CBSE Board
Class XI Mathematics
Sample Paper – 3

Time: 3 hrs **Total Marks: 100**

General Instructions:

1. All questions are compulsory.
2. The question paper consist of 29 questions.
3. Questions 1 – 4 in Section A are very short answer type questions carrying 1 mark each.
4. Questions 5 – 12 in Section B are short-answer type questions carrying 2 mark each.
5. Questions 13 – 23 in Section C are long-answer I type questions carrying 4 mark each.
6. Questions 24 – 29 in Section D are long-answer type II questions carrying 6 mark each.

SECTION – A

1. In ΔABC, a = 18, b = 24 and c = 30 and m∠C = 90°, find sin A.

2. If f(x) is a linear function of x. f: Z →Z, f(x) = a x + b. Find a and b if { (1,3) , (-1, -7) , (2, 8) (-2 , -12)} ∈f.

3. Find the domain of the function $f(x) = \frac{x^2 - 4}{x^2 - 8x + 12}$

4. With p: It is cloudy and q: Sun is shining and the usual meanings of the symbols: ⇒, ⇔, ∼, ∧, ∨, express the statement below symbolically.

'It is not true that it is cloudy if and only if the Sun is not shining.'

OR

Write negation of the : Every living person is not 150 years old.

SECTION – B

5. What are the real numbers 'x' and 'y', if (x - iy) (3 + 5i) is the conjugate of (-1 - 3i)

OR

Find modulus of (3 + 4i)(4 + i).

6. A pendulum, 36 cm long, oscillates through an angle of 10 degrees. Find the length of the path described by its extremity.

OR

The area of sector is 5.024 cm^2 and its angle is 36˚. Find the radius. (π = 3.14)

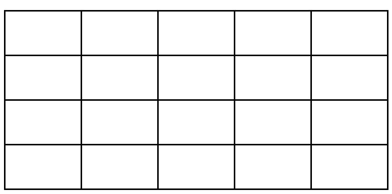
7. Find the sum of 19 terms of A.P. whose nth term is 2n+1.

8. Find the LCM of 4!, 5! and 6!

OR

Express $\frac{1}{(2+i)^2}$ in the standard form of a + ib.

9. Find the total number of rectangles in the given figure



10. Find the sum of the given sequence uptill the n^{th} term:
1.2 + 2. 3 + 3. 4 +...

11. In a group of 400 people, 250 can speak Hindi and 200 can speak English. Everyone can speak atleast one language. How many people can speak both Hindi and English?

12. If $\Sigma n = 210$, then find Σn^2.

SECTION – C

13. An equilateral triangle is inscribed in the parabola $y^2 = 4ax$, where one vertex of the triangle is at the vertex of the parabola. Find the length of the side of the triangle.

14. Prove that: (cos3x – cosx) cosx + (sin3x + sinx) sinx = 0

OR

Simplify the expression: sin7x + sinx + sin3x + sin5x

15. If the sum of an infinite geometric series is 15 and the sum of the squares of these terms is 45, find the series.

16. Let A = {a, b, c}, B = {c, d} and C = {d, e, f }. Find

(i) $A \times (B \cap C)$ (ii) $(A \times B) \cap (A \times C)$

(iii) $A \times (B \cup C)$ (iv) $(A \times B) \cup (A \times C)$

17. If $f : R \rightarrow R;\ f(x) = \frac{x^2}{x^2 + 1}$. What is the range of f?

18. What is the number of ways of choosing 4 cards from a pack of 52 playing cards? In how many of these

(i) four cards are of the same suit

(ii) four cards belong to four different suits

(iii) are face cards

(iv) two are red cards and two are black cards

19. Evaluate: $(99)^5$ using the Binomial theorem

OR

Find the ratio of the co-efficient of x^2 and x^3 in the binomial expansion $(3 + ax)^9$

20. If $x - iy = \sqrt{\frac{a\text{-}ib}{c\text{-}id}}$, find $\left(x^2 + y^2\right)^2$.

OR

Let $z_1 = 2 - i$ and $z_2 = -2 + i$, then find

(i) $\text{Re}\left[\frac{z_1 z_2}{\overline{z_1}}\right]$ (ii) $\text{Im}\left[\frac{1}{z_1 \overline{z_2}}\right]$

21. Find the roots of the equation $3x^2 - 4x + \frac{10}{7} = 0$

22. Find the domain and range of the function : $f(x) = \frac{1}{2 - \sin 3x}$

23. Plot the given linear in equations and shade the region which is common to the solution of all inequations $x \geq 0$, $y \geq 0$, $5x + 3y \leq 500$; $x \leq 70$ and $y \leq 125$.

SECTION – D

24. The scores of two batsmen A and B, in ten innings during a certain season are given below, Find which batsman is more consistent in scoring.

A	B
32	19
28	31
47	48
63	53
71	67
39	90
10	10
60	62
96	40
14	80

OR

The mean and variance of 7 observations are 8 and 16 respectively. If 5 of the observations are 2, 4, 10, 12, 14, find the remaining two observations.

25. From the digits 0, 1, 3, 5 and 7, how many 4 digit numbers greater than 5000 can be formed? What is the probability that the number formed is divisible by 5, if

(i) the digits are repeated

(ii) the digits are not repeated

26. If $x \in Q_3$ and $\cos x = -\frac{1}{3}$, then show that $\sin\frac{x}{2} = \pm\sqrt{\frac{2}{3}}$.

OR

If $\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = \tan^3\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)$ prove that $\sin\theta = \frac{3\sin\alpha + \sin^3\alpha}{1+3\sin^2\alpha}$

27. (i) Find the derivative of the given function using the first principle:

$f(x) = \cos\left(x - \frac{\pi}{16}\right)$

(ii) Evaluate: $\lim_{x\to\frac{\pi}{2}} \frac{5^{\cos x} - 1}{\frac{\pi}{2} - x}, x \neq \frac{\pi}{2}$.

