CHAPTER – 15

CIRCLE

1. Draw a circle with centre O and radius 2.5 cm. Draw two radii OA and OB such that $\angle AOB = 60^0$. Measure the length of the chord AB.

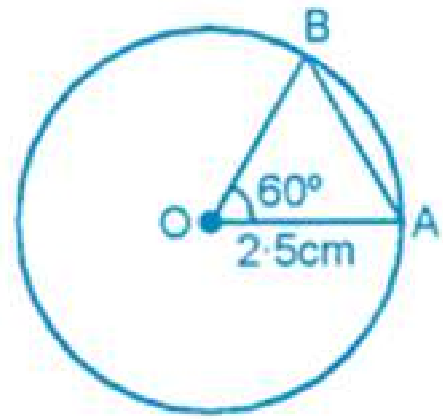
Solution:

1. Draw a circle, taking centre as O and radius equal to 2.5 cm

2. Join OA, where A is any point on the circle

3. Draw $\angle AOB$ equal to 60^0

4. Now, join AB and on measuring we get, AB = 2.5 cm



2. Draw a circle of radius 3.2 cm. Draw a chord AB of this circle such that AB = 5 cm. Shade the minor segment of the circle.

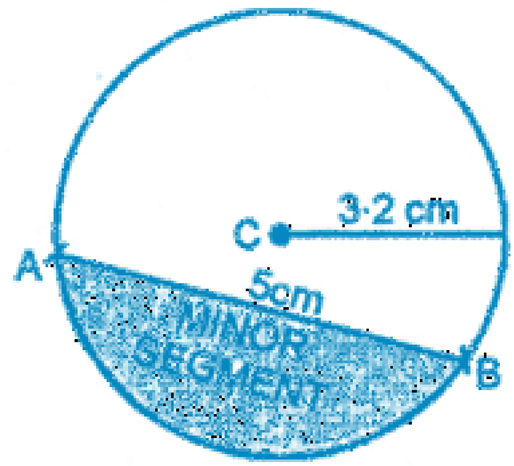
Solution:

1. Draw a circle, taking centre as O and radius = 3.2 cm

2. Take a point A on the circle

3. Taking A as centre and radius = 5 cm, draw an arc to meet the circle at point B

4. Now, join AB and shade the minor segment of the circle



3. Find the length of the tangent drawn to a circle of radius 3 cm, from a point at a distance 5 cm from the centre.

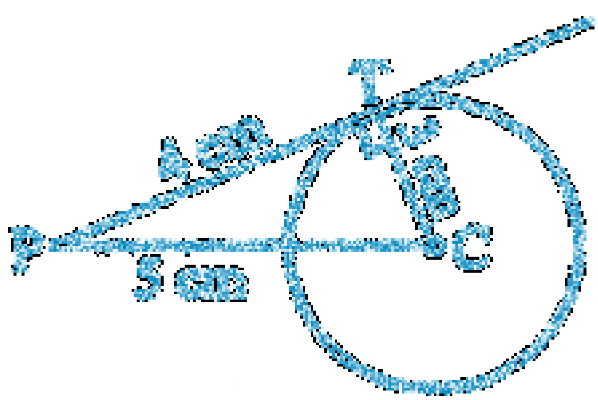
Solution:

Draw a circle, taking C as centre and radius CT = 3 cm

Let PT be the tangent, drawn from point P to a circle with centre C

Let CP = 5 cm

CT = 3 cm (given)



$\angle CTP = 90^0$ (since radius is perpendicular to tangent)

From △CPT,

$CP^2 = PT^2 + CT^2$ (By Pythagoras theorem)

$(5)^2 = PT^2 + (3)^2$

We get,

$PT^2 = 25 – 9$

$PT^2 = 16$

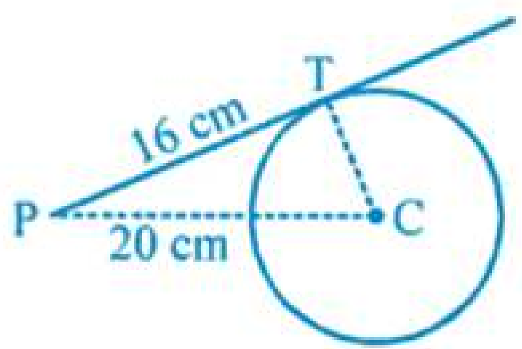
$PT = \sqrt{16}$

We get,

PT = 4

Therefore, length of tangent = 4 cm

4. In the adjoining figure, PT is a tangent to the circle with centre C. Given CP = 20 cm and PT = 16 cm, find the radius of the circle.



