Chapter - Kinetic Theory

Topic-1: Kinetic Theory of an Ideal Gas and Gas Laws

1 MCQs with One Correct Answer

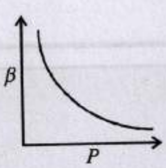
1. Two non-reactive monoatomic ideal gases have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4 : 3. The ratio of their densities is **[Adv. 2013]**

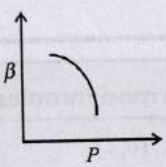
(a) 1 : 4 (b) 1 : 2 (c) 6 : 9 (d) 8 : 9

2. A real gas behaves like an ideal gas if its **[2010]**

(a) pressure and temperature are both high
(b) pressure and temperature are both low
(c) pressure is high and temperature is low
(d) pressure is low and temperature is high

3. Which of the following graphs correctly represents the variation of $\beta = -\dfrac{dV/dP}{V}$ with P for an ideal gas at constant temperature? **[2002S]**

(a)


(b)


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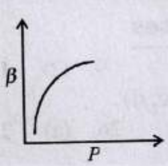
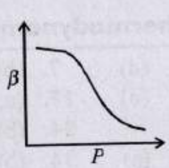


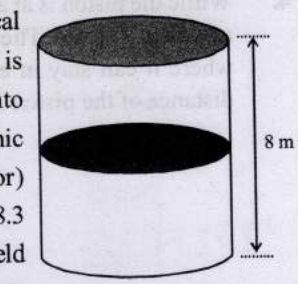
(d)


4. Two identical containers A and B with frictionless pistons contain the same ideal gas at the same temperature and the same volume V. The mass of the gas in A is m_A, and that in B is m_B. The gas in each cylinder is now allowed to expand isothermally to the same final volume 2V. The changes in the pressure in A and B are found to be ΔP and 1.5 ΔP respectively. Then **[1998S - 2 Marks]**

(a) $4m_A = 9m_B$ (b) $2m_A = 3m_B$
(c) $3m_A = 2m_B$ (d) $9m_A = 4m_B$

2 Integer Value Answer

5. A thermally isolated cylindrical closed vessel of height 8 m is kept vertically. It is divided into two equal parts by a diathermic (perfect thermal conductor) frictionless partition of mass 8.3 kg. Thus the partition is held initially at a distance of 4 m from the top, as shown in the schematic figure below. Each of the two parts of the vessel contains 0.1 mole of an ideal gas at temperature 300 K. The partition is now released and moves without any gas leaking from one part of the vessel to the other. When equilibrium is reached, the distance of the partition from the top (in m) will be _______ (take the acceleration due to gravity = 10 ms^{-2} and the universal gas constant = 8.3 J $mol^{-1}K^{-1}$).

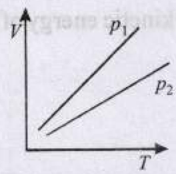


4 Fill in the Blanks

6. During an experiment, an ideal gas is found to obey an additional law VP^2 = constant, The gas is initially at a temperature T, and volume V. When it expands to a volume $2V$, the temperature becomes.......... **[1987 - 2 Marks]**

5 True / False

7. The volume V versus temperature T graphs for a certain amount of a perfect gas at two pressure p_1 and p_2 are as shown in Fig. It follows from the graphs that p_1 is greater than p_2. **[1982 - 2 Marks]**



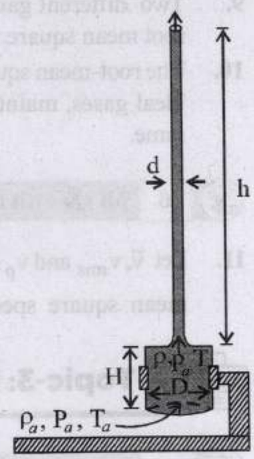
8 Comprehension/Passage Based Questions

Directions for questions no. 8 and 9:

PARAGRAPH

A cylindrical furnace has height (H) and diameter (D) both 1 m. It is maintained at temperature 360 K. The air gets heated inside the furnace at constant pressure P_a and its temperature becomes T = 360 K. The hot air with density ρ rises up a vertical chimney of diameter d = 0.1 m and height h = 9 m above the furnace and exits the chimney (see the figure). As a result, atmospheric air of density ρ_a = 1.2 kg m^{-3}, pressure P_a and temperature T_a = 300 K enters the furnace. Assume air as an ideal gas, neglect the variations in ρ and T inside the chimney and the furnace. Also ignore the viscous effects.

[Given: The acceleration due to gravity g = 10 m s^{-2} and π = 3.14]



8. Considering the air flow to be streamline, the steady mass flow rate of air exiting the chimney is ______ gm s^{-1}. **[Adv. 2023]**

9. When the chimney is closed using a cap at the top, a pressure difference ΔP develops between the top and the bottom surfaces of the cap. If the changes in the temperature and density of the hot air, due to the stoppage of air flow, are negligible then the value of ΔP is ______ N m^{-2}. **[Adv. 2023]**

Topic-2: Speed of Gas, Pressure and Kinetic Energy

1 MCQs with One Correct Answer

1. A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\frac{v_{rms}(\text{helium})}{v_{rms}(\text{argon})}\right)$ is **[2012]**

(a) 0.32 (b) 0.45 (c) 2.24 (d) 3.16

2. The ratio of the speed of sound in nitrogen gas to that in helium gas, at 300 K is **[1999S - 2 Marks]**

(a) $\sqrt{(2/7)}$ (b) $\sqrt{(1/7)}$ (c) $(\sqrt{3})/5$ (d) $(\sqrt{6})/5$

3. A closed compartment containing gas is moving with some acceleration in horizontal direction. Neglect effect of gravity. Then the pressure in the compartment is **[1999S - 2 Marks]**

(a) same everywhere (b) lower in the front side
(c) lower in the rear side (d) lower in the upper side

4. A vessel contains 1 mole of O_2 gas (relative molar mass 32) at a temperature T. The pressure of the gas is P. An identical vessel containing one mole of He gas (relative molar mass 4) at a temperature 2T has a pressure of **[1997 - 1 Mark]**

(a) P/8 (b) P (c) 2P (d) 8P

5. The temperature of an ideal gas is increased from 120 K to 480 K. If at 120 K the root-mean-square velocity of the gas molecules is v, at 480 K it becomes **[1996 - 2 Marks]**

(a) 4v (b) 2v (c) v/2 (d) v/4

6. Three closed vessels A, B and C are at the same temperature T and contain gases which obey the Maxwellian distribution of velocities. Vessel A contain only O_2, B only N_2 and C a mixture of equal quantities of O_2 and N_2. If the average speed of the O_2 molecules in vessel A is v_1 that of the N_2 molecules in vessel B is v_2, the average speed of the O_2 molecules in vessel C is **[1992 - 2 Marks]**

(a) $\frac{v_1+v_2}{2}$ (b) v_1
(c) $(v_1.v_2)^{\frac{1}{2}}$ (d) $\sqrt{\frac{3kT}{M}}$

where M is the mass of an oxygen molecule.

5 True / False

7. The root mean square (rms) speed of oxygen molecules (O_2) at a certain temperature T (degree absolute) is V. If the temperature is doubled and oxygen gas dissociates into atomic oxygen, the rms speed remains unchanged. **[1987 - 2 Marks]**

8. The ratio of the velocity of sound in Hydrogen gas ($\gamma = \frac{7}{5}$) to that in Helium gas ($\gamma = \frac{5}{3}$) at the same temperature is $\sqrt{\frac{21}{5}}$. **[1983 - 2 Marks]**

9. Two different gases at the same temperature have equal root mean square velocities. **[1982 - 2 Marks]**

10. The root-mean square speeds of the molecules of different ideal gases, maintained at the same temperature are the same. **[1981- 2 Marks]**

6 **MCQs with One or More than One Correct Answer**

11. Let $\overline{v}, v_{rms}$ and v_p respectively denote the mean speed. root mean square speed, and most probable speed of the molecules in an ideal monatomic gas at absolute temperature T. The mass of a molecule is m. Then **[1998S - 2 Marks]**
    (a) no molecule can have a speed greater than $\sqrt{2}\, v_{rms}$
    (b) no molecule can have a speed less than $v_p/\sqrt{2}$
    (c) $v_p < \overline{v} < v_{rms}$
    (d) the average kinetic energy of a molecule is $\frac{3}{4} m v_p^2$.

Topic-3: Degree of Freedom, Specific Heat Capacity and Mean Free Path

1 **MCQs with One Correct Answer**

1. An ideal gas is in thermodynamic equilibrium. The number of degrees of freedom of a molecule of the gas is n. The internal energy of one mole of the gas is U_n. and the speed of sound in the gas is v_n. At a fixed temperature and pressure, which of the following is the correct option? **[Adv. 2023]**
   (a) $v_3 < v_6$ and $U_3 > U_6$ (b) $v_5 > v_3$ and $U_3 > U_5$
   (c) $v_5 > v_7$ and $U_5 < U_7$ (d) $v_6 < v_7$ and $U_6 < U_7$

2. The average translational kinetic energy of O_2 (relative molar mass 32) molecules at a particular temperature is 0.048 eV. The translational kinetic energy of N_2 (relative molar mass 28) molecules in eV at the same temperature is **[1997 - 1 Mark]**
   (a) 0.0015 (b) 0.003
   (c) 0.048 (d) 0.768

3. From the following statements concerning ideal gas at any given temperature T, select the correct one(s) **[1995S]**
   (a) The coefficient of volume expansion at constant pressure is the same for all ideal gases
   (b) The average translational kinetic energy per molecule of oxygen gas is 3kT, k being Boltzmann constant
   (c) The mean-free path of molecules increases with increases in the pressure
   (d) In a gaseous mixture, the average translational kinetic energy of the molecules of each component is different

