weightage of S.O.M and machine of Design (M/c design)

(1) Gate - 15 marks

(2) IES - 60-70 Marks (Obj.)

65-70 Marks (Conv.)

TEXT BOOKS:-

1. Mechanical of material - Timoshenko & Gere (CBS publishers)

2. Mechanics of Solid - Popov (Pearson edn PHI publishers)

3. Mechanics of Material (Vol. 1) - Junarkar (Charotar pubns)

4. Strength of Material - Ramamrutham pubns) {IES pubn

Strength of Material

or

Mechanics of Material

or

Mechanical of Solid

or

Mechanical of structure

or

Mechanical of Deformable bodies

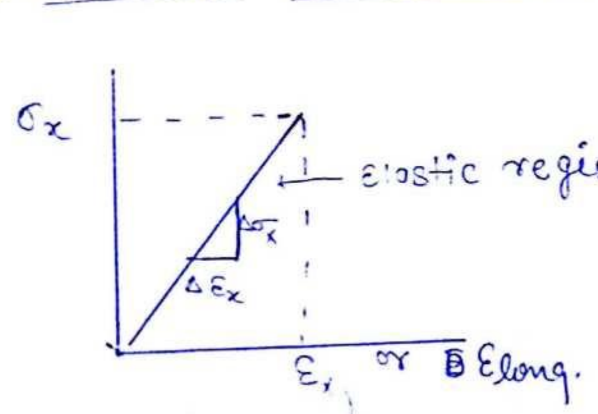
(i.e. Elastic)

Chapter - 1 & 2

→ Deal with both internal and external forces

→ Strain & Stress both are there; in rigid bodies only stress (No deformation)

Strain stress Graph for perfectly Elastic body.

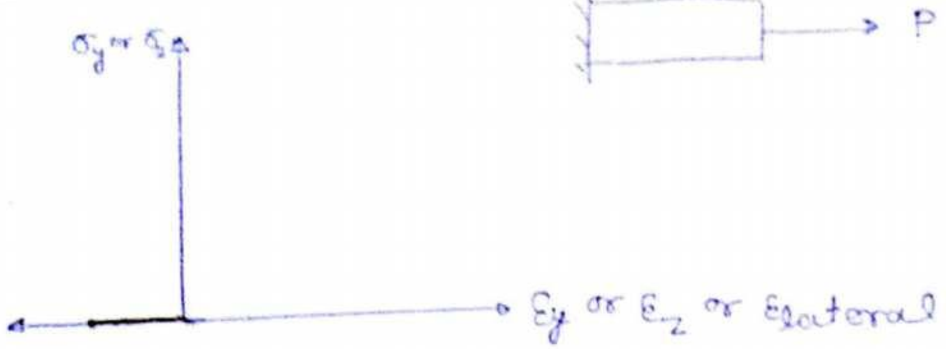


Slope of Curve = Young's modulus

$$\boxed{E = \frac{\Delta \sigma_x}{\Delta \varepsilon_x}}$$

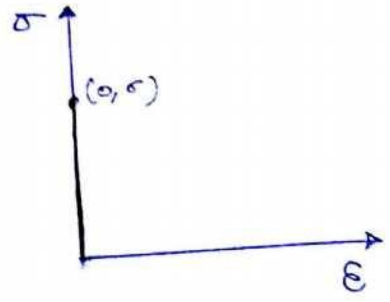
*. Perfectly Elastic bodies retain their shape in Elastic region.

Y-Z axis curve



* Fig Elastic body load applied only in x dir^n
* Diameter will decrease so $\varepsilon_{lateral}$ in -ve x-axis

For perfect rigid bodies (No deformation)



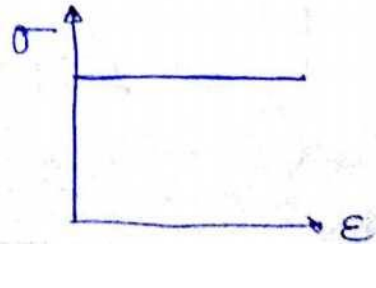
* wheather it is a rigid body or deformable body stess is always there whenever force applied
* For a deformable body when force is applied strain & stess both are there.

Hook's law

$$\sigma_x \propto (\varepsilon_x \text{ or } \varepsilon_{lateral})$$

$$\boxed{\sigma_x = E\,\varepsilon_x}$$

Perfect plastic body



Topics of E.M. w.r.t. SOM

1. Static equ^m eq^n → (3D) $\Sigma F_x = 0$ $\quad \Sigma C_x = 0$

$$\Sigma F_y = 0 \qquad \Sigma C_y = 0$$

$$\Sigma F_z = 0 \qquad \Sigma C_z = 0$$

(2D) $\Sigma H = 0$, $\Sigma V = 0$ $\quad \Sigma M = 0$

2. F.B.D.

3. Resolution of forces

☆ 4. Centroid & Moment of inertia cal^{n}s.

- Centroid - For 2D bodies eq. - Planes
- Centre of Gravity - for entire body '3D' Solid bodies

MoI

Area MOI (SOM) — Mass MOI (TOM)

5. Types of support & calculation of support reaction.

AIM of strenth of Materials

Drive expression for deformation, strain and stress which are developed under different loading conditions by using experimentaly obtain elastic ~~proble~~ properties, like Youngs Modulus & poisson ratio

⇒ Aim of design is to develope a drawing or a plan [i.e. selection of an appropiate shape, an appropiate material, calculation of appropeate dimⁿs by using som eqⁿ, selection of manufacturing process detail] in a such way that the resulting m/c component should perform its function satisfactory [i.e. without failure]

* A component said to be failure when it is unable to perform its functionality

* A Component said to be fracture when it is broken into two or more piece

Assumptions In SOM eqⁿs

1. Material is assume to homogenous and isotropic
2. Materials obeys Hooke's law (induce stresses and strain) are assume to be within the elastic region
3. Member is assume to be ~~persimatic~~ prismatic (~~throught~~ cross sections dimⁿs remain same througth out the length of member)
4. load is assume a static load
5. self weight of the component is neglacted
6. Member is assume to under static equᵐ.

Homogeneous :-

→ A material is said to be Homogeneous when it exhibits same elastic properties at any point in a given direction [Elastic properties are independent on point]

Isotropic:

A material is said to be isotropic when it exhibits same elastic properties in any dirⁿ at a given point [elastic properties are independent of direction]

~~Anis~~

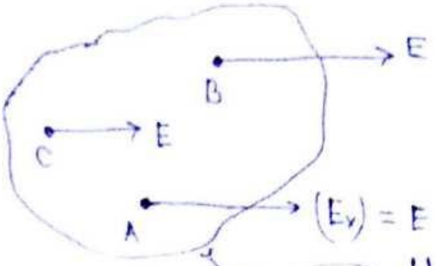
A material is said to be both homogeneous and isotropic when it exhibits same elastic properties at any point and in any direction [Elastic properties are independent of both direction and point

Every homogeneous material need not be isotropic and vice-versa but few materials are said to be both homogeneous and isotropic

Anisotropic :-

A material is said to be anisotropic when it exhibits direction dependent elastic properties
eg.- ceremic, Composite material etc.

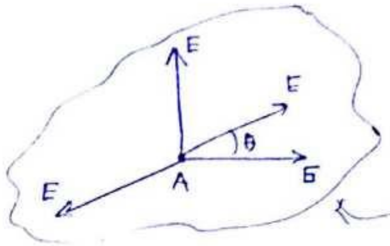
Homogeneous



To decide Homogeneous minimum two points are req.

Homogeneous material

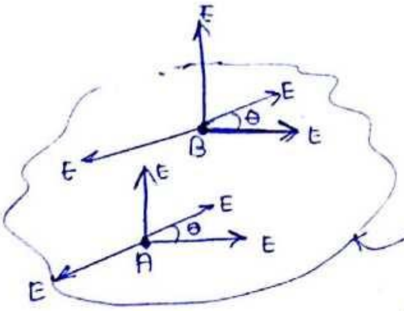
Isotropic



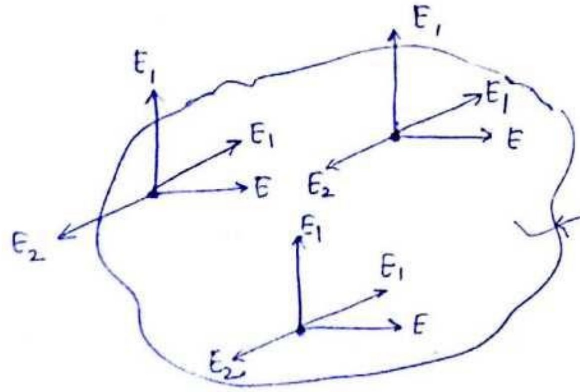
To decide isotropic one point & two dir^n req.

isotropic material

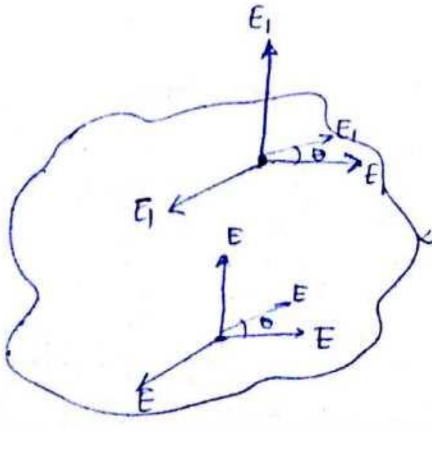
Anisotropic



both homogeneous & isotropic material (E, G, k, & μ)



Homogeneous & Anisotropic material.

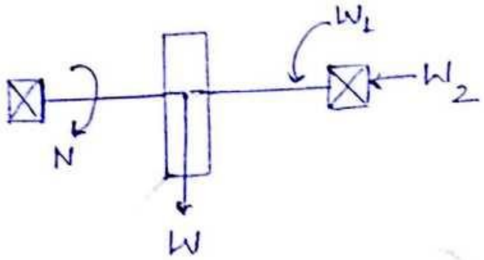


Non-homogeneous (Hetrogeneous) & isotropic material.

July 2018

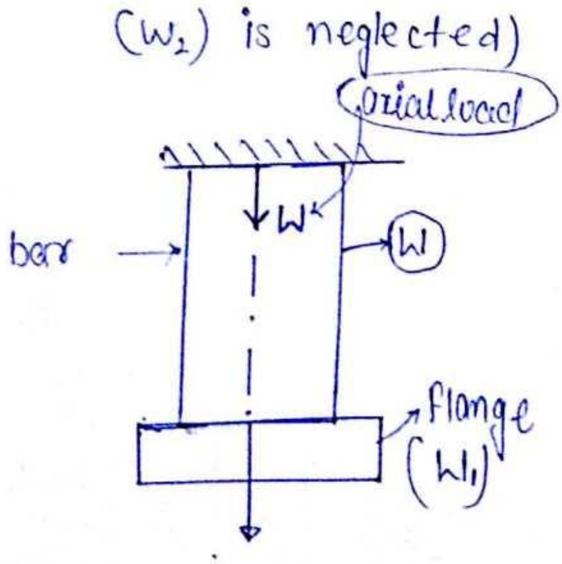
Load :- load is defined as an external force or a couple to which a component is subjected during it functionality.

e.g. - Weight of a component w.r.t. another component, centrifugal forces, inertia forces., frictional forces, wind forces, belt tension, Bending couple & twisting couples etc.

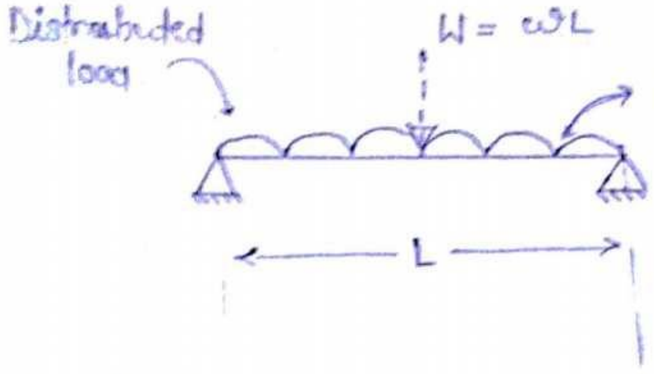


∵ self weight (i.e. weight of pully) acting transfer load. shear

- In shaft design weight of shaft will not be consider, pully weight is considered as a load
  e.g. Wt. of shaft (W_1) is neglected because it is an unknown.
- In the design of bearing. pully weight (W) and shaft weight (W_1) are considered as loads (i.e. Wt. of bearing (W_2) is neglected)

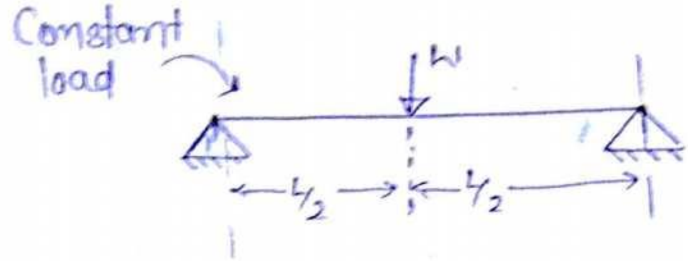


∴ In the design of bar, wt. of flange (W_1) is considered as load [∵ surface force]
(i.e. Wt of bar (W) is neglected)
body force



$w\ N/m = W/L$

① $M_{max} = \frac{wL^2}{8} = \frac{WL}{8}$

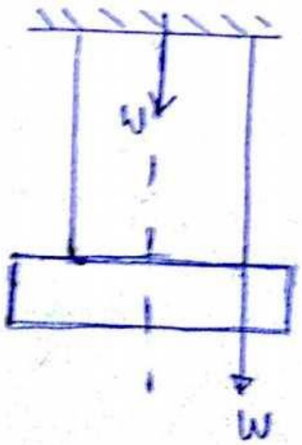


② $M_{max} = \frac{WL}{4}$

Deflection $\frac{Y_2}{Y_1} = \frac{WL^3/48EI}{\frac{5}{384}\frac{WL^3}{EI}} = 1.6$

Bending moment $\frac{(M_{max})_2}{(M_{max})_1} = \frac{WL/4}{WL/8} = 2$

* Chanse of failure in constracted load is more than uniform distributed load because of low surface area.



(ecentric load)
(Parellel to lognitude axis)

Classification of loads

- LOADS
  - w.r.t. time
    - STATIC LODS
      - Dead load
      - Gradually Applied load
    - DYNAMIC LOADS
      - Impact (or) SHOCK (or) Suddenly Applied load
      - Fatigue (or) Cyclic (or) Variable loads
  - w.r.t X-S/c
    - NORMAL LOAD ($\perp$ to X-S/c)
      - Axial loads
      - Eccentric Axial load
        - Tensile load
        - Compressive load
    - SHEAR LOAD (// el to X-S/c)
      - Transverse Shear loads
      - Eccentric transverse Shear loads
  - w.r.t. distribution of load
    - Distributed loads
      - U.D.L
      - U.V.L
      - U.D.M
    - Concentrated loads
      - Conc. Point load
      - Conc. moments

U.D.L - Uniformly distributed loads

U.V.L. - — " — varying loads

U.D.M. - — " — distributed moment

w.r.t. time

static load :- Magnituted and dir^n remain Constant w.r.t. time

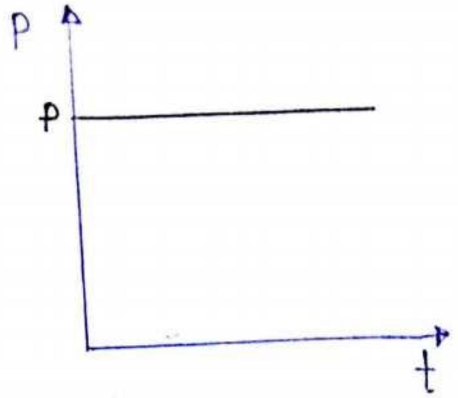


Fig:- Dead load

Wt. of structed members & M/c component

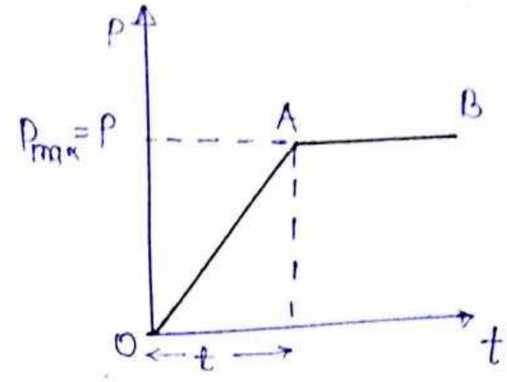
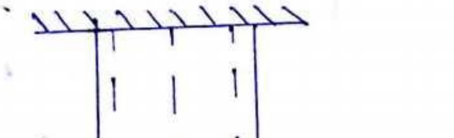


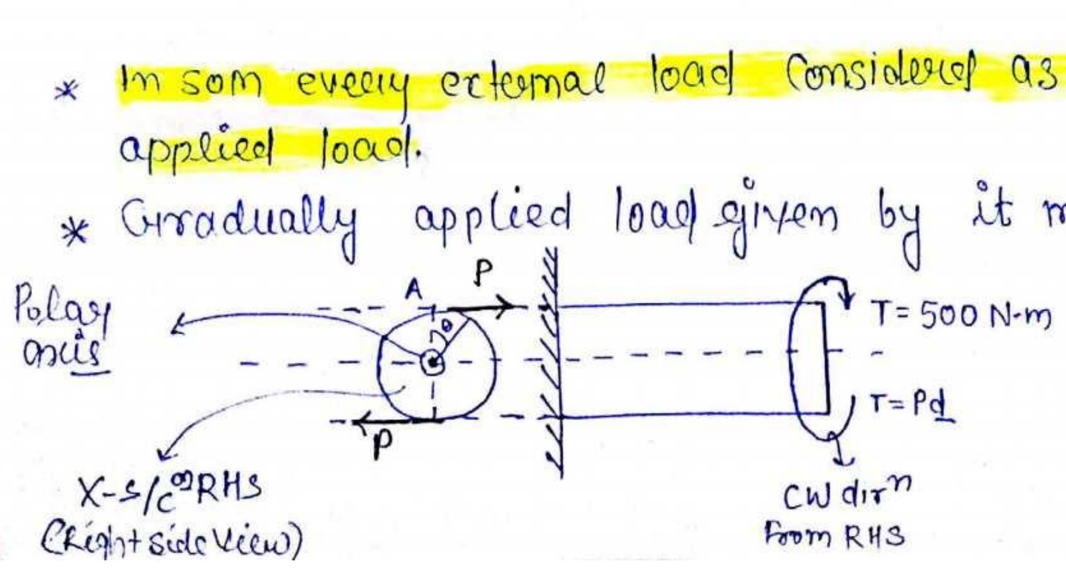
Fig:- Gradually applied load

Is a load which magnituted increase 0 to maximum and remain Constant
{0 to max. time to time variation}



W.D. by P = $\frac{1}{2}P\delta$ = Area of P-δ Graph

* In som every external load Considered as gradually applied load.
* Gradually applied load given by it max. Value "P_{max}".



Polar axis - ⊥ to X-S/c and passing throgh centriod

Work done by $T = \frac{1}{2} T \theta$

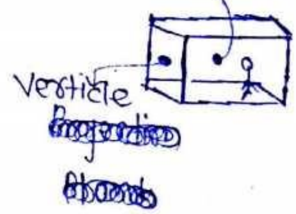
θ - Angle of twist

* Work done by Graudally applied load is average w.D. or area of P-δ Curve

* Twisting couple are applied parellel (ll) to axis and bending couple applied ⊥

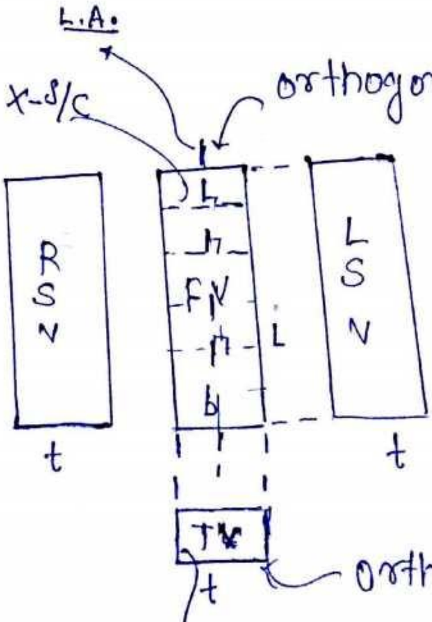
Vertical plane - In front of you.
Projection Plane - In sides of you.

Projection.



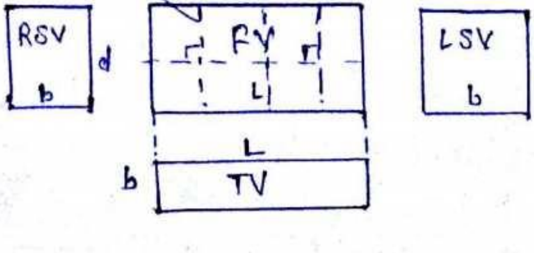
Othographic projection of Verticle member in first angle projection.

orthogonal projection of Object in vertical plane



orthogonal projection in Horizontal plane

Othographic Projection of Horizontal member

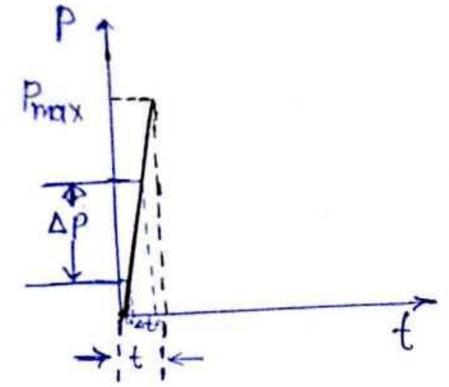


* To Get Cross Section cut w.r.t. longnitude axis

Dynamic load :-

Impact load :- Impact load is a load which are acting for short interval of time.

e.g.- springs use in shock obsorber, IC engine piston and connecting rod during power strok, spur gear thooth at higher pitch line velocities, component used in punching, foregeng and press working oper^{n}s.



stress due to impact load

$\sigma_{Impact} = \sigma_{Static} \times I.F.$ (Impact Factor (IF))

$\delta_{Impact} = \delta_{Static} \times I.F.$

$\Rightarrow \sigma_{Static}$ and δ_{Static} are obtained by using som eq^{n}s

I.F. = Impact Factor = $\boxed{IF = 1 + \sqrt{1 + \frac{2h}{\delta_{static}}}} \geq 2$

* if h is not given assume freely falleing body $h = v^2/2g$

$\boxed{\sigma_{Impact} \geq 2\,\sigma_{Static}}$

$\boxed{\delta_{Impact} \geq 2\,\delta_{Static}}$

$h \to 0$; $IF \simeq 2 \Rightarrow$ Impact load is known as Suddenly applied load (SAL) → Instenous load.

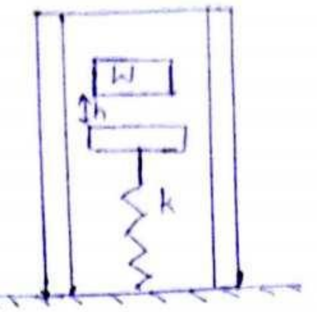
$\boxed{\sigma_{SAL} = 2\,\sigma_{Static}}$

$\boxed{\delta_{SAL} = 2\,\delta_{Static}}$

e.g:-

Find $\delta_{impact} = ?$, $k = 500\ N/mm$

$W = 2\ kN$, $h = 2\ mm$



Solⁿ:-

$$\delta_{static} = \frac{W}{k} = \frac{2000}{500} = 4\ mm$$

$$I.f. = 1 + \sqrt{1 + \frac{2h}{\delta_{st.}}} = 2.414\ mm$$

$$\delta_{Impact} = \delta_{st.} \times IF = 4 \times 2.414 = \underline{9.65\ mm}$$

Fatigue load / Cyclic / variable load :-

Fatigue loads are those loads whose mag. & dirⁿ changes w.r.t. time and they are repeatedly applied

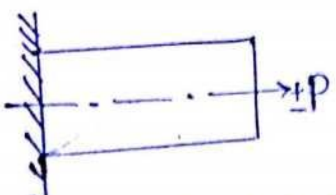


Fig:- Completely reversed fatigue loads
(i.e. only dirⁿ of load change

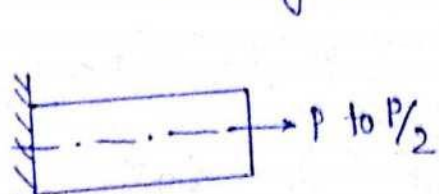


Fig: Fluctuating fatigue loads
(ie. only mag. of the load change)

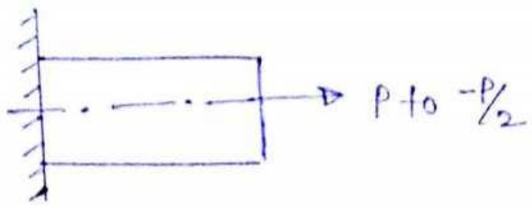


Fig.- Alternating Fatigue load
(i.e.- both dirⁿ & mag. of the load changes)

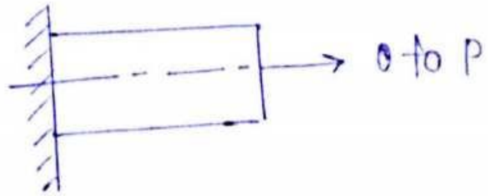
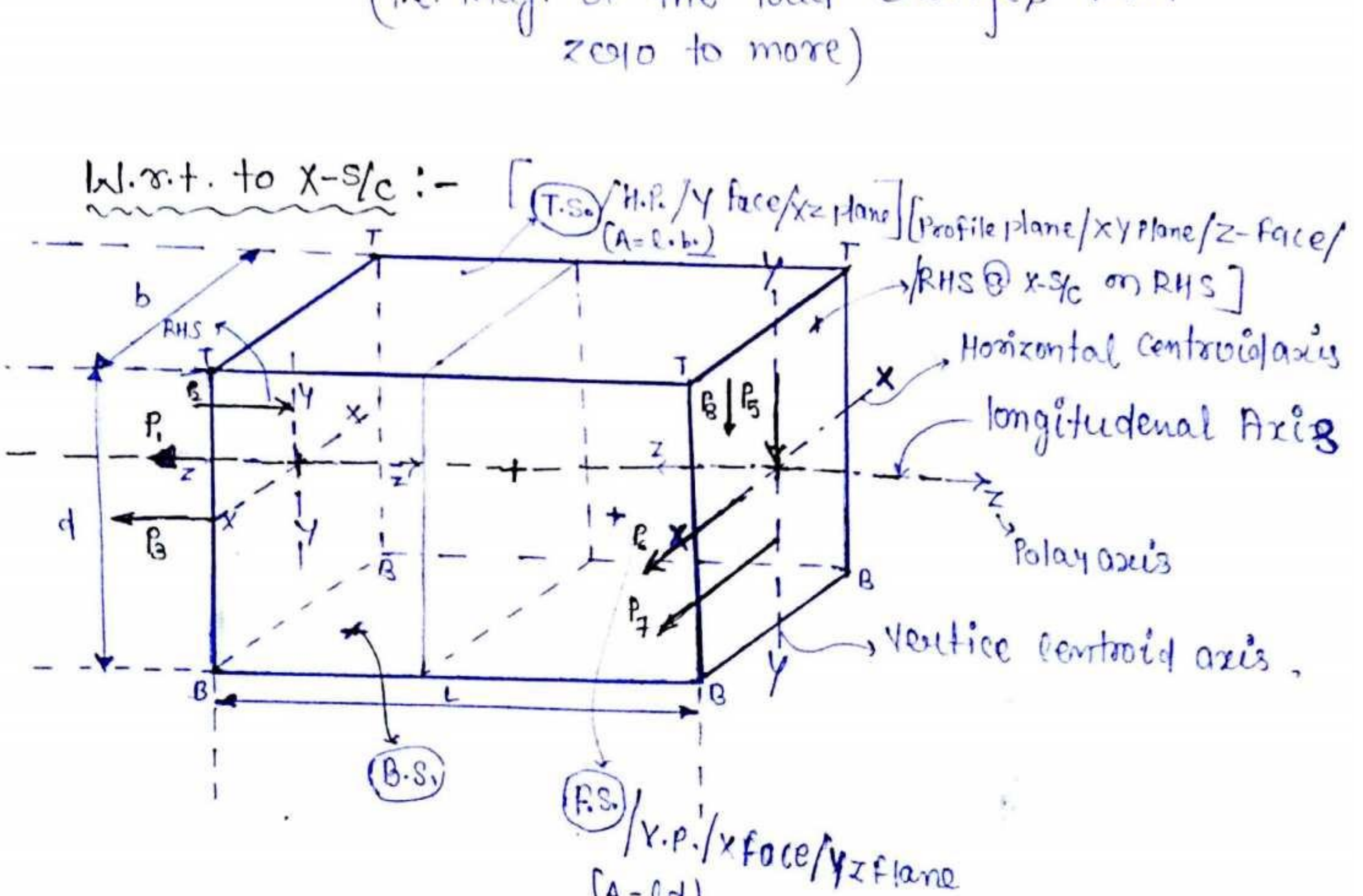


Fig:- Repeated Fatigue load changes
(i.e. mag. of the load changes from zero to more)

W.r.t. to X-S/c :-



longitudenal axis - for a member (solid)
Polar axis - for each X-S/c.

* X-X, Y-Y, Z-Z are the centroidal axis which are mutually ⊥r to each other.

* X–X & Y–Y are the C.A. which are mutually ⊥r in the plane X–S/c.

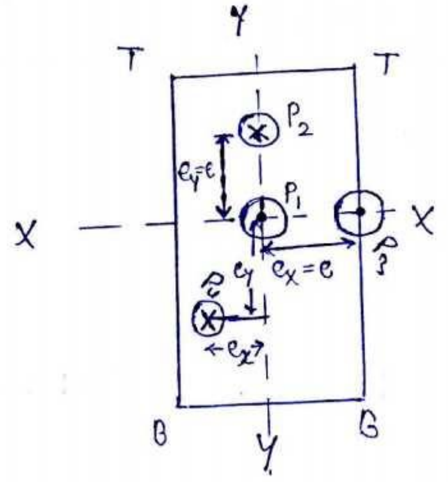
* X–X is a horizontal C.A. [i.e. H.C.A.]

* Y–Y is a vertical C.A. [i.e. V.C.A]

* Z–Z is a polar axis of X–S/c [i.e. C.A. which is ⊥r to plane of X–S/c]

* L.A. of a member is an imaginary line which is joining the centroids of every X–S/c & coincides with polar axis of every X–S/c.

Cross Section of left side (L.S.V.)



P_1 – axial tensile load

P_2 – eccentric comp. load ($e_y=e$)

P_3 – eccentric axial tensile load ($e_x=e$)

P_4 – eccentric axial comp. load (e_x, e_y)

e – ⊥r distance b/w line of action of ecc. load & centroid of the X–S/c

P

⊙ – load is acting ⊥r & away from that plane

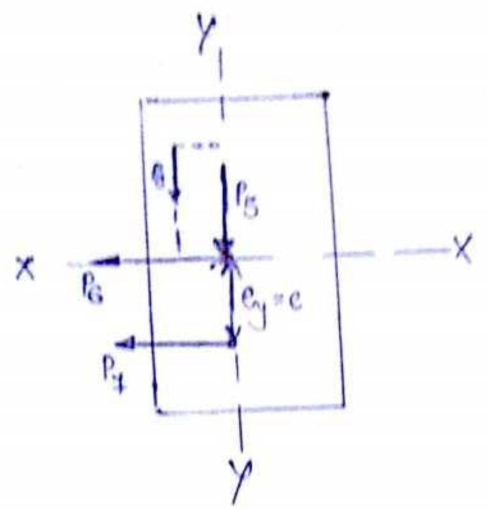
⊗ – load is acting ⊥r & towards that plane

P_5 – ∥ of X–S/c and passes to centroid {transverse shear load (vertical)

P_6 – transverse shear load (Horizontal)

P_7 – Ecc. transverse shear load

P_8 – ∥ to X–S/c but away from centroid



P_5 - V. T. S. L

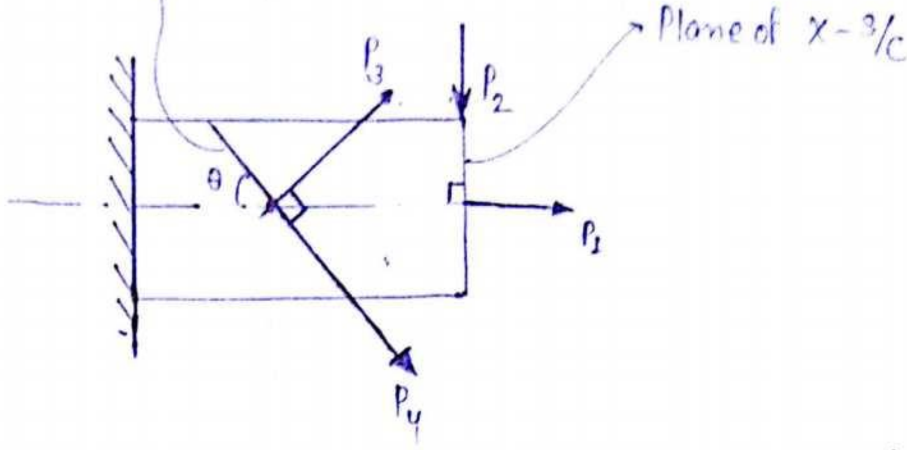
P_6 - H. T. S. L

P_7 - ecc. H. T. S. L

P_8 - Ecc. V. T. S. L.

Load	X-S/c of Member	L.A. of X-S/c	centroid axis of X-S/c
Axial load	⊥r to X-S/c & passes through centroid of the X-S/c	Along L.A.	along polar axis
E.A.L.	⊥r to X-S/c & doesn't passes through the centroid of the X-S/c	ll el to L.A	ll el to P.A.
T.S.L.	ll el to X-S/c & passes through the centroid of the X-S/c	⊥r to L.A. & Intersects L.A.	⊥r to P.A. & intersects P.A.
E.T.S.L	ll el to X-S/c & doesn't passes through centroid	⊥r & doesn't inter-sects L.A.	⊥r & doesn't intersects P.A.
Bending	⊥ to X-S/c	along L.A. of Member	about either HCA or VCA
Twisting	ll to X-S/c	⊥ to L.A. of Member	Polar axis

Oblique Plane (oblique plane Gives true shape of a plane)



Normal load — P_1 (Axial tensile load (ATL)) — P_3

Shear load — P_2 (TSL) — P_4 (

* Every axial load are normal load but not vice-versa
* Every TSL are shear load but not vice-versa.

Twisting Couple (Torque) / Twisting Moment :–

(a) A couple is said to be twisting couple when plane of the couple is parallel to the plane of cross-section of member (Twisting Couple is a shear load)

(b) A couple is said to be a twisting couple when it is acting on a plane which is perpendicular to L.A. of member,

(c) A Couple said to be twisting couple when it acting about a Centroid axis, which is ⊥er to plane of X-S/c
[Polar axis of the X-S/c]

→ In presence of twisting Couple M.O.I. Should be Considered polar axis that is polar moment of inertia. (P.M.O.I.)

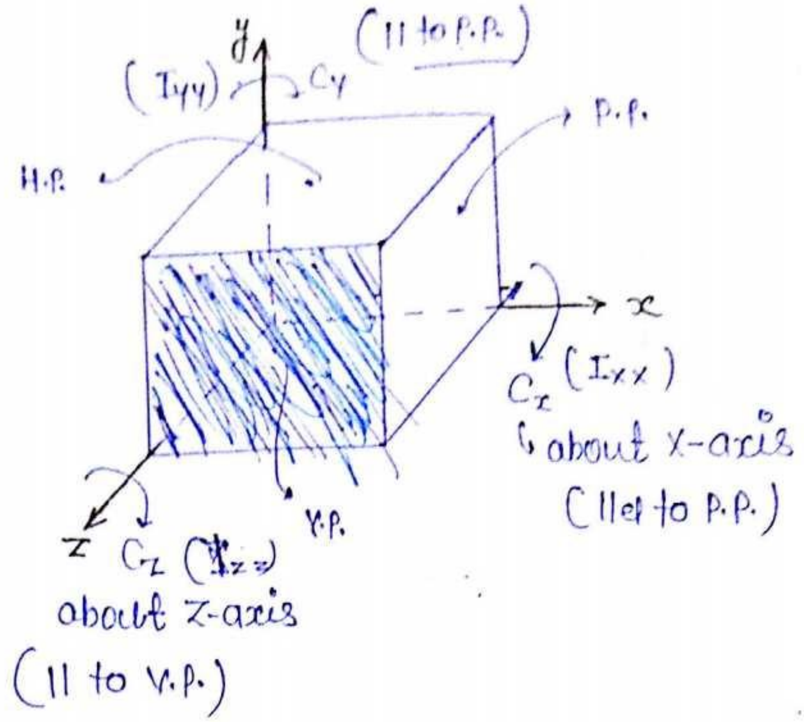
Bending Couple:—

(a) A couple is said to be a bending Couple when it is acting in a plane which is ⊥er plane of cross section of member
[bending couple is a normal load)

(b) A Couple is said to be bending Couple when it acting in a plane ~~with~~ along the L.A. of member.

(c) A couple is said to be a bending couple when it is acting about a centroidal axis which is in the plane of X-S/c [that is about a vertical or Horizonal centroidal axis]

→ In presence of bending Couple M.O.I. Should be Consider about either H.C.A. or V.C.A.
(Horizontal Centroid Axis)



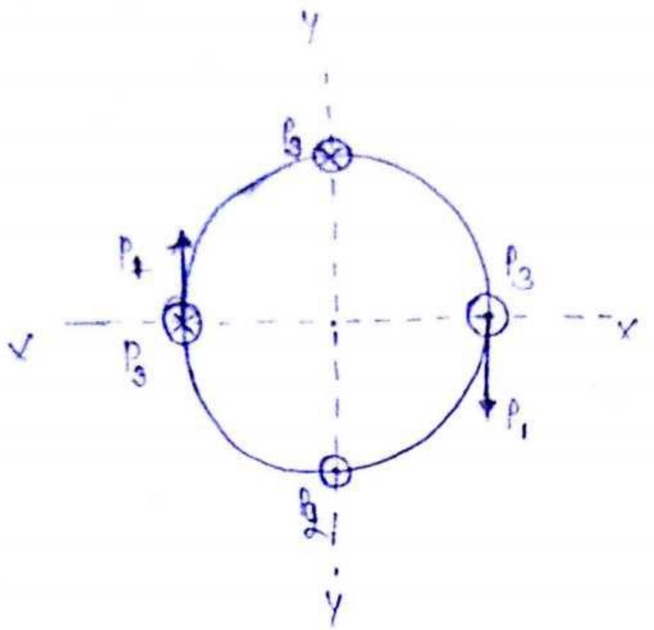
Horizontal Member $\Rightarrow C_x =$ T.C. (I_{xx})

Vertical member $\Rightarrow C_y =$ T.C. (I_{yy})

$\perp$ to plane of board $\Rightarrow C_Z =$ T.C. (I_{zz})

Plane of Couple → / Member ↓	H.P. (C_y)	V.P. (C_z)	P.P. (C_x)
Horizontal Member (X-S/C is in PP)	B.C.	B.C.	T.C.
Vertical Member (X-S/C) in H.P)	T.C.	B.C.	B.C.
$\perp$r to plane of board (X-S/c is in V.P)	B.C.	T.C.	B.C.

e.g.