28. If three lines whose equations are $y = m_1x + c_1$, $y = m_2x + c_2$ and $y = m_3x + c_3$ are concurrent, then find (i) the condition of concurrence of the three lines(ii) the point of concurrence.

OR

A beam is supported at its ends by supports which are 14 cm apart. Since the load is concentrated at its centre, there is a deflection of 5 cm at the centre and the deflected beam is in the shape of a parabola. How far from the centre is the deflection of 2 cm?

29. Prove by using the principle of mathematical induction that $(x^{2n} - y^{2n})$ is divisible by $(x + y)$.

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Sample Paper – 3 Solution

SECTION – A

1. Since $m\angle C = 90°$,therefore $\sin A = \frac{a}{c} = \frac{18}{30} = \frac{3}{5}$

2. $f(x) = ax + b$

$(1, 3) \in f \Rightarrow f(1) = a.1 + b = 3 \Rightarrow a + b = 3$

$(2, 8) \in f \Rightarrow f(2) = a.2 + b = 8 \Rightarrow 2a + b = 8$

Solving the two equations, we get a = 5, b = -2

a = 5, b = -2 also satisfy the other two ordered pairs

$f(-2) = 5(-2) – 2 = -12 \Rightarrow (-2, -12)$

$f(-1) = 5(-1) – 2 = -7 \Rightarrow (-1, -7)$

Therefore the values are a = 5 and b = -2.

3. $f(x) = \frac{x^2 - 4}{x^2 - 8x + 12}$

For f(x) to be defined, $x^2 - 8x + 12$ must be non-zero i.e. $x^2 - 8x + 12 \neq 0$

$(x - 2)(x - 6) \neq 0$

i.e. $x \neq 2$ and $x \neq 6$

Therefore domain will be $R - \{2, 6\}$

So domain of f = R - {2, 6}

4. $(\sim p \Leftrightarrow \sim q)$

OR

There exists a living person who is 150 years.

SECTION – B

5. $(x - iy)(3 + 5i) = \overline{-1 - 3i} = -1 + 3i$

$$\Rightarrow (x - iy) = \frac{-1 + 3i}{(3 + 5i)} = \frac{(-1 + 3i)(3 - 5i)}{(3 + 5i)(3 - 5i)} = \frac{-3 + 5i + 9i - 15i^2}{(9 - 25i^2)}$$

$$= \frac{-3 + 5i + 9i + 15}{9 + 25} = \frac{12 + 14i}{34} = \frac{6 + 7i}{17} = \frac{6}{17} + \frac{7i}{17}$$

$$\Rightarrow x = \frac{6}{17}; y = -\frac{7}{17}$$

OR

$(3 + 4i)(4 + i) = 12 + 3i + 16i + 4i^2 = 12 + 19i - 4 = 8 + 19i$

$$|z| = \sqrt{8^2 + 19^2} = \sqrt{64 + 361} = \sqrt{425} = 5\sqrt{17}$$

6. Length of pendulum is 36 cm long

Angle of oscillation = 10 degrees

180 degrees = π radians

so, 10 degrees $= \frac{\pi}{18}$ radians

$\Rightarrow \theta = \frac{\pi}{18}$ radians

So using this formula $l = r\theta$ and substituting the values of $r = 36$, $\theta = \frac{\pi}{18}$ radians

we get,

$\ell = 36 \text{ x } \frac{\pi}{18} = 2 \text{ x } (3.14) = 6.28$ cm

OR

Area of sector $= \frac{1}{2}r^2\theta$

$$\frac{1}{2}r^2\theta = 5.024$$

$$\frac{1}{2}r^2 \times \frac{36}{180}\pi = 5.024$$

$$r^2 = 5.024 \times \frac{180 \times 2}{36 \times 3.14}$$

$$r^2 = 16$$

$$r = 4 \text{ cm}$$

7. Let a be the first term and d be the common difference

$T_n = 2n+1$

$a = 3$.......... (T_1)

$T_2 = 5$

$d = 2$ .... $(T_2 - a)$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_n = \frac{19}{2}(2 \times 3 + (19-1) \times 2) = 399$$

8. We have 5! = 5 × 4! And 6! = 6 × 5 × 4!

LCM of 4!, 5! and 6! = LCM of {4!, 5 × 4!, 6 × 5 × 4!} = 4! 6 × 5 = 6! = 720

OR

$$\frac{1}{(2+i)^2}=\frac{1}{4+4i+i^2}$$

$$\frac{1}{(2+i)^2}=\frac{1}{4+4i-1}$$

$$\frac{1}{(2+i)^2}=\frac{1}{3+4i}$$

$$\frac{1}{(2+i)^2}=\frac{1}{3+4i}\times\frac{3-4i}{3-4i}$$

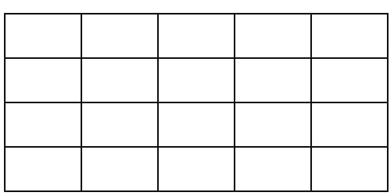
$$\frac{1}{(2+i)^2}=\frac{3-4i}{9-16i^2}$$

$$\frac{1}{(2+i)^2}=\frac{3-4i}{9+16}$$

$$\frac{1}{(2+i)^2}=\frac{3-4i}{25}$$

$$\frac{1}{(2+i)^2}=\frac{3}{25}-\frac{4i}{25}$$

9.



To make a rectangle we need to select 2 vertical lines from given 6 lines and 2 horizontal lines from given 5 line

so the number of rectangles so formed = ${}^5C_2 \times {}^6C_2$ =150

10. 1.2 + 2. 3 + 3. 4 +...

