Chapter 3 Areas Of Parallelograms And Triangles

Very Short Answer Type Questions

Question.1 Two parallelograms are on equal bases and between the same parallels. Find the ratio of their areas.

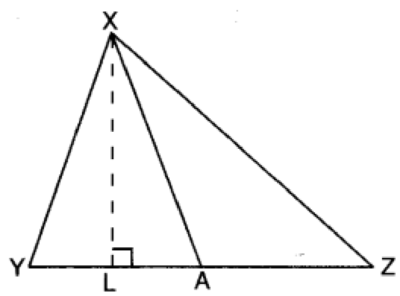
Solution. 1:1 [$\because$Two Parallelograms on the equal based and between the same parallels are equal in area.]

Question.2 If a triangle and a parallelogram are on same base and between same parallels, then find the ratio of the area of the triangle to the area of parallelogram.

Solution. 1 : 2 [$\because$ If a triangle and a parallelogram are on the same base and between the same parallels, the area of the triangle is equal to half of the parallelogram.]

Question.3

In ΔXYZ, XA is a median on side YZ. Find ratio of ar(ΔXYA) : ar(ΔXZA). [CBSE-15-6DWMWSA]



Solution.

Here, XA is the median on side YZ.

$\therefore \quad YA = AZ$

Draw $XL \perp YZ$

$$\therefore \quad ar(\Delta XYA) = \frac{1}{2} \times YA \times XL$$

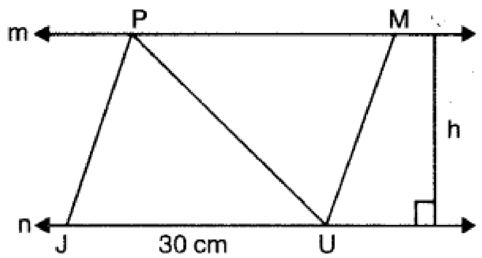
$$ar(\Delta XZA) = \frac{1}{2} \times AZ \times XL$$

$$\text{Thus, } ar\ (\Delta XYA) : ar(\Delta XZA) = \frac{1}{2} \times YA \times XL : \frac{1}{2} \times AZ \times XL$$

$$= 1 : 1 \qquad [\because YA = AZ]$$

Question.4.

In the given figure, m || n and JUMP is a parallelogram. If area of ΔPMU is 321 cm^2, then what is the value of h ?



Solution.

$$\text{area of } (\Delta PMU) = 321 \text{ cm}^2$$

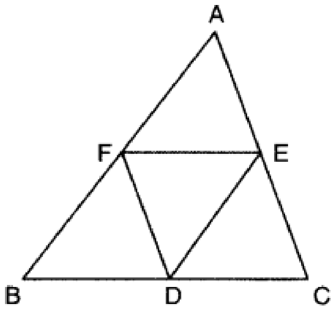
$$\Rightarrow \quad \frac{1}{2} \times PM \times h = 321$$

$$\Rightarrow \quad \frac{1}{2} \times 30 \times h = 321 \qquad [\because PM = JU = 30 \text{ cm, opp. sides of } ||^{gm}]$$

$$\Rightarrow \quad h = 21.4 \text{ cm}$$

Question.5

In ΔABC, D, E and F are the mid-points of BC, CA and AB respectively. If ar(ΔABC) = 56 cm^2, then find ar($||^{gm}$ AEDF).



Solution.

$$\Delta AEF \cong \Delta BDF \cong \Delta DEF \cong \Delta CDE$$

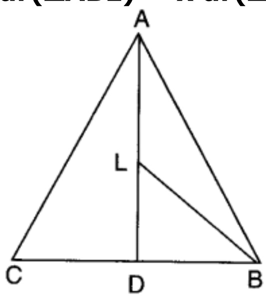
$$\therefore \quad ar(\Delta AEF) = ar(\Delta BDF) = ar(\Delta DEF) = ar(\Delta CDE)$$

$$\therefore \quad ar(\Delta AEF) = \frac{1}{4}\, ar(\Delta ABC) = \frac{1}{4} \times 56 = 14 \text{ cm}^2$$

$\because$ Diagonal of a parallelogram divide it in two triangles having same area.

$$\therefore \quad ar(||^{gm} AEDF) = 2 \times ar(\Delta AEF)$$
$$= 2 \times 14 = 28 \text{ cm}^2$$

Question.6 In the given figure, D is the mid-point of BC and L mid-is the point of AD. If ar(ΔABL) = x ar(ΔABC), then find the value of x



Solution.

In ΔABC, AD is the median

$$\therefore \quad ar(\Delta ABD) = \frac{1}{2} ar(\Delta ABC)$$

Again, in ΔABD, BL is the median

$$\therefore \quad ar(\Delta ABL) = \frac{1}{2} ar(\Delta ABD)$$

$$= \frac{1}{2} \times \frac{1}{2} ar(\Delta ABC)$$

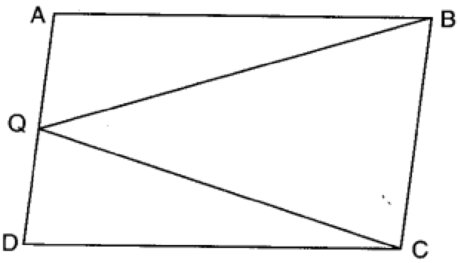
$$= \frac{1}{4} ar(\Delta ABC)$$

Hence, value of x is $\frac{1}{4}$.

Question.7

ABCD is a parallelogram and Q is any point on side AD. If ar(ΔQBC) = 10 cm^2, find ar(ΔQAB) + ar(ΔQDC).

[CBSE-15-6DWMW5A]



Solution.

Here, ΔQBC and parallelogram ABCD are on the same base BC and lie between the same parallels BC || AD.

$$\therefore \quad ar(||^{gm} ABCD) = 2\ ar(\Delta QBC)$$

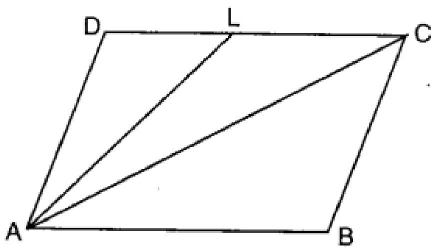
$$ar(\Delta QAB) + ar(\Delta QDC) + ar(\Delta QBC) = 2\ ar(\Delta QBC)$$

$$ar(\Delta QAB) + ar(\Delta QDC) = ar(\Delta QBC)$$

Hence, $ar(\Delta QAB) + ar(\Delta QDC) = 10\text{ cm}^2 \quad [\because ar(\Delta QBC) = 10\text{ cm}^2 \text{ (given)}]$

Question.8

In the given figure, ABCD is a parallelogram and L is the mid-point of DC. If ar(quad. ABCL) is 72 cm^2, then find ar(ΔADC).



Solution.