Solution:

We know that,

Radius is always perpendicular to tangent

i.e., CT ⊥ PT

Therefore,

$\triangle CPT$ is a right angled triangle, where CP = hypotenuse

In right angled triangle,

By Pythagoras theorem, we get,

$CP^2 = PT^2 + CT^2$

$CT^2 = CP^2 – PT^2$

$CT^2 = (20)^2 – (16)^2$

We get,

$CT^2 = 400 – 256$

$CT^2 = 144$

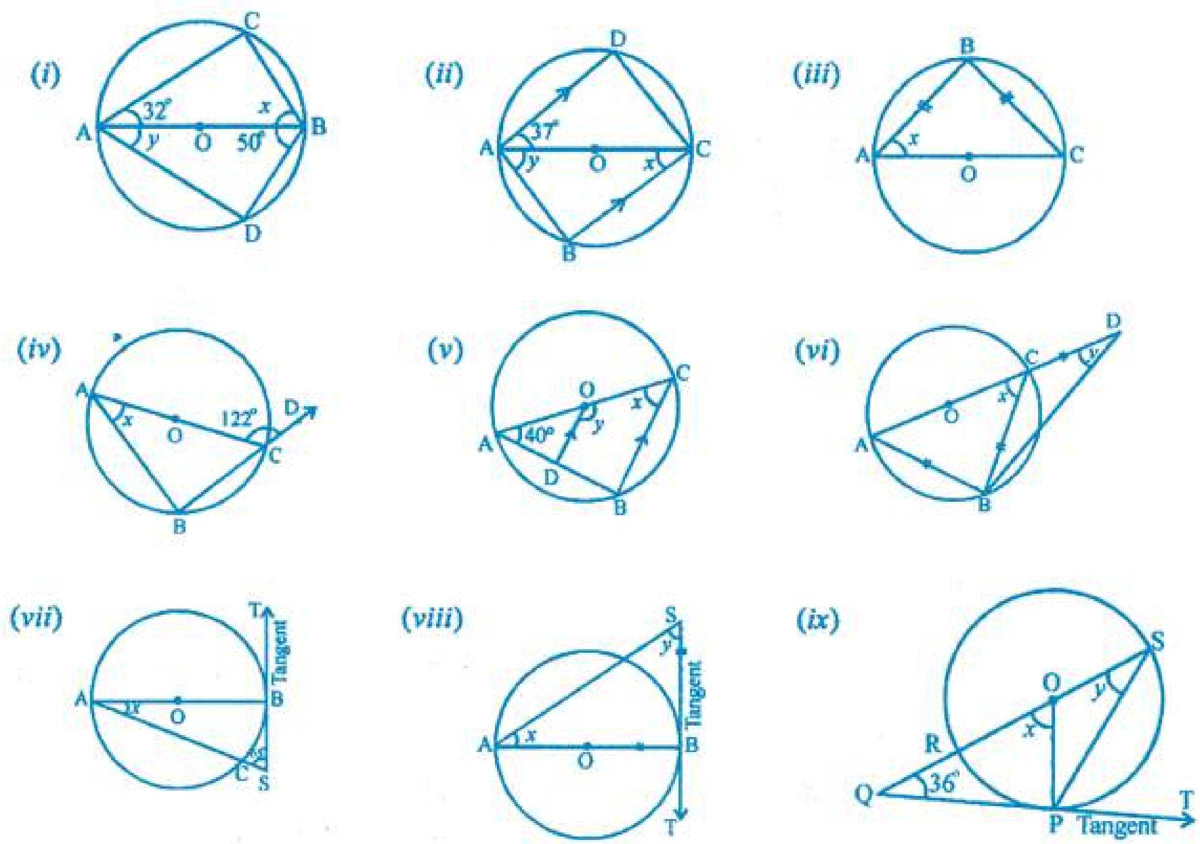
$CT = \sqrt{144}$

We get,

CT = 12 cm

Therefore, radius of circle = 12 cm

5. In each of the following figure, O is the centre of the circle. Find the size of each lettered angle:



Solution:

(i) In the given figure,

AB is the diameter and O is the centre of the circle

Given $\angle CAB = 32^0$

$\angle ABD = 50^0$

$\angle C = 90^0$ (angles in the semicircle)

By angle sum property of triangle, we get,

$\angle C + \angle CAB + \angle ABC = 180^0$

$90^0 + \angle CAB + \angle x = 180^0$

$90^0 + 32^0 + \angle x = 180^0$

$32^0 + \angle x = 180^0 - 90^0$

We get,

$\angle x = 90^0 - 32^0$

$\angle x = 58^0$

Similarly,

In right angled triangle ADB,

By angle sum property of triangle, we get,

$\angle ABD + \angle D + \angle BAD = 180^0$

$50^0 + 90^0 + \angle BAD = 180^0$

$50^0 + 90^0 + \angle y = 180^0$

$\angle y = 180^0 - 140^0$

We get,

$\angle y = 40^0$

(ii) In the figure,

AC is the diameter of circle with centre O

$\angle DAC = 37^0$

AD || BC

$\angle ACB = \angle DAC$ (Alternate angles)

Hence,

$x = 37^0$

In $\triangle ABC$,

$\angle B = 90^0$ (Angle in a semicircle)

By angle sum property of triangle, we get,

$\angle x + \angle y + \angle B = 180^0$

$37^0 + \angle y + 90^0 = 180^0$

$\angle y = 180^0 - 127^0$

We get,

$\angle y = 53^0$

(iii) In the figure,

AC is the diameter of the circle with center as O

BA = BC

Hence,

$\angle BAC = \angle BCA$ (angles of isosceles triangle)

But $\angle ABC = 90^0$ (angles in a semicircle)

In triangle ABC,

By angle sum property of triangle, we get,

$\angle BAC + \angle ABC + \angle BCA = 180^0$

$\angle BAC + \angle BCA = 180^0 - 90^0$

$\angle x + \angle x = 90^0$

$\angle 2x = 90^0$

We get,

$\angle x = 45^0$

(iv) In the figure,

AC is the diameter of the circle, with centre as O,

$\angle ACD = 122^0$

$\angle ACB + \angle ACD = 180^0$ (Linear pair)

$\angle ACB + 122^0 = 180^0$

$\angle ACB = 180^0 – 122^0$

We get,

$\angle ACB = 58^0$

In $\triangle ABC$,

$\angle ABC = 90^0$ (Angles in a semicircle)

By angle sum property of triangle, we get,

$\angle ABC + \angle BCA + \angle ACB = 180^0$

$90^0 + x + 58^0 = 180^0$

$x = 180^0 – 148^0$

We get,

$x = 32^0$

(v) In the figure,

AC is the diameter of the circle, with centre as O,

$OD \parallel CB$ and $\angle CAB = 40^0$

In $\triangle ABC$,

$\angle B = 90^0$ (Angle in a semicircle)

By angle sum property of triangle, we get,

$\angle BCA + \angle ABC + \angle BAC = 180^0$

$\angle BCA + \angle BAC + 90^0 = 180^0$

$\angle BCA + \angle BAC = 180^0 – 90^0$

$\angle BCA + \angle BAC = 90^0$

$x + 40^0 = 90^0$

$x = 90^0 – 40^0$

We get,

$x = 50^0$

∵ OD || CB

Hence,

∠AOD = ∠BCA (corresponding angles)

$\angle AOD = 50^0$

But $\angle AOD + \angle DOC = 180^0$ (Linear pair)

$50^0 + y = 180^0$

$y = 180^0 – 50^0$

We get,

$y = 130^0$

Therefore, $x = 50^0$ and $y = 130^0$

(vi) In the figure,

AC is the diameter of the circle with centre as O

BA = BC = CD

In △ABC,

$\angle ABC = 90^0$ (Angle in a semicircle)