4. If one mole of a monatomic gas $\left(\gamma = \frac{5}{3}\right)$ is mixed with one mole of a diatomic gas $\left(\gamma = \frac{7}{5}\right)$, the value of γ for mixture is **[1988 - 1 Mark]**
   (a) 1.40 (b) 1.50 (c) 1.53 (d) 3.07

5. A vessel contains a mixture of one mole of oxygen and two moles of nitrogen at 300 K. The ratio of the average rotational kinetic energy per O_2 molecule to that per N_2 molecule is **[1998S - 2 Marks]**
   (a) 1 : 1
   (b) 1 : 2
   (c) 2 : 1
   (d) depends on the moments of inertia of the two molecules

6. Two cylinders *A* and *B* fitted with pistons contain equal amounts of an ideal diatomic gas at 300 K. The piston of *A* is free to move, while that *B* is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in *A* is 30 K, then the rise in temperature of the gas in *B* is **[1998S - 2 Marks]**
   (a) 30 K (b) 18 K
   (c) 50 K (d) 42 K

7. When an ideal diatomic gas is heated at constant pressure, the fraction of the heat energy supplied which increases the internal energy of the gas is **[1990 - 2 Marks]**
   (a) $\frac{2}{5}$ (b) $\frac{3}{5}$
   (c) $\frac{3}{7}$ (d) $\frac{5}{7}$

2 **Integer Value Answer**

8. A closed container contains a homogeneous mixture of two moles of an ideal monatomic gas ($\gamma = 5/3$) and one mole of an ideal diatomic gas ($\gamma = 7/5$). Here, γ is the ratio of the specific heats at constant pressure and constant volume of an ideal gas. The gas mixture does a work of 66 Joule when heated at constant pressure. The change in its internal energy is ______ Joule. **[Adv. 2023]**

4 **Fill in the Blanks**

9. One mole of a mono-atomic ideal gas is mixed with one mole of a diatomic ideal gas. The molar specific heat of the mixture at constant volume is .... **[1984- 2 Marks]**

5 True / False

10. At a given temperature, the specific heat of a gas at constant pressure is always greater than its specific heat at constant volume. **[1987 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

11. A container of fixed volume has a mixture of one mole of hydrogen and one mole of helium in equilibrium at temperature T. Assuming the gases are ideal, the correct statement(s) is (are) **[Adv. 2015]**

(a) The average energy per mole of the gas mixture is $2RT$

(b) The ratio of speed of sound in the gas mixture to that in helium gas is $\sqrt{6/5}$

(c) The ratio of the rms speed of helium atoms to that of hydrogen molecules is 1/2

(d) The ratio of the rms speed of helium atoms to that of hydrogen molecules is $\frac{1}{\sqrt{2}}$

12. C_v and C_p denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then **[2009]**

(a) $C_p - C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(b) $C_p + C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(c) C_p / C_v is larger for a diatomic ideal gas than for a monoatomic ideal gas

(d) $C_p \cdot C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

9 Assertion and Reason Type Questions

13. **Statement-1 :** The total translational kinetic energy of all the molecules of a given mass of an ideal gas is 1.5 times the product of its pressure and its volume. **[2007]**

because

Statement-2 : The molecules of a gas collide with each other and the velocities of the molecules change due to the collision.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False

(d) Statement-2 is False, Statement-2 is True

Topic-4: Miscellaneous (Mixed Concepts) Problems

1 MCQs with One Correct Answer

1. Two moles of ideal helium gas are in a rubber balloon at 30°C. The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35°C. The amount of heat required in raising the temperature is nearly (take R = 8.31 J/mol.K) **[2012]**

(a) 62 J (b) 104 J (c) 124 J (d) 208 J

2. An ideal gas is expanding such that PT^2 = constant. The coefficient of volume expansion of the gas is – **[2008]**

(a) 1/T (b) 2/T (c) 3/T (d) 4/T

3. When an ideal diatomic gas is heated at constant pressure, the fraction of the heat energy supplied which increases the internal energy of the gas is **[1990 - 2 Marks]**

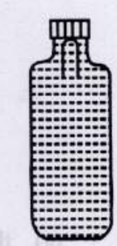
(a) $\frac{2}{5}$ (b) $\frac{3}{5}$

(c) $\frac{3}{7}$ (d) $\frac{5}{7}$

3 Numeric/New Stem Based Questions

Stem for Qs. 4-5

A soft plastic bottle, filled with water of density 1 gm/cc, carries an inverted glass test-tube with some air (ideal gas) trapped as shown in the figure. The test-tube has a mass of 5 gm, and it is made of a thick glass of density 2.5 gm/cc. Initially the bottle is sealed at atmospheric pressure $p_0 = 10^5$ Pa so that the volume of the trapped air is $v_0 = 3.3$ cc. When the bottle is squeezed from outside at constant temperature, the pressure inside rises and the volume of the trapped air reduces. It is found that the test tube begins to sink at pressure $P_0 + \Delta p$ without changing its orientation. At this pressure, the volume of the trapped air is $v_0 - \Delta v$. Let $\Delta v = X$ cc and $\Delta p = Y \times 10^3$ Pa.



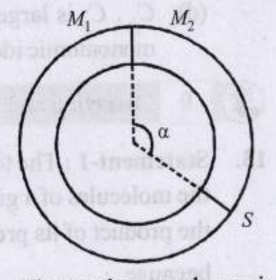
4. The value of X is ____. **[Adv. 2021]**

5. The value of Y is ____. **[Adv. 2021]**

6. One mole of a monatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 K and the universal gas constant $R = 8.0\ J\ mol^{-1}\ K^{-1}$, the decrease in its internal energy, in *Joule*, is________. **[Adv. 2018]**

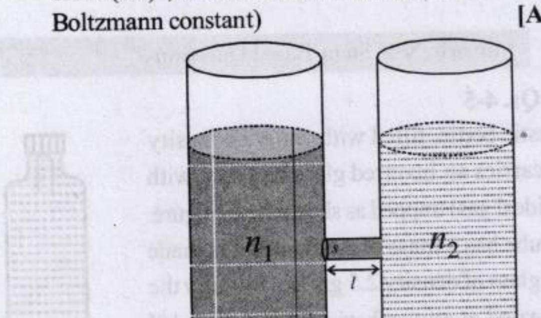
4 Fill in the Blanks

7. A ring shaped tube contains two ideal gases with equal masses and relative molar masses $M_1 = 32$ and $M_2 = 28$. The gases are separated by one fixed partition and another movable stopper S which can move freely without friction inside the ring. The angle α as shown in the figure is .... degrees. **[1997 - 2 Marks]**



6 MCQs with One or More than One Correct Answer

8. As shown schematically in the figure, two vessels contain water solutions (at temperature T) of potassium permanganate ($KMnO_4$) of different concentrations n_1 and n_2 ($n_1 > n_2$) molecules per unit volume with $\Delta n = (n_1 - n_2) << n_1$. When they are connected by a tube of small length l and cross-sectional area S, $KMnO_4$ starts to diffuse from the left to the right vessel through the tube. Consider the collection of molecules to behave as dilute ideal gases and the difference in their partial pressure in the two vessels causing the diffusion. The speed v of the molecules is limited by the viscous force $-\beta v$ on each molecule, where β is a constant. Neglecting all terms of the order $(\Delta n)^2$, which of the following is/are correct? (k_B is the Boltzmann constant) **[Adv. 2013]**



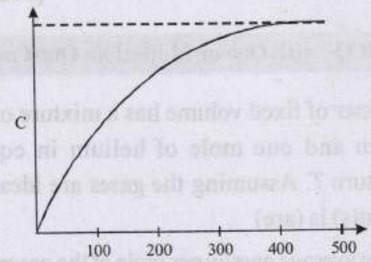
(a) the force causing the molecules to move across the tube is $\Delta n k_B TS$

(b) force balance implies $n_1 \beta v l = \Delta n k_B T$

(c) total number of molecules going across the tube per sec is $\left(\frac{\Delta n}{l}\right)\left(\frac{k_B T}{\beta}\right) S$

(d) rate of molecules getting transferred through the tube does not change with time

9. The figure below shows the variation of specific heat capacity (C) of a solid as a function of temperature (T). The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, the following statement(s) is (are) correct to a reasonable approximation. **[Adv. 2013]**



(a) The rate at which heat is absorbed in the range 0 -100 K varies linearly with temperature T.

(b) Heat absorbed in increasing the temperature from 0-100 K is less than the heat required for increasing the temperature from 400-500 K.

(c) There is no change in the rate of heat absorption in the range 400-500 K.

(d) The rate of heat absorption increases in the range 200-300 K.

10. An ideal gas is taken from the state A (pressure P, volume V) to the state B (pressure $P/2$, volume $2V$) along a straight line path in the P-V diagram. Select the correct statement (s) from the following: **[1993-2 Marks]**

(a) The work done by the gas in the process A to B exceeds the work that would be done by it if the system were taken from A to B along the isotherm.

(b) In the T-V diagram, the path AB becomes a part of a parabola

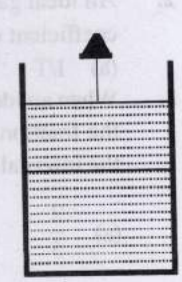
(c) In the P-T diagram, the path AB becomes a part of a hyperbola

(d) In going from A to B, the temperature T of the gas first increases to a maximum value and then decreases.

8 Comprehension/Passage Based Questions

Passage -1

In the figure, a container is shown to have a movable (without friction) piston on top. The container and the piston are all made of perfectly insulated material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting material that allows slow transfer of heat. The lower compartment of the container is filled with 2 moles of an ideal monatomic gas at 700 K and the upper compartment is filled with 2 moles of an



ideal diatomic gas at 400 K. The heat capacities per mole of an ideal monatomic gas are $C_V = \frac{3}{2}R$, $C_P = \frac{5}{2}R$, and those for an ideal diatomic gas are $C_V = \frac{5}{2}R$, $C_P = \frac{7}{2}R$.

