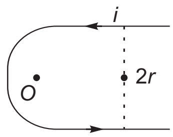
Manipal

Engineering Entrance Exam

Solved Paper 2010

Physics

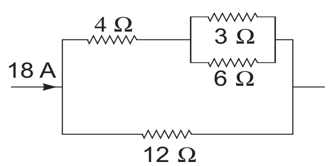
1. In the figure shown, the magnetic field induction at the point O will be



(a) $\frac{\mu_0 i}{2\pi r}$ (b) $\left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{r}\right)(\pi+2)$

(c) $\left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{r}\right)(\pi+1)$ (d) $\frac{\mu_0}{4\pi}\frac{i}{r}(\pi-2)$

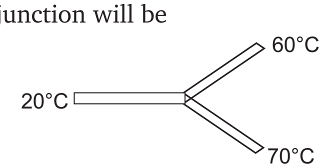
2. In the electrical network shown in the figure, the potential difference across 3 Ω resistance will be



(a) 12 V (b) 2.4 V

(c) 24 V (d) 36 V

3. Three identical thermal conductors are connected as shown in figure. Considering no heat loss due to radiation, temperature at the junction will be



(a) 40°C (b) 60°C

(c) 50°C (d) 35°C

4. Surface tension vanishes at

(a) absolute zero temperature

(b) transition temperature

(c) critical temperature

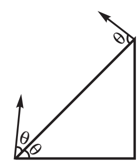
(d) None of the above

5. A transistor is working in common emitter mode. Its amplification factor (β) is 80. If the base current is 250 μA, the collector current will be

(a) 1.25 μA (b) $\frac{250}{80}$ μA

(c) 430 μA (d) 250×80 μA

6. From an inclined plane two particles are projected with same speed at same angle θ, one up and other down the plane as shown in figure, which of the following statements is/are correct?



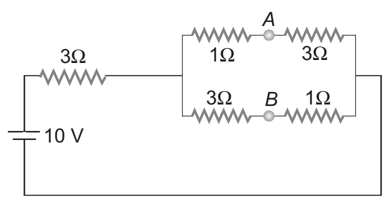
(a) The time of flight of each particle is the same

(b) The particles will collide the plane with same speed

(c) Both the particles strike the plane perpendicularly

(d) The particles will collide in mid air if projected simultaneously and time of flight of each particle is less than the time of collision

7. A battery of emf 10 V is connected to resistance as shown in figure. The potential difference $V_A - V_B$ between the points A and B is



(a) -2 V (b) 2 V

(c) 5 V (d) $\frac{20}{11}$ V

8. Solar radiation is
(a) transverse electromagnetic wave
(b) longitudinal electromagnetic wave
(c) stationary wave
(d) None of the above

9. Fundamental frequency of an open pipe is f_o, Fundamental frequency when it is half filled with water is
(a) f_o (b) $f_o/2$
(c) $2f_o$ (d) $3f_o$

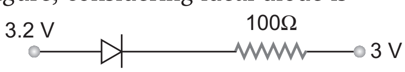
10. If the rms velocity of a gas is v, then
(a) v^2T = constant
(b) v^2/T = constant
(c) vT^2 = constant
(d) v is independent of T

11. A sounding source of frequency 500 Hz moves towards a stationary observer with a velocity 30 m/s. If the velocity of sound in air is 330 m/s, find the frequency heared by the observer.
(a) 500 Hz (b) 550 Hz
(c) 355 Hz (d) 55.5 Hz

12. At what height h above earth, the value of g becomes $g/2$? (R = Radius of earth)
(a) 3R (b) $\sqrt{2}$R
(c) $(\sqrt{2}-1)$R (d) $\frac{1}{\sqrt{2}}$R

13. A freshly prepared radioactive source of half-life 2 h emits radiation of intensity which is 64 times the permissible safe level. Calculate the minimum time after which it would be possible to work safely with this source.
(a) 12 h (b) 24 h
(c) 6 h (d) 130 h

14. The current in the circuit shown in the figure, considering ideal diode is



(a) 20 A (b) 2×10^{-3} A
(c) 200 A (d) 2×10^{-4} A

15. The tension in the string in the pulley system shown in the figure is

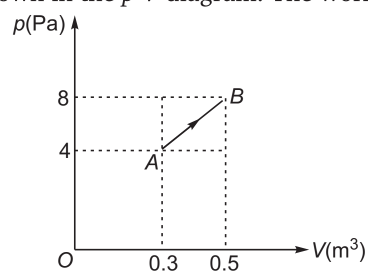
(a) 75 N (b) 80 N
(c) 7.5 N (d) 30 N

16. A glass flask having mass 390 g and an interior volume of 500 cm^3 floats on water when it is less than half filled with water. The density of material of the flask is
(a) 0.8 g/cc (b) 2.8 g/cc
(c) 1.8 g/cc (d) 0.28 g/cc

17. The angle of minimum deviation δ_m for an equilateral glass prism is 30°. Refractive index of the prism is
(a) $1/\sqrt{2}$ (b) $\sqrt{2}$
(c) $2\sqrt{2}$ (d) $1/2\sqrt{2}$

18. When the wavelength of sound changes from 1 m to 1.01 m, the number of beats heard are 4. The velocity of sound is
(a) 404 m/s (b) 4.04 m/s
(c) 414 m/s (d) 400 m/s

19. An ideal gas expands along the path AB as shown in the p-V diagram. The work done is



(a) 4×10^4 J (b) 1.2×10^5 J
(c) 2.4×10^5 J (d) None of these

20. If force is proportional to square of velocity, then the dimensions of proportionality constant is
(a) $[ML^{-1}T]$ (b) $[ML^{-1}T^0]$
(c) $[MLT^0]$ (d) $[M^0LT^{-1}]$

21. Two bodies A and B having temperatures 327°C and 427°C are radiating heat to the surrounding. The surrounding temperature is 27°C. The ratio of rates of heat radiation of *A* to that of *B* is

(a) 0.52 (b) 0.31
(c) 0.81 (d) 0.42

22. Two bulbs 40 W and 60 W and rated voltage 240 V are connected in series across a potential difference of 420 V. Which bulb will work at above its rated voltage?

(a) 40 W bulb (b) 60 W bulb
(c) Both will work (d) None of these

23. Two cars *A* and *B* move along a concentric circular path of radius r_A and r_B with velocities v_A and v_B maintaining constant distance, then $\frac{v_A}{v_B}$ is equal to

(a) $\frac{r_B}{r_A}$ (b) $\frac{r_A}{r_B}$
(c) $\frac{r_A^2}{r_B^2}$ (d) $\frac{r_B^2}{r_A^2}$

24. A body of mass 10 kg is moving with a constant velocity of 10 m/s. When a constant force acts for 4 s on it, it moves with a velocity 2 m/s in the opposite direction. The acceleration produced in it is

(a) 3 m/s^2 (b) –3 m/s^2
(c) 0.3 m/s^2 (d) 0.03 m/s^2

25. A beam of light is incident at 60° to a plane surface. The reflected and refracted rays are perpendicular to each other then refractive index of the surface is

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$
(c) $\frac{1}{2\sqrt{3}}$ (d) None of these

26. Two wires of lengths l and 2*l*, radii *r* and 2r respectively having same Young's modulus are hung with a weight mg. Net elongation is

(a) $\frac{3mgl}{\pi r^2 Y}$ (b) $\frac{2mgl}{3\pi r^2 Y}$
(c) $\frac{3mgl}{2\pi r^2 Y}$ (d) $\frac{3mgl}{4\pi r^2 Y}$

27. A ball rolls off the top of stairway with a horizontal velocity of magnitude 1.8 m/s. The steps are 0.20 m high and 0.20 m wide. Which step will the ball hit first?

(a) First (b) Second
(c) Third (d) Fourth

28. The peak value of an alternating emf *E* given by $E = E_0 \cos\omega t$ is 10 V and its frequency is 50 Hz. At a time $t = \frac{1}{600}$ s, the instantaneous value of the emf is

(a) 10 V (b) $5\sqrt{3}$ V
(c) 5 V (d) 1 V

29. A circuit area 0.01 m^2 is kept inside a magnetic field which is normal to its plane. The magnetic field changes from 2 T to 1 T in 1 ms. If the resistance of the circuit is 2 Ω. The amount of heat evolved is

(a) 0.05 J (b) 50 J
(c) 0.50 J (d) 500 J

30. A convex lens is placed between object and a screen. The size of object is 3 cm and an image of height 9 cm is obtained on the screen. When the lens is displaced to a new position, what will be the size of image on the screen?

(a) 2 cm (b) 6 cm
(c) 4 cm (d) 1 cm

31. A gas is suddenly expanded such that its final volume becomes 3 times its initial volume. If the specific heat at constant volume of the gas is 2R, then the ratio of initial to final pressures is nearly equal to

(a) 5 (b) 6.5
(c) 7 (d) 3.5

32. Two pendulums have time periods *T* and 5*T*/4. They start SHM at the same time from the mean position. What will be the phase difference between them after the bigger pendulum completed one oscillation?

(a) 45° (b) 90°
(c) 60° (d) 30°

33. A body is coming with a velocity of 72 km/h on a rough horizontal surface of coefficient of friction 0.5. If the acceleration due to gravity is 10 m/s^2, find the minimum distance it can be stopped.

(a) 400 m (b) 40 m
(c) 0.40 m (d) 4 m

34. A bullet comes out of the barrel of gun of length 2 m with a speed 80 m/s. The average acceleration of the bullet is

(a) 1.6 m/s^2 (b) 160 m/s^2
(c) 1600 m/s^2 (d) 16 m/s^2

35. A disc of radius 0.1 m is rotating with a frequency 10 rev/s in a normal magnetic field of strength 0.1 T. Net induced emf is

(a) $2\pi \times 10^{-2} V$ (b) $\pi \times 10^{-2} V$
(c) $\frac{\pi}{2} \times 10^{-2} V$ (d) None of these

36. 1 cm^3 of water at its boiling point absorbs 540 cal of heat to becomes steam with a volume of 1671 cm^3. If the atmospheric pressure $= 1.013 \times 10^5$ N/m^2 and the mechanical equivalent of heat = 4.19 J/cal, the energy spent in this process in overcoming intermolecular forces is

(a) 540 cal (b) 40 cal
(c) 500 cal (d) zero

37. A string fixed at both ends oscillates in 5 segments, length 10 m and velocity of wave is 20 m/s. What is the frequency?

(a) 5 Hz (b) 15 Hz
(c) 10 Hz (d) 2 Hz

38. When the amplitude of a body executing SHM becomes twice what happens?

(a) Maximum potential energy is doubled
(b) Maximum kinetic energy is doubled
(c) Total energy is doubled
(d) Maximum velocity is doubled

39. The time period of a geostationary satellite at a height 36000 km is 24 h. A spy satellite orbits earth at a height 6400 km. What will be the time period of spy satellite? [Radius of the earth = 6400 km]

(a) 5 h (b) 4 h
(c) 3 h (d) 12 h

40. A bomb of mass 9 kg explodes into two parts. One part of mass 3 kg moves with velocity 16 m/s, then the KE of the other part is

(a) 162 J (b) 150 J
(c) 192 J (d) 200 J

41. In the reaction

$$_7N^{14} + \alpha \longrightarrow {}_8X^{17} + {}_1p^1$$

identify *X*.

(a) O_2 (b) N_2
(c) He (d) Ar

42. Current in a coil changes from 5 A to 10 A in 0.2 s. If the coefficient of self-induction is 10 H, then the induced emf is

(a) 112 V (b) 250 V
(c) 125 V (d) 230 V

43. The force of interaction between two charges $q_1 = 6\mu C$ and $q_2 = 2\mu C$ is 12 N. If charge $q = -2\mu C$ is added to each of the charges, then the new force of interaction is

(a) 2×10^{-7} N (b) Zero
(c) 30 N (d) 2×10^{-3} N

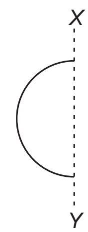
44. The number of turns in primary coil of a transformer is 20 and the number of turns in the secondary is 10. If the voltage across the primary is 220 V, what is the voltage across the secondary?

(a) 110 V (b) 130 V
(c) 190 V (d) 310 V

45. An electron of an atom transits from n_1 to n_2. In which of the following, maximum frequency of photon will be emitted?

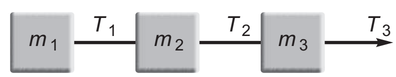
(a) $n_1 = 1$ to $n_2 = 2$ (b) $n_1 = 2$ to $n_2 = 1$
(c) $n_1 = 2$ to $n_2 = 6$ (d) $n_1 = 6$ to $n_2 = 2$

46. A rod of length *L* and mass *M* is bent to form a semicircular ring as shown in figure. The moment of inertia about XY is



(a) $\frac{1}{4}\frac{ML^2}{\pi^2}$ (b) $\frac{2ML^2}{3\pi^2}$
(c) $\frac{ML^2}{2}$ (d) $\frac{ML^2}{2}$

47. In the figure shown, $m_1 = 10$ kg, $m_2 = 6$ kg, $m_3 = 4$ kg. If $T_3 = 40$ N, $T_2 = ?$

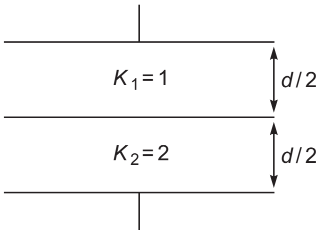


(a) 13 N (b) 32 N
(c) 25 N (d) 35 N

48. A stone is thrown at an angle θ to be horizontal reaches a maximum height *H*. Then the time of flight of stone will be

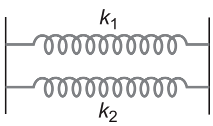
(a) $\sqrt{\frac{2H}{g}}$ (b) $2\sqrt{\frac{2H}{g}}$
(c) $\frac{2\sqrt{2H\sin\theta}}{g}$ (d) $\frac{\sqrt{2H\sin\theta}}{g}$

49. Two parallel plates of area *A* are separated by two different dielectrics as shown in figure. The net capacitance is



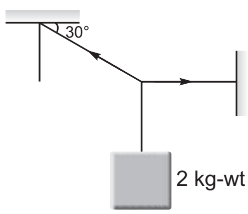
(a) $\frac{\varepsilon_0 A}{2d}$ (b) $\frac{\varepsilon_0 A}{d}$
(c) $\frac{3\varepsilon_0 A}{d}$ (d) $\frac{4\varepsilon_0 A}{3d}$

50. Two springs of force constants k_1 and k_2 are connected as shown. The effective spring constant k is



(a) $k_1 + k_2$ (b) $\frac{k_1}{k_2}$
(c) $k_1 k_2$ (d) $2k_1 k_2$

51. A body of weight 2 kg is suspended as shown in figure. The tension T_1 in the horizontal string (in kg-wt) is



(a) $2/\sqrt{3}$ (b) $\sqrt{3}/2$
(c) $2\sqrt{3}$ (d) 2

52. If the momentum of a body is increased by 100%, then the percentage increase in the kinetic energy is
(a) 150% (b) 200%
(c) 225% (d) 300%

53. In Young's double slit experiment, slit separation is 0.6 mm and the separation between slit and screen is 1.2 m. The angular width is (the wavelength of light used is 4800 Å)
(a) 30 rad (b) 8×10^{-4} rad
(c) 12 rad (d) 70.5 rad

54. X-ray of wavelength $\lambda = 2$ Å is emitted from the metal target. The potential difference applied across the cathode and the metal target is
(a) 5525 V (b) 320 V
(c) 6200 V (d) 3250 V

55. Two identical masses m moving with velocities u_1 and u_2 collide perfectly inelastically. Find the loss in energy.
(a) $m(u_1 - u_2^2)$ (b) $\frac{m}{4}(u_1 - u_2)^2$
(c) $\frac{m}{2}(u_1 - u_2)^2$ (d) $m(u_1 - u_2)^3$

56. A constant torque acting on a uniform circular wheel changes its angular momentum from A_0 to $4A_0$ in 4 s. The magnitude of this torque is
(a) $\frac{3A_0}{4}$ (b) A_0
(c) $4A_0$ (d) $12A_0$

57. When a number of small droplets combines to form a large drop, then
(a) energy is absorbed
(b) energy is liberated
(c) energy is neither liberated nor absorbed
(d) process is independent of energy

58. With what minimum acceleration can a fireman slide down a rope while breaking strength of the rope is $\frac{2}{3}$ of the weight?
(a) $\frac{2}{3}g$ (b) g
(c) $\frac{1}{3}g$ (d) Zero

59. If two waves represented by $y_1 = 4\sin\omega t$ and $y_2 = 3\sin\left(\omega t + \frac{\pi}{3}\right)$ interfere at a point. The amplitude of the resulting wave will be about
(a) 7 (b) 6
(c) 5 (d) 3.5

60. Saturated vapour is compressed to half its volume without any change in temperature, then the pressure will be
(a) doubled (b) halved
(c) the same (d) Zero

Chemistry

1. The root mean square velocity of a gas is doubled when the temperature is
(a) increased four times
(b) increased two times
(c) reduced to half
(d) reduced to one fourth

2. Acidity of phenol is due to
(a) hydrogen bonding
(b) phenolic group
(c) benzene ring
(d) resonance stabilisation of its anion

3. In which one of the following pairs the radius of the second species is greater than that of the first?
(a) Na, Mg (b) O^{2-}, N^{3-}
(c) Li^{+}, Be^{2+} (d) Ba^{2+}, Sr^{2+}

4. The radius of the first Bohr orbit of hydrogen atom is 0.529 Å. The radius of the third orbit of H^{+} will be
(a) 8.46 Å (b) 0.705 Å
(c) 1.59 Å (d) 4.79 Å

5. Both Co^{3+} and Pt^{4+} have a coordination number of six. Which of the following pairs of complexes will show approximately the same electrical conductance for their 0.001 M aqueous solutions?
(a) $CoCl_3 \cdot 4NH_3$ and $PtCl_4 \cdot 4NH_3$
(b) $CoCl_3 \cdot 3NH_3$ and $PtCl_4 \cdot 5NH_3$
(c) $CoCl_3 \cdot 6NH_3$ and $PtCl_4 \cdot 5NH_3$
(d) $CoCl_3 \cdot 6NH_3$ and $PtCl_4 \cdot 3NH_3$

6. When ice melts into water, the entropy
(a) becomes zero (b) remains same
(c) decreases (d) increases

7. Reduction of nitrobenzene in the presence of Zn/NH_4Cl gives
(a) azobenzene
(b) hydrazobenzene
(c) N-phenyl hydroxyl amine
(d) aniline

8. Which one of the following is not a protein?
(a) Wool (b) Nail
(c) Hair (d) DNA

9. Which of the following hexoses will form the same osazone when treated with excess phenyl hydrazine?
(a) D-glucose, D-fructose and D-galactose
(b) D-glucose, D-fructose and D-mannose
(c) D-glucose, D-mannose and D-galactose
(d) D-fructose, D-mannose and D-galactose

10. A silver cup is plated with silver by passing 965C of electricity. The amount of Ag deposited is
(a) 107.89 g (b) 9.89 g
(c) 1.0002 g (d) 1.08 g

11. The standard emf of a cell involving one electron change is found to be 0.591 V at 25°C. The equilibrium constant of the reaction is (F = 96,500 C mol^{-1})
(a) 1.0×10^{1} (b) 1.0×10^{5}
(c) 1.0×10^{10} (d) 1.0×10^{30}

12. E° values of Mg^{2+}/Mg is –2.37 V, of Zn^{2+}/Zn is –0.76 V and Fe^{2+}/Fe is – 0.44 V. Which of the statements is correct?
(a) Zn will reduce Fe^{2+}
(b) Zn will reduce Mg^{2+}
(c) Mg oxidises Fe
(d) Zn oxidises Fe

13. The maximum proportion of available volume that can be filled by hard spheres in diamond is
(a) 0.52 (b) 0.34
(c) 0.32 (d) 0.68

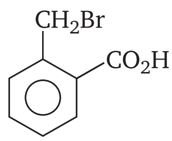
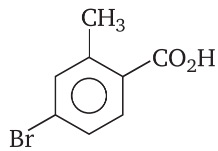
14. If we mix a pentavalent impurity in a crystal lattice of germanium, what type of semiconductor formation will occur?
(a) *p*-type
(b) *n*-type
(c) Both (a) and (b)
(d) None of the two

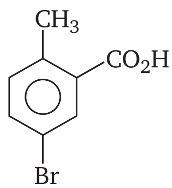
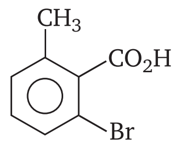
15. Which one of the following complexes is an outer orbital complex?
(Atomic numbers Mn = 25, Fe = 26, Co = 27, Ni = 28)
(a) $[Fe(CN)_6]^{4-}$ (b) $[Mn(CN)_6]^{4-}$
(c) $[Co(NH_3)_6]^{3+}$ (d) $[Ni(NH_3)_6]^{2+}$

16. The product of reaction between alcoholic silver nitrite with ethyl bromide is
(a) ethene (b) ethane
(c) ethyl nitrile (d) nitro ethane

17. *Iso*-propyl chloride undergoes hydrolysis by
(a) S_N1 mechanism
(b) S_N2 mechanisms
(c) S_N1 and S_N2 mechanisms
(d) Neither S_N1 nor S_N2 mechanism

18. *o*-toluic acid on reaction with Br_2 + Fe gives

(a) 
(b) 

(c) 
(d) 

19. Consider the acidity of the carboxylic acids

I. PhCOOH

II. *o*-$NO_2C_6H_4COOH$

III. *p*-$NO_2C_6H_4COOH$

IV. *m*-$NO_2C_6H_4COOH$

Which of the following order is correct?

