① NAMES

(i) FW TAYLOR → Father of I·E·, Productivity, sci mgt·

(ii) ADAM SMITH → scientific manu, Division of labour

(iii) HENRY L GANTT → Incentive plan, Gantt chart

(iv) GILBERTH → motion studies, Therblig Rule

(v) LHC TIPPET → work sampling

(vi) GEORGE B· DANTZING → linear programming

(vii) VON NEWMANN – Comp· strategy

(viii) A· ~~[illegible]~~ Bellman – Dyn: programing

(ix) A· Erlang - waiting line

② COST
- → F·C· don't vary with volume of producn
- → V·C· proportional to volume " "

(i) prime or direct cost → Direct lab + Direct mat + Direct Exp

(ii) Factory O·H· or Expense → Ind " + Ind " + Ind "

(iii) Factory cost → (i) + (ii)

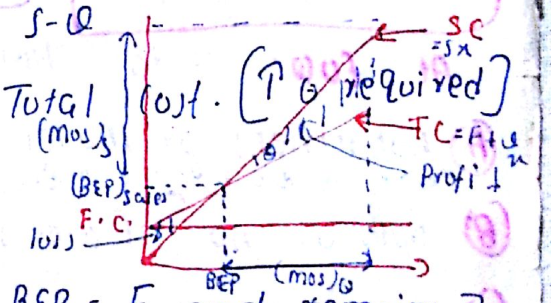
(iv) Total cost → (iii) + marketing and transportation

(v) selling " → (iv) + profit

③ BEA Analysis $S = F + V + P$ or $Sx = F + vx + P$:

Reln b/w Total cost, selling cost, producn volume

$$(X)_{BEP} = \frac{F}{S - v} \qquad (Sales)_{BEP} = \frac{FS}{S - v}.$$



(i) ∠ of incidence θ : ∠ b/w Total sales and Total cost. [↑ θ required]

(ii) contribution margin: Marginal profit

$cm: S - V = F + P$

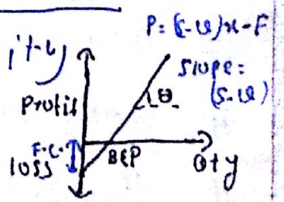
(iii) P/v Ratio (const·) : $\frac{S - V}{S} = \frac{F + P}{S}$. [At BEP = $\frac{F}{S}$ and remains constant]

(iv) margin of safety : sales or Q at $\frac{\text{Full capacity - At BEP}}{\text{Full capacity}}$

MOS = Full - At BEP

$(mos)_{\%} = \frac{\text{Full - At BEP}}{\text{Full}}$

$$(MOS)_{sales} = \frac{P}{(P/v\ ratio)} = Sx - \left(\frac{F}{S - v}\right) \cdot S$$



④ INVENTORY In hand stock having Eco value to be used later.

TYPE
- → DIRECT : Raw material, work in progress, S-finished, finished
- → IN " : (Not directly used in Product)

classification: Transit, Buffer, Seasonal, Anticipated.

② INV COST : Purchase + ordering or set up (Rs·/order) + Holding (Rs·/unit/yr·) + Stock out cost.

⑤ CHARACTERISTIC

(i) Dependent & Independent

(ii) → Q-system [Qty. fixed, Time varies] } S-S System
→ P-system [" varies, " fixed]

(iii) → Deterministic ~~EOQ~~ (d, LT const. No buffer stock)
→ probabilistic [d, LT may vary need buffer stock]

⑥ ~~(7) Mul~~ DETERMINISTIC MODELS

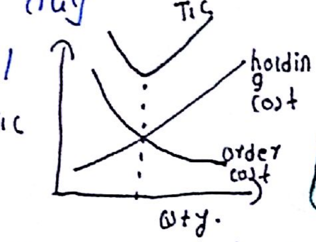
(i) EOQ model:

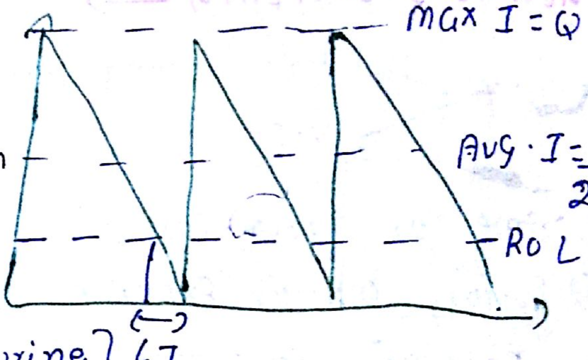
d → Avg. consumption per day

Q → max. Inventory level

T → cycle Time

$Q = T \cdot d$





ROL = LT×d [Inv. which will only last during lead time]

$$\text{Total Annual cost} = DC + \frac{Q}{2}C_h + \frac{D}{Q}C_o$$

$$\text{" Inventory "} = \frac{D}{Q}C_o + \frac{Q}{2}C_h$$

D = Annual demand (u/yr.)

C = unit purchase price (Rs./u)

C_o = ordering cost (Rs./order)

C_h = Holding " (Rs./unit/yr)

↳ may be as % of C.

$$\boxed{EOQ = Q^x = \sqrt{\frac{2DC_o}{C_h}}} \quad \boxed{TIC = \sqrt{2DC_oC_h}}$$

At EOQ: Total Holding cost = Total ordering cost.

Ⓐ Robustness or model sensitivity = $\frac{(K + 1/K)}{2}$. Here $Q = KQ^x$ $R.S = \frac{TIC(Q)}{TIC(Q^x)}$

*Proof $TIC(Q) = \frac{1}{K}\frac{D \cdot C_o}{Q^*} + \frac{K \cdot Q^*}{2} \cdot C_h$ ∵ $\frac{DC_o}{Q^*} = \frac{Q^*}{2} \cdot C_h$ ∴ $TIC(Q) = \frac{DC_o}{Q^*}\left[\frac{1}{K} + K\right]$ $TIC(Q^x) = \frac{2 \cdot D \cdot C_o}{Q^*}$

Ⓑ optimal no. of orders/yr. = $\frac{D}{Q^*}$ order/yr.

Ⓒ " time b/w 2 successive orders = $\frac{Q^x}{D}$ Yr./order.

④ Holding cost as % of unit cost = $C_h = iC$ i = % per annum

E.g. 1%/month = 12%/yr.

$C_h = .12 \times C$.

(ii) EOQ with Price break

Discount on purchasing ↑ Qty.

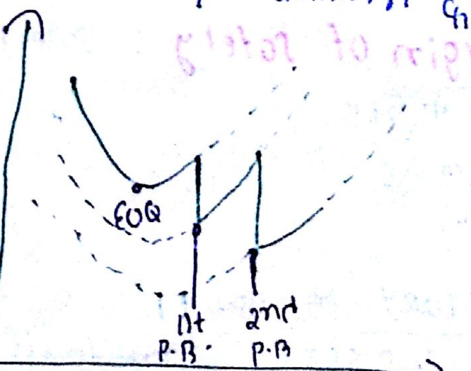
Check only at EOQ and price breaks.

1st find feasible EOQ → $\sqrt{\frac{2 \cdot D \cdot C_o}{C_h}}$ → Check beyond it

$\boxed{TC = DC + \frac{D}{Q}C_o + \frac{Q}{2}C_h}$

*Imp. At each pt. check TC not TIC.

* find EOQ feasible where and then proceed forward from it.



(iii) production or build up model

P = Production Rate

d = demand "

tp = Production time

$Q = P \cdot t_p = d \cdot T$

$Q_m = \frac{Q}{P} \cdot (P - d) = (P-d)t_p = (T - t_p) \cdot d$

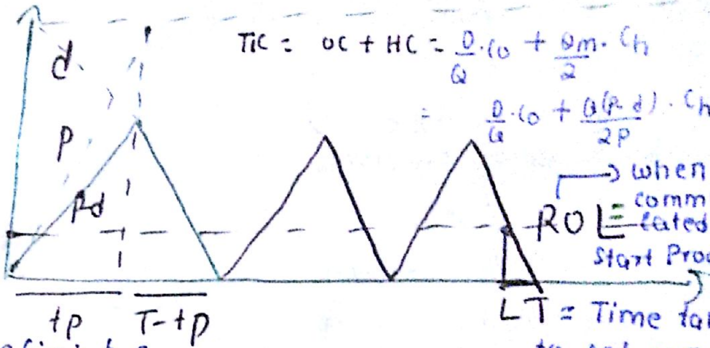
↳ MAX. INVENTORY LEVEL.