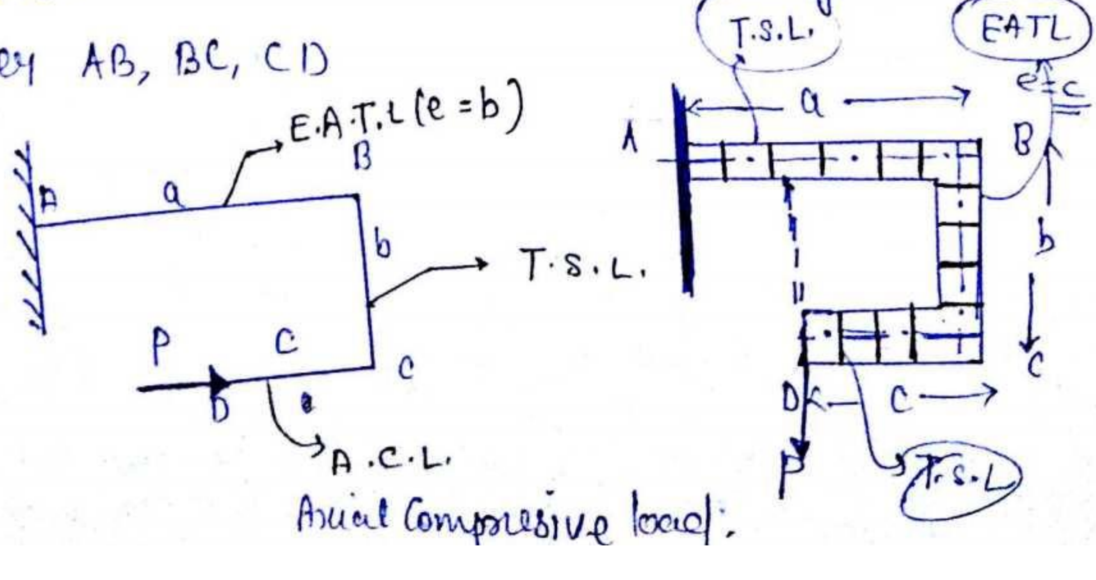
$$P_3 \text{ \& } P_3 \Rightarrow C_3 = C_y = P_3 d \text{ (H.P.)} = M_y \quad (I_{yy})$$

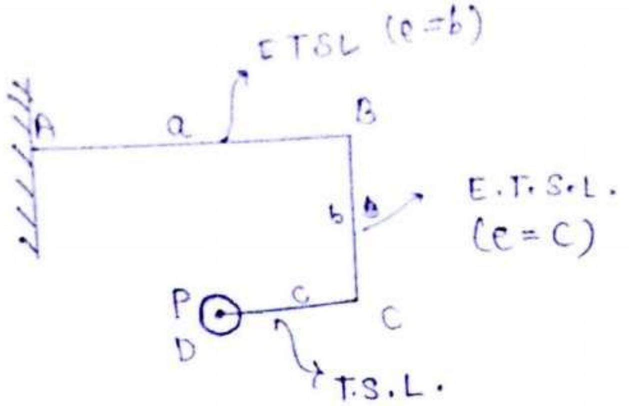
$$P_2 \text{ \& } P_2 \Rightarrow C_2 = C_x = P_2 d \text{ (P.P.)} = M_x \quad (I_{xx})$$

$$P_1 \text{ \& } P_1 \Rightarrow C_1 = C_z = P_1 d \text{ (V.P)} = T \quad (I_{zz})$$

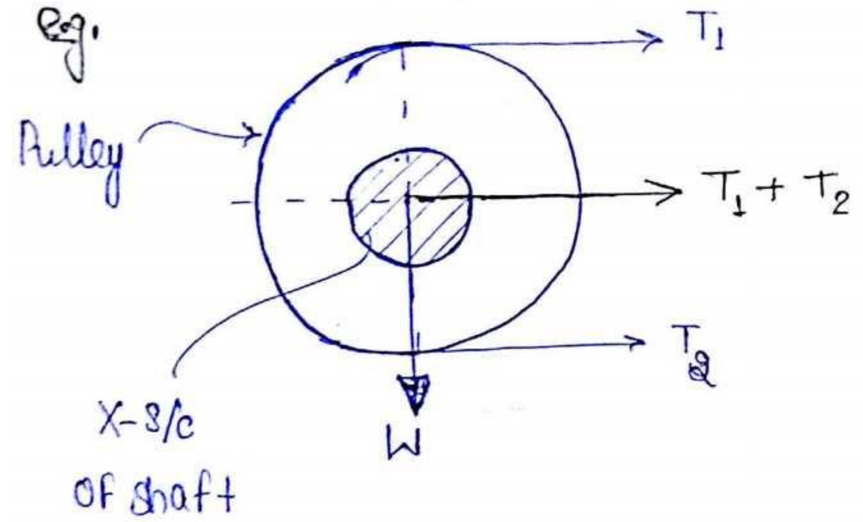
A member is subject to both B.C. & T.C. when plane of Couple is in oblique plane inclined to the L.A. of member.

Ques For the structural Member ABCD determine type of load of member AB, BC, CD





eg.



w.r.t. X-S/c of shaft,

Wt of pulley – VTSL

$T_1 + T_2$ – HTSL

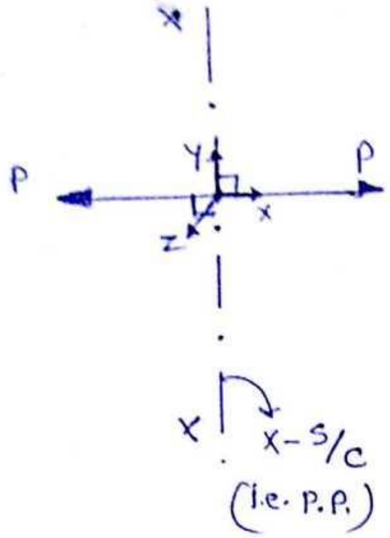
T_1 & T_2 – ETSL (e=R)

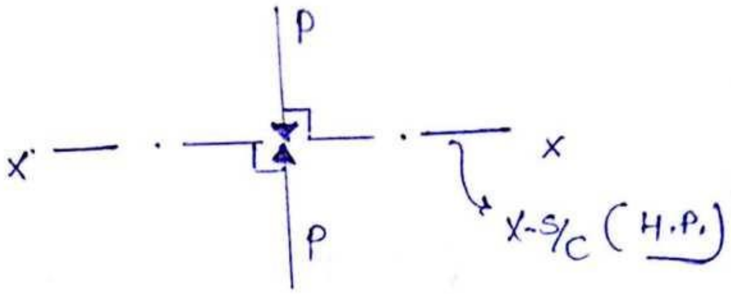
Sign Convention for the loads
Acting on the X-S/c
i.e. - A.L., S.F., B.M. & T.M.

As per method of section (M.o.S.), load acting at any arbitory X-S/c is equal to algebric sum of Corresponding loads either on the left hand side of that X-S/c or on the R.H.S of that X-S/c. (By assuming member is in Horizontal position)

Axial load and its Sign Convention :-

Axial load at any X-S/c of member is equal to algebraic sum of axial loads either on the LHS of X-S/c or R.H.S. of X-S/c





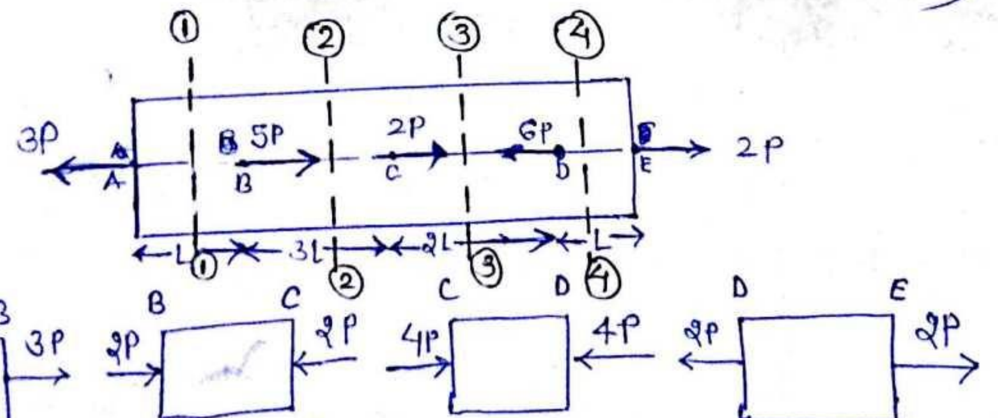
$(A.L.)_{x-x}$ = Axial tensile load = $+P$

$(A.L.)_{y-y}$ Axial Compressive load = $-P$

Axial tensile loads are considers as positive (that is the axial loads are away from X-S/c)

and axial compressive loads are considers as negative (that is the axial loads are towards X-S/c)

e.g.

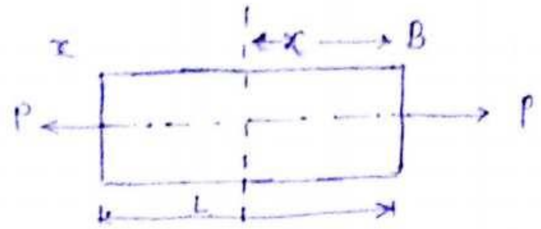


~~AB: [0 to L]~~ ~~BC: [x=L to 4L]~~

~~P_{1-1} = +3P(T) L.H.S~~ ~~P_{2-2} = -5P+3P = -2P (Comp)~~

~~= 5P+2P-6P+2P~~

#



BA : $[x = 0 \text{ to } L]$

$$(A.L.)_{X-X} = +P = \text{Constant}$$

$$(S.F.)_{X-X} = (B.M.)_{X-X} = (T.M.)_{X-X} = 0$$

Hence, bar is under Pure Axial load

$$\boxed{\sigma_A = P/A} \qquad \boxed{\delta_L = \frac{PL}{AE}}$$

* persimatic bar
* Pure axial load
* Same material

Sol^n

AB :- $[x = 0 \text{ to } L]$

$P_{1-1} = +3P\ (T)(L.H.S)$

$= 5P + 2P - 6P + 2P$

$= 3P\ (R.H.S.)$

BC :- $[x = L \text{ to } 4L]$

$P_{2-2} = -5P + 3P = -2P$

@ $2P$ (comp.) (L.H.S)

$= 2P - 6P + 2P = -2P$ (R.H.S)

CD : $[x = L \text{ to } 3L]$

$P_{3-3} = -6P + 2P$

$= -4P$ @ $4P$ (comp.) [R.H.S.]

DE : $[x = 0 \text{ to } L]$

$P_{4-4} = 2P$ (tensile) R.H.S

max axial load acting on the body $= 4P$ [comp]

Max tensile load $= 3P$

$P_A = 3P$

$P_B = 3P(T)$ [larger of P_{AB} & P_{BC}]

$P_C = 4P$ [comp.] larger of {P_{BC} & P_{CD}}

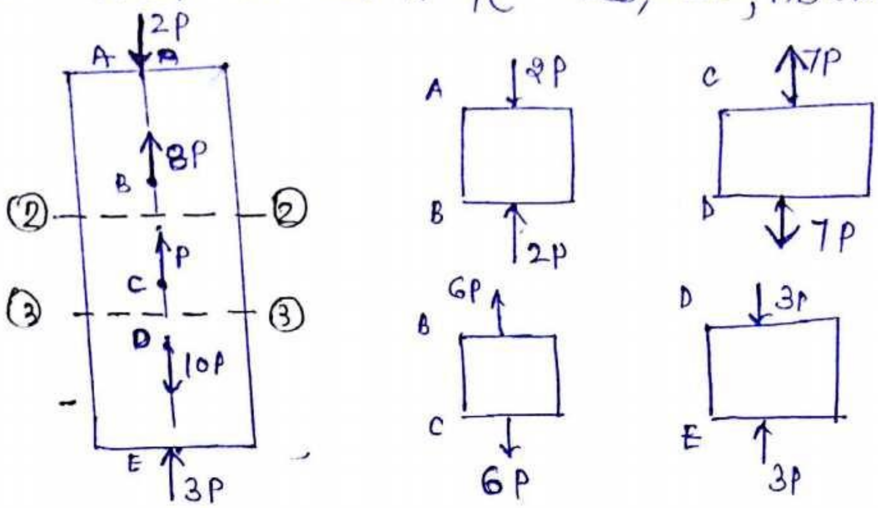
$P_D = 4P$ [comp] larger of {P_{CD} & P_{DE}}

$P_E = 2P$ (tensile)

Que. For the vertical bar as shown in fig determine

i) maximum axial tensile load & max. axial comp. load acting on bar.

ii) axial load at ~~CD~~ X-S/c ~~AB, CD~~, ABCDE.



Sol^n

$P_{AB} = P_A = 2P$ (Comp)

$P_{BC} = P_{2-2} = 8P - 2P = 6P$ (Tensile) Above

$P_{CD} = P_{3-3} = 6P + P = 7P$ (tensile) Above

$= 10P - 3P = 7P$ (tensile) Below.

$P_{DE} = P_E = -3P = 3P$ (Comp) below.

Max. Axial tensile load CD = 7P

max. axial Comp. load DE = 3P

$P_A = 2P$ [C]

$P_B = 6P$ [T]

$P_C = 7P$ [T]

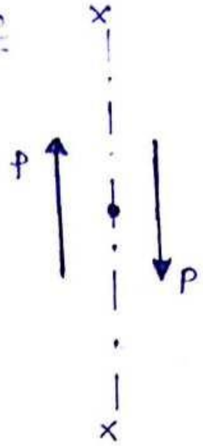
$P_D = 7P$ [T]

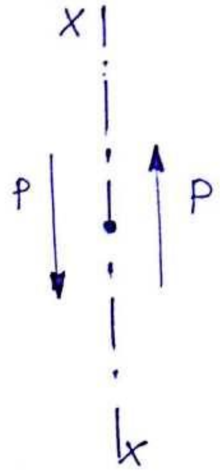
$P_E = 3P$ [C]

Shear force and its sign Convention.

Shear force at any arbitrary X-S/c of the member is equal to algebric sum of shear forces either on the L.H.S. of X-S/c or R.H.S. of X-S/c.

<u>Verticle Plane</u>

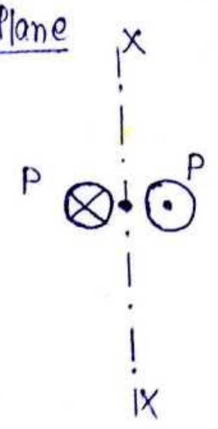


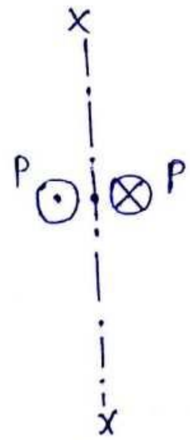


$(S.F.)_{X-X} = +P$ {Cause a couple in CW dirn}

$(S.F.)_{X-X} = -P$ {Cause a couple in ACW dirn}

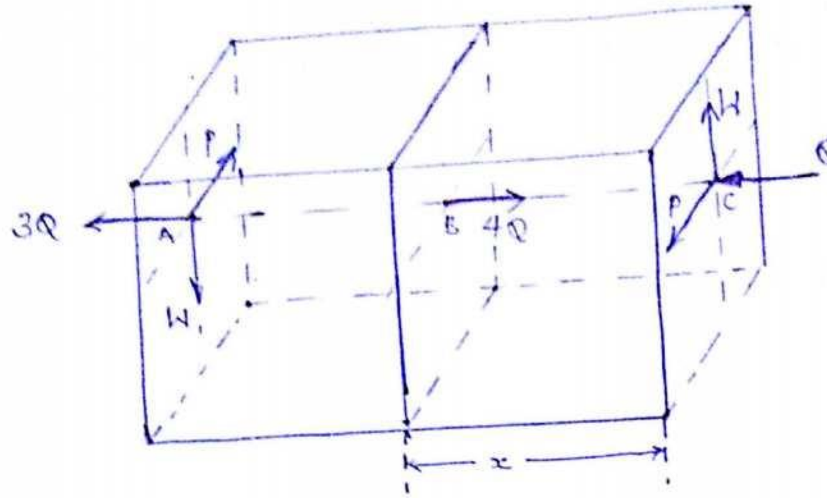
<u>Horizontal Plane</u>





$(S.F.)_{X-X} = +P$ (C.W.)

$(S.F.)_{X-X} = -P$ (A.C.W)



$$V.S.F. = -W$$
$$H.S.F.) = +P$$
$$R.S.F. = \sqrt{(-W)^2+(P)^2}$$

$$(AL)_{AB} = (AL)_A = 3Q\,(T)$$
$$(AL)_{BC} = (AL)_C = -Q = Q\,(C)$$
$$(AL)_B = 3Q\,(T).$$

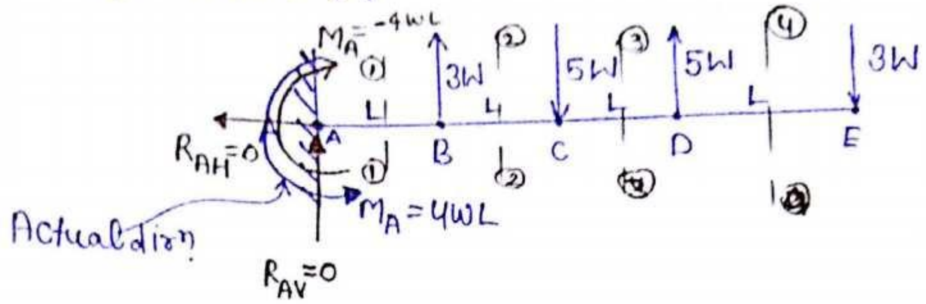
Shear force at any x-S/c of the member is said to be +ve when shear forces on either side of that x-S/c causes a couple in CW dirn and vise-versa.

Shear force sign convertation w.r.t. beams :-

shear force at any x-S/c of beam is said to +ve when it is acting in the upward dirn of the L.H.S. of x-S/c or when it is acting in the downward dirn on the ~~right~~ R.H.S. of that x-S/c and vice-versa.

shear force at any x-S/c of a beam is said to be -ve when it is acting in ~~up~~downward dirn on the LHS of x-S/c or when it is acting in upward dirn on the LHS of that x-s/c

Ques For the cantilever beam as shown in fig determine max. shear force and shear force at X-S/c ABCDE.



Sol^n

Assuming CW → −Ve

$$\Sigma M_A = 0 \Rightarrow -M_A + 3W(L) - 5W(2L) + 5W(3L) - 3W(4L) = 0$$

$$\Rightarrow M_A = -4WL \text{ @ } 4WL(\circlearrowleft)$$

$$(SF)_{AB} = (S.F)_{1-1} = +R_{AV} = 0 \quad [L.H.S.]$$

$$= -3W + 5W - 5W + 3W = 0 \quad (RHS)$$

$$(SF)_{BC} = (SF)_{2-2} = R_{AV} + 3W = 3W \,(LHS)$$

$$(S.F.)_{CD} = (SF)_{3-3} = -5W + 3W = -2W \quad (RHS)$$

$$(S.F.)_{DE} = (SF)_{4-4} = 3W \,(RHS)$$

max. SF = 3W (BC & DE)

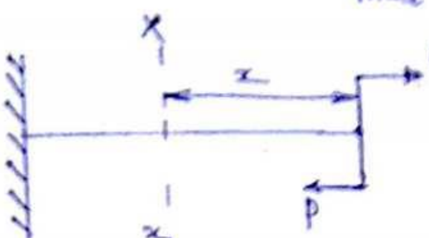
$(SF)_A = 0$ $\qquad (SF)_D = 3W$

$(SF)_B = 3W$ $\qquad (SF)_E = 3W$

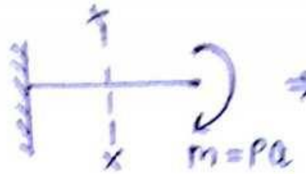
$(SF)_C = 3W$

Bending Moment & its Sign Convention! -

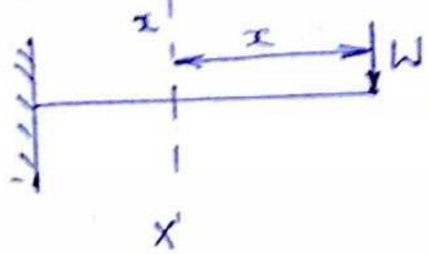
Bending moment at any x-S/c of member is equal to algebric sum of couples and moments (which are along the longitudinal axis of member) either on LHS of that x-S/c or RHS of that x-S/c.



$\equiv$



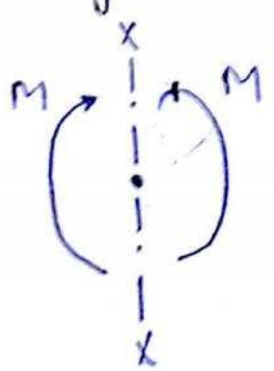
$\Rightarrow BM = -Pa$ (Const.)



$\equiv (BM)_{x-x} = -Wx$ (variable)

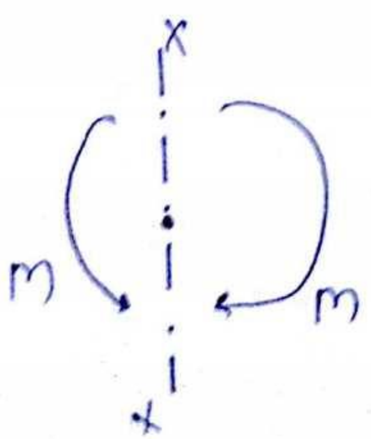
CW = -ve

B.M. sign convention



$(B.m.)_{x-x} = +M$ (i.e. Sagging bending)

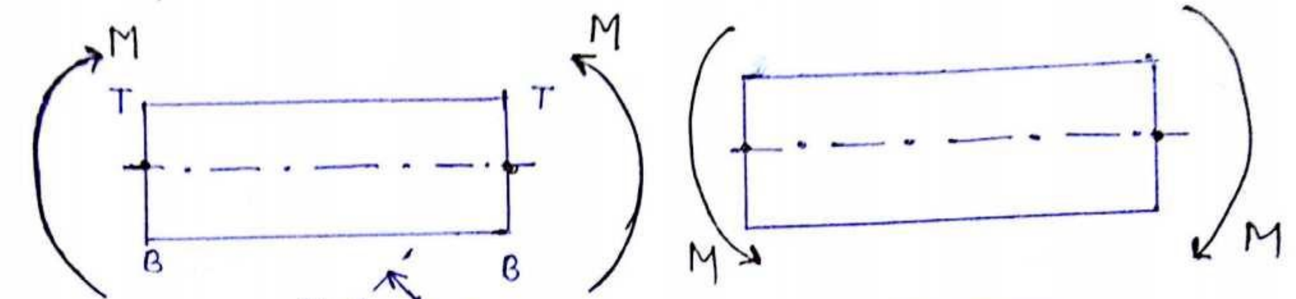
Sagging bending - Top Surface get Contracted

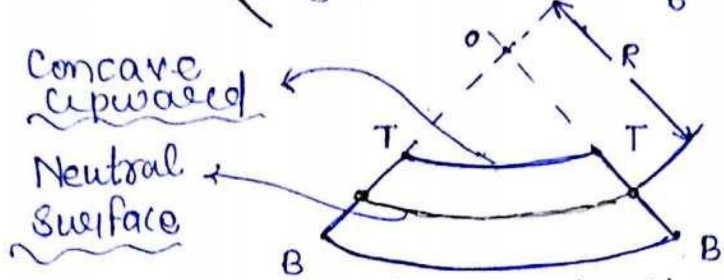


$(B.M.)_{x-x} = -M$ (i.e. - Hagging bending)

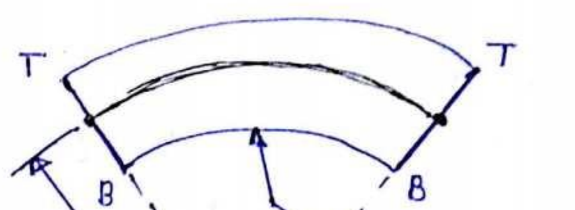
Hagging bending - Bottom Surface get Contracted.

Bending moment at a given x-s/c is said to be +ve when it is acting in clock-wise dir^n on the L.H.S. of that x-s/c or when it is acting ACW dir^n on the R.H.S. of that x-s/c and vice-versa.





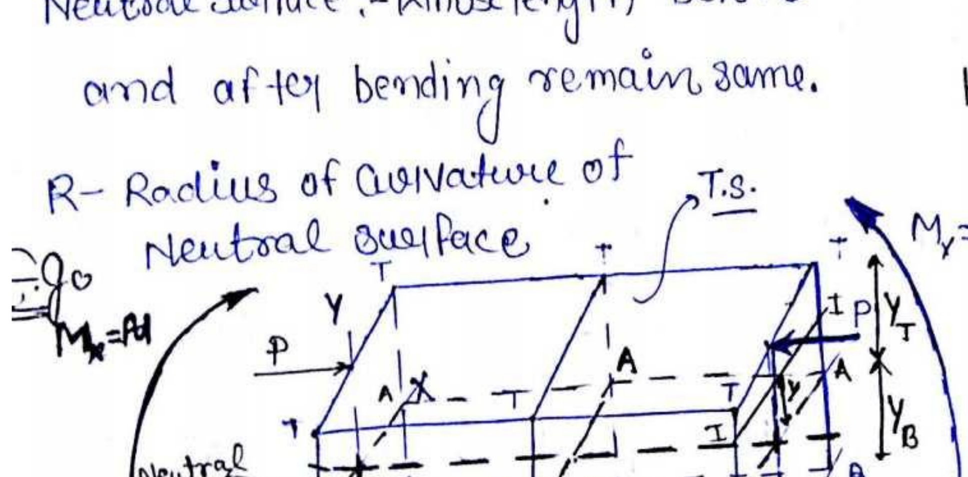
Sagging or +ve bending



Hogging / -ve bending

Neutral Surface :- Whose length before and after bending remain same.

R- Radius of curvature of Neutral surface

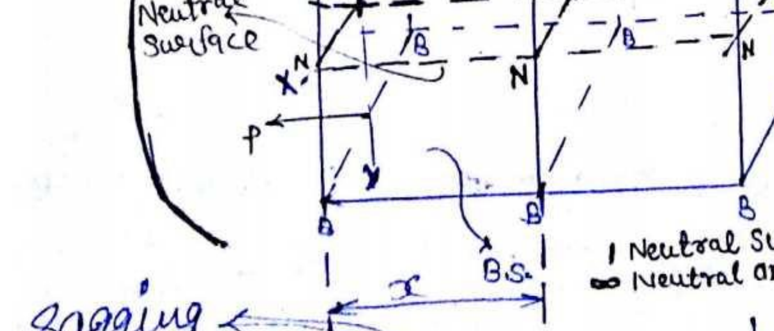


$(A.L.)_{x-x} = 0$

$(S.F.)_{x-x} = 0$

$(B.M.)_{x-x} = Pd = \text{Const.}$ (sagging)

$(T.M.)_{x-x} = 0$



* Member under Pure bending

Bending equation

$$\boxed{\frac{M}{I_{N.A.}} = \frac{\sigma_b}{Y} = \frac{E}{R}}$$ Assuming Pure bending

Neutral surface :— Neutral surface is surface whose length remain constant before and after bending.

Neutral axis :- Neutral axis is the line of intersection of neutral surface with x-S/c of the member. neutral axis coincide with one of the centroidal axis in the plan of x-S/c (that is either H.Z. or V.T centroidal axis) and about which bending couple is acting this is possible in the absence of axial load on the x-S/c.

Y = Distance of a fiber on a x-S/c from its neutral axis

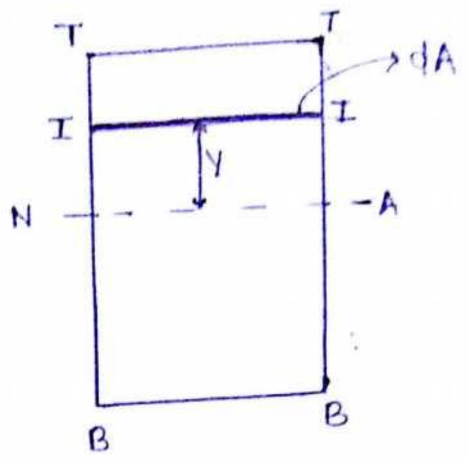
R = Radius of curvature of neutral surface.

σ_b = Bending stress of a fiber located at a distance Y

M = Bending moment acting on the x-S/c of member.

$\boxed{I_{N.A.} = \int Y^2 dA}$ = Second moment of area of the x-S/c about it N.A.

= Area MOI of the x-S/c about it N.A.



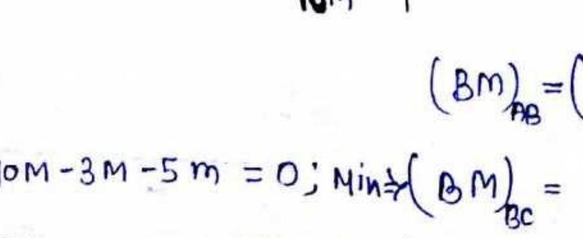
$$\frac{M}{I_{N.A.}} = \frac{\sigma_b}{Y} = \frac{E}{R}$$

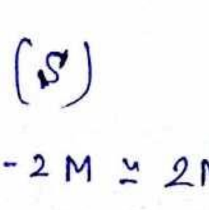
$\boxed{\sigma_b = \dfrac{MY}{I_{N.A.}}}$; σ_b varies linearly.

$\boxed{(\sigma_b)_{max} = \pm \dfrac{M Y_{max}}{I_{N.A}} = \dfrac{M}{Z_{N.A.}}}$ where $Z_{N.A.} = \dfrac{I_{N.A.}}{Y_{max.}}$

Section modulus of X-S/C about its N. A.

Ques. For the cantilever beam as shown in fig. Determine maximum sagging and hogging bending moment.





+Ve

$\Sigma M_A = 0$

$M_A - 5M + 10M - 3M - 5M = 0$; Min $\Rightarrow$ $(BM)_{BC} = 3M - 5M = -2M \simeq 2M(H)$ LHS

$M_A = +3M$

$(BM)_{AB} = (BM)_A = 3M\,(S)$

Max. $\Rightarrow$ $(BM)_{CD} = 3M + 5M = 8M(S)$ RHS

$(BM)_{DE} = 5M\,(S)$ RHS

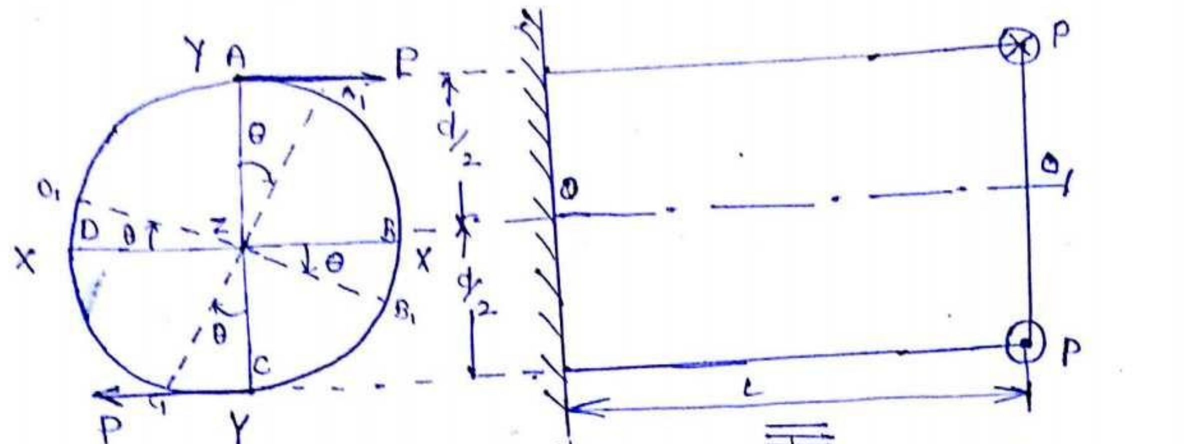
$(BM)_A = 3M(S)$; $(BM)_C = 8M(S)$

$(BM)_B = 3M(S)$ $(BM)_D = 8M(s)$

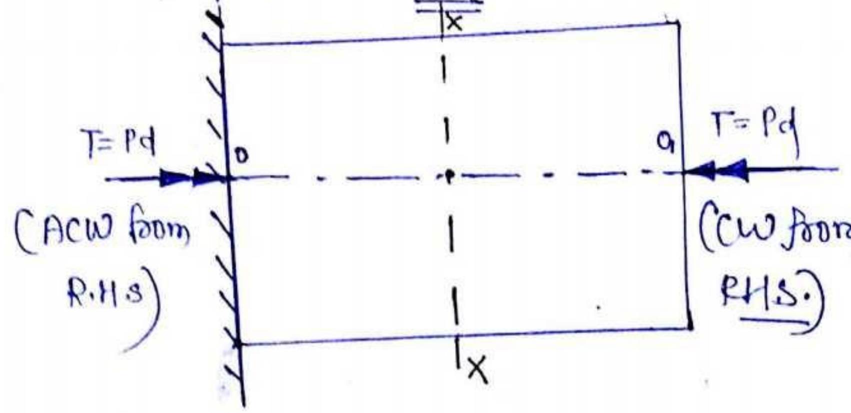
$(BM)_E = 5M(S)$

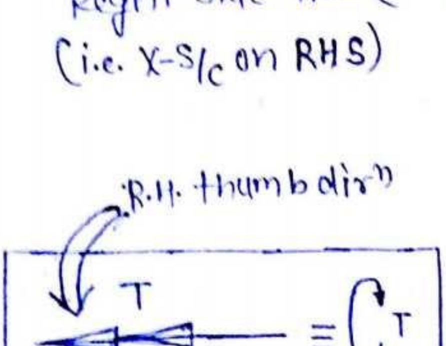
Twisting Moment & It's Sign Convention:

Twisting moment or torque at any x-S/c of the member is equal to algebric sum of couples (that is which are acting ||el to x-S/c) either on the LHS of that x-S/c or on the RHS of that x-S/c.

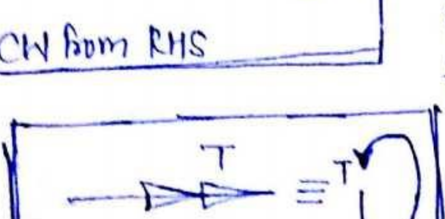


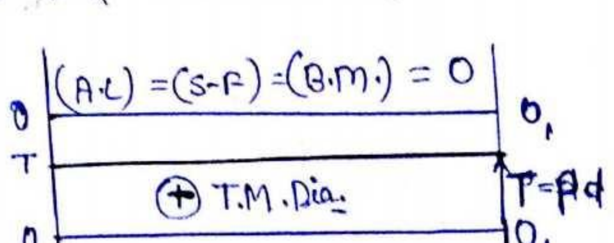
Right Side View (RSV)
(i.e. X-S/c on RHS)





Pure torsion
$$(A.L)_{x-x} = 0 \qquad (B.M.) = 0$$
$$(S.F.)_{x-x} = 0$$
$$(T.M)_{x-x} = \boxed{T = +Pd} = \text{Constant}$$





⇒ X-S/c of the member are twisted in CW dirn by observing RHS

$\boxed{\theta = \text{Angle of twist of a X-S/c at the free end}}$

Torsion equation for circular X-S/C

$$\boxed{\frac{T}{J} = \frac{\tau_{max}}{R \text{ (or) } R_o} = \frac{G\theta}{L}}$$

where

τ_{max} = max. torsion stress on the X-S/C

$$\boxed{\tau_{max} = \frac{T\,R \text{ (or) } R_o}{J}} = \frac{T}{Z_p}$$

where Z_p - Polar Section Modular of X-S/C.

$$\boxed{I_{zz} = J = \int r^2 dA}$$ = second moment of area of X-S/C about its polar axis.

= Polar Moment of Inertia of X-S/C.

$$I_{zz} = J = I_{xx} + I_{yy}$$

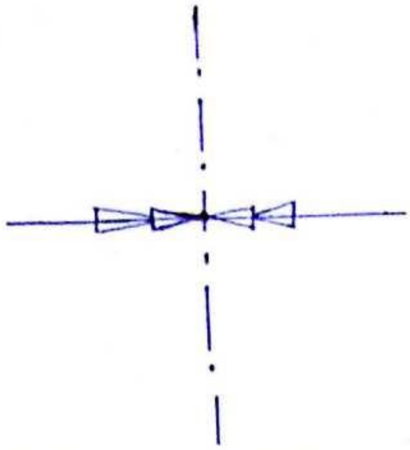
$$J = 2\left[I_{xx} \text{ or } I_{yy}\right] \qquad \boxed{I_{xx} = I_{yy} = \frac{\pi D^4}{64}}$$

$$\boxed{J = \frac{\pi D^4}{32}} \Rightarrow \text{Solid circular}$$

$$\boxed{Z_p = \frac{J}{R} = \frac{\pi d^3}{16}} \qquad \boxed{\tau_{max} = \frac{T}{Z_p} = \frac{16T}{\pi d^3}}$$

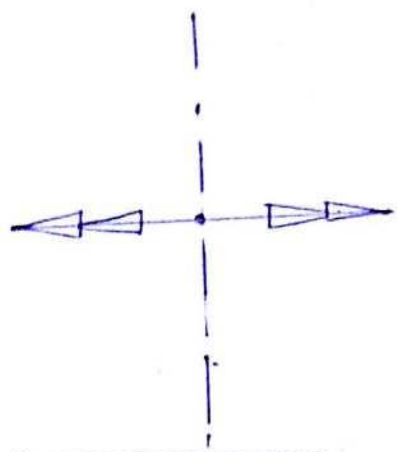
$$\frac{T}{J} = \frac{G\theta}{L} \Rightarrow \boxed{\theta = \frac{TL}{GJ} \text{ or } \frac{32\,TL}{\pi G d^4}}$$

T.M. (or) Torque Sign Convention :—



$$\boxed{(T.M.)_{X-X} = +T}$$

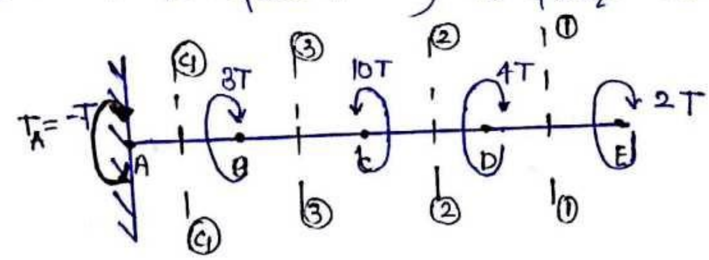
Towards Centroid +ve



$$\boxed{(TM)_{X-X} = -T}$$

Away from Centroid −ve

Ques:- For the bar as shown in Fig determine
i) maximum torque. ii) Torques at diff. Cross-Sections



$$T_{DE} = T_{1-1} = 2T$$

$$T_{CD} = T_{2-2} = 2T + 4T = 6T = T_{max}$$

$$T_{BC} = T_{33} = 6T - 10T = -4T$$

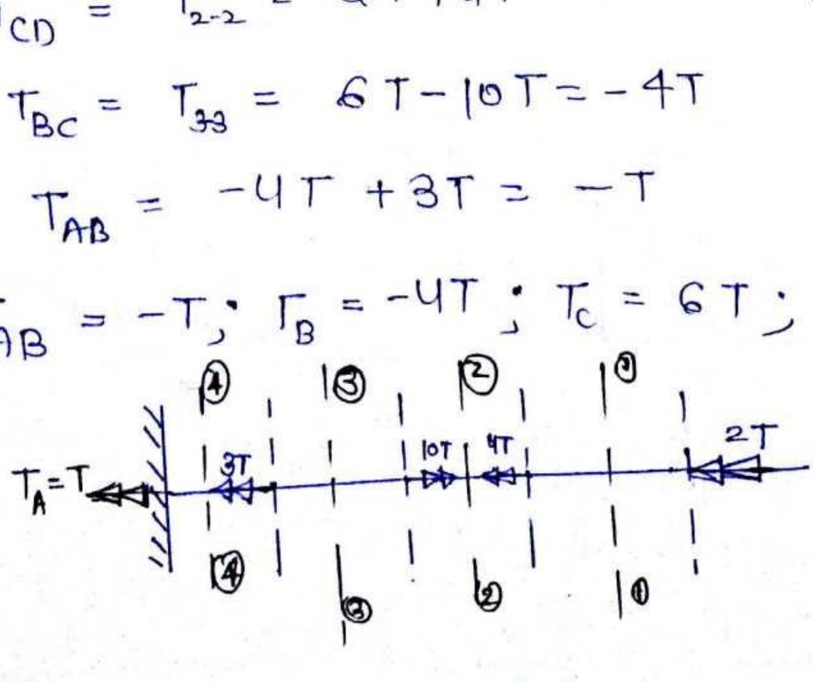
$$T_{AB} = -4T + 3T = -T$$

$$\tau_{max} = \frac{16\, T_{max}}{\pi d^3}$$

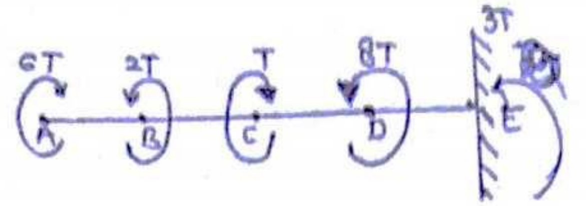
$$\tau_{max} = \frac{16 \times 6T}{\pi d^3}$$

$$\tau_{max} = \frac{96\,T}{\pi d^3}$$

$T_{AB} = -T$; $T_B = -4T$; $T_C = 6T$; $T_D = 6T$



Prob.



Determine max. torque.

