Algebra

An algebraic expression is an expression in one or more variables having different number of terms. Depending on the number of terms it may be monomials, binomials, trinomials or polynomials. Like in the case of real numbers we can also use different mathematical operations on algebraic expression. Previously we have learnt to add and subtract the algebraic expression. In this chapter we will learn, how to multiply or divide the algebraic expression. We will also learn how to find the linear factors of the algebraic expression as in the case of real numbers and how to form a linear equation in one variable and to find its solution.

Multiplication of Algebraic Expressions

When two algebraic expressions are multiplied, the result obtained is called the product. The expressions being multiplied are called factors or multiplicands. While multiplying algebraic expressions first multiply numerical coefficients, then list all the variables that occur in the terms being multiplied and add the exponents of like variables.

- **Example:**

Find the product of $\left(2x^2 - 5x + 4\right)$ and $\left(x^2 + 7x - 8\right)$.

(a) $2x^4 - 9x^3 - 47x^2 + 68x + 32$

(b) $2x^4 + 9x^3 - 47x^2 + 68x - 32$

(c) $2x^4 - 9x^3 - 47x^2 + 68x - 32$

(d) $2x^4 - 9x^3 - 47x^2 - 68x - 32$

(e) None of these

Answer (b)

Explanation: $\left(2x^2 - 5x + 4\right)\left(x^2 + 7x - 8\right)$

$$= 2x^2\left(x^2 + 7x - 8\right) - 5x\left(x^2 + 7x - 8\right) + 4\left(x^2 + 7x - 8\right)$$
$$= 2x^4 + 14x^3 - 16x^2 - 5x^3 - 35x^2 + 40x + 4x^2 + 28x - 32$$
$$= 2x^4 + 9x^3 - 47x^2 + 68x - 32$$

Division of Algebraic Expressions

The process for division of algebraic expressions is similar to the multiplication process, the only difference is that in division process we have to divide the numerical coefficients and subtract the exponents instead of adding. The following points should be remembered while dividing algebraic expressions.

- If there are numerical coefficients in the expressions to be divided, just divide the numerical coefficient and then divide the variables by using the laws of exponents.
- To divide the variables just subtract the exponents of like variables.

Example:

The simplest form of $\frac{9x^4y^7}{3x^2y^4}$ is:

(a) 3xy
(b) $3x^2y^2$
(c) $3x^2y^3$
(d) $3x^3y^3$
(e) None of these

Answer (c)

Explanation: $\frac{9x^4y^7}{3x^2y^4} = \left(\frac{9}{3}\right)(x^{4-2})(y^{7-4}) = 3x^2y^3$

Factorisation

Factorisation of an algebraic expression is the process of writing the algebraic expression as a product of two or more linear factors. Each multiple of the algebraic expression is called factors of the algebraic expression. Thus the process of splitting the given algebraic expression into the product of two or more linear factors is called factorisation.

Factor Theorem

According to the factor theorem if f(x) is polynomial which is completely divisible by another polynomial g(x) = x - a , then x - a is called the factor of the polynomial f(x) and f(a) = 0 for all values of a.

Methods of Factorisation

Different algebraic expressions can be factorised by different methods. Monomials can be easily written into their linear factors. Binomials can be factorised by using identities which you have learnt in previous classes. The quadratic equation can be factorised by spliting the middle term and cubic equation can be factorised by first dividing it by linear factors and then reducing it to the quadratic form and then spliting the middle term. The other methods of factorisation are by grouping the terms having the common coefficients or having some common variables.

Greatest Common Divisor

Greatest common divisor or simply written as GCD of a polynomial is the largest monomial which is a factor of each term of the given polynomial. The common factors of the given expression or polynomial are the GCD and the quotient thus obtained.

- **Example:**

Factorise: $y^2 + 4y + 3$

(a) (y + 1) (y + 2)
(b) (y + 1) (y + 3)
(c) (y + 2) (y + 3)
(d) (y + 3) (y + 4)
(e) None of these

Answer (b)

Explanation: $y^2 + 4y + 3 = y^2 + 3y + y + 3$

$= y(y+3)+1(y+3)=(y+3)(y+1)$

- **Example:**

Factorise: $x^2+\frac{1}{x^2}+2-2x-\frac{2}{x}$

(a) $\left(x+\frac{1}{x}\right)^2$

(b) $\left(x+\frac{1}{x}\right)\left(x+\frac{1}{x}+1\right)$

(c) $\left(x+\frac{1}{x}\right)\left(x+\frac{1}{x}+2\right)$

(d) $\left(x+\frac{1}{x}\right)\left(x+\frac{1}{x}-2\right)$

(e) None of these

Answer (d)

Explanation: $x^2+\frac{1}{x^2}+2-2x-\frac{2}{x}\left(x+\frac{1}{x}\right)^2-2\left(x+\frac{1}{x}\right)=\left(x+\frac{1}{x}\right)\left(x+\frac{1}{x}-2\right)$

Linear Equations in One Variable

A statement which states that two algebraic expressions are equal is called an equation and an equation involving any one variable is called a linear equation in one variable. The expression on the left of the sign of equality is called LHS and the expression on the right is called RHS. The value of the variable which satisfies the given equation is called solution of the linear equation. We can find the solution of the linear equation either by hit and trial method or by solving the given equation for the required variables.

Properties of an Equation

Following are some properties of an equation:

- If same quantity is added to both sides the sums on both sides remains equal.
- If same quantity is subtracted from both sides of an equation, the differences are also equal.
- If both sides of an equation are multiplied by the same quantity, then the products are equal.
- Dividing both sides of an equation by the same quantity does not alter the sign of equality.
- Changing LHS into RHS or vice-versa does not alter the sign of equality.

Transposition

We know that every equation has two sides LHS and RHS connected with the sign of equality. Sometimes both sides of the equation contain both constants and variable. In such type of cases we transpose variable to one side of the equality and constants to another side. So the process of transposing constants and variables in an equation is known as transposition.

The method of transposition involves the following steps:

Step 1: Identify the variables and constants in the equation.

Step 2: Transpose the variables on LHS and constants on RHS of the equation.

Step 3: Simplify the equation to get the solution of the equation.

- **Example:**

Solve for $x : \frac{3x-2}{4} - \frac{2x+3}{3} = \frac{2}{3} - x$

(a) 0

(b) 1

(c) 2

(d) -2

(e) None of these

Answer (c)

Explanation: We have, $\frac{3x-2}{4} - \frac{2x+3}{3} = \frac{2}{3} - x$

Multiplying both sides of the equation by 12, we get

$\Rightarrow 9x - 6 - 8x - 12 = 8 - 12x \Rightarrow x - 18 = 8 - 12x$

$\Rightarrow x + 12x = 8 + 18 \Rightarrow 13x = 26 \Rightarrow x = 2$