By angle sum property of triangle, we get,

$\angle BAC + \angle BCA + \angle ABC = 180^0$

$\angle BAC + \angle BCA + 90^0 = 180^0$

$\angle BAC + \angle BCA = 90^0$

But given that, BA = BC

Therefore, ∠BAC = ∠BCA = x

$x + x = 90^0$

$2x = 90^0$

$x = 45^0$

In △BCD,

BC = CD

Hence,

∠CBD = ∠CDB = y and

Exterior ∠ACB = Sum of interior opposite angles

∠ACB = ∠CBD + ∠CDB

$x = y + y$

Therefore,

$2y = x = 45^0$

$y = \frac{45°}{2}$

$y = 22.5^0$ or

$y = \left(22\frac{1}{2}\right)^{o}$

(vii) In the figure,

AB is the diameter of circle with centre O

ST is the tangent at point B

$\angle ASB = 65^0$

In △ABS

∵ TS is the tangent and OB is the radius

OB is perpendicular to ST or

$\angle ABS = 90^0$

But in $\triangle ASB$,

$\angle BAC + \angle ASB + \angle ABS = 180^0$

$x + 65^0 + 90^0 = 180^0$

$x + 155^0 = 180^0$

$x = 180^0 - 155^0$

We get,

$x = 25^0$

Therefore, $x = 25^0$

(viii) In the figure,

AB is the diameter of the circle with centre O

ST is the tangent to the circle at point B

AB = BS

Hence,

ST is the tangent and OB is the radius

$OB \perp ST$ or $\angle OBS = 90^0$

In $\triangle ABS$,

$\angle BAS + \angle BSA + \angle ABS = 180^0$

By angle sum property of triangle

$\angle BAS + \angle BSA + 90^0 = 180^0$

$\angle BAS + \angle BSA = 90^0$

$x + y = 90^0$

$\because AB = BS$

Hence,

x = y

Therefore, $x = y = \frac{90°}{2} = 45^0$

(ix) In the figure,

RS is the diameter of the circle with centre as O

SR is produced to Q

QT is the tangent to the circle at point P

OP is joined

$\angle Q = 36^0$

QT is the tangent and OP is the radius of the circle

Hence,

OP is perpendicular to QT

$\angle OPQ = 90^0$

In $\triangle OPQ$,

By angle sum property of triangle, we get,

$\angle OQP + \angle POQ + \angle OPQ = 180^0$

$\angle OQP + \angle POQ + 90^0 = 180^0$

Hence,

$\angle OQP + \angle POQ = 90^0$

$36^0 + x = 90^0$

$x = 90^0 - 36^0$

We get,

$x = 54^0$

In △OPS,

OP = OS (Radii of the circle)