Avg. Inventory = $\frac{Q_m}{2}$

$Q^* = \sqrt{\frac{2DC_o}{C_h} \cdot \frac{P}{(P-d)}}$ ↑ $\sqrt{\frac{P}{P-d}}$ = Producⁿ factor

$TIC(Q^*) = \sqrt{2DC_o \cdot C_h \cdot \frac{(P-d)}{P}}$ ↓

Here ∵ Internal producⁿ system
∴ Replenishment is gradual not Infinite.



$TIC = OC + HC = \frac{D}{Q} \cdot C_o + \frac{Q_m}{2} \cdot C_h$

$= \frac{D}{Q} \cdot C_o + \frac{Q(P-d)}{2P} \cdot C_h$

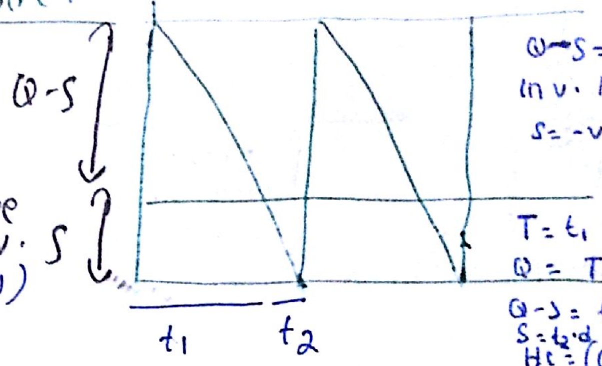
ROL → when communicated to Start Producⁿ

LT = Time taken to set up

(iv) Shortage or back order model

Shortage is planned where
(i) Shortage cost ↓
(ii) Holding " ↑

Use → ↓ OC. (By ↓ no. of orders)
→ ↓ HC.



Q-S = max inv. level
S = -ve inv.

$T = t_1 + t_2$

$Q = T \cdot d$

$Q - S = t_1 \cdot d$

$t_1 = \left(\frac{Q-S}{Q}\right) \cdot T$

$S = t_2 \cdot d$

$HC = \left(\frac{Q-S}{2}\right) \cdot t_1 \cdot C_h$

$= \frac{(Q-S)^2}{2Q} \cdot C_h$

*similarily C_b.

$$Q^* = \sqrt{\frac{2DC_o}{C_h}\left(\frac{C_h + C_b}{C_b}\right)} \qquad TIC(Q^*) = \sqrt{2DC_o \cdot C_h \cdot \frac{C_b}{(C_h + C_b)}}$$

$$TIC = \frac{D}{Q}C_o + \frac{(Q-S)^2}{2Q} \cdot C_h + \frac{S^2}{2Q} \cdot C_b .$$

$S^* = Q^*\left(\frac{C_h}{C_h + C_b}\right)$ = optimum no. of orders back ordered

max. Inv. level = $Q^* - S^*$
$= \frac{Q^* \cdot C_b}{C_h + C_b}$

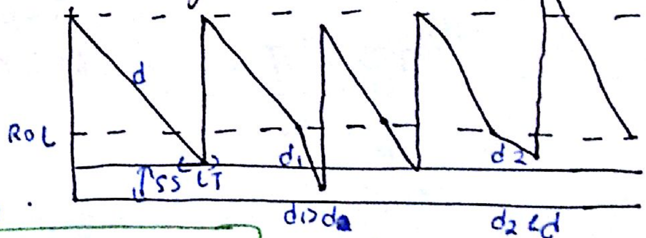
(7) PROBABILISTIC MODELS

→ d, LT, P Not const. and Not known with certainity
→ Thus Need a Safety Stock

RoL = LT · d + SS

Avg. Inventory = $SS + \frac{EOQ}{2}$

Annual cost to maintain SS = SS × C_h.



(i) Demand profit model

Order = S

P = Profit for 1 unit sold
L = loss if 1 unit wasted

$$\boxed{P(S-1) < \frac{P}{P+L} \leq P(S)}$$

P(S-1) = cumulative demand probability of S-1 units.
P(S) = " " " " S " .

At RoL qty. ordered is Q.
Q-SS used only for variations after RoP?
- b/c if v. before that then can adjust RoP left or rt. but SS is the only option after it

(ii) when stock out cost not known

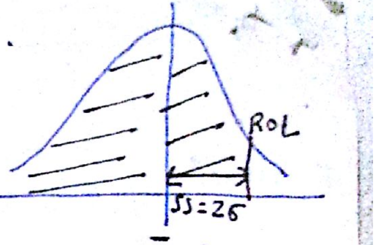
Then decision by → How much service level we wish to provide

Then assume Demand during lead time approximated by Normal dist. curve with mean ($\bar{x}$) and S.D. (σ)

RoL = $\bar{x} + Z\sigma$ $\bar{x} = d \times LT$ $Z\sigma = SS$

(i) SS = $Z\sigma$. (std. normal variate) (ii) P(1.645) = 95%. (in calculator) (iii) RoL = Demand during LT + SS

(iv) $\sigma = MAD \times 1.25$ (v) $\sigma_{net} = \sqrt{\sigma_1^2 + \sigma_2^2 + \dots \sigma_n^2}$.



$\bar{X}$ = Avg. demand
less than Avg. d ← → more than avg. d
∴ need SS

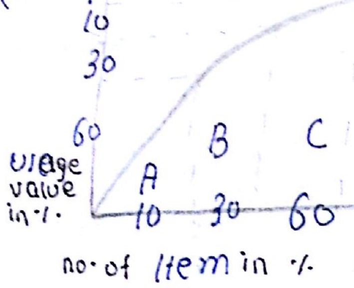
SS = max. DDLT − Avg. DDLT [if all demands given]
Demands of various LTs given

(if various LTs given)
SS = [MAX. LT − Avg. LT] × d (Avg. demand).
- various LTs given - Avg. demand/day given

⑧ INVENTORY CONTROL

(i) ABC (Always Better Control) (PARETO CURVE) (Total usage value)

(ii) VED (criticality)
(iii) HML (unit cost)
(iv) SOE (Availability)
(v) S-OS (Seasonal)



* Draw closed curve with 100%(x), 100%(Y)

[FOR A] → very strict control
→ frequent Review of their consumption

EOQ gives min: Total cost per year.

LINE BALANCING

Aim - Effective grouping of tasks to minimise unproductiveness of men, m/c.
- Tasks are grouped so that their grp. time ≤ time at each station.

(i) uniform prodn Rate (v) ↓ idle Time

(ii) less material handling (vi) ↓ work in process inventory

(iii) less congestion (vii) ↑ productivity [But not by vi]

(iv) Easy producn control

Station time = T_{Si} = sum of times of all work elements at i th station

$T_{WC} = \Sigma T_{Si}$

① B.D. = $\dfrac{nT_c - TWC}{nT_c}$ ② line η = [1-BD] = $\dfrac{TWC}{nT_c}$.

③ $n_{min} = \dfrac{TWC}{T_c}$ ④ $SI = \sqrt{\sum_{i=1}^{n} (T_{Si\,max} - T_{Si})^2}$ ⑤ $T_c \geq (T_{Si})_{max}$

⑥ idle time at each W.S. = $T_c - T_{Si}$

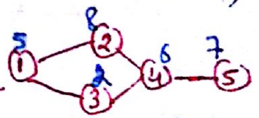
Method I - Largest candidate Rule

① Arrange in ↓ order of time

② Assign Top to bottom by using feasible elements

③ Feasibility → Follow precedence
→ Station Time ≤ T_c

METHOD II



P.W.
1-26
2-21
3 15
4-13
5-7

① Arrange in ↓ positional wt.