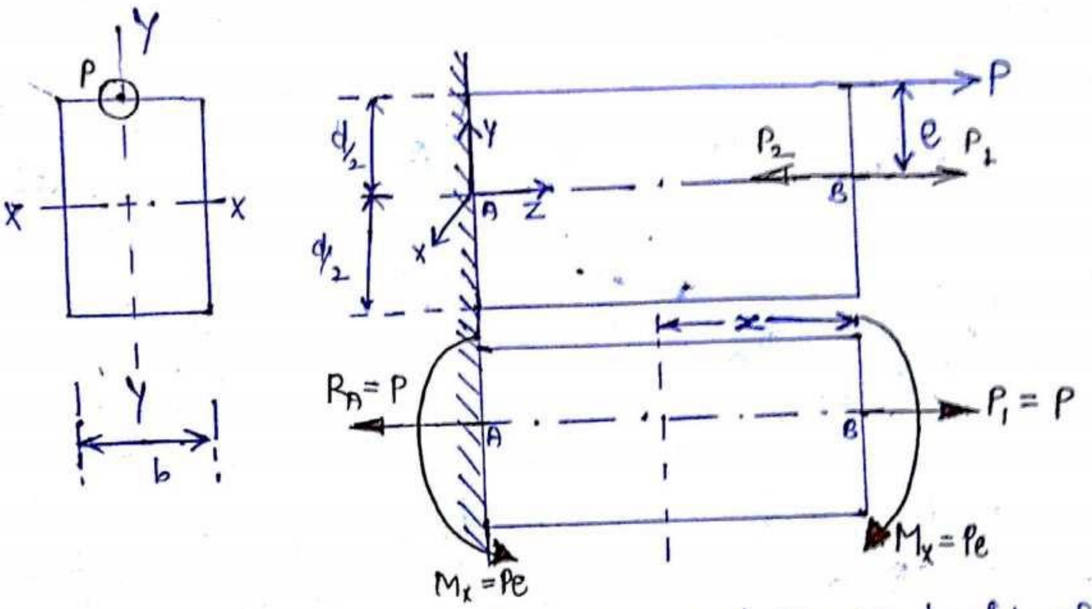
$T_{max} = 6T = T_{AB}$

$T_{AB} = 6T$; $T_{BC} = 6T - 2T = 4T$; $T_{CD} = 7T - 2T = 5T$; $T_{DE} = (1-10)T = -3T$

Eccentric Axial load :-

[i.e. Equivalent loads on the X-S/c]

$$\frac{M}{I_{N.A.}} = \frac{\sigma_b}{Y} = \frac{E}{R}$$



(1) Introduce two dummy force (i.e. P_1 & P_2) at the centroid of that x-s/c in a dirn. ||el to applied load. & ($P_1 = P_2 = P$)

(2) $e = d/2$

(3) $P \,\&\, P_2 \Rightarrow M_x = Pe = P\,d/2$ (it is a Couple)

$P_1 = P \longrightarrow$ Axial load.

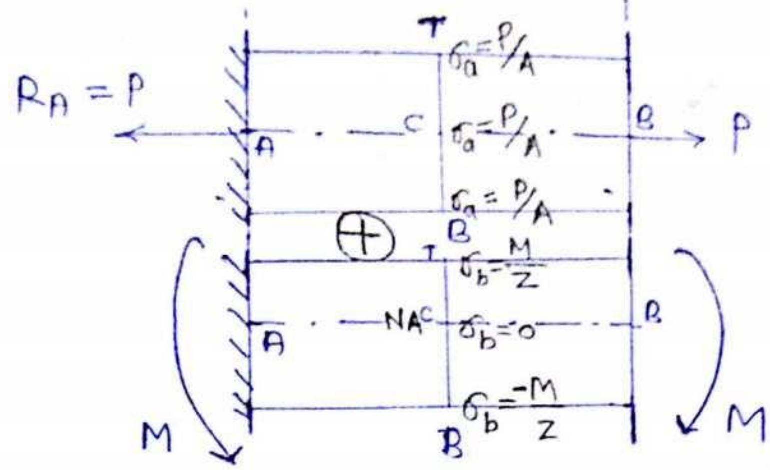
(4) Equivalent load on the X-S/c

$(AL)_{x-x} = +P =$ Const.

$(SF)_{x-x} = 0$

$(BM)_{x-x} = -Pe$ or Pe (Hogging bend) + B.M.D. is a rective.

$(TM)_{x-x} =$ zero.



Both axial load & bending moment acting simultaneously

(B) $$(\sigma_R)_{Top} = \sigma_a + \sigma_b = \frac{P}{A} + \frac{M_x}{Z_{NA}} = \frac{4P}{db}$$

$$(\sigma_R)_{Center} = \sigma_a + \cancel{\sigma_b}^{\,0} = \frac{P}{A} = \frac{P}{bd}$$

$$(\sigma_R)_{Bottom} = \sigma_a - \sigma_b = \frac{P}{A} - \frac{M}{Z_{N.A.}} = \frac{-2P}{bd}$$

(C) max resultant stress $$(\sigma_R)_{top} = \frac{P}{A} + \frac{M_x \text{ or } Pe}{Z_{N.A.} \text{ or } Z_{x-x}}$$

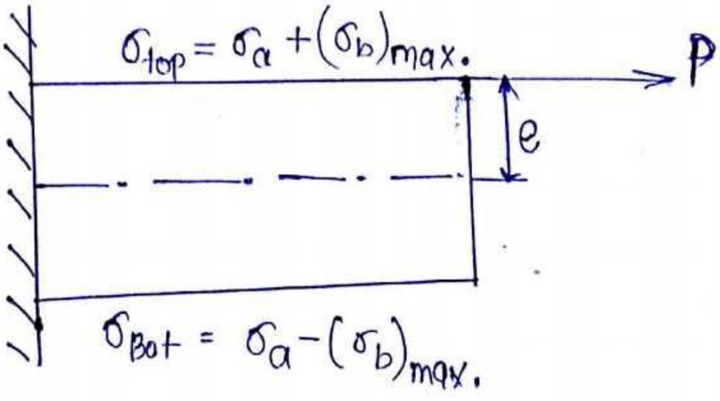
$$(\sigma_R)_{max} = \frac{P}{bd} + \frac{P(d/2)}{bd^2/6}$$

$$(\sigma_R)_{max} = \frac{4P}{bd} \quad \text{(tensile)}$$

$$\Rightarrow \boxed{Z_{x-x} = \frac{I_{x-x}}{y_{max}} = \frac{\frac{1}{12}bd^3}{d/2} = \frac{bd^2}{6}}$$

$$\Rightarrow \boxed{Z_{y-y} = \frac{I_{y-y}}{x_{max}} = \frac{\frac{1}{12}db^3}{b/2} = \frac{db^2}{6}}$$

- For this given ecc. axial loading condition neutral axis coincides with inner fiber below the centroidal axis
- In presence of ecc. axial loading Neutral axis will not coinside with centroid axis (that is either coinside with extreme fiber or inner fiber or lies outside the plane of x-s/c)



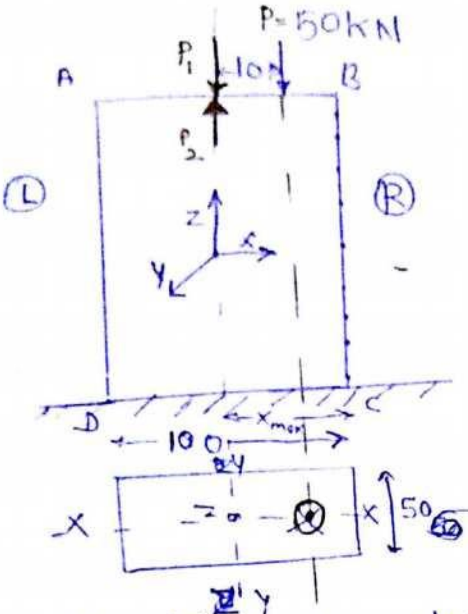
$$(\sigma_R)_{max.} = \sigma_{top} = \sigma_a + (\sigma_b)_{max}\Big]_{x-x}$$

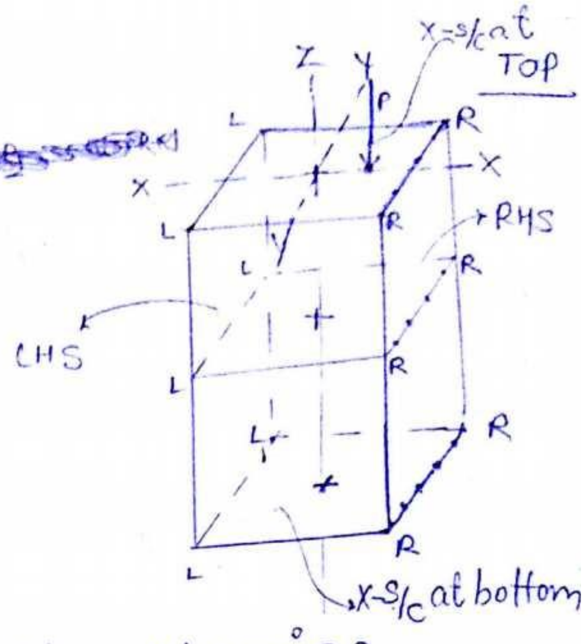
$$= \frac{P}{A} + \frac{M_x \text{ or } Pe}{Z_{x-x}}$$

$$(\sigma_R)_{min} = \sigma_{bottom} = \sigma_a - (\sigma_b)_{max}\Big]_{x-x}$$

$$= P/A - \frac{M_x}{Z_{x-x}}$$

Ques.





For the loading as shown in Fig determine max. stress develope on the x-s/c of bar.

Ans Eccentric Axial Compresive load

LHS

$$\sigma_L = -\sigma_a + (\sigma_b)_{max}$$

RHS

$$\sigma_R = \sigma_{max} = -\sigma_a - \left[(\sigma_b)_{max}\right]_{y-y}$$

$$\sigma_{max} = \sigma_R = -\left[\frac{P}{A} + \frac{M_y \text{ or } Pe}{Z_{y-y}}\right]$$

$$\sigma_{max} = \text{Comp}\left[\frac{50000}{100 \times 50} + \frac{50000 \times 10}{\frac{1}{6} \times 50 \times 100^2}\right] = 10 + 6$$

$$\sigma_{max} = 16 \text{ MPa (Comp.)}$$

$$Z_{y-y} = \frac{I_{y-y}}{x_{max}} = \frac{\frac{1}{12}(50)(100)^3}{50}$$

$$= \frac{100 \times (100)^2}{12}$$

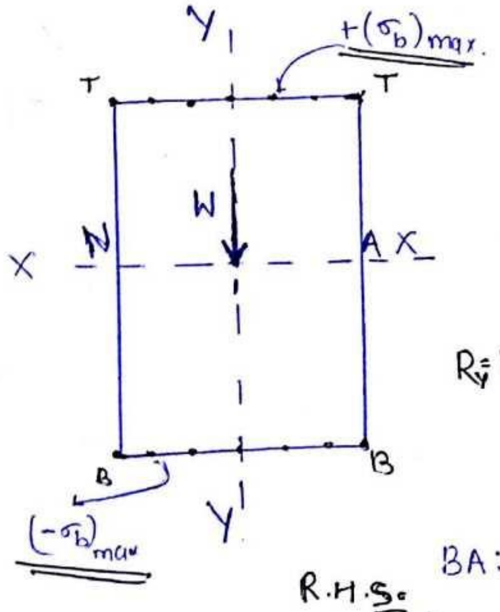
$$Z_{y-y} = \frac{50 \times 100^2}{6}$$

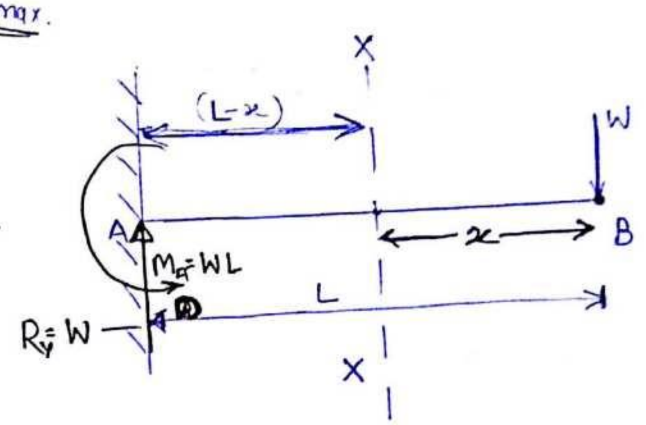
* In presence of eccentric axial load section Modulus should be consider about Horizontal centroidal axis when line of action of ecc. load lies on H.z. centroidal axis and vice-versa.

~~Transfer shear load~~ : –

Tranverse shear load :- (T.S.L)

~~[illegible]~~ Equivlant load on the x-s/c under V. T. S. L. ⇒





LHS

$$(B.M.)_{X-X} = -WL + W(L-x) = -Wx \text{ (LHS)}$$

R.H.S. BA:- $[x = 0 \text{ to } L]$

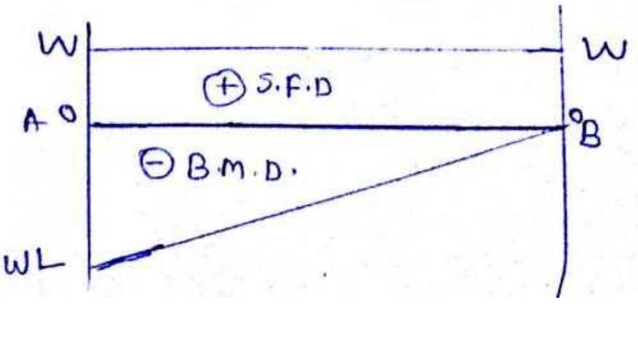
$$(A.L.)_{X-X} = 0 \qquad (B.M.)_{X-X} = -Wx \text{ (variable)}$$

$$(S.F.)_{X-X} = +W \text{ (Constant)} \qquad (T.M.)_{X-X} = 0$$

if $x = 0 \rightarrow (S.F.)_B = W \ ; \ (BM)_B = 0$

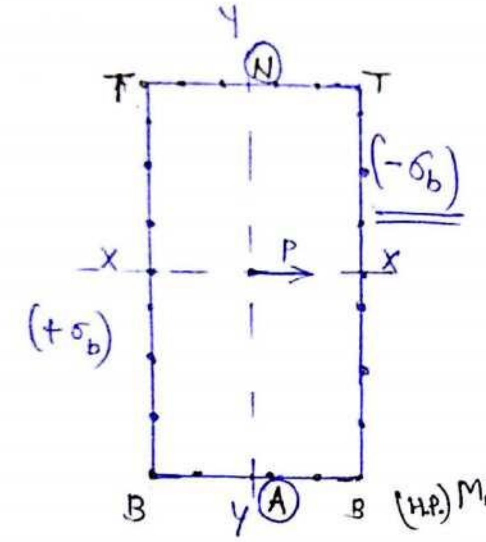
$x = L \rightarrow (SF)_A = W \ ; \ (BM)_B = -WL$

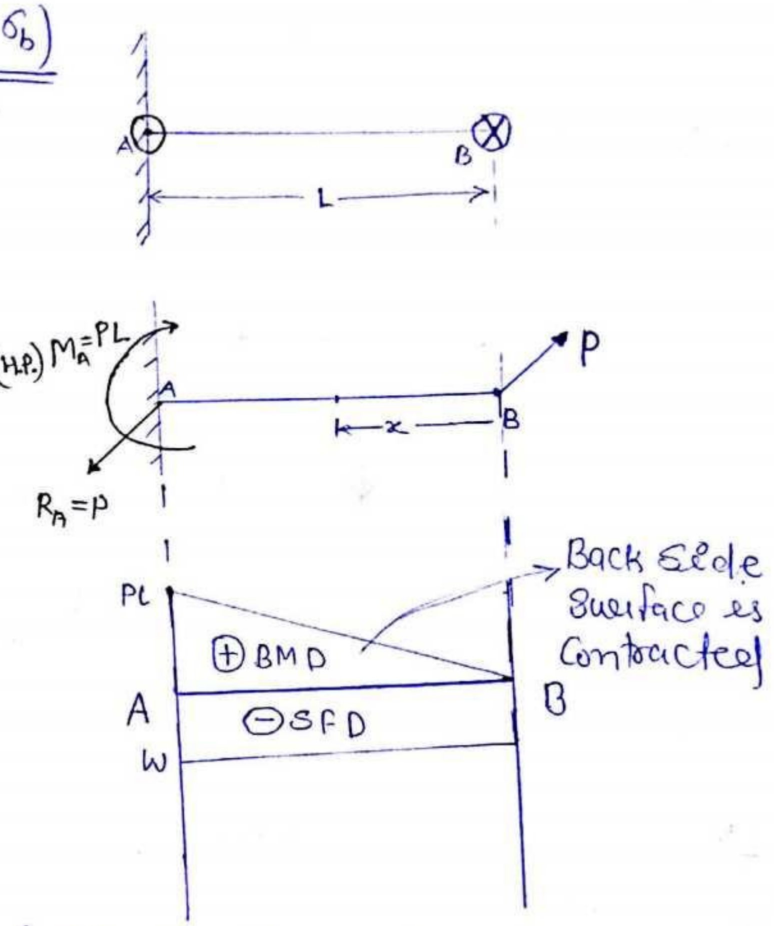
SFD & BMD



Critical X-S/C is a X-S/C where a loads are maximum.

Horizontal transverse shear load (HTSL)



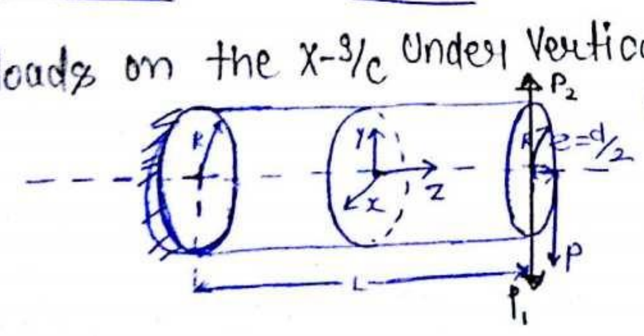


$(S.F.)_{x-x} = -P = \text{Const.} \, (ACW)$

$(B.M.)_{x-x} = Px \, (H.P.)$

* VTSL/HTSL both gives variable Bending Moment & Cont. S.F.

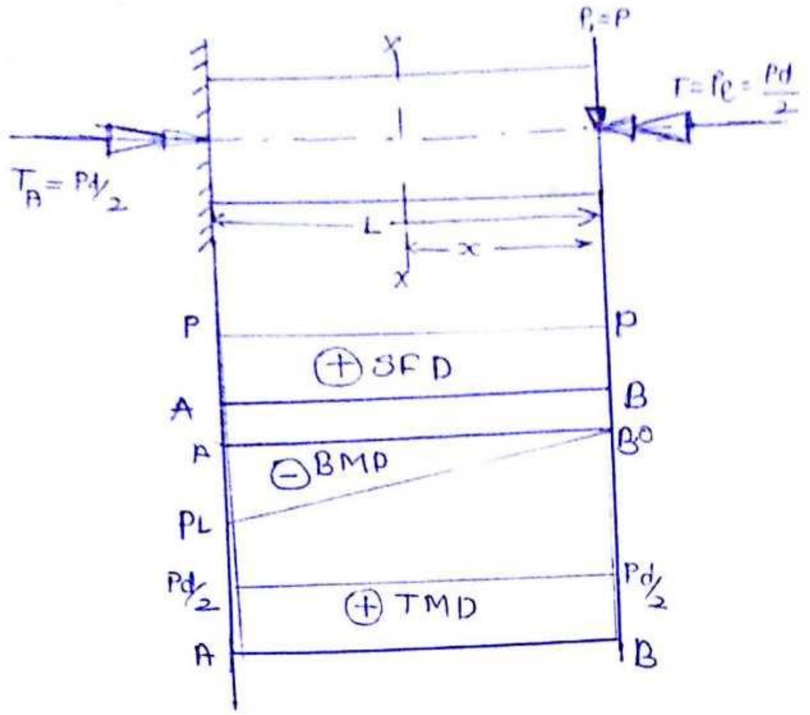
Eq. loads on the X-S/C under Vertical eccentric transverse shear load (VETSL)



$P = P_1 = P_2$

$P \,\&\, P_2 \Rightarrow T.M. = Pe = \frac{Pd}{2}$ (CW from RHS)

$P_1 = P \Rightarrow VTSL$



Equivalent load

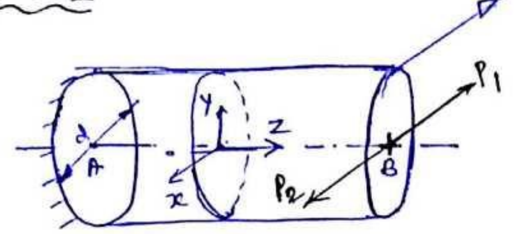
$(AL)_{X-X} = \text{Zero}$

$(SF)_{X-X} = P = \text{Const} \quad (V \cdot P.)$

$(BM)_{X-X} = -Px \, (V \cdot P.)$

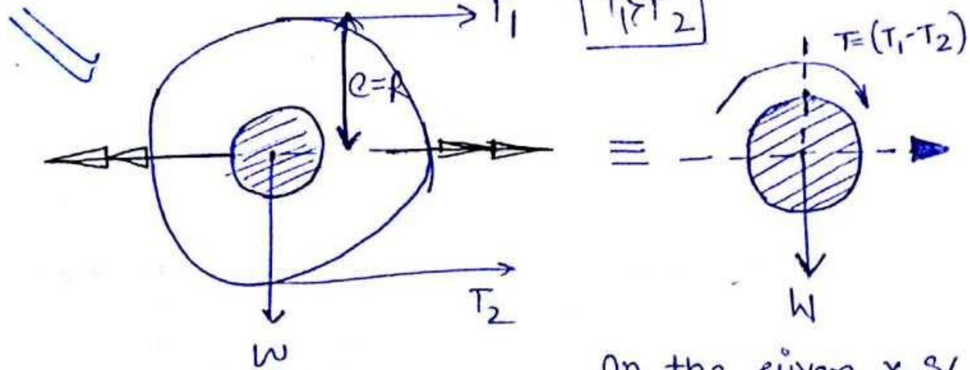
$(TM)_{X-X} = Pe = \frac{Pd}{2} = \text{Const.} \, (PP)$

H.E.T.S.L



TMD = Remain same dir^n

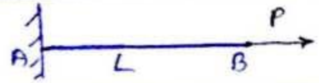
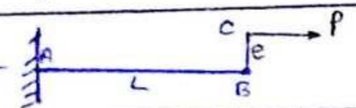
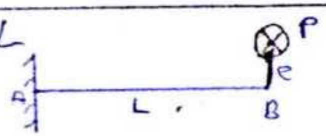
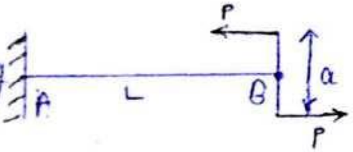
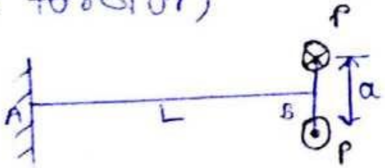
SFD & BMD = dir^n change



$\boxed{T_1 > T_2}$

On the given x-S/c shaft

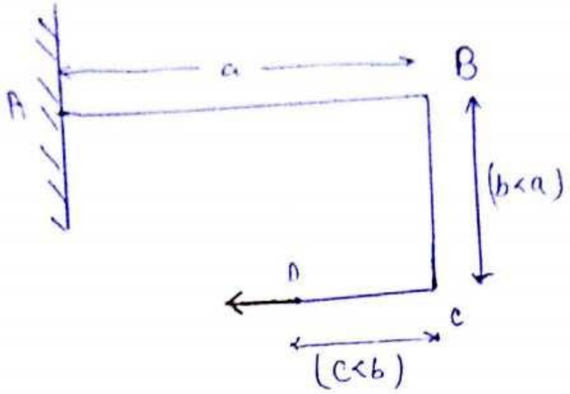
$$\left\{ \begin{array}{l} TM. = (T_1 - T_2) R \\ VSF = W \; (\downarrow) \\ HSF = (T_1 + T_2) \; (\rightarrow) \end{array} \right.$$

eq. loads on the x-s/c → Loading Cond^n ↓	A.L. $\left(\sigma_a = P/A\right)$	B.M. $\sigma_{max} = \frac{M}{Z_{N.A}}$	S.F. $\left(\tau_d = P/A\right)$	T.M. $\tau_{max} = \frac{T}{Z_P}$
PAL (Pure) 	Const. = P	0	0	0
EAL 	Const. = P	Const. = Pe	0	0
TSL 	0	variable $(BM)_B = 0$ $(BM)_A = WL$	Const. = W	0
ETSL 	0	variable $(BM)_B = 0$; $(BM)_A = PL$	Const. = P	Const. = Pe
Pure Bending 	0	Const = Pa	0	0
Pure torsion 	0	0	0	Const. = Pa

Ques For the structural member ABCD as shown in fig. Determine the following

i) Axial load, S.F, B.M., T.M. for member AB, BC, CD

ii) A.L., S.F., B.M, T.M at X-S/C A, B, C, D

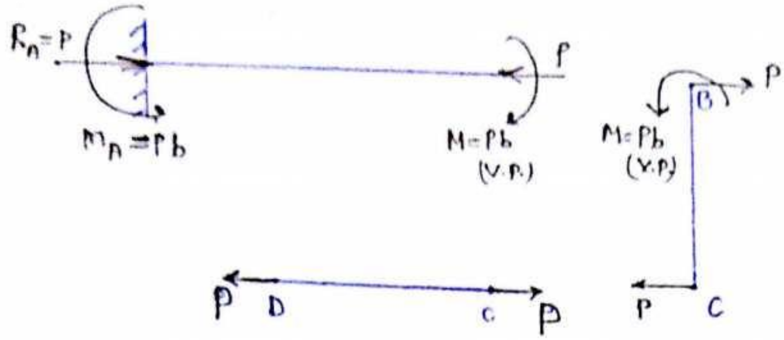


Soln

Eq. load on the X-S/C (→) / Member (↓)	AL	SF	BM	TM
DC (P.A.T.L.) Pure	Const. = P	0	0	0
BC (TSL)	0	Const = P	$M_C = 0$ $M_B = Pb$	0
AB (E.A.C.L)	Const. = -P	0	Const = Pe = Pb	0

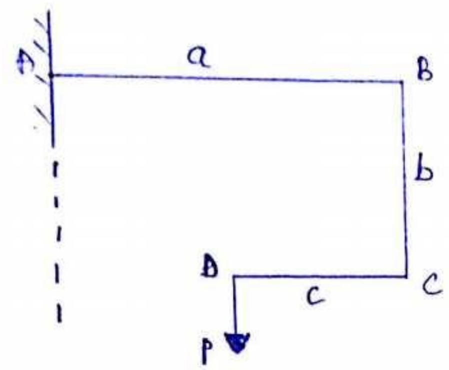
Eq. load → X-S/C (↓)	AL	SF	BM	TM
D	P	0	0	0
C	P	P	0	0
B	-P	P	Pb	0
A	-P	0	Pb	0

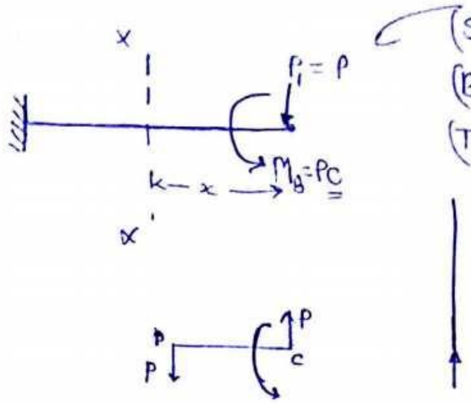
by free body diagram



$A_{x-x} = 0$
$(SF)_{x-x} = P = \text{Const}$
$(BM)_{x-x} = Pc - Px$
$(TM)_{x-x} = 0$

Ques:-

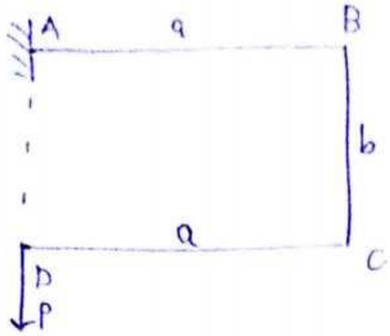




eq. load (→) on the x-s/c member (↓)	AL	SF	BM	TM
DC (TSL)	0	Const = P	$M_D = 0$ $M_C = Pc$	0
BC (ETAL) ($e = c$)	P	0	Cont = Pe = Pc	0
AB (TSL)	0	Const = P	$M_B = Pc$ $M_A = P(a-c)$	0

eq. load (→) x-s/c (↓)	AL	SF	BM	TM
D	0	P	0	0
C	P	P	Pc	0
B	P	P	Pc	0
A	0	P	P(a-c)	0

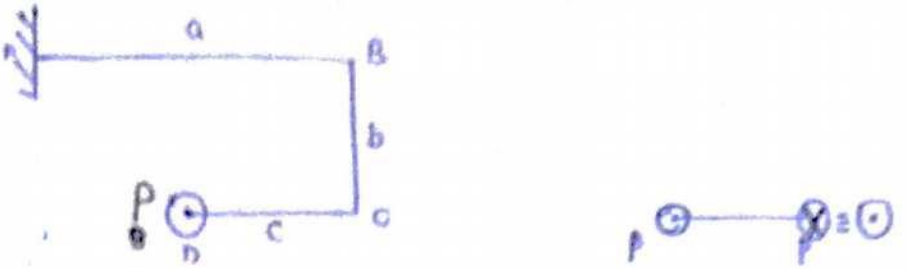
Ques



eq. load on the x-s/c (→) Member ↓	AL	SF	BM	TM
DC (TSL)	0	P	$M_D = 0$ $M_C = Pa$	0
BC (EATL)	P	0	Cont = Pe = Pa	0
AB	0	P	$M_B = Pa$ $M_A = 0$	0

eq load (→) X-s/c ↓				
D	0	P	0	0
C	P	P	Pa	0
B	P	P	Pa	0
A	0	P	0	0

Ques

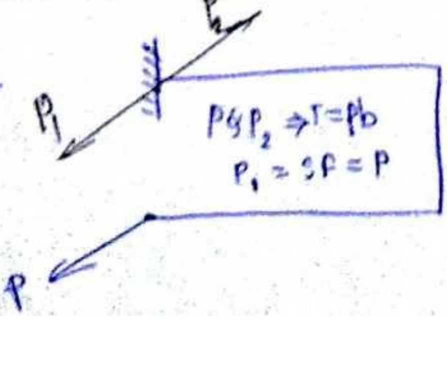


Eq. load on the x-s/c (→) Member (↓)	AL	SF	BM	TM
DC (HTSL)	0	Const = P (H.P.)	$M_D = 0$ $M_C = PC$ (H.P.)	0
BC (ETSL) (e = c)	0	Const. = P	$M_C = 0$ $M_B = Pb$ (P.P.)	Const. = Pe = Pc (H.P.)
AB	0	Const. = P	$M_B = PC$ $M_A = P(a-c)$	Const = Pe = Pb (P.P.)

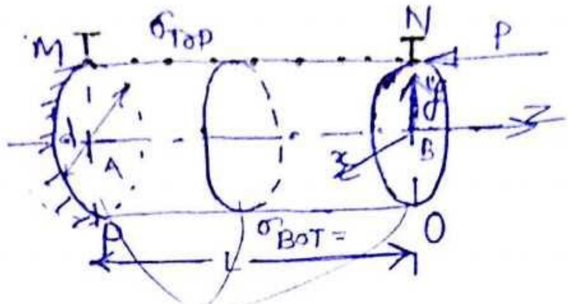
eq. load (→) x-s/c (↓)	AL.	S.F.	B.M.	T.M.
D	0	P	0	0
C	0	P	Pc	Pc
B	0	P	Pb	Pb
A	0	P	p(a-c)	Pb

H.W 5, 6, 7, 8, 9, 13. 18, 27 (chap 5)

eq



Ques: For the prismatic bar as shown below determine max. stress develope on the x-S/c of bar.



Critical x-S/c

Sol^n

$$\sigma_{Top} = \sigma_{max} = -\sigma_a - \left[(\sigma_b)_{max.}\right]_{x-x} \qquad \sigma_{BoT} = -\sigma_a + \left[(\sigma_b)_{max.}\right]_{x-x}$$

$$\sigma_{max} = -\left[\sigma_a + \{(\sigma_b)_{max}\}_{x-x}\right] \qquad \sigma_{bot} = \frac{12P}{\pi d^2} \text{ MPa}$$

$$= \text{Comp}\left[\frac{P}{A} + \frac{M_x \text{ or } Pe}{Z_{x-x}}\right]$$

$$\sigma_{max} = \text{Comp}\left[\frac{P}{\frac{\pi d^2}{4}} + \frac{P d/2}{\frac{\pi d^3}{32}}\right]$$

$$\sigma_{max.} = \frac{20P}{\pi d^2} \text{ MPa (Comp)}$$

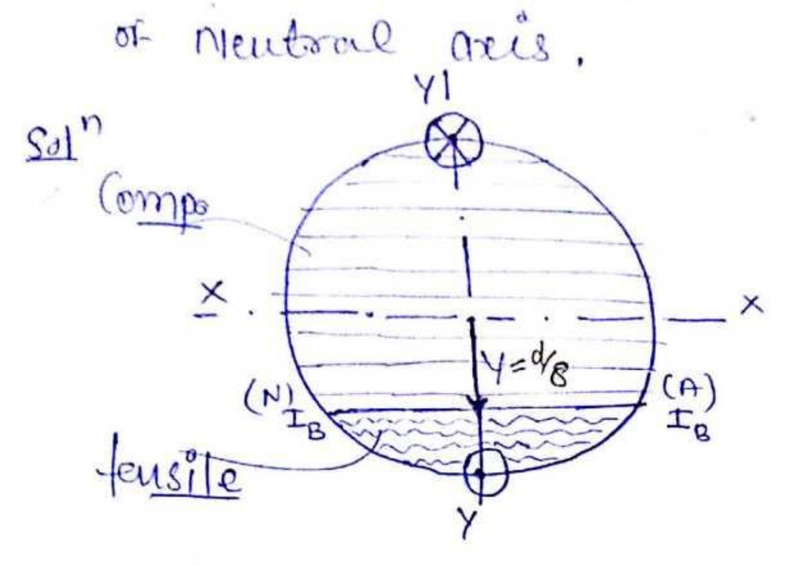
$$\left\{\begin{array}{l} I_{x-x} = I_{y-y} = \dfrac{\pi d^4}{64} \\ Z_{x-x} = Z_{y-y} = \dfrac{\pi d^3}{32} \end{array}\right. ; Y_{max} = X_{max} = \frac{d}{2}$$

* critical points = M & N

(a) ~~L~~ O
(b) ~~O~~.M
(c)
(d) ~~M~~ ⇒ Neutral axis coincide with inner fiber below the centroid axis.
(i.e. at a distance of $d/8$ from H.C.A.

Ques Repeat the above question for the location of neutral axis.

Sol^n



$$\sigma_{NA} = -\sigma_a + \sigma_b = 0$$

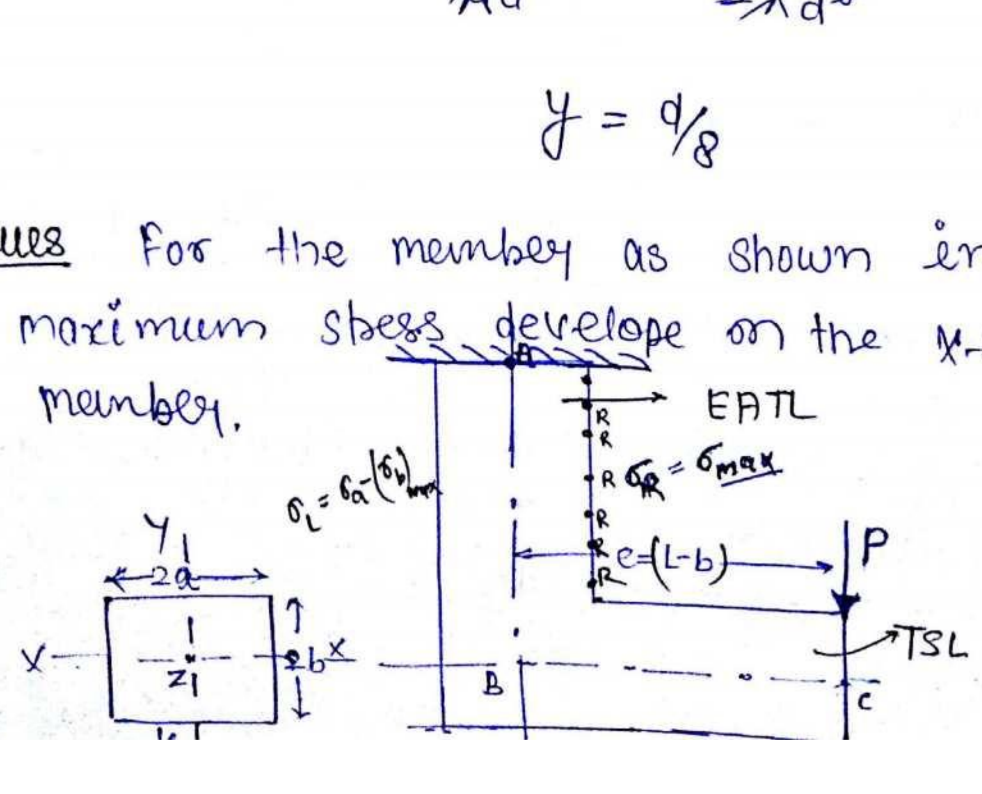
$$-\frac{P}{A} + \frac{M_x Y}{I_{x-x}} = 0$$

$$-\frac{4P}{\pi d^2} + \frac{P d/2 \; Y}{\pi d^4/64} = 0$$

$$\frac{4P}{\cancel{\pi} d^2} = \frac{\overset{8}{\cancel{32}}\cancel{64} P \cancel{d} Y}{\cancel{2}\pi d^{\cancel{3}}}$$

$$y = d/8$$

Ques For the member as shown in fig determine maximum stress develope on the X-S/e of the vertical member.



$$\sigma_R = \sigma_{max} = \sigma_a + \left[(\sigma_{max})\right]_{y-y}$$

$$\sigma_a = \frac{P}{A} = \frac{P}{(2b)^2} = \frac{P}{4b^2}$$

$$M_y = Pe = P(L-b)$$

$$\left[(\sigma_b)_{max}\right]_{y-y} = \frac{M_y \text{ or } Pe}{Z_{y-y}} = \frac{P(L-b)}{\frac{1}{6}(2b)(2b)^2} = \frac{3P(L-b)}{4b^3}$$

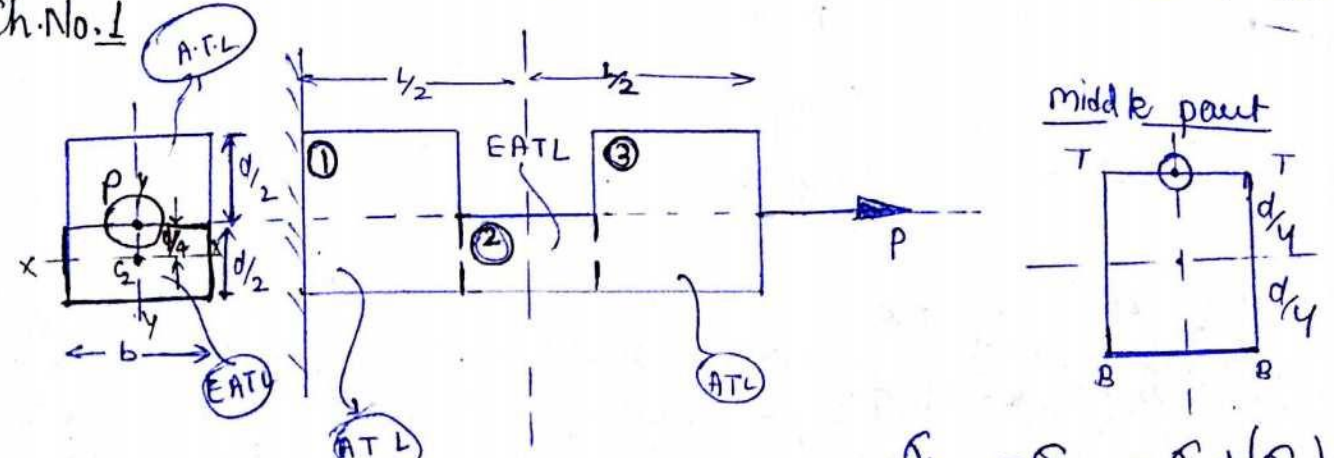
$$\sigma_{max} = \sigma_a + (\sigma_{max})_{y-y}$$

$$\sigma_{max} = \sigma_{RHS} = \frac{P}{4b^2} + \frac{3P(L-P)}{4b^3}$$

$$\sigma_{max} = \frac{P}{4b^3}(3L-2b)$$

Q.No.1 Solⁿ:- It is not a ~~pr~~ prismatic bar so cut in (3) parts

Ch.No.1



$$\sigma_{top} = \sigma_{max} = \sigma_a + (\sigma_b)_{max}$$

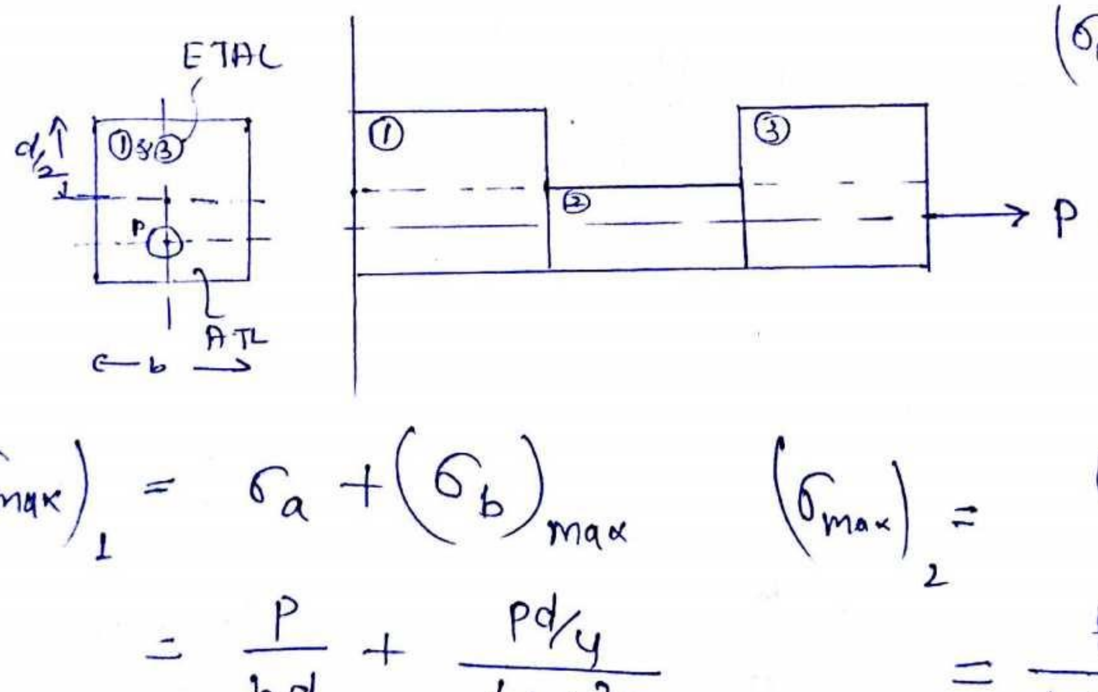
$$\sigma_{bot} = \sigma_a - (\sigma_b)_{max}$$

$$(\sigma_{max})_2 = \sigma_{top} = \frac{P}{A} + \frac{M_{xx}(Pe)}{Z_{x-x}}$$

$$= \frac{P}{bd/2} + \frac{P(d/4)}{\frac{1}{6}(b)(d/2)^2}$$

$$(\sigma_{max})_2 = \frac{8P}{bd} \text{ (tensile)}$$

Ques Repeate the above question for the maximum stress develop on the x-s/c of ① st, ② nd & ③ rd member when line of action of load is along the ~~longitudinal~~ C.A. of middle members.



$$(\sigma_{max})_1 = (\sigma_{max})_3 = \frac{5P}{2bd} = (\sigma_{BOT})$$

$$(\sigma_{max})_2 = \frac{2P}{bd}$$

$$(\sigma_{max})_1 = \sigma_a + (\sigma_b)_{max}$$

$$= \frac{P}{bd} + \frac{Pd/4}{\frac{1}{6}bd^2}$$

$$= \frac{P}{bd} + \frac{3}{2}\frac{P}{bd}$$

$$= \frac{5P}{2bd}$$

$$(\sigma_{max})_2 = \sigma_a$$

$$= \frac{P}{b \times d/2}$$

$$= \frac{2P}{bd}$$

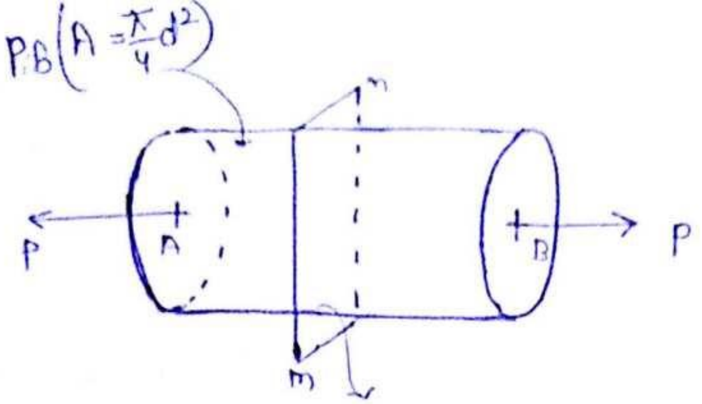
~~Stress~~

Stress:-

stress is define as the intensity or magnitude of internal resisting force (IRF) develope or Indused at a point ~~[illegible]~~ in a member under given loading condition.