$a_n = n\ (n + 1) = n^2 + n$

$$S_n = \sum_{k=1}^{n}\left(k^2 + k\right) = \sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} k$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left(\frac{(2n+1)}{3} + 1\right)$$

$$= \frac{n(n+1)(n+2)}{3}$$

11. Let H denote the set of people who can speak Hindi, and E denote the set of people who can speak English.

Given everyone can speak atleast one language,

Therefore, n(H U E) = 400 and n(H) = 250, n(E) = 200

$n(H \cup E) = n(H) + n(E) - n(H \cap E)$

$n(H \cap E) = n(H) + n(E) - n(H \cup E)$

$n(H \cap E) = 250 + 200 - 400 = 50$

50 persons can speak both Hindi and English.

12. $\Sigma n = 210$

$$\frac{n(n+1)}{2} = 210$$

$n(n + 1) = 420$

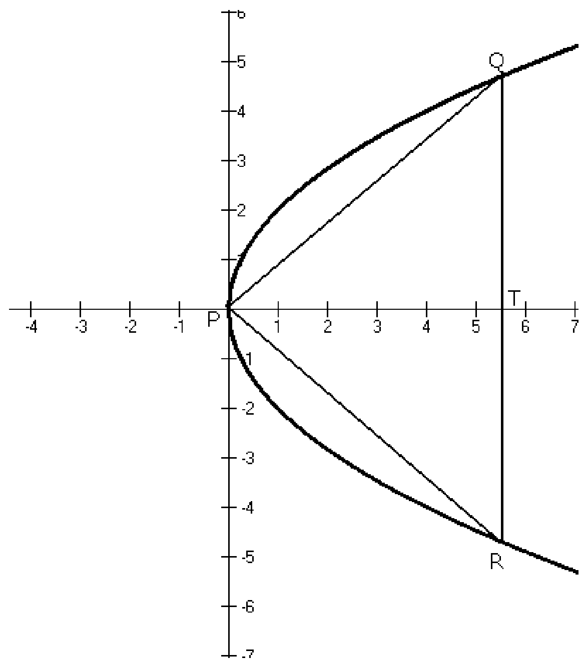
$20 \times 21 = 420$ so $n = 20$

$$\Sigma n^2 = \frac{n(n+1)(2n+1)}{6} = \frac{420 \times 41}{6} = 2870$$

SECTION – C

13. Let the two vertices of the triangle be Q and R

Points Q and R will have the same x-coordinate = k(say)



Now in the right ΔPRT, right angled at T.

$$\tan 60^\circ = \frac{k}{RT} \Rightarrow \sqrt{3} = \frac{k}{RT} \Rightarrow RT = \frac{k}{\sqrt{3}}$$

$$\Rightarrow R\left(k, \frac{k}{\sqrt{3}}\right)$$

Now R lies on the parabola : $y^2 = 4ax$

$$\Rightarrow \left(\frac{k}{\sqrt{3}}\right)^2 = 4a(k)$$

$$\Rightarrow \frac{k}{3} = 4a$$

$$\Rightarrow k = 12a$$

Length of side of the triangle = $2(RT) = 2.\frac{k}{\sqrt{3}} = 2.\frac{(12a)}{\sqrt{3}} = 8\sqrt{3}a$

14. Consider L.H.S. = (cos3x – cosx) cosx + (sin3x + sinx) sinx

$$= \left[-2\sin\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)\right]\cos x + \left[2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)\right]\sin x$$

$$= [2\sin 2x\cos x]\sin x + [-2\sin 2x\sin x]\cos x$$

$$= 2\sin x\cos x\sin 2x - 2\sin x\cos x\sin 2x = 0$$

OR

$\sin 7x + \sin x + \sin 3x + \sin 5x = (\sin 7x + \sin x) + (\sin 3x + \sin 5x)$

$(\sin 7x + \sin x) = 2\sin\left(\frac{7x+x}{2}\right)\cos\left(\frac{7x-x}{2}\right) = 2\sin\frac{8x}{2}\cos\frac{6x}{2}$

$= 2\sin 4x\cos 3x$ ......(i)

$(\sin 3x + \sin 5x) = 2\sin\left(\frac{3x+5x}{2}\right)\cos\left(\frac{3x-5x}{2}\right) = 2\sin\frac{8x}{2}\cos\frac{-2x}{2}$

$= 2\sin 4x\cos(-x) = 2\sin 4x\cos x$ ....(ii)

From (i) and (ii)

$(\sin 7x + \sin x) + (\sin 3x + \sin 5x) = 2\sin 4x\cos 3x + 2\sin 4x\cos x$

$= 2\sin 4x[\cos 3x + \cos x]$

$= 2\sin 4x\left[2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)\right] = 2\sin 4x\left[2\cos\left(\frac{4x}{2}\right)\cos\left(\frac{2x}{2}\right)\right]$

$= 4\sin 4x\cos 2x\cos x$

15. Let the infinite geometric series be a, ar, ar^2, ...

The sum of the infinite geometric series is 15.

$S_1 = \frac{a}{1-r}$

$\therefore \frac{a}{1-r} = 15$

Squaring the terms of the above infinite geometric series we get,

$a^2, a^2r^2, a^2r^4, \ldots$

Also this new series is in geometric progression.

The sum of the squares of these terms is 45.

