Chapter 7. Permutations and Combinations

Question-1

In a class there are 27 boys and 14 girls. The teacher wants to select 1 boy and 1 girl to represent a competition. In how many ways can the teacher make this selection?

Solution:

1 boy among 27 boys is being selected. Hence the number of ways of selecting boys is 27.
1 girl among 14 girls is being selected. Hence the number of ways of selecting girls is 14.
Total number of ways the teacher can make this selection is $27 \times 14 = 378$.

Question-2

Given 7 flags of different colours, how many different signals can be generated if a signal requires the use of two flags, one below the other?

Solution:

The top position can be occupied by 7 flags and the bottom position can be occupied by 6 flags. Hence the total number of signals that can be generated using two flags one below the other is $7 \times 6 = 42$.

Question-3

A person wants to buy one fountain pen, one ball pen and one pencil from a stationery shop. If there are 10 fountain pen varieties, 12 ball pen varieties and 5 pencil varieties, in how many ways can he select these articles?

Solution:

Number of ways of selecting 1 fountain pen among 10 pens is 10.
Number of ways of selecting 1 ball pen among 12 ball pens is 12.
Number of ways of selecting 1 pencil among 5 pencil pens is 5.
Therefore total number of ways that he can select these articles is $10\times 12\times 5 = 600$.

Question-4

Twelve students compete in a race. In how many ways can the first three prizes be given?

Solution:

In a race of twelve students first prize can be given to any one of the 12 students, second prize can be given to any one of the 11 students and third prize can be given to any one of the 10 students.

Therefore total number of ways that first three prizes can be given is $12 \times 11 \times 10 = 1320$

Question-5

From among the 36 teachers in a college, one principal, one vice-principal and one teacher-in charge are to be appointed. In how many ways can this be done?

Solution:

In 36 ways one principal can be appointed, 35 ways one vice – principal post can be appointed and 34 ways one teacher-incharge can be appointed.

Therefore number of ways the selection can be made is $36\times 35\times 34 = 42840$ ways.

Question-6

There are 6 multiple-choice questions in an examination. How many sequences of answers are possible, if the first three questions have 4 choices each and the next three have 2 each?

Solution:

Each of the first three questions can be answered in 4 ways. Remaining each of the three questions can be answered in 2 ways.

Therefore total number of ways of choosing the answers are $4 \times 4 \times 4 \times 2 \times 2 \times 2 = 512$

Question-7

How many numbers are there between 500 and 1000, which have exactly one of their digits as 8?

Solution:

Digits which can occupy hundreds place are 5, 6, 7, 8, 9.
If the digits that occupy hundreds place are 5, 6, 7 and 9.
If 8 takes the units place then the tens place can be occupied by 9 digits.
Similarly if 8 takes the tens place then the units place can be occupied by 9 digits.
Totally units and tens place can be represented in 18ways.
Therefore total number of ways without 8 in hundreds place are $4 \times 18 = 72$.

If 8 takes up hundreds place then units and tens place can be occupied by 9 digits.
Therefore total number of ways with 8 in hundreds place are $9 \times 9 = 81$.
Therefore total number between 500 and 1000 which have exactly one of their digits as 8 are $72 + 81 = 153$

Question-8

How many five-digit number license plates can be made if
(i) first digit cannot be zero and the repetition of digits is not allowed.
(ii) the first digit cannot be zero, but the repetition of digits is allowed?

Solution:

(i) There are totally 10 digits they are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
Ten thousands place can be filled by 9 ways
Remaining places can be filled in 9, 8, 7 and 6 ways.
Therefore total number of ways are $9 \times 9 \times 8 \times 7 \times 6 = 27216$.

(ii) There are totally 10 digits they are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
Ten thousands place can be filled by 9 ways.
Remaining places can be filled in 10, 10, 10 and 10 ways.
Therefore total number of ways are $9 \times 10 \times 10 \times 10 \times 10 = 90000$.

Question-9

How many different numbers of six digits can be formed from the digits 2, 3, 0, 7, 9, 5 when repetition of digits is not allowed?

Solution:
The lakh's place can be filled in by 5 ways. The remaining 5 places can be filled in 5, 4, 3, 2, 1 ways.
Therefore the total number of six digits that can be formed is $5 \times 5 \times 4 \times 3 \times 2 \times 1 = 600$

Question-10

How many odd numbers less than 1000 can be formed by using the digits 0, 3, 5, 7 when repetition of digits is not allowed?