In $\|^{gm}$ ABCD, AC is the diagonal

$$\therefore \quad ar(\Delta ABC) = ar(\Delta ADC) = \frac{1}{2} ar(\|^{gm} ABCD)$$

In ΔADC, AL is the median

$$\therefore \quad ar(\Delta ADL) = ar(\Delta ACL) = \frac{1}{2} ar(\Delta ADC) = \frac{1}{4} ar(\|^{gm} ABCD)$$

Now, ar(quad. ABCL) = $ar(\Delta ABC) + ar(\Delta ACL)$

$$= \frac{3}{4} ar(\|^{gm} ABCD)$$

$$72 \times \frac{4}{3} = ar(\|^{gm} ABCD)$$

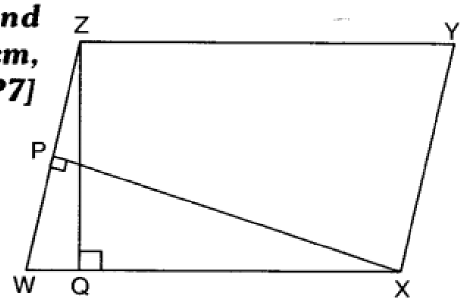
$$\Rightarrow \quad ar(\|^{gm} ABCD) = 96 \text{ cm}^2$$

$$\therefore \quad ar(\Delta ADC) = \frac{1}{2} ar(\|^{gm} ABCD)$$

$$= \frac{1}{2} \times 96 = 48 \text{ cm}^2$$

Question.9

WXYZ is a parallelogram with XP $\perp$ WZ and ZQ $\perp$ WX. If WX = 8 cm, XP = 8 cm and ZQ = 2 cm, find YX. ***[CBSE-15-NS72LP7]***



Solution.

$$ar(\|^{gm} WXYZ) = ar(\|^{gm} WXYZ)$$

$$WX \times ZQ = WZ \times XP$$

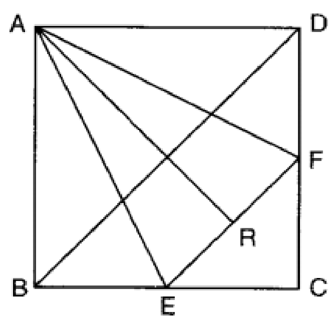
$$8 \times 2 = WZ \times 8$$

$$\Rightarrow \quad WZ = 2 \text{ cm}$$

Now, $YX = WZ = 2$ cm [$\because$ opposite sides of parallelogram are equal]

Question.10

ABCD is a square. E and F are respectively the mid-points of BC and CD. If R is the mid-point of EF. Prove that : ar(ΔAER) = ar(ΔAFR).



Solution.

Since R is the mid-point of EF.

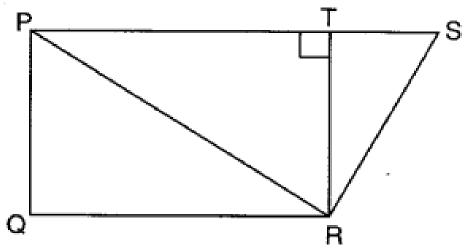
$\therefore$ AR is the median in ΔAEF.

As, a median of a triangle divides it into two triangles of equal area.

$\therefore \quad ar(\Delta AER) = ar(\Delta AFR)$

Question.11 In figure, TR ⊥ PS, PQ // TR and PS // QR. If QR = 8 cm, PQ = 3 cm and SP = 12 cm, find arquad. PQRS).[CBSE-14-17DIG1U]

Solution.



Here, $PS \parallel QR$ [given]

$\therefore$ PQRS is a trapezium

Now, $TR \perp PS$ and $PQ \parallel TR$ [given]

$\Rightarrow PQ \perp PS$

$\therefore PQ = TR = 3$ cm [given]

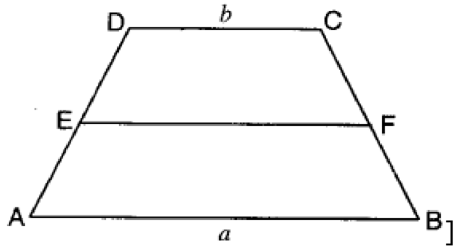
Now, $ar(quad.\ PQRS) = \frac{1}{2}(PS + QR) \times PQ = \frac{1}{2}(12 + 8) \times 3$

$= 30\ cm^2$

Question.12

ABCD is a trapezium with parallel sides AB = a cm and DC = b cm (fig.). E and F are the mid-points of the non-parallel sides. Find the ratio of ar(ABFE) and ar(EFCD).

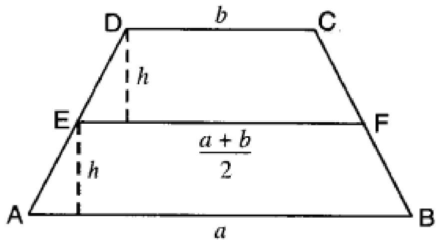
[NCERT Exemplar Problem]



Solution.

Clearly, $EF = \frac{AB + DC}{2} = \frac{a+b}{2}$

Let h be the height, then



$ar(Trap.\ ABFE) : ar(Trap.\ EFCD)$

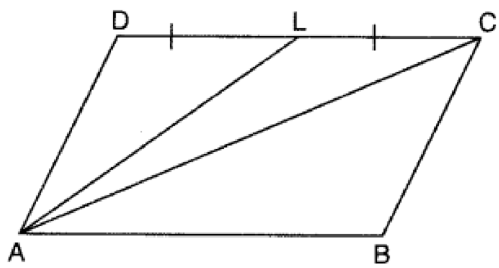
$\Rightarrow \frac{1}{2}\left[a+\left(\frac{a+b}{2}\right)\right] \times h : \frac{1}{2}\left[b+\left(\frac{a+b}{2}\right)\right] \times h$

$\Rightarrow \frac{2a+a+b}{2} : \frac{2b+a+b}{2}$

$\Rightarrow 3a + b : 3b + a$

Question.13

In given figure, ABCD is a parallelogram and L is the mid-point of DC. If ar(||^{gm} ABCD) = 84 cm^2, then find ar(ΔACL).



Solution.

AC is the diagonal of parallelogram ABCD

$$\therefore \quad ar(\Delta ACD) = \frac{1}{2} ar(||^{gm} ABCD) \quad ...(i)$$

Now, L is the mid-point of DC

$$\therefore \quad ar(\Delta ACL) = \frac{1}{2} ar(\Delta ACD) \quad ...(ii)$$

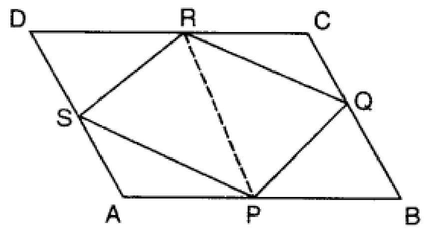
From (*i*) and (*ii*), we have

$$ar(\Delta ACL) = \frac{1}{4} ar(||^{gm} ABCD)$$

$$= \frac{1}{4} \times 84 = 21 \text{ cm}^2$$

Question.14

If P, Q, R, S are respectively the mid-points of the sides of a parallelogram ABCD, if ar($||^{gm}$ PQRS) = 32.5 cm^2, then find ar($||^{gm}$ ABCD).



Solution.

Join PR.