Pressure	Stress
→ Due to external force.	→ due to ~~external force~~ IRF
→ Act Normal to Surface	→ Normal or ll el depends on load Cond^n
→ mag. ~~vary~~ Cont.	→ mag. Vary Plan to Plan
→ due to pressure stress develop.	→ but due to stress Pressure May or May not develope
→ Measure by P. gauge	→ Stress Can't measure but strain can Measure
→ Pressure Scalar quantity	→ stress is 2nd order tensor

$$\left\{\begin{array}{l}\text{Scalar} - 0 \text{ order tensor}\\ \text{Vector} - 1\text{st order tensor}\\ \text{Stress} - 2^{nd} \text{ order tensor.}\end{array}\right.$$

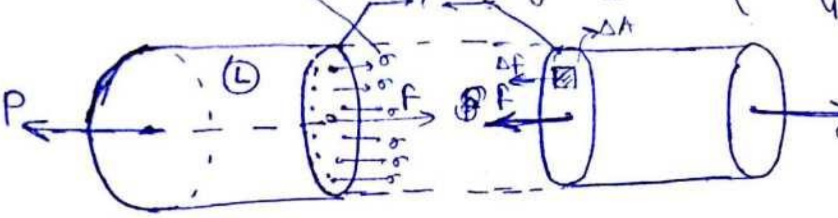


SP or CP

Section point — Cutting Point

Uniformaly dist.

Resisting x-s/c Area's $\left(A = \frac{\pi}{4} d^2\right)$



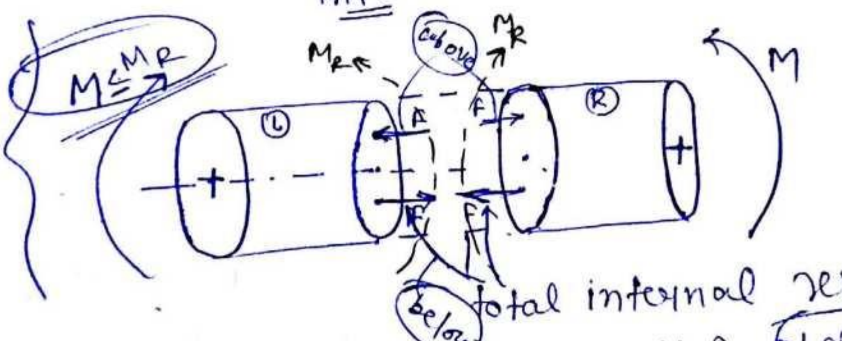
$P \leq F$ At equ^m $\underline{P = F}$

$P_1 \rightarrow F_1$
$P_2 \rightarrow F_2$
$P_n \rightarrow F_n = F_{max} = F$

$$\boxed{\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}}$$ → Mag. of Normal IRF developed at a point

$\Rightarrow P\uparrow \rightarrow F\uparrow$ (Gradual deformation)
if $P > F$ Sudden def^n occur fracture occur.
P_{n+1}

$M \leq M_R$



At equ^m ⇒ $M = M_R$

total internal resisting force develope on ~~N.A.~~ below the N.A.
above

$$\sigma = \frac{dF}{dA}$$

$$\Rightarrow \int dF = \int \sigma \, dA$$ ⇒ Assuming stresses are uniform distributed on x-s/c bar

$$\Rightarrow F = \sigma \int dA \Rightarrow \boxed{F = \sigma . A}$$ $$\boxed{\sigma_{avg} = \frac{F}{A} = \frac{P}{A}}$$

* Resisting force always depends on applied load.

$$\boxed{\sigma_{avg} = \frac{F}{A} = \frac{P}{A}}$$

$$\boxed{\begin{aligned} \sigma_x &= \sigma = P/A \\ \sigma_y &= 0 \\ \sigma_z &= 0 \end{aligned}}$$ ⇒ Uniaxial State of Stress at a point.

$$\boxed{\sigma_{avg} = \frac{P}{A}}$$

eg (a) bar subjected to pure Axial load

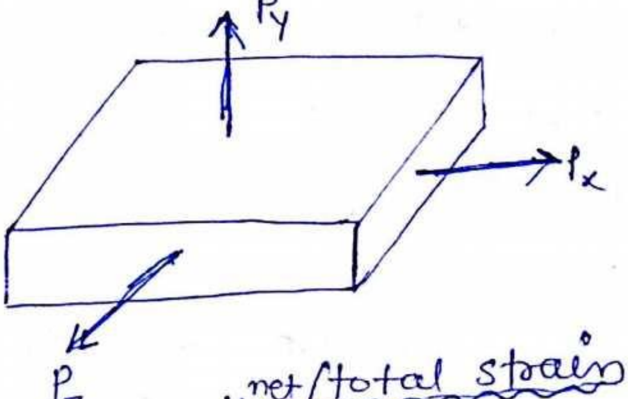
(b) Beam subjected to Pure bending

(c) bar — " — E.A.L.

$$\left.\begin{aligned} \varepsilon_x &= \varepsilon_{long.} = \frac{\sigma_x}{E} \neq 0 \\ \varepsilon_y &= \varepsilon_{lateral} \neq 0 \\ \epsilon_z &= \epsilon_{lateral} \neq 0 \end{aligned}\right\}$$ ⇒ Tri-axial state of strain at a point

⇒ Hook's law gives rel^n in the dir^n of load only.

⇒



$\sigma_x \neq 0$

$\sigma_y \neq 0$

$\sigma_z \neq 0$

$\epsilon_x \neq 0$

$\epsilon_y \neq 0$

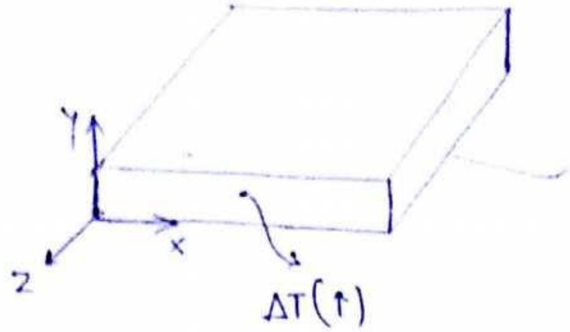
$\epsilon_z \neq 0$

net/total strain

$$\left.\begin{aligned} \sigma_x &\neq 0 \\ \sigma_y &\neq 0 \\ \sigma_z &\neq 0 \end{aligned}\right\} \Rightarrow \text{Triaxial state of stress} \quad \left.\begin{aligned} \epsilon_x &\neq 0 \\ \epsilon_y &\neq 0 \\ \epsilon_z &\neq 0 \end{aligned}\right\} \Rightarrow \text{Tri-axial State of strain}$$

$$E = \frac{\sigma_x}{(\varepsilon_{long})_x} = \frac{\sigma_y}{(\varepsilon_{long})_y} = \frac{\sigma_z}{(\varepsilon_{long})_z}$$

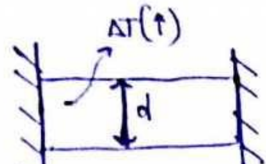
$\Rightarrow$



$$\sigma_x = \sigma_y = \sigma_z = 0 \quad \text{(No load is applied)}$$

$$\varepsilon_x = \varepsilon_y = \varepsilon_z = \varepsilon_{th} = \alpha T$$

$\Rightarrow$



$$\left.\begin{aligned} &\Rightarrow \sigma_x \neq 0 \\ &\Rightarrow \sigma_y = 0 \\ &\Rightarrow \sigma_z = 0 \end{aligned}\right\} \Rightarrow \text{Uni-axial state of stress.}$$

$$\Rightarrow (\varepsilon_x)_{total} = 0 \;;\; (\varepsilon_y)_{total} \neq 0 \;;\; (\varepsilon_z)_{total} \neq 0$$

$$\Downarrow \qquad\qquad \Downarrow \qquad\qquad \Downarrow$$

$$\underline{\varepsilon_{long} + \varepsilon_{th}} \qquad \underline{\varepsilon_{lat} + \varepsilon_{th}} \qquad \underline{\varepsilon_{th} + \varepsilon_{lat}}$$

Strength :- strength is define as the max. or limiting value of stress that a material can withstand without a failure or fracture.

for Mild steel

Yield strength in tension $S_{yt} = 250\,MPa$ $\Rightarrow \sigma_{induced} \leq Y.S. \Rightarrow$ Yielding will not occur (i.e. permanent def^n will not occur)

Ultimate strength in tension $S_{ut} = 400\,MPa$ $\Rightarrow \sigma_{ind} \leq U.S. \Rightarrow$ Fracture will not occur

strength which can withstand without fracture.

i) Ductile material exhibits parmanent defⁿ before fracture but brittle material can not exhibits permanent defⁿ

ii) Ductile material can wear Y.S. & U.S. but brittle material can only wear U.S..

iii) For ductile $S_{yc} > S_{yt} > S_{ys}$

Brittle $S_{uc} > S_{us} > S_{ut}$

iv) Ductile material $S_{yc} \simeq S_{yt}$ (Same in Compression & tension) equally strong in elastic region)

Brittle material $S_{yc} >>> S_{yt}$ (weak in tension) (Strong in Compression)

v) ductile $S_{ys} = \frac{S_{yt}}{2}$ or $\frac{S_{yt}}{\sqrt{3}}$

brittle $S_{uc} = (3 \text{ to } 4)\, S_{ut}$ for cast iron = 600 to 700 MPa

vi) ductile $\delta_{total} = \delta_{E.D} + \delta_{E.P.D.} + \delta_{P.D.}$

Total defⁿ.

Elasto-plastic defⁿ

$$\delta_{P.D} >> \delta_{E.D.} >> \delta_{E.P.D.}$$

Brittle

$$\delta_{total} = \delta_{E.D} + \delta_{E.P.D} \quad (\because \delta_{P.D.} = 0)$$

cast iron & Mild steel.

$(\delta_{E.D})_{CI} > (\delta_{E.D.})_{MS}$ because $\boxed{E_{CI} < E_{ms}}$ Young's Modulas

— in elastic region

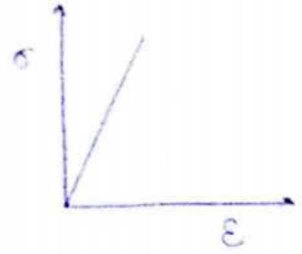
$E_{MS} = 200$ GPa

$E_{CI} = 100$ GPa

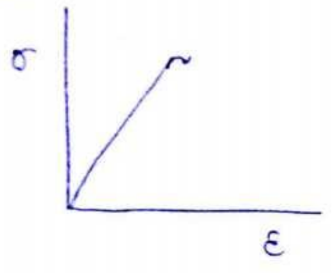
$(\delta_{total})_{CI} < (\delta_{total})_{MS}$ $\quad \delta_L = \frac{PL}{AE}$

For perfect brittle material (i.e. Ceramics)

(No permanent defⁿ



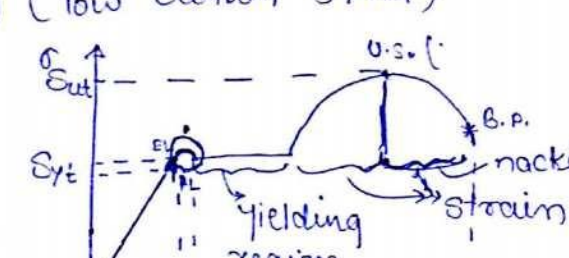
For Cast iron (it exhibits little bit permanent defⁿ.
(cast iron tensile test)



Ductile

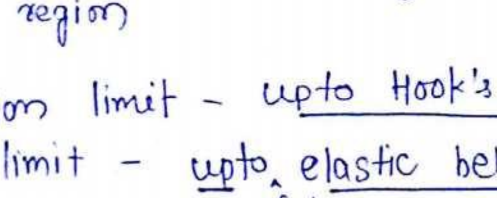
Mild steel (low Carbon steel)

①' - elastoplastic region



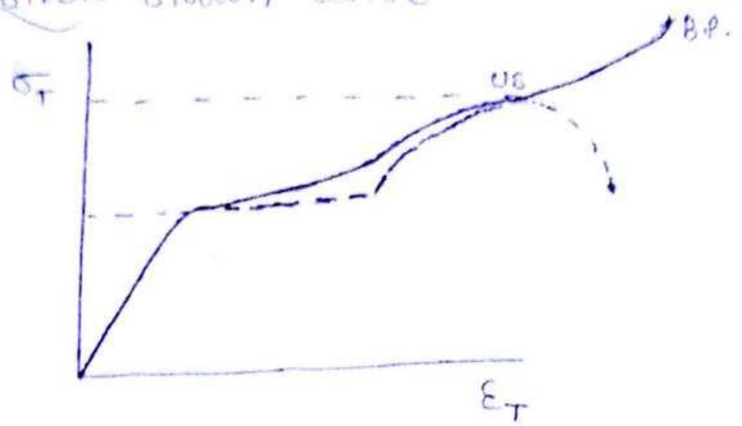
P.L. - proportion limit - upto Hook's low valid (linear relⁿ)

E.L - elastic limit - upto perfect elastic behaviour



* upto to elastic limit Engineering & true Stress, strain are equal.

True stress strain curve



* strain hardening offer more resistance to deformation

Units of stress & strength :-

Pa, MPa, GPa, kgf/cm^2

kgf/cm^2 → mks unit

Pa → SI unit

$1 \; Pa = 1 \; N/m^2$

$1 \; kgf/cm^2 = 0.1 \; MPa$

if $1 \; kgf \simeq 10N$

$1 \; MPa = 10^6 \; N/m^2 = 1 \; N/mm^2$

$1 \; GPa = 10^9 \; (N/m^2 \text{ or } Pa)$

$= 10^3 \; (N/mm^2 \text{ or } MPa)$

for mild steel

$S_{Yt} = 250 \; MPa$

$E = 200 \; GPa$

$S_{Yt} = 250 \; MPa$

$= 250 \; N/mm^2$

$= 250 \times 10^6 \; N/m^2$

$E = 200 \; GPa$

$= 200 \times 10^3 \; N/mm^2$

$= 200 \times 10^9 \; (N/m^2 \text{ or } Pa)$

Design Criterion! — { In Academic - Calculate dim^ns
In actual/practicle - Draw design

① Strength Criterion! - dim^ns are Calculated by using permissible stress/allowable stress/ /safe/design/working stress.

* Permissible stress not a property it ~~can~~ is determine by using Factor of safety (F.O.S.)

② Rigidity Criterion:- dim^ns are Calculated by using permissible deformation.

Cond^n for safe design w.r.t. strength Criterion:—

$\boxed{\text{(Max. stress induced)} \leq \text{(permissible stress)}}$

↳ by using SOM eq^ns ↳ By using F.O.S.

* Applicable only when normal stress ~~&~~ or shear stress or B.M.. if both are given Not applicable

Cond^n for safe design wrt Rigidity Criterion

$\boxed{\text{Max. def}^n \text{ induced} \leq \text{Permissible def}^n}$

By using SOM eq^n ↳ Design Standard

Factor of safety :→ (N)

$$\boxed{N = \frac{\text{failure stress}}{\text{per. stress}}}$$

it is define as the ratio of failure stress to permissible stress.

$$\boxed{\text{Permissible Stress} = \frac{\text{Failure stress}}{N}}$$

* Failure stress = Yield strength (Ductile) } static loading
= Ultimate strength (Brittle)
= Endorance limit ⇒ fatigue loading

* if per. stress Given not use fos

* For a given material & type of load,

$$\text{per. stress} \propto \frac{1}{N} \quad [\because \text{failure stress} = \text{Const.}]$$

$N(\uparrow) \rightarrow$ per. stress $(\downarrow) \rightarrow$ dim $(\uparrow)$
safety $(\uparrow)$ ✓ So don't go higher fos
cost $(\uparrow)$ ✗

$N(\downarrow) \rightarrow$ per. stress $(\uparrow) \rightarrow$ $\dim^n (\downarrow)$
safety $(\downarrow)$ ✗ Don't go very less fos.
Cost $(\downarrow)$ ✓

* An optimum value of fos require. and it can obtain by experiance
(Based on experiance)

$\boxed{\text{For safe design, } N > 1}$

$\boxed{\text{For failure design, } N = 1}$

Endurance limit :- Endu. limit is define as the max. value of <u>Completaly</u> <u>reverse stress</u> that a material can withstand ~~without a~~ for an infinite No. of cycle without a fatigue failure (without a crack initiation). Nacking is indication of failure

Fatigue life – <u>No. of revolution</u> before crack is start.

For Mild steel

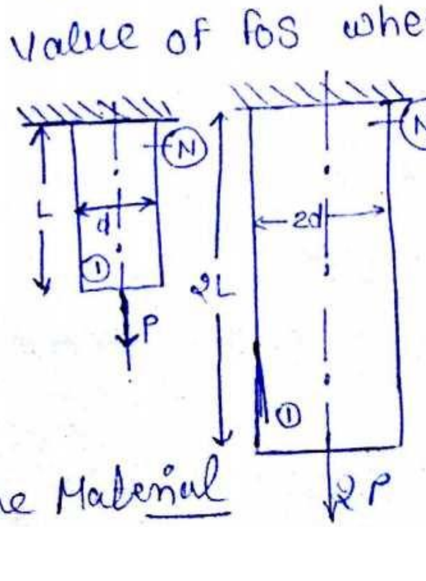
$$E.L. = 200\,MPa = 0.50\, S_{yt}$$

if $\sigma_{ind} \le E.L. \Rightarrow$ Infinite life ($\ge 10^6$ cycles)

$\sigma_{ind} > E.L. \Rightarrow$ finite life (10^3 to 10^6 cycles)

Ques. If all the dim^n of a ~~pars~~ prismatic bar and axial load become double, ~~when bar is subject~~ what is the value of fos when bar is subjected to pure axial load.

Sol^n



Same Material

$$\sigma_1 = \frac{P}{A} \qquad \sigma_2 = \frac{P_2}{A_2} = \frac{1}{2}\left[\frac{P}{A}\right]$$

$$\sigma_2 = \frac{\sigma_1}{2}$$

$$\sigma_{ind} = \sigma_{per}$$

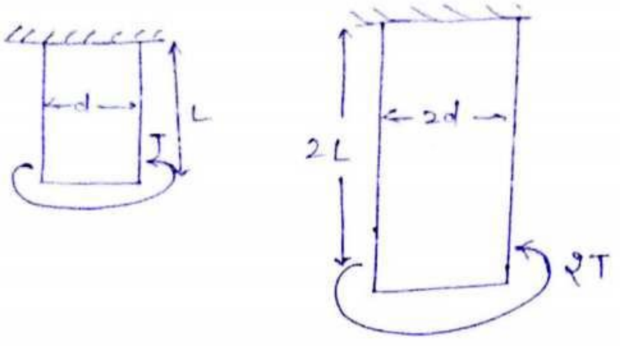
$$\sigma_{ind} = \frac{\sigma_{Failure}}{N}$$ For a given material type of load $\sigma_{ind} \propto \frac{1}{N}$

$$\frac{N_2}{N_1} = \frac{\sigma_1}{\sigma_2} \Rightarrow \frac{N_2}{N_1} = 2 \Rightarrow N_2 = 2N_1 = 2N$$

eg. if bar is subjected to pure torsion $T \rightarrow 2T$

$d \rightarrow 2d$

$L \rightarrow 2L$



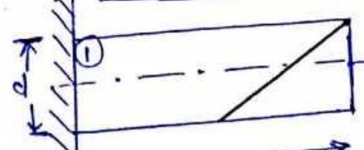
$$\tau_1 = \frac{16T}{\pi d^3} \qquad \tau_2 = \frac{16(2T)}{\pi (2d)^3}$$

$$\tau_2 = \frac{1}{4}\left(\frac{16T}{\pi d^3}\right) = \frac{\tau_1}{4}$$

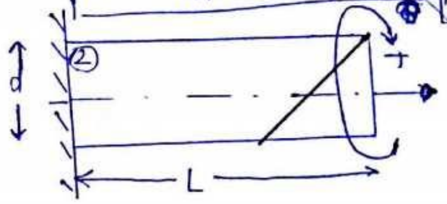
$$\frac{N_2}{N_1} = \frac{\tau_1}{\tau_2} \Rightarrow N_2 = 4N_1 = \underline{\underline{4N}}$$

Expression for normal & shear stress on an oblique plane (O.P.) under axial loading :-

$$\boxed{\tau_{max} = \frac{1}{2}(\sigma_t)_{max}}$$



$$\boxed{(\sigma_t)_{max} = \tau_{max} = \frac{16T}{\pi d^3} = (\sigma_c)_{max}}$$



(1) Det. Max. σ_t induced in bar

$\left(\frac{4P}{\pi d^2}\right)$ ✓ (on the X-S/C)

(2) Det. max τ_s induced in bar

~~zero~~ (X), $\frac{2P}{\pi d^2}$ ✓

3) Det. σ_t & τ_s on the X-S/C of bar? $\sigma_t = \frac{P}{A} = \frac{4P}{\pi d^2}$, $\tau_s = 0$

(i) Det. max τ_s induced in the bar

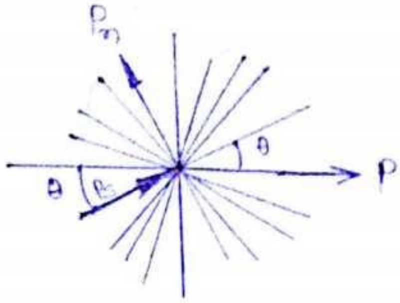
$\left(\frac{16T}{\pi d^3}\right)$ ✓ $\theta = 0° \rightarrow$ plane of X-S/C

(2) Det. max. σ_t induced in the bar

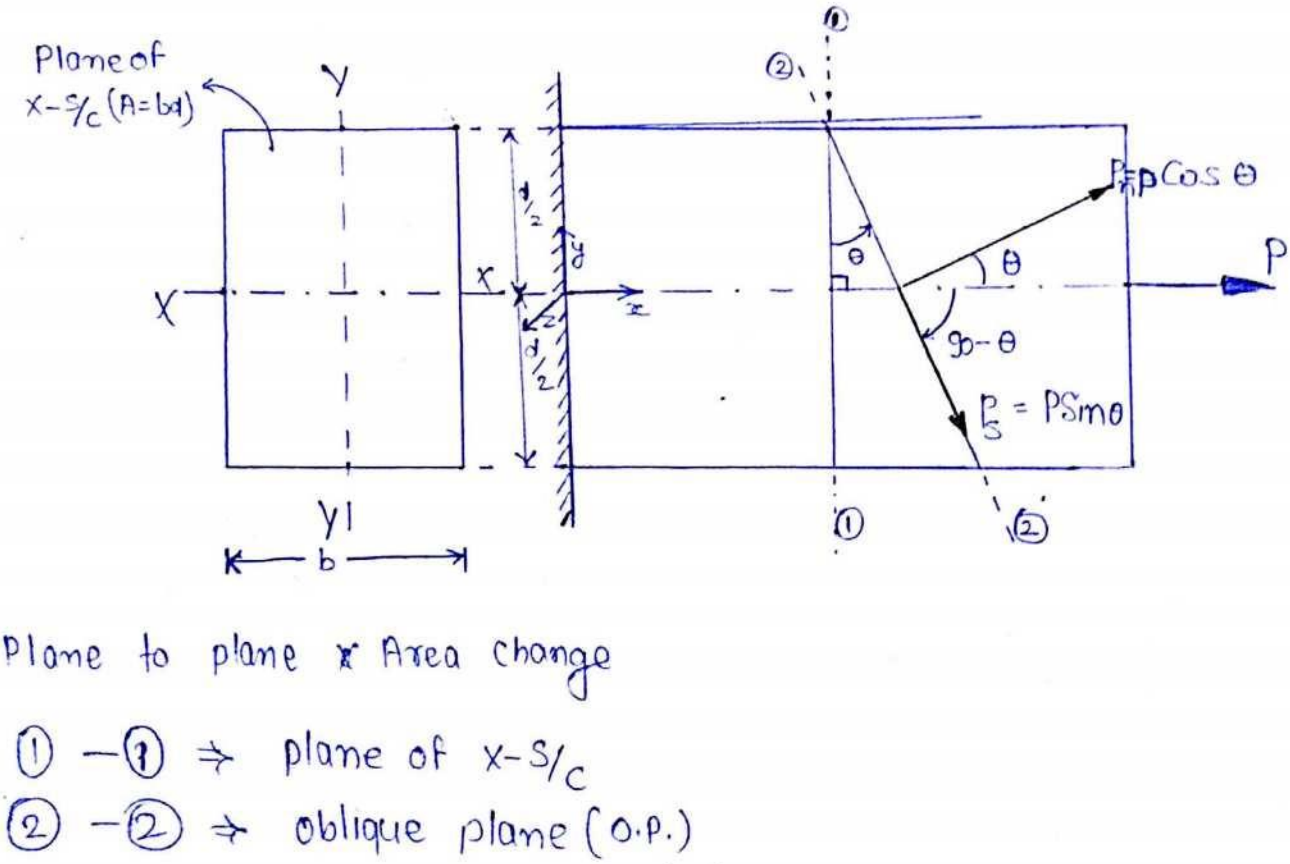
~~zero~~ (X), $\frac{16T}{\pi d^3}$ ✓

(3) Det (torsional) Shear stress & Normal stress on X-S/C of bar.?

$$\tau_t = \frac{16T}{\pi d^3}, \quad \sigma_t = 0$$



* Stresses are induced on point, at a point there are infinite plane so there are both Normal as well as shear stress induced in both case
To Calculate then input data require and to find max. stress differenciate it and equal to zero



* Plane to plane ~~x~~ Area change

① − ① ⇒ plane of x-S/c
② − ② ⇒ oblique plane (O.P.)
θ = Angle made by oblique plane with the plane of x-S/c in ACW dirn.

$$A_{1-1} = A = bd \qquad -①$$

$$A_{2-2} = A' = b'd' = \frac{bd}{\cos\theta}$$

$$A' = \frac{A}{\cos\theta}$$

Stresses developed on plane of x-S/c

$$(P_n)_{1-1} = P \; ; \; (P)_{1-1} = \text{Zero}$$

$$(\sigma_n)_{1-1} = \sigma = \frac{P}{A} \quad \text{— (2)}$$

$$(\tau_s)_{1-1} = \tau = \frac{P_s}{A} = \text{zero}$$

Stresses developed on O.P. :-

$$\left.\begin{aligned}(P_n)_{2-2} &= P_n = P\cos\theta \\ (P_s)_{2-2} &= P_s = P\sin\theta\end{aligned}\right\} \text{— (3)}$$

$$(\sigma_n)_{2-2} = \sigma_n = \frac{P_n}{A'} = \frac{P\cos\theta}{A/\cos\theta}$$

$$\sigma_n = \frac{P}{A}\cos^2\theta = \sigma\cos^2\theta \quad \text{— (4)}$$

$$(\tau_s)_{2-2} = \tau_s = \frac{P_s}{A'} = \frac{P\sin\theta}{A/\cos\theta}$$

$$\tau_s = \frac{P}{A}\sin\theta\cos\theta \times \frac{2}{2}$$

$$\tau_s = \frac{\sigma}{2}\sin 2\theta \quad \text{— (5)}$$

$$\boxed{\begin{aligned}\sigma_n &= \sigma\cos^2\theta \\ \tau_s &= \frac{\sigma}{2}\sin 2\theta \\ \text{where } \sigma &= P/A\end{aligned}}$$

* Using these eq^n in uni state of stress we can determine Normal & shear stress in any oblique plane.
* Bi-axial state of stress - when stress develope in Normal.

$$\boxed{\begin{aligned}\sigma_n &= \sigma \cos^2\theta \\ \tau_s &= \frac{\sigma}{2}\sin 2\theta\end{aligned}}$$ where $\because \sigma = P/A$

from above two equation (4) & (5) following conclusion can be made.

(1) Stress at a point is a 2nd order tensor

(2) Normal & shear stress on any oblique plane passing through a point under uni axial state stress can be determine when stress is develope in plane of x-s/c is know.

(3) Similarly to determine normal & shear stress on any oblique plane passing through a plane under Bi-axial state of stress. (i.e. $\sigma_x \neq 0$; $\sigma_y \neq 0$; $\tau_{xy} \neq 0$) one should know stress develope on two mutual ⊥r plane passing through that point.

$$(\sigma_t)_{max} = \boxed{\begin{aligned}(\sigma_n)_{max} &= \sigma \quad \text{when } \theta = 0^\circ \\ (\sigma_n)_{min} &= \text{zero} \quad \text{when } \theta = 90^\circ\end{aligned}}$$ (plane of x-s/c)

$$\boxed{\begin{aligned}\tau_{max} &= \frac{\sigma}{2} \quad \text{when } \theta = 45^\circ \\ &= -\frac{\sigma}{2} \quad \text{when } \theta = 135^\circ\end{aligned}}$$ — dirn change, mag. Unchanged

Cast Iron

Axial loading (Tension)

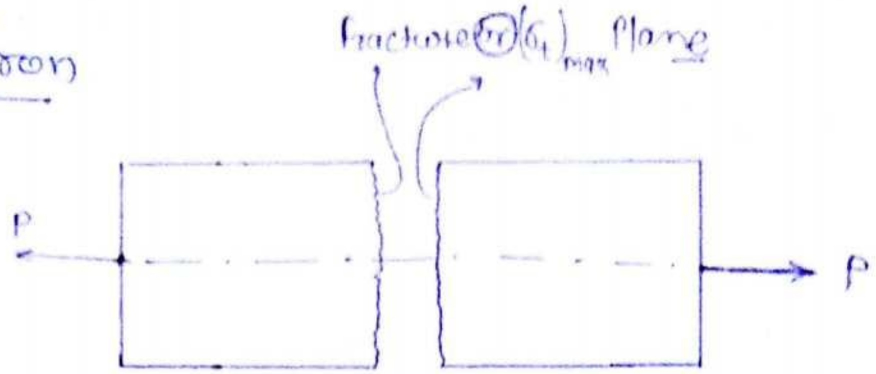


Fig: Granular transverse fracture of Cast iron bar under tension

Mild steel

Axial loading (tension)

Granular
दानेदार

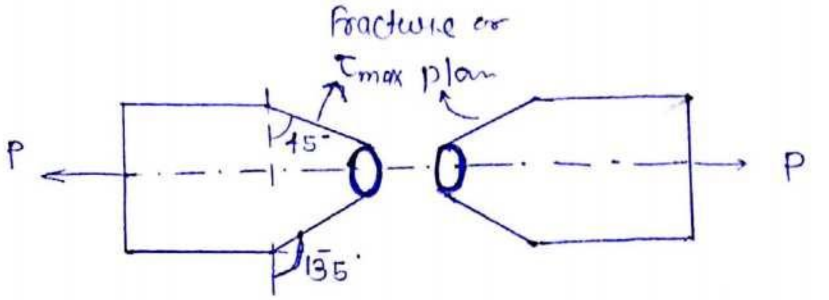


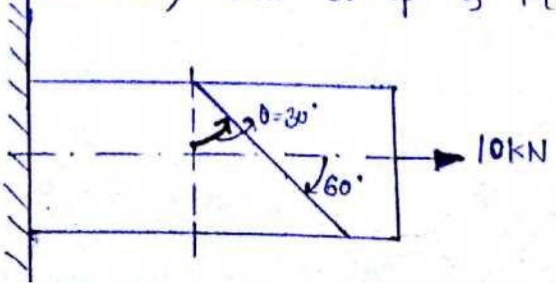
Fig. Cup & Cone fracture of M.S. Bar under tension

Ques Determine the following when a prismatic bar Subjected to an axial tensile load. Assume X-S/c. Area is 200 mm^2 and axial load is 10 kN.

i) Normal & shear stress on a oblique plane inclined at an angle of 60° with l.A. in clockwise dirn

ii) Resultant stress on max. shear stress plane

iii) Max. tensile, Max. Comp. & Max Shear stress in bar.



$P = 10 kN$

$A = 200 \, mm^2$

$$\Rightarrow \sigma = \frac{P}{A} = \frac{10 \times 10^3}{200} = 50 \, MPa$$

(i) Normal stress on plane $\theta = 30^\circ$ $\quad \cos 30 = \frac{\sqrt{3}}{2}$
& shear stress

$$\sigma_n = \sigma \cos^2\theta$$
$$= 50\left(\frac{\sqrt{3}}{2}\right)^2$$
$$\sigma_n = \frac{150}{4} = 37.5 \, MPa$$

$$\tau_s = \frac{\sigma}{2} \sin 2\theta$$
$$= \frac{50}{2} \times \sin 60$$
$$\tau_s = 21.65 \, MPa.$$

(ii) Max. shear stress plane $\theta = 45^\circ, 135^\circ$

$$(\sigma_n)_{\theta = 45 \& 135} = 50 \cos^2 45 = \underline{25 \, MPa} = \frac{\sigma}{2}$$

$$(\tau_s)_{max} = (\tau_s)_{\theta = 45 \& 135} = \pm \frac{50}{2} \sin 90 = \pm \frac{\sigma}{2}$$
$$= \underline{\pm 25 \, MPa}$$

$$(\sigma_r)_{\tau_{max} \, plan} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \left(\frac{\sigma}{2}\right)^2} = \frac{\sigma}{\sqrt{2}} = \frac{50}{\sqrt{2}} = 35.35 \, \underline{MPa}$$

(iii) $(\sigma_t)_{max, \theta = 0} = \sigma = 50 \, MPa$ $\quad (\tau_s)_{max, \theta = 45 \& 135} = \pm \frac{\sigma}{2} = \pm 25 \, \underline{MPa}$

$(\sigma_c)_{max} = 0$

Oblique Plane (θ)	$(\sigma_n)_\theta = \sigma \cos^2\theta$	$(\tau_s)_\theta = \frac{\sigma}{2} \sin 2\theta$
$\theta = 0^\circ$ Major Principle Plane or Plane of zero τ_s or Plane of Max. σ_t / σ_n	$(\sigma_n)_{max} = \sigma = (\sigma_t)_{max} = \sigma_1$ = Major Principle stress	(τ_s) = zero
$\theta = 45^\circ$ Max. τ_s plane	$\sigma_n^* = \frac{\sigma}{2} = \frac{\sigma_1+\sigma_2}{2}$	$\tau_{max} = \frac{\sigma}{2}$
$\theta = 90^\circ$ Minor Principle Plane or Plane of zero τ_s or Plane of min σ_n	$(\sigma_n)_{min}$ = zero $= \sigma_2$ = Minor principle stress.	τ_s = zero
$\theta = 135^\circ$ Max τ_s plane	$\sigma_n^* = \frac{\sigma}{2} = \frac{\sigma_1+\sigma_2}{2}$	$\tau_{max} = -\frac{\sigma}{2}$

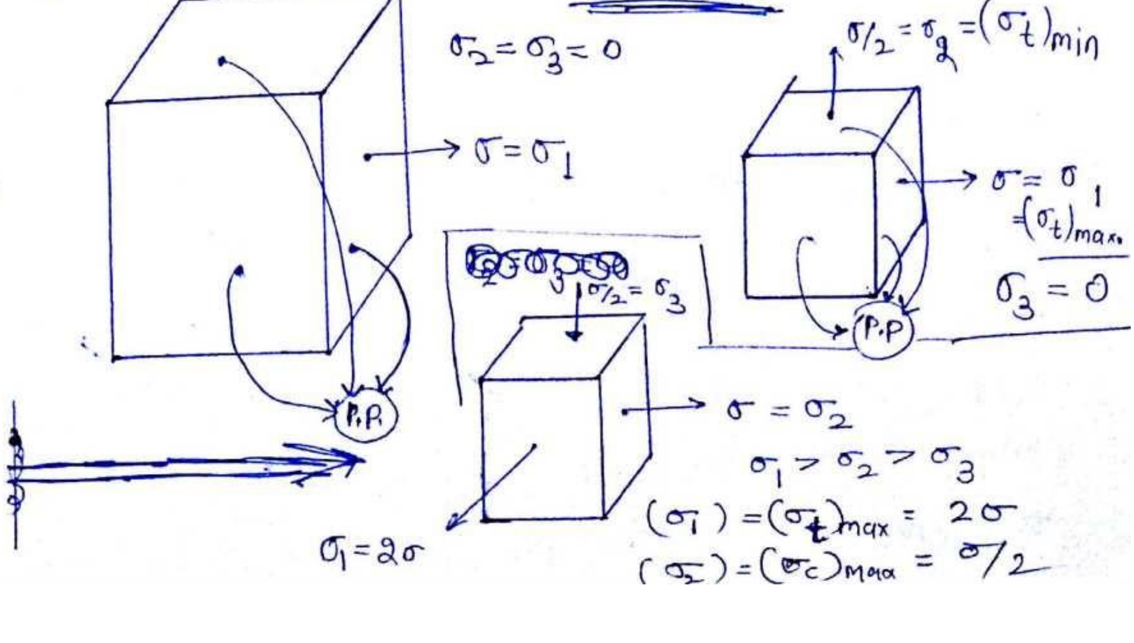
* where shear stress zero Normal stress should be max. or min vice versa not true.
* Angle b/w Principle plane allways 90° & angle b/w principle plane & $(\tau_s)_{max}$ Plane always 45° or 135°

Uni-oxial →
State of stress

P.P. - Principle plane.