Solution:
Number of odd digits with one digit are 3.
Number of odd digits with two digits are $3 \times 2 = 6$.
Number of odd digits with three digits are $3 \times 3 \times 2 = 18$
Therefore total number of odd numbers less than 1000 that can be formed by using the digits 0, 3, 5, 7 when repetition of digits is not allowed = $3 + 6 + 18 = 27$.

Question-11

In how many ways can an examinee answer a set of 5 true/false type questions?

Solution:
For each set there are two answers, true/false.
Total number of answers an examinee can get are $2 \times 2 \times 2 \times 2 \times 2 = 32$

Question-12

How many 4-digit numbers are there?

Solution:
The thousands place cannot be filled by zeros.
Therefore the number of ways thousands place can be filled is 9.
The remaining three places can be filled by 10 ways.
Hence the total number of ways 4-digit numbers can be formed = $9 \times 10 \times 10 \times 10 = 9000$

Question-13

How many three-letter words can be formed using a, b, c, d, e if:
(i) repetition is allowed (ii) repetition is not allowed?

Solution:
(i) Total number of ways of forming three-letter word are $5 \times 5 \times 5 = 125$
(ii) Total number of ways of forming three-letter word without repetition are
$5 \times 4 \times 3 = 60$

Question-14

A coin is tossed five times and outcomes are recorded. How many possible outcomes are there?

Solution:
Each time a coin is tossed there are two outcomes head/tail.
Total number of possible outcomes = $2 \times 2 \times 2 \times 2 \times 2 = 32$

Question-15

Evaluate the following:
(i) $_5P_3$
(ii) $_{15}P_3$
(iii) $_5P_5$
(iv) $_{25}P_{20}$
(v) $_9P_5$

Solution:
:(i) $_5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{(5 \times 4 \times 3) \times 2!}{2!} = 60$
(ii) $_{15}P_3 = \frac{15!}{(15-3)!} = \frac{15!}{12!} = \frac{(15 \times 14 \times 13) \times 12!}{12!} = 2730$
(iii) $_5P_5 = \frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{1} = 120$
(iv) $_{25}P_{20} = \frac{25!}{(25-20)!} = \frac{25!}{5!} = \frac{25 \times 24 \times 23!}{5 \times 4 \times 3 \times 2} = 5 \times 23!$
(v) $_9P_5 = \frac{9!}{(9-5)!} = \frac{9!}{4!} = \frac{9 \times 8 \times 7!}{4 \times 3 \times 2 \times 1} = 3 \times 7!$

Question-16

If $_nP_4 = 20. _nP_3$, find n.

Solution:

$_nP_4 = 20. _nP_3$

$\frac{n!}{(n-4)!} = 20. \frac{n!}{(n-3)!}$

$\frac{n!}{(n-4)!} = 20. \frac{n!}{(n-3)(n-4)!}$

$n - 3 = 20$

$n = 23$

Question-17

If $_{10}P_r = 5040$, find the value of r.

Solution:

$_{10}P_r = 5040$

$\frac{10!}{(10-r)!} = 7!$

$\frac{10!}{(10-r)!} = \frac{10!}{6!}$

$(10 - r)! = 6!$

$10 - r = 6$

$r = 4$

Question-18

If $_{56}P_{r+6} : _{54}P_{r+3} = 30800 : 1$,find r.

Solution:

$_{56}P_{r+6} : _{54}P_{r+3} = 30800 : 1$

$\frac{56!}{(56-r-6)!} : \frac{54!}{(54-r-3)!} = 30800$

$\frac{56!}{(56-r-6)!} \times \frac{(54-r-3)!}{54!} = 30800$

$\frac{56 \times 55 \times 54!}{(50-r)!} \times \frac{(51-r)!}{54!} = 30800$

$56 \times 55\ (51 - r) = 30800$

$51 - r = 10$

$r = 41$

$= 1 + [2! - 1!] + [3! - 2!] \ldots\ldots [+(n+1)! - n!]$

$= 1 + (n+1)! - 1$

$=(n + 1)!$

Question-20

Prove that $_nP_r = {}_{(n-1)}P_r + r.{}_{(n-1)}P_{(r-1)}$.

Solution:

$_{(n-1)}P_r + r.{}_{(n-1)}P_{(r-1)} = \frac{(n-1)!}{(n-1-r)!} + \frac{r.(n-1)!}{((n-1)-(r-1))!}$

$= \frac{(n-1)!}{(n-1-r)!} + \frac{r.(n-1)!}{(n-r)!}$

$= \frac{(n-r)(n-1)!}{(n-r)(n-1-r)!} + \frac{r.(n-1)!}{(n-r)!}$

$= \frac{(n-r)(n-1)!}{(n-r)!} + \frac{r.(n-1)!}{(n-r)!}$

$= \frac{(n-1)! \times (n-r+r)}{(n-r)!}$

$= \frac{(n-1)! \times n}{(n-r)!}$

$= \frac{n!}{(n-r)!}$

$= {}_nP_r$

Question-21

Three men have 4 coats, 5 waistcoats and 6 caps. In how many ways can they wear them?