$\because$ ΔPSR and $||^{gm}$ APRD are on the same base and between same parallel lines.

$$ar(\Delta PSR) = \frac{1}{2} ar(||^{gm} APRD)$$

Similarly, $$ar(\Delta PQR) = \frac{1}{2} ar(||^{gm} PBCR)$$

$$ar(||^{gm} PQRS) = ar(\Delta PSR) + ar(\Delta PQR)$$

$$= \frac{1}{2} ar(||^{gm} APRD) + \frac{1}{2} ar(||^{gm} PBCR)$$

$$= \frac{1}{2} ar(||^{gm} ABCD)$$

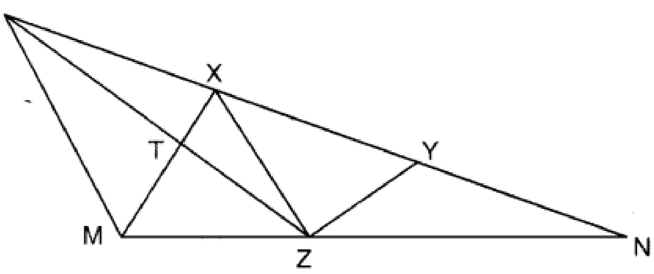
$$\Rightarrow \quad ar(||^{gm} ABCD) = 2 \times ar(||^{gm} PQRS) = 2 \times 32.5 = 65 \text{ cm}^2$$

SHORT ANSWER QUESTIONS TYPE-I

Question.15

X and Y are points on the side LN of the triangle LMN, such that LX = XY = YN. Through X, a line is drawn parallel to LM to meet MN at Z (see figure).

Prove that : ar(ΔLZY) = ar(quad. MZYX).



Solution.

Here, ΔXZM and ΔXZL are on the same base (XZ) and lie between the same parallels (XZ || LM).

$\therefore \quad ar(\Delta XZL) = ar(\Delta XZM)$

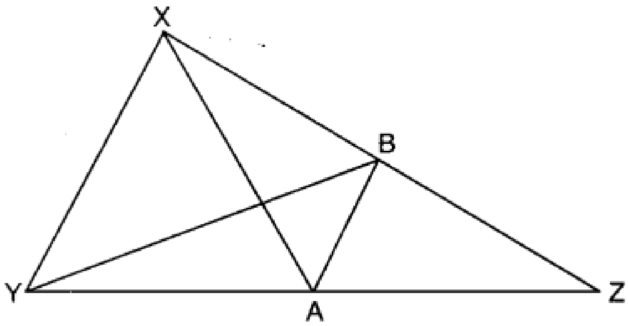
Adding ar(ΔXZY) on both sides, we have

$ar(\Delta XZL) + ar(\Delta XZY) = ar(\Delta XZM) + ar(\Delta XZY)$

$\Rightarrow \quad ar(\Delta LZY) = ar(\text{quad. } MZYX)$

Question.16

In the given figure of ΔXYZ, XA is a median and AB // YX. Show that YB is also median. [CBSE-14-NS72LP7



Solution.

Here, in ΔXYZ, AB || YX and XA is a median.

∴ A is the mid-point of YZ.

Now, AB is a line segment from mid-point of one side (YZ) and parallel to another side (AB || YX), therefore it bisects the third side XZ.

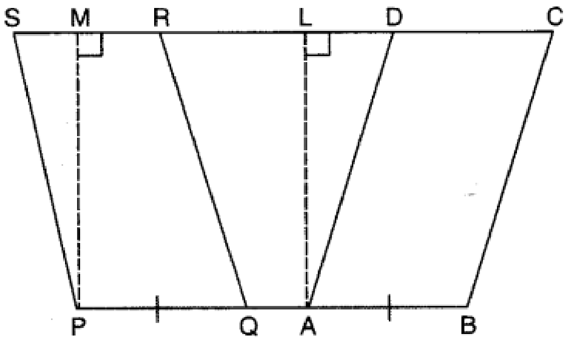
⇒ B is the mid-point of XZ.

Hence, YB is also a median of ΔXYZ.

Question.17 Prove that parallelogram on equal bases and between the same parallels are equal in area. [CBSE March 2012]

Solution.

Suppose AL and PM are the altitudes corresponding to equal bases AB and PQ of $||^{gms}$ ABCD and PQRS respectively.



Since the $||^{gms}$ are between the same parallels PB and SC.

$\therefore \quad AL = PM$

Now, $ar(||^{gm} ABCD) = AB \times AL$

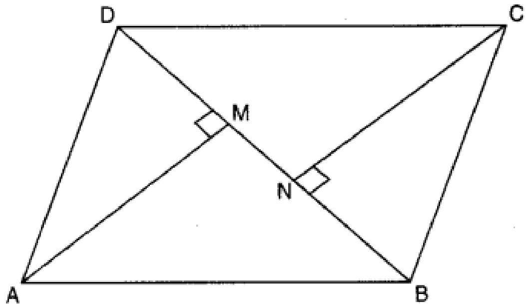
$ar(||^{gm} PQRS) = PQ \times PM$

But, $AB = PQ$ [given]

$AL = PM$ [proved]

$\therefore \quad ar(||^{gm} ABCD) = ar(||^{gm} PQRS)$

Question.18



BD is one of the diagonals of a quadrilateral ABCD. AM and CN are the perpendiculars from A and C respectively on BD. Show that :

ar(quad. ABCD) = $\frac{1}{2}$BD. (AM + CN).

[CBSE March 2012]

Solution.

We know that area of a triangle $= \frac{1}{2} \times \text{base} \times \text{altitude}$

$\therefore \quad ar(\Delta ABD) = \frac{1}{2} \times BD \times AM$

and $\quad ar(\Delta BCD) = \frac{1}{2} \times BD \times CN$

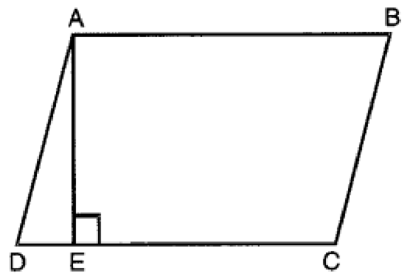
Now, $ar(\text{quad. } ABCD) = ar(\Delta ABD) + ar(\Delta BCD)$

$$= \frac{1}{2} \times BD \times AM + \frac{1}{2} \times BD \times CN$$

$$= \frac{1}{2} \times BD \times (AM + CN)$$

Question.19

In the given figure, ABCD is a parallelogram and AE ⊥ DC. If AB is 20 cm and the area of parallelogram ABCD is 80 cm^2, find AE. [CBSE March 2012]



[opposite sides of a $||^{gm}$]

Solution.

$\because$ $AB = 20$ cm

$AB = CD$

$\therefore$ $CD = 20$ cm

Now, $ar(||^{gm} ABCD) = \text{base} \times \text{height}$

$80 = 20 \times AE$

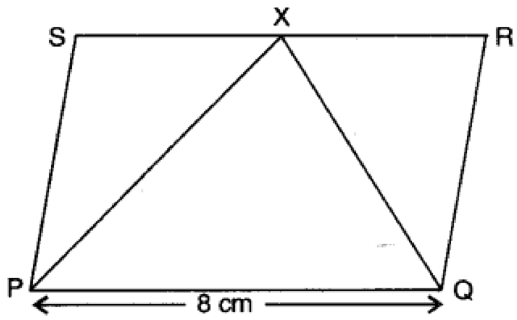
$\Rightarrow \dfrac{80}{20} = AE$

$\Rightarrow AE = 4$ cm

Question.20

In the figure, PQRS is a parallelogram with PQ = 8 cm and ar(ΔPXQ) = 32 cm². Find the altitude of $||^{gm}$ PQRS and hence its area.

[CBSE-14-17DIG1U]



Solution.