Hence,

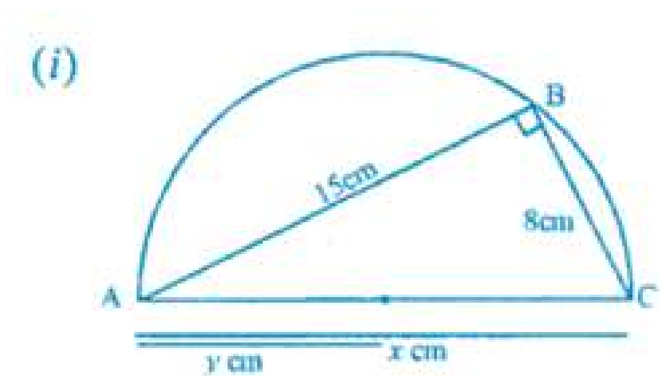
∠OPS = ∠OSP = y and

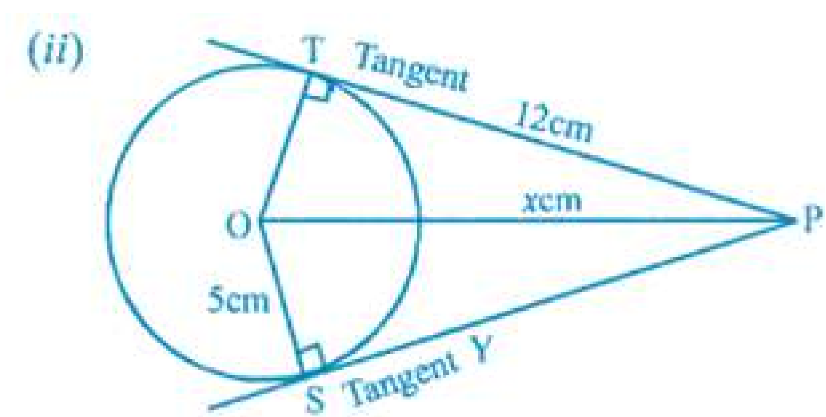
Exterior angle ∠POQ = ∠OPS + ∠OSP

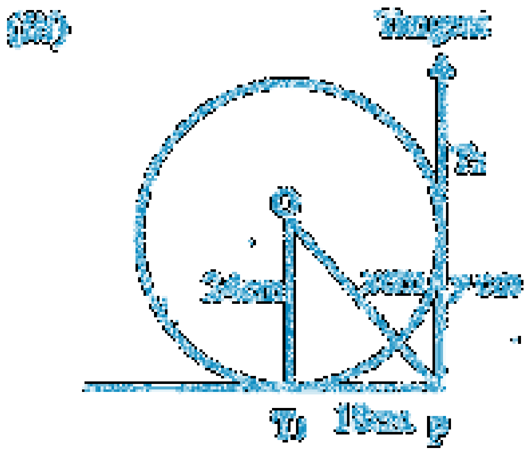
x = y + y

$x = 2y = 54^0$

6. In each of the following figures, O is the centre of the circle. Find the values of x and y.







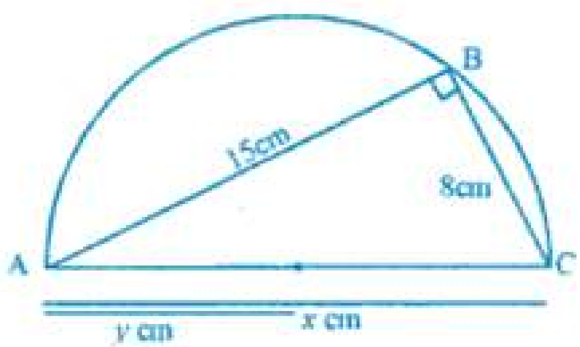
Solution:

(i) Given

O is the centre of the circle

AB = 15 cm,

BC = 8 cm



$\angle ABC = 90^0$ (Angles in a semicircle)

By Pythagoras Theorem,

$AC^2 = AB^2 + BC^2$

$AC^2 = (15)^2 + (8)^2$

$AC^2 = 225 + 64$

$AC^2 = 289$

$AC^2 = (17)^2$

AC = 17 cm

x = 17 cm

$y = \frac{1}{2}$

(Since AC is the diameter and AO is the radius of the circle)