$S_2 = \frac{a^2}{1-r^2}$

$\therefore \frac{a^2}{1-r^2} = 45$

Consider $\frac{S_1}{S_2}$:

$\frac{S_1}{S_2} = \frac{\frac{a}{1-r}}{\frac{a^2}{1-r^2}}$

$\Rightarrow \frac{15}{45} = \frac{1-r^2}{a(1-r)}$

$\Rightarrow \frac{1}{3} = \frac{(1+r)}{a}$

$\Rightarrow a = 3(1+r)$

Substitute the value of a in S_2, we have,

$$S_2 = \frac{a^2}{1-r^2} = \frac{\left(3\left(1+r\right)\right)^2}{1-r^2}$$

$$\Rightarrow 45 = \frac{9\left(1+r\right)^2}{1-r^2}$$

$$\Rightarrow r = \frac{2}{3}$$

So the series is $5, \frac{10}{3}, \frac{20}{9}, \ldots$

16. A = {a, b, c} B = {c, d} C ={d, e, f}

(i) $(B \cap C)$ = {d}

$\Rightarrow A \times (B \cap C)$ = {(a, d), (b, d), (c, d)}

(ii) A × B = {(a, c), (a, d), (b, c), (b, d), (c, c), (c, d)}

A × C = {(a, d), (a, e), (a, f), (b, d), (b, e), (b, f), (c, d), (c, e), (c, f)}

$(A \times B) \cap (A \times C)$ = {(a, d),(b, d),(c, d)}

(iii) $(B \cup C)$ = {c, d, e ,f}

$A \times (B \cup C)$ = {(a, c), (a, d), (a, e), (a, f), (b, c), (b, d), (b, e), (b, f), (c, c), (c, d), (c, e), (c, f)}

(iv) $(A \times B) \cup (A \times C)$ = {(a, c), (a, d), (a, e), (a, f), (b, c), (b, d), (b, e), (b, f), (c, c), (c, d), (c, e), (c, f)}

17. Let $y = f(x) = \frac{x^2}{x^2+1}$

$x^2 \geq 0 \Rightarrow x^2+1 \geq 1 \Rightarrow \text{Denominator} \geq \text{Numerator} \Rightarrow y \leq 1$

Now, $y = \frac{x^2}{x^2+1} \Rightarrow y\left(x^2+1\right) = x^2 \Rightarrow yx^2 + y = x^2 \Rightarrow x^2(y-1) = -y$

$$\Rightarrow x^2 = \frac{y}{1-y}$$

$\Rightarrow 1 - y \neq 0$

$\Rightarrow y \neq 1$

Now, $x^2 = \frac{y}{1-y} \geq 0$

Case1 : $y \geq 0; 1-y \geq 0$

$\Rightarrow y \geq 0; 1 \geq y$ or $y \leq 1$, but $y \neq 1$

i.e $y \in [0,1)$

Case2 : $y \leq 0; 1-y \leq 0$

$\Rightarrow y \leq 0; 1 \leq y$ or $y \geq 1$

Not possible $\therefore$ Range of f(x) = [0,1)

18. The number of ways of choosing 4 cards from a pack of 52 playing cards

$= {}^{52}C_4 = \frac{52!}{4!48!} = \frac{52.51.50.49}{1.2.3.4}$

$= 270725$

(i) The number of ways of choosing four cards of any one suit

$= {}^{13}C_4 = \frac{13!}{4!9!} = \frac{13.12.11.10}{1.2.3.4} = 715$

Now, there are 4 suits to choose from, so

The number of ways of choosing four cards of one suit = 4×715 = 2860

(ii) Four cards belong to four different suits, i.e., one card from each suit.

The number of ways of choosing one card from each suit

$= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 = 13 \times 13 \times 13 \times 13 = 13^4$

(iii) Are face cards

There are 12 face cards

The number of ways of choosing four face cards from 12

$= {}^{12}C_4$

=495

(iv) two are red cards and two are black cards,

There are 26 red cards and 26 black cards,

The number of ways of choosing 2 red and 2 black from 26 red and 26 black cards

$= {}^{26}C_2 \times {}^{26}C_2 = \frac{26!}{2!24!} \times \frac{26!}{2!24!} = \frac{26.25}{2} \times \frac{26.25}{2} = 13 \times 25 \times 13 \times 25$

=105625

19. $(99)^5$

To be able to use binomial theorem, let us express 99 as a binomial: 99 = 100 – 1

$(99)^5 = (100 - 1)^5 = {}^5C_0(100)^5(-1)^0 + {}^5C_1(100)^4(-1)^1 + {}^5C_2(100)^3(-1)^2 + {}^5C_3(100)^2(-1)^3$

$+ {}^5C_4(100)^1(-1)^4 + {}^5C_5(100)^0(-1)^5$

$= 1.(100)^5 - 5(100)^4 + 10.(100)^3 - 10(100)^2 + 5.(100) - 1$

= (10000000000) - 5(100000000) + 10.(1000000) - 10(10000) + 5.(100) - 1

= (10000000000) - (500000000) + (10000000) - (100000) + (500) - 1

= 10010000500 - 500100001

= 9509900499

OR

Given: $(3 + ax)^9$

General term in the expansion of $(3 + ax)^9$

$t_{r+1} = {}^9C_r (ax)^r (3)^{9-r}$

Coefficient of $x^r = {}^9C_r (a)^r (3)^{9-r}$

Coefficient of $x^2 = {}^9C_2 (a)^2 (3)^{9-2} = {}^9C_2 a^2 3^7$

Coefficient of $x^3 = {}^9C_3 (a)^3 (3)^{9-3} = {}^9C_3 a^3 3^6$

$$\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3} = \frac{{}^9C_2 a^2 3^7}{{}^9C_3 a^3 3^6} = \frac{3.{}^9C_2}{a.{}^9C_3} = \frac{3.3}{7a} = \frac{9}{7a}$$

20. $x - iy = \sqrt{\frac{a\text{-}ib}{c\text{-}id}} \Rightarrow (x - iy)^2 = \left[\sqrt{\frac{a\text{-}ib}{c\text{-}id}}\right]^2$