Solution:

Total number of ways that the three men can wear = $4 \times 5 \times 6 = 120$

Question-22

How many 4-letter words, with or without meaning, can be formed, out of the letters of the word, 'LOGARITHMS', if repetition of letters is not allowed?

Solution:

Total number of 4-letter words that can be formed with or without meaning are $_{10}P_4$

$= \frac{10!}{6!}$

$= 10 \times 9 \times 8 \times 7$

$= 5040$

Question-23

How many 3-digit numbers are there, with distinct digits, with each digit odd?

Solution:

Total number of 3-digit numbers that can be formed with distinct odd digit numbers = $_5P_3$

= $\frac{5!}{2!}$

= 5× 4× 3

= 60

Question-24

Find the sum of all the numbers that can be formed with the digits 2, 3, 4, 5 taken all at a time.

Solution:

There are 4 digits and 4 places. Therefore number of permutation is $4P_4$ = 4! = 24.

There are six numbers ending with 2, 3, 4, 5.(Each of the digits 2, 3, 4, 5 occurs in 3! times in the unit's place)

Therefore totality of unit places = (6× 2) + (6× 3) + (6× 4) + (6× 5) = 6(2 + 3 + 4 + 5) = 84

Similarly the sum in the ten's place, hundreds place and thousands place each is 84.

Therefore total 84 + 840 + 8400 + 84000 = 93324.

Question-25

How many different words can be formed with the letters of the word 'MISSISSIPPI'?

Solution:

Number of letters = 11

Number of I's, S's and P's are 4, 4 and 2.

Therefore number of permutation = 11!/4!4!2!

Question-26

(i) How many different words can be formed with letters of the word 'HARYANA'?
(ii) How many of these begin with H and end with N?

Solution:
(i) Total number of letters = 7
Number of A's = 3
Therefore required number of permutation = 7!/3! = 840

(ii) Let the first place be H and the last place be N.
The remaining letters are five namely ARYAA
Number of A's = 3
Therefore number of permutation = 5!/3! = 20.

Question-27

How many 4-digit numbers are there, when a digit may be repeated any number of times?

Solution:
The thousands place cannot be filled by zeros.(Then it becomes a three digit number.)
Therefore the number of ways thousands place can be filled is 9.
The remaining three places can be filled by 10 ways.
Hence the total number of ways 4-digit numbers can be formed = 9 × 10 × 10 × 10 = 9000

Question-28

In how many ways can 5 rings of different types be worn in 4 fingers?

Solution:
Number of permutation is 5^4 = 625

Question-29

In how many ways can 9 students are seated in a (i) line (ii) circle?

Solution:
(i) The number of ways in which 9 students can be arranged in a line = ${}_9P_9$ = 9!
(ii) The number of ways in which 9 students can be arranged in a circle = (9 - 1)! = 8!

Question-30

In how many ways can a garland of 20 flowers be made?

Solution:

Number of ways a garland of 20 flowers can be made is 19!/2.

Question-31

Evaluate the following:

(i) $_{10}C_8$

(ii) $_{100}C_{98}$

(iii) $_{75}C_{75}$

Solution:

(i) $_{10}C_8 = \frac{10!}{8!2!} = \frac{10\times 9\times 8!}{8!2!} = 45.$

(ii) $_{100}C_{98} = \frac{100!}{98!2!} = \frac{100\times 99\times 98!}{98!2!} = 4950.$

(iii) $_{75}C_{75} = \frac{75!}{75!0!} = 1$

Question-32

If $_nC_{10} = {_nC_{12}}$, find $_{23}C_n$.

Solution:

$_nC_{10} = {_nC_{12}}$

$n = 10 + 12 = 22$

$_{23}C_n = {_{23}C_{22}} = \frac{23!}{22!1!} = 23$

Question-33

If $_8C_r - {_7C_3} = {_7C_2}$, find r.

Solution:

$_8C_r - {_7C_3} = {_7C_2}$

$_8C_r = {_7C_3} + {_7C_2}$

$_8C_r = {_8C_3}$ (Since $_nC_r + {_nC_{r-1}} = {_{n+1}C_r}$)

$r = 3$

Question-34

If $_{16}C_4 = {}_{16}C_{r+2}$, find $_rC_2$.