$= \frac{1}{2} \times 17$

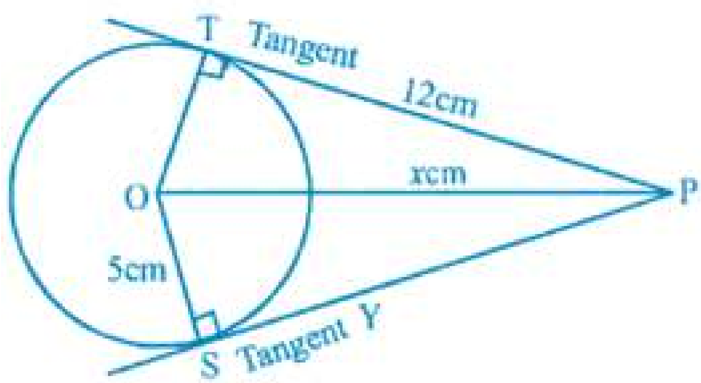
$= \frac{17}{2}$ cm

= 8.5 cm

(ii) O is the centre of the circle

PT and PS are the tangents to the circle from point P

OS and OT are the radii of the circle



Hence, $\angle OSP = \angle OTP = 90^0$

OS = OT = 5 cm and

PT = PS = 12 cm

Now,

In right angled triangle OTP,

By Pythagoras Theorem

$OP^2 = OT^2 + PT^2$

$= (5)^2 + (12)^2$

$= 25 + 144$

$= 169$

We get,

$OP^2 = (13)^2$

Therefore, OP = 13 cm

i.e., x = 13 cm

Since PS = PT = 12 cm

Therefore,

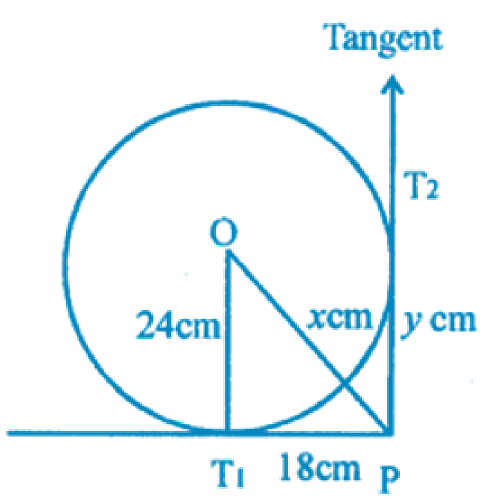
y = 12 cm

(iii) O is the centre of the circle

OT_1 is the radius of the circle

PT_1 and PT_2 are the tangents of the circle from point P

OT_1 = 24 cm and PT_1 = 18 cm



Here,

OT_1 is the radius and PT_1 is the tangent,

Hence,

$OT_1 \perp PT_1$

Now,

In right angled triangle OPT,

By Pythagoras Theorem

$OP^2 = OT_1^2 + PT_1^2$

$OP^2 = (24)^2 + (18)^2$

$OP^2 = 576 + 324$

We get,

$OP^2 = 900$

$OP^2 = (30)^2$

$OP = 30$ cm

$x = 30$ cm

Since, PT_1 and PT_2 are the tangents from point P

Therefore,

$PT_1 = PT_2 = 18$ cm

i.e., $y = 18$ cm

Mental Maths

Question 1: Fill in the blanks:

(i) A chord of a circle is a line segment with its end points ……………

(ii) A diameter of a circle is a chord that …………… the centre of circle.

(iii) A line meets a circle at most in …………… points.

(iv) One-half of the whole arc of a circle is called a …………… of the circle.

(v) The angle subtended by an arc of a circle at the centre of the circle is called the …………… by the arc.

(vi) A line which meets a circle in one and only one point is called a …………… to the circle.

(vii) The tangent at any point of a circle and the radius through that point are …………… to each other.

(viii)From a point outside the circle … tangents can be drawn to the circle.

(ix) The measure of an angle in a semicircle is ……………

Solution:

(i) A chord of a circle is a line segment with its endpoints on the circle.

(ii) A diameter of a circle is a chord that passes through the centre of the circle.

(iii) A line meets a circle almost in two points.

(iv) One-half of the whole arc of a circle is called a semicircle of the circle.

(v) The angle subtended by an arc of a circle at the centre of the circle is called the angle subtended by the arc.

(vi) A line which meets a circle in one and only one point is called a tangent to the circle.

(vii) The tangent at any point of a circle and the radius through that point are perpendicular to each other.

(viii)From a point outside the circle, two tangents can be drawn to the circle.
(ix) The measure of an angle in a semicircle is right angle.

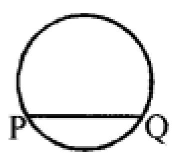
Question 2: State whether the following statements are true (T) or false (F):
(i) A line segment with its end-points lying on a circle is called a radius of the circle.
(ii) Diameter is the longest chord of the circle.
(iii) The end-points of a diameter of a circle divide the circle into two parts; each part is called a semicircle.
(iv) A diameter of a circle divides the circular region into two parts; each part is called a semicircular region.
(v) The diameters of a circle are concurrent. The centre of the circle is the point common to all diameters.
(vi) Every circle has unique centre and it lies inside the circle.
(vii) Every circle has unique diameter.
(viii)From a given point in the exterior of a circle, two tangents can be drawn to it and these two tangents are equal in length.
Solution:
(i) A line segment with its end-points lying on a
circle is called a radius of the circle. False
Correct:
It is called a chord of the circle.
(ii) Diameter is the longest chord of the circle. True
(iii) The end-points of a diameter of a circle divide the circle
into two parts; each part is called a semicircle. True
(iv) A diameter of a circle divides the circular region
into two parts; each part is called a semicircular region. True
(v) The diameters of a circle are concurrent.
The centre of the circle is the point common to all diameters. True

(vi) Every circle has unique centre and it lies inside the circle. True
(vii) Every circle has unique diameter. False
Correct:
It has infinite number of diameters.
(viii) From a given point in the exterior of a circle,
two tangents can be drawn to it and these two tangents are equal in length. True

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 6):

Question 3: If P and Q are any two points on a circle, then the line segment PQ is called a
(a) radius of the circle
(b) diameter of the circle
(c) chord of the circle
(d) secant of the circle
Solution:
P and Q are two points on a circle.