Now, $(x - iy)^2 = |x - iy|^2$

$\therefore (x - iy)^2 = |x - iy|^2 = \left[\left|\sqrt{\frac{a\text{-}ib}{c\text{-}id}}\right|\right]^2$

But $|x - iy| = \sqrt{x^2 + y^2}$

$\Rightarrow |x - iy|^2 = \left[\sqrt{x^2 + y^2}\right]^2 = x^2 + y^2$ ...(i)

$$\left[\left|\sqrt{\frac{a\text{-}ib}{c\text{-}id}}\right|\right]^2 = \left|\frac{a\text{-}ib}{c\text{-}id}\right| = \frac{|a\text{-}ib|}{|c - id|} = \frac{\sqrt{a^2 + (-b)^2}}{\sqrt{c^2 + (-d)^2}} = \frac{\sqrt{a^2 + b^2}}{\sqrt{c^2 + d^2}} \text{ ...(ii)}$$

From (i) and (ii), we have

$$x^2 + y^2 = \frac{\sqrt{a^2 + b^2}}{\sqrt{c^2 + d^2}}$$

$$\Rightarrow \left(x^2 + y^2\right)^2 = \left[\frac{\sqrt{a^2 + b^2}}{\sqrt{c^2 + d^2}}\right]^2 = \frac{a^2 + b^2}{c^2 + d^2}$$

OR

We have,

$z_1 = 2 - i$ and $z_2 = -2 + i$

(i)

$$\frac{z_1 z_2}{\bar{z}_1} = \frac{(2-i)(-2+i)}{(2+i)} = \frac{-(4+i^2-4i)}{(2+i)}$$

$$= -\frac{3-4i}{2+i} = -\frac{3-4i}{2+i} \times \frac{2-i}{2-i}$$

$$= -\frac{6-3i-8i+4(i)^2}{4+1}$$

$$= -\frac{6-11i-4}{5} = -\frac{2-11i}{5} = \frac{-2+11i}{5}$$

$$\text{Re}\left[\frac{z_1 z_2}{\bar{z}_1}\right] = \text{Re}\left[\frac{-2+11i}{5}\right] = \text{Re}\left[\frac{-2}{5} + \frac{11i}{5}\right] = \frac{-2}{5}$$

(ii)

$$\left[\frac{1}{z_1 \bar{z}_2}\right] = \frac{1}{(2-i)(-2-i)} = \frac{1}{-4+2i-2i+(i)^2}$$

$$= \frac{1}{-4-1} = -\frac{1}{5}$$

$$\therefore \text{Im}\left[\frac{1}{z_1 \bar{z}_2}\right] = \text{Im}\left(-\frac{1}{5}\right) = 0$$

21. $3x^2 - 4x + \frac{10}{7} = 0$

$\Rightarrow 21x^2 - 28x + 10 = 0$

$D = (28)^2 - 4 \times 21 \times 10 = 784 - 840 = -56 < 0$

The equation has complex roots

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-28) \pm \sqrt{-56}}{2(21)}$

$= \frac{28 \pm \sqrt{56}i}{42} = \frac{28 \pm 2\sqrt{14}i}{42}$

$= \frac{14 + \sqrt{14}i}{21}, \frac{14 - \sqrt{14}i}{21}$

22. $f(x) = \frac{1}{2 - \sin 3x}$

We know that

$-1 \le \sin 3x \le 1$ for all $x \in R$

$-1 \le -\sin 3x \le 1$ for all $x \in R$

$1 \le 2 - \sin 3x \le 3$ for all $x \in R$

$2 - \sin 3x \ne 0$ for any $x \in R$

$f(x) = \frac{1}{2 - \sin 3x}$ is defined for all $x \in R$

Hence, domain (f) = R

Range of f : As discussed above

$1 \le 2 - \sin 3x \le 3$ for all $x \in R$

$\frac{1}{3} \le \frac{1}{2 - \sin 3x} \le 1$ for all $x \in R$

$\frac{1}{3} \le f(x) \le 1$ for all $x \in R$

$f(x) \in \left[\frac{1}{3}, 1\right]$

Range of (f) = $\left[\frac{1}{3}, 1\right]$

23. System of inequations

$x \geq 0, y \geq 0, 5x + 3y \leq 500; x \leq 70$ and $y \leq 125$.

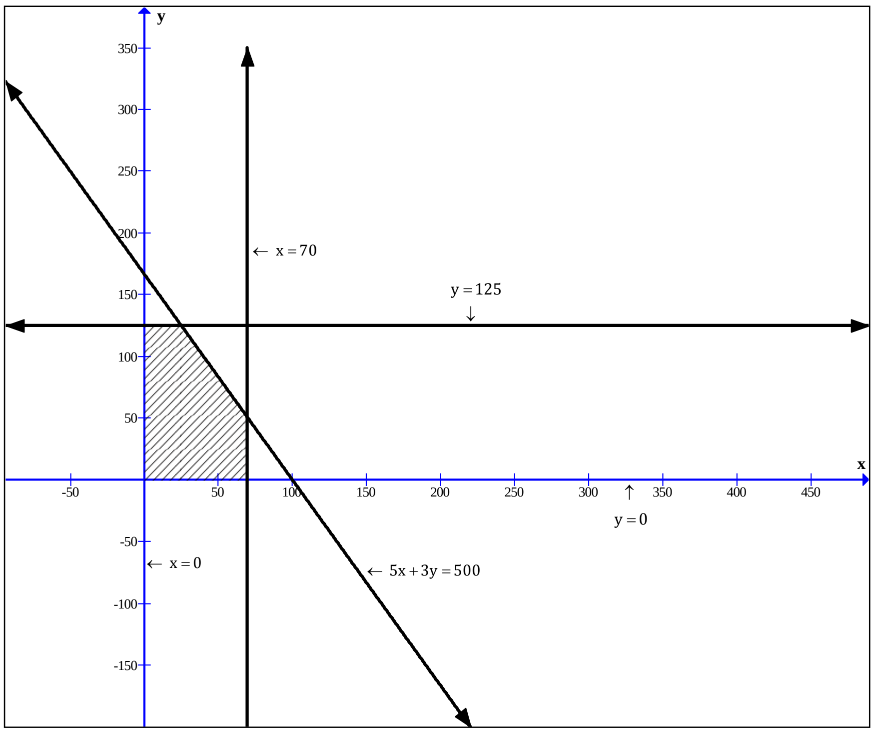
Converting inequations to equations

$5x + 3y = 500 \Rightarrow y = \dfrac{500 - 5x}{3}$

x	100	40	-80
y	0	100	300

$x \leq 70$ is $x = 70$ and $y \leq 125$ is $y = 125$.