(a) I > II > III > IV (b) II > IV > III > I

(c) II > IV > I > III (d) II > III > IV > I

20. Which of the following does not answer iodoform test?

(a) *n*-butyl alcohol

(b) Acetophenone

(c) Acetaldehyde

(d) Ethylmethyl ketone

21. Which one of the following undergoes reaction with 50% sodium hydroxide solution to give the corresponding alcohol and acid?

(a) Phenol (b) Benzaldehyde

(c) Butanal (d) Benzoic acid

22. The IUPAC name of

$$CH_3-\underset{\underset{OH}{|}}{CH}-CH=\underset{\underset{CH_3}{|}}{C}-CHO \text{ is}$$

(a) 4-hydroxy-1-methylpentanal

(b) 4-hydroxy-2-methylpent-2-en-1-al

(c) 2-hydroxy-4-methylpent-3-en-5-al

(d) 2-hydroxy-3-methylpent-2-en-5-al

23. The activation energy of exothermic reaction $A \rightarrow B$ is 80 kJ mol^{-1}. The heat of reaction is 200 kJ mol^{-1}. The activation energy for the reaction $B \rightarrow A$ (in kJ mol^{-1}) will be

(a) 80 (b) 120

(c) 40 (d) 280

24. Which of the following electrolytes is least effective in coagulating ferric hydroxide solution?

(a) KBr (b) K_2SO_4

(c) K_2CrO_4 (d) $K_4[Fe(CN)_6]$

25. Of the following outer electronic configurations of atoms, the highest oxidation state is achieved by which one of them?

(a) $(n-1)d^8, ns^2$ (b) $(n-1)d^5, ns^1$

(c) $(n-1)d^3, ns^2$ (d) $(n-1)d^5, ns^2$

26. The relative lowering of vapour pressure of a dilute aqueous solution containing non volatile solute is 0.0125. The molality of the solution is about

(a) 0.70 (b) 0.50

(c) 0.90 (d) 0.80

27. When hydrogen peroxide is added to acidified potassium dichromate, a blue colour is produced due to formation of

(a) CrO_3 (b) Cr_2O_3

(c) CrO_5 (d) CrO_4^{2-}

28. In the following reaction,

$NaOH + S \longrightarrow A + Na_2S + H_2O$; A is

(a) Na_2SO_4 (b) Na_2SO_3

(c) Na_2S (d) $Na_2S_2O_3$

29. On igniting Fe_2O_3 at 1400°C, the product obtained is

(a) Fe_2O_3 melt (b) FeO

(c) Fe_3O_4 (d) metallic iron

30. Sulphuric acid has great affinity for water because

(a) it hydrolyses the acid

(b) it decomposes the acid

(c) acid forms hydrates with water

(d) acid decomposes water

31. Helium-oxygen mixture is used by deep sea divers in preference to nitrogen-oxygen mixture because

(a) helium is much less soluble in blood than nitrogen

(b) nitrogen is much less soluble in blood than helium

(c) due to high pressure deep under the sea nitrogen and oxygen react to give poisonous nitric oxide

(d) nitrogen is highly soluble in water

32. One mole of CO_2 contains
(a) 3 g atoms of CO_2
(b) 18.1×10^{23} molecules of CO_2
(c) 6.02×10^{23} atoms of O
(d) 6.02×10^{23} atoms of C

33. The equivalent weight of $KMnO_4$ for acid solution is
(a) 79 (b) 52.16
(c) 158 (d) 31.6

34. Which has the highest weight?
(a) 1 m^3 of water
(b) A normal adult man
(c) 10 L of Hg
(d) All have same weight

35. Which has the highest *e/m* ratio?
(a) He^{2+} (b) H^+
(c) He^+ (d) D^+

36. The ionisation potential order for which set is correct?
(a) Cs < Li < K (b) Cs > Li > B
(c) Li > K > Cs (d) B > Li < K

37. The oxidation state of Fe in Fe_3O_4 is
(a) +3 (b) 8/3
(c) +6 (d) +2

38. For a first order reaction, the concentration changes from 0.8 to 0.4 in 15 min. The time taken for the concentration to change from 0.1 M to 0.025 M is
(a) 30 min (b) 15 min
(c) 7.5 min (d) 60 min

39. When phenol is treated with excess of bromine water, it gives
(a) *m*-bromophenol
(b) *o*-and *p*-bromophenols
(c) 2, 4-dibromophenol
(d) 2, 4, 6-tribromophenol

40. Which reaction is suitable for the preparation of α-chloroacetic acid?
(a) Hell-Volhard Zelinsky reaction
(b) Nef reaction
(c) Stephen's reaction
(d) Perkin condensation

41. Which one of the following compounds will dissolve in an alkali solution after it has undergone reaction with Hinsberg reagent?
(a) CH_3NH_2 (b) $(CH_3)_3N$
(c) $(C_2H_5)_2NH$ (d) $C_6H_5NHC_6H_5$

42. Angle strain in cyclopropane is
(a) 24° 44′ (b) 9° 44′
(c) 44′ (d) – 5° 16′

43. A group of atoms can function as a ligand only when
(a) it is a small molecule
(b) it has an unshared electron pair
(c) it is a negatively charged ion
(d) it is a positively charged ion

44. The bond order in NO is 2.5 while that in NO^+ is 3. Which of the following statements is true for these two species?
(a) Bond length in NO^+ is greater than in NO
(b) Bond length in NO is greater than in NO^+
(c) Bond length in NO^+ is equal to that in NO
(d) Bond length is unpredictable

45. In a homonuclear molecule which of the following set of orbitals is degenerate?
(a) $\sigma 2s$ and $\sigma 1s$ (b) $\pi 2p_x$ and $\pi 2p_y$
(c) $\pi 2p_x$ and $\sigma 2p_z$ (d) $\sigma 2p_z$ and $\pi^* 2p_x$

46. The pH of a neutral water sample is 6.5. Then the temperature of water
(a) is 25°C
(b) is more than 25°C
(c) is less than 25°C
(d) can be more or less than 25°C

47. Which is Lewis acid?
(a) BF_3 (b) NF_3
(c) Cl^- (d) H_2O

48. The pH of 10^{-10} M NaOH solution is nearest to
(a) 4 (b) – 10
(c) 4 (d) 7

49. Which of the following is not true for carbanions?
(a) The carbon carrying the charge has eight valence electrons
(b) They are formed by heterolytic fission
(c) They are paramagnetic
(d) The carbon carrying the charge is sp^3 hybridised

50. Which among the following statements is correct with respect to the optical isomers?
(a) Enantiomers are non-superimposable mirror images
(b) Diastereomers are superimposable mirror images
(c) Enantiomers are superimposable mirror images
(d) *Meso* forms have no plane of symmetry

51. Inductive effect involves
(a) delocalisation of σ-electrons
(b) displacement of σ-electrons
(c) delocalisation of n-electrons
(d) displacement of π-electrons

52. The solubility of AgCl in 0.2 M NaCl solution is (K_{sp} of AgCl $= 1.20 \times 10^{-10}$)
(a) 6.0×10^{-10} M (b) 0.2 M
(c) 1.2×10^{-10} M (d) 0.2×10^{-10} M

53. Lipids are
(a) nucleic acids occurring in plants
(b) proteins occurring in animals
(c) carbohydrates occurring in plants
(d) fats of natural origin

54. The enzyme pepsin hydrolyses
(a) proteins to amino acids
(b) fats to fatty acids
(c) glucose to ethyl alcohol
(d) polysaccharides to monosaccharides

55. Carbon cannot be used in the reduction of Al_2O_3 because
(a) it is an expensive proposition
(b) the enthalpy of formation of CO_2 is more than that of Al_2O_3
(c) pure carbon is not easily available
(d) the enthalpy of formation of Al_2O_3 is too high

56. Oxalic acid when heated with conc H_2SO_4, gives
(a) H_2O_2 and CO_2 (b) CO and CO_2
(c) H_2O_2 and CO (d) CO_2 and H_2S

57. At 25°C, the highest osmotic pressure is exhibited by 0.1 M solution of
(a) urea (b) glucose
(c) KCl (d) $CaCl_2$

58. Ammonia is dried over
(a) slaked lime
(b) calcium chloride
(c) phosphorus pentoxide
(d) quick lime

59. $C_6H_6 \xrightarrow[350\text{ K}]{H_2SO_4} A \xrightarrow[\text{Fusion}]{\text{Alkali}} B \xrightarrow[H_2O]{Br_2} C$

In the above sequence, *C* is
(a) *o*-bromophenol
(b) *p*-bromophenol
(c) *m*-bromophenol
(d) 2, 4, 6-tribromophenol

60. Collin's reagent is used to convert
(a) $>C{=}O \longrightarrow >CHOH$
(b) $—CH_2OH \longrightarrow —CHO$
(c) $—CHO \longrightarrow —COOH$
(d) $—CHO \longrightarrow —CH_2OH$

Mathematics

1. The range of the function $f(x) = \log_e(3x^2 - 4x + 5)$ is
(a) $\left(-\infty, \log_e \frac{11}{3}\right]$
(b) $\left[\log_e \frac{11}{3}, \infty\right)$
(c) $\left[-\log_e \frac{11}{3}, \log_e \frac{11}{3}\right]$
(d) None of the above

2. The domain of the function $f(x) = \dfrac{\sqrt{9 - x^2}}{\sin^{-1}(3 - x)}$ is
(a) (2, 3) (b) [2, 3)
(c) (2, 3] (d) None of these

3. The numbers a_n's are defined by
$$a_0 = 1,\ a_{n+1} = 3n^2 + n + a_n,\ (n \ge 0)$$
Then, a_n is equal to
(a) $n^3 + n^2 + 1$ (b) $n^3 - n^2 + 1$
(c) $n^3 - n^2$ (d) $n^3 + n^2$

4. Nishi has 5 coins each of the different denomination. The number different sums of money she conform
(a) 32 (b) 25
(c) 31 (d) None of these

5. There are P copies of *n*-different books. The number of different ways in which a non-empty selection can be made from them, is
(a) $(P + 1)^n - 1$ (b) $P^n - 1$
(c) $(P + 1)^{n-1} - 1$ (d) None of these

6. Out of 18 points in a plane no three are in the same straight line except five points which are collinear. The number of straight lines that can be formed joining them, is

(a) 143 (b) 144
(c) 153 (d) None of these

7. The term independent of x in $\left[\sqrt{\left(\frac{x}{3}\right)}+\sqrt{\left(\frac{3}{2x^2}\right)}\right]^{10}$ is

(a) 1 (b) ${}^{10}C_1$
(c) $\frac{5}{12}$ (d) None of these

8. The greatest coefficient in the expansion of $(1+x)^{2n}$ is

(a) ${}^{2n}C_n$ (b) ${}^{2n}C_{n-1}$
(c) ${}^{2n}C_{n-2}$ (d) None of these

9. The number of terms in the expansion of $(\sqrt{5}+\sqrt[4]{11})^{124}$ which are integers, is equal to

(a) 0 (b) 30 (c) 31 (d) 32

10. The constant term in the expansion of $(1+x)^m\left(1+\frac{1}{x}\right)^n$ is

(a) ${}^{m+n}C_{m-1}$ (b) ${}^{m+n}C_n$
(c) ${}^{m+n}C_{m-n}$ (d) None of these

11. If $a>1$, roots of the equation

$$(1-a)x^2+3ax-1=0 \text{ are}$$

(a) one positive and one negative
(b) both negative
(c) both positive
(d) both non-real complex

12. The number of values of the triplet (a, b, c) for which $a\cos 2x+b\sin^2 x+c=0$ is satisfied by all real x, is

(a) 0 (b) 2
(c) 3 (d) infinite

13. If α, β, γ are such that $\alpha+\beta+\gamma=2$, $\alpha^2+\beta^2+\gamma^2=6$, $\alpha^3+\beta^3+\gamma^3=8$, then $\alpha^4+\beta^4+\gamma^4$ is

(a) 5 (b) 18
(c) 12 (d) 36

14. If a, b, c are three distinct positive real numbers of real roots of $ax^2+2b|x|-c=0$ is

(a) 4 (b) 2
(c) 0 (d) None of these

15. If A is a skew-symmetric matrix, then trace of A is

(a) 1 (b) –1
(c) 0 (d) None of these

16. If the matrix $A=\begin{bmatrix} y+a & b & c \\ a & y+b & c \\ a & b & y+c \end{bmatrix}$ has rank 3, then

(a) $y \neq (a+b+c)$
(b) $y \neq 1$
(c) $y=0$
(d) $y \neq -(a+b+c)$ and $y \neq 0$

17. For positive numbers x, y, z the numerical value of the determinant $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ is

(a) 0 (b) 1
(c) 2 (d) None of these

18. If A is an orthogonal matrix, then

(a) det A = not exist (b) det A = 0
(c) det A = ± 1 (d) None of these

19. Let a, b, c be real numbers with $a \neq 0$ and let α, β be the roots of the equation $ax^2+bx+c=0$, then $a^3x^2+abcx+c^3=0$ has roots

(a) $\alpha^2\beta, \beta^2\alpha$ (b) α, β^2
(c) $\alpha^2\beta, \beta\alpha$ (d) $\alpha^3\beta, \beta^3\alpha$

20. If n_1, n_2 are positive integers, then $(1+i)^{n_1}+(1+i^3)^{n_1}+(1+i^5)^{n_2}+(1+i^7)^{n_2}$ is a real number if and only if

(a) $n_1=n_2+1$
(b) $n_1=n_2$
(c) n_1, n_2 are any two negative integers
(d) n_1, n_2 are both any positive integers

21. The equation $|z+i|-|z-i|=k$ represent a hyperbola, if

(a) $-2<k<2$ (b) $k>2$
(c) $0<k<2$ (d) None of these

22. The period of the function $f(x)=|\sin 4x|+|\cos 4x|$ is

(a) $\pi/2$ (b) $\pi/8$
(c) $\pi/4$ (d) None of these

23. If $\cos x=\frac{2\cos y-1}{2-\cos y}$, where $x, y \in (0, \pi)$, then $\tan\frac{x}{2}\cdot\cot\frac{y}{2}$ is equal to

(a) $\sqrt{2}$ (b) $\sqrt{3}$
(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$

24. The value of $\cos\frac{2\pi}{15}\cdot\cos\frac{4\pi}{15}\cdot\cos\frac{8\pi}{15}\cdot\cos\frac{16\pi}{15}$ is equal to

(a) $\frac{1}{16}$ (b) $\frac{1}{32}$

(c) $\frac{1}{64}$ (d) $\frac{1}{8}$

25. The sum of all the solutions of the equation $\cos x\cdot\cos\left(\frac{\pi}{3}+x\right)\cdot\cos\left(\frac{\pi}{3}-x\right)=\frac{1}{4}$, $x\in[0, 6\pi]$ is

(a) 15π (b) 30π

(c) $\frac{110\pi}{3}$ (d) None of these

26. $2\tan^{-1}(\operatorname{cosec}\tan^{-1}x-\tan\cot^{-1}x)$ is equal to

(a) $\cot^{-1}x$ (b) $\cot^{-1}\frac{1}{x}$

(c) $\tan^{-1}x$ (d) None of these

27. The value of

$$\sin^{-1}\left[\cos\left(\sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}}+\cos^{-1}\frac{\sqrt{12}}{4}+\sec^{-1}\sqrt{2}\right)\right]$$ is

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

28. In a Δ ABC, $\angle B=90°$, then $\tan^2\left(\frac{A}{2}\right)$ is

(a) $\frac{b-c}{b+c}$ (b) $\frac{b+c}{b-c}$

(c) $\frac{b-2c}{b+c}$ (d) None of these

29. In a triangle, if $r_1>r_2>r_3$, then

(a) $a>b>c$ (b) $a<b<c$

(c) $a>b$ and $b<c$ (d) $a<b$ and $b>c$

30. A and B are two points on one bank of a straight river and C, D are two other points on the other bank of river, if direction from A to B is same as that from *C* to *D* and $AB=a$, $\angle CAD=\alpha$, $\angle DAB=\beta$, $\angle CBA=\gamma$, then CD is equal to

(a) $\frac{a\sin\beta\cdot\sin\gamma}{\sin\alpha\cdot\sin(\alpha+\beta+\gamma)}$

(b) $\frac{a\sin\alpha\cdot\sin\gamma}{\sin\beta\cdot\sin(\alpha+\beta+\gamma)}$

(c) $\frac{a\sin\alpha\cdot\sin\beta}{\sin\gamma\cdot\sin(\alpha+\beta+\gamma)}$

(d) None of the above

31. In a trapezoid of the vector $\vec{\mathbf{BC}}=\lambda\,\vec{\mathbf{AD}}$. We will, then find that $\vec{\mathbf{P}}=\vec{\mathbf{AC}}+\vec{\mathbf{BD}}$ is collinear with $\vec{\mathbf{AD}}$. If $\vec{\mathbf{P}}=\mu\,\vec{\mathbf{AD}}$, then

(a) $\mu=\lambda+1$ (b) $\lambda=\mu+1$

(c) $\lambda+\mu=1$ (d) $\mu=2+\lambda$

32. If $P(\vec{\mathbf{p}})$, $Q(\vec{\mathbf{q}})$, $R(\vec{\mathbf{r}})$ and $S(\vec{\mathbf{s}})$ be four points such that $3\vec{\mathbf{p}}+8\vec{\mathbf{q}}=6\vec{\mathbf{r}}+5\vec{\mathbf{s}}$, then the lines PQ and *RS* are

(s) skew (b) intersecting

(c) parallel (d) None of these

33. Let $\vec{\mathbf{a}}=2\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}$ and $\vec{\mathbf{b}}=\hat{\mathbf{i}}+\hat{\mathbf{j}}$, if $\vec{\mathbf{c}}$ is a vector such that $\vec{\mathbf{a}}\cdot\vec{\mathbf{c}}=|\vec{\mathbf{c}}|, |\vec{\mathbf{c}}-\vec{\mathbf{a}}|=2\sqrt{2}$ and the angle between $\vec{\mathbf{a}}\times\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ is 30°, then $|(\vec{\mathbf{a}}\times\vec{\mathbf{b}})\times\vec{\mathbf{c}}|$ is equal to