11. Consider the partition to be rigidly fixed so that it does not move. When equilibrium is achieved, the final temperature of the gases will be **[Adv. 2014]**

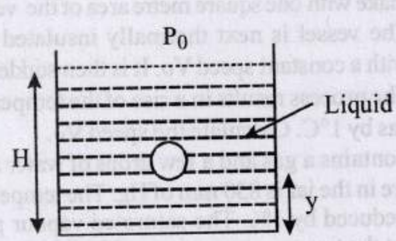
(a) 550 K (b) 525 K
(c) 513 K (d) 490 K

12. Now consider the partition to be free to move without friction so that the pressure of gases in both compartments is the same. The total work done by the gases till the time they achieve equilibrium will be **[Adv. 2014]**

(a) $250R$ (b) $200R$
(c) $100R$ (d) $-100R$

Passage - 2

A small spherical mono - atomic ideal gas bubble ($\gamma = 5/3$) is trapped inside a liquid of density ρ (see figure). Assume that the bubble does not exchange any heat with the liquid. The bubble contains n moles of gas. The temperature of the gas when the bubble is at the bottom is T_0, the height of the liquid is H and the atmospheric pressure is P_0 (Neglect surface tension). **[2008]**



13. As the bubble moves upwards, besides the buoyancy force the following forces are acting on it

(a) Only the force of gravity
(b) The force due to gravity and the force due to the pressure of the liquid
(c) The force due to gravity, the force due to the pressure of the liquid and the force due to viscosity of the liquid
(d) The force due to gravity and the force due to viscosity of the liquid

14. When the gas bubble is at a height y from the bottom, its temperature is –

(a) $T_0\left(\frac{P_0+\rho_\ell gH}{P_0+\rho_\ell gy}\right)^{2/5}$ (b) $T_0\left(\frac{P_0+\rho_\ell g(H-y)}{P_0+\rho_\ell gH}\right)^{2/5}$

(c) $T_0\left(\frac{P_0+\rho_\ell gH}{P_0+\rho_\ell gy}\right)^{3/5}$ (d) $T_0\left(\frac{P_0+\rho_\ell g(H-y)}{P_0+\rho_\ell gH}\right)^{3/5}$

15. The buoyancy force acting on the gas bubble is (Assume R is the universal gas constant)

(a) $\rho_\ell nRgT_0\frac{(P_0+\rho_\ell gH)^{2/5}}{(P_0+\rho_\ell gy)^{7/5}}$

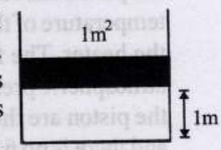
(b) $\frac{\rho_\ell nRgT_0}{(P_0+\rho_\ell gH)^{2/5}[P_0+\rho_\ell g(H-y)]^{3/5}}$

(c) $\rho_\ell nRgT_0\frac{(P_0+\rho_\ell gH)^{3/5}}{(P_0+\rho_\ell gy)^{8/5}}$

(d) $\frac{\rho_\ell nRgT_0}{(P_0+\rho_\ell gH)^{3/5}[P_0+\rho_\ell g(H-y)]^{2/5}}$

10 Subjective Problems

16. A diatomic gas is enclosed in a vessel fitted with massless movable piston. Area of cross section of vessel is 1 m². Initial height of the piston is 1 m (see the figure). The initial temperature of the gas is 300 K. The temperature of the gas is increased to 400 K, keeping pressure constant, calculate the new height of the piston. The piston is brought to its initial position with no heat exchange. Calculate the final temperature of the gas. You can leave answer in fraction. **[2004 - 2 Marks]**



17. An insulated container containing monoatomic gas of molar mass *m* is moving with a velocity v_0. If the container is suddenly stopped, find the change in temperature. **[2003 - 2 Marks]**

18. A cubical box of side 1 meter contains helium gas (atomic weight 4) at a pressure of 100 N/m². During an observation time of 1 second, an atom travelling with the root-mean-square speed parallel to one of the edges of the cube, was found to make 500 hits with a particular wall, without any collision with other atoms. Take $R = \frac{25}{3}$ J/mol-K and $k = 1.38 \times 10^{-23}$ J/K **[2002 - 5 Marks]**

(a) Evaluate the temperature of the gas.
(b) Evaluate the average kinetic energy per atom.
(c) Evaluate the total mass of helium gas in the box.

19. A gaseous mixture enclosed in a vessel of volume *V* consists of one mole of a gas *A* with $\gamma (= C_p/C_v) = 5/3$ and another gas *B* with $\gamma = 7/5$ at a certain temperature *T*. The relative molar masses of the gases *A* and *B* are 4 and 32, respectively. The gases *A* and *B* do not react with each other and are assumed to be ideal. The gaseous mixture follows the equation $PV^{19/13}$ = constant, in adiabatic processes. **[1995 - 10 Marks]**

(a) Find the number of moles of the gas *B* in the gaseous mixture.
(b) Compute the speed of sound in the gaseous mixture at *T* = 300 K.
(c) If *T* is raised by 1K from 300 K, find the % change in the speed of sound in the gaseous mixture.

(d) The mixture is compressed adiabatically to 1/5 of its initial volume V. Find the change in its adiabatic compressibility in terms of the given quantities.

20. A closed container of volume 0.02 m^3 contains a mixture of neon and argon gases, at a temperature of 27°C and pressure of $1 \times 10^5 \text{ Nm}^{-2}$. The total mass of the mixture is 28 g. If the molar masses of neon and argon are 20 and 40 g mol^{-1} respectively, find the masses of the individual gases in the container assuming them to be ideal (Universal gas constant $R = 8.314 \text{ J/mol} - K$). **[1994 - 6 Marks]**

21. An ideal monatomic gas is confined in a cylinder by a spring-loaded piston of cross-section $8.0 \times 10^{-3} \text{ m}^2$. Initially the gas is at 300 K and occupies a volume of $2.4 \times 10^{-3} \text{ m}^3$ and the spring is in its relaxed (unstretched, uncompressed) state, fig. The gas is heated by a small electric heater until the piston moves out slowly by 0.1 m. Calculate the final temperature of the gas and the heat supplied (in joules) by the heater. The force constant of the spring is 8000 N/m, atmospheric pressure is $1.0 \times 10^5 \text{ Nm}^{-2}$. The cylinder and the piston are thermally insulated. The piston is massless and there is no friction between the piston and the cylinder. Neglect heat loss through lead wires of the heater. The heat capacity of the heater coil is negligible. Assume the spring to be massless. **[1989 - 8 Mark]**

22. Two moles of helium gas ($\gamma = 5/3$) are initially at temperature 27°C and occupy a volume of 20 litres. The gas is first expanded at constant pressure until the volume is doubled. Then it undergoes an adiabatic change until the temperature returns to its initial value. **[1988 - 6 Marks]**

(i) Sketch the process on a p-V diagram.

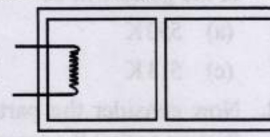
(ii) What are the final volume and pressure of the gas ?

(iii) What is the work done by the gas ?

23. A thin tube of uniform cross-section is sealed at both ends. It lies horizontally, the middle 5 cm containing mercury and the two equal end containing air at the same pressure P. When the tube is held at an angle of 60° with the vertical direction, the length of the air column above and below the mercury column are 46cm and 44.5 cm respectively. Calculate the pressure P in centimeters of mercury. (The temperature of the system is kept at 30°C). **[1986 - 6 Marks]**

24. Two glass bulbs of equal volume are connected by a narrow tube and are filled with a gas at 0°C and a pressure of 76 cm of mercury. One of the bulbs is then placed in melting ice and the other is placed in a water bath maintained at 62°C. What is the new value of the pressure inside the bulbs? The volume of the connecting tube is negligible. **[1985 - 6 Marks]**

25. The rectangular box shown in Fig has a partition which can slide without friction along the length of the box. Initially each of the two chambers of the box has one mole of a mono-atomic ideal gas ($\gamma = 5/3$) at a pressure P_0, volume V_0 and temperature T_0. The chamber on the left is slowly heated by an electric heater. The walls of the box and the partition are thermally insulated. Heat loss through the lead wires of the heater is negligible. The gas in the left chamber expands pushing the partition until the final pressure in both chambers becomes $243 P_0/32$. Determine (i) the final temperature of the gas in each chamber and (ii) the work done by the gas in the right chamber. **[1984- 8 Marks]**



26. One gram mole of oxygen at 27° and one atmospheric pressure is enclosed in a vessel. **[1983 - 8 Marks]**

(i) Assuming the molecules to be moving with V_{rms}, Find the number of collisions per second which the molecules make with one square metre area of the vessel wall.

(ii) The vessel is next thermally insulated and moved with a constant speed V_o. It is then suddenly stopped. The process results in a rise of the temperature of the gas by 1°C. Calculate the speed V_o.