Plotting these lines and determining the area of each line we get



SECTION – D

24. Coefficient of variation is used for measuring dispersion. In that case the batsman with the smaller dispersion will be more consistent.

Cricketer A			Cricketer B		
x	= x - 50		x	= x - 50	
32	-18	324	19	-31	961
28	-22	484	31	-19	361
47	-3	9	48	-2	4
63	13	169	53	3	9
71	21	441	67	17	289
39	-11	121	90	40	1600
10	-40	1600	10	-40	1600
60	10	100	62	12	144
96	46	2116	40	-10	100
14	-36	1296	80	30	900
Total	-40	6660	Total	0	5968

For cricketer A:

$$\text{Mean} = 50 + \left(\frac{-40}{10}\right) = 46$$

$$\text{S.D.} = \sqrt{\frac{6660}{10} - \left(\frac{-40}{10}\right)^2} = \sqrt{650} = 25.5$$

$$\therefore \text{C.V.} = \left(\frac{25.5}{46}\right) \times 100 = 55$$

For Cricketer B:

$$\text{Mean} = 50 + \left(\frac{0}{10}\right) = 50$$

$$\text{S.D.} = \sqrt{\frac{5968}{10} - \left(\frac{0}{10}\right)^2} = \sqrt{596.8} = 24.4$$

$$\therefore \text{C.V.} = \left(\frac{24.4}{50}\right) \times 100 = 49$$

Since the C.V for cricketer B is smaller, he is more consistent in scoring.

OR

Let x and y be the remaining two observations. Then,

Mean = 8

$$\frac{2+4+10+12+14+x+y}{7}=8$$

$$42+x+y=56$$

$$x+y=14$$

Variance = 16

$$\frac{1}{7}\left(2^2+4^2+10^2+12^2+14^2+x^2+y^2\right)-8^2=16$$

$$\frac{1}{7}\left(4+16+100+144+196+x^2+y^2\right)-64^2=16$$

$$x^2+y^2=100$$

$(x + y)^2 + (x – y)^2 = 2(x^2 + y^2)$

$196 + + (x – y)^2 = 2 \times 100$

$(x – y)^2 = 4$

$x – y = \pm 2$

If x – y = 2 then x + y = 14 and x – y = 2 give x = 8, y = 6

If x – y = -2 then x + y = 14 and x – y = -2 give x = 6 and y = 8.

25. There are 4 places to be filled

Th	H	T	U
4	3	2	1

The number has to be greater than 5000, so in place 4 only 5 or 7 out of 0, 1, 3, 5, and 7 can be used

(i) When repetition of digits is allowed

The number of choices for place 4 = 2

The number of choices for place 3 = 5

The number of choices for place 2 = 5

The number of choices for place 1 = 5

Total number of choices = $2 \times 5 \times 5 \times 5 = 250$

Now, for the number to be divisible by 5, there should be 0 or 5 in the units place

The number of choices for place 4 = 2

The number of choice for place 3 = 5

The number of choice for place 2 = 5

The number of choice for place 1 = 2

The number of choice = $2 \times 5 \times 5 \times 2 = 100$

$$P\left(\text{number divisible by 5 is formed when digits are repeated}\right)=\frac{100}{250}=\frac{2}{5}$$

(ii) When repetition of digits is not allowed

The number of choices for place 4 = 2

The number of choices for next place = 4

The number of choices for next place = 3

The number of choices for next place = 2

The number of choices = $2 \times 4 \times 3 \times 2 = 48$

For the number to be divisible by 5 there should be either 0 or 5 in the units place, giving rise to 2 cases.

Case I: there is 0 in the units place

The number of choices for place 4 = 2

Total number of choices for remaining places = $3 \times 2 \times 1=6$

Total number of choices = $2 \times 3 \times 2 \times 1 = 12$

Case II: there is 5 in the units place

Then there is 7 in place 4

The number of choices for place 4 = 1

The number of choice for remaining places = $3 \times 2 \times 1 = 6$

Total number of choices = 6

From case I and II:

Total number of choices = 6 + 12 = 18

$P\left(\text{a number divisible by 5 is formed when repetition of digits is not allowed} = \frac{18}{48} = \frac{3}{8}\right)$