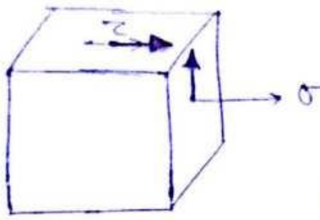
Bi-oxial

Tri-axial



* whenever there is no shear stress. Normal stresses directly become principle plane

⇒



→ Principle Planes (P.P.) are not known

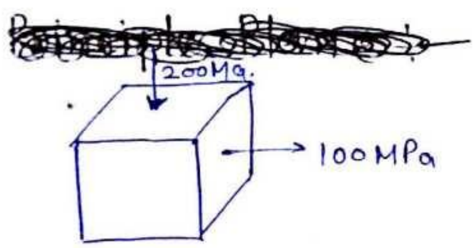
* Calculation are require

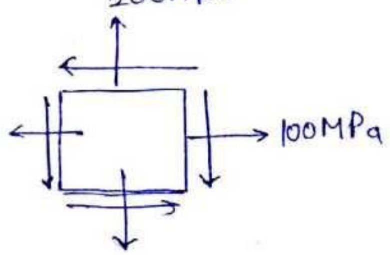
$(\sigma_t)_{max} = ?$ $\tau_{max} = ?$

$(\sigma_c)_{max} = ?$

∵ we can't tell because P.P. are not known & shear is there.

* Out of infinite plane only three plane shear stress zero (except hydrostatic ~~as~~ Case)





$\Rightarrow (\sigma_c)_{max} = 200\,MPa$

$\Rightarrow (\sigma_t)_{max} = 100\,MPa$

stress is present so we Can't ~~can~~ determine principle plane without calculation.

Principle Planes :-

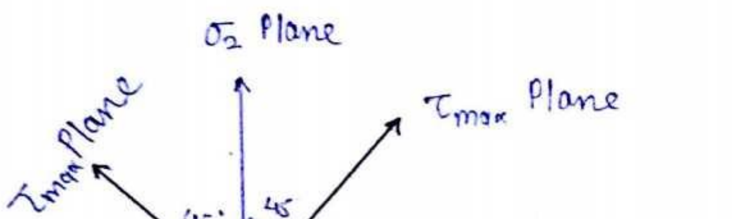
Principle planes are those complimentory oblique plane on which shear stress is zero but normal stress ~~is~~ becomes either max./min. hence they are also known as planes of zero shear stress or plance of pure complimentory Normal stress or plane of max./min. Normal Stress or planes Max. tensile ~~or~~ plane of Max. Comp.

- When principle ~~stress~~ plane are like in nature they are known as max. normal stress and min. normal stress plane.

- Principle stress also known as max. tensile / Max. Comp. planes when they are unlike in nature

* <u>Max. shear stress plane</u> :- $(\tau_s)_{max}$ $\underline{(\tau_s)_{max}}$ those complimentory planes oblique plane on which <u>shear stress is max.</u>
and <u>normal stress are non zero</u> and <u>average</u> of <u>major</u> and <u>minor principle stress</u>.

- Max. shear stress plane are mutually ⊥r to each other and they are inclined at an angle of 45° to both the principle plane



when major & minor principle stress are equal and unlike in nature ($\sigma_1 = -\sigma_2$) then max. shear stress plane also know as plane of pure shear or plane of zero normal stress.

Location of Principle Plane (P.P.)

$$(\tau_s)_\theta = 0 \quad \text{or} \quad \frac{d}{d\theta}(\sigma_n)_\theta = 0$$

Location of τ_{max} Plane

$$\frac{d}{d\theta}(\tau_s)_\theta = 0$$

* Complimentory shear stress at a point are equal in magnitude and unlike in nature.

$$\text{i.e} \left[(\tau_s)_\theta = -(\tau_s)_{90+\theta} \right]$$

State of Stress at a point:-

State of stress at a point is use to ~~diffe~~ define different stresses which are develope on three mutually $\perp^r$ planes passing through that point.

State of stress can be represented by following two base.

(1) Graphical representation

(2) matrix representation (i.e. stress tensor)

Stress Notations :—

$\perp^r$ to the given Plane @ face

$$\left[(\sigma_{xx}) \text{ or } \sigma_x \right]$$

Face

Dirn of the load

llel to given face

$$(\tau_{xy})$$

Face

dirn of the load

Graphical

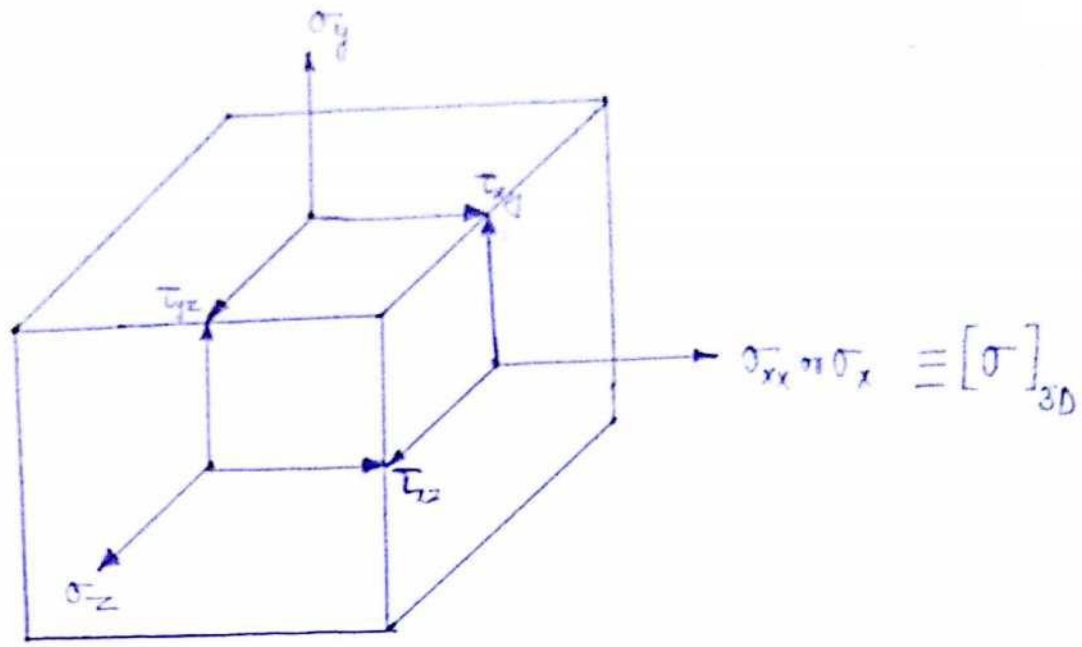


Fig → Tri-axial state of stress at a point

i.e. $\Sigma f_x = 0$; $\Sigma C_x = 0$

$\Sigma f_y = 0$; $\Sigma C_y = 0$

$\Sigma f_z = 0$; $\Sigma C_z = 0$

Matrix

Stress tensor at a point in 3D (Square & Symmetric matrix)

$$[\sigma]_{3D} = \begin{bmatrix} \sigma_x & \cancel{\sigma}\tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}_{3\times3} \begin{array}{l} \to x\text{ face} \\ \to y\text{ face} \\ \to z\text{ face} \end{array} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

Uni-axial $[\ ]_{1\times1}$

Bi-axial $[\ ]_{2\times2}$

Triaxial $[\ ]_{3\times3}$

$\because \tau_{xy} = \tau_{yx}$

$\tau_{yz} = \tau_{zy}$

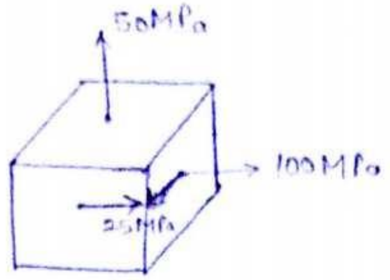
$\tau_{xz} = \tau_{zx}$

V.P. $\Sigma C_x = 0 \Rightarrow \tau_{xy} = -\tau_{yx}$

H.P. $\Sigma C_y = 0 \Rightarrow \tau_{xz} = -\tau_{zx}$

P.P. $\Sigma C_z = 0 \Rightarrow \tau_{yz} = -\tau_{zy}$

Complimentory Shear stress are always equal & ~~opposite~~ Unlike eq.

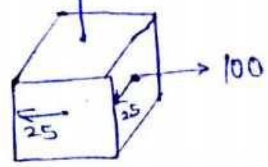


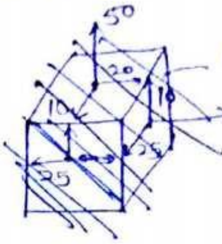
$\sigma_x = 100\ MPa$ $\qquad \tau_{xy} = 0 \ ; \ \tau_{yz} = 0$

$\sigma_y = 50\ MPa$ $\qquad \tau_{xz} = 25\ MPa$

$\sigma_z = 0$ $\qquad \tau_{zx} = -25\ MPa$

eq



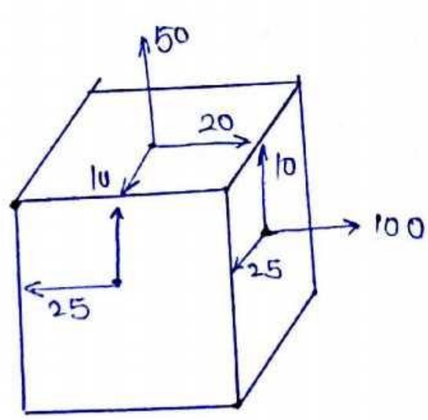


Couple are non zero.

→ sum of couple is not equal to zero so complimentory stress not unlikely so. it is not in static equ^m

∵ Complinentory stess are should be equal in mag. & opposite in dir^n.

eq

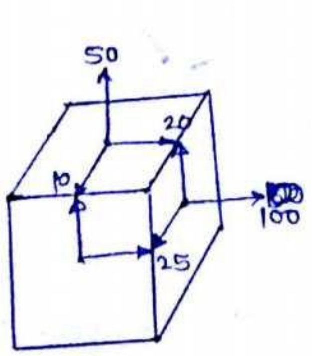


$\Sigma C_y \neq 0 \left[\because \text{dir}^n \text{ of Comp. } \tau_s \text{ on X-Z plane are Incorrect} \right]$

$\therefore \Sigma C_z \neq 0 \left[\because \text{mag. of Comp. } \tau_s \text{ on X-Y plane are incorrect} \right]$

$\Sigma_x = 0 \left[\because \text{Mag \& dir}^n \text{ of Comp. } \tau_s \text{ on Y-Z plane are Correct} \right]$

eq



$\sigma_x = 100\ MPa$ $\qquad \tau_{xy} = -20\ MPa$

$\sigma_y = 50\ MPa$ $\qquad \tau_{xz} = 25\ MPa$

$\sigma_z = 0\ MPa$ $\qquad \tau_{yz} = -10\ MPa$

08 July 2016

Bi-axial state of stress problems:

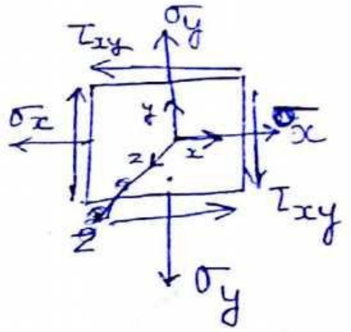
(or) Plane stress (or) 2D problem

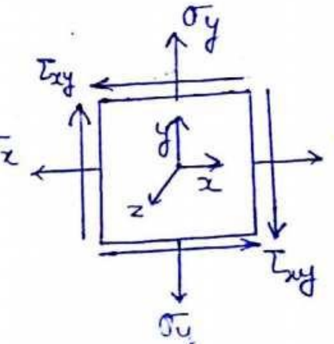
Plane stress problems are those ~~proplems~~ in which stress in any one of the mutual $\perp^r$ plan passing through a point are assumed as zero.

stress developed on z-face are assumed as zero $\left[\cancel{\text{ie. } \sigma_z\ \sigma_{zx}\ \sigma_{zy}}\right.$

$$\begin{bmatrix} \text{i.e. } \sigma_z = \tau_{zx} = \tau_{zy} = 0 \\ \tau_{xz} = \tau_{yz} = 0 \end{bmatrix}$$

$$[\sigma]_{2D} = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix}$$

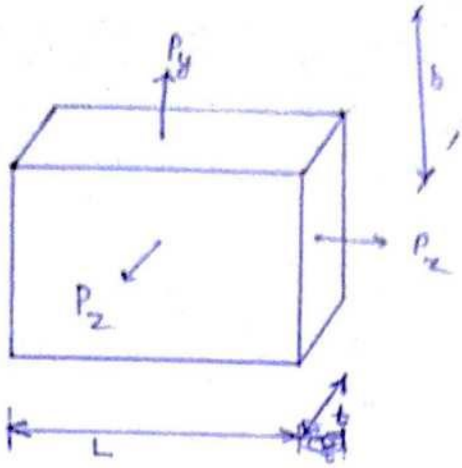




⇒ Fig: Bi-axial state of stress at a plane

$$\Sigma H = 0 \text{ or } \Sigma F_x = 0$$
$$\Sigma V = 0 \text{ or } \Sigma F_y = 0$$
$$\Sigma M = 0 \text{ or } \Sigma C_z = 0$$

* load in one dir^n very small as compaire to others assumed zero (or) dimension in that dir^n are very small.



$$\sigma_x = \frac{P_x}{bt}$$

$$\sigma_y = \frac{P_y}{Lt}$$

$$\sigma_z = \frac{P_z}{Lb}$$

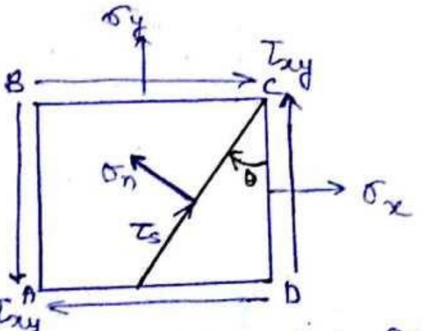
$$t <<< (L \& b)$$

$$\& \quad P_z <<< (P_x \& P_y)$$

So $\quad \sigma_z <<< \sigma_x \& \sigma_y$

Hence, bi-axial state of stress is assumed (i.e. σ_z is neglected)

Oblique Plane at an angle θ



Normal and shear stress in any oblique plane under Bi-axial st of stress

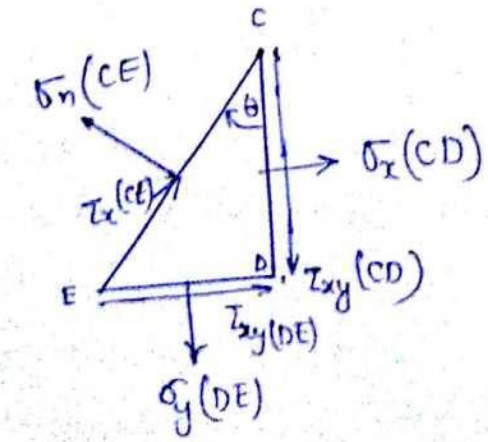
$$\Rightarrow \boxed{(\sigma_n)_\theta = \frac{1}{2}\left[\sigma_x + \sigma_y\right] + \frac{1}{2}\left[\sigma_x - \sigma_y\right]\cos 2\theta + \tau_{xy}\sin 2\theta} \quad -(1)$$

$$\Rightarrow \boxed{(\tau_s)_\theta = -\frac{1}{2}\left[\sigma_x - \sigma_y\right]\sin 2\theta + \tau_{xy}\cos 2\theta} \quad -(2)$$

for uniaxial $\sigma_x = \sigma$; $\sigma_y = \tau_{xy} = 0$; $\theta = -\theta$

$(\sigma_n)_\theta = \sigma\cos^2\theta$; $(\tau_s)_\theta = \frac{\sigma}{2}\sin 2\theta$

Forces



Strain tensor! -

Strain tensor is used to define state of strain of state at a point [different strain develope on three mutual $\perp^r$ planes passing through a point]

$$\sigma \longrightarrow \epsilon \text{ (i.e. Normal strain)} \quad * \ \sigma = E\epsilon$$

$$\tau \longrightarrow \gamma \text{ (i.e. shear strain)} \quad * \ \tau = G\gamma$$

Strain tensor at a point in 3D

$$[\epsilon]_{3D} = \begin{bmatrix} \epsilon_x & \frac{\gamma_{xy}}{2} & \frac{\gamma_{xz}}{2} \\ \frac{\gamma_{xy}}{2} & \epsilon_y & \frac{\gamma_{yz}}{2} \\ \frac{\gamma_{xz}}{2} & \frac{\gamma_{yz}}{2} & \epsilon_z \end{bmatrix}_{3\times3}$$

$\gamma_{xy} = -\gamma_{yx}$

$\gamma_{xz} = -\gamma_{zx}$

$\gamma_{yz} = -\gamma_{zy}$

Comp. shear strain are equa & unlike

Bi-axial/2D

$$[\epsilon]_{2D} = \begin{bmatrix} \epsilon_x & \frac{\gamma_{xy}}{2} \\ \frac{\gamma_{xy}}{2} & \epsilon_y \end{bmatrix}_{2\times2}$$

Normal & shear strain in oblique plane at an angle θ

$$\boxed{(\epsilon_n)_\theta = \frac{1}{2}[\epsilon_x + \epsilon_y] + \frac{1}{2}[\epsilon_x - \epsilon_y]\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta} \quad -(3)$$

$$\left(\frac{\gamma_s}{2}\right)_\theta = -\frac{1}{2}[\epsilon_x - \epsilon_y]\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\boxed{(\gamma_s)_\theta = -[\epsilon_x - \epsilon_y]\sin 2\theta + \gamma_{xy}\cos 2\theta} \quad -(4)$$

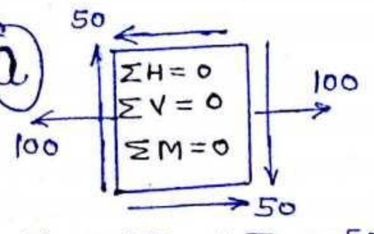
- Equation ① & ② are used to determine normal stress and shear stress on oblique plane passing through a plane under biaxial state of stress and uni-axial state of stress [$\sigma_x = \sigma$; $\sigma_y = \tau_{xy} = 0$]

- equations ③ & ④ are used to determine normal strain and shear strain on an oblique plane passing through a point under biaxial state of strain or plane strain

- Strain analysis at a point is similar to stress analysis at that point that is to obtain strain analysis equation Normal stress should be replaced by corresponding normal strain & shear stress replaced by half of the corresponding shear strain.

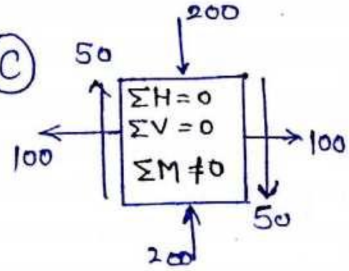
Que Which of the following figures represent biaxial state of stress at a point.

(a)



$\sigma_x = \sigma_y = 100$; $\sigma_y = 0$; $\tau_{xy} = -50\,MPa$

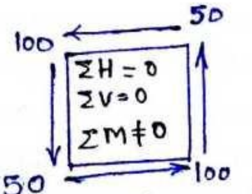
(c)



Ans (A)

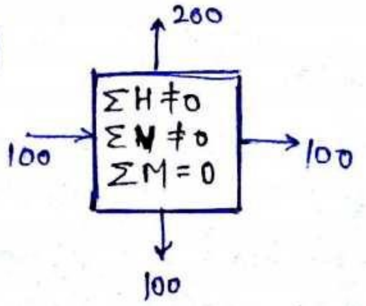
→ (in C No. Comp. shear stress)

(b)



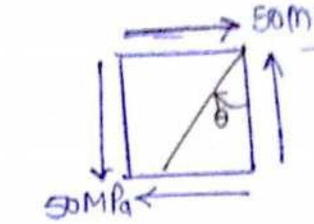
(Dirn & Mag. of Comp τ_s is incorrect.)

(d)



Dirn of σ_x & Mag. of σ_y are incorrect

⇒



$\sigma_x = \sigma_y = 0$

$\tau_{xy} = -50$ MPa

$(\sigma_n)_\theta \neq 0$

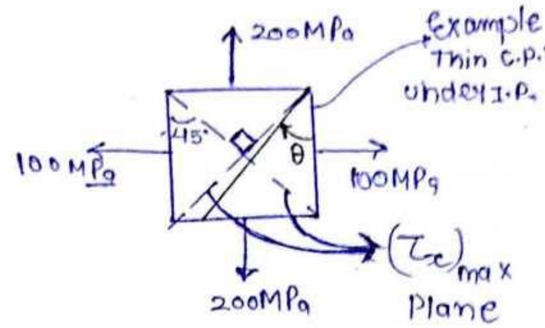
(a) $[\sigma]_{1D} = [\sigma_x]_{1\times1}$

$(\tau_s)_\theta \neq 0$

(a) bar under PAL
(b) beam under pure bending
(c) bar under EAL

} Principle plane {Directly we can tell bcoz $\tau_{xy} = 0$

Fig. Pure shear state of stress at a point.



$\sigma_x = 100$ MPa

$\sigma_y = 200$ MPa

$\tau_{xy} = 0$

$(\tau_s)_\theta \neq 0$

Fig. - Bi-axial pure normal state of stress at a point

Conclusion:-

1. State of stress at a point represent different stress develope on three mutual $\perp^r$ plane passing through a point
2. State of stress can be represented by following two methods (1) Graphical representation (2) Matrix rep^n.
3. Matrix rep^n of state of stress at a point is know as stress tensor.
4. every shear stress associated a comp. shear stress of equal mag. & unlike in nature. *Complimentory
5. Stress tensor is a square and symmetric matrix
6. Symetry of stress tensor is obtained by using moment or couple equ^m eq^n.
7. Complimentory shear stress also known as cross shear

(8) State of stress at a point should be in such a way that point should be in static equ^n

(9) No. of stress component in a stress tensor for a point under tri-axial state of stress are nine (9) [3 Normal & 6 shear stress]

(10) No. of stress component in a stress tensor for a point bi-axial state of stress are four [$\sigma_x, \sigma_y, \tau_{xy}$ & τ_{yx}] ie 2 Normal & 2 shear stress

(11) No. of stress component ~~in a stress tensor for~~ to define triaxial state of stress at a point are six ($\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{xz}$ & τ_{yz})

(12) No. of stress comp ~~in a stress tensor for a point~~ to define bi-axial state of stress at a point are three (σ_x, σ_y & τ_{xy})

Types of STRAIN:-

- strain
  - Normal strain (ϵ)
    - long. strain
    - lateral strain (or) transverse strain
    - Volumetric strain
      - (each of the above →) Tensile strain
      - (each of the above →) Compressive strain
  - Shear strain (γ)

$$\text{Normal strain } (\varepsilon) = \frac{\text{change in dim}^n}{\text{original dim}^n} = \frac{\Delta L}{L}$$

ΔL +ve → tensile

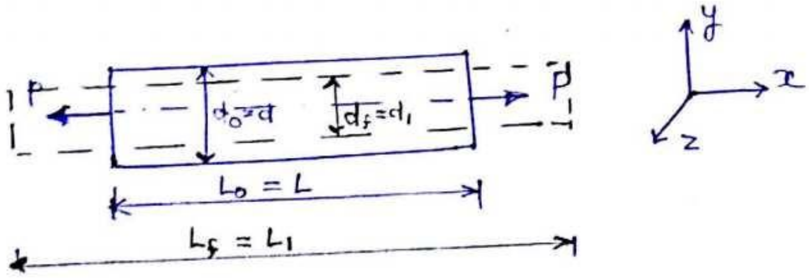
ΔL −ve → Compresive

* long. strain is the normal strain in the dir^n of applied load.

* lateral strain is the normal strain in a dir^n which is $\perp^r$ to the dir^n of applied load

* ~~[illegible]~~

* Every long. strain is associated with two lateral strains
* long. & lateral strain are unequal and unlike in nature.
* Under tri-axial loading, total nine normal strains are developed in three mutual ⊥r dirn (i.e. three long. & six. lateral strain)
* Under tri-axial loading, total normal strain in each mutual ⊥r dirn is equal to sum of one longitudinal & two lateral strains in that dirn.



$$\epsilon_{long.} = \epsilon_x = \frac{\delta L}{L_o} = \frac{L_1 - L_o}{L_o} \quad \Rightarrow \text{ E require}$$

$$\epsilon_{lateral} = \epsilon_y = \frac{\delta d}{d_o} = \frac{d_1 - d_o}{d_o} \quad \Rightarrow \text{ E \& } \mu \text{ require}$$

$$\epsilon_{lateral} = \epsilon_z = \frac{\delta d}{d_o} = \frac{d_1 - d_o}{d_o}$$

Bulk Modulus req. in case of hydrostatic

$$\epsilon_{volumetric} = \frac{\delta V}{V_o} = \epsilon_x + \epsilon_y + \epsilon_z$$

Rectangular bar!— $V = lbt$ —(1)

$$\delta V = \delta l[bt] + \delta b[lt] + \delta t[lb] \quad \text{—(2)}$$

$$\epsilon_V = \frac{\delta V}{V} = \underbrace{\frac{\delta l}{l}}_{\epsilon_x} + \underbrace{\frac{\delta b}{b}}_{\epsilon_y} + \underbrace{\frac{\delta t}{t}}_{\epsilon_z} \quad \text{—(3)}$$

Cylindrical bar :- $V = \frac{\pi}{4} d^2 l$ —①

$$\delta V = \frac{\pi}{4}\left[(\delta l) d^2 + l(2d)\,\delta d\right] \quad -②$$

$$\epsilon_v = \frac{\delta V}{V} = \frac{\delta l}{l} + 2\left(\frac{\delta d}{d}\right)$$

Cylindrical Pressure vassel (CPV) :

$$\epsilon_v = \frac{\delta V}{V} = (\epsilon)_{long} + 2(\epsilon)_{Hoop} = \frac{\delta L}{L} + 2\,\frac{\delta D}{D}$$

Spherical body :- $V = \frac{4}{3}\pi R^3$ or $\frac{\pi D^3}{6}$

$$\delta V = \frac{\pi}{6}\left[3D^2(\delta D)\right]$$

$$\epsilon_v = \frac{\delta V}{V} = 3\left[\frac{\delta D}{D}\right]$$

Spherical pressure Vassel :- (S.P.V.)

$$e_v = \frac{\delta V}{V} = 3\left[\epsilon_{Hoop}\right]$$

where $\epsilon_{Hoop} = \frac{\delta D}{D}$

eg

$P = 50kN$
$L_0 = 1m$
$d_0 = 50M$

we can't determine strains
so to determine we require
→ ϵ_{long} → Young's Modulus (E)
→ $\epsilon_{lateral}$ → Young's Modulus & Poission ration (E & μ)

eg.

Now if $P = 100\,kN$, $L_0 = 1\,m$, $d_0 = 50\,mm$

$E = 200\,GPa$, $\mu = 0.3$

find $L_f = ?$, $d_f = ?$, $V_f = ?$

Sol^n

$$\sigma_a = \frac{P}{A} = \frac{4P}{\pi d^2} = 50.92958\,MPa$$

$$E = \frac{\sigma_x}{\epsilon_x = \epsilon_{long}} \Rightarrow \epsilon_x / \epsilon_{long} = \frac{\sigma_x}{E} = 2.54647 \times 10^{-4}$$

$$\epsilon_{long} = \frac{\delta L}{L} = \frac{L_1 - L}{L} \Rightarrow L_1 = 1000.254\cancel{6}4\,mm$$

$$\boxed{\mu = -\frac{\epsilon_{lateral}}{\epsilon_{long}}} \Rightarrow \epsilon_{lateral} = -\mu\,\epsilon_{long} = -7.64 \times 10^{-5}$$

$$\epsilon_{lateral} = \epsilon_y = \epsilon_z = \frac{\delta d}{d} = \frac{d_1 - d}{d} = d_1 = 49.99618\,mm$$

$$\delta d = 0.003$$

$$\epsilon_V = \epsilon_x + \epsilon_y + \epsilon_z$$

$$= \epsilon_{long} + 2\epsilon_{lateral} = \epsilon_{long} - 2\mu\,\epsilon_{long} = 1.0185 \times 10^{-4}$$

$$\epsilon_V = \frac{\delta V}{V} = \frac{V_1 - V}{V}$$

Within the elastic Region

$$\boxed{\underset{\text{engineering}}{\sigma_E} \simeq \underset{\text{true}}{\sigma_T}}$$

$\because$ (change in x-s/c dim$^{ns} \simeq 0$
i.e. elastic def^n are very small)

Within the plastic Region

$$\boxed{\sigma_E \neq \sigma_T}$$

($\because$ change in x-s/c dim$^{ns} \neq 0$
i.e. Plastic deformation are very large.)

Under tensile loading Condition

$$\boxed{\sigma_T > \sigma_E} \qquad \boxed{\epsilon_T < \epsilon_E}$$

Under comp. loading Conditions

$$\boxed{\sigma_T < \sigma_E} \qquad \boxed{\epsilon_T > \epsilon_E}$$

$$\sigma_E = \frac{P}{A_o} \qquad \sigma_T = \frac{P_i}{A_i} \;; \quad \epsilon_E = \frac{L_f - L_o}{L_o}$$

$$\boxed{\sigma_T = \sigma_E(1+\epsilon_E)} \qquad \boxed{\epsilon_T = \ln(1+\epsilon_E)}$$

Assuming Change in Vol. = zero (i.e. $\mu = 0.5$)

eg

Tensile

$L_0 = L$

$L_f = 2L$

$\epsilon_E = \frac{2L-L}{L}$ $\quad \sigma_T = 2\sigma_E$

$\epsilon_E = 1$

$\epsilon_T = \ln(1+1)$

$\epsilon_T = 0.69$

$\boxed{\epsilon_E > \epsilon_T}$

$\boxed{\sigma_T > \sigma_E}$

Compressive

$L_0 = L$

$L_f = L/2$

$\epsilon_E = -0.5$ $\quad \sigma_T = \frac{\sigma_E}{2}$

$\epsilon_T = -0.69$

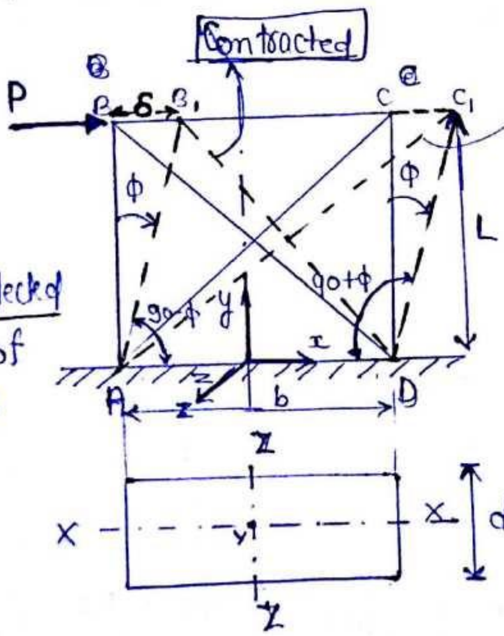
$\boxed{\epsilon_T > \epsilon_E}$

$\boxed{\sigma_E > \sigma_T}$

* Non of strain determine without elastic constant.

Shear Strain (γ):– shear strain is define as the change in initial right angle between two line elements which are ||el to X & Y axis.

Assumption
only TSL
effect of BM neglected because length of Block very less



ϕ = Shear Angle

$\boxed{\gamma = \phi}$

shear strain = Shear Angle

ΔABB_1

$\tan\phi = \frac{BB_1}{AB} = \frac{\delta}{L} \Rightarrow \phi = \frac{\delta}{L} \Rightarrow$ shear strain $\boxed{\gamma = \phi = \frac{\delta}{L}}$

For smaller Angle $\tan\phi \simeq \phi$

δ can't determine without knowing γ

To determine $\gamma \Rightarrow$ Shear Modulus req.

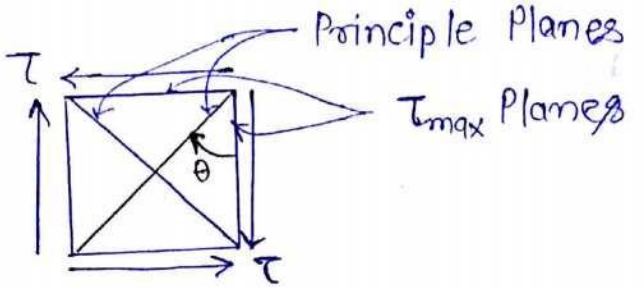
Theoretically

(1) $\tau_d = \dfrac{P}{A}$

(2) $G = \dfrac{\tau}{\gamma} \Rightarrow \gamma = \dfrac{\tau}{G}$ ← shear Modulus

(3) $\phi = \gamma \Rightarrow \boxed{\delta = \phi L}$

<u>State of Shear</u>



$$[\sigma]_{2D} = \begin{bmatrix} 0 & \tau \\ \tau & 0 \end{bmatrix}$$

$$[\epsilon]_{2D} = \begin{bmatrix} 0 & \gamma/2 \\ \gamma/2 & 0 \end{bmatrix}$$

ϵ_x, $\dfrac{\gamma_{xy}}{2}$, ϵ_y

$$(\epsilon_n)_{\theta=45^\circ} = \frac{1}{2}[0+0] + \frac{1}{2}[0-0]\cos 90^\circ + \left(\frac{\gamma}{2}\right)\sin 90^\circ$$

$(\epsilon_n)_{\theta=45^\circ} = \gamma/2$ (tensile)

$(\epsilon_n)_{\theta=135} = -\gamma/2$ (comp.)

$(\gamma_s)_{\theta=45} = -[0-0]\sin 90^\circ + \gamma\cos 90^\circ = 0$

$(\gamma_s)_{\theta=135} = 0$ Shear strain is zero so these Plane are called principle plane

Elastic Constant :- (E.C.)

→ Elastic constant are used to determine strain theoretically

→ Elastic constant are used to obtained stress - strain relationship.

→ For a homogeneous and isotropic material the number of elastic Constant are four

[i.e. E - Young Modulas

G or C - Shear Modulas / Modulas of rigidity

K - Bulk Modulas

μ or $\frac{1}{m}$ - Poission ratio]

→ For a homogeneous & isotropic materials the no. of independent elastic Constants are two '2'

[i.e. E & μ or $1/m$]

Materials	Independents E.C.
isotropic	2
orthotropic	9
Anisotropic	21

ORthotropic :- A material said to orthotropic when it exhibits diff. elastic properties in orthogonal dir^n (mutual $\perp^n$) at a given point.

eg. thin layered Material - Graphite, plywood

Relationship between Elastic Constants :-

$$E = 2G(1+\mu) \Rightarrow G = \frac{E}{2(1+\mu)}$$

$$E = 3k(1-2\mu) \Rightarrow K = \frac{E}{3(1-2\mu)}$$

$$E = \frac{9kG}{3k+G}$$

For any engineering materials $E, k, G > 0$

but $\mu \geq 0$

* $\boxed{\text{For cork, } \mu = 0}$

If k should be +ve, $1 - 2\mu \geq 0$

$\Rightarrow \mu \leq \frac{1}{2}$

So for any engineering material

$$\boxed{0 \leq \mu \leq \frac{1}{2}}$$

Upper and lower limits for 'G' & 'k' in terms of 'E'

μ	$G = \frac{E}{2(1+\mu)}$	$k = \frac{E}{3(1-2\mu)}$
0	$E/2$	$E/3$
$1/2$	$E/3$	∞

$\mu(\uparrow) \rightarrow G(\downarrow) \rightarrow k(\uparrow)$ $\boxed{\frac{E}{3} \leq G \leq \frac{E}{2}}$ $\boxed{\frac{E}{3} \leq k \leq \infty}$

For Metals :- $\boxed{E > K > G}$

μ	G	K
0.25	0.4E	0.67E
$\frac{1}{3}$	0.375 E	E

if $\mu < \frac{1}{3} \Rightarrow E > K$

$\mu = \frac{1}{3} \Rightarrow E = K$

$\mu > \frac{1}{3} \Rightarrow K > E$

Matetrial	Poission Ration (μ)
Cork	0
Concrete	0.1 to 0.2
Metals	$\frac{1}{4}$ to $\frac{1}{3}$
Rubber, Paraffin, WAX	0.5 $\Rightarrow \delta V = 0$ $\Downarrow$ Perfect plastic deformation

Definition of E.C. :-

As per Hook's law, up to proportional limit (P.L.)

$\Rightarrow \sigma \propto \epsilon_{long} \Longrightarrow \sigma = E\,\epsilon_{long}$

measure of elasticity

<u>Young's Modulas</u> $\boxed{E = \frac{\sigma}{\epsilon_{long}}}$ — Under Uni-axial state of stress.

ratio of stress & strain

$\Rightarrow$ slope of σ Vs ϵ_{long}. Curve up to P.L.

$\Rightarrow \tau \propto \gamma \Longrightarrow \tau = G\gamma$

<u>Shear Modulas</u> $\Rightarrow \boxed{G = \frac{\tau}{\gamma} = \frac{\tau}{\phi}}$ — under pure shear state of stress.

$\Rightarrow$ slope of τ Vs γ Curve upto P.L.

$\Rightarrow \sigma \propto \epsilon_v \Longrightarrow \sigma = K\epsilon_v$

<u>Bulk Modulas</u> $\boxed{K = \frac{\sigma}{\epsilon_v} \text{ or } \frac{-P}{\epsilon_v}}$ — Under hydro static state of stress.

i.e. $\sigma_x = \sigma_y = \sigma_z = \sigma$

$\tau_{xy} = \tau_{xz} = \tau_{yz} = 0$

$\Rightarrow$ slope of σ Vs ϵ_v Curve up to P.L.

<u>Poission Ratio</u>

$$\boxed{\mu = -\left(\frac{\text{lateral strain}}{\text{long. strain}}\right) = \frac{-\epsilon_y}{\epsilon_x} \text{ or } \frac{-\epsilon_z}{\epsilon}}$$

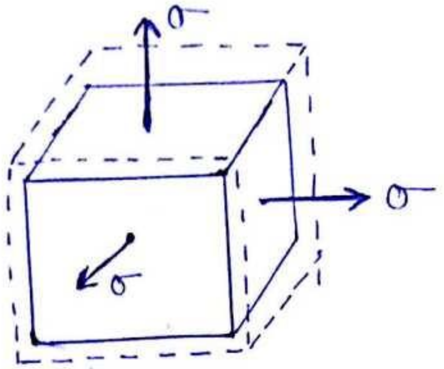
lateral strain $= -\mu$ (longitudinal strain)

$$\boxed{\epsilon_{lat.} = -\mu\,\epsilon_{long.}}$$

Under pure axial tensile load. (Tension test)

$$\mu = -\frac{\epsilon_{lateral}}{\epsilon_{long.}} = -\left[\frac{\left(\frac{d_f - d_0}{d_0}\right)}{\left(\frac{L_f - L_0}{L_0}\right)}\right]$$

Hydrostatic state of stress

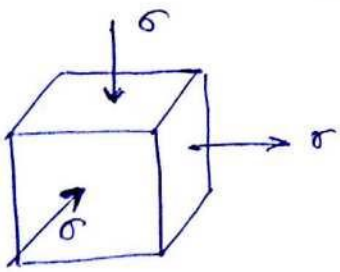


$$[\sigma]_{3D} = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \end{bmatrix}$$

⇒ "only Volume change i.e. No distorsion bcoz No shear stress.

⇒ Every planes are principle plane (No shear stress) i.e. τ_s on every oblique planes zero.