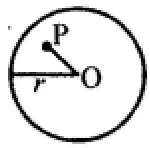
Then line segment PQ is called a chord of the circle. (c)

Question 4: If P is a point in the interior of a circle with centre O and radius r, then
(a) OP = r
(b) OP > r
(c) OP > r
(d) OP < r

Solution:

P is a point in the interior of a circle with centre O, r is the radius

∴ OP < r (d)



Question 5: If AB = 12 cm, BC = 16 cm and AB is perpendicular to BC, then the radius of the circle passing through the points A, B and C is

(a) 6 cm

(b) 8 cm

(c) 10 cm

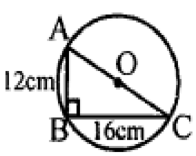
(d) 20 cm

Solution:

AB = 12 cm, BC = 16 cm

AC is the diagonal of ΔABC

and AC is the diameter of the circle (∵ ∠B = 90°)



Now AC = $\sqrt{(AB)^2 + (BC)^2} = \sqrt{12^2 + 16^2}$

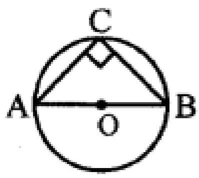
$= \sqrt{144 + 256} = \sqrt{400} = 20$ cm

∴ Radius of the circle = $\frac{\text{Diameter}}{2}$

$= \frac{20}{2}$ cm = 10 cm (c)

Question 6: In the given figure, AB is a diameter of the circle. If AC = BC, then ∠CAB is equal to

(a) 30°
(b) 45°
(c) 60°
(d) 90°



Solution:
In circle with centre O, AB is its diameter.
∴ ∠C = 90° (Angle in a semi-circle)
By ∠ sum property of Δ
∠A + ∠B +∠C = 180°
∠A + ∠B + 90°= 180°
∴ ∠A + ∠B = 90°
∵ CA = CB
∴ ∠A = ∠B = $\frac{90}{2}$ = 45°
∴ ∠CAB = 45° (b)

Check Your Progress

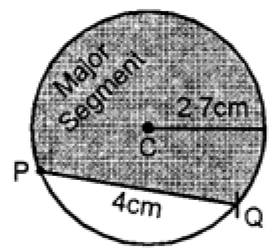
Question 1: Draw a circle of radius 2·7 cm. Draw a chord PQ of length 4 cm of this circle. Shade the major segment of this circle.

Solution:

(i) Draw a circle of radius = 2·7 cm.

(ii) Take a point P anywhere on the circle.

(iii) With P as centre and 4 cm as radius, draw an arc which cuts the circle at Q.



(iv) Join PQ which is the required chord.

(v) Shade the major segment.

Question 2: Draw a circle of radius 3·2 cm and in it make a sector of angle.

(i) 30°

(ii) 135°

(iii) $2\frac{2}{3}$ right angles

Draw separate diagrams and shade the sectors.

Solution:

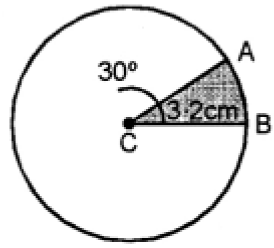
(i) 30°

Steps :

(a) Draw a circle with centre C and radius CB = 3.2 cm

(b) From C, make an angle of 30°.

(c) Shade the region enclosed in ABC.



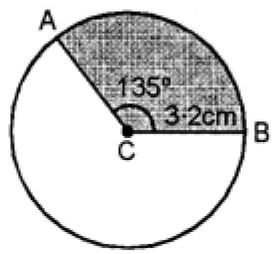
(ii) 135°

Steps :

(a) Draw a circle with centre C and radius CB = 3.2 cm.

(b) From C, make an angle of 135°.

(c) Shade the region enclosed in ACB.