② P.wt. = max Time from beg. of element to end of network

③ Assign → ↓ order of P.W.
→ By station time
→ predecedence Automatically covered.

Imp → [AON] Diagram

*TABLE * S.No. Time Precedence Work Stn. S.No. Ti Tsi idle = $T_c - T_{Si}$
* // '' Pos. wt.

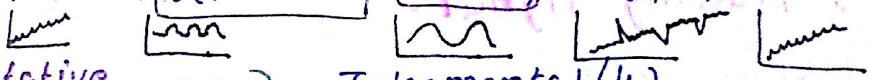
$SI = \sqrt{\Sigma (idle)^2}$

FORECASTING

① **Forecasting:** # To predict future sales # Basis for Req. of men and mat.
'A projection based on human Judgement & Past data.

② **Types of variations**

(Upward or downward) TREND, (Short-term Regular variation) * SEASONAL, (Time period ↑ *) CYCLIC, (Unusual circumstance) IRREGULAR, (Residual variation) RANDOM



③ **Classification**

$$\text{Classification} \begin{cases} \text{Qualitative} \longrightarrow \text{Judgemental (4)} \\ \text{Quantitative} \begin{cases} \text{TIME SERIES (4)} \\ \text{CAUSAL (2)} \end{cases} \end{cases}$$

Qualitative : Subjective, 2-5 yrs, long term, No past data, New product
Quantitative : Objective, 1-3 mon, short ″, Past data

(4) **Judgemental-** OPINION SURVEY (↓ cost), MARKET TRIAL (↑ cost ford. ″ product), MARKET RESEARCH (EXT AGENCY), DELPHI (BEST) (Sequential Questions)

(5) **TIME SERIES -** FOR CYCLIC TRENDS [WHY].
- PAST DATA: Chrono order, D (dep var), T (ind var)

A- Past Avg. → F.C. = Avg. of previous demand,

B- Moving ″ → ″ = moving Avg. of constant period

C- wt. ″ ″ → ″ = ″ ″ ″ with weights. $\left[w_n = \frac{n}{\Sigma n}\right]$
(Latest given Test wt.)

D- Exp. Smoothing → $F_t = F_{t-1} + \alpha \{D_{t-1} - F_{t-1}\}$. [↑ wt. to Recent value]

↳ No need of large past data, Assign wt. to all pre. data, Pattern of wts. is exponential $\Rightarrow F_t = \alpha \cdot D_{t-1} + (1-\alpha) F_{t-1}$

- General Term: $F_t = \alpha \cdot D_{t-1} + \alpha(1-\alpha) D_{t-2} + \alpha(1-\alpha)^2 D_{t-3} + \dots$

→ α ≡ Equivalent to moving avg. of N periods $\boxed{\alpha = \frac{2}{n+1}}$

→ [↑ wt. to Recent, wt. to older periods ↓ exponentially]

⑥ **if initial period F.C. Not given**

(i) NAIVE METHOD: Initial F.C. = Actual demand for that period. $\boxed{F_1 = D_1}$

(ii) F_1 = Avg. of all the demands given.

TABLE	S.No	D	F	error

⑦ **FORECAST ERROR** $e_i = D = D_i - F_i$.

(i) $MAD = \frac{1}{N} \Sigma |e_i|$ (gives magnitude but not direction)

(ii) $MFE = BIAS = \frac{1}{N} \Sigma e_i$ [+ve underestimated F.C. / -ve over ″ ″]

(iii) $RSFE = \Sigma e_i$ Running sum F. Error.

(iv) $MSE = \frac{1}{N} \Sigma e_i^2$ [Amplifies large error]

(v) $\sigma = \sqrt{MSE}$.

(vi) $MAPE = \frac{100}{N} \Sigma \left| \frac{D_i - F_i}{D_i} \right| = \frac{100}{N} \Sigma \left| \frac{e_i}{D_i} \right|$

* To find Tracking signal:

$$\frac{RSFE}{MAD} = \frac{\Sigma e_i}{\Sigma |e_i|} \times N$$

find for each activity with $N = 1, 2, 3, \ldots\ldots$

vii) Tracking signal = $\frac{RSFE}{MAD}$ [± 4-5 permit]

- Denotes how well forecast is Predicting actual values
- TS = 0 is good.

e_i	Σe_i	$\Sigma \lvert e_i \rvert$	N

$$TS = \frac{\Sigma e_i}{\Sigma |e_i|} \times N$$

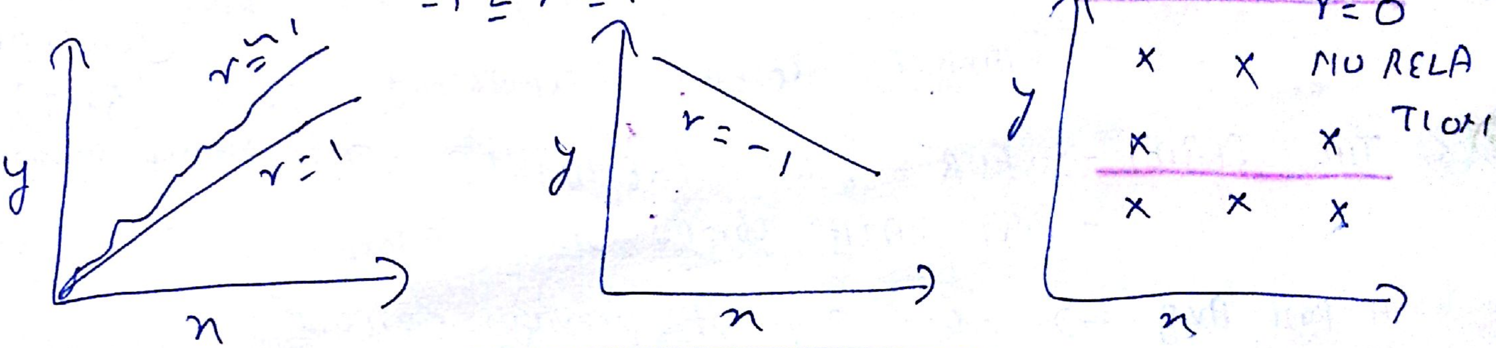
(8) CAUSAL OR ECOCENTRIC

\# Establish cause and Effect relation b/w D and some other factor

\# Can be used for Trend variation

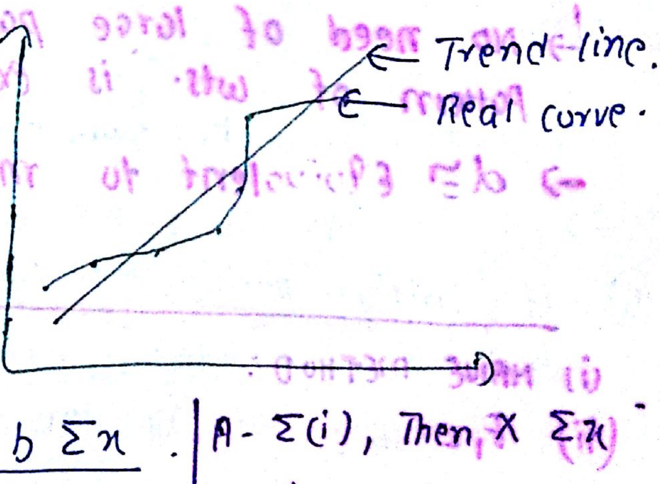
(i) Correlation Analysis r = Degree of closeness b/w 2 variable,

$-1 \leq r \leq 1$



$$\boxed{r = \frac{\Sigma (x - \bar{x})(y - \bar{y})}{\sqrt{\Sigma (x - \bar{x})^2 . (y - \bar{y})^2}}}$$

(ii) linear Regression



\# mathematical method of obtaining line of best fit b/w the dep. var. (usually demand) and an ind. variable.

$Y = a + bx$ —(i)

$$b = \frac{n \Sigma xy - \Sigma x . \Sigma y}{n \Sigma x^2 - [\Sigma x]^2} \qquad a = \frac{\Sigma y - b \Sigma x}{n}.$$

A - Σ(i), Then, × Σx

B - (i)×x, Σ, × n.

