
42-44	43	21	1	21	21
45-47	46	27	2	54	108
		$\Sigma f_i = 107$		$\Sigma f_i u_i = 22$	$\Sigma f_i u_i^2 = 216$

Here $N = \Sigma f_i = 107$, $\Sigma f_i u_i = 22$, $\Sigma f_i u_i^2 = 216$, $A = 40$ and, $h = 3$

$\therefore$ Var (X) $= h^2 \left\{ \left(\frac{1}{N}\Sigma f_i u_i^2\right) - \left(\frac{1}{N}\Sigma f_i u_i\right)^2 \right\} = 9 \left\{ \frac{216}{107} - \left(\frac{22}{107}\right)^2 \right\}$

$\Rightarrow$ Var (X) $= 9\,(2.0187 - 0.0420) = 9 \times 1.9767 = 17.7903$

$\therefore$ S.D. $= \sqrt{17.7903} = 4.2178$

OR

Here A = 55, h = 10

Calculation of Mean and Standard Deviation

Age	mid-values (x_i)	Number of persons (f_i)	$u_i = \frac{x_i - 55}{10}$	$f_i u_i$	u_i^2	$f_i u_i^2$
20-30	25	3	-3	-9	9	27
30-40	35	51	-2	-102	4	204
45-50	45	122	-1	-122	1	122
50-60	55	141	0	0	0	0
60-	65	130	1	130	1	130

70	65	130	1	130	1	130
70-80	75	51	2	102	4	204
80-90	85	2	3	6	9	18
		$N = \Sigma f_i = 500$		$\Sigma f_i u_i = 5$		$\Sigma f_i u_i^2 = 705$

Here, $N = \Sigma f_i = 500$, $\Sigma f_i u_i = 5$, $\Sigma f_i u_i^2 = 705$

$\therefore \overline{X} = A + h\left(\frac{1}{N}\Sigma f_i u_i\right) = 55 + 10\left(\frac{5}{500}\right) = 55.1$

and, $\sigma^2 = h^2\left\{\left(\frac{1}{N}\Sigma f_i u_i^2\right) - \left(\frac{1}{N}\Sigma f_i u_i\right)^2\right\}$

$\Rightarrow \sigma^2 = 100\left\{\frac{705}{500} - \left(\frac{5}{500}\right)^2\right\} = 100 \times 1.4099 = 140.99$

$\Rightarrow$ Standard Deviation, $\sigma = \sqrt{140.99} = 11.8739$

38. We have,

$4x + 3y \geq 12$ ...(i)

$4x - 5y \geq -20$ ...(ii)

Take inequality (i),

$4x + 3y \geq 12$

Convert inequality (i) into linear equation i.e. $4x + 3y = 12$,

x	0	3
y	4	0

Thus, line $4x + 3y = 12$ passes through points (0, 4) and (3, 0)

On putting $x = 0$ and $y = 0$ in inequality (i), we get

$4(0) + 3(0) \geq 12$

$\Rightarrow 0 \geq 12$

which is false.

$\therefore$ The region represented by $4x + 3y \geq 12$ is the region which does not contain the origin.

Take inequality (ii),

4x - 5y $\geq$ - 20

Convert inequality (ii) into linear equation i.e., 4x - 5y = - 20

x	0	- 5
y	4	0

Thus, line 4x - 5y = - 20 passes through points (0, 4) and (- 5, 0)

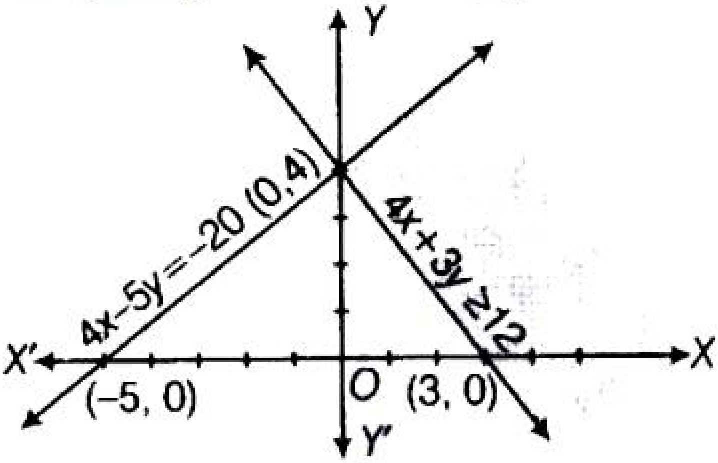
On putting x = 0 and y = 0 in inequality (ii), we get

4 (0) - 5 (0) $\geq$ - 20

$\Rightarrow$ 0 $\geq$ - 20

which is true.

$\therefore$ The region represented by 4x - 5y $\geq$ - 20 which is region contain origin.



Hence, the common shaded region represents a solution of the given system of inequalities.

OR

The graphical representation is given by the common region in the given below.

$x + 2y \leq 100$ ..... (i)

$2x + y \leq 120$ ..... (ii)

$x \geq 0$ ..... (iii)

$y \geq 0$ ..... (iv)

$x + y \leq 70$ ..... (v)

Inequality (i) represents the region below line x + 2y = 100 (including all points on the line x + 2y = 100).

Inequality (ii) represents the region below line 2x + y = 120 (including all points on the line 2x + y = 120).

Inequality (iii) represents the region in front of line x = 0 (including all points on the line x = 0).

Inequality (iv) represents the region above line y = 0 (including all points on the line y = 0).

Inequality (v) represents the region below line x + y = 70 (including the line x + y = 70).

Therefore, every point in the common shaded region including the points on the respective lines represents the solution for the given inequalities.

This can be represented as follows;