Since parallelogram PQRS and ΔPXQ are on the same base PQ and lie between the same parallels PQ || SR

$\therefore$ Altitude of the ΔPXQ and $||^{gm}$ PQRS is same.

Now, $\dfrac{1}{2} PQ \times \text{altitude} = ar(\Delta PXQ)$

$\Rightarrow \dfrac{1}{2} \times 8 \times \text{altitude} = 32$

$\text{altitude} = 8$ cm

$ar(||^{gm} PQRS) = 2\, ar(\Delta PXQ)$

$= 2 \times 32$

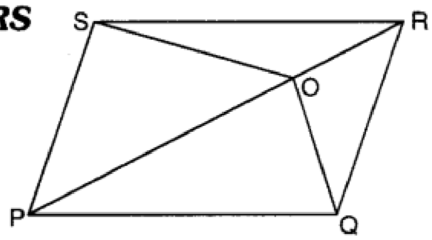
$= 64 \text{ cm}^2$

Hence, the altitude of parallelogram PQRS is 8 cm and its area is 64 cm^2.

Question. 21

O is any point on the diagonal PR of a parallelogram PQRS (see figure).

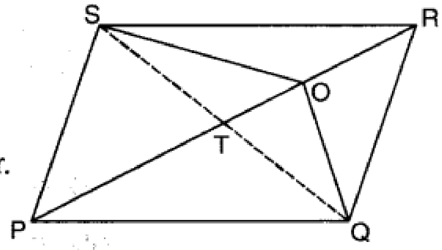
Prove that : ar(ΔPSO) = ar(ΔPQO).



Solution.

Join QS.

Let diagonals PR and QS intersect each other at T.



We know, that diagonals of a parallelogram bisect each other.

$\therefore$ T is the mid-point of QS.

Since a median of a triangle divides it into two triangles of equal area.

$\therefore$ In ΔPQS, PT is its median.

$\Rightarrow \quad ar(\Delta PTS) = ar(\Delta PQT) \quad ...(i)$

In ΔSQO, OT is its median.

$\Rightarrow \quad ar(\Delta STO) = ar(\Delta QTO) \quad ...(ii)$

Adding (i) and (ii), we have

$ar(\Delta PTS) + ar(\Delta STO) = ar(\Delta PQT) + ar(\Delta QTO)$

$\Rightarrow \quad ar(\Delta PSO) = ar(\Delta PQO)$

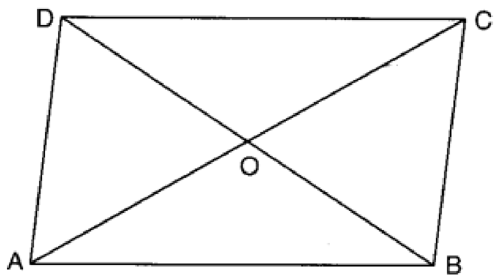
Question.22 ABCD is a parallelogram and O is the point of intersection of its diagonals. If ar(ΔAOD) = 4 cm^2, find area of parallelogram ABCD. [CBSE-14-GDQNI3W]

Solution.

Here, ABCD is a parallelogram in which its diagonals AC and BD intersect each other in O.

$\therefore$ O is the mid-point of AC as well as BD.

Now, in ΔADB, AO is its median



$\therefore \quad ar(\Delta ADB) = 2\ ar(\Delta AOD)$

[$\because$ median divides a triangle into two triangles of equal areas]

So, $\quad ar(\Delta ADB) = 2 \times 4 = 8\ cm^2$

Now, Δ ADB and ||^{gm} ABCD lie on the same base AB and lie between same parallels AB and CD

$\therefore \quad ar(ABCD) = 2\ ar(\Delta ADB)$

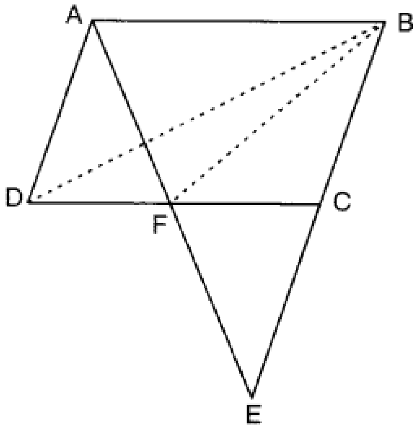
$= 2 \times 8$

$= 16\ cm^2$

Question.23

ABCD is a parallelogram in which BC is produced to E such that CE = BC (figure). AE intersects CD at F.

If ar(ΔDFB) = 3 cm², find the area of the parallelogram ABCD.



Solution.

In ΔADF and ΔECF, we have

$$\angle ADF = \angle ECF \quad \text{[alt. int. } \angle\text{s]}$$

$$AD = EC \quad [\because AD = BC \text{ and } BC = EC]$$

$$\angle DFA = \angle CFE \quad \text{[vert. opp. } \angle\text{s]}$$

∴ By AAS congruence rule,

$$\Delta ADF \cong \Delta ECF$$

$\Rightarrow$ $DF = CF$ [c.p.c.t.]

$\Rightarrow$ $ar(\Delta ADF) = ar(\Delta ECF)$

Now, $DF = CF$

$\Rightarrow$ BF is a median in ΔBDC

$\Rightarrow$ $ar(\Delta BDC) = 2\, ar(\Delta DFB)$

$= 2 \times 3 = 6 \text{ cm}^2$ $[\because ar(\Delta DFB) = 3 \text{ cm}^2]$

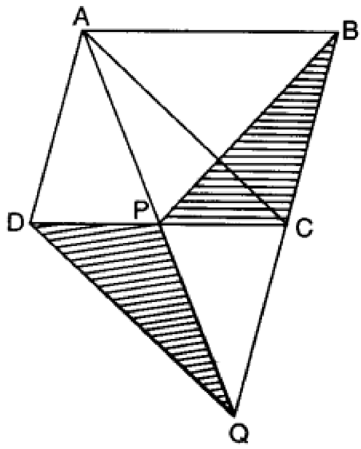
Thus, $ar(\|^{gm} ABCD) = 2\, ar(\Delta BDC)$

$= 2 \times 6 = 12 \text{ cm}^2$

Question.24

ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ (fig.). If AQ intersects DC at P, show that ar(ΔBPC) = ar(ΔDPQ).

[NCERT Exemplar Problem]



Solution.

In ||^{gm} ABCD,

$$ar(\Delta APC) = ar(\Delta BCP) \quad .$$

[$\because$ triangles on the same base and between the same parallels have equal ar

Similarly, $$ar(\Delta ADQ) = ar(\Delta ADC) \quad ..$$

Now, $$ar(\Delta ADQ) - ar(\Delta ADP) = ar(\Delta ADC) - ar(\Delta ADP)$$

$$ar(\Delta DPQ) = ar(\Delta ACP) \quad ...$$

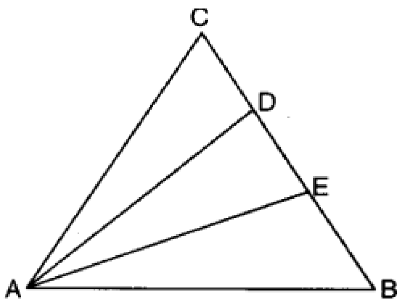
From (*i*) and (*iii*), we have

$$ar(\Delta BCP) = ar(\Delta DPQ)$$

or $$ar(\Delta BPC) = ar(\Delta DPQ)$$

Question.25

In ΔABC, D and E are points on side BC such that $CD = DE = EB$. If $ar(\Delta ABC) = 27\ cm^2$, find $ar(\Delta ADE)$ [CBSE - 14-GDQNI3W]



Solution.