\# special least sq. method

if Independent var. is linear and can be written $\Sigma x = 0$

Then $b = \frac{\Sigma xy}{\Sigma x^2}$ $a = \frac{\Sigma y}{n}$.

note (i) fluctuation is due to Randomness: ↓α.

	x
00	-2
01	-1
02	0
03	1
04	2

SEQUENCING #Imp

Job	machine

\# Beware of sequence A-C-B

① **SEQUENCING** To decide the order of doing task, Effective utilisation of man, mat. and m/c

② (i) Job flow time : Time for a Job from some ~~its~~ start pt. (i.e. 0) to End of that Job

(ii) make span " : Time for start of 1st Job to complete of last Job

(iii) Tardiness : Amt. of time for completing after due date.

(iv) Avg. No. of Jobs in system : $\frac{\text{Total flow time of all Jobs}}{\text{make span time}}$

(v) mean flow time = $\frac{\Sigma \text{Job flow times}}{\text{no. of Jobs}}$

③ **N Jobs on 1 M/C**

SPT (Shortest processing time) ↑ order (minimises mean flow time, mean Tardiness, Avg. no of Jobs)

EDD (Earliest due date) " (minimise max. Tardiness)

CRR (critical ratio Rule) " $\left(\frac{\text{Due date}}{\text{process time}}\right)$

STR (Slack Time Remaining) " (DD - P.T.)

④ **N Jobs on 2 m/c** A B

(i) min A 1st (ii) min B last (iii) min A_i = min B_J $\underline{i}$ _ _ _ $\underline{J}$

(iv)

	A	B
i	8	6
J	9	6

_ $\underline{J}$ $\underline{i}$

(v)

	A	B
i	5	8
J	5	9

$\underline{i}$ $\underline{J}$ _

(vi)

	A	B
i	4	4

$\underline{i}$ \- \- \- _

\- \- \- $\underline{i}$

⑤ **N Jobs on 3 m/c**

A B C

1
2
:
n

if min(A) ≥ max(B) Then

or min(C) ≥ max B

A+B B+C

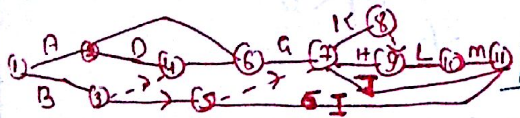
Apply Johnson

⑥ **N Jobs on m m/C** similar.

Gantt chart

m/c A	Shade the area
m/c B	when m/c is busy

→ Time



PERT / CPM

*E.g. of Network drawing A, B, C(A), D(A), E(B), F(B,D), G(C,F) H(G,E) I(E) J(G,E) K(G,E) L(H,K) m(L)

① (i) project : A grp. of interrelated activities having some logical sequence.

(ii) Event : They are pts. in time denoting beg. and End of activities. Don't consume time and resources.

E.g. A, B, C, D, E(A), F(B), G(CF), H(DG), I(EG), J(HI), K(D), L(JK), m(L)

(iii) Activity : physically identifiable part of project using time and resources.

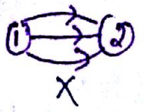
② **NETWORK DIAGRAM**

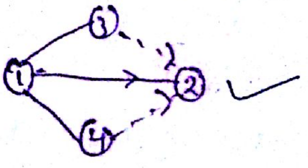
(i) first finish preceeding activity

(ii) length and direcn of Arrow Indicative

(iii) Time flows from left to Right

(iv)

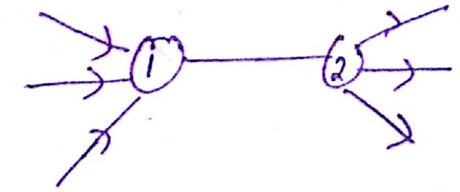




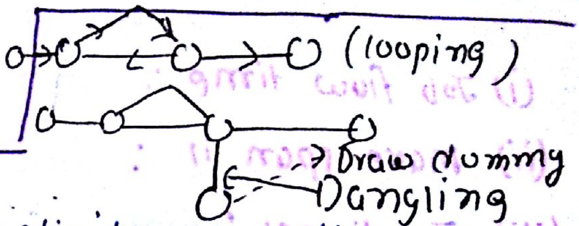
No 2 activities have same head and tail Events.

(v) Dummy activity → Just to show the dependency of 1 activity over other but does not consume any time of resource.

(vi)

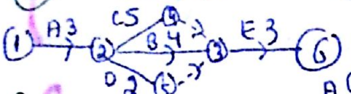


① merge Event
② Burst Event



(vii) No looping or dangling → when activity other than final activity do not have any successive activity.

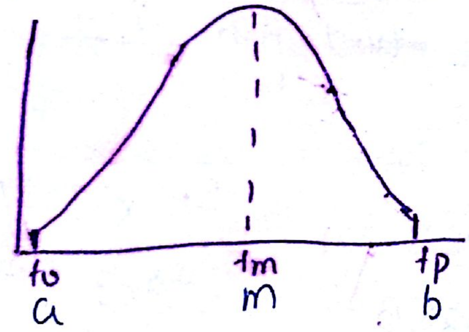
③ DIAGRAM → Event on Node or Activity on Arrow.
→ Activity on Node [No dummy activity]



④ PERT: 3 Times → T_o = optimistic time; → T_m = most likely "; → T_p = pessimistic "] PROJECT EVALUATION AND REVIEW TECHNIQUE

Activity Time : By B-distribution (skewed about mean line)
Project " : " N- " (Symmetric " " ")

B-distribution :



Assumption
① t_o, t_m, t_p formed the End pts.
② $P(m) = 4\,P(a) = 4\,P(b)$

Thus, ① $T_E = \dfrac{T_o + 4T_m + T_P}{6}$.

② $\sigma^2 = var = \left(\dfrac{t_p - t_o}{6}\right)^2$. $\sigma = \left(\dfrac{t_p - t_o}{6}\right)$

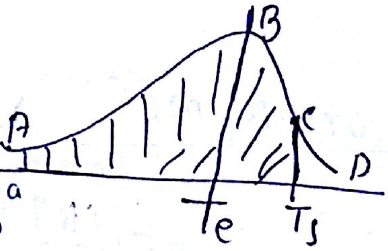
③ $\sigma_{equivalent} = \sqrt{\Sigma\sigma^2}$
$= \sqrt{\Sigma var}$

⑤ P (of meeting scheduled dates)

(i) $Z = \dfrac{T_S - T_E}{\sigma}$ (ii) $P = P(Z)$ (using calc)

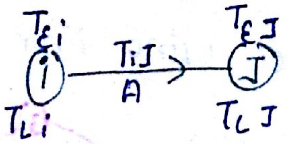
(iii) $P(T_S) = \dfrac{Ar \cdot ABC}{Ar \cdot ABD}$.

Critical path = path in a network which takes max. time



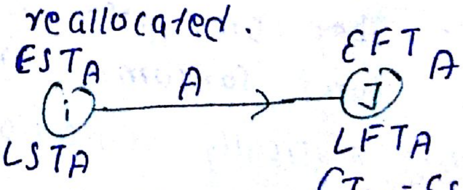
P (i) < ·3 Replan (ii) ·3 - ·5 close Exami. (iii) ·5 - ·65 Satisfactory
(iv) > ·65 Excess Resources need to be reallocated.

⑥ (i) EVENT



if, $T_{Li} - T_{Ei} = 0$
$T_{LJ} - T_{EJ} = 0$
$T_{LJ} - T_{Li} = T_{EJ} - T_{Ei} = T_{iJ}$] CRITICAL

(ii) Activity



($T_{Ei} = ESTA$)

if,
$LFT - EFT = 0$
$LST - EST = 0$
$LFT - LST = EFT - EST = T_{iJ}$] CRITICAL

⑦ (i) Slack or float Amt. of time which a particular Event or activity can be delayed without affecting the schedule of project.