⇒

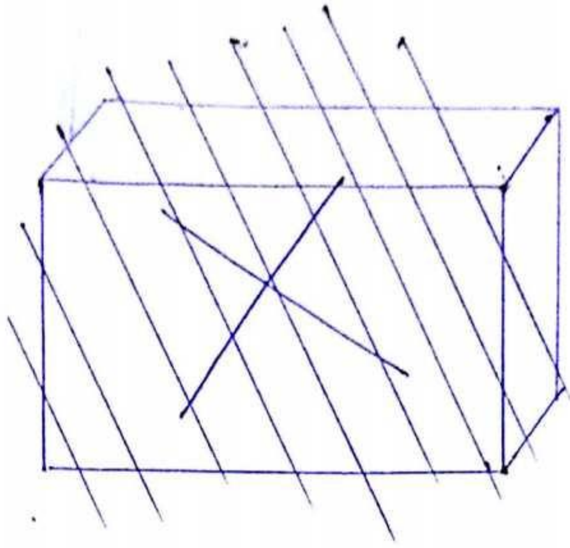


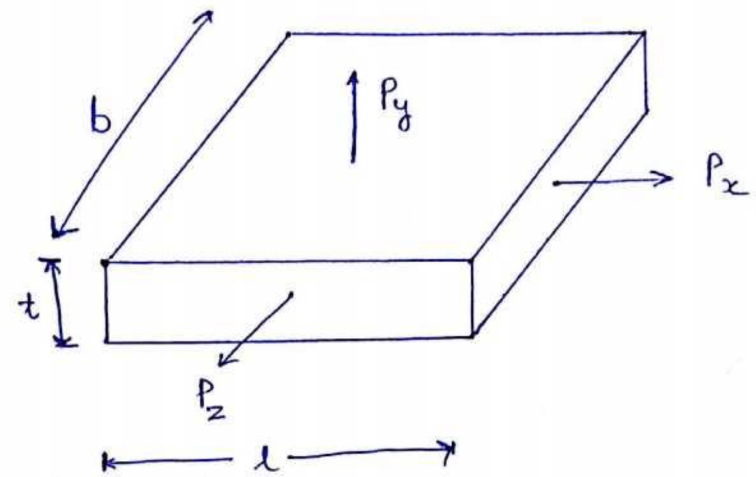
⇒ only three P.P.

⇒ volume change as well distortion occur.

July 2016

Expression for Vol. Metric Strain (ϵ_v) Under Tri-axial loading





$$\sigma_x = \frac{P_x}{bt} \quad ; \quad \sigma_y = \frac{P_y}{lb} \quad ; \quad \sigma_z = \frac{P_z}{lt}$$

$$\epsilon_v = \frac{\delta V}{V} = \epsilon_x + \epsilon_y + \epsilon_z \quad \text{— (1)}$$

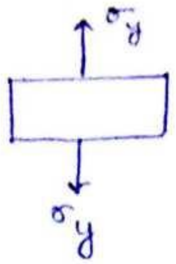
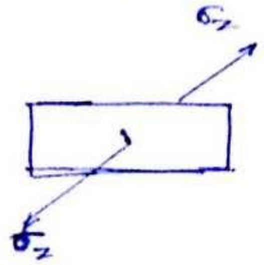
$$\epsilon_x = \frac{\sigma_x}{E} \qquad\qquad \epsilon_x = \frac{\sigma_x}{E}$$

$$\epsilon_y = \frac{-\mu\sigma_x}{E} \qquad\qquad \epsilon_y = \frac{\sigma_y}{E}$$

$$\epsilon_z = \frac{-\mu\sigma_x}{E} \qquad\qquad \epsilon_z = \frac{\sigma_z}{E}$$

Valid Under Uni-axial state of stress

Valid under $\mu = 0$ triaxial State of stress.

Strain	X-dir^n	Y-dir^n	Z-dir^n
Load	ϵ_{long}	$\epsilon_{lateral}$	$\epsilon_{lateral}$
	$\frac{\sigma_x}{E}$	$\frac{-\mu\sigma_x}{E}$	$\frac{-\mu\sigma_x}{E}$
	$\frac{-\mu\sigma_y}{E}$	$\frac{\sigma_y}{E}$	$\frac{-\mu\sigma_y}{E}$
	$\frac{-\mu\sigma_z}{E}$	$\frac{-\mu\sigma_z}{E}$	$\frac{\sigma_z}{E}$

Total strain in x-y-z dir^n

$$\left.\begin{aligned}
(\epsilon_x)_{total} &= \frac{1}{E}\left[\sigma_x - \mu(\sigma_y+\sigma_z)\right] = \frac{\delta l}{l} \\
(\epsilon_y)_{total} &= \frac{1}{E}\left[\sigma_y - \mu(\sigma_x+\sigma_z)\right] = \frac{\delta t}{t} \\
(\epsilon_z)_{total} &= \frac{1}{E}\left[\sigma_z - \mu(\sigma_x+\sigma_y)\right] = \frac{\delta b}{b}
\end{aligned}\right\} \quad -②$$

Above equ^n are used to determine total strain in three mutual $\perp^r$ dimensions passing through a point and change in dim^{ns} under any state of stess condition.

by sub. eqns (2) in eqn (1)

$$\boxed{\epsilon_v = \frac{\delta V}{V} = \left(\frac{1-2\mu}{E}\right)\left[\sigma_x + \sigma_y + \sigma_z\right]} \quad -(3)$$

⇒ In hydro static case

$$\sigma_x = \sigma_y = \sigma_z = \sigma$$

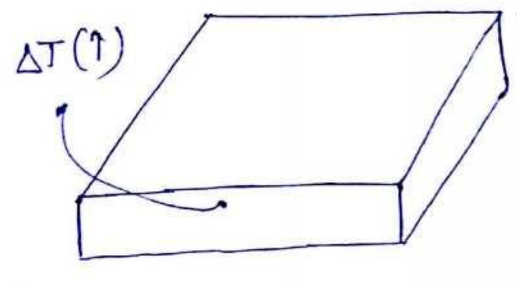
$$\boxed{\epsilon_v = \frac{\delta V}{V} = \frac{3\sigma(1-2\mu)}{E}}$$

$$E = 3k(1-2\mu)$$

$$k = \frac{E}{3(1-2\mu)}$$

$$\frac{E}{3(1-2\mu)} = \frac{\sigma}{\epsilon_v} \Rightarrow \boxed{k = \frac{\sigma}{\epsilon_v}}$$

⇒



To det. ϵ_v only eqn (1) should be used

$\because \sigma_x = \sigma_y = \sigma_z = 0$

From eqn (3) $\delta V = 0$ if $\mu = 0.5$

(or) $\sigma_x + \sigma_y + \sigma_z = 0$

→ If poission ratio of a material 0.5 change in volume equal to under any state of stess condition

$\mu = 0.5$

→ If $\mu < 0.5$ for a material change in volume is equal to zero when sum of the stess in 3 mutual $\perp^r$ dirn is equal to zero (i.e. ⇒ $\sigma_x + \sigma_y + \sigma_z = 0$)

For uniaxial ~~stress~~ state of stress condⁿ

$\sigma_x = \sigma \; ; \; \sigma_y = 0 \; ; \; \sigma_z = 0$

$$\boxed{\epsilon_v = \left(\frac{1-2\mu}{E}\right)(\sigma)}$$

if $\mu = 0.5 \Rightarrow \delta V = 0$

if $\mu \neq 0.5 \Rightarrow \delta V \neq 0$

A bar under Pure axial ~~force~~ load

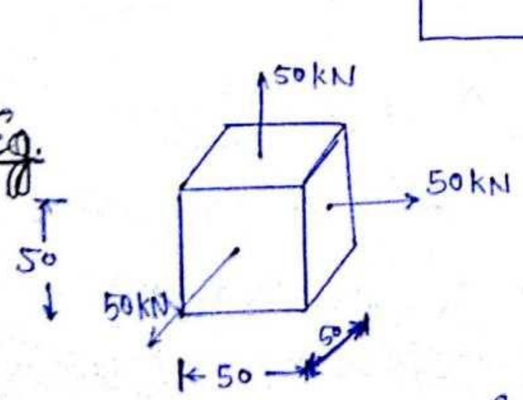
$$\sigma_x = \sigma = P/A$$

$$\Rightarrow \epsilon_v = \frac{\delta V}{V} = \frac{(1-2\mu)}{E}(\sigma)$$

$$\frac{\delta V}{A \times L} = \frac{(1-2\mu)}{E}\left(P/A\right)$$

Change in Volume. $$\boxed{\delta V = \frac{(1-2\mu)(PL)}{E}}$$

Eg.



Determine $\delta V = ?$ $E = 200\,GPa$, $\mu = 0.25$

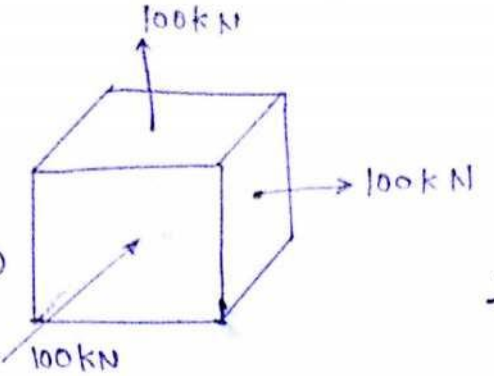
$$\sigma_x = \sigma_y = \sigma_z = \sigma = \frac{P}{A} = \frac{50000}{50 \times 50} = 20\,MPa$$

$$\epsilon_v = \frac{\delta V}{V} = \frac{3\sigma(1-2\mu)}{E} \Rightarrow \delta V = \frac{V\,3\sigma(1-2\mu)}{E}$$

$$\delta V = \frac{(50)^3 \times 20 \times (1-2\times 0.25)}{200 \times 10^3} \Rightarrow \delta V = 18.75\,mm^3$$

Ex

Cube of Side = 50mm



Cube of Side = 50MM

Det. $\delta V = ?$

if $K = 100$ GPa, $\mu = 0.25$

Sol^n

$$E = 3K(1-2\mu)$$

$$= 300 \times \tfrac{1}{2}$$

$$\underline{E = 150 \text{ GPa}}$$

$$\sigma_x = \sigma_y = \frac{100 \times 10^3}{50 \times 50} = 40 \text{ MPa}$$

$$\sigma_z = -\sigma_x = -40 \text{MPa}$$

$$\varepsilon_v = \frac{(1-2\mu)}{E.}(\cancel{\sigma_x} + \sigma_y \cancel{+ \sigma_z})$$

$$\varepsilon_v = \frac{(1-2 \times 0.25)}{150 \times 10^3} \times 40$$

$$\varepsilon_v = \frac{\delta V}{V} = \frac{20}{150 \times 10^3}$$

$$\delta V = \frac{\cancel{50} \times \cancel{50} \times 50 \times 2\cancel{0}}{3\,\cancel{150} \times \cancel{10^3}}$$

$$\delta V = \frac{50}{3} \text{ mm}^3$$

Ques. Determine the following when a prismatic bar having length $\underline{1\ m}$ & 30 mm diameter is subjected to Axial tensile load of ~~500 kN~~ $\underline{5000\ kN}$

i) $\mu = ?$ & $E = ?$

ii) $\Delta l = ?$ & $\Delta V = ?$ assume final diameter $\Rightarrow d = 29.891\ mm$

& $G = 70\ GPa$

Soln

$$\sigma_x = \frac{P}{A} = \sigma = \frac{500 \times 10^3}{\frac{\pi}{4}(30)^2} = 707.35\ MPa$$

$$\Delta l = \frac{PL}{AE} \qquad E = 2G(1+\mu)$$

$$\mu = \frac{-\Delta d/d}{\Delta l/l} \Rightarrow \mu = \frac{0.109}{30} \times \frac{1000}{\Delta l}$$

$$\mu = \frac{109}{30 \times \Delta l}$$

$$\frac{109}{30\mu} = \frac{5000 \times 10^3 \times 1000}{\underbrace{706.83} \times 2 \times 70 \times \cancel{10^3} \times (1+\mu)}$$

$$\frac{1+\mu}{\mu} = \frac{[illegible]}{[illegible]}$$

$$\frac{1+\mu}{\mu} = 13.9$$

$$12.9 = \frac{1}{12.9}$$

$$.0775$$

$$\mu = 0.0775$$

$$\epsilon_{lateral} = -\mu \, \epsilon_{long}$$

$$\frac{d_1 - d_0}{d_0} = -\mu \frac{\sigma}{E}$$

$$\frac{29.891 - 30}{30} = \frac{-\mu}{E} \frac{500 \times 10^3 \times 4}{\pi \times (30)^2}$$

$$\frac{\mu}{E} = 5.13650 \times 10^{-6}$$

$$E = 2G(1+\mu)$$

$$\mu = 0.0775$$

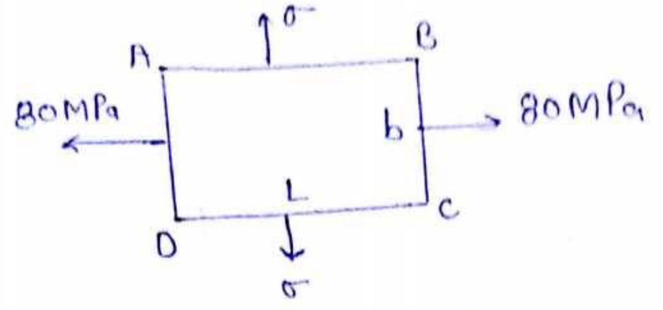
$$E = 150.84 \text{ MPa}$$

$$\delta L = \frac{PL}{AE} = \frac{4 \times 5000 \times 10^3 \times 10^3}{\pi (30)^2 \times 150.84 \times 10^3} = 46.89 \text{ mm}^2$$

$$\delta V = \frac{1-2\mu}{E}(PL) = \left[\frac{(1 - 2 \times 0.077)}{150.84 \times 10^3}\right] \times (8000 \times 10^6)$$

$$\delta V =$$

eg The state of stress on an element as shown in fig $E = 200$ GPa, $\mu = 0.3$ then mag. of stress σ for no strain BC is



$$\epsilon_x = \frac{\delta L}{L} \quad ; \quad \epsilon_y = \frac{\delta b}{b}$$

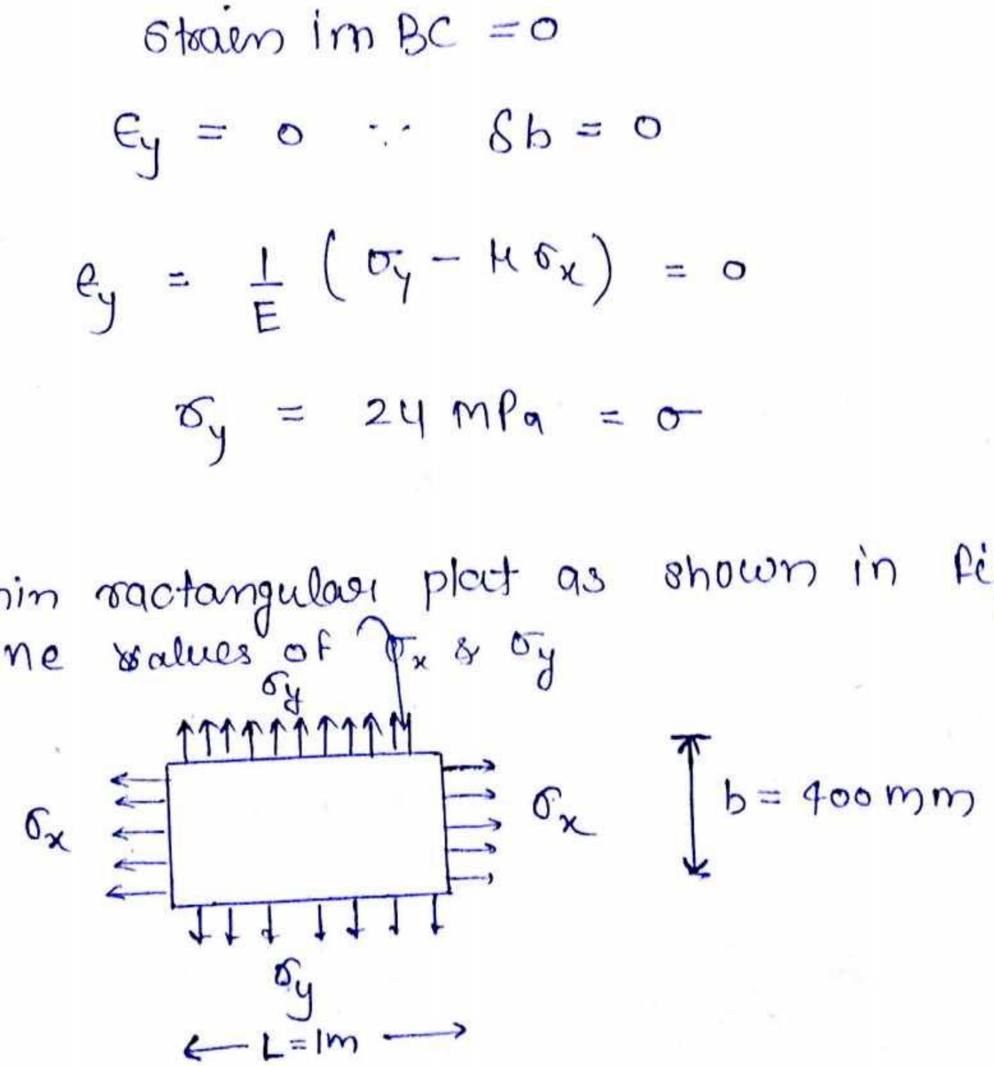
Strain in BC $= 0$

$$\epsilon_y = 0 \quad \because \quad \delta b = 0$$

$$e_y = \frac{1}{E}\left(\sigma_y - \mu\sigma_x\right) = 0$$

$$\sigma_y = 24 \text{ MPa} = \sigma$$

Ques For thin rectangular plat as shown in fig determine values of σ_x & σ_y



Assume $\delta L = 0.5$ mm

$\delta b = 0.1$ mm

$E = 200$ GPa

$\mu = 0.25$

$\underline{Sol^n}$

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\mu \sigma_y}{E}$$

$$\epsilon_y = \frac{-\mu \sigma_y}{E} + \frac{\sigma_x}{E}$$

θ.

$$\epsilon_x = \frac{\Delta l}{l} = \frac{1}{E}\left(\sigma_x - \mu \sigma_y\right)$$

$$\frac{0.5}{1000} = \frac{1}{200\times10^3}\left(\sigma_x - 0.25\,\sigma_y\right)$$

$$\sigma_x - 0.25\,\sigma_y = 100 \text{ MPa} \quad — (1)$$

$$\epsilon_y = \frac{\Delta b}{b} = \frac{1}{E}\left(\sigma_y - \mu \sigma_x\right)$$

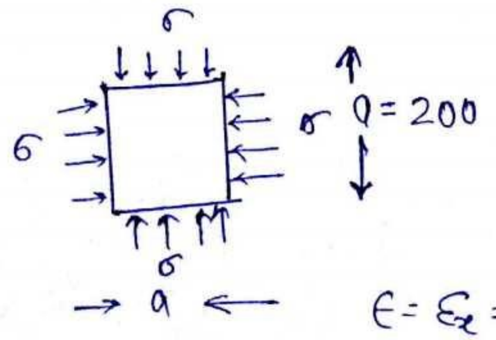
$$\frac{0.1}{400\times10} = \frac{1}{200\times10^3}\left(\sigma_y - 0.25\,\sigma_x\right)$$

$$\sigma_y - 0.25\,\sigma_x = \cancel{100}\ 50 \text{ MPa} \quad — (2)$$

from (1) & (2) $\sigma_x = 120$ MPa

$\sigma_y = 80$ MPa

$\underline{eg}$



get $\sigma = ?$

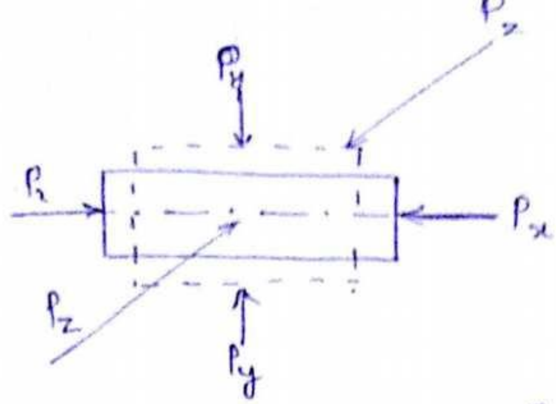
$\delta a = 0.2$ MM

$E = 200$ GPa $\quad \mu = 0.25$

$$\epsilon = \epsilon_x = \epsilon_y = \frac{\delta a}{a} = \frac{0.2}{200} = \frac{1}{E}\left(\sigma - \mu\sigma\right)$$

$$\sigma = 266.67 \text{ MPa}$$

Q.13
work
book



$\sigma_y = \sigma_z = ?$ if $\epsilon_y = \epsilon_z = 0$

$$\epsilon_y = \frac{1}{E}\left(\sigma_y - \mu\sigma_x - \mu\sigma_z\right)$$

$$\sigma_y - \mu\sigma_x - \mu\sigma_y = 0$$

$$\sigma_y = \frac{\mu\sigma_x}{(1-\mu)}$$

Q.24
work book

$$\cancel{\epsilon_x = \frac{1}{E}\left(\sigma_1 - \mu(2\sigma_2)\right)}$$

$$\cancel{\epsilon_y = \frac{1}{E}\left(\sigma_2 - \mu(\sigma_1+\sigma_2)\right)}$$

$$\frac{1}{\cancel{8}E}\left(\sigma_1 - 2\mu\sigma_2\right) = \cancel{\frac{1}{2E}\left(\sigma_2 - \mu\sigma_1 - \sigma_2\mu\right)}$$

$$\cancel{\sigma_1 - 2\mu\sigma_2 = 2\sigma_2(1-\mu) - 2\mu\sigma_1}$$

$$\cancel{\sigma_1(1+2\mu) = 2\sigma_2 - 2\sigma_2\mu + 2\sigma_2\mu}$$

$$\cancel{\sigma_2 = \frac{\sigma_1}{2}}$$

$$\cancel{2\sigma_1 - 4\mu\sigma_2 = \sigma_2 - \mu\sigma_1 - \sigma_2\mu}$$

Q.24

$\underline{\sigma_y = \sigma_z = ?}$ if $\epsilon_y = \epsilon_z = \frac{1}{2}$ [strain in y & z dirn when P_x acting alone

$$\epsilon_y = \frac{1}{E}\left(\sigma_y - \mu(\sigma_x + \sigma_z)\right) = \frac{1}{2}\left(-\frac{\mu\sigma_x}{E}\right)$$

$$\sigma_y - \mu(\sigma_x + \sigma_z) = -\frac{\mu\sigma_x}{2}$$

$$\sigma_y = \frac{\mu\sigma_x}{2(1-\mu)}$$

$$\sigma_2 = \frac{\mu\sigma_1}{2(1-\mu)}$$

* BARS IN SERIES & PARALLEL :-

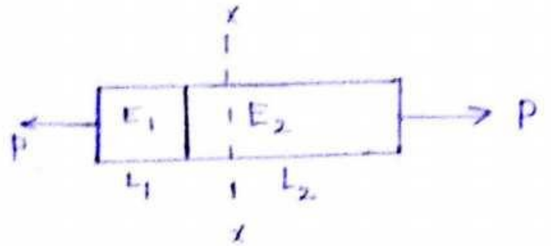
$$\boxed{\sigma_a = \frac{P}{A} = \frac{4P}{\pi d^2}}$$

$$\boxed{\delta L = \frac{PL}{AE} = \frac{4PL}{\pi d^2 E}}$$

Condition to be satisfied for above eqns :-

1. bar should be a prismatic bar
2. bar should be under pure axial loading
3. bars should be made of same material.

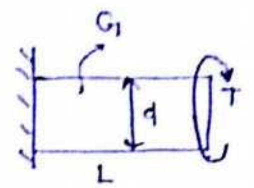
e.g.

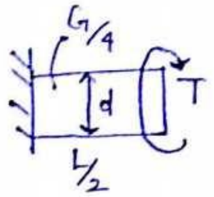


$$\boxed{\sigma_{x-x} = \sigma_{1-1} = \sigma_{2-2} = P/A}$$

$$\boxed{(\delta L)_{1-1} \neq (\delta L)_{2-2}}$$

eg





$\tau_1 = 100$ mpa

$\tau_2 = ? = \tau_1 = 100$ MPa
$[\because T_1 = T_2 = T]$

* Load & X-S/C are same so stresses are same

$\tau_2 = 100$ MPa

$[\because T_1 = T_2 = T]$ & $[\because d_1 = d_2 = d]$

* In static loading ~~condition~~ Cond^n stress is only dependend on load and X-S/C (dim^s), Not depend on material.

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$$

Series :-

Conditions.

① Axial deformations are cumulativ

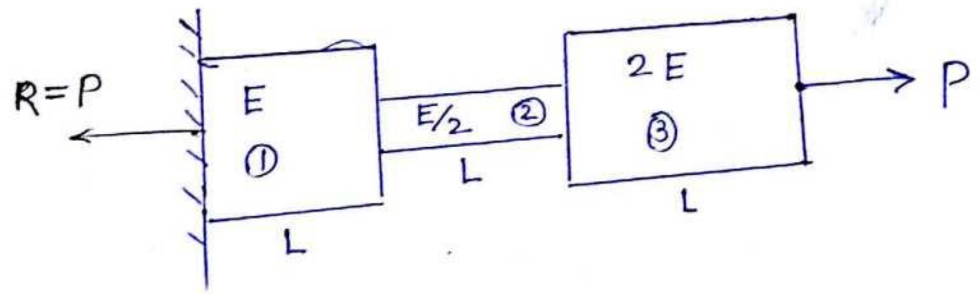
$$\text{i.e.} \left[(\delta L)_{total} = (\delta L)_1 + (\delta L)_2 + (\delta L)_3 \right]$$

② Axial loads are equal & like in nature

$$\text{i.e} \left[P_1 = P_2 = P_3 \cdots = P_n = P \right]$$

Valid when axial loads are applied at the extreme ends only.

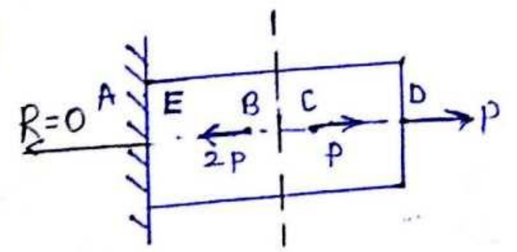
eq.



loads in every bar is equal

$$P_1 = P_2 = P_3 = P$$

eq



$$P_{AB} = 0$$

$$P_{BC} = 2P$$

$$P_{CD} = P$$

Parallel :- (composite bars) {Statically Indeterminate bars}

Condⁿs

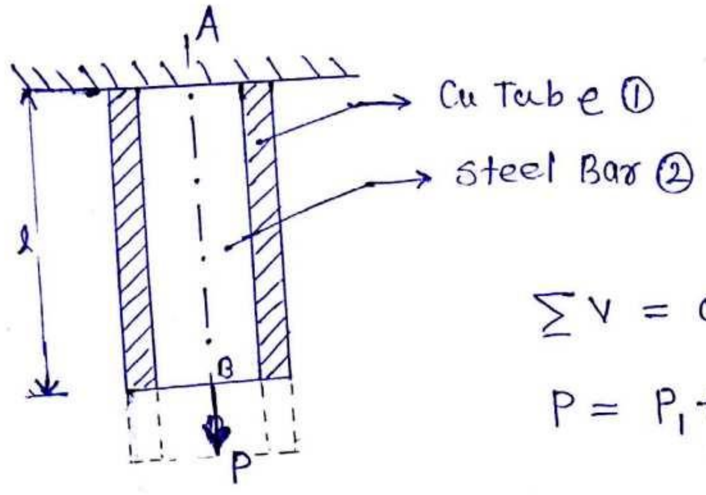
(1) Axials loads are cumulative

i.e $\left[P = P_1 + P_2\right]$

(2) Axial deformation are equal & like in nature

i.e. $\left[\delta L = \delta L_1 = \delta L_2\right]$

e.g.



$$\sum V = 0$$

$$P = P_1 + P_2 \quad -①$$

Statically indeterminate bar

[Total no. of reacⁿs or unknowns] > [No. of useful static equᵐ eqⁿs]

So use compatability eqⁿs

$$(\delta L) = (\delta L)_1 = (\delta L)_2 \quad -②$$

$$\frac{P_1 L}{A_1 E_1} = \frac{P_2 L}{A_2 E_2}$$

$$(\epsilon_{long})_1 = (\epsilon_{long})_2 \quad [\because L_1 = L_2]$$

Case I

load P given, det. $\sigma_{1,2} = ?$

From eqn (2) $\delta L_1 = \delta L_2$

$$\frac{P_1 L_1}{A_1 E_1} = \frac{P_2 L_2}{A_2 E_2}$$

$$\frac{P_1}{P_2} = \frac{A_1 E_1}{A_2 E_2} \quad -(a) \qquad P = P_1 + P_2 \quad -(b)$$

$$\boxed{\sigma_1 = P_1/A} \quad \boxed{\sigma_2 = P_2/A}$$

* AE = Axial rigidity of the x-s/c
- $EI_{N.A.}$ = Flexural rigidity of the x-s/c
- GJ = Torsional rigidity of the x-s/c

⇒ Higher the rigidity, lower the deflection or deformation.

$$\delta L = \delta L_1 \text{ or } \delta L_2 = \frac{P_1 L_1}{A_1 E_1} \text{ OR } \frac{P_2 l_2}{A_2 E_2}$$

change in length of Composite bar

Similarly in torsion

$$\boxed{(\delta L)_{C.B.} = \frac{PL}{\sum_{i=1}^{2} A_i E_i}}$$

⇒ $P = P_1 + P_2$

$$P = \frac{A_1 E_1 \delta L_1}{L_1} + \frac{A_2 E_2 \delta L_1}{L_2} \Rightarrow P = \frac{\delta L}{L}(A_1 E_1 + A_2 E_2)$$

$$\Rightarrow \boxed{\delta L = \frac{PL}{A_1 E_1 + A_2 E_2}}$$

Case-(2) Either σ_1 (or) σ_2 is given

load $P = ?$

from (a) $\frac{P_1}{P_2} = \frac{A_1 E_1}{A_2 E_2}$

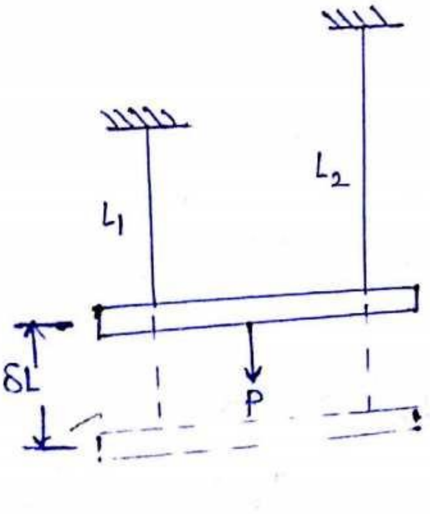
$$\boxed{\frac{\sigma_1}{\sigma_2} = \frac{E_1}{E_2}} \quad -(c)$$

$$P = P_1 + P_2 \quad -(d)$$

by using eqn (c) & (d) load P can be determine

$$\delta L = \frac{PL}{A_1E_1 + A_2E_2}$$

<u>Ex.</u>



Bar should remain H.Z.

So both bars equally elongated

$$(\delta L)_1 = (\delta L)_2$$

$$\frac{P_1 L_1}{A_1 E_1} = \frac{P_2 L_2}{A_2 E_2}$$

$$\frac{\sigma_1}{\sigma_2} = \frac{E_1}{E_2} \times \frac{L_2}{L_1}$$

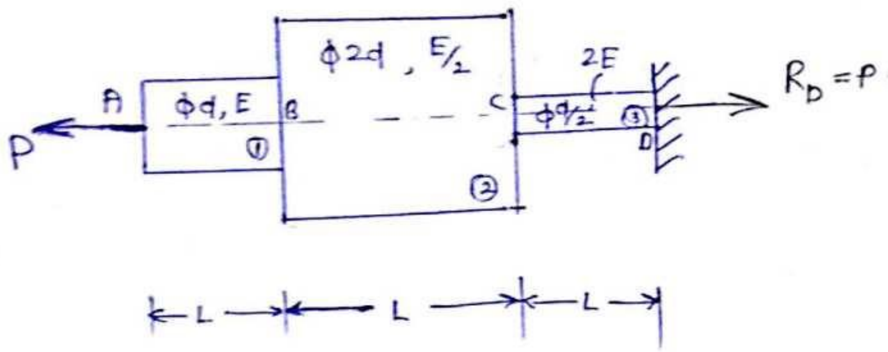
Now we can find stress in Another string series

Case -1

Ques. For the stappered bar as shown in fig determine

(i) max^n ~~stress~~ stress induced on x-S/C of bar

(ii) elongation of the bar



$$P_1 = P_2 = P_3 = P$$

$$\sigma_{max} = \sigma_3 = \frac{4P_3}{\pi d_3^2} = \frac{4P}{\pi (d/2)^2} = \frac{16P}{\pi d^2}$$

$$\sigma_1 = \frac{4P_1}{\pi d_1^2} = \frac{4P}{\pi d^2}$$

$$\sigma_{min} = \sigma_2 = \frac{4P_2}{\pi (d_2)^2} = \frac{4P}{\pi (2d)^2} = \frac{P}{\pi d^2}$$

$$\frac{\sigma_{max}}{\sigma_{min}} = \frac{\sigma_3}{\sigma_2} = \frac{A_2}{A_3} = \frac{(2d)^2}{(d/2)^2} = 16$$

For a given Axial load (P)

$$\boxed{\sigma_{axial} \propto \frac{1}{A} \text{ (OR) } \frac{1}{d^2}}$$

$$(\delta L)_{total} = (\delta L)_1 + (\delta L)_2 + (\delta L)_3$$

$$\underset{\substack{\swarrow \quad \searrow \\ \text{Free end} \quad \text{fixed end}}}{\delta_{AD}} = \delta_{AB} + \delta_{BC} + \delta_{CD}$$

$$(\delta_L)_{total} = PL\left[\frac{1}{\frac{\pi}{4}d^2E} + \frac{1}{\frac{\pi}{4}(2d)^2\cdot\frac{E}{2}} + \frac{1}{\frac{\pi}{4}(d/2)^2 2E}\right]$$

$$= \frac{4PL}{\pi d^2 E}\left[1 + \frac{1}{2} + 2\right]$$

<u>Note</u>

$$\delta_B = \delta = \delta_{BD} = \delta_{BC} + \delta_{CD}$$

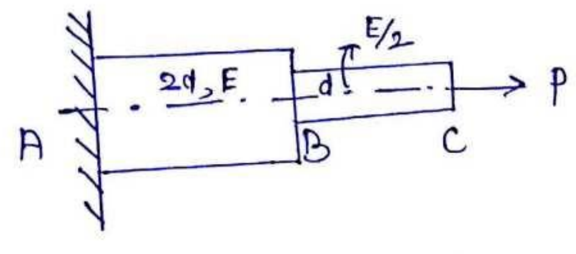
$$\delta_A = \delta_{AD} = \delta_{AB} + \underbrace{\delta_{BC} + \delta_{CD}}_{\delta}$$

$$\frac{\delta_{AB}}{\delta_{BC} + \delta_{CD}} = \frac{\left(PL/AE\right)_{AB}}{\left(\frac{PL}{AE}\right)_{BC} + \left(\frac{PL}{AE}\right)_{CD}}$$

Alway w.r.t. fixed end

$$(\delta L)_{total} = \frac{14PL}{\pi d^2 E} \text{ mm (elongation)}$$

<u>Ex.</u>



$\delta_C = ?$ if $\delta_B = \delta$

$$\delta_B = \delta_{BA} = \delta$$

$$\delta_C = \underset{\substack{\Downarrow \\ ?}}{\delta_{CB}} + \underset{\substack{\Downarrow \\ \delta}}{\delta_{BA}}$$

$$\frac{\delta_{CB}}{\delta_{BA}} = \frac{\frac{4PL}{\pi d^2 E/2}}{\frac{4PL}{\pi (2d)^2 E}} = 4 \times 2 = 8$$

$$\delta_{CB} = 8\,\delta_{BA} = 8\delta$$

$$\delta_C = 8\delta + \delta = \underline{\underline{9\delta}}$$

Case-2

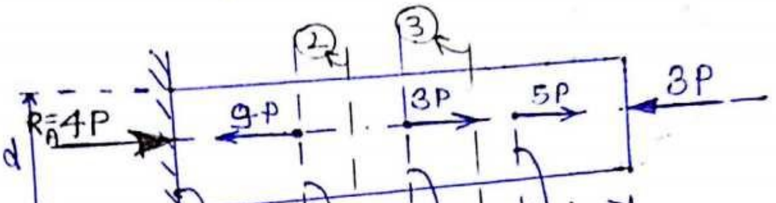
Ques. For the prismatic as shown in Fig Determine-

i) Max. axial tensile load and max. axial compressive load

ii) Expression for the diameter of bar if permisable stress equal to σ MPa

$\sigma_{per} = \sigma$ MPa

iii) Change in length of the bar.



$\Sigma H = 0$

$-3P + 5P + 3P - 9P + R_A = 0$

$\boxed{R_A = 4P}$

M.O.S.

i) $P_1 = P_A = 4P$ [Comp]

$P_2 = 9P - 4P = 5P$ [tensile]

$P_3 = 5P - 3P = 2P$ [tensile]

$P_4 = P_E = 3P$ [Comp.]

max. tensile load $P_2 = 5P$

max. Comp. load $P_1 = 4P$

ii) d:- $\sigma_{max} = \sigma_2 = \frac{P_2}{A_2} = \frac{4P_2}{\pi d_2^2} = \frac{20P}{\pi d^2}$ MPa

safe Cond^n for desing

$(\sigma_{max})_{ind} \leq \sigma_{per}$

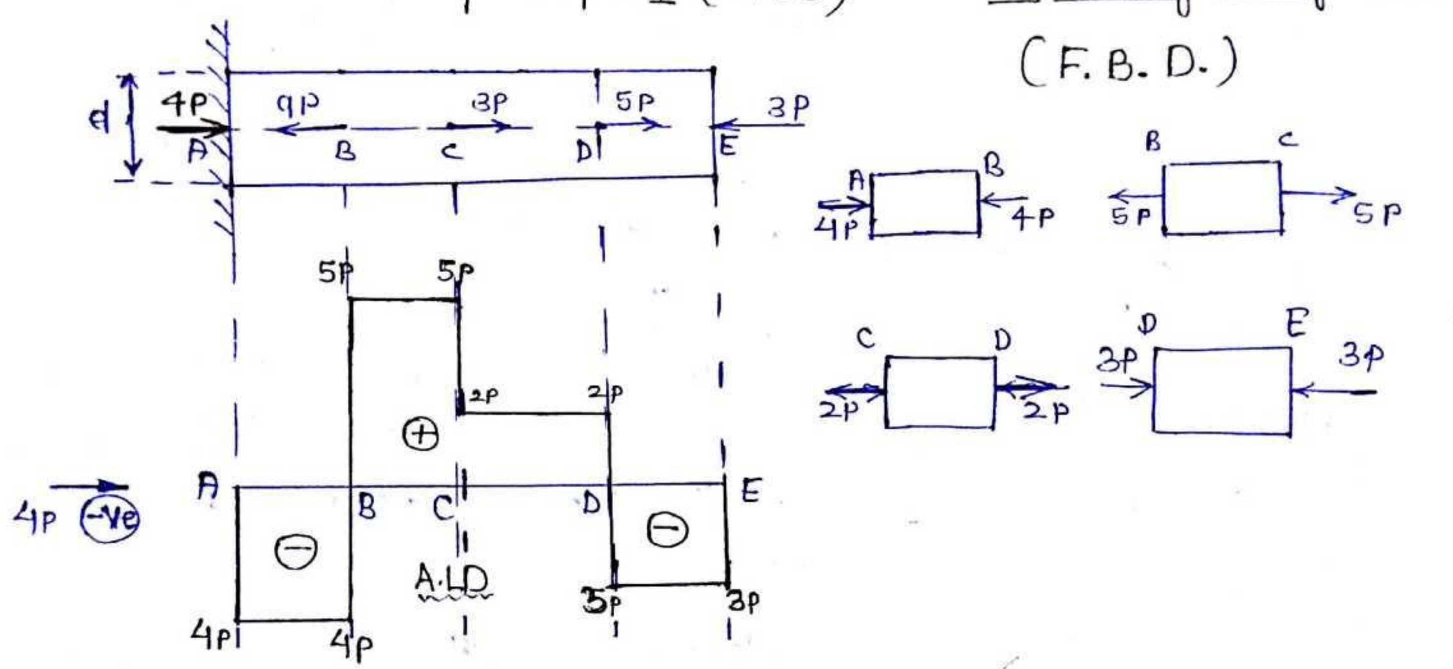
$\frac{20P}{\pi d^2} \leq \sigma \Rightarrow d \geq \sqrt{\frac{20P}{\pi \sigma}}$ mm

(iii) $(\delta L)_{total} = (\delta L)_1 + (\delta L)_2 + (\delta L)_3 + (\delta L)_4$

$$= \frac{L}{AE}\left[-4P + 5P + 2P - 3P\right]$$

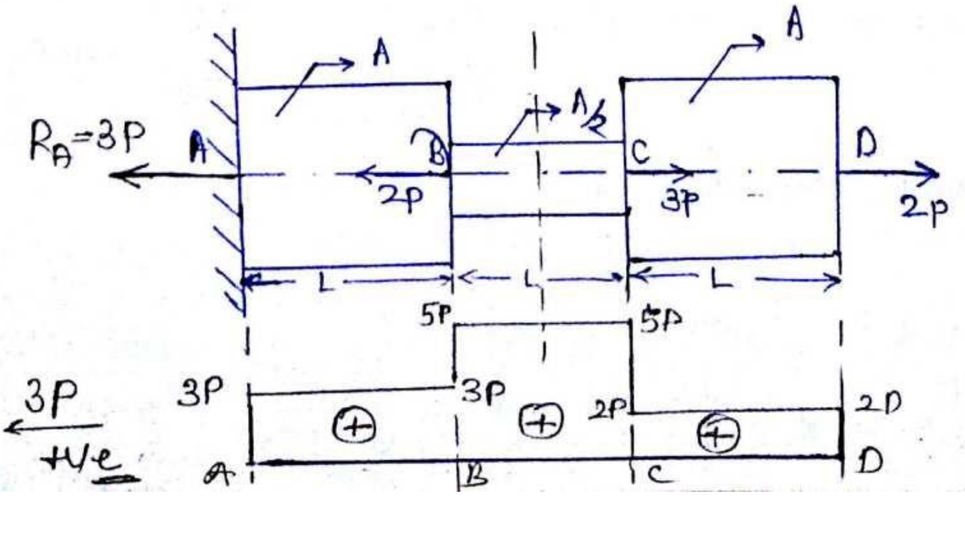
$$(\delta L)_{total} = \text{zero}.$$

Axial loading diagram (ALD)   Free body Diagram (F.B.D.)