Since in ΔAEC, $CD = DE$, AD is a median.

$\therefore$ $$ar(\Delta ACD) = ar(\Delta ADE) \quad ... (i)$$

[$\because$ median divides a triangle into two triangles of equal areas]

Now, in ΔABD, $DE = EB$, AE is a median

$\therefore$ $$ar(\Delta ADE) = ar(\Delta AEB) \quad ... (ii)$$

From (*i*), (*ii*), we obtain

$$ar(\Delta ACD) = ar(\Delta ADE) = ar(\Delta AEB) = \frac{1}{3} ar(\Delta ABC)$$

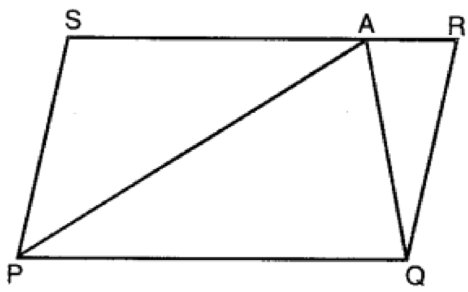
$\therefore$ $$ar(\Delta ADE) = \frac{1}{3} \times 27 = 9\ cm^2$$

SHORT ANSWER QUESTI-IIONS TYPE

Question. 26 A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the field is divided ? What are the shapes of these parts ? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it ?

Solution.

From the adjoining figure, we have



The field PQRS is divided into three parts, ΔPAQ, ΔAPS and ΔAQR.

Now, ΔPAQ and $||^{gm}$ PQRS are on the same base and lie between the same parallels.

$$\therefore \quad ar(\Delta PAQ) = \frac{1}{2} ar(||^{gm} PQRS)$$

$$\text{and } ar(\Delta APS) + ar(\Delta AQR) = \frac{1}{2} ar(||^{gm} PQRS)$$

Hence, she can sow wheat in ΔAPS and ΔAQR, pulses in ΔPAQ or vice-versa.

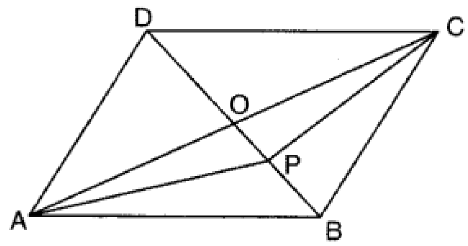
Question.27

ABCD is a parallelogram whose diagonals intersect at O. If P is any point on BO, prove that :

(i) ar(ΔADO) = ar(ΔCDO)

(ii) ar(ΔABP) = ar(ΔCBP)

[CBSE March 2012]



Solution.

(*i*) Since diagonals of a parallelogram bisect each other.

$\therefore$ O is the mid-point of AC as well as BD.

$\therefore$ In ΔADC, OD is a median.

$$\therefore \quad ar(\Delta ADO) = ar(\Delta CDO)$$

[$\because$ A median of a triangle divide it into two triangles of equal area]

(*ii*) Since O is the mid-point of AC

$\therefore$ OB and OP are medians of ΔABC and ΔAPC respectively.

$$\therefore \quad ar(\Delta AOB) = ar(\Delta BOC) \quad \ldots (i)$$

$$\text{and} \quad ar(\Delta AOP) = ar(\Delta COP) \quad \ldots (ii)$$

[$\because$ A median of a triangle divide it into two triangles of equal area]

Subtracting (*ii*) from (*i*), we have

$$ar(\Delta AOB) - ar(\Delta AOP) = ar(\Delta BOC) - ar(\Delta COP)$$

$$\Rightarrow \quad ar(\Delta ABP) = ar(\Delta CBP)$$

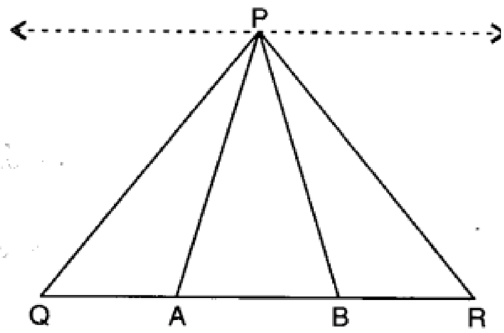
Question.28 InΔPQR, A and B are points on side QR such that they trisect QR. Prove that ar(ΔPQB) = 2ar(ΔPBR).

Solution.

Here, in ΔPQR, A and B are points on side QR such that $QA = AB = BR$.

Through P, draw a line l parallel to QR

Now, ΔPQA, ΔPAB and ΔPBR on the equal bases and between the same parallels $l \parallel QR$



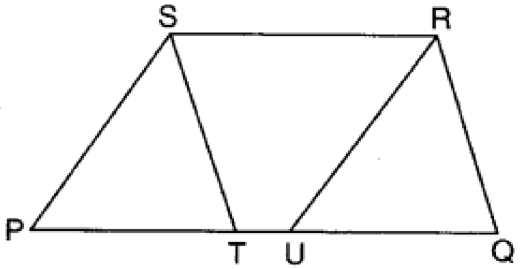
$\Rightarrow \quad ar(\Delta PQA) = ar(\Delta PAB) = ar(\Delta PBR) \quad .... (i)$

Now, $\quad ar(\Delta PQB) = ar(\Delta PQA) + ar(\Delta PAB)$

$= 2ar(\Delta PQA) \quad$ [using (i)]

$= 2ar(\Delta PBR) \quad$ [using (i)]

Question.29 For the given figure, check whether the following statement is true or false. Also justify your answer. PQRS is a trapezium with PQ // SR, PS // RU and ST // II RQ, then ar(PURS) = ar(TQRS)
[CBSE-14-ERFKZ8H]



Solution.

Since $ST \parallel RQ$ and $SR \parallel TQ$ [given]

$\Rightarrow$ STQR is a $\|^{gm}$

Similarly $PS \parallel UR$ and $SR \parallel PU$ [given]

$\Rightarrow$ PSRU is a $\|^{gm}$

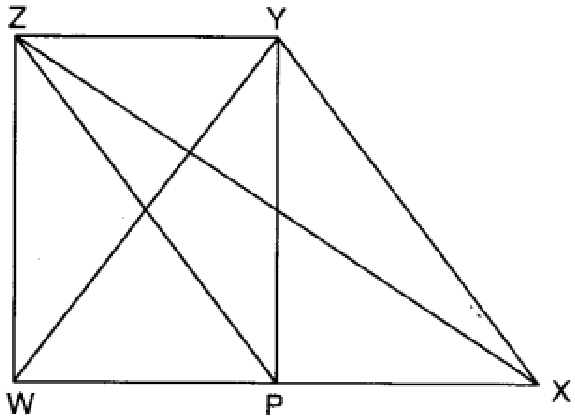
Also, similarly $\|^{gm}$ STQR and $\|^{gm}$ PSRU lie on same base SR and between same parallels PQ and SR.

$\therefore ar(\|^{gm} STQR) = ar(\|^{gm} PSRU)$

Hence, the given statement is true.