$\#$ Slack $= T_{Li} - T_{Ei}$

$\#$ Total float $=$ LFT $-$ EFT $=$ LST $-$ EST

] +ve: Surplus Resources and can be Reallocated.
0: Just sufficient to complete.
-ve: Insufficient resources.

(ii) Free float = TF - Head Event slack

(iii) Independent " = FF - Tail " "

$\boxed{TF \geq FF \geq IF}$

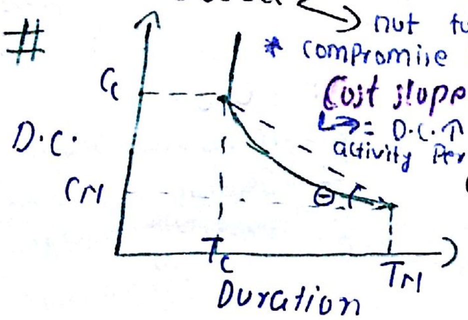
⑧ CRASHING OR TIME COST MODEL

(i) I.C. $\propto$ Duration of project.

(ii) D.C. $\propto \dfrac{1}{\text{Duration " "}}$.

Object < To minimise cost (D+I)
> not to minimise time

* compromise b/w time & cost.



Cost slope $= \tan\theta = \dfrac{C_c - C_{rl}}{T_{rl} - T_c}$.

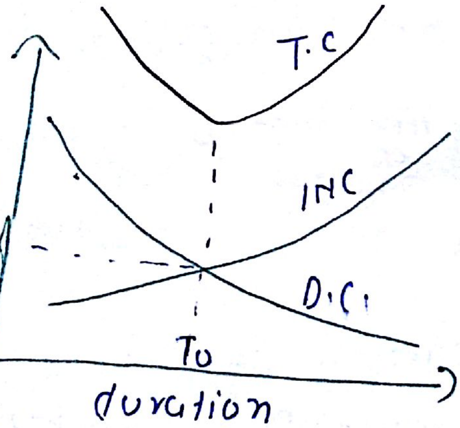
↳ = D.C. ↑ for crashing activity per unit time.

C_c, T_c cost, time for crash table

C_{rl}, T_{rl} cost, time for normal table

T_c = crash time is the min. activity duration to which an activity can be compressed by ↑ resources hence by ↑ direct cost.

⑨



⑩ STEPS (i) Find critical path and duration

(ii) find cost slope for each activity in the critical path

(iii) Crash activity of ↓ cost slope to max. possible Extent i.e. Reduce the duration of this activity in time units

(iv) we get critical path parallel

(v) simultaneously crash " critical paths.

Initially DC & IDC/day given
→crashing : DC ↑es) DC + crash cost × t
IDC ↓es) IDC/day × t

⑩ PERT: Probabilistic, 3 times, No cost Analysis, Event, New Job - Research

CPM: Deterministic, 1 time, cost Analysis, Activity, Already " - Construction

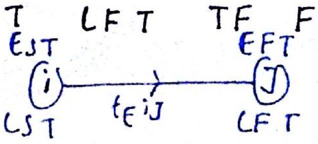
* Forward pass computation - T_{Ej} = max. of all $(T_{Ei} + t_{Eij})$ For merged Event

* Back ward " " - T_{Li} = min. of $(T_{Lj} - t_{Eij})$ For Burst "

* Float (CPM activity) * Slack (PERT Event)

Q- To find floats. Table

Activity	t_{Eij}	EST	EFT	LST	LFT	TF	FF	IF



- In diagram do FW, BW pass computation
- In Table for EST → EFT follow normally
- if more than 1 way to reach i then for EST for $i \lessgtr^{j}_{k}$ use max. of EFT.
- similarily for BW → For LST & LFT start from last LFT.

QUEING MODELS

① CHARACTERISTICS

(i) Arrival pattern - poisson distrobuted (Inter-arrival -ve e).

(ii) service " - exponentially " (Inter- service -ve P) .

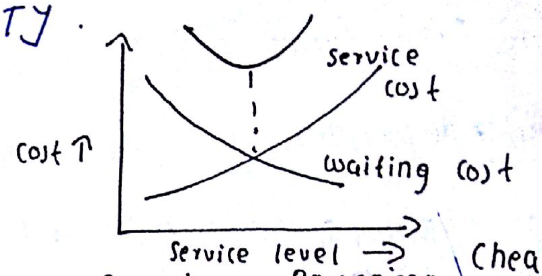
(iii) service Rule - FIFO, LIFO, SIRO, PRIORITY .

(iv) service Channel and server - 4 types

(v) customer in system - Finite or ∞ .

calling Popu. ∞ when no effect on future incoming.

(vi) customer attitude → Patient / → Impatient [Jockeying, Balking , Reneging, Cheating]



② KENDALL REPRESENTATION (a/b/c) : (d/e/f)

For a,b
M = Markovian Poisson, e
GI = General independant
E = Erlang/ r distribution
D = Deterministic

a = Arrival pattern

b = service pattern

C = No. of servers

d = Service Rule

e = max. allowed customers in system (N or ∞)

f = Calling population (N or ∞)

λ, μ = customers/time

③ (i) $\rho = \frac{\lambda}{\mu}$ Utilisation factor / % time server is busy / Probability that a customer has to wait

(ii) $P_0 = 1-\rho$ P of system being empty.
P that customer does not have to wait.

(iii) $P_N = \rho^n \cdot P_0 = \rho^n(1-\rho) =$ P of having exactly N cust in system.

(iv) Avg. no. of customers in system: $L_s = \sum_{i=1}^{n} n \cdot P_n$.

$$L_s = \frac{\rho}{1-\rho} .$$

$$L_q = \frac{\rho^2}{1-\rho} .$$

$W_s = \frac{L_s}{\lambda}$ → Littles law

Avg. wating time in sys in que

$W_q = \frac{L_q}{\lambda}$

(i) $W_q = W_s - \frac{1}{\mu}$

(ii) Avg. length of Non-Empty Que $= \frac{\mu}{\mu-\lambda} = \frac{1}{1-\rho}$.

(v) P (of n Arrivals in time T) $= \frac{e^{-\lambda T} \cdot (\lambda T)^n}{n!}$ → Governing eqn. of P. dist.

E.g. 6 customers in 10 mins

(vi) P(more than T period to serve a customer) $= e^{-\mu T}$.

(vii) P(Not " " " " i.e. $\leq T$) $= 1 - e^{-\mu T}$

(viii) $P(W_q > T) = \rho \cdot e^{\left(\frac{-T}{W_s}\right)}$.

(ix) $P(W_s > T) = e^{\left(\frac{-T}{W_s}\right)}$.

- * if system capacity = N $P_0 + P_1 + P_2 + \dots P_N = 1$
- * " " ∞ $P_0 + P_1 + P_2 + \dots P_\infty = 1$
- * P(<3 customers) $P_0 + P_1 + P_2$
- * P(≥ 3 ") $1-(P_0 + P_1 + P_2)$

$\boxed{P_N = \rho^N(1-\rho)}$

WORK STUDY < METHOD STUDY / TIME STUDY

① Work study : Study of work (Thus by rearrangement ↑ productivity)

② purpose :

i) ↑ productivity

(ii) Better workplace condition

(iii) Efficient mat. handling

(iv) No backtracking

(v) ↓ fatigue to workers

(vi) ↑ product quality

(vii) Efficient Resource use

③ method study : SREDIMP

Select → Record → Examine → Develop — INSTALL — MAINTAIN — ↑ PRODUCTIVITY

④ Recording Technique :

(i) **process charts** : showing sequence of operations.

a- outline PC → only little detail, only □, ○ used

b- flow PC → (i) man Type, (ii) mat. " , (ii) m/c Type] # All symbols used # can add remarks like time

Detailed version

OF 2H

c- 2 handed PC → Record L, R hand activity relative to each other. synchronise for repititive and short cycle operations)

PC SYMBOLS

○ : operation

□ : inspection

△ : Storage

D : Delay or T. storage

⇨ : Transportation

(ii) TIME SERIES CHARTS : Using a time scale. more than 1 item (m, mat, m/c)

a- multiple activity chart → Activities of more than 1 item to show Interrelationship recorded on common time scale to show Interrelation ship. □ idle ▨ working.