Ques For the steppered bar as shown in figure
i) Determine maximum value for load P if permisable stress in AB, BC & CD are 100 MPa, 150 MPa & 50 MPa.

$A = 200\,mm^2$



$P_1 = 3P \ (T)$

$P_2 = 5P \ (T)$

$P_3 = 2P \ (T)$

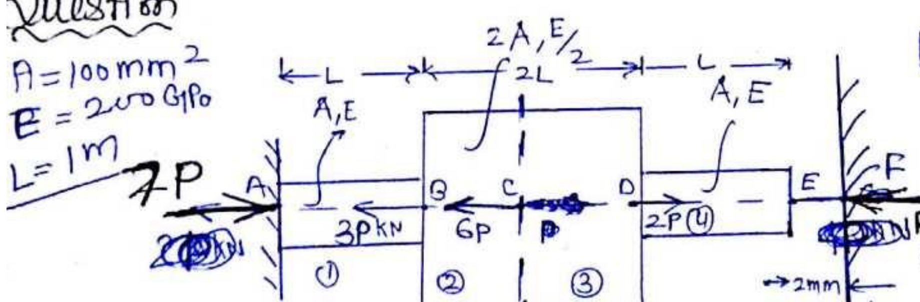
Safe Cond^n For AB	Safe Cond^n for BC	Safe Cond^n for CD
$\sigma_1 \le \sigma_{per}$	$\sigma_2 \le \sigma_{per}$	$\sigma_3 \le \sigma_{per}$
$\frac{P_1}{A_1} \le 100$	$\frac{P_2}{A_2} \le 150$	$\frac{P_3}{A_3} \le 50$
$\frac{3P}{200} \le 100$	$\frac{5P}{100} \le 150$	$\frac{2P}{200} \le 50$
$P \le 6.67\text{ kN}$	$P \le 3\text{ kN}$	$P \le 5\text{ kN}$

Max. safe value for P = min of the above values

$$P = 3\text{ kN}$$

Question

$A = 100\text{ mm}^2$
$E = 200\text{ GPa}$
$L = 1\text{ m}$



Determine load P if the gap b/w bar & fixed end should be zero

Sol^n $\cancel{P_1 = P_{AB} = 3P}$ $\quad \cancel{P_3 = 4P}$

$\cancel{P_2 = 5P}$ $\quad \cancel{P_4 = 4P}$

$$(\delta L)_{total} = \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2} + \frac{P_3 L_3}{A_3 E_3} + \frac{P_4 L_4}{A_4 E_4}$$

if $\delta_E = (\delta_L)_{bar} \le 2\text{mm} \Rightarrow R_F = 0 \Rightarrow$ Static determinate bar

if $(\delta_L)_{bar} > 2\text{ mm} \Rightarrow R_F \ne 0 \Rightarrow$ Static indet. bar

$\Sigma H = 0 \qquad R_H - 3P - 6P + 2P = 0$

$$R_H = 7P$$

$P_1 = -7P$

$P_2 = -7P + 3P = -4P$

$P_3 = 2P$

$P_4 = R_A = 0$

$\cancel{\frac{-7RL}{AE}}$

$(\delta_L)_{bar} = \text{Gap}$

$$\frac{P_1L_1}{A_1E_1} + \frac{P_2L_2}{A_2E_2} + \frac{P_3L_3}{A_3E_3} + \frac{P_4L_4}{A_4E_4} = 2 \text{ mm}$$

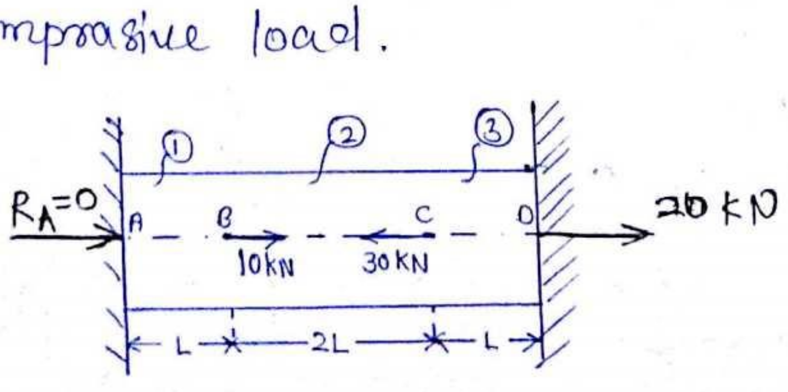
$$\frac{-7P\times1000}{100\times200\times10^3} + \frac{-4P\times1000}{200\times100\times10^3} + \frac{2P\times1000}{200\times100\times10^3} + 0 = 2$$

$$P = -4.44 \text{ kN}$$

Imp ☆☆

Case-3 Bar Fixed at Both the ends :-
(Statically Indeterminate bar)

Ques For the prismatic bar as shown in figure (i) Ratio of Reaction (ii) max. tensile load & max. compressive load.



For objective Exam ($A_1E_1 = A_2E_2 = A_3E_3$)

① Net axial load = 20 kN (←)

② Introduce reactions at one fixed end in a dir^n opposite direction of net axial load

③ $$R_A = \frac{(-10)(3L) + (30)(L)}{4L}$$

→ if load in same dir^n of rxn load ⇒ −ve (−10)
→ if load in opposite dir^n of rxn load ⇒ +ve (+30)
→ Distance from another fixed end (3L & L)
$\quad\quad\quad\quad\quad\quad$ 10kN $\quad$ 30kN

$$R_A = zero$$

④ $P_1 = -R_A = 0$

$P_2 = -R_A - 10 = -10\ kN$

$P_3 = P_D = R_D = 20\ kN$

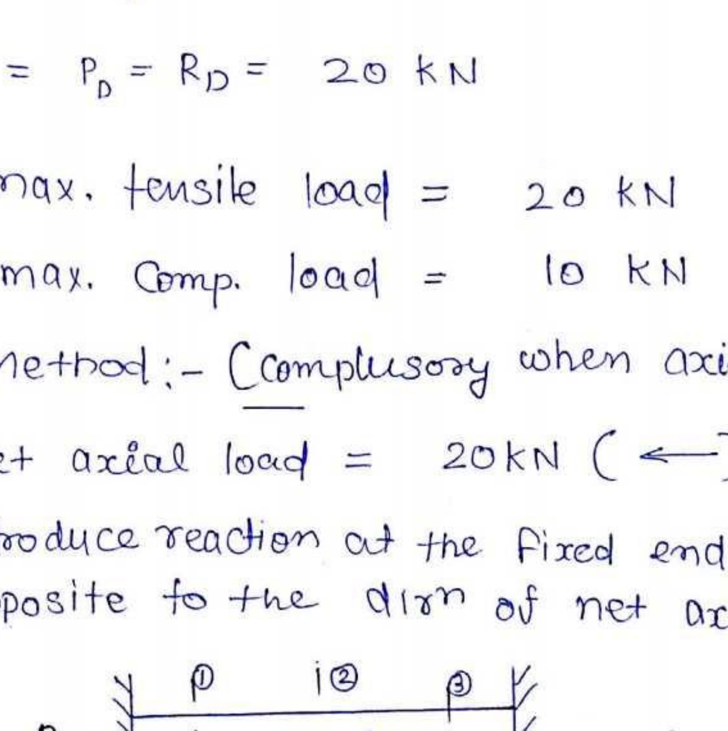
max. tensile load = 20 kN

max. Comp. load = 10 kN

Actual Method :- (Complusory when axial rigidities are unequal)

① Net axial load = 20kN (←)

② Introduce reaction at the fixed ends in a dir^n opposite to the dir^n of net axial load.



③ $P_1 = -R_A$; $P_2 = -R_A - 10$; $P_3 = R_D$ or $-R_A - 10 + 30$
$= 20 - R_A$

④ $\Sigma H = 0 \Rightarrow -R_A - 10 + 30 - R_D = 0$

$$R_A + R_D = 20\ kN \quad -①$$

⑤ Compatibility eqn

$$(\delta L)_{total} = (\delta L)_1 + (\delta L)_2 + (\delta L)_3 = 0 \quad (2)$$

⑥ From ②

$$\frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2} + \frac{P_3 L_3}{A_3 E_3} = 0$$

$$\frac{1}{AE}\left(-R_A L + (-R_A - 10)(2L) + (20 - R_A) L\right) = 0$$

$$\frac{L}{AE}\left(-R_A - 2R_A - \cancel{20} + \cancel{20} - R_A\right) = 0$$

$$\frac{L}{AE} \neq 0$$

$$4R_A = 0$$

$$R_A = 0$$

From eq ① $R_D = 20 - R_A$

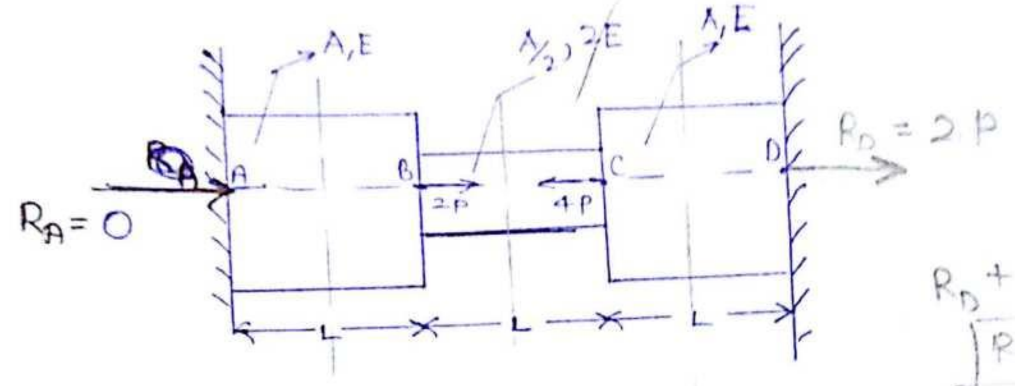
$$R_D = 20 \text{ kN}$$

$P_1 = -R_A = 0$

$P_2 = -R_A - 10 = -10 \text{ kN} = 10 \text{ kN (comp)}$ max. comp.

$P_3 = 20 - R_A = 20 \text{ kN}$ max. tensile

Question For stappered bar as shown in fig Determine i) max. axial load, ratio max. & mini axial stress and deformation at B & C.



$R_D + 2P = 4P$

$\boxed{R_D = 2P}$

$R_A = 0$

Net axial load $= 2P(\leftarrow)$

$$R_A = \frac{(-2P)(2L) + (4P)(L)}{3L}$$

$$R_A = 0$$

$P_{AB} = 0$ $\quad P_{CD} = 2P$

$P_{BC} = -2P$

max ~~axial~~ Comp load $= -2P$

max. tensile load $= 2P$

$$\sigma_{max} = \sigma_{BC} = \left(\frac{P}{A}\right)_{BC} = \frac{-2P}{A/2} = \frac{4P}{A} \text{ (comp.)}$$

$$\frac{\sigma_{max}}{\sigma_{min}} = \frac{\sigma_{BC}}{\sigma_{AB}} = \infty$$

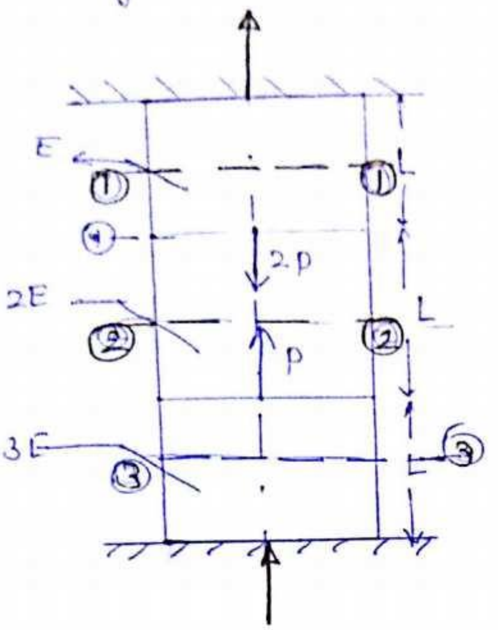
$\delta_B = \delta_{BA}$ or $\delta_{BD} = \delta_{BC} + \delta_{CD}$

$\delta_B = \delta_{BA} = 0$ (load is zero.

$\delta_C = \delta_{CA}$ or δ_{CD} $\quad \delta_C = \delta_{CD} = \frac{PL}{AE} = \frac{2P \times L}{AE} = \frac{2PL}{AE}$ mm $(\leftarrow)$

$\delta_{CB} + \delta_{BA}$

Ques:- For the vertical stepped bar as shown in fig. ~~Find out net axial load~~ Determine (a) $R_A/R_D = ?$ (b) $\sigma_{max} = ?$ (c) $\frac{\sigma_{max}}{\sigma_{min}} = ?$



Sol^n

1.) Net axial load $= P(\downarrow)$

2)

3) $P_1 = R_A$

$P_2 = R_A - 2P$

$P_3 = R_A - 2P + P = R_A - P$

(4) $\Sigma V = 0 \Rightarrow R_A - 2P + P + R_D = 0$

$$R_A + R_D = P$$

(5) $(\delta_L)_{total} = \delta_1 + \delta_2 + \delta_3 = 0$

$$\frac{L}{A}\left[\frac{R_A}{E} + \frac{R_A - 2P}{2E} + \frac{R_A - P}{3E}\right] = 0$$

$$\frac{L}{6AE}\left(6R_A + 3R_A - 6P + 2R_A - 2P\right) = 0$$

$$\frac{L}{6AE} \neq 0 \qquad R_A = \frac{8}{11}P\ (\uparrow)$$

$$R_D = \frac{3}{11}P\ (\uparrow)$$

(6) $P_1 = \frac{8P}{11}\ (T)$

$P_2 = \frac{8P}{11} - 2P = -\frac{14}{11}P$ (OR) $\frac{14P}{11}\ (C) \Rightarrow \sigma_{max} = \frac{14P}{11A}$

$P_3 = \frac{8P}{11} - P = -\frac{3P}{11}$ or $\frac{3P}{11}\ (C) \Rightarrow \sigma_{min} = \frac{3P}{11A}$ $\quad \frac{\sigma_{max}}{\sigma_{min}} = \frac{14}{3}$

$$\sigma_2 = \sigma_{max} = \frac{14P}{11A}$$

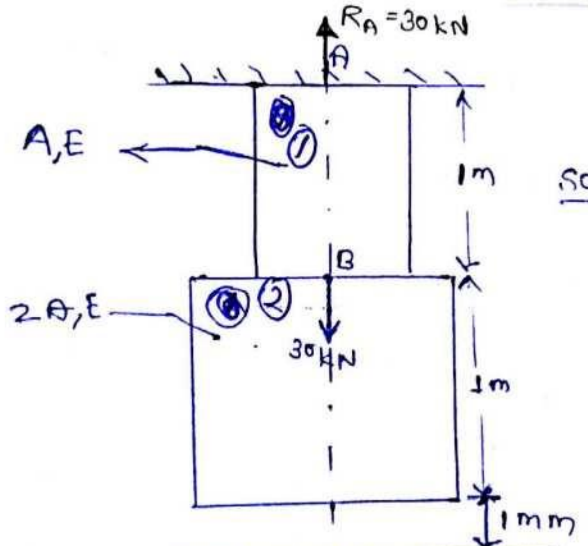
$$\sigma_3 = \sigma_{min} = \frac{3P}{11A}$$

$$\frac{\sigma_{max}}{\sigma_{min}} = \frac{14}{3}$$

(8) $$\delta_B = \delta_{BA} = \delta_1 = \frac{P_1 L_1}{A_1 E_1} = \frac{8PL}{11AE} \text{ mm } (\downarrow)$$

$$\delta_C = \delta_{CD} = \delta_3 = \frac{P_3 L_3}{A_3 E_3} = \frac{-PL}{11AE} \text{ mm or } \frac{PL}{11AE} (\downarrow)$$

Ques Vertical stepped bar shown in fig det. max. induced stress in bar.



$A = 100 \text{ mm}^2$
$E = 100 \text{ GPa}$

Sol^n Assuming $R_D = 0$ ($\because \delta_{bar} \le 1$ mm

$P_1 = 30kN$

$P_2 = $ zero

$$\delta_{bar} = \delta_1 + \delta_2$$

$$\delta_{bar} = \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2}$$

$$= \frac{30 \times 10^3 \times 1000}{1000 \times 100 \times 10^3} + 0 = 3 \text{ mm}$$

$$\delta_{bar} = 3 \text{ mm}$$

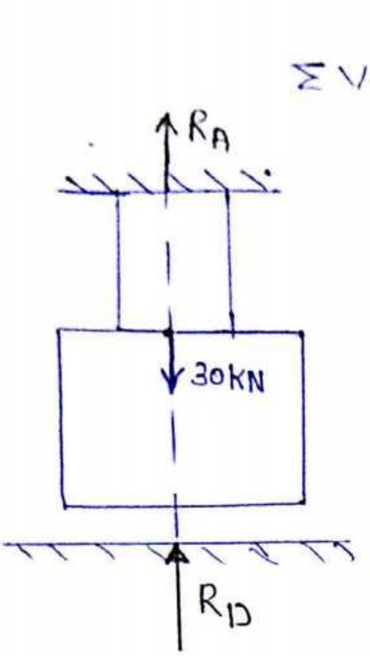
deformation of bar $\delta_{bar} > 1$

$$\Rightarrow R_D \neq 0$$

Hence, bar is a statical indeterminate bar

$P_1 = R_A \; ; \; P_2 = -R_D$ or $R_A - 30$

$\Sigma V = 0 \rightarrow R_A + R_D = 30 \quad \text{---} (1)$



$$\delta_{bar} = \delta_1 + \delta_2 = 1 mm$$

$$\frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2} = 1 \text{ mm}$$

$$\frac{R_A \times 1000}{100 \times 100 \times 10^3} + \frac{(R_A - 30000)(1000)}{200 \times 100 \times 10^3} = 1$$

$$R_A = 16.67 \text{ KN } (\uparrow)$$

From eqn (1) $\quad R_D = 13.33 \text{ kN } (\uparrow)$

$$P_1 = R_A = 16.67 \text{ KN } (T)$$
$$P_2 = -R_D = 13.33 \text{ kN } (C)$$

$$\sigma_{max.} = \sigma_1 = \frac{P_1}{A_1} = \frac{16.67 \times 10^3}{100}$$

$$\sigma_{max.} = 166.7 \text{ MPa } (T)$$

If Applied load was 10 kN

So. $P_1 = 10$ kN

$P_2 =$ zero

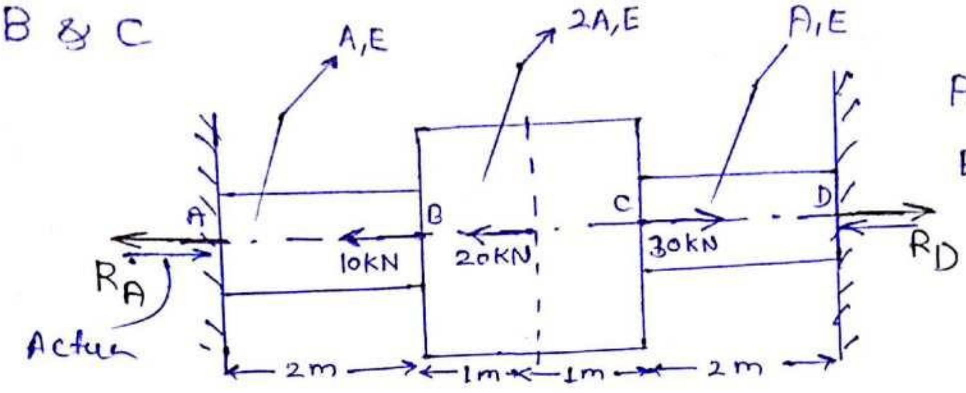
$$\sigma_{max} = \frac{10 \times 10^3}{100}$$

$$\sigma_{max} = 100 \text{ MPa}$$

$$\delta_{bar} = \frac{P_1 L_1}{A_1 E_1} + 0 = \frac{10 \times 10^3 \times 1000}{100 \times 100 \times 10^3} = 1 \text{ mm}$$

Ques

for Non prismatic bar as shown in fig determine max. tensile and max. comp. ~~least~~ stress and def^n at B & C



$A = 100\ mm^2$

$E = 100\ GPa$

$P_1 = R_A$

$P_2 = R_A + 10$

$P_3 = R_A + 30$

$P_4 = R_A + 30 - 30 = R_A = R_D$

$\Sigma H = R_A + 10 + 20 - 30 + R_D = 0$

$R_A = R_D$

equal & opposite

$$\delta_{total} = \delta_1 + \delta_2 + \delta_3 + \delta_4 = 0$$

$$\frac{R_A(2000)}{100 \times 100 \times 10^3} + \frac{(R_A+10)(1000)}{2 \times 100 \times 10^3 \times 100} + \frac{(R_A+30)\times(1000)}{2 \times 100 \times 10^3 \times 100} + \frac{R_A \times 2000}{100 \times 10^2 \times 10^3} = 0$$

$$\frac{1}{10^5}\left[\frac{R_A \times 2000}{100} + \frac{(R_A+10)(1000)}{200} + \frac{(R_A+30)\,1000}{200} + \frac{R_A \times 2000}{100}\right] = 0$$

$R_A = -4\ KN$ @ $4\ KN\ (\rightarrow)$

$R_D = -4\ KN$ @ $4\ KN\ (\leftarrow)$

$P_1 = 4\ KN\ (C)$; $P_3 = 26\ KN\ (T)$

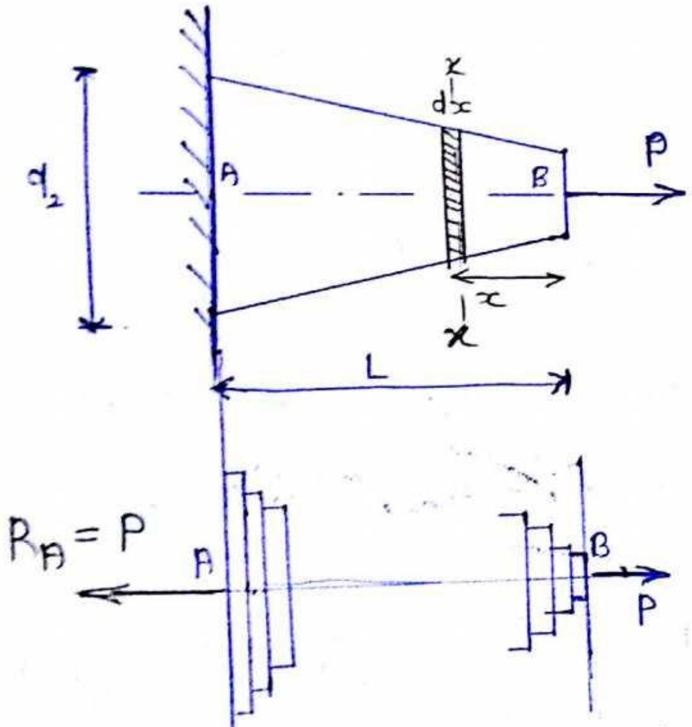
$P_2 = 6\ KN\ (T)$ $P_4 = 4\ KN\ (C)$

$$\delta_B = \delta_{BA} = \frac{-P_1 L_1}{A_1 E_1} = \frac{-4\times10^3 \times 2000}{100 \times 100 \times 10^3}$$

$$\delta_B = -0.8 \text{ mm} \text{ (or) } 0.8\text{mm (Towards left)}$$

$$\delta_c = \delta_B = 0.8 \text{ mm (towards right)}$$

Elongation of A Tapered Bar Under axial load :-



* A tapered bar treated as assembly of 'n' bars of diff diameter which are in series.

$$P_1 = P_2 = P_3 - - = P_n$$

$$\sigma_{max} = \sigma_B = \frac{4P_B}{\pi d_B^2} = \frac{4P}{\pi d_1^2} = \frac{4P}{\pi d_{smaller}^2}$$

$$\frac{\sigma_{max}}{\sigma_{min}} = \frac{\sigma_B}{\sigma_A} = \left(\frac{d_A}{d_B}\right)^2 = \left(\frac{d_2}{d_1}\right)^2 = \left(\frac{d_{larger}}{d_{smaller}}\right)^2$$

$$(\delta L)_{T.B.} = \delta_1 + \delta_2 + - - + \delta_n$$

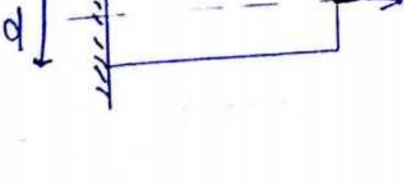
$$(\delta L)_{T.B.} = \int_0^L (\delta L)_{strip} = \int_0^L \frac{(P_{x-x})\,dx}{(AE)_{x-x}}$$

$$= \int_0^L \frac{4P\,dx}{\pi d^2 E}$$

$$d = \left[d_1 + (d_2-d_1)(x/L)\right]$$

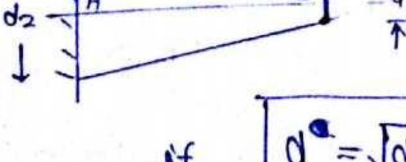
$$(\delta L)_{T.B.} = \int_0^L \frac{4P\,dx}{\pi\left[d_1 + (d_2-d_1)\frac{x}{L}\right]^2 E}$$

$$\boxed{(\delta L)_{T.P.} = \frac{4PL}{\pi d_2 d_1 E}}$$



$$(\delta L)_{P.B.} = \frac{4PL}{\pi d^2 E}$$

$$(\delta L)_{T.B} = \frac{4PL}{\pi d_1 d_2 E}$$



If $\boxed{d = \sqrt{d_1 d_2}} \rightarrow (\delta L)_{P.B.} = (\delta L)_{T.B.}$

↳ Geometric mean of Dia. of tapered bar.

Elongation of a prismatic bar under axial load is equal to elongation of a taper bar when dia of P.B. bar is G.M. of diameters of T.B.

i.e. $d^2 = \sqrt{d_1 d_2}$

Ques

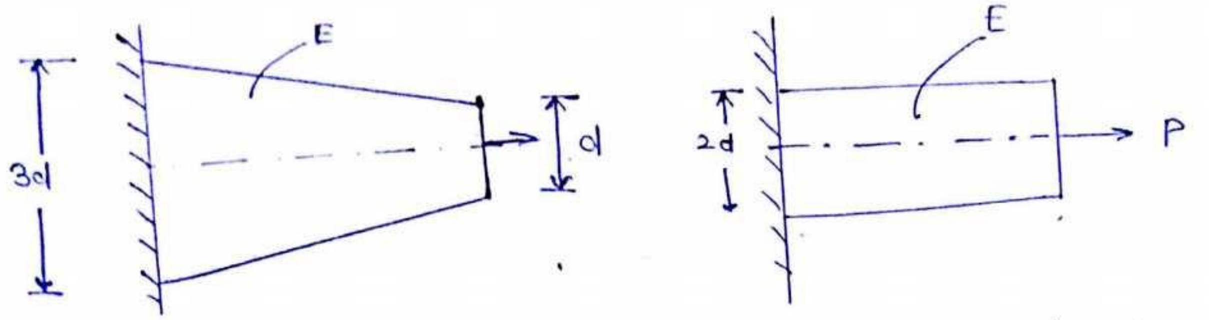


Fig.- Actual bar   Fig:- Assumed bar for cal^n

Det (a) % of error in calculated elongation ?
(b) % of error in calculated max. stress ?

$$\delta_{actual} = (\delta L)_{T.B.} = \frac{4PL}{\pi d_1 d_2 E} = \frac{1}{3}\left[\frac{4PL}{\pi d^2 E}\right]$$

$$\delta_{cal} = (\delta L)_{P.B.} = \frac{4PL}{\pi d^2 E} = \frac{1}{4}\left[\frac{4PL}{\pi d^2 E}\right]$$

$$\left\{\begin{matrix}\text{\% of error} \\ \text{in Cal. elong}^n\end{matrix}\right\} = \frac{\delta_{actual} - \delta_{cal}}{\delta_{actual}} \times 100$$

$$= \frac{(1/3) - (1/4)}{(1/3)} \times 100$$

% error = 25 %

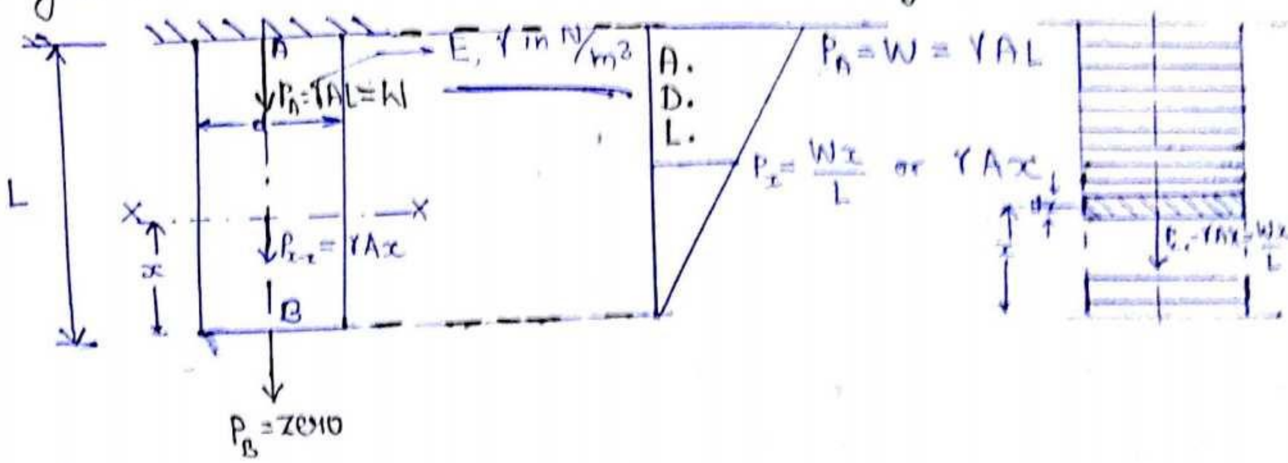
$$\left(\sigma_{actual}\right)_{max} = \left(\sigma_{max}\right)_{T.B.} = \frac{4P}{\pi d^2_{smaller}} = \frac{4P}{\pi d^2}$$

$$\left(\sigma_{cal}\right)_{max} = \left(\sigma_{max}\right)_{PB} = \frac{4P}{\pi d^2} = \frac{1}{4}\left(\frac{4P}{\pi d^2}\right)$$

$$\left(\begin{array}{c}\text{\% of error in} \\ \text{cal. max. stress}\end{array}\right) = \frac{\left(\sigma_{actual}\right)_{max} - \left(\sigma_{cal}\right)_{max}}{\left(\sigma_{actual}\right)_{max}} \times 100$$

$$(\text{\% error.}) = \frac{1-\frac{1}{4}}{1} \times 100 = 75\%.$$

Elongation of P.B. Under its Self weight :-



$$\left(\text{Axial load}\right)_{x-x} = P_{x-x} = \gamma A x \text{ or } \frac{Wx}{L} \quad -①$$

γ - N/m^3

where W = total self wt of bar

$W = \gamma AL$

$$\left(\sigma_{axial}\right)_{x-x} = \frac{P_{x-x}}{A_{x-x}} = \gamma x \quad - ②$$

$$\left(\sigma_{max}\right)_{axial} = \sigma_A = \underline{\gamma L} = \frac{W}{A}$$

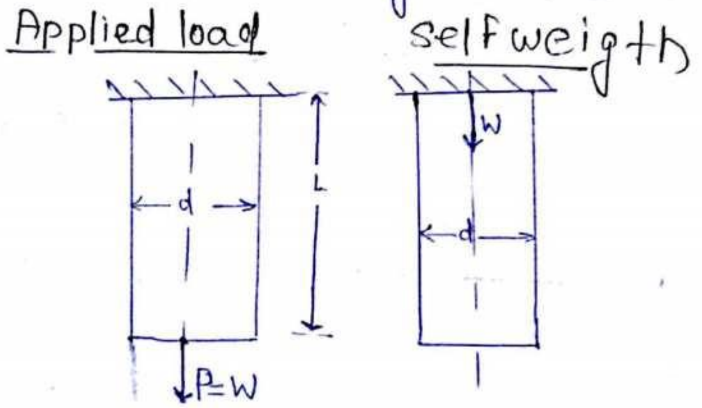
$$(\delta L)_{P.B.} = \delta_1 + \delta_2 + \cdots + \delta_n$$

or $$(\delta L)_{P.B.} = \int_0^L (\delta L)_{strip} = \int_0^L \frac{P_{x-x}\, dx}{(AE)_{x-x}}$$

$$(\delta L)_{P.B.} = \frac{1}{AE}\int_0^L \gamma A x\, dx = \frac{\gamma L^2}{2E}$$

$$\boxed{(\delta L)_{P.B} = \frac{\gamma L^2}{2E} \text{ or } \frac{WL}{2AE}} \qquad \boxed{W = \gamma AL}$$

* Elongation of a P.B. under its self weight is equal to half of the elongation of a Indentical P.B. under axial loading



Applied load	Self weight
1) A.L.D. is a rectangle	(1) A.L.D. is triangle
2) stress $\sigma = P/A$ = Const.	(2) $\sigma_{x-x} = \gamma x$; $\sigma_{max} = \gamma L$
3) $\sigma \propto \frac{1}{A}$ & indepent of material & L	(3) $\sigma_{max} \propto L$, indepent of A
4) $\delta L = \frac{WL}{AE}$	(4) $\delta L = \frac{\gamma L^2}{2E}$ or $\frac{WL}{2AE}$
5) $\delta L \propto L$	(5) $\delta L \propto L^2$
6) $\delta L \propto 1/A$	(6) δL is indepent of x-s/c Area

if All the dimentations become double

$\sigma_2 = \frac{\sigma_1}{4}$ $\qquad\qquad$ $(\sigma_{max})_2 = 2(\sigma_{max})_1$

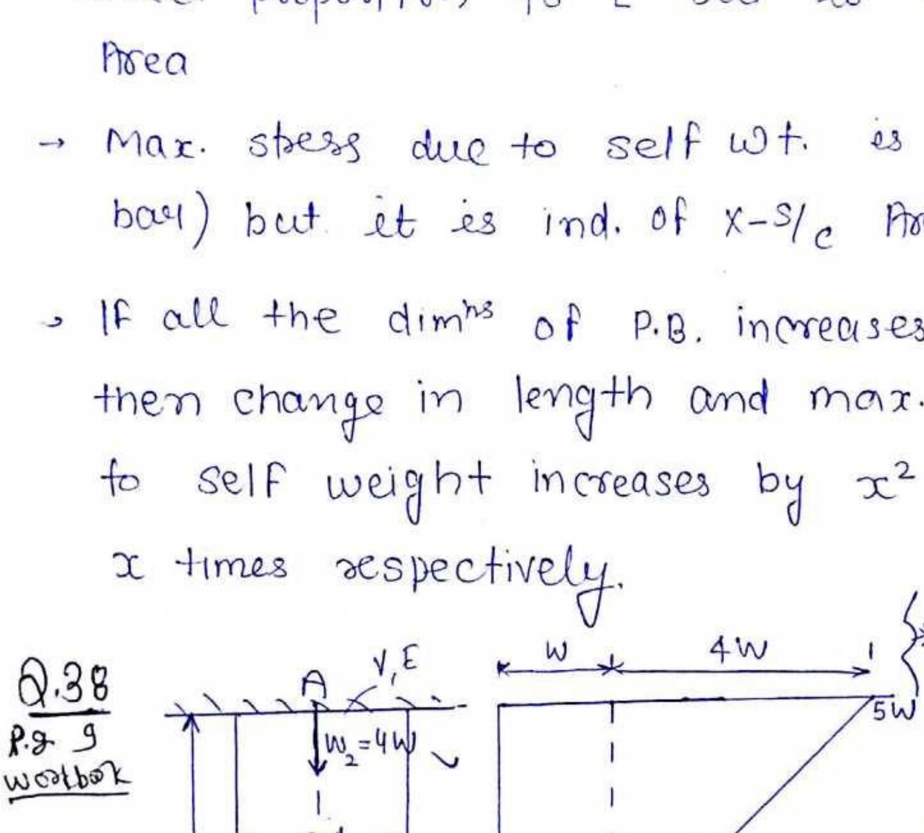
$(\delta L)_2 = \frac{(\delta L)_1}{2}$ $\qquad\qquad$ $(\delta L)_2 = 4(\delta L)_1$

→ Elongation of a prismatic bar under its self wt. direct proportion to L^2 but it independent of X-S/c Area

→ Max. stress due to self wt. is $\propto L$ (length of bar) but it is ind. of X-S/c Area.

→ If all the dim^ns of P.B. increases by x time then change in length and max. stress due to self weight increases by x^2 times and x times respectively.

Q.38
P.g 9
workbook



{* Wt of 1st bar acts as a PAL for 2nd bar

$W_1 = W = \gamma AL$

$W_2 = \gamma(4AL) = 4W_1 = 4W$

$(\delta L)_1$ = change in length of 1st bar due to its self wt.

$$(\delta L)_1 = \frac{WL}{2AE}$$

$(\delta L)_2$ = total change in length of 2nd bar

$$= (\delta L)_{P.A.L.} + (\delta L)_{S.W.}$$

$$= \frac{P_2 L_2}{A_2 E_2} + \frac{W_2 L_2}{2A_2 E_2} = \frac{WL}{4AE} + \frac{4WL}{8AE}$$

So Total elongation

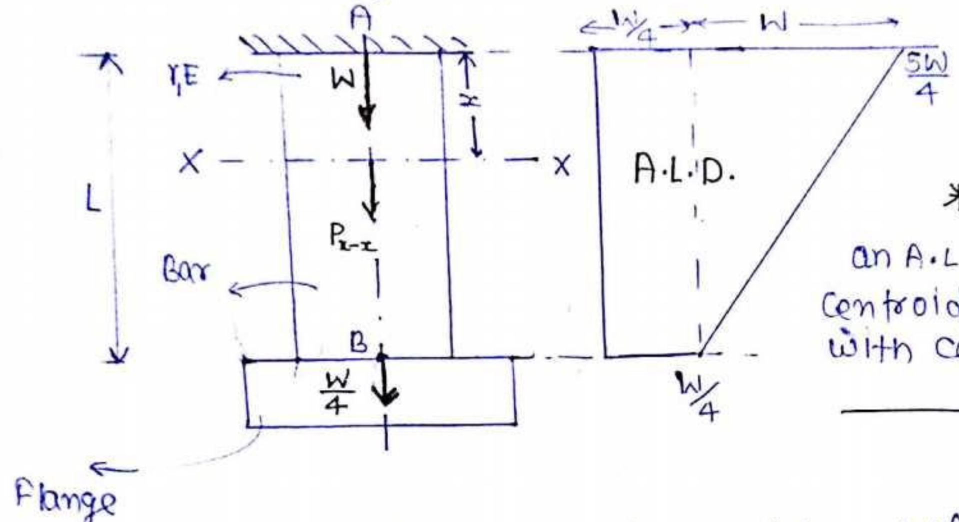
$$\delta L = \frac{WL}{AE}\left\{\frac{1}{2} + \frac{1}{2} + \frac{1}{4}\right\} = \frac{5}{4}\frac{WL}{AE} \text{ Ans}$$

Ques For the prismatic bar as shown in fig determine following

i) Axial load at x-s/c x – x

ii) max. stress on the x-s/c of the bar

iii) Elongation of bar if W is the the weigth of the bar and $W/4$ is the weight of Flange.