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$

(c) 2 (d) 3

34. Consider a tetrahedron with faces F_1, F_2, F_3, F_4. Let $\vec{\mathbf{V}}_1, \vec{\mathbf{V}}_2, \vec{\mathbf{V}}_3, \vec{\mathbf{V}}_4$ be the vectors whose magnitudes are respectively equal to areas of F_1, F_2, F_3, F_4 and whose directions are perpendicular to these faces in outward direction, then $|\vec{\mathbf{V}}_1+\vec{\mathbf{V}}_2+\vec{\mathbf{V}}_3+\vec{\mathbf{V}}_4|$ equals

(a) 1 (b) 4

(c) $\vec{0}$ (d) None of these

35. If *V* is the volume of the parallelopiped having three coterminus edges as $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$, then the volume of the parallelopiped having three coterminus edge as

$$\vec{\alpha}=(\vec{\mathbf{a}}\cdot\vec{\mathbf{a}})\vec{\mathbf{a}}+(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}})\vec{\mathbf{b}}+(\vec{\mathbf{a}}\cdot\vec{\mathbf{c}})\vec{\mathbf{c}}$$

$$\vec{\beta}=(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}})\vec{\mathbf{a}}+(\vec{\mathbf{b}}\cdot\vec{\mathbf{b}})\vec{\mathbf{b}}+(\vec{\mathbf{b}}\cdot\vec{\mathbf{c}})\vec{\mathbf{c}}$$

$$\vec{\gamma}=(\vec{\mathbf{a}}\cdot\vec{\mathbf{c}})\vec{\mathbf{a}}+(\vec{\mathbf{b}}\cdot\vec{\mathbf{c}})\vec{\mathbf{b}}+(\vec{\mathbf{c}}\cdot\vec{\mathbf{c}})\vec{\mathbf{c}}$$ is

(a) V^3 (b) 3V

(c) V^2 (d) 2V

36. Let us define the length of a vector $a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$ as $|a| + |b| + |c|$. This definition coincides with the usual definition of length of a vector $a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$, iff

(a) $a = b = c = 0$
(b) any two of a, b and *c* are zero
(c) any one of a, b and *c* is zero
(d) $a + b + c = 0$

37. Let $\vec{\mathbf{u}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$, $\vec{\mathbf{v}} = \hat{\mathbf{i}} - \hat{\mathbf{j}}$ and $\vec{\mathbf{w}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$, if $\hat{\mathbf{n}}$ is a unit vector such that $\vec{\mathbf{u}} \cdot \hat{\mathbf{n}} = 0$ and $\vec{\mathbf{v}} \cdot \hat{\mathbf{n}} = 0$, then $|\vec{\mathbf{w}} \cdot \hat{\mathbf{n}}|$ is equal to

(a) 3 (b) 0
(c) 1 (d) 2

38. For two events *A* and *B*, it is given that $P(A) = P\left(\frac{A}{B}\right) = \frac{1}{4}$ and $P\left(\frac{B}{A}\right) = \frac{1}{2}$. Then

(a) *A* and *B* are mutually exclusive events
(b) *A* and *B* are dependent events
(c) $P\left(\frac{A}{B}\right) = \frac{3}{4}$
(d) None of the above

39. The least number of times a fair coin must be tossed so that the probability of getting atleast one head is atleast 0.8 is

(a) 7 (b) 6
(c) 5 (d) 3

40. If x follows a binomial distribution with parameters $n = 100$ and $p = \frac{1}{3}$, then $p(X = r)$ is maximum when r equals

(a) 16 (b) 32
(c) 33 (d) None of these

41. If $\bar{x}_1$ and $\bar{x}_2$ are the means of two distributions such that $\bar{x}_1 < \bar{x}_2$ and $\bar{x}$ is the mean of the combined distribution, then

(a) $\bar{x} < \bar{x}_1$ (b) $\bar{x} > \bar{x}_2$
(c) $\bar{x} = \frac{\bar{x}_1 + \bar{x}_2}{2}$ (d) $\bar{x}_1 < \bar{x} < \bar{x}_2$

42. If the axes be turned through an angle $\tan^{-1} 2$. What does the equation $4xy - 3x^2 = a^2$ become?

(a) $X^2 - 4Y^2 = a^2$ (b) $X^2 + 4Y^2 = a^2$
(c) $X^2 + 4Y^2 = -a^2$ (d) None of these

43. If t_1, t_2 and t_3 are distinct, the points $(t_1, 2at_1 + at_1^3)$, $(t_2, 2at_2 + at_2^3)$, $(t_3, 2at_3 + at_3^3)$ are collinear, if

(a) $t_1 t_2 t_3 = 1$
(b) $t_1 + t_2 + t_3 = t_1 t_2 t_3$
(c) $t_1 + t_2 + t_3 = 0$
(d) $t_1 + t_2 + t_3 = -1$

44. The distance of the point (2, 3) from the line $2x - 3y + 9 = 0$ measured along a line $x - y + 1 = 0$ is

(a) $4\sqrt{2}$ (b) $2\sqrt{2}$
(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

45. The equation of the straight line which passes through the intersection of the lines $x - y - 1 = 0$ and $2x - 3y + 1 = 0$ and is parallel to *x*-axis, is

(a) $y = 3$ (b) $y = -3$
(c) $x + y = 3$ (d) None of these

46. If the straight line $a_1x + b_1y + c_1 = 0$, $a_1x + b_1y + c_2 = 0$, $a_2x + b_2y + d_1 = 0$ and $a_2x + b_2y + d_2 = 0$ are the sides of rhombus, then

(a) $(a_2^2 + b_2^2)(c_1 - c_2)^2 = (a_1^2 + b_1^2)(d_1 - d_2)^2$
(b) $(a_1^2 + b_1^2)|d_1 - d_2| = (a_2^2 + b_2^2)|c_1 - c_2|$
(c) $(a_2^2 + b_2^2)(d_1 - d_2)^2 = (a_1^2 + b_1^2)(c_1 - c_2)^2$
(d) $(a_1^2 + b_1^2)|c_1 - c_2| = (a_2^2 + b_2^2)|d_1 - d_2|$

47. The equation $3x^2 + 7xy + 2y^2 + 5x + 5y + 2 = 0$ represents

(a) a pair of straight lines
(b) a circle
(c) an ellipse
(d) a hyperbola

48. To the lines $ax^2 + 2hxy + by^2 = 0$ the lines $a^2x^2 + 2h(a + b)xy + b^2y^2 = 0$, are

(a) equally inclined
(b) perpendicular
(c) bisector of the angle
(d) None of the above

49. Consider four circles $(x \pm 1)^2 + (y \pm 1)^2 = 1$, then the equation of smaller circle touching these four circle is

(a) $x^2 + y^2 = 3 - \sqrt{2}$
(b) $x^2 + y^2 = 6 - 3\sqrt{2}$
(c) $x^2 + y^2 = 5 - 2\sqrt{2}$
(d) $x^2 + y^2 = 3 - 2\sqrt{2}$

50. The locus of the point of intersection of tangents to the circle $x = a\cos\theta$, $y = a\sin\theta$ at the points, whose parametric angles differe by $\frac{\pi}{2}$, is
(a) a straight line
(b) a circle
(c) a pair of straight line
(d) None of the above

51. The equation of the circle passing through (1, 0) and (0, 1) and having the smallest possible radius is
(a) $x^2 - y^2 - x - y = 0$
(b) $x^2 + y^2 - x - y = 0$
(c) $x^2 + y^2 + x + y = 0$
(d) $x^2 + y^2 - 2x - 2y = 0$

52. The length of the common chord of the two circles $(x-a)^2 + (y-b)^2 = c^2$ and $(x-b)^2 + (y-a)^2 = c^2$ is
(a) $\sqrt{4c^2 + 2(a-b)^2}$ (b) $\sqrt{4c^2 - (a-b)^2}$
(c) $\sqrt{4c^2 - 2(a-b)^2}$ (d) $\sqrt{2c^2 - 2(a-b)^2}$

53. If $\left(m_i, \frac{1}{m_i}\right)$ are four distinct points on a circle, then
(a) $m_1m_2m_3m_4 = 1$
(b) $m_1m_2m_3m_4 = -1$
(c) $m_1m_2m_3m_4 = \frac{1}{2}$
(d) $\frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{m_3} + \frac{1}{m_4} = \frac{1}{4}$

54. If the normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$ cuts the parabola again at $(aT^2, 2aT)$, then
(a) $-2 \le T \le 2$
(b) $T \in (-\infty, -8) \cup (8, \infty)$
(c) $T^2 < 8$
(d) $T^2 \ge 8$

55. The number of real tangents that can be drawn to the ellipse $3x^2 + 5y^2 = 32$ passing through (3, 5) is
(a) 0 (b) 1
(c) 2 (d) infinite

56. If the equation $lx^2 + 2mxy + ny^2 = 0$ represents a pair conjugate diameter of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then
(a) $la^2 + nb^2 = 0$ (b) $la^2 = nb^2$
(c) $2la^2 = nb^2$ (d) None of these

57. If e is the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and θ is the angle between the asymptotes, then $\cos\left(\frac{\theta}{2}\right)$ is equal to
(a) $\frac{1}{e}$ (b) $\frac{-1}{e}$
(c) e (d) $\frac{2}{e}$

58. The tangent and normal to a rectangular hyperbola $xy = c^2$ at a point cuts off intercepts a_1 and a_2 on one axis and b_1, b_2 on the other, then $a_1a_2 + b_1b_2$ is equal to
(a) 1 (b) 2
(c) 3 (d) 0

59. The direction cosines of any normal to the xy-plane are
(a) 1, 0, 0 (b) 0, 1, 0
(c) 1, 1, 0 (d) 0, 0, 1

60. The locus of the equation $xy + yz = 0$ is
(a) a pair of straight lines
(b) a pair of parallel lines
(c) a pair of perpendicular planes
(d) None of the above

61. The reflection of the point (2, – 1, 3) in the plane $3x - 2y - z = 9$ is
(a) $\left(\frac{26}{7}, \frac{15}{7}, \frac{17}{7}\right)$ (b) $\left(\frac{26}{7}, \frac{-15}{7}, \frac{17}{7}\right)$
(c) $\left(\frac{15}{7}, \frac{26}{7}, \frac{-17}{7}\right)$ (d) $\left(\frac{26}{7}, \frac{17}{7}, \frac{-15}{7}\right)$

62. $\lim_{x \to 0} \frac{\sqrt{1 - \cos 2x}}{\sqrt{2} \cdot x}$ is
(a) 1 (b) −1
(c) zero (d) does not exist

63. If $f(x) = \begin{cases} ax^2 + b, & 0 \le x < 1 \\ x + 3, & 1 < x \le 2 \\ 4, & x = 1 \end{cases}$, then the value of (a, b) for which $f(x)$ cannot be continuous at $x = 1$ is
(a) (2, 2) (b) (3, 1)
(c) (4, 0) (d) (5, 2)

64. If $f(x) = \log_x(\log x)$, then $f'(x)$ at $x = e$ is
(a) $1/e$ (b) e
(c) $-1/e$ (d) 0

65. Radius of the circle

$$\vec{r}^2 + \vec{r} \cdot (2\hat{i} - 2\hat{j} - 4\hat{k}) - 19 = 0,$$

$$\vec{r} \cdot (\hat{i} - 2\hat{j} + 2\hat{k}) + 8 = 0, \text{ is}$$

(a) 5 (b) 4
(c) 3 (d) 2

66. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{\log x}{(1+\log x)^2}$ (b) $\frac{x-y}{(1+\log x)}$
(c) $\frac{x-y}{(1+\log x)^2}$ (d) $\frac{1}{(1+\log x)}$

67. If $y = \sin(m \sin^{-1} x)$, then $(1-x^2)y'' - xy'$ is equal to

(a) $m^2 y$ (b) my
(c) $-m^2 y$ (d) None of these

68. If $f(x) = (ax+b)\sin x + (cx+d)\cos x$, then the values of a, b, c and d such that $f'(x) = x\cos x$ for all x, are

(a) $b = c = 0, a = d = 1$
(b) $b = d = 0, a = c = 1$
(c) $c = d = 0, a = b = 1$
(d) None of the above

69. If a particle is moving such that the velocity acquired is proportional to the square root of the distance covered, then its acceleration is

(a) a constant (b) $\propto s^2$
(c) $\propto \frac{1}{s^2}$ (d) $\propto \frac{1}{s}$

70. The point in the interval $[0, 2\pi]$, where $f(x) = e^x \sin x$ has maximum slope, is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) π (d) $\frac{3\pi}{2}$

71. The function $f(x) = \frac{ax+b}{(x-1)(x-4)}$ has a local maxima at $(2, -1)$, then

(a) $b = 1, a = 0$ (b) $a = 1, b = 0$
(c) $b = -1, a = 0$ (d) $a = -1, b = 0$

72. If $z = \tan(y+ax) + \sqrt{y-ax}$, then $z_{xx} - a^2 z_{yy}$ is equal to

(a) 0 (b) 2
(c) $z_x + z_y$ (d) $z_x z_y$

73. If $\int f(x)dx = F(x)$, then $\int x^3 f(x^2)dx$ is equal to

(a) $\frac{1}{2}[x^2\{F(x)\}^2 - \int \{F(x)\}^2 dx]$
(b) $\frac{1}{2}[x^2 F(x^2) - \int F(x^2)d(x^2)]$
(c) $\frac{1}{2}[x^2 F(x) - \frac{1}{2}\int \{F(x)\}^2 dx]$
(d) None of the above

74. $\int \frac{x^2+4}{x^4+16} dx$ is equal to

(a) $\frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{x^2+4}{2x}\right) + c$
(b) $\frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{x^2-4}{2\sqrt{2}x}\right) + c$
(c) $\frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{x^2+4}{2\sqrt{2}x}\right) + c$
(d) $\frac{1}{2} \tan^{-1}\left(\frac{x^2-4}{2x}\right) + c$

75. Evaluate $\int \frac{1}{(x+1)\sqrt{x^2-1}} dx$

(a) $\sqrt{\frac{x+1}{x-1}} + c$ (b) $\sqrt{\frac{x-1}{x+1}} + c$
(c) $\sqrt{\frac{1}{x+1}} + c$ (d) None of these

76. The value of the integral $\int_{\pi/2}^{3\pi/2} [\sin x]\, dx$, where $[\cdot]$ denotes the greatest integer function, is

(a) $\frac{\pi}{2}$ (b) $-\frac{\pi}{2}$
(c) 0 (d) π

77. The area of the loop the curve $ay^2 = x^2(a-x)$ is

(a) $\frac{8a^2}{15}$ sq unit (b) $\frac{4a^2}{15}$ sq unit
(c) $\frac{2a^2}{15}$ sq unit (d) None of these

78. Solution of the differential equation

$$x = 1 + xy\frac{dy}{dx} + \frac{(xy)^2}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{(xy)^3}{3!}\left(\frac{dy}{dx}\right)^3 + \ldots \text{ is}$$

(a) $y = \log_e(x) + c$
(b) $y = (\log_e x)^2 + c$
(c) $y = \pm\sqrt{(\log_e x)^2 + 2c}$
(d) $xy = x^y + k$

79. If the solution of the differential equation $\frac{dy}{dx} = \frac{ax + 3}{2y + f}$ represents a circle, then the value of *a* is

(a) 2 (b) −2
(c) 3 (d) −4

80. The approximate value of $\int_1^5 x^2 dx$ using trapezoidal rule with n = 4 is

(a) 41 (b) 41.5
(c) 41.75 (d) 42

General English and Aptitude

Directions (1-4) *In each of the following questions, choose the alternative which best expresses the meaning of the word given in capital letters.*

1. MOSQUERADE
(a) to provide support
(b) to go in disguise
(c) to mesmerise
(d) marathon race

2. PREPOSTEROUS
(a) careful (b) casual
(c) absurd (d) deterrent

3. SOLITUDE
(a) musical composition
(b) aloneness
(c) true statement
(d) single mindedness

4. PROPITIOUS
(a) favourable (b) clean
(c) nearby (d) patriotic

Directions (5-8) *In each of the following questions, choose the alternative which is opposite in meaning to the word given in capital letters.*

5. RELENTLESS
(a) merciless (b) yielding
(c) monotonous (d) incisive

6. FORBEARANCE
(a) patience (b) self-control
(c) intolerance (d) preference

7. PALTRY
(a) absolete (b) cautious
(c) random (d) plentiful

8. EXODUS
(a) influx (b) return
(c) home coming (d) restoration

Directions (9-12) *In each of the following questions, choose the most suitable alternative to fill in the blank.*

9. The........argument put forth for not disclosing the facts did not impress anybody.
(a) intemperate (b) spurious
(c) specious (d) convincing

10. Director, he said, would.......the matter at once.
(a) invigilate (b) explore
(c) investigate (d) survey

11. Everyone was.........by surprise when she announced her plan to marry that boy.
(a) moved (b) shaken
(c) taken (d) prevailed

12. Your case is so unique that I am not getting any........to support it.
(a) reason (b) help
(c) happening (d) precedent

Directions (13-16) *In each of the following questions, choose the alternative which can be substituted for the given sentence.*

13. The writing or compiling of dictionaries
(a) Lexicography (b) Numismatics
(c) Cytology (d) Demography

14. The study of Insects
(a) Chromatics (b) Dactylology
(c) Calligraphy (d) Entomology

15. Murder of one's brother
(a) Regiside (b) Foeticide
(c) Fratricide (d) Uxoricide

16. Government by one person
(a) Autonomy (b) Autocracy
(c) Plutocracy (d) Theocracy

Directions (17-20) *In each of the following questions, choose the alternative which can best improve the given sentence by substituting the italicised portion. If the sentence is correct as it is, your answer is (d).*

17. She cut a *sad figure* in her first performance on the stage.
(a) made a sorry figure
(b) cut a sorry face
(c) cut a sorry figure
(d) no improvement

18. *No sooner I saw* the tiger, than I ran away.
(a) as soon as I saw
(b) no sooner I had seen
(c) no sooner did I see
(d) no improvement

19. If he *had time* he will call you.
(a) would have (b) would have had
(c) has (d) no improvement

20. *All his answers* were correct.
(a) his all answers (b) his every answers
(c) all of his answers (d) no improvement

Directions (21-25) *In the following questions, choose the group of words that shows the same relationship as given at the top of every question.*

21. Jam : Jelly : Pickles
(a) Butter : Marmalade : Grapes
(b) Granite : Basalt : Coke
(c) Cow : Dry : Draft
(d) Bleat : Bray : Grunt

22. Mountain : Height : Climber
(a) River : Length : Water
(b) Land : Farmer : Crop
(c) College : Building : Student
(d) Sea : Depth : Diver

23. Head : Brain : Think
(a) Eyes : Lashes : See
(b) Skin : Sweat : Touch
(c) Feet : Dance : Toe
(d) Mouth : Teeth : Chew

24. Jute : Cotton : Wool
(a) Potato : Carrot : Bean
(b) Canada : Chile : Asia
(c) Liver : Heat : Blood
(d) Shark : Cod : Eel

25. Sial : Sima : Mantle
(a) Core : Asteroid : Comet
(b) Pneumonia : Tetanus : Hepatitis
(c) Calcite : Magnesium : Zinc
(d) Wrestling : Karate : Boxing

Directions (26-30) *In the following questions, choose the option which shows common feature in the relationship given in each question.*

26. Rourkela : Bokaro : Durgapur
(a) They have steel plants
(b) They have coal mines
(c) They have atomic power plants
(d) They have the best technical colleges