Solution:

$_{16}C_4 = {}_{16}C_{r+2}$

$r + 2 + 4 = 16$ (Since $_nC_x = {}_nC_y$ and $x \neq y$, then $x + y = n$)

$r = 10$

$_rC_2 = {}_{10}C_2 = \frac{10!}{2!8!} = \frac{10 \times 9 \times 8!}{2!8!} = 45$

Question-35

Find n if

(i) $_nC_3 = \frac{20}{3}\ {}_nC_2$

(ii) $_nC_{(n-4)} = 70$

Solution:

(i) $_nC_3 = \frac{10}{3}\ {}_nC_2$

$\frac{nC3}{nC2} = \frac{10}{3}$

$\frac{n-3+1}{3} = \frac{10}{3}$ (Since $\frac{nCr}{nCr\text{-}1} = \frac{n-r+1}{r}$)

$n - 2 = 10$

$n = 12.$

(ii) $_nC_{(n-4)} = 70$

$\frac{n!}{(n-4)!4!} = 70$

$\frac{n(n-1)(n-2)(n-3) \times (n-4)!}{(n-4)!4!} = 70$

$n(n\text{-}1)(n\text{-}2)(n\text{-}3) = 70 \times 24$

$(n^2 - 3n)(n^2 - 3n + 2) = 1680$

$n^2 - 3n = y$

$y(y + 2) = 1680$

$y^2 + 2y - 1680 = 0$

$y(y + 42) - 40(y + 42) = 0$

$(y + 42)(y - 40) = 0$

Therefore $y = -42$ or 40

therefore $n^2 - 3n = -42$

$n^2 - 3n + 42 = 0$

$n = \frac{3 \pm \sqrt{9-168}}{2} = \frac{3 \pm \sqrt{-159}}{2}$

$n^2 - 3n = 40$

$n^2 - 3n - 40 = 0$

$n^2 - 8n + 5n = 40$

$n^2 - 3n - 40 = 0$

$n^2 - 8n + 5n = 40$

$n(n - 8) + 5(n - 8) = 0$

$(n + 5)(n - 8) = 0$

$n = 8$ or -5

Therefore n = 8.

Question-36

If $_{(n+2)}C_8 : {}_{(n-2)}P_4 = 57 : 16$, find n.

Solution:

$$\frac{(n+2)!}{8!(n-6)!} \times \frac{(n-6)!}{(n-2)!} = \frac{57}{16}$$

$$\frac{(n+2)(n+1)n(n-1)}{8!} = \frac{57}{16}$$

$$(n+2)(n+1)n(n-1) = 57 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times \frac{2}{16}$$

$$(n+2)(n+1)n(n-1) = 57 \times 7 \times 6 \times 5 \times 4 \times 3$$

$$(n^2 + n)(n^2 + n - 2) = 143640$$

$$y = n^2 + n$$

$$y(y - 2) = 143640$$

$$y^2 - 2y - 143640 = 0$$

$$y = \frac{2 \pm \sqrt{4 + 4(143640)}}{2}$$

$$= \frac{2 \pm \sqrt{574564}}{2}$$

$$= \frac{2 \pm 758}{2}$$

$$= \frac{760}{2}$$

$$\text{or } \frac{-756}{2} = 380 \text{ or } -378$$

$$n^2 + n - 380 = 0$$

$$\text{or } n^2 + n + 378 = 0$$

$$(n + 20)(n - 19) = 0$$

$$\text{or } n^2 + n + 378 = 0$$

$$n = -20 \text{ or } 19$$

Therefore n = 19

$n^2 + n + 378 = 0$

Since the roots are imaginary, n = 19.

Question-37

Prove that the product of 2n consecutive negative integers is divisible by (2n)!

Solution:

Let the numbers be $-(x + 1), -(x + 2), \ldots\ldots - (x + 2n)$

Their product is $(-1)^{2n} (x + 1)(x + 2) \ldots\ldots\ldots\ldots(x + 2n)$

$= (x + 1)(x + 2) \ldots\ldots\ldots\ldots(x + 2n)$

$= \frac{1.2.3\ldots\ldots x.(x+1)\ldots\ldots(x+2n)}{1.2.3\ldots\ldots x}$

$= \frac{(x+2n)!}{x!}$

$\frac{\text{Product}}{2n!} = \frac{(x+2n)!}{x!2n!} = x + 2nC_x$

Product is divisible by (2n)!

Question-38

If $_{28}C_{2r} : {}_{24}C_{2r-4} = 225 : 11$, find r.