* Wt. of Flange acts an A.L. for the bar if centroid of flange coincide with centroid of bar

$W = \gamma A L$

$$(P)_{x-x} = (\text{Axial load})_{x-x} = \frac{W}{4} + \frac{W(L-x)}{L}$$

$$x = 0 \Rightarrow P_A = \frac{5W}{4} = P_{max}$$

$$x = L \Rightarrow P_B = W/4$$

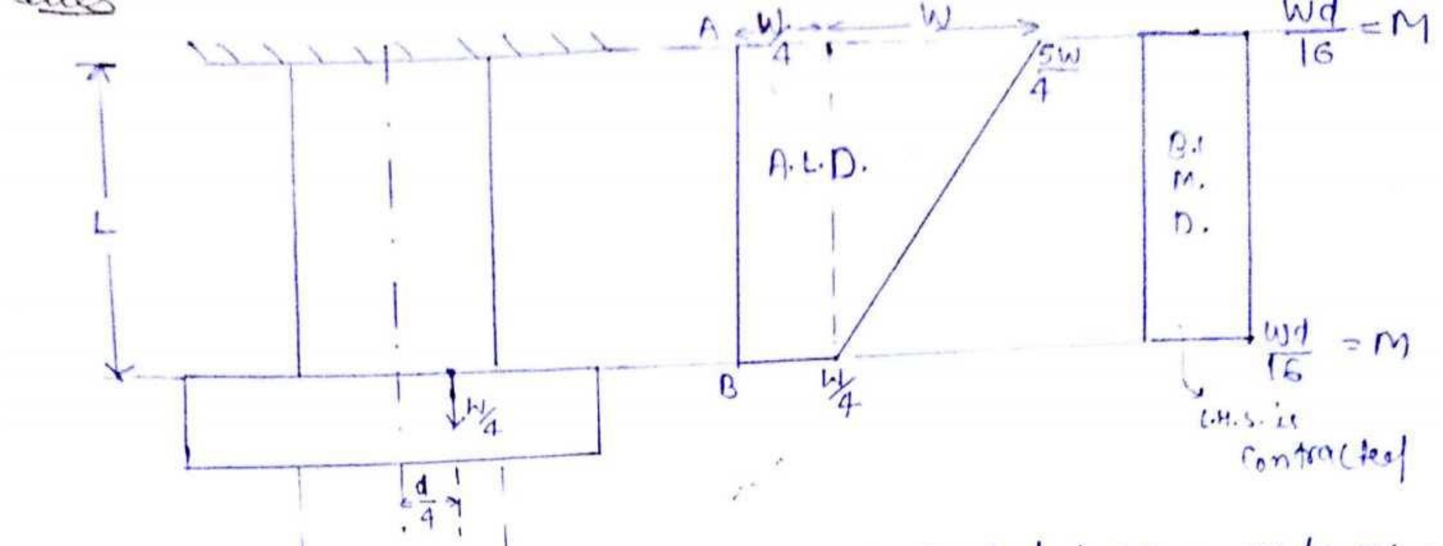
$$(\sigma_{max})_{max} = \sigma_A = \frac{5W/4}{\pi d^2/4} = \frac{5W}{\pi d^2}$$

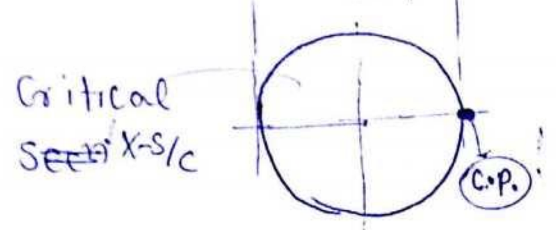
$$(\delta L)_{PB} = (\delta L)_{P.A.L} + (\delta L)_{S.W.}$$

$$= \frac{W/4 \; L}{AL} + \frac{5WL}{2AE}$$

$$(\delta L)_{P.B.} = \frac{3}{4}\frac{WL}{AE}$$

Ques





T.S. is contracted ⇒ B.M.D = +ve (sagging)

Backside surface is contracted ⇒ BMD = +ve

CH surface contracted BMD +ve.

X-S/c at the TOP

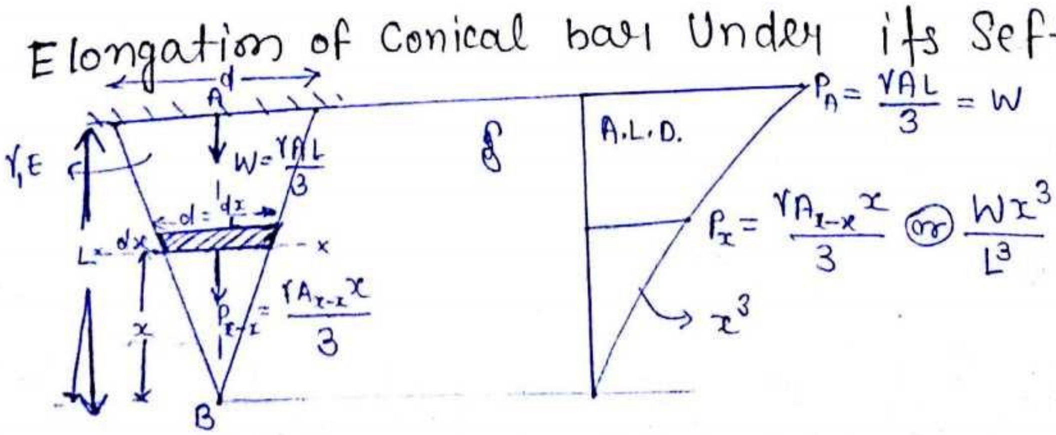
~~Elongation~~ $\sigma_{max} = \sigma_a + \sigma_b$

$$= \frac{P}{A} + \frac{M \text{ or } Pe}{Z} =$$

$$= \frac{5W}{4A} + \frac{\frac{Wd}{16}}{\frac{\pi d^3}{32}} \qquad Z = \frac{\pi d^2}{32}$$

$$\sigma_{max} = \frac{7W}{\pi d^2}$$

Elongation of Conical bar Under its Self weight



$$(\delta L)_{C.B.} = \frac{\gamma L^2}{6E} \text{ or } \frac{WL}{2AE} \quad ; W = \frac{\gamma A L}{3}$$

d = dia. of C.B. at Fixed end [i.e. $x = L$)

d_{x-x} = dia of C.B. at $x-x$ $= \frac{dx}{L}$

A = area of X-S/C of C.B. at fixed end $= \frac{\pi}{4} d^2$

A_{x-x} = ——"—— at $x-x$ $= \frac{\pi}{4} (d_{x-x})^2$

$$A_{x-x} = \frac{\pi}{4}\left(\frac{d.x}{L}\right)^2 = \frac{A x^2}{L^2}$$

$$P_A = (A.L)_A = \text{Wt. of C.B.} = W = \frac{\gamma A L}{3}$$

$$(AL)_{x-x} = P_{x-x} = \frac{\gamma A_{x-x} x}{3} \text{ (or) } W\left(\frac{x^3}{L^3}\right)$$

$$(\sigma_{axial})_{x-x} = \frac{P_{x-x}}{(A)_{x-x}} = \frac{\gamma x}{3}$$

$$(\sigma_{max})_{axial} = \sigma_A = \frac{\gamma L}{3} \text{ (or) } \frac{W}{A}$$

$$(\delta L)_{C.B.} = \int_0^L (\delta L)_{strip} = \int \frac{P_{x-x}\, dx}{(AE)_{x-x}}$$

$$(\delta L)_{CB} = \int_0^L \frac{\gamma A_{x-x} x}{3} \frac{1}{(A_{x-x})E} dx$$

$$= \int_0^L \frac{\gamma x}{3E} dx$$

$$\boxed{(\delta L)_{C.B.} = \frac{\gamma L^2}{6E} \text{ (or) } \frac{WL}{2AE}}$$ $W = \frac{\gamma A L}{3}$

→ Elongation of a Conical bar under its self weight is equal to one third ($\frac{1}{3}$) of elongation of an Identical bar under its self weight

Strain Energy, Resilience and Toughness.

Strain Energy is define as the energy absorbed by a member when workdone by the load deforms that member

Resilience is define as the energy absorbed by a member withing the elastic region

Resilience = Area of load vs defⁿ curve within the elastic ~~[illegible]~~ limit.

(Assume static load

Proof Resilience:-(PR) is define as the max. energy absorbe by a component within elastic region

PR = Area of load vs defⁿ Curve upto E.L.

$$\boxed{PR = \frac{1}{2} P_{E.L.} \delta_{E.L.} = \frac{(\sigma_{E.L})^2}{2E}(Vol.) = \frac{1}{2}(\sigma_{E.L.} \epsilon_{E.L})Vol.}$$

P.R. (↑) ⇒ (a) lower 'E'
(b) higher 'E.L'
(c) Vol. (↑)/More Vol.

Modulas of resilience :- is define as the max. energy absorved by a Component within the elastic region per unit its Volume.

$$\boxed{M.R. = \frac{P.R.}{Vol.}} = \frac{\sigma_{EL}^2}{2E} = \frac{1}{2}\sigma_{E.L.}\epsilon_{E.L.}$$

M.R. = Area of σ vs ϵ Curve upto E.L.

* modulas of resilience is property of a material.

Thoughness:- Thoughness is define as the energy stored by a member just before its fracture.

thoughness = total area load vs def^n curve.

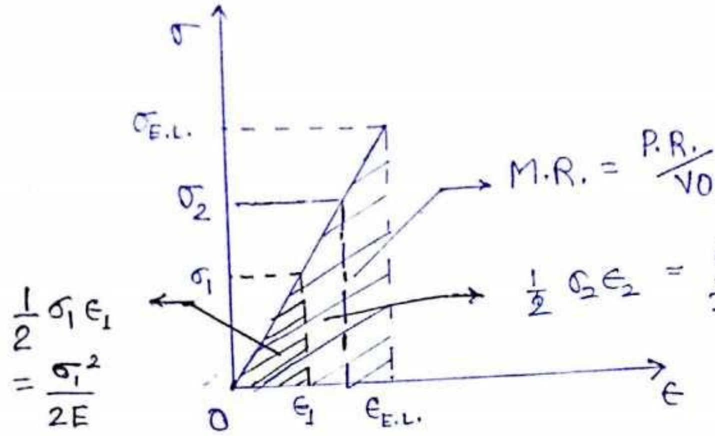
$$\text{Modulas of thoughness} = \frac{\text{Thoughness}}{\text{Volume}}$$

Total area of stess vs strain Curve.

* Modulas of Resilience & Modulas of thoughness are two imp. properties when it subjected to impact or shock loading.

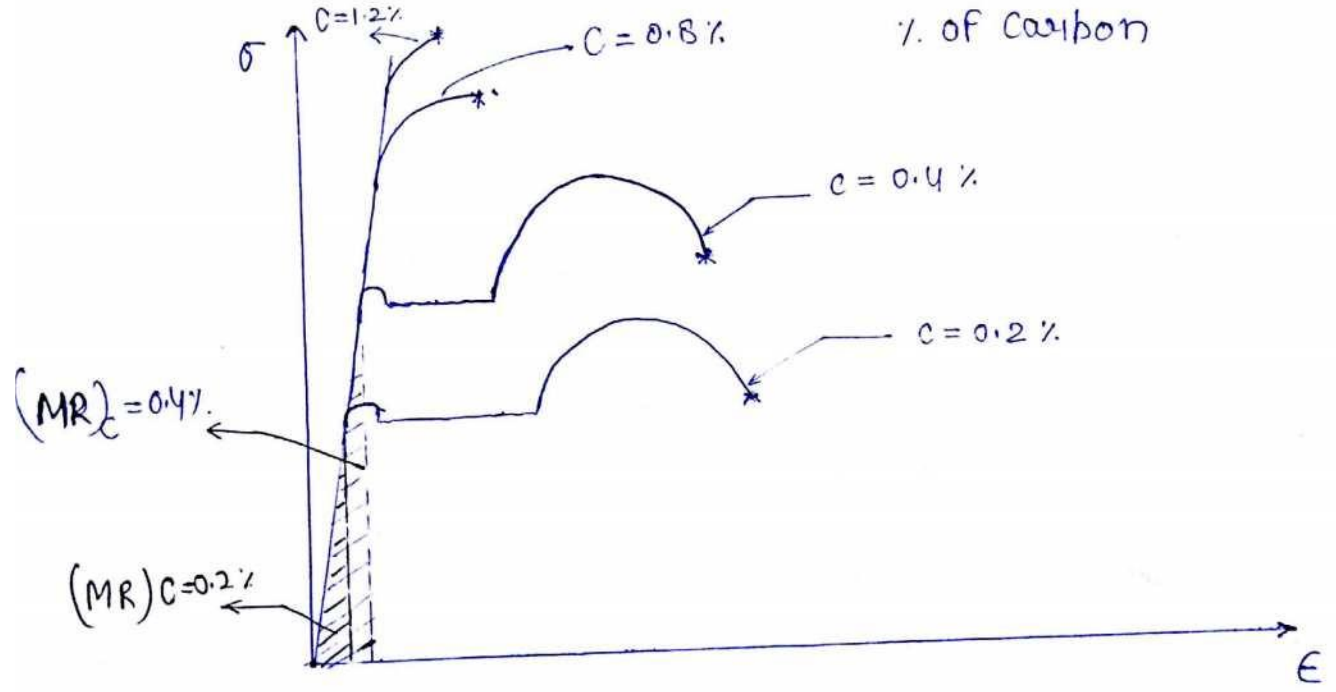
⇒ M.R. Should be Consider in the design of a Component when it is undergoing elastic def^n due to impact load acting on it.

* M.T. should be considered in the design of a component when it is going on permanent defⁿ due to impact load acting on it.



$$M.R. = \frac{P.R.}{Vol.} = \frac{1}{2}\sigma_{E.L.}\epsilon_{E.L.} \text{ or } \frac{\sigma_{E.L.}^2}{2E}$$

$\frac{1}{2}\sigma_2\epsilon_2 = \frac{\sigma_2^2}{2E}$ = Resilience/Vol. Corresponds to σ_2



Persantage of Carbon (↑) increase

Properties (↑)

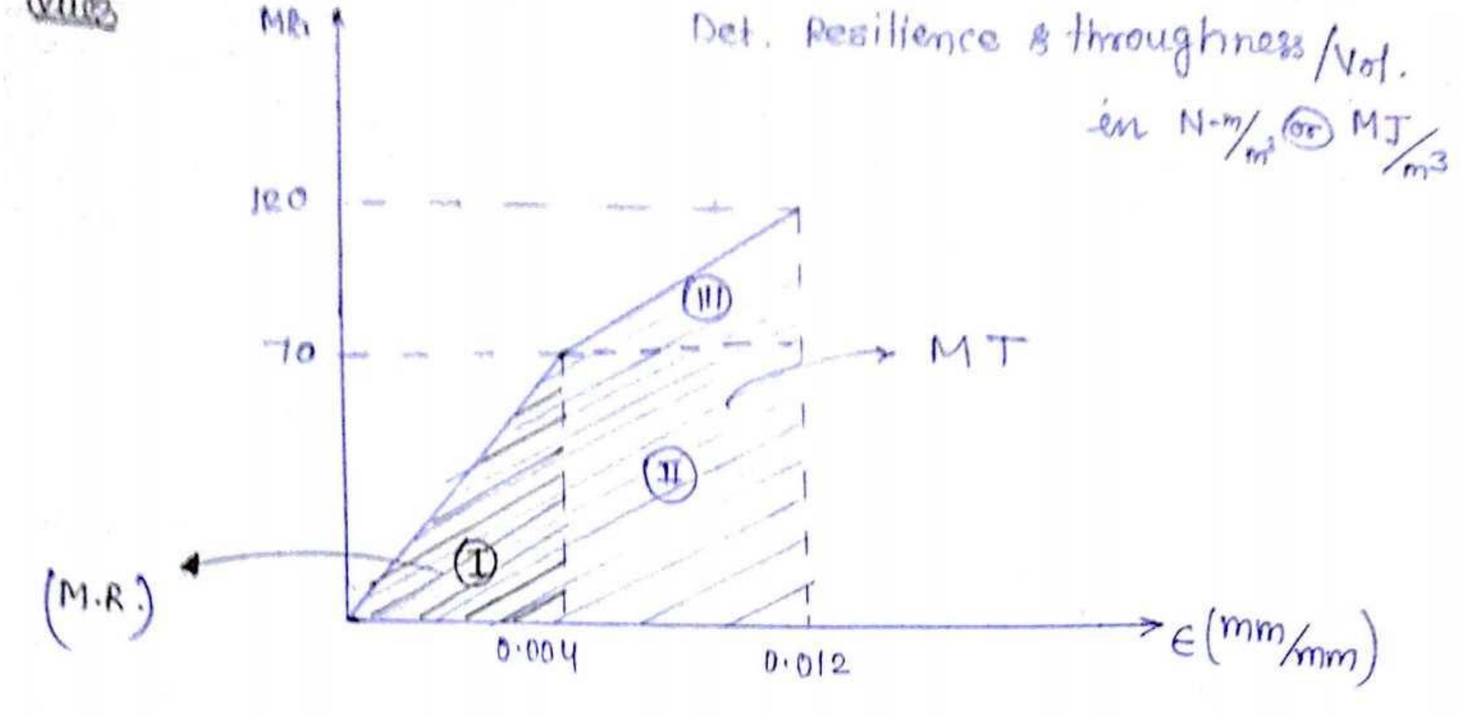
(a) strength
(b) M.R.
(c) hardness
(d) wear resistance
(e) Cost↑

Properties (↓)

(a) M.T.
(b) Ductility
(c) Formability
(d) Machinability
(e) Mellability

'E' Remain Constant

Ques



Solⁿ

$$M.R. = \text{Area } 1 = \frac{1}{2} \times 70 \times 0.004 \times 10^6 = 14 \times 10^4 \ N\text{-}m/m^3$$

$$M.R. = 0.14 \ MJ/m^3$$

Modulas to thoughness

$$M.T. = (14 \times 10^4) + (70 \times 10^6 \times 0.008) + \frac{1}{2} \times 50 \times 10^6 \times 0.008$$

$$MT = 90 \times 10^4 \ N\text{-}m/m^3 = 0.90 \ MJ/m^3$$

Strain Energy Under axial loading:-

let U = Strain Energy of a bar under Axial load.

$$U = \int_a^b \frac{(P_{x-x})^2 dx}{2(AE)_{x-x}}$$ ⇒ du = strain energy of small strip.

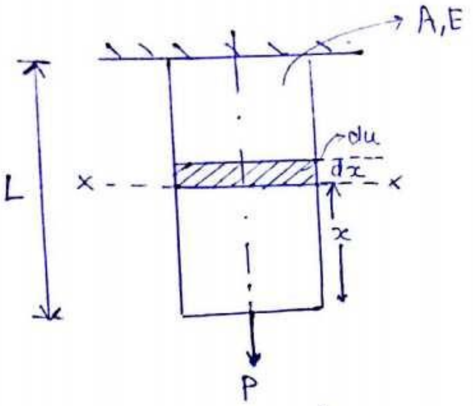
U = S.E. of a beam Under bending

$$U = \int_a^b \frac{(M_{x-x})^2 dx}{2(EI_{N-A})_{x-x}}$$ ⇒ By neglecting S.E. due to shear force

U = S.E. of a shaft under Torsion

$$U = \int_a^b \frac{(T_{x-x})^2 dx}{2(GJ)_{x-x}}$$

U = S.E. of a prismatic bar under Pure axial load ⇒



$x = 0$ to L

$P_{x-x} = P = \text{Const.}$

$A_{x-x} = A = \text{Const.}$

$E_{x-x} = E = \text{Const.}$

$$U = \int_0^L \frac{P^2 dx}{2AE} \Rightarrow \boxed{U = \frac{P^2 L}{2AE}}$$

$$U = \frac{(\sigma_a)^2}{2E} \times (\text{Vol.}) \text{ or } \frac{1}{2}(\sigma_a)(\epsilon_a)\text{ Vol.}$$

where $\sigma_a = P/A$; $\epsilon_a = \frac{\sigma_a}{E}$

when $\sigma_a = \sigma_{E.L.} \Rightarrow$ 'U' is known as Proof Resiliance of the bar.

U = S.E. of a P.B. under its self weight ($W = \gamma AL$)

$x = 0$ to L

$P_{x-x} = \gamma A x$ or $\frac{Wx}{L}$

$A_{x-x} = A$; $E_{x-x} = E$

$$U = \int_0^L \frac{\left(\frac{Wx}{L}\right)^2 dx}{2AE} = \frac{W^2 L}{6AE} \text{ or } \frac{\gamma^2 A L^3}{6E} \Rightarrow \boxed{U = \frac{W^2 L}{6AE} \text{ or } \frac{\gamma^2 A L^3}{6E}}$$

U = S.E. of a C.B. Under its Self Wt $\left(W = \frac{\gamma AL}{3}\right)$

$x = 0$ to L

$$P_{x-x} = \frac{\gamma A_{x-x} x}{3} \quad \text{(OR)} \quad \frac{Wx^3}{L^3}$$

$$A_{x-x} = A\left[\frac{x^2}{L^2}\right]$$

$$E = E = \text{Cont.}$$

$$U = \int_0^L \frac{\left(\frac{Wx^3}{L^3}\right)^2 dx}{2\left(\frac{Ax^2}{L^2}\right)E} = \frac{W^2L}{10AE}$$

$$\boxed{U = \frac{W^2L}{10AE} = \frac{\gamma^2AL^3}{90E}}$$

Under A.L. :-

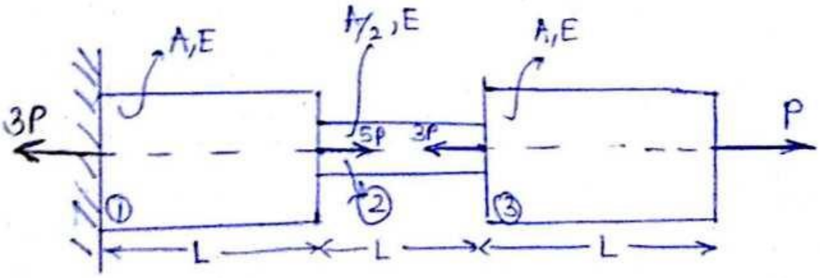
$$\boxed{U = \left(\frac{1}{k}\right)\frac{P^2L}{AE}}$$

$k = 2 \Rightarrow$ P.B. Under PAL $(P = P)$

$k = 6 \Rightarrow$ P.B. Under its self Wt. $[P = W = \gamma AL]$

$k = 10 \Rightarrow$ C.B. Under its self Wt $\left[P = W = \frac{\gamma AL}{3}\right]$

Quest For the Stappered bar as shown in fig strain energy



$P_1 = 3P$

$P_2 = -2P$

$P_3 = P$

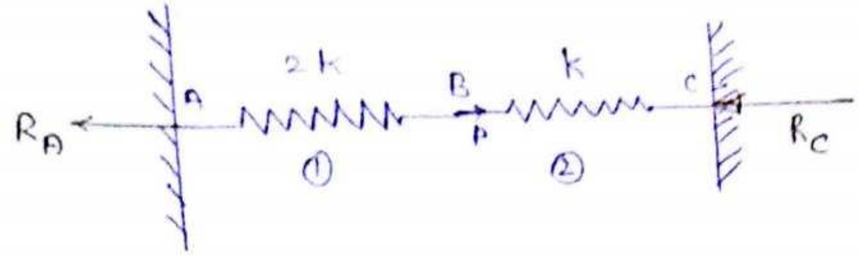
$$U = U_1 + U_2 + U_3$$

$$U = \frac{L}{2E}\left[\frac{P_1^2}{A_1^2} + \frac{P^2}{A_2^2} + \frac{P^3}{A_3^2}\right]$$

$$U = \frac{L}{2E}\left[\frac{9P^2}{A} + \frac{4P^2}{A/2} + \frac{P^2}{A}\right]$$

$$U = \frac{9P^2L}{AE} \text{ N-m}$$

Ques Determine the strain energy of each spring.



* Prob. same a P.B. in series.

$$P_1 = R_A$$

$$P_2 = R_A - P$$

$$\delta_1 + \delta_2 = 0 \quad \Rightarrow \quad \frac{P_1}{k_1} + \frac{P_2}{k_2} = 0$$

$$\frac{R_A}{2k} + \frac{(R_A - P)}{k} = 0$$

$$\Rightarrow \quad \frac{1}{2k}\left(R_A + 2R_A - 2P\right)$$

$$R_A = \frac{2P}{3} \quad ; \quad R_D = P/3$$

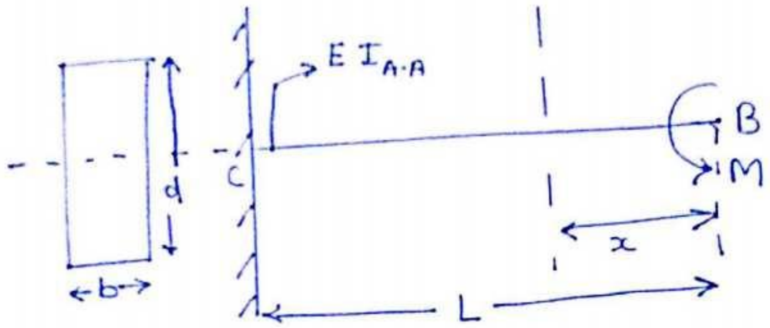
$$P_1 = R_A = \frac{2P}{3} \quad (T)$$

$$P_2 = -R_C = P/3 \quad (C)$$

$$U_1 = \frac{1}{2} P_1 \delta_1 = \frac{P_1^2}{2K_1} = \frac{(2P/3)^2}{2 \times 2k} = \frac{P^2}{9K}$$

$$U_2 = \frac{P_2^2}{2k_2} = \frac{P^2}{18k}$$

U = S.E. of a prismatic beam under pure bending :-



$$BC : [x = 0 \text{ to } L]$$

$$M_{x-x} = M = \text{Constant}$$

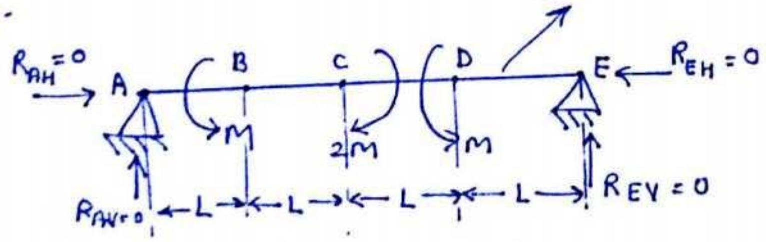
$$E_{x-x} = E = \text{Constant}$$

$$I_{x-x} = I = \text{Constant}$$

$$U = \int_a^b \frac{(M_{x-x})^2 dx}{2(E I_{x-x})} = \int_0^L \frac{M^2 dx}{2 E I_{N.A}}$$

$$\boxed{U = \frac{M^2 L}{2 E I_{N.A.}}} \quad \text{or} \quad \boxed{U = \frac{6 M^2 L}{E b d^3}}$$

Ques.



Strain Energy = ?

$$\Sigma H = 0 \rightarrow R_{AH} = R_{EH} = 0$$

$$\Sigma V = 0 \rightarrow \& \ \Sigma M = 0 \rightarrow R_{AV} = R_{EV} = 0$$

$M_{AB} = 0$

$M_{BC} = -M \quad (LHS)$

$M_{CD} = -M + 2M = M \quad (LHS)$

$M_{DE} = 0 \quad (RHS)$

$= -M + 2M + M = 0 \quad (LHS)$

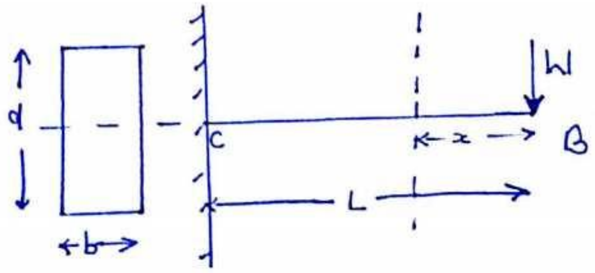
$$U = U_{AB} + U_{BC} + U_{CD} + U_{DE}$$

$$U = 0 + \frac{(-M)^2 L}{2EI} + \frac{M^2 L}{2EI} + 0$$

$$U = \frac{M^2 L}{EI}$$

U = S.E. under transverse shear load

⇒



BC :- $[x = 0 \text{ to } L]$

$M_{x-x} = -Mx$

$E_{x-x} = E \;;\; I_{x-x} = I$ Const.

$$U = \int \frac{M_{x-x}^2 \, dx}{2(EI)_{x-x}} = \int_0^L \frac{(-Wx)^2 \, dx}{2(EI)_{x-x}}$$

$$\boxed{U = \frac{W^2 L^3}{6EI_{N.A.}}}$$

by neglacting Strain Energy due to Shear force

$$U = \left(\frac{M^2 L}{2EI_{N.A}}\right)_{AB} + \int_0^L \left(\frac{M_{x-x}^2 \, dx}{2EI}\right)_{BC}$$

$$U = \frac{(Pe)^2 L}{2EI} + \int_0^L \frac{(Px)^2 \, dx}{2EI}$$

$$U = \frac{P^2 L^3}{2EI} + \frac{P^2 L^3}{6EI} \Rightarrow U = \frac{2}{3}\left[\frac{P^2 L^3}{EI_{N.A.}}\right]$$

$$U = \frac{2}{3}\left[\frac{P^2 L^3}{EI_{N.A.}}\right]$$

$\left\{\right.$ ∵ By neglecting S.E. of AB due to P.A.L. & S.E. of BC due to S.F.

$\left\{\right.$ e is very large so S.E. due to B.M. very high.

Under EAL

$$\frac{U_{Purbending}}{U_{P.A.L.}} = \frac{\left(\frac{M^2 L}{2EI}\right)}{\frac{P^2 L}{2AE}} = \frac{\frac{(Pe)^2 \cancel{L}}{2\cancel{E}I}}{\frac{P^2 \cancel{L}}{2A\cancel{E}}}$$

$$= \left(\frac{A}{I}\right) \times e^2 = \frac{\frac{\pi}{4}d^2}{\frac{\pi}{64}d^4} \times e^2 = 16\left[\frac{e}{d}\right]^2$$

* When eccentricity is very large incomparison to x-s/c dim^{ns}, then strain energy due to Pure axial load is neglected

$$\because \boxed{U_{P.A.L} <<<< U_{P.bending}}$$

⇒ Circular x-s/c

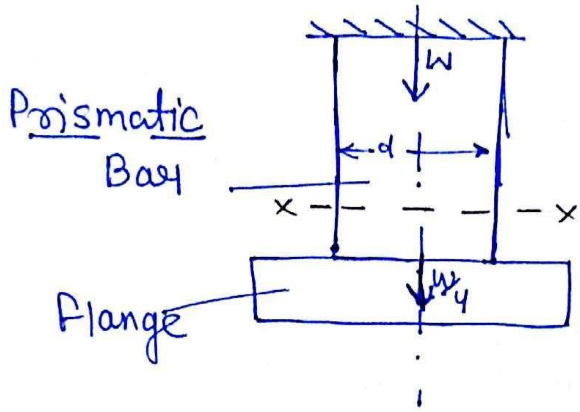
if. $d = 100\,mm$; $L = 1\,m$

$$\frac{U_{P.B.}}{U_{PAL}} = 16\left(\frac{1000}{100}\right)^2 = 1600$$

⇒ In Column $d = 100\,mm$; $e_{max} = d/2 = 50$

$$\frac{U_{P.B.}}{U_{PAL}} = 16\left(\frac{50}{100}\right)^2 = 4$$

Ques



Flange - load $= W/4$

Bar load $= W$

Find Strain energy $U = ?$

Soln : One load is constant and one load is varying so we need integration.

$x = 0$ to L

$$(\text{L.})_{x-x} = \frac{W}{4} + \frac{Wx}{L}$$

$$(A.E.)_{x-x} = AE = \text{Const.}$$

$$U = \int_0^L \frac{\left(\frac{W}{4} + \frac{Wx}{L}\right)^2 dx}{2AE} = \cancel{\frac{1}{2AE}\left[2\left(\frac{W}{4} + \frac{Wx}{L}\right)\left(\frac{W}{L}\right)\right]_0}$$

$$U = \frac{W^2}{2AE}\int_0^L \left(\frac{1}{16} + \frac{x}{2L} + \frac{x^2}{L^2}\right)dx$$

$$U = \frac{W^2 L}{2AE}\left(\frac{1}{16} + \frac{1}{4} + \frac{1}{3}\right)$$

$$\left(\frac{3+12+16}{48}\right)$$

$$U = \left(\frac{31}{96}\right)\left[\frac{W^2 L}{AE}\right]$$

U = S.E. of a hollow Circular (Pure torsion)

$$U = \frac{T^2 L}{2GJ} \quad \text{or} \quad \frac{\tau_{max}^2}{4G} \times \text{Vol.}\left[1+k^2\right]$$

$$\text{where} \quad k = \frac{d_i}{d_o} = \frac{d}{D} < 1$$

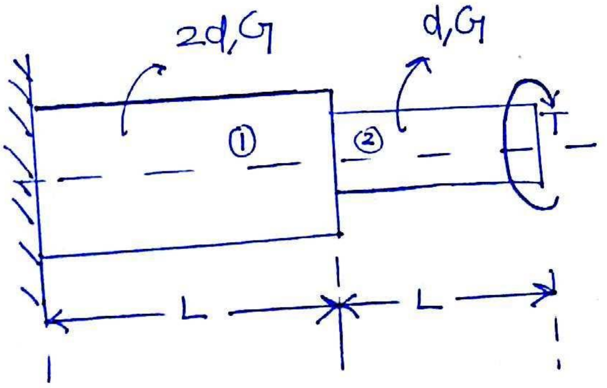
$$\tau_{max} = \frac{T}{Z_p} = \frac{16T}{\pi D^3(1-k^4)}$$

$$\boxed{T = \frac{\pi D^3}{16} \tau_{max}(1-k^4)}$$

$$\Rightarrow J = \frac{\pi}{32}\left[D^4 - d^4\right] = \frac{\pi D^4}{32}(1-k^4)$$

$$\Rightarrow A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi D^2}{4}(1-k^2)$$

eg



Strain Energy $U = ?$

$T = 500$ N-m

$L = 0.5$ m

$G = 80$ GPa ; $d = 40$ mm

$$U = U_1 + U_2$$

$$U = \left(\frac{T^2 L}{2GJ}\right)_1 + \left(\frac{T^2 L}{2GJ}\right)_2$$

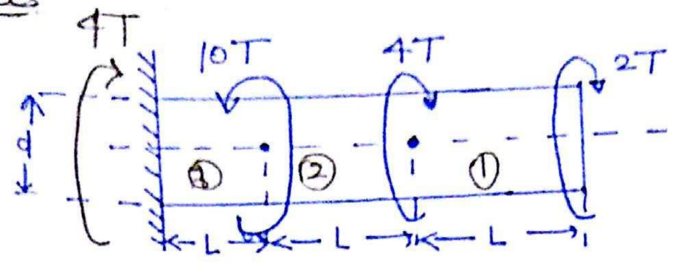
$$U = \frac{T^2 L}{2G\frac{\pi}{32}(2d)^4} + \frac{T^2 L}{2G\frac{\pi}{32}d^4}$$

$$U = \frac{T^2 L}{\pi G d^4} + \frac{16 T^2 L}{\pi G d^4}$$

$$\boxed{U = 17\frac{T^2 L}{\pi G d^4}}$$

Ques

G, J



U = ?

$$T_1 = 2T \; ; \; T_2 = \cancel{2}\,4T + 2T = 6T$$

$$T_3 = 6T - 10T = -4T$$

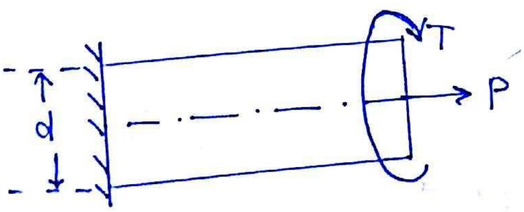
$$U = U_1 + U_2 + U_3 \qquad \because U = \frac{T^2 L}{2GJ}$$

$$U = \frac{L}{2GJ}\left[T_1^2 + T_2^2 + T_3^2\right]$$

$$= \frac{16L}{\pi d^4 G}\left[(2T)^2 + (-4T)^2 + (6T)^2\right]$$

$$U = (896)\frac{T^2 L}{\pi G d^4}$$

Ques



Find strain Energy Under P.A.L. & P. Torsion. Both are Constant

$$U = U_{A.L.} + U_{P.T}$$

$$U = \frac{P^2 L}{2AE} + \frac{T^2 L}{2GJ}$$

$$U = \frac{2P^2 L}{\pi d^2 E} + \frac{16 T^2 L}{\pi G d^4}$$

$$U = \frac{2P^2 L}{\pi d^2 E} + \frac{16 T^2 L}{\pi d^4 \frac{E}{2(1+\mu)}}$$

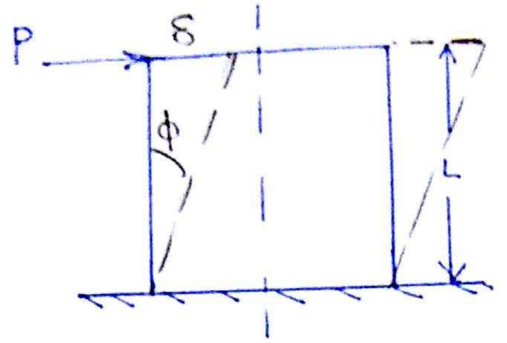
$$U = \frac{2P^2 L}{\pi d^2 E} + \frac{32 T^2 L}{\pi E d^4}(1+\mu)$$

$$U = \frac{32 L}{\pi d^2 E}\left[\frac{P^2}{16} + \frac{T^2(1+\mu)}{d^2}\right]$$

if P.T = Pd

$$\Rightarrow U = \frac{32 P^2 L}{\pi d^2 E}\left[\frac{1}{16} + \mu + 1\right]$$

U = strain energy of a rectangular block under a shear force (P)



$$\boxed{U = \frac{(\tau_d)^2}{2G} \times Vol.}$$ → By neglecting shear energy due to Bending moment

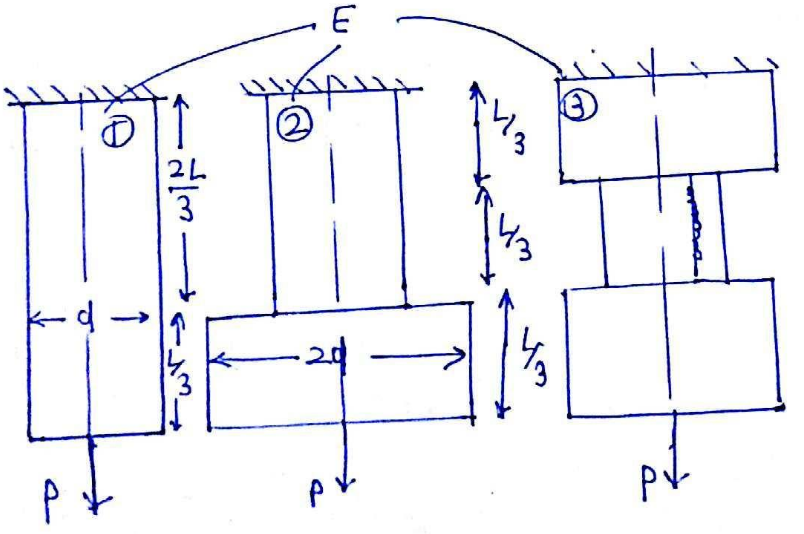
where $\boxed{\tau_d = P/A}$

$$U = \frac{1}{2} \times P \times \delta$$

$$U = \frac{1}{2} \times P \times \phi L$$

$$U = \frac{1}{2} \times PL \frac{\tau_d}{G} \times \frac{A}{A} \Rightarrow U = \frac{(\tau_d)^2}{2G} \times Vol.$$

Ques (H.W.)



Find $\frac{U_1}{U_2} = ?\ \frac{4}{3}$

$\frac{U_2}{U_3} = ?\ \frac{3}{2}$

which bar has max strain energy