27. Chlorine : Fluorine : Iodine
(a) These are names of inert gases
(b) These are gases at room temperature
(c) These are transition elements
(d) These are halogens

28. Petrol : Phosphorus : Cooking gas
(a) They are fuels
(b) They are highly inflamable
(c) They can be sold without permit
(d) India has to import them

29. Green : Violet : Orange
(a) They are primary colours
(b) These colours occur together in a rainbow
(c) They are made by mixing other colours
(d) These colours are not found in butterflies

30. Supernova : Protostar : Red Giant
(a) These are kinds of stars
(b) These are members of galaxies
(c) These are stages in the life of a star
(d) These move about the Sun

31. The classical dance, Kathakali belongs to
(a) Tamil Nadu (b) Karnataka
(c) Orissa (d) Kerala

32. The highest civilian award in India is
(a) Paramveer Chakra
(b) Bharat Ratna
(c) Bhatnagar Award
(d) Kalinga Award

33. Who is called 'Nightingale of India'?
(a) Sarojini Naidu (b) Lata Mangeshkar
(c) Amir Khushro (d) Asha Bhosale

34. Nehru Trophy is related with
(a) Cricket (b) Volleyball
(c) Hockey (d) Badminton

35. The headquarters of NATO is situated at
(a) Brussels (b) Geneva
(c) Newyork (d) Paris

36. The first Indian woman to swim across the English channel is
(a) Bachendri Pal (b) Santosh Yadav
(c) Nirja Mishra (d) Aarti Saha

37. Which of the following river is related with Bhakra-Nagal Project?
(a) Ravi (b) Satluj
(c) Gandak (d) Mahanadi

38. Namdhapha National Park is located in
(a) Rajasthan
(b) Orissa
(c) Arunachal Pradesh
(d) UP

39. Which of the following is known as 'evening star'?
(a) Venus (b) Mercury
(c) Saturn (d) Jupiter

40. Which of the following was the founder of Indian National Congress?
(a) Jawahar Lal Nehru
(b) Mahatma Gandhi
(c) Moti Lal Nehru
(d) A. O. Hume

Answers

Physics

1.	(b)	**2.**	(c)	**3.**	(c)	**4.**	(c)	**5.**	(d)	**6.**	(a)	**7.**	(b)	**8.**	(a)	**9.**	(a)	**10.**	(b)
11.	(b)	**12.**	(c)	**13.**	(a)	**14.**	(b)	**15.**	(a)	**16.**	(b)	**17.**	(b)	**18.**	(a)	**19.**	(b)	**20.**	(b)
21.	(a)	**22.**	(b)	**23.**	(b)	**24.**	(b)	**25.**	(a)	**26.**	(c)	**27.**	(d)	**28.**	(b)	**29.**	(a)	**30.**	(d)
31.	(a)	**32.**	(b)	**33.**	(b)	**34.**	(c)	**35.**	(b)	**36.**	(c)	**37.**	(a)	**38.**	(d)	**39.**	(b)	**40.**	(c)
41.	(a)	**42.**	(b)	**43.**	(b)	**44.**	(a)	**45.**	(b)	**46.**	(a)	**47.**	(b)	**48.**	(b)	**49.**	(d)	**50.**	(a)
51.	(c)	**52.**	(d)	**53.**	(b)	**54.**	(c)	**55.**	(b)	**56.**	(a)	**57.**	(b)	**58.**	(c)	**59.**	(b)	**60.**	(c)

Chemistry

1.	(a)	**2.**	(d)	**3.**	(b)	**4.**	(d)	**5.**	(c)	**6.**	(d)	**7.**	(c)	**8.**	(d)	**9.**	(b)	**10.**	(d)
11.	(c)	**12.**	(a)	**13.**	(b)	**14.**	(b)	**15.**	(d)	**16.**	(d)	**17.**	(c)	**18.**	(c)	**19.**	(d)	**20.**	(a)
21.	(b)	**22.**	(b)	**23.**	(d)	**24.**	(a)	**25.**	(d)	**26.**	(a)	**27.**	(c)	**28.**	(b)	**29.**	(d)	**30.**	(c)
31.	(a)	**32.**	(d)	**33.**	(d)	**34.**	(a)	**35.**	(b)	**36.**	(c)	**37.**	(b)	**38.**	(a)	**39.**	(d)	**40.**	(a)
41.	(a)	**42.**	(a)	**43.**	(b)	**44.**	(b)	**45.**	(b)	**46.**	(b)	**47.**	(a)	**48.**	(d)	**49.**	(c)	**50.**	(a)
51.	(b)	**52.**	(a)	**53.**	(d)	**54.**	(a)	**55.**	(d)	**56.**	(b)	**57.**	(d)	**58.**	(d)	**59.**	(d)	**60.**	(b)

Mathematics

1.	(b)	**2.**	(b)	**3.**	(c)	**4.**	(c)	**5.**	(a)	**6.**	(b)	**7.**	(d)	**8.**	(a)	**9.**	(d)	**10.**	(b)
11.	(c)	**12.**	(d)	**13.**	(b)	**14.**	(b)	**15.**	(c)	**16.**	(d)	**17.**	(a)	**18.**	(c)	**19.**	(a)	**20.**	(d)
21.	(a)	**22.**	(b)	**23.**	(b)	**24.**	(a)	**25.**	(b)	**26.**	(c)	**27.**	(a)	**28.**	(a)	**29.**	(a)	**30.**	(b)
31.	(a)	**32.**	(b)	**33.**	(b)	**34.**	(c)	**35.**	(a)	**36.**	(b)	**37.**	(a)	**38.**	(c)	**39.**	(d)	**40.**	(c)
41.	(d)	**42.**	(a)	**43.**	(c)	**44.**	(a)	**45.**	(a)	**46.**	(a)	**47.**	(a)	**48.**	(a)	**49.**	(d)	**50.**	(b)
51.	(b)	**52.**	(c)	**53.**	(a)	**54.**	(d)	**55.**	(c)	**56.**	(b)	**57.**	(a)	**58.**	(d)	**59.**	(d)	**60.**	(c)
61.	(b)	**62.**	(d)	**63.**	(d)	**64.**	(a)	**65.**	(b)	**66.**	(a)	**67.**	(c)	**68.**	(a)	**69.**	(a)	**70.**	(b)
71.	(b)	**72.**	(a)	**73.**	(b)	**74.**	(b)	**75.**	(b)	**76.**	(b)	**77.**	(a)	**78.**	(c)	**79.**	(b)	**80.**	(d)

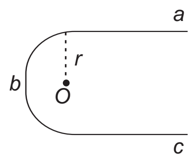
General English and Aptitude

1.	(b)	**2.**	(c)	**3.**	(b)	**4.**	(a)	**5.**	(b)	**6.**	(c)	**7.**	(d)	**8.**	(c)	**9.**	(c)	**10.**	(b)
11.	(d)	**12.**	(c)	**13.**	(a)	**14.**	(d)	**15.**	(c)	**16.**	(b)	**17.**	(c)	**18.**	(c)	**19.**	(c)	**20.**	(d)
21.	(d)	**22.**	(d)	**23.**	(d)	**24.**	(d)	**25.**	(d)	**26.**	(d)	**27.**	(d)	**28.**	(b)	**29.**	(b)	**30.**	(c)
31.	(d)	**32.**	(b)	**33.**	(a)	**34.**	(c)	**35.**	(a)	**36.**	(d)	**37.**	(b)	**38.**	(c)	**39.**	(a)	**40.**	(d)

Hints & Solutions

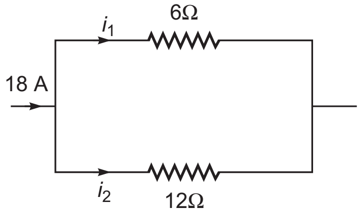
Physics

1. Field due to a straight wire of infinite length is $\frac{\mu_0 i}{4\pi r}$ if the point is on a line perpendicular to its length while at the centre of a semicircular coil is $\frac{\mu_0 \pi i}{4\pi r}$.



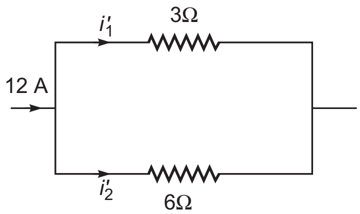
$$\therefore \quad B = B_a + B_b + B_c$$
$$= \frac{\mu_0}{4\pi}\frac{i}{r} + \frac{\mu_0}{4\pi}\frac{\pi i}{r} + \frac{\mu_0}{4\pi}\frac{i}{r}$$
$$= \frac{\mu_0}{4\pi}\frac{i}{r}(\pi + 2) \text{ out of the page}$$

2. As $3\,\Omega$ and $6\,\Omega$ resistances are in parallel their equivalent resistance will be $2\,\Omega$. Here $2\,\Omega$ and $4\,\Omega$ are in series, their equivalent resistance will be $6\,\Omega$.



From current distribution law

$$i_1 = \frac{12 \times 18}{18} = 12\text{ A}$$
$$i_2 = \frac{6 \times 18}{18} = 6\text{ A}$$



Now 12 A current is entering in parallel combination of $3\,\Omega$ and $6\,\Omega$ again from current distribution law

$$i_1' = \frac{6 \times 12}{9} = 8\text{ A}$$
$$i_2' = \frac{3 \times 12}{9} = 4\text{ A}$$

$\therefore$ Potential difference across $3\,\Omega$ resistance

$$= 8 \times 3 = 24\text{ V}$$

3. Let the temperature of junction be θ. In equilibrium, rate of flow of heat through rod 1 is equal to sum of rate of flow of heat through rods 2 and 3, *ie,*

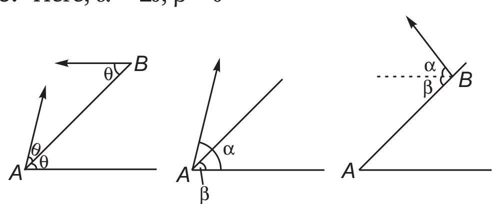
$$\left(\frac{dQ}{dt}\right)_1 = \left(\frac{dQ}{dt}\right)_2 + \left(\frac{dQ}{dt}\right)_3$$
$$\therefore \quad \frac{KA(\theta - 20)}{l} = \frac{KA(60 - \theta)}{l} + \frac{KA(70 - \theta)}{l}$$
$$\Rightarrow \quad \theta - 20 = 130 - 2\theta$$
$$\text{or} \quad 3\theta = 150$$
$$\therefore \quad \theta = 50°\text{C}$$

4. The surface tension of liquids decreases with increase of temperature. For small temperature difference it decreases almost linearly. The surface tension of a liquid becomes zero at a particular temperature, called the critical temperature of that liquid.

5. Current amplification factor, $\beta = \frac{i_C}{i_B}$

$$\therefore \quad i_C = i_B \times \beta$$
$$= 250 \times 80\,\mu\text{A}$$

6. Here, $\alpha = 2\theta, \beta = \theta$



Time of flight of *A* is,

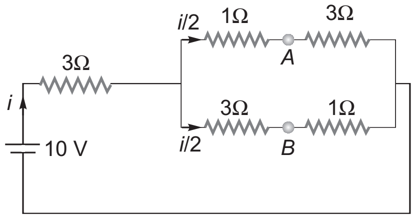
$$T_1 = \frac{2u\sin(\alpha - \beta)}{g\cos\beta}$$
$$= \frac{2u\sin(2\theta - \theta)}{g\cos\theta}$$
$$= \frac{2u}{g}\tan\theta$$

Time of flight of B is, $T_2 = \frac{2u\sin\theta}{g\cos\theta}$

$$= \frac{2u}{g}\tan\theta$$

So, $T_1 = T_2$. The acceleration of both the particles is g downwards. Therefore, relative acceleration between the two is zero or relative motion between the two is uniform. The relative velocity of A w.r.t. B is towards AB, therefore collision will take place between the two in mid air.

7. $R_{eq} = 5\Omega$, current $i = \frac{10}{5} = 2\,A$ and current in each branch = 1 A. Potential difference between C and A,



$$V_C - V_A = 1 \times 1 = 1\,V \quad \ldots(i)$$

Potential difference between C and B

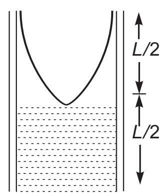
$$V_C - V_B = 1 \times 3 = 3\,V \quad \ldots(ii)$$

On solving Eqs. (i) and (ii) we get

$$V_A - V_B = 2V$$

8. Solar radiations are transverse electromagnetic waves. The central core of the sun emits a continuous electromagnetic spectrum.

9. If an open pipe is half submerged in water, it will become a closed organ pipe of length half that of open pipe as shown in figure. So, its frequency will become,



$$f_c = \frac{v}{4\left(\frac{L}{2}\right)} = \frac{v}{2L} = f_o$$

ie, equal to that of open pipe *ie,* frequency will remain unchanged.

10. Rms velocity is given by

$$v = \sqrt{\frac{3kT}{m}} \quad \text{or} \quad v^2 = \frac{3kT}{m}$$

For a gas, k and m are constants.

$$\therefore \quad \frac{v^2}{T} = \text{constant}$$

11. $v_s = 30$ m/s

Source • Observer

$v_o = 0$

Here, $n = 500$ Hz, $v_o = 0$

$v_s = 30$ m/s, $v = 330$ m/s

$\therefore$ From, $$n' = n\left(\frac{v - v_o}{v - v_s}\right)$$

$$= 500\left(\frac{330}{330 - 30}\right)$$

$$= 550 \text{ Hz}$$

12. The value of acceleration due to gravity at height h (when h is not negligible as compared to R)

$$g' = g\frac{R^2}{(R + h)^2}$$

Here, $$g' = \frac{g}{2}$$

$$\therefore \quad \frac{g}{2} = g\frac{R^2}{(R + h)^2}$$

or $$\frac{1}{2} = \frac{R^2}{(R + h)^2}$$

or $$\sqrt{\frac{1}{2}} = \frac{R}{R + h}$$

or $$R + h = \sqrt{2}R$$

$$\therefore \quad h = (\sqrt{2} - 1)R$$

13. For working safely, the activity must reduce to $\frac{1}{64}$.

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \frac{1}{64}$$

$$\therefore \quad n = 6$$

Thus, $$t = nT = 6 \times 2 = 12\,h$$

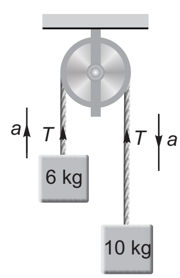
14. Potential difference across diode

$$= 3.2 - 3 = 0.2\,V$$

$\therefore$ Current through diode

$$i = \frac{0.2}{100} = 2 \times 10^{-3}\,A$$

15.



$$10g - T = 10a \quad \ldots(i)$$

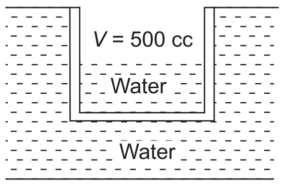
$$T - 6g = 6a \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$T = 75\,N$$

16. As the flask floats in water when less than half filled with water, it will float just fully submerged when half filled. In this situation, mass of flask + mass of water in it = $V\sigma$

ie, $390 + 250 = V \times 1$ (as $\sigma = 1$ g/cc)



ie, outer volume of flask

$$V = 640 \text{ cc}$$

Now as inner volume of flask is given to be 500 cc, so the volume of the material of flask = 640 – 500 = 140 cc. But as mass of flask is 390 g, so density of material of flask

$$\rho = \frac{m}{V} = \frac{390}{140} = 2.8 \text{ g/cc}$$

17. $A = 60°, \delta_m = 30°$

As $$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$\therefore$ $$\mu = \frac{\sin\frac{(60° + 30°)}{2}}{\sin\left(\frac{60°}{2}\right)} = \frac{\sin 45°}{\sin 30°}$$

$$= \frac{1/\sqrt{2}}{1/2} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

18. Let n_1 and n_2 be the frequencies of the two notes of wavelengths λ_1 and λ_2.

Here, $\lambda_1 = 1$ m, $\lambda_2 = 1.01$ m

If v is velocity of sound, then

$$n_1 = \frac{v}{1} \quad \text{and} \quad n_2 = \frac{v}{1.01}$$

If the number of beats produced per second is b, then

$$b = n_1 - n_2$$

$$4 = \frac{v}{1} - \frac{v}{1.01}$$

$\therefore$ $$v = \frac{4.04}{0.01} = 404 \text{ m/s}$$

19. Work done = area of trapezium

$$= \frac{1}{2} \times (8 \times 10^5 + 4 \times 10^5) \times 0.2$$

$$= 1.2 \times 10^5 \text{ J}$$

20. As $F \propto v^2$

or $$F = kv^2$$

$\therefore$ $$k = \frac{F}{v^2}$$

$$[k] = \frac{[F]}{[v^2]} = \frac{[MLT^{-2}]}{[L^2T^{-2}]}$$

$$= [ML^{-1}T^0]$$

21. If temperature of surrounding is considered, then net loss of energy of a body by radiation

$$Q = Ae\sigma(T^4 - T_0^4) \Rightarrow Q \propto (T^4 - T_0^4)$$

$\therefore$ $$\frac{Q_1}{Q_2} = \frac{T_1^4 - T_0^4}{T_2^4 - T_0^4}$$

$$= \frac{(273 + 327)^4 - (273 + 27)^4}{(273 + 427)^4 - (273 + 27)^4}$$

$$= \frac{(600)^4 - (300)^4}{(700)^4 - (300)^4} = 0.52$$

22. Resistance of 40 W bulb $= \frac{240 \times 240}{40}$

$$= 1440\ \Omega$$

Its safe current $= \frac{240}{1440} = 0.167$ A

Resistance of 60 W bulb $= \frac{240 \times 240}{60}$

$$= 960\ \Omega$$

Its safe current $= \frac{240}{960} = 0.25$ A

When connected in series to 420 V supply, then the current

$$i = \frac{420}{1440 + 960} = \frac{420}{2400}$$

$$= 0.175 \text{ A}$$

Thus, current is greater for 40 W bulb, so it will fuse.

23. Angular velocity ω is constant.

$\therefore$ $$v \propto r \quad \text{or} \quad \frac{v_A}{v_B} = \frac{r_A}{r_B}$$

24. According to equation of motion

$$v = u + at$$

$\Rightarrow$ $$-2 = 10 + a \times 4$$

$\therefore$ $$a = -3 \text{ m/s}^2$$

25. According to Brewster's law

$$\mu = \tan i_p = \tan 60° = \sqrt{3}$$

26. Young's modulus, $Y = \frac{mgl}{a_1 l_1}$

$\therefore$ $$l_1 = \frac{mgl}{Y\pi r^2} \quad \ldots(i)$$

and $$Y = \frac{mg(2l)}{a_2 l_2} = \frac{mg(2l)}{\pi(2r)^2 l_2}$$

or $$l_2 = \frac{mgl}{2Y\pi r^2} \quad \dots(ii)$$

From Eqs. (i) and (ii), we have

$$\therefore \quad l_1 + l_2 = \frac{mgl}{Y\pi r^2} + \frac{mgl}{2Y\pi r^2} = \frac{3}{2}\frac{mgl}{Y\pi r^2}$$

27. Given, x = 0.20 m, y = 0.20 m, u = 1.8 m/s.
Let the ball strikes the *n*th step of stairs,
Vertical distance travelled

$$= ny = n \times 0.20 = \frac{1}{2}gt^2$$

Horizontal distance travelled, nx = ut

or $$t = \frac{nx}{u}$$

$$\therefore \quad ny = \frac{1}{2} g \times \frac{n^2x^2}{u^2}$$

or $$n = \frac{2u^2}{g}\frac{y}{x^2} = \frac{2 \times (1.8)^2 \times 0.20}{9.8 \times (0.20)^2} = 3.3 \approx 4$$

28. $E = E_0 \cos\omega t = 10\cos(2\pi \times ft)$

$$= 10\cos\left(2\pi \times 50 \times \frac{1}{600}\right)$$

$$= 10\cos\left(\frac{\pi}{6}\right) = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}\ V$$

29. Induced emf produced in coil

$$e = \frac{-d\phi}{dt} = \frac{-d}{dt}(BA)$$

$$\therefore \quad |e| = A\frac{dB}{dt} = 0.01 \times \frac{1}{1 \times 10^{-3}}$$

$$|e| = 10\ V$$

Current produced in coil,

$$i = \frac{|e|}{R} = \frac{10}{2} = 5\,A$$

Heat evolved $= i^2Rt = (5)^2 \times (2) \times 1 \times 10^{-3}$

$$= 0.05\ J$$

30. From displacement method size of object, $O = \sqrt{I_1 I_2}$

Here, $O = 3\,cm$, $I_1 = 9\,cm$,

$$\therefore \quad 3 = \sqrt{9I_2}$$

or $$I_2 = 1\ cm$$

31. As gas is suddenly expanded so it is an adiabatic process,

ie, $$pV^\gamma = \text{constant}$$

or $$p_1V_1^\gamma = p_2V_2^\gamma$$

Given $V_2 = 3V_1$, $C_V = 2R$

$$\therefore \quad C_p = 2R + R = 3R$$

$$\Rightarrow \quad \gamma = \frac{C_p}{C_V} = \frac{3R}{2R} = 1.5$$

$$\therefore \quad \frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^\gamma = (3)^{1.5} = 5.1 \approx 5$$

32. When bigger pendulum of time period $\frac{5T}{4}$ completes one vibration, the smaller pendulum will complete $\left(\frac{5}{4}\right)$ vibrations. It means the smaller pendulum will be leaded the bigger pendulum by phase $\frac{T}{4}$ sec $= \frac{\pi}{2}$ rad = 90°.