Most popular → man m/c chart.

b- SIMO CHART

c- PMTS

MASIPM

Gang process chart : A type of MA chart. → wrt diff. members of a grp

(iii) DIAGRAMS : FCTCS

Scaled diagram showing location of various activities - Either man or material type.

a- flow diagram → Scaled diagram showing location of various activities identified by numbered symbols.

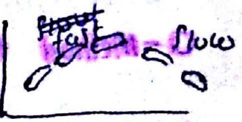
b- String " → Scaled plan using a thread to measure the path of worker or material.

c- Travel chart → Tabular record to represent quantitative data about movements of man or material.

[Each sq. element is a work station]

d- cycle graph → In dark room, light source attached to the hands of worker and recorded by camera.

d- chrono " " → Intermittent light source. series of Pear shaped dots. Direction, speed known



⑤ Principles of motion Economy : - To reduce the worker fatigue. - minimise Time & Energy consumption for a Job.

(i) use of human body (iii) Design of tools and equipments.

(ii) Arrangement of work place

⑥ MICROMOTION STUDY: used for operations having short cycle.

Human activity is divided into groups of micro motion (Therbligs) and studies.

Therblig: (18) fundamental hand motions

SIMO CHART: -micromotion form of 2-handed P.C.
- Activities Recorded in terms of Therbligs
- Time for these also Recorded

-Film Analysis

wink count,	LH activity T. time	RH activity T. time

⑦ TIME STUDY:

Direct → Stop watch, work sampling

Indirect → synthetic or standard data, Analytical Estimation, PMTS → MTM, WF

Q- if cycles with elements within it given? Find avg. of elements and not cycle. Then use RF for Element avg. Then add all to get avg. NT for a cycle.

(i) Observed time: The time measured or Observed.

(ii) Normal time: Time for doing a Job by Avg. worker

$$\boxed{NT = OT \times RF}$$

(iii) Standard time

$$\boxed{ST = NT + \text{Allowanes}}$$

Allowance could be % of ST or NT.

Allowances - Extra time a qualified worker would need above NT. (Rest, personal, Policy, contigency, fatigue)

⑧ work sampling: A large no. of observations are made at random.

No. of observations N

$$\boxed{N = \frac{Z^2}{L^2} P(1-P)}$$

if frequency given

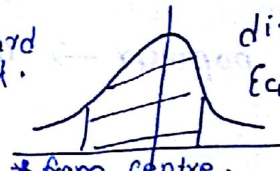
$$N = \left\{ \frac{Z}{L} \frac{\sqrt{n \Sigma f x^2 - (\Sigma f x)^2}}{\Sigma f x} \right\}^2$$

Z = % confidence level in terms of standard variant.

L = limit of accuracy = 5% = .05.

P = fraction of occurence of an activity

95% Z = 1.96   99.74% = 3.0



95.45% → Z = 2

⑨ (i) synthetic and Standard data → set of carefully compiled data for normal time for diff. elements of Job. * Build NT by adding these.
origin: small or local, can't be used everywhere.

(ii) Analytical Estimation → In this time determined partially by standard data and partially by expert Estimator.

(iii) PMTS → predetermined motion Time Study. similar to (i) but universal in nature. * Records minute motions.

a- Method time measurement (MTM): 19 fundamental motions
8 manual
9 Pedal
2 Eye

* NT = Σ elemental times from catalogue
* ST = NT + All.

1 MTU = 10^{-5} hr

b- work factor system (WF): Easy, Also consider mental process, for experienced workers, 1 TMU = 10^{-4} min. (Time measurement unit)

c- motion time Analysis

d- " " " data for assembly work.

Q- Total obs: 2000 Idle = 500
∴ working = 1500/2000 = 75%.
Time = 60 hrs work time = 45 hrs
manual: mlc = 3 : 1 33.75, 11.25 hrs
Rating: 85%. All = 15%. NT = 28.6875 + 11.25 = 39.9375
ST = 1.15 NT = 45.928 hrs

LINEAR PROGRAMMING

① L.P. → To find optimal solⁿ for most efficient use of resources

② **Requirement of L.P.**

(i) Obj. funcⁿ : MIN or MAX (cost or profit)

(ii) constraints : limited Resources.

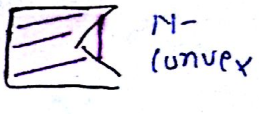
(iii) (i), (ii) must be linear.

③ **General statement of L.P** (x_1 x_2) Decision variable, Profit coefficient, (c_1 c_2)

$\text{Max } Z = c_1 x_1 + c_2 x_2$ (a) Technological coeff, Resource value (b)

$a_1 x_1 + a_2 x_2 \le b_1$

④ **1st method Graphical** [only if 2 variables]



 # feasible region : space which satisfies all constraints should be convex to solve L.P.

 # Note : In exam always first show ~~evaluation~~. formulation.

⑤ **FOR OPTIMALITY**

(i) Conv: one of the corner pt. or vertex pt. gives optimal solⁿ

why → ∵ slope of obJ. is constant, draw ∥ obJ. functions and shift it towards const. at vertex pt. it will be tangent.

(ii) OBJ : Intersecⁿ of those const. give optimal in b/w whose slopes lies the slope of obJ. funcⁿ.

can't be used → when one of the const. is Redundant. (when it does not form boundary of feasible region thus Inc. or Exc. don't affect)

 # Binding const → whose Resources are fully utilised.

⑥ **Special cases in Graphical** * imp. Q. in copy.

(i) ∞ or multi optimal solⁿ * check if slope of objective funcⁿ = slope of constraint. forming boundary of feas region
→ obJ. funcⁿ ∥ to one of the const. (i.e. it is binding const)
→ " " optimises in direcⁿ of that const.

(ii) No solution No common feasible regions, No optimal solⁿ.

(iii) Unbounded " " max. limit of " " , " " .

- Binding const : Resources fully utilised
- Non " " : " not " "

⑦ **SIMPLEX** [BY DANTZIG] [Helpful for >2 variable]

step by step process, start with initial feasible solⁿ, then Improve.

Conditions (i) $b_i \ge 0$ (ii) $x_J \ge 0$ (iii) convert inequality to equality
(use slack, surplus, variable) #Include in obJ. by multiplying 0.
(Added) (Subtracted)
(iv) obJ $<$ min or max

⑧ **Algorithm** n var, m const:

\# put (n-m) var = 0 [Non B var] , m variables (Basic)

\# This ↓ no. of alternatives. $\boxed{max : {}^{n}C_{m}}$

⑨ **STRUCTURE**

e_i	Basis	x_1	x_2	S_1	S_2	bi	θ_i
Coefficient of basic variable in objective funcn	Basic variable	first	constraint			—	—
	Basic variable	2nd	constraint			—	—
	C_J	Objective	function				

$$Z_J = \sum a_{iJ} \cdot e_i$$

$$\Delta_J = C_J - Z_J$$

\# non-basic variable – which put 0 in initial soln (x_1, x_2)

\# Basic variable – which not " " " " (S_1, S_2)
↳ Those in the coloumn of Basis.

(i) max. ObJ → Till all Δ_J become 0 or -ve

→ choose max. +ve Δ_J as Incoming variable

→ $\theta_i = \dfrac{bi}{\text{corresponding Incoming col}}$ → min +ve θ_i = Out going variable

→ Replace out going by Incoming [# i.e. change it in ei coloumn - the coeff. from obJ. funcn]

→ Make key element = 1 and correspondingly divide whole row even bi.

→ make corresponding in same coloumn of key element = 0 correspondingly all other also bi.

(ii) min ⓐ multiply obJ by (-1) and solve as before

ⓑ → Till all Δ_J become 0 or +ve

→ For incoming choose most -ve

→ θ_i = Replacement Ratio (same) (min +ve).

(iii) Infeasible, no soln → Artificial variable (A) in basis of final soln.