27. A jar contains a gas and a few drops of water at T°K. The pressure in the jar is 830 mm of Hg. The temperature of the jar is reduced by 1%. The saturated vapour pressures of water at the two temperatures are 30 and 25 mm of Hg. Calculate the new pressure in the jar. **[1980]**

28. Given samples of 1 c.c. of hydrogen and 1c.c. of oxygen, both at N.T.P. which sample has a larger number of molecules? **[1979]**

Answer Key

Topic-1 : Kinetic Therory of an Ideal Gas and Gas Laws

1. (d) **2.** (d) **3.** (a) **4.** (c) **5.** (6) **6.** ($\sqrt{2}T$) **7.** (False) **8.** (47.10) **9.** (18)

Topic-2 : Speed of Gas, Pressure and Kinetic Energy

1. (d) **2.** (c) **3.** (b) **4.** (c) **5.** (b) **6.** (b) **7.** (False) **8.** (False) **9.** (False)
10. (False) **11.** (c, d)

Topic-3 : Degree of Freedom, Specific Heat Capacity and Mean Free Path

1. (c) **2.** (c) **3.** (a) **4.** (b) **5.** (a) **6.** (d) **7.** (d) **8.** (121) **10.** (True) **11.** (a, b, d)
12. (b, d) **13.** (b)

Topic-4 : Miscellaneous (Mixed Concepts) Problems

1. (d) **2.** (c) **3.** (d) **4.** (0.30) **5.** (10) **6.** (900J) **8.** (a,b,c) **9.** (b,c,d) **10.** (a, b, d)
11. (d) **12.** (d) **13.** (d) **14.** (b) **15.** (b)

Hints & Solutions

Topic-1: Kinetic Theory of an Ideal Gas and Gas Laws

1. **(d)** $P_1M_1 = P_1RT$ and $P_2M_2 = P_2RT$

$\therefore \frac{P_1}{P_2} \times \frac{M_1}{M_2} = \frac{P_1}{P_2}$

$\frac{4}{3} \times \frac{2}{3} = \frac{P_1}{P_2} \quad \therefore \frac{P_1}{P_2} = \frac{8}{9}$

2. **(d)** A real gas behaves as an ideal gas at low pressure and high temperature.

3. **(a)** According to Boyle's law, PV = constant.

$\therefore \quad PdV + VdP = 0$

$$\Rightarrow \quad \frac{PdV}{dP} = -V; \beta = -\left(\frac{1}{V}\right)\left(\frac{dV}{dP}\right) = \left(\frac{1}{P}\right) \Rightarrow \beta \times P = 1$$

Hence graph between β and P will be a rectangular hyperbola.

4. **(c)**

Container A	Container B
Mass of gas = m_A	Mass of gas = m_B
Change in pressure = ΔP	Change in pressure = $1.5\Delta P$
$P_AV = \frac{m_A}{M}RT$	$P_BV = \frac{m_B}{M}RT$
$P'_A(2V) = \frac{m_A}{M}RT$	$P'_B(2V) = \frac{m_B}{M}RT$

$$\Rightarrow P_A - P'_A = \frac{m_ART}{MV} - \frac{m_ART}{M(2V)}$$

$$\Rightarrow \quad \Delta P = \frac{m_ART}{2MV} \quad \text{...(i)}$$

$$\text{and } P_B - P'_B = \frac{m_BRT}{MV} - \frac{m_BRT}{M(2V)}$$

$$1.5\,\Delta P = \frac{m_BRT}{2MV} \quad \text{... (ii)}$$

Dividing eq. (ii) by (i)

$$\frac{1.5\Delta P}{\Delta P} = \frac{m_B}{M_A} \Rightarrow \frac{3}{2} = \frac{m_B}{m_A} \Rightarrow 3m_A = 2m_B$$

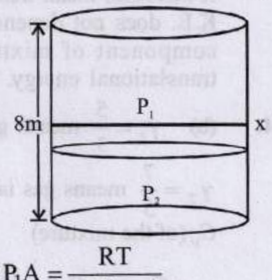
5. **(6)** Initially partition is held at a distance of 4m from the top.

Temperature, T = 300 K

$PV = PAh = nRT$

$P_04A = 0.1R \times 300$ (For both)

Let piston shifts by $\times$ meter and final temerature is T, then



$P_2A(4-x) = 0.1RT$

$$\therefore P_2A = \frac{0.1RT}{4-x} = \frac{RT}{10(4-x)} \text{ and } P_1A = \frac{RT}{10(4+x)}$$

$$\text{Finally } P_2A - P_1A = mg = 83 = \frac{RT}{10}\left(\frac{1}{4-x} - \frac{1}{4+x}\right)$$

$$\Rightarrow \frac{RT}{10}\left(\frac{2x}{16-x^2}\right) = 83$$

$$0.2C_v \times 300 + mg\,.\,x = 0.2C_vT$$

$$\Rightarrow \frac{3}{2}R \times \frac{2}{10} \times 300 + 83x = \frac{2}{10} \times \frac{3}{2}RT$$

$$\Rightarrow \left(\frac{900R}{10} + 83x\right) = 3\left(\frac{RT}{10}\right)$$

$$\Rightarrow \left(\frac{300R}{10} + \frac{83}{3}x\right) = \frac{RT}{10} \Rightarrow \left(30R + \frac{83x}{3}\right)\left(\frac{2x}{16-x^2}\right) = 83$$

$$\Rightarrow 83\left(3 + \frac{x}{3}\right)\left(\frac{2x}{16-x^2}\right) = 83 \Rightarrow \frac{(9+x)}{3}\frac{(2x)}{(16-x^2)} = 1$$

$$\Rightarrow 18x + 2x^2 = 48 - 3x^2 \Rightarrow 5x^2 + 18x - 48 = 0$$

$\therefore x = 1.78 \approx 2$

Hence distance from top when reached equilibrium = 4 + 2 = 6 m

6. **($\sqrt{2}T$)** $PV = nRT$ (Ideal gas equation)

$$\Rightarrow \quad P = \frac{nRT}{V} \quad \text{...(i)}$$

Given VP^2 = const ...(ii)

From (i) and (ii)

$$\therefore \quad \frac{T^2}{V} = \text{constant} \quad \text{or,} \quad T \propto \sqrt{v}$$

$\therefore \quad V_1 = 2V$ and $T_1 = T$

$$\therefore \quad \frac{T_1^2}{V_1} = \frac{T_2^2}{V_2} \Rightarrow T_2 = T_1\sqrt{\frac{V_2}{V_1}} = T\sqrt{\frac{2V}{V}} = \sqrt{2}T$$

7. **(False)** For a particular temperature T, $V \propto \frac{1}{P}$

Volume is greater for pressure $P_1 \quad \therefore \quad P_1 < P_2$

8. **(47.10)** Since, pressure P = constant $\rho_aT_a = \rho T$

$\Rightarrow = 1.2 \times 300 = \rho(360) \therefore \rho = 1\ kg/m^3$

Applying Bernoullis theorem between upper and bottom point

Assuming velocity of hot air inside the furnace $\simeq 0$

$$P_a + 0 + 0 = P_a - \rho_ag(h) + \rho g(h) + \frac{1}{2}\rho V^2$$

$$\therefore V = \sqrt{\frac{2(\rho_a - \rho)g \times 9}{\rho}} = \sqrt{2(0.2)90} = 6$$

Therefore the steady mass flow rate of air existing the chimney

$$Q = \rho\pi\left(\frac{d^2}{4}\right)V = 1 \times 3.14\frac{\times(0.1)^2}{4} \times 6$$

$$= 0.0471\ kg/s = 47.10\ gms^{-1}$$

9. **(18)** Pressure $P_a = P_{in} + \rho g (h)$

$\Rightarrow P_a = P_{inside} + \rho g (9)$

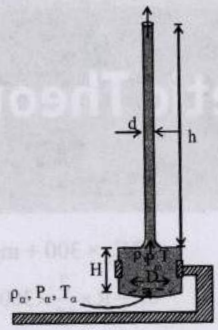
$\Rightarrow P_{inside} = P_a - \rho g (9)$

And, $P_{outside} = P_a - \rho_a g(9)$

$\therefore \Delta P = P_{inside} - P_{outside}$

$= (\rho_a - \rho) g \times 9$

$= (1.2 - 1) \times 10 \times 9 = 18 \text{ N/m}^2$



Topic-2: Speed of Gas, Pressure and Kinetic Energy

1. **(d)** Using $V_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow V_{rms} \propto \frac{1}{\sqrt{M}}$

$$\frac{v_{rms\,(helium)}}{v_{rms\,(argon)}} = \sqrt{\frac{M_{argon}}{M_{helium}}} = \sqrt{\frac{40}{4}} = \sqrt{10} \approx 3.16$$

2. **(c)** Velocity of sound in a gas $v = \sqrt{\frac{\gamma p}{p}} = \sqrt{\frac{\gamma RT}{M}}$

$$\frac{V_{N_2}}{V_{He}} = \sqrt{\frac{\gamma_{N_2} M_{He}}{\gamma_{He} M_{N_2}}} = \sqrt{\frac{7/5 \times 4}{5/3 \times 28}} = \frac{\sqrt{3}}{5}$$

3. **(b)** When a enclosed gas is accelerated in the positive x-direction then the pressure of the gas decreases along the positive x-axis and follows the equation

$\Delta P = -\rho\, a\, dx$

So more pressure on the rear side and less pressure on the front side.

4. **(c)** From, $PV = nRT \Rightarrow P = \frac{nRT}{V}$ or $P \propto T$

($\because$ V and n are same.)

Therefore, if T is doubled, pressure also becomes two times, i.e., 2P.

5. **(b)** $\frac{(V_{rms})_1}{(V_{rms})_2} = \sqrt{\frac{T_1}{T_2}} \Rightarrow \frac{V}{(V_{rms})_2} = \sqrt{\frac{120}{480}}$

$\Rightarrow \frac{V}{(V_{rms})_2} = \frac{1}{2} \Rightarrow (V_{rms})_2 = 2V$

6. **(b)** According to Maxwell's distribution of speed, average speed of molecules of an ideal gas $v \propto \sqrt{T}$.

$\therefore$ The velocity of oxygen molecules will be same in A as well as C as the temperature of *A* and *C* are same

7. **(False)** We know that, rms speed

$$v = \sqrt{\frac{3RT}{M}} \text{ then } v' = \sqrt{\frac{3R(2T)}{M/2}} \quad \therefore v' = 2v$$

8. **(False)** $$\frac{(C_{H_2})_1}{(C_{H_e})_2} = \frac{\sqrt{\frac{\gamma_1 RT}{M_1}}}{\sqrt{\frac{\gamma_2 RT}{M_2}}} = \sqrt{\frac{\gamma_1}{\gamma_2} \times \frac{M_2}{M_1}}$$

$$= \sqrt{\frac{7/5}{5/3} \times \frac{4}{2}} = \sqrt{\frac{7}{5} \times \frac{3}{5} \times 2} = \sqrt{\frac{42}{25}}$$

9. **(False)** For a particular termperature root mean square speed,

$V_{rms} \propto \frac{1}{\sqrt{M}}$

i.e., V_{rms} will have different values for different gases.