33. Given, u = 72 km/h = 20 m/s

$a = \mu g = 0.5 \times 10\ m/s^2$

From $\quad v^2 = u^2 - 2as$

$$\therefore \quad (0)^2 = (20)^2 - 2 \times 0.5 \times 10 \times s$$

$$\therefore \quad s = \frac{20 \times 20}{2 \times 0.5 \times 10}$$

or $$s = 40\ m$$

34. Given, s = 2 m, u = 80 m/s, v = 0

From $$v^2 = u^2 - 2as$$

$$\therefore \quad (0)^2 = (80)^2 - 2 \times a \times 2$$

or $$a = \frac{80 \times 80}{4} = 1600\ m/s^2$$

35. Net induced emf due to disc is

$$\varepsilon = \frac{1}{2}B\omega l^2$$

$$= \frac{1}{2} \times 0.1 \times 2\pi \times 10 \times 0.1 \times 0.1$$

$$= \pi \times 10^{-2} V$$

36. According to first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

$$\therefore \quad \Delta U = \Delta Q - \Delta W$$

$$= 540 - \frac{p(V_2 - V_1)}{J}$$

$$= 540 - \frac{1.013 \times 10^5 \times [(1671 - 1) \times 10^{-6}]}{4.2}$$

$$= 540 - 40 = 500\ cal$$

37. Frequency of vibration is given by

$$n = \frac{p}{2l}\sqrt{\frac{T}{m}} \quad \left(\text{where } v = \sqrt{\frac{T}{m}}\right)$$

$$\therefore \quad n = \frac{pv}{2l} = \frac{5 \times 20}{2 \times 10} = 5\ Hz$$

38. $v_{max} = A\omega$

when A becomes twice v_{max} is also doubled.

39. From Kepler's third law of planetary motion

$$T^2 \propto R^3$$

$$\therefore \quad \frac{T_2^2}{T_1^2} = \frac{R_2^3}{R_1^3}$$

$$\text{or} \quad \frac{T_2^2}{(24)^2} = \left(\frac{6400 + 6400}{36000 + 6400}\right)^3$$

$$\text{or} \quad T_2^2 = (24)^2 \times \left(\frac{16}{53}\right)^3$$

$$\Rightarrow \quad T_2 = 4\,\text{h}$$

40. From the law of conservation of momentum

$$3 \times 16 + 6 \times v = 9 \times 0$$

$$\text{or} \quad v = -8 \text{ m/s}$$

$$\Rightarrow \quad v = 8 \text{ m/s} \quad \text{(numerically)}$$

Therefore, its kinetic energy

$$K = \frac{1}{2} \times 6 \times (8)^2 = 192 \text{ J}$$

41. In the reaction

$${}_7N^{14} + \alpha \longrightarrow {}_8X^{17} + {}_1p^1$$

8 is the atomic number of oxygen molecule.

So, here X is oxygen (O_2) molecule.

42. Induced emf in the coil is given by

$$e = \frac{Ldi}{dt}$$

$$\text{or} \quad e = 10 \times \frac{(10-5)}{0.2}$$

$$\Rightarrow \quad e = 250 \text{ V}$$

43. New force $F = \dfrac{1}{4\pi\varepsilon_0} \dfrac{(6-2)(2-2)}{r^2} \times 10^{-12}$

$$= 0$$

44. Given, $N_P = 20$, $N_S = 10$, $e_P = 220$ V

$\therefore$ Transformation ratio, k

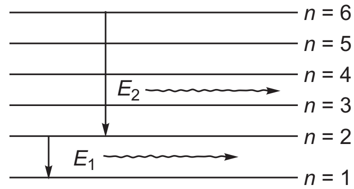
$$\frac{e_S}{e_P} = \frac{N_S}{N_P}$$

$$\text{or} \quad e_S = \frac{N_S}{N_P} \times e_P$$

$$= \frac{10}{20} \times 220 = 110 \text{ V}$$

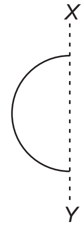
45. As $\quad E_1 > E_2$

$\therefore \quad \nu_1 > \nu_2$



ie, photons of higher frequency will be emitted if transition takes place from n = 2 to n = 1.

46. Here,



$$L = \pi R \quad \text{or} \quad R = \frac{L}{\pi}$$

$\therefore$ Moment of inertia about XY

$$= \frac{1}{2}\left(\frac{1}{2}MR^2\right)$$

$$= \frac{1}{4}\frac{ML^2}{\pi^2}$$

47. Let a be the acceleration of each block. Then,

$$T_3 = (m_1 + m_2 + m_3)a \quad \ldots\text{(i)}$$

$$\text{and} \quad T_2 = (m_1 + m_2)a \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$T_2 = \left(\frac{m_1 + m_2}{m_1 + m_2 + m_3}\right) \times T_3$$

$$= \left(\frac{10 + 6}{10 + 6 + 4}\right) \times 40 = 32 \text{ N}$$

48. $H = \dfrac{u^2 \sin^2\theta}{2g}$ and $T = \dfrac{2u\sin\theta}{g}$

$$\Rightarrow \quad T^2 = \frac{4u^2\sin^2\theta}{g^2}$$

$$\therefore \quad \frac{T^2}{H} = \frac{8}{g}$$

$$\Rightarrow \quad T = \sqrt{\frac{8H}{g}} = 2\sqrt{\frac{2H}{g}}$$

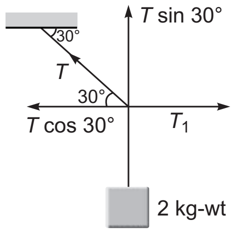
49. The net capacitance $C = \dfrac{\varepsilon_0 A}{\dfrac{d_1}{K_1} + \dfrac{d_2}{K_2}}$

$$= \frac{\varepsilon_0 A}{\dfrac{d}{2}\left(\dfrac{1}{1} + \dfrac{1}{2}\right)} = \frac{4\varepsilon_0 A}{3d}$$

50. Effective spring constant of parallel combination

$$k_e = k_1 + k_2$$

51. $T \sin 30° = 2\,\text{kg-wt}$



$\Rightarrow \quad T = 4\,\text{kg-wt}$

$T_1 = T\cos 30°$

$= 4\cos 30° = 2\sqrt{3}$

52. $KE = \dfrac{p^2}{2m} \Rightarrow KE_2 = KE_1\left(\dfrac{p_2}{p_1}\right)^2 = KE_1\left(\dfrac{2p}{p}\right)^2$

$\Rightarrow \quad KE_2 = 4KE_1$

$= KE_1 + 3KE_1 = KE_1 + 300\%\text{ of }KE_1$

53. Angular width $\theta = \dfrac{\beta}{D}$

$= \dfrac{\lambda}{d} = \dfrac{4800\times10^{-10}}{0.6\times10^{-3}}$

$= 8\times10^{-4}$ rad

54. Potential difference $V = \dfrac{hc}{e\lambda}$

$= \dfrac{6.6\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}\times2\times10^{-10}}$

$= 6200$ volt

55. Loss in kinetic energy

$= \dfrac{1}{2}\dfrac{m_1m_2(u_1-u_2)^2}{(m_1+m_2)}$

$= \dfrac{1}{2}\dfrac{m\cdot m(u_1-u_2)^2}{(m+m)}$

$= \dfrac{m}{4}(u_1-u_2)^2$

56. Torque $\vec{\tau} = \dfrac{d\vec{\mathbf{L}}}{dt} = \dfrac{L_2-L_1}{\Delta t} = \dfrac{4A_0-A_0}{4} = \dfrac{3A_0}{4}$

57. When a number of small droplets coalesce to form a bigger drop surface energy is released because its surface area decreases.

58. If man slides down with some acceleration, then its apparent weight decreases. For critical condition rope can bear only 2/3 of his weight. If a is the minimum acceleration, then

tension in the rope $= m(g-a)$

$=$ breaking strength

$\Rightarrow \quad m(g-a) = \dfrac{2}{3}mg$

$\Rightarrow \quad a = g - \dfrac{2g}{3} = \dfrac{g}{3}$

59. Resulting amplitude of two interfering waves

$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$

Here, $\phi = \pi/3$, $a_1 = 4$, $a_2 = 3$

$\Rightarrow \quad A \simeq 6$

60. The saturation vapour pressure is the static pressure of a vapour when the vapour pressure of some material is in equilibrium with the liquid phase of that material. The saturation vapour pressure of any material is solely dependent on the temperature of that material. Since, temperature change does not occur in the given case, hence pressure will be the same.

Chemistry

1. $v_{rms} = \sqrt{\dfrac{3RT}{M}}$

$\therefore \quad v_{rms} \propto \sqrt{T}$

$\therefore$ At two different temperatures,

$\dfrac{v_{rms}}{v'_{rms}} = \sqrt{\dfrac{T}{T'}}$

Given, $\quad v'_{rms} = 2v_{rms}$

$\dfrac{1}{2} = \sqrt{\dfrac{T}{T'}}$

or $\quad \dfrac{1}{4} = \dfrac{T}{T'}$

$\therefore \quad T' = 4T$

$\therefore$ v_{rms} gets doubled, when the temperature is increased four times.

2. All the reaction proceed by stable ions. After the lose of H^+ ion, phenol forms phenoxide ion. The phenoxide ion is resonance stabilized, thus makes the phenol more acidic.

3. O^{2-} and N^{3-} both are isoelectronic but differ in the charge possessed by them. As the negative charge increases, the electrons are held less and less tightly by the nucleus, therefore ionic radii increases. Hence, ionic radii of N^{3-} is greater than O^{2-}.

In a period from left to right atomic radii decreases but in a group on moving downwards it increases.

4. According to Bohr model,
radius of hydrogen atom

$$(r_n) = \frac{0.529 \times n^2}{Z} \text{ Å}$$

(where, n = number of orbit, Z = atomic number)

$$r_3 = \frac{0.529 \times (3)^2}{1} = 4.761 \text{ Å}$$

5. Co^{3+} and Pt^{4+} have 6 coordination number.
$CoCl_3 \cdot 6NH_3$ and $PtCl_4 \cdot 5NH_3$

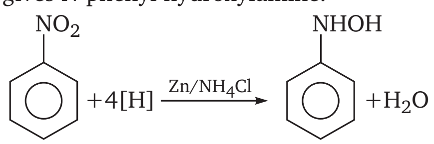
$$[Co(NH_3)_6]Cl_3 \xrightarrow{\text{In solution}} [Co(NH_3)_6]^{3+} + 3Cl^-$$

$$[PtCl(NH_3)_5]Cl_3 \xrightarrow{\text{In solution}} [PtCl(NH_3)_5]^{3+} + 3Cl^-$$

Number of ionic species are same in the solution of both complexes, therefore their equimolar solutions will show same conductance.

6. Entropy is the measure of randomness. In liquids randomness is more than solids.
$\because$ When ice melts, randomness increases, (solid $\longrightarrow$ liquid)
$\therefore$ Entropy increases.

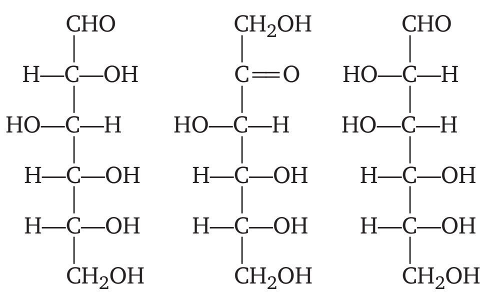
7. Reduction of nitrobenzene by Zn and NH_4Cl gives N-phenyl hydroxylamine.



N-phenyl hydroxylamine

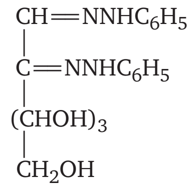
8. Proteins are the polymer of amino acids and contain polypeptide bonds. DNA, however, are the polymers of nucleotide and contain no polypeptide bond. Hence, DNA is not a protein.

9. D-glucose, D-fructose and D-mannose form the same osazone when treated with excess of phenyl hydrazine because they differ only in Ist and 2nd carbon atoms which are transformed to the same form.



D-glucose D-fructose D-mannose

They form the following osazone



10.
$$Ag^+ + e^- \longrightarrow Ag$$

$\because$ 96500 C are required to deposite Ag
$= 108$ g
$\therefore$ 965 C are required to deposite Ag

$$= \frac{108}{96500} \times 965$$

$$= 1.08 \text{ g}$$

11.
$$E^\circ_{cell} = \frac{2.303\,RT}{nF} \log K_{eq}$$

$$E^\circ_{cell} = \frac{0.0591}{n} \log K_{eq} \quad \text{(At 298 K)}$$

$$0.591 = \frac{0.0591}{1} \log K_{eq}$$

$\therefore$ $\log K_{eq} = 10$
$\therefore$ $K_{eq} = 1 \times 10^{10}$

12. Higher the negative value of E°, more is the reducing power. The order of E° values (negative value) is

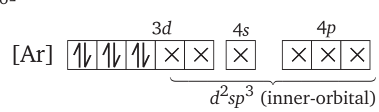
$$\underset{(Mg)}{-2.37} > \underset{(Zn)}{-0.76} > \underset{(Fe)}{-0.44}$$

$\therefore$ Mg can reduce both Zn^{2+} and Fe^{2+}. Zn can reduce Fe^{2+}, but not Mg^{2+}. Fe cannot reduce Mg and Zn but can oxidise them.

13. In diamond,
the maximum proportion of available volume that can be filled by hard spheres $= \frac{\pi\sqrt{3}}{16}$
$= 0.34$

14. On adding a pentavalent impurity with germanium, we get n-type of semiconductors because excess of electrons is responsible for conduction.

15. The complex in which nd orbitals are used in hybridisation, are called outer orbital complex.
(a) $[Fe(CN)_6]^{4-}$ =



(b) $[Mn(CN)_6]^{4-}$ =

$$[Ar]\ \underset{3d}{\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\times}\boxed{\times}}\ \underset{4s}{\boxed{\times}}\ \underset{4p}{\boxed{\times}\boxed{\times}\boxed{\times}}$$

d^2sp^3 (inner-orbital)

(c) $[Co(NH_3)_6]^{3+}$ =

$$[Ar]\ \underset{3d}{\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\times}\boxed{\times}}\ \underset{4s}{\boxed{\times}}\ \underset{4p}{\boxed{\times}\boxed{\times}\boxed{\times}}$$

d^2sp^3 (inner-orbital)

(d) $[Ni(NH_3)_6]^{2+}$ =

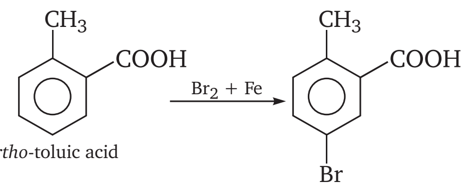
$$[Ar]\ \underset{3d}{\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}}\ \underset{4s}{\boxed{\times}}\ \underset{4p}{\boxed{\times}\boxed{\times}\boxed{\times}}\ \underset{4d}{\boxed{\times}\boxed{\times}\boxed{\ }\boxed{\ }\boxed{\ }}$$

sp^3d^2 (outer-orbital)

16. $C_2H_5Br + AgNO_2\ (alc.) \rightarrow \underset{\text{nitro ethane}}{C_2H_5NO_2} + AgBr$

17. *Iso*-propyl chloride is a 2° halide and 2° halides can undergo hydrolysis either by S_N1 or S_N2 mechanism depending upon the nature of solvent used.

18.



ortho-toluic acid

(∵ In the product, —Br is *para* to —CH_3 and *meta* to —COOH.)

19. —NO_2 group at any position shows electron withdrawing effect, thus acid strength is increased. But o-nitro benzoic acid believed to have *ortho* effect. As a result, resonance gets prevented. Hence, its acid strength is maximum, thus, the order of acid strength (II) < (III) > (IV) > (I)

(The effect is more at *para* position than *meta*.)

20. Iodoform test is given by the compounds containing either CH_3CO—group or $CH_3\overset{|}{C}HOH$ group.

The structures of the given compounds are as

(a) $CH_3CH_2CH_2CH_2OH$

(b) $CH_3COC_6H_5$

(c) CH_3CHO

(d) $CH_3COC_2H_5$

∴ n butyl alcohol does not give iodoform test because it does not possess the CH_3CO—or $CH_3\overset{|}{C}HOH$ group.

21. A + NaOH ⟶ alcohol + acid

Thus, it is Cannizaro reaction.

A is thus aldehyde without H at α-carbon.

(like C_6H_5CHO, HCHO)

$$\underset{\text{benzaldehyde}}{2C_6H_5CHO} + NaOH \longrightarrow C_6H_5CH_2OH + C_6H_5COONa$$

22.
$$\overset{5}{C}H_3-\underset{\underset{OH}{|}}{\overset{4}{C}H}-\overset{3}{C}H=\underset{\underset{CH_3}{|}}{\overset{2}{C}}-\overset{1}{C}HO$$

4-hydroxy-2-methylpent-2-en-1-al

23. $E_a\ (A \rightarrow B) = 80\ kJ\ mol^{-1}$

Heat of reaction $(A \rightarrow B) = 200\ kJ\ mol^{-1}$

For $(B \rightarrow A)$ backward reaction,

$E_a\ (B \rightarrow A) = E_a\ (A \rightarrow B) +$ heat of reaction

$= 80 + 200 = 280\ kJ\ mol^{-1}$

24. $Fe(OH)_3$ is a positively charged sol, thus coagulated by negative ion (anion). Smaller the charge on anion, smaller is its coagulating power or higher is its flocculation value.

(a) $KBr \longrightarrow K^+ + Br^-$

(b) $K_2SO_4 \longrightarrow 2K^+ + SO_4^{2-}$

(c) $K_2CrO_4 \longrightarrow 2K^+ + CrO_4^{2-}$

(d) $K_4[Fe(CN)_6] \longrightarrow 4K^+ + [Fe(CN)_6]^{4-}$

∵ Br^- has smaller charge.

∴ KBr is least effective in coagulating $Fe(OH)_3$ sol.