⑩ (i) infinite or multi optimal: if non basic variable has $\Delta_J = 0$

(ii) Infinite but unbounded : if all θ_i -ve or ∞

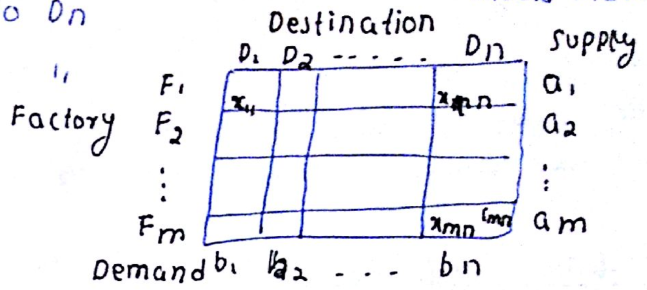
Transportation: - special case of simplex minimisation - All demand, supply Should match.

x_{mn} = supply from F_m to D_n

C_{mn} = supply cost " "

$\Sigma b = \Sigma a$

$$\min z = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} \cdot C_{ij}$$



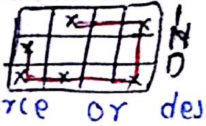
Basic feasible if Total allocations = $m+n-1$

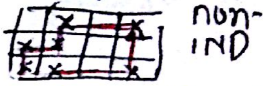
Non-degenerate " " (i) " = $m+n-1$

(ii) All allocations at Independent positions. (if Joining them does not form loop)

if this then only → Can Check optimality

Thus if not $m+n-1$ then add ε at min. cost place so that at Ind. position.

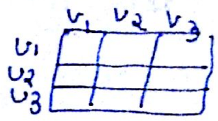




if unbalanced problem - Balance with dummy source or destination with 0 cost.

INITIAL Soln: ① N-W corner: ↓ → ↘ ② Row minima (1st Row) ③ column minima (1st coloumn) ④ least cost method ⑤ vogels approx method - * in every Row and Coloumn and take diff. of lowest, 2nd lowest * Among these select Best from overall * Then in that R or C allocate max. possible at min. cost [only 1 at a time] * Remove fulfilled R or C * Repeat procedure. * when only 2 left then Select min.

Check optimality: U-V method: ① write cost matrix for allocated cells. ② Start with $V_1 = 0$ ③ find all U, V ④ By using all U, V develop matrix for non allocated cells ⑤ Now subtract this matrix of non-allocated cells from original cost matrix. This gives cell evaluation matrix.



- if all +ve Then this is opt. soln.
- if any -ve minimise further.

① Identify most -ve entry ② Trace a path with start and End at this cell and other 3 corners at allocated cells. ③ +ve, -ve, +ve, -ve ④ Eliminate from -ves and add to +ves ⑤ Now cell with 0 allocation leaves the matrix.

ASSIGNMENT

- minimisation - Hungarian method
- max. " - Subtract all elements from highest. Then use H. method.

* should be a sq. matrix. * Develop opportunity cost matrix

- Subtract lowest Element in Every Row from all elements
- " " " Coloumn " "

* Now atleast 1, 0 in Every Row and Coloumn

Now assign: * moving Row wise find where single 0, assign it and X all other 0 in that coloumn

* After examining all rows move to colomns.
* Do (i), (ii) until all assigned or crossed or Remaining unmarked lies at least 2 in Each row & coloumn. (Then use hit and trial)

Optimality - if no. of assignments = order of matrix. Then opt. soln.

① Condition to apply simplex $b \geq 0$

(i) All Resource values (b_i) i.e. RHS of each constraint ≥ 0

(ii) Each dec. variable ≥ 0 i.e. $x_1, x_2 \geq 0$

(iii) convert IneQ to equality.

(iv) obj. funcn either min or max.

② (i) Unbounded soln (infinite soln but No optimal soln)

→ For outgoing variable we Choose min +ve value of Replacement Ratio θ_i. # For this condn all θ_i values either −ve or ∞.

(ii) Infeasible soln (No solution)

→ if in final soln an artificial variable ($A_1, A_2, \dots$) remains in basis.

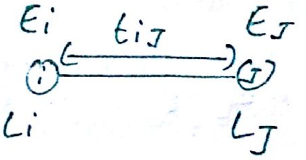
→ if infeasible then detected in Φ-1 of 2-Φ method.

(iii) infinite or multi optimal soln (slopes =)

if Non-basic variable (Not in basis coloumn) has $\Delta_J = 0$.

(iv) Degeneracy : Tie b/w outgoing (θ_i same).

③ Total ~~float~~ float of activity i-J



$= L_J - E_i - t_{i-J}$

④ KANBAN : A method of Inventory control

⑤ Degeneracy in L.P. → one of the constraints is Redundant

⑥ (i) SPT → minimises MFT, Avg. No. of Jobs in system

(ii) EDD → 〃 the max. Tardiness

(iii) moores procedure → 〃 No. of Tardy Jobs

⑦

	↓ α	↑ α
(i)	more wt. to past figures	more wt. to Recent figures
(ii)	For stable series	For fluctuating series

⑧ ~~Earliest~~ Additional Time available for an activity provided all activities start at their Earliest start time: Free float.

optimality in assignment: (i) mark all rows ✓ where assignment NOT done. (ii) mark coloumns ✓ which hv unassigned 0s in marked Rows ↑. (iii) mark Rows ✓ 〃 〃 assigned 0s in marked coloumns. (iv) complete this till Possible (v) Draw min. no. of lines thru unmarked rows / marked col. to cover all 0s. no. of lines = no. of assignments made (vi) Select min. element from those which don't have line thru it (i) subtract from those w/o line (ii) Add to Intersecn (iii) leave others.

-RCR
-UUA
UR, MC

(i) $\bar{x}$-chart : $\bar{\bar{x}} \pm \sigma_x Z$, $\bar{\bar{x}} \pm \frac{\sigma}{\sqrt{n}} \cdot 3$, $\bar{x} \pm \frac{3\bar{R}}{d_2\sqrt{n}}$, $\boxed{x \pm A_2\bar{R}}$

- measures shift in central tendanty

use $\bar{R} = d_2 \cdot \sigma$ (Plot $\bar{x}$ Pts.) ($\because \pm 3\sigma$ limit) $\boxed{\sigma = \frac{\bar{R}}{d_2}}$ $\boxed{A_2 = \frac{3}{d_2\sqrt{n}}}$

$\bar{\bar{x}}$ = same as avg. of all indl. values

$\frac{\bar{x}_1, R_1}{\boxed{n}}_1$ $\frac{\bar{x}_2, R_1}{\boxed{n}}_2$ $\frac{\bar{x}_3, R_3}{\boxed{n}}_3 \ldots \frac{\bar{x}_N, R_N}{\boxed{n}}_N$

$\bar{\bar{x}} = \frac{\Sigma \bar{x}_i}{N}$ $\bar{R} = \frac{\Sigma R_i}{n}$

(ii) R- chart : $\bar{R} \pm 3\sigma d_3$, $\bar{R} \cdot D_4$ and $\bar{R} D_3$ $\boxed{D_3 = 0 \text{ for } n \in 7}$ $\bar{R} \pm 3\bar{R}\frac{d_3}{d_2}$ $\bar{R}\left(1 \pm \frac{3d_3}{d_2}\right)$

measures dispersion (Plot R Pts.) of Process

(iii) P- Chart : $\bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}}$ $\bar{p} = \frac{\Sigma P_i}{N}$, $\bar{n} = \frac{\Sigma n_i}{N}$

- proportion or fraction - both sample size, no. of defects if known.
defect chart
- sample size diff.

sample sizes : $n_1, n_2, \ldots n_N$ $P_i = \frac{d_i}{n_i}$ $\bar{p} = \frac{\Sigma P_i}{N}$.
defets in it : $d_1, d_2, \ldots d_N$

(iv) np-chart : $n\bar{p} \pm 3\sqrt{n\bar{p}(1-\bar{p})}$

- similar to P-chart
- Here all sample size n is same
- multiply P-chart by n - $\bar{n} = n$

V) c -chart : $\bar{c} \pm 3\sqrt{\bar{c}}$. $\bar{c}$ = Avg. no. of defects Poison [A special case of Binumial when $n\uparrow, P\downarrow$]

↳ In poisson mean = variance $\therefore \sigma = \sqrt{\text{variance}} = \sqrt{\text{mean}}$

① process layout : Job or Batch Type production
various smaller components

② product or line layout : For assembly

③ combination : TV, music system, fridge

④ Fixed : Ship, Air craft.