10. **(False)** RMS speed, $V_{rms} = \sqrt{\frac{3RT}{M}}$

i.e., V_{rms} depends on temperature and molar mass and hence rms speed will be different for different ideal gases.

11. **(c, d)** We know that

$$\bar{v} = \sqrt{\frac{8RT}{\pi M}}; v_{rms} = \sqrt{\frac{3RT}{M}} \text{ and } v_p = \sqrt{\frac{2RT}{M}}$$

From these expressions, we can conclude that

$v_p < \bar{v} < v_{rms}$

Also, $V_{rms} = \sqrt{\frac{3}{2}} V_P$

And, average kinetic energy of gaseous molecules

$$\bar{E} = \frac{1}{2} m v_{rms}^2 = \frac{1}{2} m \left(\frac{3}{2} v_p^2\right) = \frac{3}{4} m v_p^2$$

Topic-3: Degree of Freedom, Specific Heat Capacity, and Mean Free Path

1. **(c)** The internal energy of one mole of the gas,

$$U_n = \frac{1 \times n \times RT}{2} = \frac{nRT}{2}$$

$\therefore U_7 > U_5 > U_3$ and $U_7 > U_6 > U_3$

Speed of the sound in the gas (of number of degree of freedom of a molecule = n) v_n

$$= \sqrt{\frac{\gamma RT}{M}} = \sqrt{\left(\frac{2}{n} + 1\right)\frac{RT}{M}}$$

Clearly, more 'n', less 'v'

$\therefore v_5 > v_7$

2. **(c)** Average translational kinetic energy of an ideal gas molecule is $\frac{3}{2}kT$ which depends on temperature only. Therefore, if temperature is same, translational kinetic energy of O_2 and N_2 both will be equal.

3. **(a)** For an ideal gas PV = nRT

Coefficient of volume expansion

$$\left(\frac{\Delta V}{\Delta T}\right)_p = \frac{nR}{P} = \text{Constant}$$

Average translation K.E. for $O_2 = \frac{3}{2}kT$

(Three degrees of freedom for translational motion). Now decrease in pressure increases the volume. It increases mean free path of the molecules. Also average K.E. does not depend on the gas, so molecules of each component of mixture of gases have same average translational energy.

4. **(b)** $\gamma_1 = \frac{5}{3}$ means gas is monoatomic or $C_{v_1} = \frac{3}{2}R$

$\gamma_2 = \frac{7}{5}$ means gas is diatomic or $C_{v_2} = \frac{5}{2}R$

C_v (of the mixture)

$$= \frac{n_1 C_{v_1} + n_2 C_{v_2}}{n_1 + n_2} = \frac{(1)\left(\frac{3}{2}R\right) + (1)\left(\frac{5}{2}\right)R}{1+1} = 2R$$

C_p (of the mixture) $= C_v + R = 3R$

$\therefore \gamma_{mixture} = \frac{C_p}{C_v} = \frac{3R}{2R} = 1.5$

5. (a) According to the law of equipartition of energy the average kinetic energy associated with each degree of freedom per molecule is $\frac{1}{2}$k.T. In this case, O_2 and N_2 both have two degrees of rotational kinetic energy and since the temperature is also same, the ratio of the average rotational kinetic energy $\frac{\frac{2 \times KT}{2}}{\frac{2 \times KT}{2}}$ *i.e.*, 1 : 1.

6. (d) Piston *A* is free to move, therefore heat will be supplied at constant pressure

$\therefore \quad \Delta Q_A = nC_p\Delta T_A$ ... (i)

Piston *B* is held fixed, therefore heat will be supplied at constant volume.

$\therefore \quad \Delta Q_B = nC_v\Delta T_B$ ... (ii)

But $\Delta Q_A = \Delta Q_B$ (given)

$\therefore \quad nC_p\Delta T_A = nC_v\Delta T_B \quad \therefore \Delta T_B = \left(\frac{C_p}{C_v}\right)\Delta T_A$

$\Delta T_B = \gamma(\Delta T_A)$ [$\gamma = 1.4$ (diatomic)]

$= (1.4)(30\text{ K})$

$\therefore \quad \Delta T_B = 42\text{ K}$

7. (d) $\frac{\Delta U}{Q_p} = \frac{nC_v\Delta T}{nC_p\Delta T} = \frac{C_v}{C_p} = \frac{1}{\gamma} = \frac{1}{7/5} = \frac{5}{7}$

8. (121) At constant pressure, work done

$W = P\,\Delta V = nR\Delta T = 66$

$\Delta U = n(C_V)_{mix}\Delta T$

$(C_V)_{mix} = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1 + n_2}$

$\Rightarrow (C_V)_{mix} = \frac{2 \times \frac{3}{2}R + 1 \times \frac{5}{2}R}{3} = \frac{11}{6}R$

$\Delta U = \frac{11}{6}(nR\Delta T) = \frac{11}{6} \times 66 = 121\text{J}$

9. Molar specific heat of the mixture at constant volume

$\bar{C}_v = \frac{n_1C_{v_1} + n_2C_{v_2}}{n_1 + n_2} = \frac{1 \times \frac{3}{2}R + 1 \times \frac{5}{2}R}{1+1} = 2R$

10. (True) Clearly, $C_p > C_v$ $C_p - C_v = R \Rightarrow C_p = C_V + R$

11. (a, b, d) According to question no. of mole of hydrogen = no. of mole of helium = 1

Total internal energy, u

$= \frac{f_1}{2}nRT + \frac{f_2}{d}nRT \Rightarrow u = \frac{3}{2}RT + \frac{5}{2}RT = 4RT$

$\therefore$ Average internal energy per mole $= \frac{u}{2n} = \frac{4RT}{2} = 2RT$

We know that $V_{sound} = \sqrt{\frac{\gamma RT}{M}}$

$\frac{n_1 + n_2}{\gamma_{mix} - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1} \Rightarrow \frac{2}{\gamma_{mix} - 1} = \frac{1}{\frac{5}{3} - 1} + \frac{1}{\frac{7}{5} - 1}$

$\frac{2}{\gamma_{mix} - 1} = \frac{3}{2} + \frac{5}{2} = 4 \Rightarrow \gamma_{mix} - 1 = \frac{1}{2} \therefore \gamma_{mix} = \frac{3}{2}$

$\frac{(V_s)_{mix}}{(V_s)_{He}} = \sqrt{\frac{\gamma_{mix}}{M_{mix}} \times \frac{M_{He}}{\gamma_{He}}} = \sqrt{\frac{\frac{3}{2} \times 4}{3 \times \frac{5}{3}}} = \sqrt{\frac{6}{5}}$

$\left[\because M_{mix} = \frac{1 \times 2 + 1 \times 4}{2} = 3\right]$

We know that $V_{rms} = \sqrt{\frac{3RT}{M}}$

$\therefore \frac{(V_{rms})_{He}}{(V_{rms})_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{H_E}}} = \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$

12. (b, d) We known for all gases $C_p - C_v = R$

For monoatomic gas : $C_v = \frac{3}{2}R$; $C_p = \frac{5}{2}R$; $\gamma = \frac{5}{3}$

$\therefore C_p.C_v = \frac{15R^2}{4}$ and $C_p + C_v = 4R$

For diatomic gas : $C_v = \frac{5}{2}R$; $C_p = \frac{7}{2}R$; $\gamma = \frac{7}{5}$

$\therefore C_p.C_v = \frac{35R^2}{4}$ and $C_p + C_v = 6R$

13. (b) The total translational kinetic energy of n moles of gas $= \frac{3}{2}$nRT 1.5 PV $(\because PV = nRT)$

Yes, the molecules of a gas collide with each other and the velocities of the molecules change due to collision.

Topic-4: Miscellaneous (Mixed Concepts) Problems

1. (d) The heat is supplied at constant pressure. *i.e.*, the process is isobaric

$\therefore Q = n\,C_p\,\Delta t$

$= 2\left[\frac{5}{2}R\right] \times \Delta t = 2 \times \frac{5}{2} \times 8.31 \times 5 = 208\text{ J}$

$\left(\because C_p = \frac{5}{2}R \text{ for mono-atomic gas}\right)$

2. (c) pT^2 = constant (given)

$\therefore \left(\frac{nRT}{V}\right)T_2$ = constant or T^3V^{-1} = constant

$(\therefore PV = nRT)$

Differentiating the equation, we get

$\frac{3T^2}{V}.dT - \frac{T^3}{V^2}dV = 0$ or $3dT = \frac{T}{V}.dV$ ...(i)

From the equation

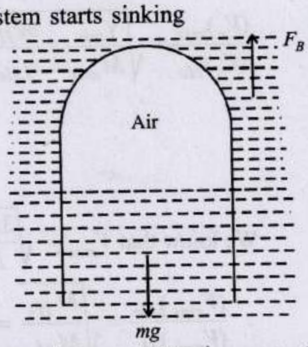
$dV = V\gamma\, dT$

γ = coefficient of volume expansion of gas $= \frac{dV}{V.dT}$

From eq. (i) $\gamma = \frac{dV}{V.dT} = \frac{3}{T}$

3. (d) $\frac{\Delta U}{Q_p} = \frac{nC_v \Delta T}{nC_p \Delta T} = \frac{C_v}{C_p} = \frac{1}{\gamma} = \frac{1}{7/5} = \frac{5}{7}$

4. **(0.30)** When tube + air system starts sinking



$F_B = mg$

$\Rightarrow \rho_0 (V_{glass} + V_{gas}) = m$

$1(2 + V_{gas}) = 5$

$\Rightarrow V_{gas} = 3$ cc

Hence $\Delta V = V_0 - V_{gas}$

$= 3.3cc - 3cc =$ **0.3 cc.**

$\therefore \; x = \Delta V = = 0.3$

5. **(10.00)** Isothermal process for air, temperature is constant.