25. Maximum oxidation state exhibited by d block elements (O.S) = no. of ns electrons + no. of (n – 1) d electrons.

(a) O. S. = 2 + 2 = 4 (b) O. S. = 5 + 1 = 6

(c) O. S. = 3 + 2 = 5 (d) O. S. = 5 + 2 = 7

∴ $(n-1)d^5ns^2$ configuration will achieve the highest oxidation state.

26. Relative lowering of vapour pressure = mole fraction of solute **(Raoult's law)**

$$\frac{p - p_S}{p} = \chi_2$$

$$\frac{p - p_S}{p} = \frac{wM}{mW}$$

$$0.0125 = \frac{wM}{mW}$$

or $$\frac{w}{mW} = \frac{0.0125}{18} = 0.00070$$

Hence, $$\text{molality} = \frac{w}{mW} \times 1000$$

$$= 0.0007 \times 1000 = 0.70$$

27. H_2O_2 oxidises the acidified potassium dichromate solution into blue peroxide of chromium, CrO_5.

$$Cr_2O_7^{2-} + 2H^+ + 4H_2O_2 \longrightarrow 2CrO_5 + 5H_2O$$

28. $NaOH + S \longrightarrow \underset{\text{'A'}}{Na_2SO_3} + Na_2S + H_2O$

29. On igniting at 1400°C, Fe_2O_3 get reduced to metallic Fe.

$$3Fe_2O_3 + CO \longrightarrow 2Fe_3O_4 + CO_2$$
$$Fe_3O_4 + CO \longrightarrow 3FeO + CO_2$$
$$FeO + CO \longrightarrow Fe + CO_2$$

30. H_2SO_4 forms hydrate with water. That's why it has great affinity towards water.

31. In blood, He is much less soluble than nitrogen, hence He—O_2 mixture is used by deep sea divers in preference to N_2—O_2 mixture.

32. One mole $= 6.02 \times 10^{23}$ molecules/atoms/ions.

$\because$ CO_2 molecule has one C and two oxygen atoms.

$\therefore$ CO_2 contains 6.023×10^{23} atoms of C and 6.023×10^{23} molecules of O_2.

33. In acidic medium following reaction takes place.

$$8H^+ + 5e^- + MnO_4^- \longrightarrow Mn^{2+} + 4H_2O$$

$\therefore$ Equivalent weight of $KMnO_4$ in acidic medium $= \dfrac{\text{molecular weight of } KMnO_4}{5}$

$$= \frac{158}{5}$$
$$= 31.6$$

34. (a) Density of water $= 1 \text{ g cm}^{-3}$

Mass of water $= 1 \text{ m}^3 = 10^6 \text{ cm}^{-3}$

$$\text{Mass} = \text{volume} \times \text{density}$$
$$= 10^6 \text{ cm}^{-3} \times 1 \text{ g cm}^{-3}$$
$$= 10^6 \text{ g}$$
$$= \frac{10^6}{10^3} \text{ kg}$$
$$= 1000 \text{ kg}$$

(b) Mass of normal adult man = 65 kg

(c) Density of Hg $= 13.6 \text{ g cm}^{-3}$

Volume of Hg $= 10 \text{ L} = 10 \times 1000 \text{ cm}^{-3}$

$\therefore$ Mass of Hg $= 13.6 \times 10 \times 1000$

$$= 136000 \text{ g}$$
$$= 13.6 \text{ kg}$$

$\therefore$ Mass of 1 m^3 water is highest.

35. e/m ratio for $He^{2+} = \dfrac{2}{4}$

e/m ratio for $H^+ = \dfrac{1}{1}$

e/m ratio for $He^+ = \dfrac{1}{4}$

e/m ratio for $D^+ = \dfrac{1}{2}$

$\therefore$ The e/m is highest for hydrogen.

36. The ionisation potential decreases down the group (due to increase in size of atom) and increases in a period from left to right.

$\therefore$ Out of the given choices $Li > K > Cs$ is correct.

37. Let the oxidation state of Fe in $Fe_3O_4 = x$

$\therefore$ $3x + 4 \times (-2) = 0$

or $3x - 8 = 0$

$\therefore$ $x = \dfrac{8}{3}$

38. $T_{50} = 15$ min

$$k = \frac{2.303 \log 2}{T_{50}} = \frac{2.303 \log 2}{15}$$

$a = 0.1$ M

$(a - x) = 0.025$ M

For first order reaction,

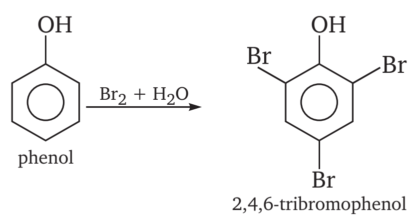
$$k = \frac{2.303}{t} \log\left(\frac{a}{a-x}\right)$$

$$\frac{2.303 \log 2}{15} = \frac{2 \times 2.303}{t} \log \frac{0.1}{0.025}$$
$$= \frac{2.303}{t} \log 4$$

$\therefore$ $\dfrac{2.303 \log 2}{15} = \dfrac{2 \times 2.303 \log 2}{t}$

$\therefore$ $t = 30$ min

39.



40. $CH_3COOH \xrightarrow[\Delta]{Cl_2/\text{Red P}} \underset{\alpha\text{-chloroacetic acid}}{CH_2ClCOOH}$

This reaction is called Hell-Volhard-Zelinsky reaction.

41. $\underset{1^\circ \text{ amine}}{CH_3NH_2} + \underset{\text{Hinsberg's reagent}}{C_6H_5SO_2Cl} \longrightarrow \underset{N\text{-methyl benzene sulphonamide}}{C_6H_5SO_2NHCH_3}$

$\xrightarrow[\text{(alkali)}]{\text{NaOH}} \underset{\text{soluble sodium salt}}{C_6H_5SO_2N(Na)CH_3}$

42. Angle strain, $\alpha = \frac{1}{2}[109°28' - \theta]$

In case of cyclopropane,

$$\theta = 60°$$

$$\therefore \quad \alpha = \frac{1}{2}(109°28' - 60°)$$

$$= 24°44'$$

43. An species or group of atoms can act as ligand only when it carries an unshared pair, *ie,* lone pair of electrons.

44. Bond order $\propto \frac{1}{\text{bond length}}$

$$\text{BO of NO} < \text{BO of } NO^+$$

$\therefore$ Bond length of NO is greater than the bond length of NO^+.

45. $\pi\ 2p_x$ and $\pi\ 2p_y$ or $\overset{*}{\pi}\ 2p_x$ and $\overset{*}{\pi}\ 2p_y$ orbitals have nearly equal energy and thus, are called degenerate orbitals.

46. pH varies inversely with temperature. Thus, if the pH of neutral water is 6.5, the temperature of water is more than 25°C.

47. Electron acceptors are Lewis acids. They are electron deficient compounds. BF_3 is Lewis acid because B has only 6 electrons in its valence shell and it can accept electrons. NF_3, Cl^- and H_2O have lone pair of electrons. Thus, they are electron donors and Lewis bases.

48. Given, concentration of NaOH $= 10^{-10}$ M

$$\underset{10^{-10}}{NaOH} \longrightarrow \underset{10^{-10}}{Na^+} + \underset{10^{-10}}{OH^-}$$

$$\therefore \quad [OH]^- \text{ from NaOH} = 10^{-10}$$

$$\therefore \quad \text{Total } [OH^-] = 10^{-7} + 10^{-10}$$

$$= 10^{-7}(1 + 0.001)$$

$$= 10^{-7}\left(\frac{1001}{1000}\right)$$

$$= 10^{-10} \times 1001$$

$$pOH = -\log[OH^-]$$

$$= -\log(1001 \times 10^{-10})$$

$$= -3.004 + 10$$

$$= 6.9996$$

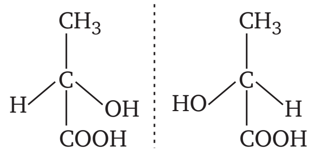
$$\because \quad pH + pOH = 14$$

$$\therefore \quad pH = 14 - 6.9996$$

$$= 7.0004 \approx 7$$

49. Carbanions contain even number of valence electrons and thus, show diamagnetic behaviour.

50. Enantiomers are non-superimposable mirror images. *eg,* lactic acid



Diastereomers are non-superimposable and are not the mirror images of each other. Moreover, *meso* form has plane of symmetry.

51. Inductive effect involves only displacement (and not delocalisation) of σ-electrons.

52. Given, concentration of NaCl = 0.2 M

$$K_{sp}(AgCl) = 1.20 \times 10^{-10}$$

Let the solubility of AgCl in NaCl = x

$$AgCl \longrightarrow \overset{x}{Ag^+} + \overset{x}{Cl^-}$$

$$\text{Solubility} \quad \underset{0.2}{NaCl} \longrightarrow \underset{0.2}{Na^+} + \underset{0.2}{Cl^-}$$

$$\therefore \quad [Ag^+] = x \text{ and } [Cl^-] = (x + 0.2)$$

$$\therefore \quad K_{sp}(AgCl) = [Ag^+][Cl^-]$$

$$= x(x + 0.2)$$

$$= x^2 + 0.2x$$

$$\therefore \quad K_{sp} = 0.2x \ (x^2 << 1)$$

$$\text{or} \quad 1.2 \times 10^{-10} = 0.2x$$

$$\therefore \quad x = 6 \times 10^{-10}$$

53. Naturally occurring fats are called lipids.

54. Pepsin hydrolyses proteins into amino acids as

$$\text{Proteins} \xrightarrow{\text{Pepsin}} \text{amino acids.}$$

55. The enthalpy of formation of Al_2O_3 is very high and hence, it is not possible to reduce it by carbon.

56. $\underset{\text{oxalic acid}}{\begin{matrix} COOH \\ | \\ COOH \end{matrix}} + \text{conc. } H_2SO_4 \longrightarrow CO + CO_2 + H_2O$

Concentrated H_2SO_4 is a strong dehydrating agent.

57. Osmotic pressure is a colligative property *ie,* depends only upon the number of particles or ions in solution. More the number of ions in solution, more will be the osmotic pressure of solution.

(i) 0.1 M urea and 0.1 M glucose will have same number of molecules in solution as they don't ionise.

(ii) $KCl \longrightarrow K^+ + Cl^-$ (2 ions)

(iii) $CaCl_2 \longrightarrow Ca^{2+} + 2Cl^-$ (3 ions)

$\therefore CaCl_2$ produces maximum number of ions.

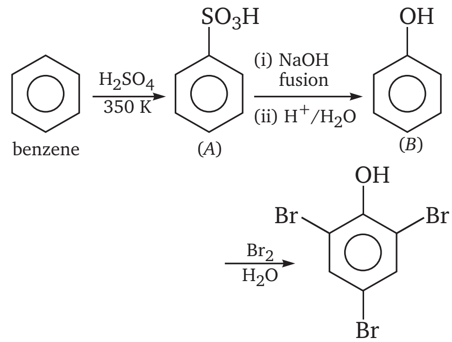
$\therefore$ It will have highest osmotic pressure.

58. Quick lime, CaO is used to dry ammonia, as with other given dehydrating agents ammonia reacts.

$$4NH_3 + CaCl_2 \longrightarrow CaCl_2 \cdot 4NH_3$$
$$4NH_3 + 2P_2O_5 \longrightarrow 4NH_4PO_3$$

$Ca(OH)_2$ is never used as dehydrating agent.

59.



2,4,6-tribromophenol

(C)

60. Collin's reagent is used to convert

$$-CH_2OH \longrightarrow -CHO.$$

Mathematics

1. f(x) is defined, if $3x^2 - 4x + 5 > 0$

$$\Rightarrow \quad 3\left[x^2 - \frac{4}{3}x + \frac{5}{3}\right] > 0$$

$$\Rightarrow \quad 3\left[\left(x - \frac{2}{3}\right)^2 + \frac{11}{9}\right] > 0$$

which is true for all real *x*.

$\therefore$ Domain of $f = (-\infty, \infty)$

Let $y = \log_e(3x^2 - 4x + 5) \Rightarrow e^y = 3x^2 - 4x + 5$

$\Rightarrow 3x^2 - 4x + (5 - e^y) = 0$

For *x* to be real,

$$16 - 12(5 - e^y) \geq 0$$

$$\Rightarrow \quad 12e^y \geq 44$$

$$\Rightarrow \quad e^y \geq \frac{11}{3}$$

$$\Rightarrow \quad y \geq \log_e \frac{11}{3}$$

Range of $f = \left[\log_e \frac{11}{3}, \infty\right)$

2. $\sqrt{9 - x^2}$ is defined for

$$9 - x^2 \geq 0 \Rightarrow (3 - x)(3 + x) \geq 0$$

$$\Rightarrow \quad (x - 3)(x + 3) \leq 0 \quad \ldots(i)$$

$$\Rightarrow \quad -3 \leq x \leq 3$$

$\sin^{-1}(3 - x)$ defined for

$$-1 \leq 3 - x \leq 1$$

$$\Rightarrow \quad -4 \leq -x \leq -2$$

$$\Rightarrow \quad 2 \leq x \leq 4 \quad \ldots(ii)$$

Also, $\sin^{-1}(3 - x) \neq 0$

$$\Rightarrow \quad 3 - x \neq 0 \text{ or } x \neq 3 \quad \ldots(iii)$$

From Eqs. (i), (ii) and (iii), we get

The domain of

$$f = ([-3, 3] \cap [2, 4]) - \{3\} = [2, 3)$$

3. Given, $a_0 = 1$

and $\quad a_{n+1} = 3n^2 + n + a_n$

$$\Rightarrow \quad a_n = 3(n-1)^2 + (n-1) + a_{n-1}$$

Now, put n = 1, 2, 3, …, n

We get,

$$a_1 = 0 + a_0 = 0 + 1 = 1$$
$$\Rightarrow \quad a_2 = 4 + a_1$$
$$\Rightarrow \quad a_3 = 14 + a_2$$
$$\Rightarrow \quad a_4 = 30 + a_3$$

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$$\Rightarrow \quad a_{n-1} = 3(n-2)^2 + (n-2) + a_{n-2}$$
$$\Rightarrow \quad a_n = 3(n-1)^2 + (n-1) + a_{n-1}$$

On adding, we get

$$a_n = (1 + 4 + 14 + 30 + \ldots + 3(n-1)^2 + (n-1))$$

$$\Rightarrow \quad a_n = \Sigma\, 3(n^2 + 1 - 2n) + (n - 1)$$

$$\Rightarrow \quad a_n = \Sigma(3n^2 + 3 - 6n + n - 1)$$

$$\Rightarrow \quad a_n = \Sigma(3n^2 - 5n + 2)$$

$$\Rightarrow \quad a_n = 3\Sigma n^2 - 5\Sigma_n + 2\Sigma 1$$

$$\Rightarrow \quad a_n = \frac{3n(n+1)(2n+1)}{6} - \frac{5n(n+1)}{2} + 2n$$

$$\Rightarrow \quad a_n = \frac{n}{2}(n+1)\{2n+1-5\} + 2n$$

$$\Rightarrow \quad a_n = \frac{n}{2}(n+1)(2n-4) + 2n$$

$$= n(n+1)(n-2) + 2n$$

$$\Rightarrow \quad a_n = (n^2+n)(n-2) + 2n$$

$$= n^3 - 2n^2 + n^2 - 2n + 2n$$

$$\Rightarrow \quad a_n = n^3 - n^2 \qquad (\because n \geq 0)$$

4. Required number of ways

$$= {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5 = 2^5 - 1 = 31$$

5. Number of selections of any number of copies of a book = P + 1, (because copies of the same book are identical things). Similarly, for each book.

$\therefore$ Total number of selections

$$= (P+1)(P+1)\ldots \text{ to } n \text{ factors}$$

$$= (P+1)^n$$

But this includes a selections which is empty *ie,* zero copy of each is selected. Excluding this, the required number of non-empty selections

$$= (P+1)^n - 1$$

6. The number of straight lines

$$= {}^{18}C_2 - ({}^5C_2 - 1) = 144$$

7. The general term is

$$T_{r+1} = {}^{10}C_r \left(\sqrt{\frac{x}{3}}\right)^{10-r} \left(\sqrt{\frac{3}{2x^2}}\right)^r$$

$$= {}^{10}C_r \left(\frac{1}{3}\right)^{5-(r/2)} \left(\frac{3}{2}\right)^{r/2} \cdot x^{5-(3r/2)}$$

for term independent of *x*.

$5 - \frac{3r}{2} = 0 \Rightarrow r = \frac{10}{3}$, which is not a positive integer. Hence, there is no term independent of *x*.

8. $\frac{\text{The coefficient of } t_{r+1}}{\text{The coefficient of } t_r} = \frac{{}^{2n}C_r}{{}^{2n}C_{r-1}}$

$$= \frac{2n-r+1}{r}$$

The coefficient of $t_{r+1} \geq$ The coefficient of t_r, provided

$$\frac{2n-r+1}{r} \geq 1 \text{ or } 2n+1 \geq 2r$$

or $\quad r \leq \frac{2n+1}{2}$ or $r \leq n + \frac{1}{2}$

Hence, the greatest coefficient

= The coefficient of $(n+1)$th term

$$= {}^{2n}C_n = \frac{(2n)!}{(n!)^2}$$

9. The general term in the expansion of $(\sqrt{5} + 4\sqrt{11})^{124}$ is

$$T_{r+1} = {}^{124}C_r (\sqrt{5})^{124-r} (\sqrt[4]{11})^r$$

$$= {}^{124}C_r \cdot 5^{(124-r)/2} \cdot 11^{r/4}$$

For integer terms r = 0, 4, 8, 12, …, 124

$\therefore$ Number of terms which are integers = 32

10. We have,

$$(1+x)^m \left(1 + \frac{1}{x}\right)^n = (1+x)^m \left(\frac{x+1}{x}\right)^n$$

$$= \frac{(1+x)^{m+n}}{x^n} = x^{-n}(1+x)^{m+n}$$

$\therefore$ Required term independent of *x*

= Coefficient of x^0 in $x^{-n}(1+x)^{m+n}$

= Coefficient of x^n in $(1+x)^{m+n}$

$= {}^{m+n}C_n$

11. Let α, β are the roots of the equation $(1-a)x^2 + 3ax - 1 = 0$, then $\alpha + \beta = \frac{3a}{a-1}$, $\alpha\beta = \frac{1}{a-1}$

As $a > 1$, $\alpha + \beta > 0$ and $\alpha\beta > 0$ and $D = 9a^2 + 4(1-a)$

$= 9\left(a^2 - \frac{4}{9}a + \frac{4}{9}\right) = 9\left\{a\left(a - \frac{4}{9}\right) + \frac{4}{9}\right\} > 0$, as $a > 1$

$\therefore$ The equation has real and positive roots.

12. $a \cos 2x + b \sin^2 x + c = 0$

$$\Rightarrow \quad a(1 - 2\sin^2 x) + b \sin^2 x + c = 0$$

ie, $\quad (b - 2a)\sin^2 x + (a + c) = 0$

It is an identity, if $b - 2a = 0$, $a + c = 0$, so

$$\frac{a}{1} = \frac{b}{2} = \frac{c}{-1}$$

Thus, number of triplet (a, b, c) are infinite.