Types of C. charts

① variable CC : - continuous data
- Normal distribution (i) $\bar{x}$ chart
- can be measured (ii) R chart

② Attribute CC : - can be counted
- cant be measured in fracn or decimal
- determines only presence or absence of something (i) P-chart
- BINOMIAL (ii) nP- "
(iii) c- chart

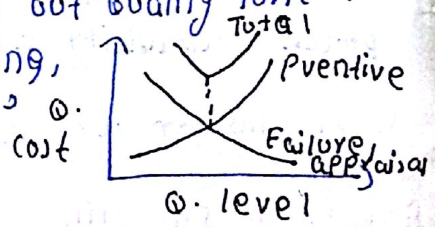
Quality : Refers to fitness for use

Quality cost : cost to carry out quality funcn.

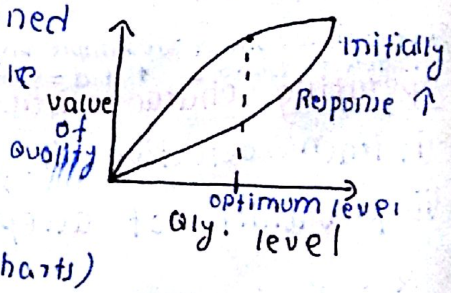
(i) Failure cost <
→ Internal
→ External

(ii) Appraisal cost : measuring, discovering, auditing to identify the defectives Q.

(iii) preventive cost : To minimise (i) and (ii)



VALUE OF QUALITY : The return direct or indirect gained by a company due to its Qly. control activities like ↑ mkt. share, customer retention, etc.



Quality control vs Inspection :

Statiscal Quality control :
→ Descriptive statistics [$\bar{x}$, σ, var]
→ Statiscal Process control (control charts)
→ Acceptance Sampling

Variations : 1- Chance, Common or random 2- Assignable Cause

Errors : 1- TYPE I : No shift but we still think that there is assignable cause

2- Type II : There is shift " " no " "

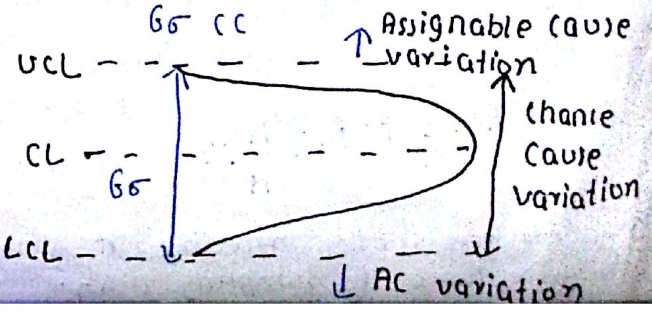
↳ graph showing how a process changes over time.

Control Chart : σ = process SD. if 6σ chart then 99.74% parts within specified limit.

$\pm 3\sigma$ 99.74%.
$\pm 2\sigma$ 95.4%.
$\pm \sigma$ 68.4%.

σ = Process SD

σ_x = Sample SD = $\frac{\sigma}{\sqrt{n}}$ n=sample size.



① WORK Study is mainly Aimed at- Developing Standard method And Standard Time of a Job. (∵ WORK Study $\begin{cases} \rightarrow \text{method Study} \\ \rightarrow \text{Time Study} \end{cases}$)

② In plant layout design- operation P C, man m/c PC, Travel chart.

③ Best determination of S T by- Analysis of Standard data System.

④ Objective of value Analysis
1- To ↓ cost ✓
2- To ↑ Quality ✓
3- To ↑ profit } X
4- Scrap, useless parts ↓ } ✓

⑤ VALUE ENGG.
1- Funcn oriented approach
2- multi disciplined Team "
3 - System oriented "

⑥ Use of Inventory: Elimination of possibility of duplicate ordering. } X

Process capability: It is the min. spread of a specific measurement variation which will include 99.74% of the measurement in a given process.

Process capability $= 6\sigma$ P.C. Index $= \dfrac{X_{max} - X_{min}}{6\sigma}$ X_{max} = max. spec. limit
X_{min} = min. spec. limit

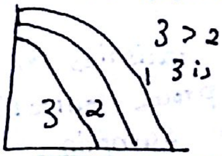
$C_p > 1$ (no or very less defective) $C_p = 1$ (.26% defective) $C_p < 1$ (Defective).

Acceptance sampling:
→ Single (Economic, ↓ time, if destructive ↓ staff)
→ Double (might Reject good, might accept bad)
→ multiple

Single: (i) Lot size N (ii) sample picked n (iii) defects found d * if $d \leq c$ (Accept) * $d > c$ (Reject)

Double: (i) (ii) (iii) same $d \leq c_1$ (accept) $d \geq c_2$ (Reject) $c_1 < d < c_2$ → 2nd sample (n', d') → $d + d' \leq c'$ (A) $d + d' > c'$ (R) $c_2 > c$

Operating characteristic curve: for an attribute sampling is a graph plot b/w:
(i) fracn defective in a lot
(ii) probability of acceptance

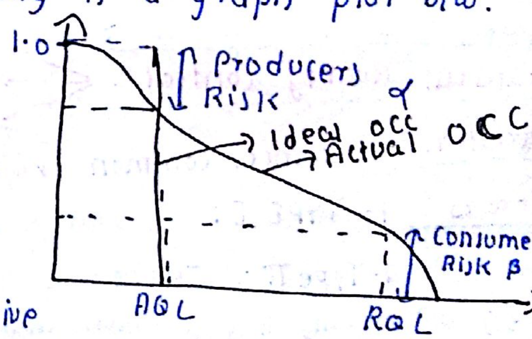


3 > 2 > 1
3 is better

α = P of Rejecn at AQL
β = " " accepting " RQL

$$\boxed{P_a = \frac{e^{-nP} \cdot (nP)^x}{x!}}$$

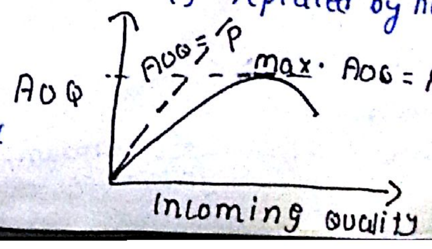
P of acceptance



AOQ- Avg. outgoing Quality- Represents avg. % defective in the outgoing products including (i) all accepted (ii) All Rejected which have be 100% Inspected and defectives replaced by non-defectives.

$$\boxed{AOQ = P \cdot P_a \cdot \frac{(N-n)}{N}}$$

P = % of defectives
P_a = P of acceptance.
N =



OCC- For any fraction defective, it shows the probability of its acceptance by sampling plan.

① The outbreak of Natural Calamity Necessiates updating X. wrong.

② Social Acceptability > Economic viability > Technical feasibility

③ Diversification → To Satisfy more customers.

④

Distribution

(i) Demand during lead time when Stock-out cost Not Known - Normal distribution

(ii) ProJect duration in CPM and PERT - Normal

(iii) Activity time in PERT - β - [Can be skewed abt mean].

(iv) Arrival pattern in Queing - poisson

(v) Service " " " - Exponential

(vi) Control Chart for variables - Normal

(vii) " " " Attributes - Binomial

(vii) " " " C-chart - poisson

For variables (X, R Chart) - About those which can be continuously measured in terms of their value.

For Attributes - These have discrete values like Yes or No, Good or bad, Correct or defective. They are counted.

p-chart : Both sample, defective measured [Sample size vary]

np-chart : " " " " [" " const.]

Count of defects chart: only No. of defects known. Sample size very large. Thus cannot compute the proportion that is defective.