$\therefore$ From $P_1V_1 = P_2V_2$

$10^5 \times (3.3) = P_2 (3)$

$\Rightarrow P_2 = 1.1 \times 10^5$

$\Delta P = P_2 - P_1 = 1.1 \times 10^5 - 10^5 = 0.1 \times 10^5$

or, $\Delta P = 10 \times 10^3$ Pascal

$= Y \times 10^3$ Pascal

$\therefore$ **$Y = 10$**

6. **(900J)** Given: $T_i = 100k$, $V_f = 8V_i$

For and adiabatic process, $TV^{\gamma-1}$ = constant

or, $T_iV_i^{\gamma-1} = T_fV_f^{\gamma-1}$

$\Rightarrow \frac{T_i}{T_f} = \left(\frac{V_f}{V_i}\right)^{\gamma-1} \Rightarrow \frac{T_i}{T_f} = \left(\frac{8V_i}{V_i}\right)^{\gamma-1}$

For monoatomic gas $\gamma = \frac{5}{3}$

$\therefore T_f = \frac{T_i}{\left(8^{5/3-1}\right)} = \frac{T_i}{4}$

Change in internal energy $\Delta u = nC_V\Delta T$

$= 1 \times \frac{3}{2}R\left(\frac{T_i}{4} - T_i\right) = \frac{3}{2} \times 8\left(\frac{-3}{4}\right) \times 100 = -900J$

7. The movable stopper will adjust to a position with equal pressure on either sides. Therefore, $P_1 = P_2$

$P_1 = \frac{n_1RT}{V_1} = \frac{m}{M_1}RT$, $P_2 = \frac{n_2RT}{V_2} = \frac{m}{M_2}RT$

$\Rightarrow \frac{V_2}{V_1} = \frac{M_1}{M_2} = \frac{32}{28} = \frac{8}{7} \quad \therefore \; \alpha = \frac{360°}{(8+7)} \times 8 = 192°$

8. **(a, b, c)** Force = Pressure × Area

$p_1 = \frac{n_1RT}{N_A}$ and $p_2 = \frac{n_2RT}{N_A}$

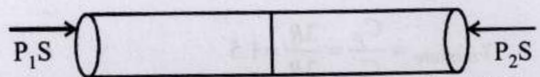
$F = \Delta p.A = \left(\frac{n_1RT}{N_A} - \frac{n_2RT}{N_A}\right)S$

$F = (n_1 - n_2)k_BTS = \Delta nk_BTS$

$\left(\because \frac{R}{N_A} = K_B \text{ (Boltzmann constant)}\right)$

Hence, option (a) is correct.

$V = \frac{\Delta nk_B TS}{\beta}$



Force balance = pressure × area = total number of molecules × βv

$\Delta n \, k_BTS = \ell n_1 S\beta v$ [βv = viscous force (given)]

$\Rightarrow n_1\beta v\ell = \Delta nk_BT$

So option (b) is correct.

Total number of molecules/sec, $\frac{\Delta N}{\Delta t} = \frac{(n_1 v dt)S}{\Delta t}$

$= n_1vS = \frac{\Delta nk_BTvS}{\beta v\ell} = \left(\frac{\Delta n}{\ell}\right)\left(\frac{k_BT}{\beta}\right)S$

Option (c) is correct.

As Δn will decrease with time so rate of molecules getting transferred through the tube decreases with time.

Hence option (d) is incorrect.

9. **(b, c, d)** (a) As we know, $Q = mc\,\Delta T$

$\Rightarrow \frac{dQ}{dt} = mc\frac{dT}{dT}$ or, $\frac{dQ}{dt} \propto C$ i.e., rate of heat absorption ∝ C.

In the range 0 to 100K from the graph, C increases with temperature but not linearly therefore the rate at which heat is absorbed varies with temperature. But not linearly.

(b) As the value of C is greater in the temperature range 400-500K, the heat absorbed in increasing the temperature from 0 - 100K is less than the heat required for increasing the temperature from 400 - 500K.

(c) From the graph the value of C does not change in the temperature range 400-500K, therefore there is no change in the rate of heat absorption in this range.

(d) As the value of C increases from 200-300K, the rate of heat absorption increases in the range 200-300K.

10. **(a, b, d)**

(a)

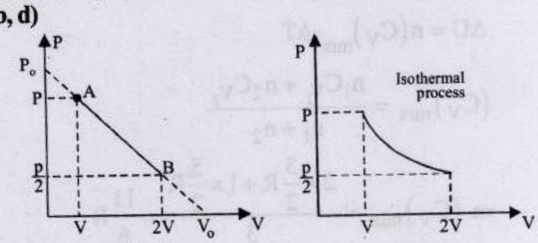


Fig (i) Fig (ii)

Workdone = Area under $P - V$ curve and V-axis

$\therefore W_1 > W_2$

(b) To study V-T diagram.

In the given process, AB is a straight line. It has a negative slope and a positive intercept.

The equation of line is $P = -\alpha V + \beta$ ...(i)

Where α and β are positive constants.

$\because PV = nRT$ ...(ii)

$\therefore P = \frac{nRT}{V} \quad \therefore \frac{nRT}{V} = -\alpha V + \beta$

or $T = \frac{-\alpha V^2}{nR} + \frac{\beta V}{nR}$ ...(iii)

This represents a parabola in terms of T and V.

∴ The path AB becomes a part of parabola.

(c) For P-T diagram, eliminate V in (i) and (ii)

$\therefore P = -\alpha\frac{nRT}{P} + \beta$ or $P^2 - P\beta = -\alpha nRT$

or $T = \frac{-P^2}{\alpha nR} + \frac{P\beta}{\alpha nR}$ ...(iv)

This represents a parabola in terms of T and P.
$\therefore$ The path AB becomes a part of parabola.
(d) Variation of T along AB

From $eq(iii), T = \frac{-\alpha V^2}{nR} + \frac{\beta V}{nR}$

$$\therefore \frac{dT}{dV} = \frac{-2\alpha V}{nR} + \frac{\beta}{nR} \quad ...(v)$$

When $\frac{dT}{dV} = 0, V = \frac{\beta}{2\alpha}$ ...(vi)

$$\frac{d^2T}{dV^2} = \frac{-2\alpha}{nR} + 0 = \frac{-2\alpha}{nR}$$

$\frac{d^2T}{dV^2}$ is negative. It means T has some maximum value.

$V = \frac{\beta}{2\alpha}$ is the value of maxima of temperature.

Also $P_AV_A = P_BV_B, RT_A = RT_B$ or $T_A = T_B$
In going from A to B, the temperature of the gas first increases of maximum at $V = \beta/2\alpha$
Then the temperature decreases and restored to original value.

11. **(d)** Let T be the final temperature of the gases when equilibrium is achieved.
Heat lost by monatomic gas at constant volume
= Heat gained by diatomic gas at constant pressure

$$\therefore nC_{v_1}(700 - T) = nC_{p_2}(T - 400)$$

$$\frac{3}{2}R(700 - T) = \frac{7}{2}R(T - 400)$$

$\Rightarrow \quad 2100 - 3T = 7T - 2800 \Rightarrow 10T = 4900 \ \therefore T = 490\,K$

12. **(d)** As the pressure of gases in both compartments is the same

$$\therefore nC_{p_1}(700 - T) = nC_{p_2}(T - 400)$$

$$\frac{5}{2}R(700 - T) = \frac{7}{2}R(T - 400)$$

$3500 - 5T = 7T - 2800 \Rightarrow 12T = 6300 \quad \therefore T = 525\,K$

Applying first law of thermodynamics

$$\Delta W_1 + \Delta U_1 = \Delta Q_1$$

and $\Delta W_2 + \Delta U_2 = \Delta Q_2$

$\Delta Q_1 + \Delta Q_2 = 0$

or, $-(\Delta W_1 + \Delta W_2) = \Delta U_1 + \Delta U_2$

$= nC_{v_1}(525 - 700) + n_2C_{v_2}(525 - 400)$

$= -2 \times \frac{3R}{2} \times 175 + 2 \times \frac{5R}{2} \times 125 = -525R + 625R = -100R$

Therefore, total work done = -100 R

13. **(d)** The forces acting besides buoyancy force which is the force due to pressure of the liquid are
(i) Force of gravity (vertically downwards)
(ii) Viscous force (vertically downwards)

14. **(b)** As the bubble does not exchange any heat with the liquid. So, the process is adiabatic.
Applying relation, $T^{\gamma}P^{1-\gamma}$ = constant

$$T_2 = T_1\left[\frac{P_1}{P_2}\right]^{\frac{1-\gamma}{\gamma}}$$

Here $T_1 = T_0, P_1 = P_0 + H\rho_\ell g, T_2 = T, P_2 = P$

$P = P_0 + (H - y)\rho_\ell g, \ \gamma = \frac{5}{3}$

$$\therefore \quad T = T_0\left[\frac{P_0 + H\rho_\ell g}{P_0 + (H - y)\rho_\ell g}\right]^{1-\frac{5}{3}/5/3}$$

$$= T_0\left[\frac{P_0 + H\rho_\ell g}{P_0 + (H - y)\rho_\ell g}\right]^{\frac{-2}{3}\times\frac{3}{5}}$$

$$T = T_0\left[\frac{P_0 + (H - y)\rho_\ell g}{P_0 + H\rho_\ell g}\right]^{\frac{2}{5}}$$