13. We have,

$$(\alpha + \beta + \gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \gamma\alpha)$$

$$\Rightarrow \quad 4 = 6 + 2(\alpha\beta + \beta\gamma + \gamma\alpha)$$

$$\Rightarrow \quad \beta\gamma + \gamma\alpha + \alpha\beta = -1 \qquad \ldots(i)$$

Also, $\quad \alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma$

$$= (\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha)$$

$\Rightarrow \quad 8 - 3\alpha\beta\gamma = 2(6+1)$

$\Rightarrow \quad 3\alpha\beta\gamma = 8 - 14 = -6$

or $\quad \alpha\beta\gamma = -2 \quad \ldots$(ii)

Now, $\quad (\alpha^2+\beta^2+\gamma^2)^2 = \alpha^4+\beta^4+\gamma^4 + 2(\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2)$

$= (\alpha^4+\beta^4+\gamma^4) + 2[(\alpha\beta+\beta\gamma+\gamma\alpha)^2 - 2\alpha\beta\gamma(\alpha+\beta+\gamma)]$

$\Rightarrow \quad (\alpha^4+\beta^4+\gamma^4) = 36 - 2[(-1)^2 - 2(-2)(2)] = 18$

14. $ax^2 + 2b|x| - c = 0$

$\Rightarrow \quad a|x|^2 + 2b|x| - c = 0$

$\therefore \quad |x| = \dfrac{-2b \pm \sqrt{4b^2+4ac}}{2} = -b \pm \sqrt{b^2+ac}$

Since, a, b, c are positive. So,

$$|x| = -b + \sqrt{b^2+ac}$$

$\therefore$ x has two real values, neglecting

$$|x| = -b - \sqrt{b^2+ac}, \text{ as } |x| > 0$$

15. Since, the diagonal elements of a skew-symmetric matrix are always zero.

$\therefore$ Trace of A = 0

16. Here, the rank of A is 3.

Therefore, the minor of order 3 of A $\neq 0$

$$\Rightarrow \quad \begin{vmatrix} y+a & b & c \\ a & y+b & c \\ a & b & y+c \end{vmatrix} \neq 0$$

[Applying $C_1 \to C_1 + C_2 + C_3$ and taking $(y+a+b+c)$ common from C_1]

$$\Rightarrow \quad (y+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & y+b & c \\ 1 & b & y+c \end{vmatrix} \neq 0$$

[Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$]

$$\Rightarrow \quad (y+a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & y & 0 \\ 0 & 0 & y \end{vmatrix} \neq 0$$

Expanding along C_1

$\Rightarrow \quad (y+a+b+c)(y^2) \neq 0$

$\Rightarrow \quad y \neq 0$ and $y \neq -(a+b+c)$

17. We have, $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$

$$= \begin{vmatrix} 1 & \dfrac{\log y}{\log x} & \dfrac{\log z}{\log x} \\ \dfrac{\log x}{\log y} & 1 & \dfrac{\log z}{\log y} \\ \dfrac{\log x}{\log z} & \dfrac{\log y}{\log z} & 1 \end{vmatrix}$$

$$= \frac{1}{\log x \cdot \log y \cdot \log z}\begin{vmatrix} \log x & \log y & \log z \\ \log x & \log y & \log z \\ \log x & \log y & \log z \end{vmatrix} = 0$$

[$\because$ all rows are identical]

18. Since, A is an orthogonal matrix, therefore,

$AA' = I \Rightarrow |A \cdot A'| = |I| \Rightarrow |A||A'| = 1$

$\Rightarrow \quad |A||A| = 1 \quad (\because |I| = 1)$

$\Rightarrow \quad |A|^2 = 1 \quad (\because |A'| = |A|)$

$\Rightarrow \quad |A| = \pm 1$

19. Dividing the equation,

$a^3x^2 + abcx + c^3 = 0$ by c^2, we get

$$a\left(\frac{ax}{c}\right)^2 + b\left(\frac{ax}{c}\right) + c = 0$$

$\Rightarrow \quad \dfrac{ax}{c} = \alpha, \beta$ are roots.

$\Rightarrow \quad x = \dfrac{c}{a}\alpha$ and $x = \dfrac{c}{a}\beta$

$\Rightarrow \quad x = \alpha^2\beta$ and $x = \alpha\beta^2$

[$\because \alpha\beta = \dfrac{c}{a}$ as α, β are the roots of $ax^2 + bx + c = 0$]

are roots of above equation.

20. Expression can be written as

$= (1+i)^{n_1} + (1-i)^{n_1} + (1+i)^{n_2} + (1-i)^{n_2}$

$$= 2^{n_1/2}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)^{n_1} + 2^{n_1/2}\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)^{n_1} + 2^{n_2/2}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)^{n_2} + 2^{n_2/2}\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)^{n_2}$$

$$= 2^{n_1/2}\left\{\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^{n_1} + \left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)^{n_1}\right\} + 2^{n_2/2}\left\{\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^{n_2} + \left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)^{n_2}\right\}$$

$$= n^{n_1/2}\left\{\cos\frac{n_1\pi}{4} + i\sin\frac{n_1\pi}{4} + \cos\frac{n_1\pi}{4} - i\sin\frac{n_1\pi}{4}\right\}$$

$$+ 2^{n_2/2}\left\{\cos\frac{n_2\pi}{4} + i\sin\frac{n_2\pi}{4} + \cos\frac{n_2\pi}{4} - \sin\frac{n_2\pi}{4}\right\}$$

$$= 2^{n_1/2}\cdot 2\cos\frac{n_1\pi}{4} + 2^{n_2/2}\cdot 2\cos\frac{n_2\pi}{4} = \text{real}$$

ie, n_1, n_2 are any two positive integers.

21. Let $z = x + iy$, then $|z+i| - |z-i| = k$ becomes

$\therefore \quad \sqrt{x^2+(y+1)^2} - \sqrt{x^2+(y-1)^2} = k \quad \ldots(i)$

or $\quad x^2+(y+1)^2 - x^2 - (y-1)^2$

$= k\{\sqrt{x^2+(y+1)^2} + \sqrt{x^2+(y-1)^2}\}$

$\therefore \quad \sqrt{x^2+(y-1)^2} + \sqrt{x^2+(y+1)^2} = \frac{4y}{k} \quad \ldots(ii)$

From Eqs. (i) and (ii),

$$2\sqrt{x^2+(y+1)^2} = k + \frac{4y}{k}$$

$$\Rightarrow \quad 4x^2 + 4y^2 + 8y + 4 = k^2 + \frac{16y^2}{k^2} + 8y$$

$$\Rightarrow \quad 4x^2 + \left(4 - \frac{16}{k^2}\right)y^2 = k^2 - 4$$

For an hyperbola, $\frac{4k^2-16}{k^2} < 0 \Rightarrow k^2 - 4 < 0$

$\Rightarrow \quad |k| < 2 \text{ or } -2 < k < 2$

22. Since, $|\sin x| + |\cos x|$ is a periodic function with period $\pi/2$.

$\therefore \quad f(x) = |\sin 4x| + |\cos 4x|$ is a periodic function with period $\frac{1}{4}\cdot\frac{\pi}{2}$ *ie,* $\frac{\pi}{8}$.

23. $\cos x = \frac{2\cos y - 1}{2 - \cos y}$

$$\Rightarrow \quad \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{2\left(\frac{1-\tan^2\frac{y}{2}}{1+\tan^2\frac{y}{2}}\right) - 1}{2 - \left(\frac{1-\tan^2\frac{y}{2}}{1+\tan^2\frac{y}{2}}\right)}$$

$$\Rightarrow \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{2 - 2\tan^2\frac{y}{2} - 1 - \tan^2\frac{y}{2}}{2 + 2\tan^2\frac{y}{2} - 1 + \tan^2\frac{y}{2}}$$

$$\Rightarrow \quad \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{1 - 3\tan^2\frac{y}{2}}{1 + 3\tan^2\frac{y}{2}}$$

$$\Rightarrow 1 + 3\tan^2\frac{y}{2} - \tan^2\frac{x}{2} - 3\tan^2\frac{x}{2}\tan^2\frac{y}{2}$$

$$= 1 - 3\tan^2\frac{y}{2} + \tan^2\frac{x}{2} - 3\tan^2\frac{x}{2}\tan^2\frac{y}{2}$$

$$\Rightarrow \quad 6\tan^2\frac{y}{2} = 2\tan^2\frac{x}{2}$$

$$\Rightarrow \quad \tan\frac{x}{2}\cdot\cot\frac{y}{2} = \sqrt{3}$$

24. $\cos\frac{2\pi}{15}\cdot\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{16\pi}{15}$

$$= \frac{1}{2\sin\frac{2\pi}{15}}\cdot 2\sin\frac{2\pi}{15}\cdot\cos\frac{2\pi}{15}\cdot\cos\frac{4\pi}{15}\cdot\cos\frac{8\pi}{15}\cdot\cos\frac{16\pi}{15}$$

$$\Rightarrow \frac{1}{2\sin\frac{2\pi}{15}}\cdot\sin\frac{4\pi}{15}\cdot\cos\frac{4\pi}{15}\cdot\cos\frac{8\pi}{15}\cdot\cos\frac{16\pi}{15}$$

$$\Rightarrow \quad \frac{1}{4\sin\frac{2\pi}{15}}\cdot\sin\frac{8\pi}{15}\cdot\cos\frac{8\pi}{15}\cdot\cos\frac{16\pi}{15}$$

$$= \frac{1}{8\sin\frac{2\pi}{15}}\cdot\sin\frac{16\pi}{15}\cdot\cos\frac{16\pi}{15}$$

$$= \frac{1}{16\sin\frac{2\pi}{15}}\cdot\sin\frac{32\pi}{15}$$

$$= \frac{1}{16\sin\frac{2\pi}{15}}\cdot\sin\left(2\pi + \frac{2\pi}{15}\right) = \frac{1}{16\sin\frac{2\pi}{15}}\cdot\sin\frac{2\pi}{15}$$

$$= \frac{1}{16}$$

25. We have, $\cos x\cdot\cos\left(\frac{\pi}{3}+x\right)\cos\left(\frac{\pi}{3}-x\right) = \frac{1}{4}$

$$\Rightarrow \quad \cos x\left(\frac{1}{4}\cos^2 x - \frac{3}{4}\sin^2 x\right) = \frac{1}{4}$$

or $\quad \frac{\cos x}{4}(4\cos^2 x - 3) = \frac{1}{4}$

$\Rightarrow \quad 4\cos^3 x - 3\cos x = 1$

$(\because \cos 3x = 4\cos^3 x - 3\cos x)$

or $\quad \cos 3x = 1$

$\Rightarrow \quad 3x = 2n\pi$

$\Rightarrow x = \frac{2n\pi}{3}$, where $n = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$

$(\because x \in [0, 6\pi])$

$\therefore$ The required sum $= \frac{2\pi}{3}\sum_{n=0}^{9} n = 30\pi$

26. The given expression

$$= 2\tan^{-1}[\operatorname{cosec}\tan^{-1}x - \tan\cot^{-1}x]$$

$$= 2\tan^{-1}\left[\operatorname{cosec}\left\{\operatorname{cosec}^{-1}\frac{\sqrt{1+x^2}}{x}\right\} - \tan\left\{\tan^{-1}\left(\frac{1}{x}\right)\right\}\right]$$

$$= 2\tan^{-1}\left[\frac{\sqrt{1+x^2}}{x} - \frac{1}{x}\right] = 2\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$$

$$= 2\tan^{-1}\left[\frac{\sec\theta - 1}{\tan\theta}\right] \qquad [\text{putting } x = \tan\theta]$$

$$= \tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right]$$

$$= 2\tan^{-1}\left[\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}\right]$$

$$= 2\tan^{-1}\tan\frac{\theta}{2} = 2\cdot\frac{\theta}{2} = \theta = \tan^{-1}x$$

27. We have,

$$\sin^{-1}\left[\cos\left(\sin^{-1}\sqrt{\left(\frac{2-\sqrt{3}}{4}\right)} + \cos^{-1}\frac{\sqrt{12}}{4} + \sec^{-1}\sqrt{2}\right)\right]$$

$$= \sin^{-1}\left[\cos\left(\sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) + \cos^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\frac{1}{\sqrt{2}}\right)\right]$$

$$\left[\because \left(\frac{2-\sqrt{3}}{4}\right)^{1/2} = \frac{\sqrt{3}-1}{2\sqrt{2}}\right]$$

$$= \sin^{-1}[\cos(15^\circ + 30^\circ + 45^\circ)]$$

$$= \sin^{-1}[\cos 90^\circ] \qquad \left(\because \sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}\right)$$

$$= \sin^{-1}0$$

$$= 0$$

28. In given right angled triangle, we have

$$\cos A = c/b$$

$$\Rightarrow \frac{1-\tan^2\frac{A}{2}}{1+\tan^2\frac{A}{2}} = c/b$$

$$\Rightarrow \frac{\left(1+\tan^2\frac{A}{2}\right) - \left(1-\tan^2\frac{A}{2}\right)}{\left(1+\tan^2\frac{A}{2}\right) + \left(1-\tan^2\frac{A}{2}\right)} = \frac{(b-c)}{(b+c)}$$

(by componendo and dividendo rule)

Hence, $\tan^2\frac{A}{2} = \frac{b-c}{b+c}$

29. We have, $r_1 > r_2 > r_3$

$$\Rightarrow \frac{\Delta}{s-a} > \frac{\Delta}{s-b} > \frac{\Delta}{s-c}$$

$$\Rightarrow \frac{s-a}{\Delta} < \frac{s-b}{\Delta} < \frac{s-c}{\Delta}$$

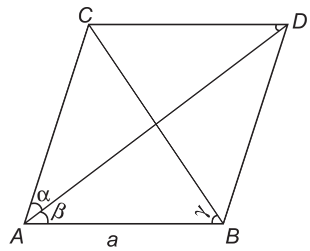
$$\Rightarrow s-a < s-b < s-c$$

$$\Rightarrow -a < -b < -c \Rightarrow a > b > c$$

30. Clearly,

$\angle CDA = \beta$, $\angle DCB = \gamma$, $\angle ACB = \pi - (\alpha + \beta + \gamma)$

Using since rule in ΔCAB, we get



$$\frac{AB}{\sin(\pi - (\alpha+\beta+\gamma))} = \frac{CA}{\sin\gamma}$$

$$\Rightarrow CA = \frac{a\cdot\sin\gamma}{\sin(\alpha+\beta+\gamma)}$$

Now, using sine rule in ΔCAD,

we get $\frac{CA}{\sin\beta} = \frac{CD}{\sin\alpha}$

$$\Rightarrow CD = \frac{CA\cdot\sin\alpha}{\sin\beta} = \frac{a\sin\alpha\cdot\sin\gamma}{\sin\beta\cdot\sin(\alpha+\beta+\gamma)}$$

31. We have, $\vec{\mathbf{P}} = \vec{\mathbf{AC}} + \vec{\mathbf{BD}} = \vec{\mathbf{AC}} + \vec{\mathbf{BC}} + \vec{\mathbf{CD}}$

$$= \vec{\mathbf{AC}} + \lambda\,\vec{\mathbf{AD}} + \vec{\mathbf{CD}} = \lambda\,\vec{\mathbf{AD}} + (\vec{\mathbf{AC}} + \vec{\mathbf{CD}})$$

$$= \lambda\,\vec{\mathbf{AD}} + \vec{\mathbf{AD}} = (\lambda + 1)\,\vec{\mathbf{AD}}$$

But, $\vec{\mathbf{P}} = \mu\,\vec{\mathbf{AD}}$

$$\therefore \quad \mu = \lambda + 1$$

32. Given, $3\vec{\mathbf{p}} + 8\vec{\mathbf{q}} = 6\vec{\mathbf{r}} + 5\vec{\mathbf{s}}$

$$\Rightarrow \frac{3\vec{\mathbf{p}} + 8\vec{\mathbf{q}}}{8+3} = \frac{6\vec{\mathbf{r}} + 5\vec{\mathbf{s}}}{6+5}$$

$\Rightarrow$ The point which divides PQ in ratio 8 : 3 in the same as the point which divides RS in the ratio 5 : 6. Hence, the line PQ and RS intersect.

33. We have, $\vec{\mathbf{a}}\cdot\vec{\mathbf{c}} = |\vec{\mathbf{c}}|$ and $|\vec{\mathbf{c}} - \vec{\mathbf{a}}| = 2\sqrt{2}$

$$\Rightarrow \vec{\mathbf{a}}\cdot\vec{\mathbf{c}} = |\vec{\mathbf{c}}| \text{ and } |\vec{\mathbf{c}}|^2 + |\vec{\mathbf{a}}|^2 - 2(\vec{\mathbf{a}}\cdot\vec{\mathbf{c}}) = 8$$

$$\Rightarrow |\vec{\mathbf{c}}|^2 + 9 - 2|\vec{\mathbf{c}}| = 8$$

$$\Rightarrow \quad (|\vec{c}|-1)^2 = 0$$

$$\Rightarrow \quad |\vec{c}| = 1$$

$$\Rightarrow \quad |(\vec{a}\times\vec{b})\times\vec{c}| = |\vec{a}\times\vec{b}||\vec{c}| \sin 30^\circ$$

$$= \frac{1}{2}|\vec{a}\times\vec{b}| = \frac{3}{2}$$

$$[\because \quad \vec{a}\times\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}]$$

34. We have,

$$\vec{V}_1 = \frac{1}{2}(\vec{a}\times\vec{b}),\ \vec{V}_2 = \frac{1}{2}(\vec{b}\times\vec{c}),\ \vec{V}_3 = \frac{1}{2}(\vec{c}\times\vec{a})$$

and $\vec{V}_4 = \frac{1}{2}\{(\vec{c}-\vec{a})\times(\vec{b}-\vec{a})\}$

$$\therefore \vec{V}_1+\vec{V}_2+\vec{V}_3+\vec{V}_4 = \frac{1}{2}(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$$

$$+\vec{c}\times\vec{b}-\vec{c}\times\vec{a}-\vec{a}\times\vec{b}) = \vec{0}$$

$$|\vec{V}_1+\vec{V}_2+\vec{V}_3+\vec{V}_4| = \vec{0}$$

35. We have,

$$|[\vec{a}\,\vec{b}\,\vec{c}]| = V$$

Let V_1 be the volume of parallelopiped formed by the vectors $\vec{\alpha}$, $\vec{\beta}$ and $\vec{\gamma}$. Then,

$$V_1 = |[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]|$$

$$\Rightarrow \quad V_1 = \begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{a}\cdot\vec{b} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c} \\ \vec{a}\cdot\vec{c} & \vec{b}\cdot\vec{c} & \vec{c}\cdot\vec{c} \end{vmatrix} [\vec{a}\,\vec{b}\,\vec{c}]$$

$$\Rightarrow \quad V_1 = [\vec{a}\,\vec{b}\,\vec{c}]^2[\vec{a}\,\vec{b}\,\vec{c}]$$

$$\Rightarrow \quad V_1 = [\vec{a}\,\vec{b}\,\vec{c}]^3$$

$$\Rightarrow \quad V_1 = V^3$$

36. We have, $|a\hat{i}+b\hat{j}+c\hat{k}| = |a|+|b|+|c|$

$\Rightarrow \sqrt{a^2+b^2+c^2} = |a|+|b|+|c|$

$\Rightarrow a^2+b^2+c^2$

$= a^2+b^2+c^2+2(|a||b|+|b||c|+|c||a|)$

$\Rightarrow |a||b|+|b||c|+|c||a| = 0$

$\Rightarrow ab = bc = ca = 0$

$\Rightarrow$ Any two of a, b and c are zero.