Question.30 In the given figure, WXYZ is a quadrilateral with a point P on side WX. If ZY // WX, show that :
(i) ar(ΔZPY) = ar(ΔZXY)
(ii)ar(ΔWZY) = ar(ΔAZPY)

(iii)ar(ΔZWX) = ar(ΔXWY) [CBSE-14-ERFKZ8H]



Solution.

ΔZPY and ΔZXY lie on same base ZY and between same parallels ZY and WX

$\therefore$ ar (ΔZPY) = ar(ΔZXY)

Again, (ΔWZY) amd (ΔZPY) lie on same base ZY and between same parallels ZY and WX

$\therefore$ ar(ΔWZY) = ar(ΔZPY)

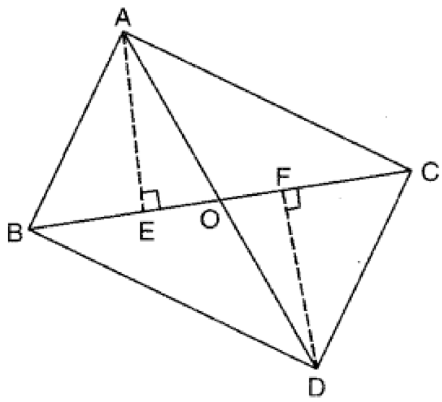
Also, ΔZWX and ΔXWY lie on same base XW and between same parallels ZY and WX

$\therefore$ ar(ΔZWX) = ar(ΔXWY)

Question.31. Triangles ABC and DBC are on the same base BC and A, D on the opposite sides of BC, such that ar(ΔABC) = ar(ΔDBC), show that BC bisect AD.

Solution.

Draw $AE \perp BC$ and $DF \perp BC$



$$\text{ar}(\Delta ABC) = \frac{1}{2}(BC \times AE)$$

Also, $$\text{ar}(\Delta DBC) = \frac{1}{2}(BC \times DF)$$

Now, $$\text{ar}(\Delta ABC) = \text{ar}(\Delta DBC)$$

$$\Rightarrow \quad \frac{1}{2}(BC \times AE) = \frac{1}{2}(BC \times DF)$$

$$\Rightarrow \quad AE = DF$$

Now, in ΔAEO and ΔDFO

$\angle AEO = \angle DFO$ [each = 90°]

$\angle AOE = \angle DOF$ [vertically opp. angles]

$AE = DF$ [proved above]

$\Delta AEO \cong \Delta DFO$ [AAS congruency]

$AO = DO$ [c.p.c.t.]

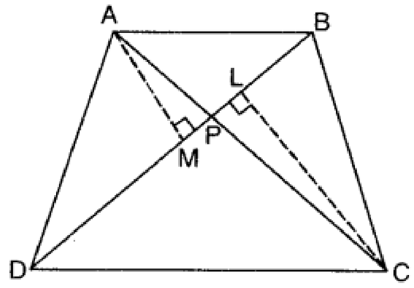
$\Rightarrow$ BC bisects AD

Question.32 Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that ar(ΔAPB) x ar(ΔCPD) = ar(ΔAPD) X ar(ΔBPC).

Solution.

Draw AM $\perp$ BD and CL $\perp$ BD.

Now, ar(ΔAPB) $\times$ ar(ΔCPD)



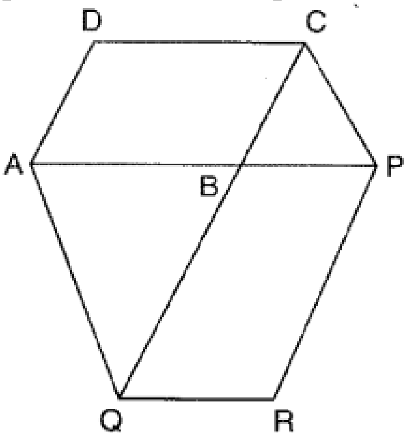
$$= \left\{\frac{1}{2}PB \times AM\right\} \times \left\{\frac{1}{2}DP \times CL\right\}$$

$$= \left\{\frac{1}{2}PB \times CL\right\} \times \left\{\frac{1}{2}DP \times AM\right\}$$

$$= ar(\Delta BPC) \times ar(\Delta APD)$$

Hence, ar(ΔAPB) $\times$ ar(ΔCPD) = ar(ΔAPD) $\times$ ar(ΔBPC)

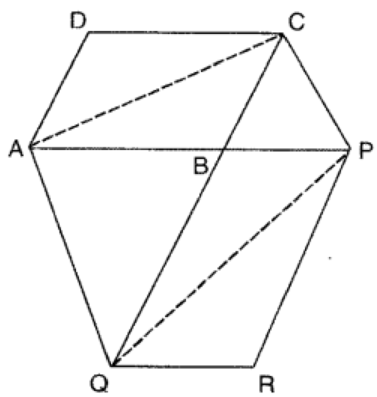
Question.33 The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q, then parallelogram PBQR is completed (see figure). Show that ar$(\|^{gm} ABCD)$ = ar$(\|^{gm} ABCD)$.
[CBSE March 2012]



Solution.

Join AC and QP, also it is given that AQ || CP

$\therefore$ ΔACQ and ΔAPQ are on the same base AQ and lie between the same parallels AQ || CP.



$\therefore$ ar(ΔACQ) = ar(ΔAPQ)

or ar(ΔABC) + ar(ΔABQ) = ar(ΔBPQ) + ar(ΔABQ)

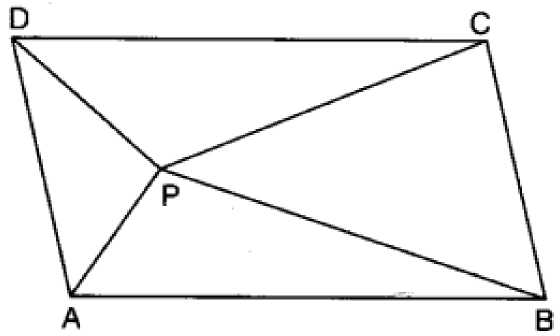
or ar(ΔABC) = ar(ΔBPQ)

or $\frac{1}{2}$ ar($\|^{gm}$ ABCD) = $\frac{1}{2}$ ar($\|^{gm}$ PBQR)

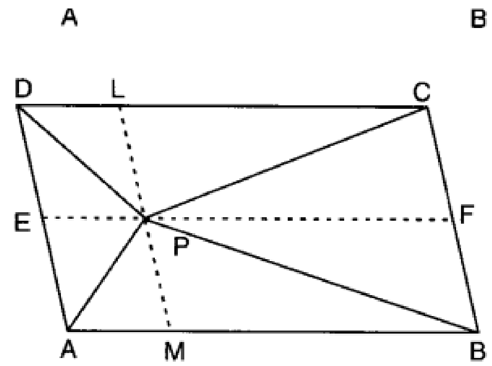
or ar($\|^{gm}$ ABCD) = ar($\|^{gm}$ PBQR)

Long Answer Type Questions

Question.34. ΔBCD is a parallelogram and P is any point in its interior. Show that ar(ΔAPB) + ar(ΔCPD)= ar(ΔBPC) + ar(ΔAPD) [CBSE-15-6DWMW5A)



Solution.



Through P, draw a line LM || DA and EF || AB

Since ΔAPB and $||^{gm}$ ABFE are on the same base AB and lie between the same parallels AB and EF.