15. **(b)** $\because PV = nRT$

$$\Rightarrow V = \frac{nRT}{P} = \frac{nRT}{P_0 + (H - y)\rho_\ell g}$$

Where P is pressure of the bubble at an arbitrary location distant y from the bottom.
Substituting the value of P and T from above we get

$$V = \frac{nR}{[P_0 + (H - y)\rho_\ell g]} \times \frac{T_0[P_0 + (H - y)\rho_\ell g]^{\frac{2}{5}}}{[P_0 + H\rho_\ell g]^{\frac{2}{5}}}$$

$$= \frac{nRT_0}{[P_0 + (H - y)\rho_\ell g]^{\frac{3}{5}}\,[P_0 + H\rho_\ell g]^{\frac{2}{5}}}$$

$\therefore$ Buoyancy force

$$= V\rho_\ell g = \frac{nRT_0\rho_\ell g}{[P_0 + (H - y)\rho_\ell g]^{\frac{3}{5}}\,[P_0 + H\rho_\ell g]^{\frac{2}{5}}}$$

16. At constant pressure, $V \propto T$ or, $\frac{T_1}{V_1} = \frac{T_2}{V_2}$

$$\therefore \quad \frac{T_1}{Ah_1} = \frac{T_2}{Ah_2} \quad (\because V = A \times h)$$

$$\Rightarrow \quad h_2 = \frac{T_2h_1}{T_1} = \frac{400}{300} \times 1 = \frac{4}{3}\,\text{m}$$

Gas is compressed without heat exchange, So process is adiabatic
From $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$$\therefore T'_1 = T_2\left(\frac{V_2}{V_1}\right)^{\gamma-1} = 400\left(\frac{4}{3}\right)^{2/5} = 448.8\,K$$

17. When container is stopped, velocity decreases by v_0.

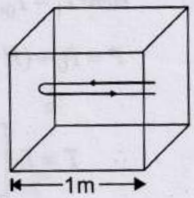
Hence kinetic energy decreases by $\frac{1}{2}MV_0^2$

Decrease in kinetic energy = Increase in internal energy of the gas

$$\frac{1}{2}mv_0^2 = nC_V\Delta T$$

$$\frac{1}{2}mv_0^2 = \frac{m}{M}\left(\frac{3}{2}R\right)\Delta T \ \therefore \Delta T = \frac{mv_0^2}{3R}$$

18. Let T be temperature of the gas
The distance travelled by an atom of helium in $\frac{1}{500}$ s i.e., time between two successive collisions $= 2l = 2m$ ($\because l = 1$ m)



$$V_{rms} = \frac{\text{distance}}{\text{time}} = \frac{2}{1/500} = 1000 \text{ m/s}$$

(a) $V_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow 1000 = \sqrt{\frac{3 \times 25/3 \times T}{4 \times 10^{-3}}} \quad \therefore T = 160\,K$

(b) Average kinetic energy of an atom of a monoatomic helium gas $K.E = \frac{3}{2}kT$

$$\therefore \quad K.E = \frac{3}{2}kT = \frac{3}{2} \times \left(1.38 \times 10^{-23}\right) \times 160 = 3.312 \times 10^{-21}$$

(c) From ideal gas equation $PV = nRT = \frac{m}{M}RT$

$$\therefore m = \frac{PVM}{RT} = \frac{100 \times 1 \times 4}{25/3 \times 160} \Rightarrow m = 0.3012 \text{ gm}$$

Hence mass of helium gas in the box, m = 0.3012 g

19. (a) Let the number of moles of gas $B = n$
The number of moles of gas A = 1

$$\therefore U = \frac{nRT}{\gamma - 1} \quad m = \frac{19}{13}$$

$$\therefore U_m = U_A + U_B$$

$$\therefore \frac{(n_A + n_B)RT}{\gamma_m - 1} = \frac{n_A RT}{\gamma_A - 1} + \frac{n_B RT}{\gamma_B - 1}$$

$$\text{or } \frac{1+n}{\left(\frac{19}{13} - 1\right)} = \frac{1}{\left(\frac{5}{3} - 1\right)} + \frac{n}{\left(\frac{7}{5} - 1\right)}$$

$$\text{or } \frac{13(1+n)}{6} = \frac{3}{2} + \frac{5n}{2} \text{ or } 13 + 13n = 9 + 15n$$

or $4 = 2n$ or $n = 2$

(b) Speed of sound $v = \sqrt{\frac{\gamma RT}{M}}$

For mixture, $M = \frac{n_A M_A + n_B M_B}{n_A + n_B}$

$$M = \frac{(1 \times 4) + (2 \times 32)}{1 + 2} = \frac{68}{3} g \text{ mol}^{-1}$$

$$\therefore v = \sqrt{\frac{19}{13} \times \frac{8.31 \times 300 \times 3}{68 \times 10^{-3}}} \text{ or } v = 401 \text{m/s}$$

(c) Velocity of sound $v \propto \sqrt{T}$ $\therefore v =$ (constant k) $T^{1/2}$

$$\text{or } \frac{dv}{dT} = \frac{1}{2}k\,T^{-1/2}$$

or $dv = \frac{kdT}{2\sqrt{T}}$ where $dT = 1K, T = 300K$.

$$\text{or } \frac{dv}{v} = \frac{kdT}{2\sqrt{T}} \times \frac{1}{k\sqrt{T}} = \frac{1}{2}\left(\frac{dT}{T}\right)$$

$$\therefore \frac{dv}{v} \times 100 = \frac{1}{2}\frac{dT}{T} \times 100$$

$$\text{or \% change in } v = \frac{1}{2} \times \left(\frac{1}{300}\right) \times 100 = \frac{1}{6} = 0.167\%$$

(d) For adiabatic change. PV^γ = constant

$$\therefore V^\gamma(dP) + P\gamma V^{\gamma-1}(dV) = 0$$

$$\text{or } \frac{dP}{dV} = -\frac{\gamma P}{V} \text{ or } -\frac{dPV}{dV} = \gamma P$$

or Bulk modulus = E = γP

$$\because \text{Compressibility}(C) = \frac{1}{\text{Bulk modulus }(E)}$$

$$\therefore C = \frac{1}{\gamma P} \therefore \Delta C = C_2 - C_1$$

$$\text{or } \Delta C = \frac{1}{\gamma P_2} - \frac{1}{\gamma P_1} = \frac{1}{\gamma}\left(\frac{1}{P_2} - \frac{1}{P_1}\right) \quad \text{...(i)}$$

$$\text{Again, } P_1 = \frac{(n_A + n_B)RT}{V}$$

$$\text{or } P_1 = \frac{(1+2) \times 8.31 \times 300}{V} \text{ or } P_1 = \frac{7479}{V} \quad \text{...(ii)}$$

$$\text{Again, } \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma = \left(\frac{5V_1}{V_1}\right)^\gamma = 5^\gamma$$

$$\text{or } P_2 = P_1 \times 5^\gamma \left(\text{where } \gamma = \frac{19}{13} \text{ given}\right) \quad \text{...(iii)}$$

Putting value of P_1, P_2 in eq (i)

$$\therefore \Delta C = \frac{1}{\gamma}\left(\frac{1}{P_1 \times 5^\gamma} - \frac{1}{P_1}\right) = \frac{1}{\gamma P_1}\left[\frac{1}{5^\gamma} - 1\right]$$

$$\text{or } \Delta C = \frac{13V}{19 \times 7479}\left[\frac{1}{5^{19/13}} - 1\right]$$

$$= \frac{13V}{19 \times 7479}[0.1 - 1] = -\frac{(13 \times 0.90)V}{19 \times 7479} = \frac{(13 \times 1)V}{19 \times 8310}$$

$$= -0.0000827V = -8.27 \times 10^{-5}V$$

20. The total pressure exerted by the mixture $P = 10^5$ Nm^{-2}
Temperature $T = 300$ K ; Volume = 0.02 m^3
Let there be x gram of Ne. ∴ mass of Ar = $(28 - x)g$

Number of gram moles of Neon, $n_1 = \frac{x}{20}$;

Number of gram moles of Argon, $n_2 = \frac{28 - x}{40}$

But according to Dalton's law of partial pressure

$$P = p_1 + p_2 = \frac{n_1 RT}{V} + \frac{n_2 RT}{V}$$

$$\text{or, } 10^5 = \frac{xRT}{20V} + \frac{(28 - x)RT}{40V}$$

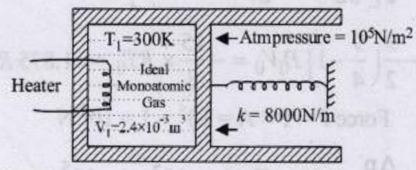
$$\Rightarrow \frac{10^5 \times 40 \times 0.02}{8.314 \times 300} = x + 28 \Rightarrow x = 4.074\,g$$

∴ Mass of Neon = 4.074 g and mass of Argon = 28 − 4.074 = 23.926 g

21. Final pressure

$$P_f = P_{atm} + \frac{kx}{A}$$

$$\Rightarrow P_f = 10^5 + \frac{8000 \times 0.1}{8 \times 10^{-3}} = 2 \times 10^5 \text{ N/m}^2$$



Final volume,

$$V_f = V_1 + xA = 2.4 \times 10^{-3} + 0.1 \times 8 \times 10^{-3} = 3.2 \times 10^{-3} \text{ m}^3$$

Applying $\frac{P_iV_i}{T_i} = \frac{P_fV_f}{T_f} \Rightarrow T_f = \frac{P_fV_fT_i}{P_iV_i}$

or, $T_f = \frac{2 \times 10^5 \times 3.2 \times 10^{-3} \times 300}{10^5 \times 2.4 \times 10^{-3}} = 800\text{K}.$

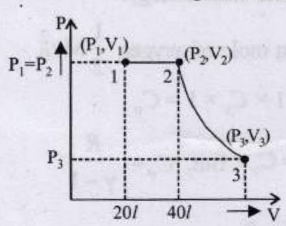
Here heat supplied by the heater is used for expansion of the gas, increasing its temperature and storing potential energy in the spring.

$$Q = P\Delta V + nC_v\Delta T + \frac{1}{2}kx^2$$

$$= 10^5[0.8 \times 10^{-3}] + \frac{P_1V_1}{RT_1}C_v\Delta T + \frac{1}{2}kx^2$$

$$= 80 + \frac{10^5 \times 2.4 \times 10^{-3}}{2 \times 300} \times \frac{3}{2} \times 2 \times 500 + \frac{1}{2} \times 8000 \times 0.1 = 720 \text{ J}$$

22. (i) The process on $P - V$ diagram is as shown below.