26. $x \in Q_3$ III quadrant and $\cos x = -\frac{1}{3}$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\Rightarrow \cos x = 2\cos^2\frac{x}{2} - 1$$

$$\Rightarrow -\frac{1}{3} + 1 = 2\cos^2\frac{x}{2} \Rightarrow \frac{2}{3\times 2} = \cos^2\frac{x}{2}$$

$$\Rightarrow \cos\frac{x}{2} = \pm\sqrt{\frac{1}{3}}$$

Now, $x \in Q_3$

$$\Rightarrow 2n\pi + \pi < x < 2n\pi + \frac{3\pi}{2}$$

$$\Rightarrow \frac{2n\pi + \pi}{2} < \frac{x}{2} < \frac{2n\pi + \frac{3\pi}{2}}{2}$$

$$\Rightarrow n\pi + \frac{\pi}{2} < \frac{x}{2} < n\pi + \frac{3\pi}{4}$$

$$\text{Case I:When n is even} = 2k(\text{say})$$

$$\Rightarrow (2k)\pi + \frac{\pi}{2} < \frac{x}{2} < (2k)\pi + \frac{3\pi}{4}$$

$$\Rightarrow \frac{x}{2} \in Q_2$$

$$\text{Case I:When n is odd} = 2k+1(\text{say})$$

$$\Rightarrow (2k+1)\pi + \frac{\pi}{2} < \frac{x}{2} < (2k+1)\pi + \frac{3\pi}{4}$$

$$\Rightarrow (2k)\pi + \pi + \frac{\pi}{2} < \frac{x}{2} < (2k)\pi + \pi + \frac{3\pi}{4}$$

$$\Rightarrow (2k)\pi + \frac{3\pi}{2} < \frac{x}{2} < (2k)\pi + \frac{7\pi}{4}$$

$$\Rightarrow \frac{x}{2} \in Q_4$$

$$\sin\frac{x}{2} = \pm\sqrt{1-\left(\cos\frac{x}{2}\right)^2} = \pm\sqrt{1-\left(\pm\sqrt{\frac{1}{3}}\right)^2} = \pm\sqrt{1-\frac{1}{3}} = \pm\sqrt{\frac{2}{3}}$$

$$\text{In} Q_2 \sin\frac{x}{2} = \sqrt{\frac{2}{3}}$$

$$\text{In} Q_4 \sin\frac{x}{2} = -\sqrt{\frac{2}{3}}$$

$$\text{So } \sin\frac{x}{2} = \pm\sqrt{\frac{2}{3}}$$

OR

$$\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = \tan^3\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)$$

$$\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = \tan^3\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)$$

$$\frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}} = \left(\frac{1+\tan\frac{\alpha}{2}}{1-\tan\frac{\alpha}{2}}\right)^3$$

$$\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}} = \left(\frac{\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}}\right)^3$$

$$\left(\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}\right)^2 = \left(\frac{\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}}\right)^{3\times 2}$$

$$\frac{1+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{1-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}=\left(\frac{1+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{1-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)^3$$

$$\frac{1+\sin\theta}{1-\sin\theta}=\left(\frac{1+\sin\alpha}{1-\sin\alpha}\right)^3$$

$$\frac{1+\sin\theta-(1-\sin\theta)}{1+\sin\theta+(1-\sin\theta)}=\frac{(1+\sin\alpha)^3-(1-\sin\alpha)^3}{(1+\sin\alpha)^3+(1-\sin\alpha)^3}$$

$$\frac{2\sin\theta}{2}=\frac{6\sin\alpha+2\sin^3\alpha}{2+6\sin^2\alpha}$$

$$\sin\theta=\frac{3\sin\alpha+\sin^3\alpha}{1+3\sin^2\alpha}$$

27. (i) $f(x) = \cos\left(x-\frac{\pi}{16}\right)$

$$f(x+\delta x)=\cos\left(x+\delta x-\frac{\pi}{16}\right)$$

$$f(x+\delta x)-f(x)=\cos\left(x+\delta x-\frac{\pi}{16}\right)-\cos\left(x-\frac{\pi}{16}\right)$$

$$=-2\sin\frac{\left(x+\delta x-\frac{\pi}{16}+x-\frac{\pi}{16}\right)}{2}\sin\frac{\left(x+\delta x-\frac{\pi}{16}-\left(x-\frac{\pi}{16}\right)\right)}{2}$$

$$=-2\sin\frac{\left(2x+\delta x-\frac{\pi}{8}\right)}{2}\sin\frac{\delta x}{2}$$

$$\frac{f(x+\delta x)-f(x)}{\delta x}=-\frac{2\sin\frac{\left(2x+\delta x-\frac{\pi}{8}\right)}{2}\sin\frac{\delta x}{2}}{\delta x}=\frac{\sin\left(x+\frac{\delta x}{2}-\frac{\pi}{16}\right)\sin\frac{\delta x}{2}}{\frac{\delta x}{2}}$$

$$\lim_{\delta x\to 0}\frac{f(x+\delta x)-f(x)}{\delta x}=-\lim_{\delta x\to 0}\sin\left(x+\frac{\delta x}{2}-\frac{\pi}{16}\right)\lim_{\delta x\to 0}\frac{\sin\frac{\delta x}{2}}{\frac{\delta x}{2}}$$

$$=-\sin\left(x-\frac{\pi}{16}\right)$$

(ii) $\lim_{x\to\frac{\pi}{2}} \frac{5^{\cos x}-1}{\frac{\pi}{2}-x} = \lim_{y\to 0} \frac{5^y-1}{\frac{\pi}{2}-\cos^{-1} y}$ [Let cosx=y]

$$= \lim_{y\to 0} \frac{5^y-1}{\sin^{-1} y}$$

$$= \frac{\lim_{y\to 0} \frac{5^y-1}{y}}{\lim_{y\to 0} \frac{\sin^{-1} y}{y}}$$

$$= \frac{\ln 5}{1}$$

$$= \ln 5$$

28. The three lines whose equations are $y = m_1x + c_1$...(1), $y = m_2x + c_2$...(2) and $y = m_3x + c_3$ ....(3) are given

The point of intersection of (1) and (2) can be obtained by solving

$y = m_1x + c_1$, $y = m_2x + c_2$

$\Rightarrow m_1x + c_1 = m_2x + c_2$

$\Rightarrow m_1x + c_1 - m_2x - c_2 = 0$

$\Rightarrow (m_1 - m_2)x = c_2 - c_1$

$\Rightarrow x = \frac{c_2 - c_1}{m_1 - m_2}$

$y = m_1\left(\frac{c_2 - c_1}{m_1 - m_2}\right) + c_1 = \frac{m_1c_2 - m_2c_1}{m_1 - m_2}$