$\therefore$ ar(ΔAPB) $= \frac{1}{2}$ ar($||^{gm}$ ABFE) ... (*i*)

Similarly, ΔCPD and parallelogram DCFE are on the same base DC and between the same parallels DC and EF.

$\therefore$ ar(ΔCPD) $= \frac{1}{2}$ ar($||^{gm}$ DCFE) .... (*ii*)

Adding (*i*) and (*ii*), we have

$$\text{ar}(\Delta APB) + \text{ar}(\Delta CPD) = \frac{1}{2}\text{ar}(||^{gm}\text{ ABFE}) + \frac{1}{2}\text{ar}(||^{gm}\text{ DCFE})$$
$$= \frac{1}{2}\text{ar}(||^{gm}\text{ ABCD}) \quad \text{... }(iii)$$

Since ΔAPD and parallelogram ADLM are on the same base AB and between the same parallels AD and ML

$$\therefore \quad \text{ar}(\Delta APD) = \frac{1}{2}\text{ar}(||^{gm}\text{ ADLM}) \quad \text{.... }(iv)$$

Similarly
$$\text{ar}(\Delta BPC) = \frac{1}{2}\text{ar}(||^{gm}\text{ BCLM}) \quad \text{.... }(v)$$

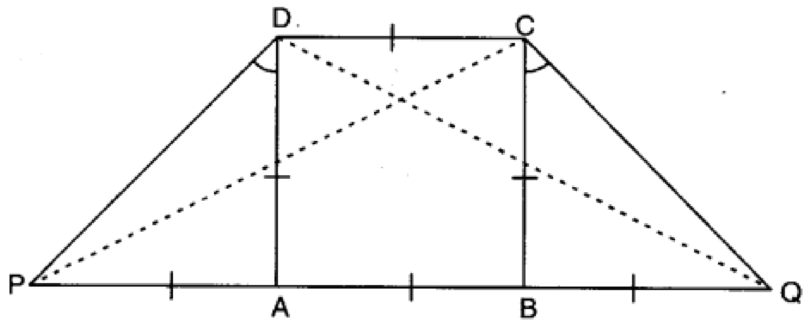
Adding (*iv*) and (*v*), we have

$$\text{ar}(\Delta APD) + \text{ar}(\Delta BPC) = \frac{1}{2}\text{ar}(||^{gm}\text{ ABCD}) \quad \text{.... }(vi)$$

From (*iii*) and (*vi*),we obtain

$$\text{ar}(\Delta APB) + \text{ar}(\Delta CPD) = \text{ar}(\Delta APD) + \text{ar}(\Delta BPC)$$

Question.35 In the given figure, ABCD is a square. Side AB is produced to points P and Q in such a way that PA = AB = BQ. Prove that DQ = CP.[CBSE-15-NS72LP7]



Solution.

In ΔPAD, $\angle A = 90^\circ$ and $DA = PA = AB$

$\Rightarrow \angle ADP = \angle APD = \frac{90^\circ}{2} = 45^\circ$

Similarly, in ΔQBC, $\angle B = 90^\circ$ and $BQ = BC = AB$

$\Rightarrow \angle BCQ = \angle BQC = \frac{90^\circ}{2} = 45$

In ΔPAD and ΔQBC, we have

$PA = BQ$ [giver

$\angle A = \angle B$ [each $= 90$

$AD = BC$ [sides of a square

$\Rightarrow \Delta PAD \cong \Delta QBC$ [by SAS congruence rule

$\Rightarrow PD = QC$ [c.p.c.t

Now, in ΔPDC and ΔQCD

$DC = DC$ [commor

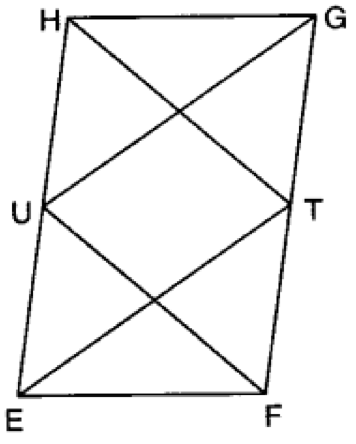
$PD = QC$ [prove above

$\angle PDC = \angle QCD$ [each $= 90^\circ + 45^\circ = 135$

$\Rightarrow \Delta PDC \cong \Delta QCD$ [by SAS congruence rule

$\Rightarrow PC = QD$ *or* $DQ = CP$

Question.36 EFGH is a parallelogram and U and T are points on sides EH and GF respectively. If ar(ΔEHT) = 16 cm^2, find ar(ΔGUF) [CBSE-15-NS72LP7]



Solution.

ΔEHT and parallelogram EFGH are on the same base HE and lie between the same parallels HE and GF

$\therefore$ ar(ΔEHT) = $\frac{1}{2}$ ar($||^{gm}$ EFGH) .... (i)

Similarly, ΔGUF and parallelogram EFGH are on the same base GF and lie between th same parallels GF and HE

$\therefore$ ar(ΔGUF) = $\frac{1}{2}$ ar($||^{gm}$ EFGH) .... (ii)

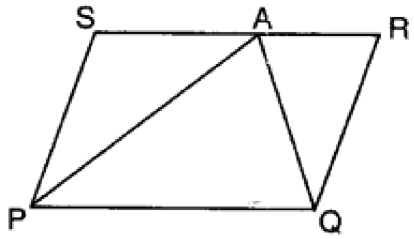
From (i) and (ii), we have

ar(ΔGUF) = ar(ΔEHT)

= 16 cm^2 [$\because$ ar(ΔEHT) = 16 cm^2] (given)

Value Based Questions (Solved)

Question.1 A farmer having afield in the form of parallelogram PQRS.
He planned to built a home for old persons of the village in the field leaving open portion equal to portion covered by the home. For this he divided the field by taking a point A on RS and joining AP, AQ respectively as shown in figure.
(i) How should he do it ?
(ii)What values are depicted in his plan ?



Solution.

(i) From the given figure :

The field PQRS is divided into three parts, ΔPAQ, ΔAPS and ΔAQR.

Now, ΔPAQ and $||^{gm}$ PQRS are on the same base and lie between the same parallels.

$\therefore$ ar(ΔPAQ) = $\frac{1}{2}$ ar($||^{gm}$ PQRS)

and ar(ΔAPS) + ar(ΔAQR)= $\frac{1}{2}$ ar($||^{gm}$ PQRS)

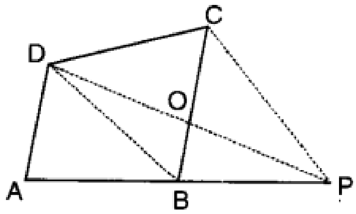
He should build the home in portion ΔAPQ and should leave open ΔAPS and ΔAQR.

(ii) We should respect our elders.

Question.2 Naveen was having a plot in the shape of a quadrilateral. He decided to donate some portion of it to construct a home for orphan girls. Further he decided to buy a land in lieu of his donated portion of his plot so as to form a triangle.
(i) Explain how this proposal will be implemented ?
(ii)Which mathematical concept is used in it ?
(iii)What values are depicted by Naveen ?

Solution.

(i) Let ABCD be the plot and Naveen decided to donate some portion to construct a home for orphan girls from one corner say C of plot ABCD. Now, Naveen also purchases equal amount of land in lieu of land CDO, so that he may have triangular form of plot. BD is joined. Draw a line through C parallel to DB to meet AB produced in P.