(ii) To find the final volume and pressure of the gas.

Applying $P_1V_1 = nRT_1$

$\therefore \quad P_1 \times 20 \times 10^{-3} = 2 \times 8.3 \times 300$

$P_1 = 2.49 \times 10^5 \text{ Nm}^{-2}$

For process $1 \to 2$ (isobaric)

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \text{ or, } \frac{20}{300} = \frac{40}{T_2} \Rightarrow T_2 = 600 \text{ K}$$

For process, 2 → 3 (adiabatic)

$$T_2V_2^{\gamma-1} = T_3V_3^{\gamma-1}$$

$$\therefore \quad V_3 = V_2\left[\frac{T_2}{T_3}\right]^{\frac{1}{\gamma-1}} = 40\left[\frac{600}{300}\right]^{\frac{1}{\frac{5}{3}-1}} = 113\ell$$

$[\because \gamma = \frac{5}{3}$ for mono atomic gas$]$

Now, $P_3V_3 = nRT_3$

$$\therefore \quad P_3 = \frac{nRT_3}{V_3} = \frac{2 \times 8.3 \times 300}{113 \times 10^{-3}} = 0.44 \times 10^5 \text{ N/m}^2$$

$\because \quad T_3 = T_1$ given

(iii) Work done by the gas $W = W_{12} + W_{23}$

$$= P_1(V_2 - V_1) + \frac{nR}{\gamma - 1}(T_2 - T_3)$$

$$= 2.49 \times 10^5 (40 - 20) 10^{-3} + \frac{2 \times 8.3}{\frac{5}{3} - 1}(600 - 300)$$

$$= 4980 + 7470 = 12450 \text{ J}$$

23. Let A be the area of cross-section of the tube.

Since temperature is the same, applying Boyle's law i.e., PV = constant in two sides of mercur column

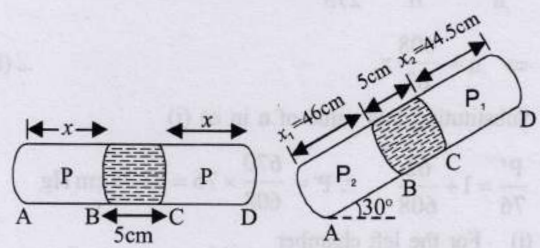
$P \times (x \times A) = P_2 \times (x_2 \times A)$

and $P \times (x \times A) = P_1 \times (x_1 \times A)$

$\therefore \quad P_1 \times (x_1 \times A) = P_2 \times (x_2 \times A)$

or, $P_1x_1 = P_2x_2$ ...(i)

where $P_2 = P_1$ + Pressure due to mercury column



Pressure due to mercury column

$$P = \frac{F}{A} = \frac{mg\sin 30°}{A} = \frac{Vdg\sin 30°}{A}$$

$$= \frac{(A \times 5) \times dg\sin 30°}{A} = 5 \sin 30° \text{ cm of Hg}$$

$$P_2 = P_1 + 5\sin 30° = P_1 + 2.5$$

Substituting this value in (iii)

$P_1 \times x_1 = [P_1 + 2.5] \times x_2$

or, $P_1 \times 46 = [P_1 + 2.5] \times 44.5$

$$\therefore \quad P_1 = \frac{44.5 \times 2.5}{1.5}$$

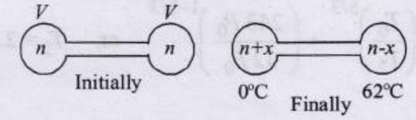
Putting this value of P_1 in $px = P_1x_1$

$$P \times x = \frac{44.5 \times 2.5}{1.5} \times 46$$

$$\Rightarrow P \times \left[\frac{46 + 44.5}{2}\right] = \frac{44.5 \times 2.5}{1.5} \times 46$$

$$[\because x = \frac{x_1 + x_2}{2}] \Rightarrow P = 75.4 \text{ cm}$$

24. Let x moles transfer from high temperature side to low temperature side bulb.



$\because$ Applying $PV = nRT$

For left bulb

$76 \times V = nR \times 273$ Initially

$P' \times V = (n+x) R \times 273$ Finally

Dividing, we get

$$\frac{P'}{76} = \frac{n+x}{n} \quad ...(i)$$

For right bulb

$76 \times V = nR \times 273$ Initially

$P' \times V = (n-x) R \times 335$ Finally

Again dividing, we get

$$\frac{P'}{76} = \frac{n-x}{x} \times \frac{335}{273} \quad ...(ii)$$

From eq. (i) and (ii)

$$\frac{n+x}{n} = \frac{n-x}{n} \times \frac{335}{273}$$

$$\Rightarrow n = \frac{608}{62} x. \quad ...(iii)$$

Substituting this value of n in eq (i)

$$\frac{P'}{76} = 1 + \frac{62}{608} \quad \therefore P' = \frac{670}{608} \times 76 = 83.75 \text{ cm Hg}$$

25. (i) For the left chamber

$$\frac{P_0 V_0}{T_0} = \frac{P_0 \times 243}{32 \times T_1} \times V_1$$

$$\Rightarrow T_1 = \frac{243}{32} \times \frac{V_1 T_0}{V_0} \quad ...(i)$$

For the right chamber, adiabatic compression occurs

$$\therefore P_0 V_0^{\gamma} = P_0 \times \frac{243}{32} \times V_2^{\gamma}$$

$$\Rightarrow \frac{V_2}{V_0} = \left(\frac{32}{243}\right)^{3/5} \Rightarrow V_2 = \frac{8}{27} V_0$$

But $V_1 + V_2 = 2V_0$

$$\therefore V_1 = 2V_0 - V_2 = 2V_0 - \frac{8}{27} V_0 = \frac{46}{27} V_0 \quad ...(ii)$$

From eq (i) and (ii) $T_1 = \frac{243}{32} \times \frac{46 \times V_0}{V_0 \times 27} \times T_0$

or, $T_1 = \frac{207}{16} T_0 = 12.9 T_0$

Again using P–T equation for right chamber i.e., $P_{1-\gamma}\, T_{\gamma} =$ constant

$$\left(\frac{T_1}{T_2}\right)^{\gamma} = \left(\frac{P_2}{P_1}\right)^{1-\gamma}$$

$$\Rightarrow \left(\frac{T_0}{T_2}\right)^{5/3} = \left(\frac{243 P_0}{32 P_0}\right)^{1-5/3} \quad \text{or,} \quad T_2 = 2.25\, T_0$$

(ii) Work done by the gas in right chamber (adiabatic process)

$$W = \frac{1}{1-\gamma}(P_2 V_2 - P_0 V_0)$$

$$= -\frac{3}{2}\left[\frac{243}{32} P_0 \times \frac{8}{27} V_0 - P_0 V_0\right]$$

$$= -\frac{3}{2}\left(\frac{9}{4} - 1\right) P_0 V_0 = -\frac{15}{8} \times RT_0 = -1.875\, RT_0$$

26. (i) Force $F = P \times A = 10^5 \times 1 = 10^5$ N

$$F = \frac{\Delta p}{\Delta t} \Rightarrow \Delta p = F \times \Delta t = 10^5 \times 1 = 10^5 \quad ...(i)$$

Now, momentum change per second

$(\Delta p) = n \times 2mv$ ...(ii)

Where n = number of collisions per second per square metre area

From (i) and (ii)

$$n \times 2mv = 10^5 \quad \therefore n = \frac{10^5}{2mv}$$

Root mean square velocity

$$v = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.314 \times 300}{32/1000}} = 483.4 \text{ m/s}$$

$$\therefore \quad n = \frac{10^5 \times 6.023 \times 10^{23}}{2 \times 32 \times 483.4} = 1.97 \times 10^{27}$$

(ii) Kinetic energy of motion of oxygen molecules will be converted into heat energy.

K.E. of 1 gm mole of oxygen $\frac{1}{2} M v_0^2$

$= nC_v \Delta T = 1 \times C_v \times 1 = C_v$

$$\therefore \frac{1}{2} M v_0^2 = C_v \quad \text{But, } C_v = \frac{R}{\gamma - 1}$$

$$\therefore \frac{1}{2} M v_0^2 = \frac{R}{\gamma - 1}$$

$$\text{or, } v_0 = \sqrt{\frac{2R}{M(\gamma-1)}} = \sqrt{\frac{2 \times 8.314}{\frac{32}{100} \times (1.41 - 1)}} = 35.6 \text{ ms}^{-1}$$

27. $P_1 = 830 - 30 = 800$ mm Hg ; P_2 ?

$V_1 = V$; $V_2 = V$; $T_1 = T$; $T_2 = T - 0.01\, T = 0.99\, T$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \therefore P_2 = \frac{P_1 T_2}{T_1} = \frac{800 \times 0.099 T}{T} = 792 \text{ mmHg}$$

$\therefore$ Total pressure in the jar = 792 + 25 = 817 mm Hg

28. From, PV = nRT

When P, T are same $n \propto V$

As volumes are same, i.e., 1c.c of each hydrogen and oxygen, So both samples will have equal number of molecules