(iii) $2\frac{2}{3}$ right angles

Steps :

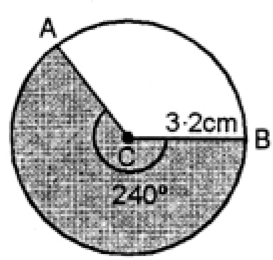
(a) Draw a circle with centre C and radius 3.2 cm.

(b) From C, make an $\angle\ 2\frac{2}{3}$ of right angle

$= 2\frac{2}{3} \times 90°$

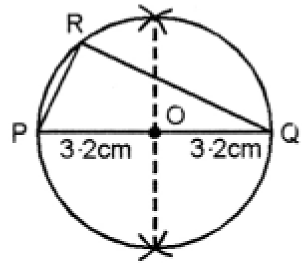
$= \frac{8}{3} \times 90° = 240°$

(c) Shade the region enclosed in ACB.



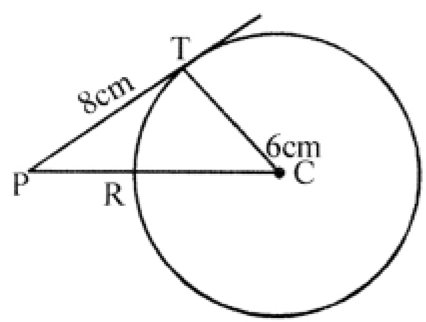
$= \frac{8}{3} \times 90° = 240°$

Question 3: Draw a line segment PQ = 6·4 cm. Construct a circle on PQ as diameter. Take any point R on this circle and measure ∠PRQ.
Solution:
(i) Draw a line segment PQ = 6·4 cm.
(ii) Draw ⊥ bisector of PQ.
(iii) With O as centre and OP or OQ as radius draw a circle which passes through P as well as through Q.



(iv) Take point R on the circle.
(v) Join PR and QR.
(vi) Measure ∠PRQ, we get ∠PRQ = 90°.

Question 4: In the adjoining figure, the tangent to a circle of radius 6 cm from an external point P is of length 8 cm. Calculate the distance of the point P from the nearest point of the circumference.



Solution:
C is the centre of the circle
PT is the tangent to the circle from P.
CT is the radius

∴ CT ⊥ PT

CT = CR = 6 cm, PT = 8 cm

Now in right ΔCPT (By Pythagoras Theorem)

$CP^2 = PT^2 + CT^2 = (8)^2 + (6)^2$

$= 64 + 36 = 100 = (10)^2$

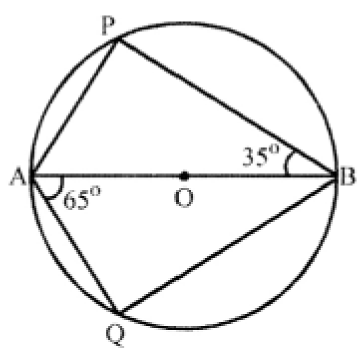
∴ CP = 10 cm

Now PR = CP – CR = 10 – 6 = 4 cm

Question 5: In the given figure, O is the centre of the circle. If ∠ABP = 35° and ∠BAQ = 65°, find

(i) ∠PAB

(ii) ∠QBA



Solution:

In the figure,

AB is the diameter of the circle with centre O

∠ABP = 35° and ∠BAQ = 65°

(i) ∠APB = 90° (Angle in a semicircle)

In ΔAPB, By ∠sum property of Δ

∠PAB + ∠P + ∠ABP = 180°

∠PAB + 90° + ∠ABP = 180°

∴ ∠PAB + ∠ABP = 90°

⇒ ∠PAB + 35° = 90° ⇒ ∠PAB = 90° – 35° ∠PAB = 55°

(ii) Similarly $\angle AQB = 90°$ (Angle in a semicircle)
In ΔAOB, By angle sum property of Δ
$\angle BAQ + \angle Q + \angle QBA = 180°$
$\angle BAQ + \angle QBA + 90° = 180°$
$\therefore \angle BAQ + \angle QBA = 90°$
$\Rightarrow 65° + \angle QBA = 90°$
$\Rightarrow \angle QBA = 90° - 65° = 25°$
Hence $\angle PAB = 55°$ and $\angle QBA = 25°$