Solution:

$\frac{28!}{2r!(28-2r)!} \times \frac{(2r-4)!(24-2r+4)!}{24!} = \frac{225}{11}$

$\frac{28!}{2r!(28-2r)!} \times \frac{(2r-4)!(28-2r)!}{24!} = \frac{225}{11}$

$\frac{28 \times 27 \times 26 \times 25}{2r(2r-1)(2r-2)(2r-3)} = \frac{225}{11}$

$9(2r)(2r - 1)(2r - 2)(2r - 3) = 28 \times 27 \times 26 \times 11$

$2r(2r - 1)(2r - 2)(2r - 3) = 28 \times 3 \times 26 \times 11$

Put $x = 2r$

$x(x - 1)(x - 2)(x - 3) = 28 \times 3 \times 26 \times 11$

$(x^2 - 3x)(x^2 - 3x + 2) = 28 \times 3 \times 26 \times 11$

Let $y = x^2 - 3x$

$y (y + 2) = 24024$

$y^2 + 2y - 24024 = 0$

$(y + 156)(y - 154) = 0$

$y = -156$ or 154

$x^2 - 3x = -156$

$x^2 - 3x + 156 = 0$

$x = \frac{3 \pm \sqrt{9-624}}{2}$

$x^2 - 3x = 154$

$x^2 - 3x - 154 = 0$

$(x - 14)(x - 11) = 0$

$x = 14$ or -11

Therefore $2r = 14$

$r = 7$.

Question-39

If there are 12 persons in a party, and if each two of them shake hands with each other, how many handshakes happen in the party?

Solution:

Number of persons in a party = 12.

Number of handshakes = ${}_{12}C_2 = \frac{12!}{10!2!} = \frac{12\times 11\times 10!}{10!2!} = 66$

Question-40

In how many ways a committee of 5 members can be selected from 6 men and 5 women, consisting of 3 men and 2 women?

Solution:

Number of men = 6

Number of women = 5∴ Required number of ways = ${}_6C_3 \times {}_5C_2 = \frac{6!}{3!3!} \times \frac{5!}{2!3!} = \frac{6\times 5\times 4\times 3!}{3\times 2\times 3!} \times \frac{5\times 4\times 3!}{2\times 3!} = 20\times 10 = 200.$

Question-41

How many triangles can be obtained by joining 12 points, five of, which are collinear?

Solution:

Number of points = 12

Number of points collinear = 5

∴ Number of triangles = ${}_{12}C_3 - {}_5C_3 = \frac{12\times 11\times 10}{1\times 2\times 3} - \frac{5\times 4\times 3}{1\times 2\times 3} = 220 - 10 = 210$

Question-42

A box contains 5 different red and 6 different white balls. In how many ways can 6 balls be selected so that there are atleast two balls of each other?

Solution:

Number of red balls = 5

Number of white balls = 6

∴ Required number of ways = ${}_5C_4 \times {}_6C_2 + {}_5C_3 \times {}_6C_3 + {}_5C_2 \times {}_6C_4$

$= \frac{5!}{4!1!} \times \frac{6!}{2!4!} + \frac{5!}{3!2!} \times \frac{6!}{3!3!} + \frac{5!}{2!3!} \times \frac{6!}{4!2!}$

$= 5 \times \frac{6\times 5\times 4!}{2!4!} + \frac{5\times 4\times 3!}{3!2!} \times \frac{6\times 5\times 4\times 3!}{3!3!} + \frac{5\times 4\times 3!}{2!3!} \times \frac{6\times 5\times 4!}{4!2!}$

$= 5 \times 15 + 10 \times 20 + 10 \times 15$

$= 75 + 200 + 150$

$= 425$

Question-43

In how many ways can a cricket team of eleven be chosen out of a batch of 15 players if
(i) there is no restriction on the selection
(ii) a particular player is always chosen;
(iii) a particular player is never chosen?

Solution:

Number of players = 15

(i) Number of ways team selected when there is no restriction on the selection = ${}_{15}C_{11}$

$$= \frac{15!}{11!4!} = \frac{15\times14\times13\times12\times11!}{11!4\times3\times2} = 1365.$$

(ii) Number of ways team selected when a particular player is always chosen = ${}_{14}C_{10}$

$$= \frac{14!}{10!4!} = \frac{14\times13\times12\times11\times10!}{10!4\times3\times2} = 1001.$$

(iii) Number of ways team selected when a particular player is never chosen = ${}_{14}C_{11}$

$$= \frac{14!}{11!3!} = \frac{14\times13\times12\times11!}{11!3\times2} = 364.$$