$\therefore$ The point of intersection $= \left(\frac{c_2 - c_1}{m_1 - m_2}, \frac{m_1c_2 - m_2c_1}{m_1 - m_2}\right)$

If this point also lies on line (3), then the three lines are concurrent and it is the point of concurrence

We substitute $\left(\frac{c_2 - c_1}{m_1 - m_2}, \frac{m_1c_2 - m_2c_1}{m_1 - m_2}\right)$ into $y = m_3x + c_3$ ...(3), to verify

$\frac{m_1c_2 - m_2c_1}{m_1 - m_2} = m_3\left(\frac{c_2 - c_1}{m_1 - m_2}\right) + c_3$

So $m_1c_2 - m_2c_1 = m_3(c_2 - c_1) + c_3(m_1 - m_2)$

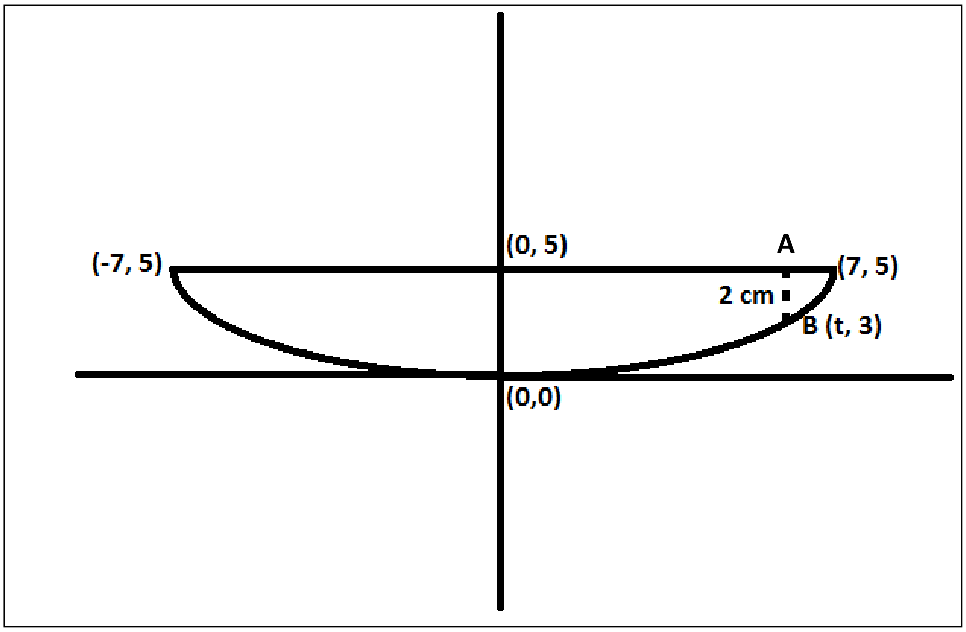
$\Rightarrow m_1c_2 - m_2c_1 = (m_3c_2 - m_3c_1) + (c_3m_1 - c_3m_2)$

$\Rightarrow m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$

is the required condition for concurrence.

(ii) Point of concurrency is $\left(\frac{c_2 - c_1}{m_1 - m_2}, \frac{m_1c_2 - m_2c_1}{m_1 - m_2}\right)$

OR



Let the vertex be at the origin and the vertical axis be along the y-axis. Therefore general equation is of the form $x^2 = 4ay$.

Since deflection in the centre is 5 cm, so (7, 5) is a point on the parabola

Therefore it must satisfy the equation of parabola i.e 49 = 20a

i.e a = 49/20

So the equation of parabola becomes

$x^2 = 49/5y = 9.8\ y$

Let the deflection of 2 cm be t cm away from the origin. Let AB be the deflection of beam,

So the co-ordinates of point B will be (t, 3)

Now, since the parabola passes through (t, 3), it must satisfy the equation of parabola,

Therefore $t^2 = 9.8 \times 3 = 29.4$

$\Rightarrow t = 5.422$ cm

Therefore distance of the deflection from the centre is 5.422 cm

29. Let $P(n)$: $x^{2n} - y^{2n}$ is divisible by $x + y$

$P(1) = x^2 - y^2 = (x + y)(x - y)$

So $P(1)$ is divisible by $(x + y)$

Now we assume $P(k)$: $x^{2k} - y^{2k}$ is divisible by $x + y$

To Prove : $P(k+1)$: $x^{2(k+1)} - y^{2(k+1)}$ is divisible by $x + y$

$x^{2(k+1)} - y^{2(k+1)} = x^{2k+2} - y^{2k+2}$

$= x^2 x^{2k} - x^2 y^{2k} + x^2 y^{2k} - y^2 y^{2k}$

$= x^2 \left(x^{2k} - y^{2k}\right) + y^{2k} \left(x^2 - y^2\right)$

$\left(x^{2k} - y^{2k}\right)$ is divisible by $x + y$ from $P(k)$

$x^2 \left(x^{2k} - y^{2k}\right)$ is divisible by $x + y$ from $P(k)$

$x^2 - y^2$ is divisible by $x + y$ from $P(1)$

$y^{2k} \left(x^2 - y^2\right)$ is divisible by $x + y$ from $P(1)$

$\therefore x^2 \left(x^{2k} - y^{2k}\right) + y^{2k} \left(x^2 - y^2\right)$ is divisible by $x + y$

So, $P(k+1)$: $x^{2(k+1)} - y^{2(k+1)}$ is divisible by $x + y$

Hence by principle of mathematical induction it is proved that $x^{2n} - y^{2n}$ is divisible by $(x + y)$.