Join DP to intersect BC at O.

Now, ΔBCD and ΔBPD are on the same base and between same parallels $CP \parallel DB$.

$\Rightarrow \quad ar(\Delta BCD) = ar(\Delta BPD)$

$\Rightarrow \quad ar(\Delta COD) + ar(\Delta DBO) = ar(\Delta BOP) + ar(\Delta DBO)$

$\Rightarrow \quad ar(\Delta COD) = ar(\Delta BOP)$

$\Rightarrow \quad ar(\text{quad. } ABCD) = ar(\text{quad. } ABOD) + ar(\Delta COD)$

$= ar(\text{quad. } ABOD) + ar(\Delta BOP)$

$[\because ar(\Delta COD) = ar(\Delta BOP)]$ proved above

(ii) Area of parallelogram and triangle and mid-point theorem.

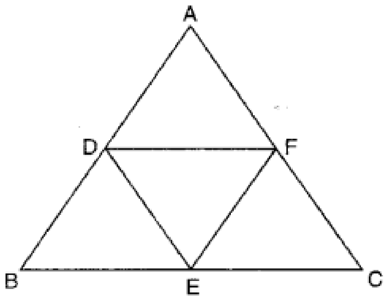
(iii) Every child, boy or girl have equal right, so avoid discrimination in boy and girl.

Question.3 Mr Sharma explains his four children two boys and two girls about distribution of his property among them by a picture of A ABC such that D, E, F are mid-points of sides AB, BC, CA respectively are joined to divide A ABC in four triangles as shown in figure.

(i) If total property is equal to area of A ABC and share of each child is equal to area of each of four triangles, what does each child has share ?

(ii)Which mathematical concept is used in it ?

(iii)Which values are depicted in Mr Sharma's plan ?



Solution.

(*i*) $\because$ D, E and F are mid-points of AB, BC and CA respectively.

$\therefore$ By mid-point theorem, we have

$$DF \parallel BC \text{ and } EF \parallel AB$$

$$\Rightarrow \quad DF \parallel BE \text{ and } EF \parallel BD$$

$\Rightarrow$ BEFD is a parallelogram.

$\because$ The diagonal of a parallelogram divide it into two congruent triangles.

$\therefore \quad \Delta DEF \cong \Delta BED$

Similarly, $\quad \Delta DEF \cong \Delta ADF$

And $\quad \Delta DEF \cong \Delta CEF$

$\therefore \ \Delta DEF \cong \Delta BED \cong \Delta ADF \cong \Delta CEF$

$\Rightarrow \quad ar(\Delta DEF) = ar(\Delta BED) = ar(\Delta ADF) = ar(\Delta CEF)$

$\because \quad ar(\Delta DEF) + ar(\Delta BED) + ar(\Delta ADF) + ar(\Delta CEF) = ar(\Delta ABC)$

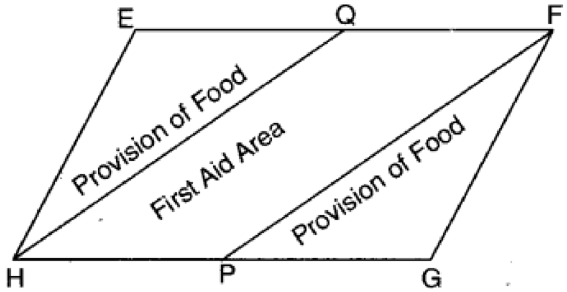
Hence, $ar(\Delta DEF) = ar(\Delta BED) = ar(\Delta ADF) = ar(\Delta CEF) = \frac{1}{4} ar(\Delta ABC)$

Each child will get equal share of property.

(*ii*) Area of parallelogram and triangle and mid-point theorem.

(*iii*) Every child, boy or girl have equal right, so avoid discrimination in boy and girl.

Question.4 A flood relief camp was organized by state government for the people affected by the natural calamity near a city. Many school students volunteered to participate in the relief work. In the camp, the food items and first aid centre kits were arranged for the flood victims.
The piece of land used for this purpose is shown in the figure.
(a) If EFGH is a parallelogram with P and Q as mid-points of sides GH and EF respectively, then show that area used for first aid is half of the total area.
(b) What can you say about the student volunteers working for the relief work ?
[CBSE-14-GDQNI3W]



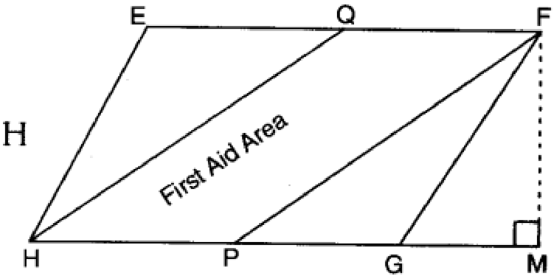
Solution.

(*a*) Here, EFGH is a $||^{gm}$

$\therefore \quad EF = GH$ and $EF \parallel GH$

i.e., $\frac{1}{2}EF = \frac{1}{2}GH$ and $\frac{1}{2}EF \parallel \frac{1}{2}GH$

$\Rightarrow \quad QF = PH$ and $QF \parallel PH$



Thus, HPFQ is a $||^{gm}$

Now, $FM \perp HM$

And $ar(||^{gm}\ HPFQ) = HP \times FM$

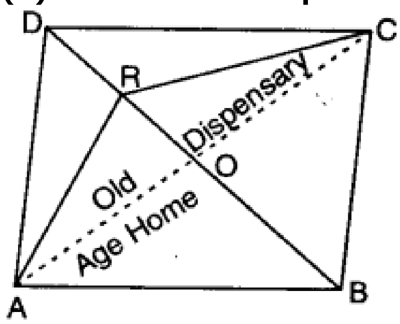
$= \frac{1}{2} HG \times FM$ [$\because$ P is the mid-point of HG]

$= \frac{1}{2}\{ar(||^{gm}\ EFGH)\}$

Hence, area used for first aid is half of the total area.

(*b*) Students working for the noble cause show compassion towards the affected people. They also realize their social responsibility to work for helping the ones in need.

Question.5 Sunita has a plot of land which she decides to use for building an old age home and a dispensary for the needy. Her plot is shown in the figure. Plot ABCD is a parallelogram
(a) If R is point on diagonals BD. Show that equal areas allotted for building old age home and the dispensary.
(b) What value is depicted by the above situation ?



Solution.

(*a*) Here, ABCD is a parallelogram. Join AC.

Let diagonals AC and BD intersect at O.

Since diagonals of a $||^{gm}$ bisects each other.

$\therefore \qquad AO = OC$

Now, in ΔABC, BO is a median.

$\therefore \qquad ar(\Delta ABO) = ar(\Delta CBO) \qquad .... (i)$

[$\because$ median divides a triangle into two triangles of equal areas]

Also, in ΔARC, RO is a median

$\therefore \qquad ar(\Delta ARO) = ar\ (CRO) \qquad .... (ii)$

Adding (*i*) and (*ii*), we have

$ar(\Delta ABO) + ar(\Delta ARO) = ar(\Delta CBO) + ar(\Delta CRO)$

$\Rightarrow ar(\Delta ARB) = ar(\Delta CRB)$

Thus, equal areas are used for building old age home and dispensary.

(*b*) Value depicted : Respect for human beings (relations) and compassion towards aged and needy people or any relevant value.