37. We have, $\vec{u}\cdot\hat{n} = 0$ and $\vec{v}\cdot\hat{n} = 0$

$$\Rightarrow \quad \hat{n}\perp\vec{u} \text{ and } \hat{n}\perp\vec{v}$$

$$\Rightarrow \quad \hat{n} = \pm\frac{\vec{u}\times\vec{v}}{|\hat{u}\times\vec{v}|}$$

Now, $\vec{u}\times\vec{v} = (\hat{i}+\hat{j})\times(\hat{i}-\hat{j}) = -2\hat{k}$

$$\hat{n} = \pm\hat{k}$$

Hence, $|\vec{w}\cdot\hat{n}| = |(\hat{i}+2\hat{j}+3\hat{k})\cdot(\pm\hat{k})| = 3$

38. We have, $P(A) = P\left(\frac{A}{B}\right) = \frac{1}{4}$. This shows that A and B are independent events. So,

$$P(B) = P\left(\frac{B}{A}\right) = \frac{1}{2}$$

Now, $\quad P\left(\frac{A}{B}\right) = \frac{1}{4} \Rightarrow P\left(\frac{\bar{A}}{B}\right) = 1-\frac{1}{4} = \frac{3}{4}$

39. Suppose the coin is tossed n times. Let X be the number of the heads obtained. Then, X follows a binomial distribution with parameters n and $p = \frac{1}{2}$

Now, $P(X\ge 1)\ge 0.8 \Rightarrow 1-P(X=0)\ge 0.8$

$$\Rightarrow \quad 1-{}^nC_0p^0(1-p)^n \ge 0.8$$

$$\Rightarrow \quad \left(\frac{1}{2}\right)^n \le 0.2 \text{ or } \left(\frac{1}{2}\right)^n \le \frac{1}{5}$$

$$\Rightarrow \quad 2^n \ge 5$$

$$\Rightarrow \quad n \ge 3$$

Hence, the least value of n is 3.

40. Since, $(n+1)p = \frac{101}{3}$ is not an integer.

Therefore, $p(X=r)$ is maximum when $r = \left[\frac{101}{3}\right] = 33$

41. Let n_1 and n_2 be the number of observations in two groups having means $\bar{X}_1$ and $\bar{X}_2$ respectively.

Then $\quad \bar{X} = \frac{n_1\bar{X}_1+n_2\bar{X}_2}{n_1+n_2} - \bar{X}_2$

Now, $\quad \bar{X}-\bar{X}_1 = \frac{n_1\bar{X}_1+n_2\bar{X}_2}{n_1+n_2} - \bar{X}_1$

$$= \frac{n_2(\bar{X}_2-\bar{X}_1)}{n_1+n_2} > 0 \quad [\because \bar{X}_2 > \bar{X}_1]$$

$$\Rightarrow \quad \bar{X} > \bar{X}_1 \quad \ldots(i)$$

And $\quad \bar{X}-\bar{X}_2 = \frac{n(\bar{X}_1-\bar{X}_2)}{n_1+n_2} < 0 \quad [\because \bar{X}_2 > \bar{X}_1]$

$$\Rightarrow \quad \bar{X} < \bar{X}_2 \quad \ldots(ii)$$

From Eqs. (i) and (ii), $\bar{X}_1 < \bar{X} < \bar{X}_2$

42. Here, $\tan\theta = 2$,

So, $\quad \cos\theta = \frac{1}{\sqrt{5}},\ \sin\theta = \frac{2}{\sqrt{5}}$

For x and y, we have

$$x = X\cos\theta - Y\sin\theta = \frac{X - 2Y}{\sqrt{5}}$$

and $$y = X\sin\theta + Y\cos\theta = \frac{2X + Y}{\sqrt{5}}$$

The equation $4xy - 3x^2 = a^2$ reduces to

$$\frac{4(X-2Y)}{\sqrt{5}}\cdot\frac{(2X+Y)}{\sqrt{5}} - 3\left(\frac{X-2Y}{\sqrt{5}}\right)^2 = a^2$$

$$\Rightarrow \quad 4(2X^2 - 2Y^2 - 3XY) - 3(X^2 - 4XY + 4Y^2) = 5a^2$$

$$\Rightarrow \quad 5X^2 - 20Y^2 = 5a^2$$

$$\Rightarrow \quad X^2 - 4Y^2 = a^2$$

43. The given points are collinear, if

$$\begin{vmatrix} t_1 & 2at_1 + at_1^3 & 1 \\ t_2 & 2at_2 + at_2^3 & 1 \\ t_3 & 2at_3 + at_3^3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad a\begin{vmatrix} t_1 & 2t_1 + t_1^3 & 1 \\ t_2 & 2t_2 + t_2^3 & 1 \\ t_3 & 2t_3 + t_3^3 & 1 \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$, we get

$$\begin{vmatrix} t_1 & 2t_1 + t_1^3 & 1 \\ t_2 - t_1 & 2(t_2 - t_1) + (t_2^3 - t_1^3) & 0 \\ t_3 - t_1 & 2(t_3 - t_1) + (t_3^3 - t_1^3) & 0 \end{vmatrix} = 0$$

$$\Rightarrow (t_2 - t_1)(t_3 - t_1)\begin{vmatrix} t_1 & 2t_1 + t_1^3 & 1 \\ 1 & 2 + t_2^2 + t_1^2 + t_2 t_1 & 0 \\ 1 & 2 + t_3^2 + t_1^2 + t_3 t_1 & 0 \end{vmatrix} = 0$$

$$\Rightarrow (t_2 - t_1)(t_3 - t_1)(t_3 - t_2)(t_3 + t_2 + t_1) = 0$$

$$\Rightarrow \quad t_1 + t_2 + t_3 = 0 \quad [\because t_1 \neq t_2 \neq t_3]$$

44. The slope of the line $x - y + 1 = 0$ is 1. So, it makes an angle of 45° with x-axis.

The equation of a line passing through (2, 3) and making an angle of 45°

$$\frac{x-2}{\cos 45^\circ} = \frac{y-3}{\sin 45^\circ} = r$$

Coordinates of any point on this line are

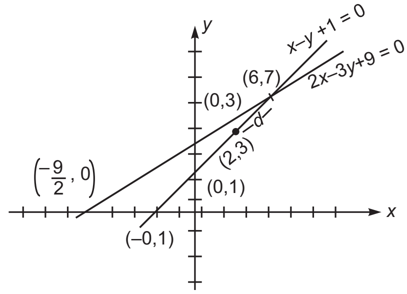
$$(2 + r\cos 45^\circ,\ 3 + r\sin 45^\circ)$$

or $$\left(2 + \frac{r}{\sqrt{2}},\ 3 + \frac{r}{\sqrt{2}}\right)$$

If this point lies on the line $2x - 3y + 9 = 0$, then $4 + \sqrt{2}\,r - 9 - \frac{3r}{\sqrt{2}} + 9 = 0$

$$\Rightarrow \quad r = 4\sqrt{2}$$

Alternate Method



Since the point (2, 3) lies on the line $x - y + 1 = 0$. Therefore the distance from (2, 3) to the line $2x - 3y + 9 = 0$ along the line $x - y + 1 = 0$ is equal to the distance between the points (2, 3) and intersection point of $2x - 3y + 9 = 0$ and $x - y + 1 = 0$ *ie,* (6, 7).

Hence required distance

$$d = \sqrt{(6-2)^2 + (7-3)^2} = \sqrt{32}$$

$$d = 4\sqrt{2}$$

45. The equation of any line through the point of intersection of the lines $x - y - 1 = 0$ and $2x - 3y + 1 = 0$ is

$$(x - y - 1) + \lambda(2x - 3y + 1) = 0$$

$$\Rightarrow \quad (2\lambda + 1)x - y(3\lambda + 1) + (\lambda - 1) = 0 \quad \ldots(i)$$

The line in Eq. (i) will be parallel to x-axis, if it is of the form $y =$ constant, therefore coefficient of x in Eq. (i) $= 0$

ie, $$2\lambda + 1 = 0 \Rightarrow \lambda = -\frac{1}{2}$$

On putting $\lambda = -\frac{1}{2}$ in Eq. (i), we get $y = 3$

This is the equation of the required line.

46. Since, given straight lines are the sides of a rhombus. Therefore, distance between the parallel lines

$a_1x + b_1y + c_1 = 0$, $a_1x + b_1y + c_2 = 0$ and $a_2x + b_2y + d_1 = 0$, $a_2x + b_2y + d_2 = 0$ must be equal

$$\therefore \quad \frac{|c_1 - c_2|}{\sqrt{a_1^2 + b_1^2}} = \frac{|d_1 - d_2|}{\sqrt{a_2^2 + b_2^2}}$$

$$\Rightarrow (a_2^2 + b_2^2)(c_1 - c_2)^2 = (a_1^2 + a_1^2)(d_1 - d_2)^2$$

47. The given equation is of the form $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, on comparing the given equation with it

We obtain $a = 3$, $h = \frac{7}{2}$, $b = 2$, $g = \frac{5}{2}$, $f = \frac{5}{2}$ and $c = 2$

Now, $abc + 2fgh - af^2 - bg^2 - ch^2$

$$= 12 + 2 \times \frac{5}{2} \times \frac{5}{2} \times \frac{7}{2} - 3\left(\frac{5}{2}\right)^2 - 2\left(\frac{5}{2}\right)^2 - 2\left(\frac{7}{2}\right)^2$$

$= 0$

Hence, the given equation represents a pair of straight lines.

48. If the two pairs of straight lines have the same bisectors, then the two pairs are equally inclined.

The equation of the bisectors of the angle between the lines given by $ax^2 + 2hxy + by^2 = 0$ is

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h} \quad \ldots(i)$$

The equation of the bisectors of the angle between the lines given by

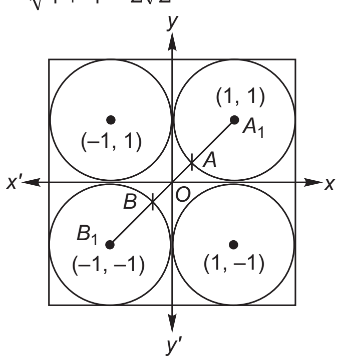
$a^2x^2 + 2h(a + b)xy + b^2y^2 = 0$ is

$$\frac{x^2 - y^2}{a^2 - b^2} = \frac{xy}{h(a + b)}$$

$$\Rightarrow \quad \frac{x^2 - y^2}{a - b} = \frac{xy}{h} \quad \ldots(ii)$$

Clearly Eqs. (i) and (ii) are the same. The two pairs of straight lines are equally inclined.

49. $A_1B_1 = \sqrt{4 + 4} = 2\sqrt{2}$



$$AB = 2\sqrt{2} - 2 = 2(\sqrt{2} - 1)$$

$\Rightarrow OA = OB = \frac{AB}{2} = \sqrt{2} - 1$ which is the radius of required circle.

Thus, equation of required circle is

$$x^2 + y^2 = (\sqrt{2} - 1)^2 = 3 - 2\sqrt{2}$$

50. We have, the given circle $x = a\cos\theta,\ y = a\sin\theta$

$$\Rightarrow \quad x^2 + y^2 = a^2 \quad \ldots(i)$$

Tangents to the circle (i) at θ and $\left(\theta + \frac{\pi}{2}\right)$ are

$x\cos\theta + y\sin\theta = a$

and $x\cos\left(\theta + \frac{\pi}{2}\right) + y\sin\left(\theta + \frac{\pi}{2}\right) = a$

or $\quad x\cos\theta + y\sin\theta = a$

and $\quad -x\sin\theta + y\cos\theta = a$

Squaring and adding, we get

$$x^2 + y^2 = 2a^2$$

which is a circle.

51. Let the equation of the required circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \ldots(i)$$

This passes through A (1, 0) and B(0, 1), therefore,

$$1 + 2g + c = 0 \text{ and } 1 + 2f + c = 0$$

$$\Rightarrow \quad g = -\left(\frac{c + 1}{2}\right) \text{ and } f = -\left(\frac{c + 1}{2}\right)$$

Let r be the radius of circle (i), then

$$r = \sqrt{g^2 + f^2 - c}$$

$$\Rightarrow \quad r = \sqrt{\left(\frac{c + 1}{2}\right)^2 + \left(\frac{c + 1}{2}\right)^2 - c}$$

$$\Rightarrow \quad r = \sqrt{\frac{c^2 + 1}{2}}$$

$$\Rightarrow \quad r^2 = \frac{1}{2}(c^2 + 1)$$

Clearly, r is minimum when $c = 0$ and the minimum value of r is $\frac{1}{\sqrt{2}}$.

For $c = 0$, we have

$$g = -\frac{1}{2} \text{ and } f = -\frac{1}{2}$$

On substituting the values of g, f and c in Eq. (i), we get,

$$x^2 + y^2 - x - y = 0$$

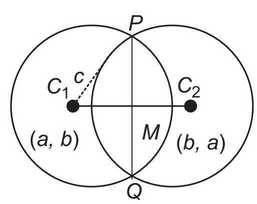
This is equation of the required circle.

52. The equations of two circles are

$$S_1 \equiv (x - a)^2 + (y - b)^2 = c^2 \quad \ldots(i)$$

and $\quad S_2 \equiv (x - b)^2 + (y - a)^2 = c^2 \quad \ldots(ii)$

The equation of the common chord of these circle is



$$S_1 - S_2 = 0$$

$\Rightarrow (x-a)^2-(x-b)^2+(y-b)^2-(y-a)^2=0$

$\Rightarrow (2x-a-b)(b-a)+(2y-b-a)(a-b)=0$

$\Rightarrow 2x-a-b-2y+b+a=0$

$\Rightarrow x-y=0$

Let C_1 and C_2 be the centres of circles (i) and (ii) respectively. Then, the coordinates of C_1 and C_2 are (a, b) and (b, a) respectively.

$$C_1M=\left|\frac{a-b}{\sqrt{1+1}}\right|=\frac{|a-b|}{\sqrt{2}}$$

In right ΔC_1PM, we have,

$$PM=\sqrt{C_1P^2-C_1M^2}=\sqrt{c^2-\frac{(a-b)^2}{2}}$$

$$\therefore \quad PQ=2PM=2\sqrt{c^2-\frac{(a-b)^2}{2}}$$

$$=\sqrt{4c^2-2(a-b)^2}$$

53. Let the points $\left(m_i, \frac{1}{m_i}\right)$, $i=1, 2, 3, 4$ lie on the circle $x^2+y^2+2gx+2fy+c=0$

Then,

$$\Rightarrow m_i^2+\frac{1}{m_i^2}+2gm_i+\frac{2f}{m_i}+c=0,\ i=1,2,3,4$$

$$\Rightarrow \quad m_i^4+2gm_i^3+cm_i^2+2fm_i+1=0,$$
$$i=1,2,3,4$$

$\Rightarrow$ m_1, m_2, m_3 and m_4 are the roots of the equation

$$m^4+2gm^3+cm^2+2fm+1=0$$

$$\Rightarrow \quad m_1m_2m_3m_4=\frac{1}{1}=1$$

54. Equation of the normal of the parabola $y^2=4ax$ at the point $(at^2, 2at)$ is

$$y+tx=2at+at^3 \qquad \ldots(i)$$

$\because$ Eq. (i) cuts the parabola again at $(aT^2, 2aT)$.

Then, $\quad 2aT+taT^2=2at+at^3$

$$\Rightarrow \quad 2a(T-t)=-at(T^2-t^2)$$

$$\Rightarrow \quad 2=-t(T+t) \qquad (\because t\neq T)$$

$$\Rightarrow \quad t^2+tT+2=0$$

$\because$ *t* is real, $\therefore T^2-4\cdot1\cdot2\geq 0$

$$\Rightarrow \quad T^2\geq 8$$

55. Since, $3(3)^2+5(5)^2-32>0$. So, the given point lies outside the ellipse. Hence, two real tangents can be drawn from the point to the ellipse.

56. Let $y=m_1x$ and $y=m_2x$ be the lines represented by $lx^2+2mxy+ny^2=0$, then

$$m_1m_2=\frac{l}{n} \qquad \ldots(i)$$

But $y=m_1x$ and $y=m_2x$ are conjugate diameters of the hyperbola.

$$\therefore \quad m_1m_2=\frac{b^2}{a^2} \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{l}{n}=\frac{b^2}{a^2}\Rightarrow la^2=nb^2$$

57. The equations of the asymptotes of the hyperbola

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \text{ are } y=\pm\frac{b}{a}\ ie,$$

$$y=\frac{b}{a}x \text{ and } y=\frac{-b}{a}x$$

If θ is the angle between the two asymptotes, then,

$$\tan\theta=\frac{\frac{b}{a}+\frac{b}{a}}{1-\frac{b}{a}\cdot\frac{b}{a}}=\frac{2b}{a}\times\frac{a^2}{a^2-b^2}=\frac{2ab}{a^2-b^2}$$

From this, we get $\cos\theta=\frac{a^2-b^2}{a^2+b^2}$

$$\Rightarrow \quad 2\cos^2\frac{\theta}{2}-1=\frac{a^2-b^2}{a^2+b^2}$$

$$\Rightarrow \quad 2\cos^2\frac{\theta}{2}=1+\frac{a^2-b^2}{a^2+b^2}$$

$$\Rightarrow \quad 2\cos^2\frac{\theta}{2}=\frac{2a^2}{a^2+b^2}$$

$$\Rightarrow \quad \cos^2\frac{\theta}{2}=\frac{a^2}{a^2+b^2}$$

But $b^2=a^2(e^2-1)=a^2e^2-a^2\Rightarrow a^2+b^2=a^2e^2$

$$\therefore \quad \cos^2\left(\frac{\theta}{2}\right)=\frac{a^2}{a^2e^2}=\frac{1}{e^2}$$

$$\Rightarrow \quad \cos\left(\frac{\theta}{2}\right)=\frac{1}{e}$$

58. Let the point be $P\left(ct, \frac{c}{t}\right)$

The equation of the tangent to the hyperbola $xy=c^2$ at *P* is

$$x\frac{c}{t}+cty=2c^2$$

$$\Rightarrow \quad x+yt^2-2ct=0 \qquad \ldots(i)$$

Then, putting $y = 0, x = 0$ successively in Eq. (i), we get $a_1 = 2ct$ and $b_1 = \frac{2c}{t}$

($\because$ intercepts of tangent on the axes are a_1 and b_1)

Again the equation of the normal to the hyperbola $xy = c^2$ at P is

$$xt^3 - yt - ct^4 + c = 0$$

As before, $a_2 = ct - \frac{c}{t^3}$ and $b_2 = -ct^3 + \frac{c}{t}$

$$\therefore a_1a_2 + b_1b_2 = 2ct\left(ct - \frac{c}{t^3}\right) + \frac{2c}{t}\left(-ct^3 + \frac{c}{t}\right)$$

$$= 0$$

59. The equation of xy-plane is $z = 0$

$\therefore$ Direction cosines of its normal are 0, 0, 1.

60. $xy + yz = 0 \Leftrightarrow (x + z)y = 0 \Leftrightarrow y = 0$

or $\qquad x + z = 0$

The equations $y = 0$ and $x + z = 0$ represents planes which are at right angles as their altitudes numbers are $< 0, 1, 0 >$ and $< 1, 0, 1 >$ respectively.

61. By image formula,

$$\frac{x_2 - x_1}{a} = \frac{y_2 - y_1}{b} = \frac{z_2 - z_1}{c}$$

$$= -\frac{2(ax_1 + by_1 + cz_1 + d)}{a^2 + b^2 + c^2}$$

$$\Rightarrow \quad \frac{x_2 - 2}{3} = \frac{y_2 + 1}{-2} = \frac{z_2 - 3}{-1}$$

$$= -2\frac{(6 + 2 - 3 - 9)}{9 + 4 + 1}$$

$$\Rightarrow \quad \frac{x_2 - 2}{3} = \frac{y_2 + 1}{-2} = \frac{z_2 - 3}{-1} = \frac{-2(-4)}{14}$$

$$\therefore \quad x_2 = \frac{26}{7}, y_2 = \frac{-15}{7}, z_2 = \frac{17}{7}$$

62. $\lim\limits_{x \to 0} \frac{\sqrt{1 - \cos 2x}}{\sqrt{2} \cdot x} = \lim\limits_{x \to 0} \frac{\sqrt{1 - (1 - 2\sin^2 x)}}{\sqrt{2} \cdot x}$

$$= \lim_{x \to 0} \frac{\sqrt{2\sin^2 x}}{\sqrt{2} \cdot x} = \lim_{x \to 0} \frac{|\sin x|}{x}$$

Let $f(x) = \frac{|\sin x|}{x}$

Then,

$$f(0 + 0) = \lim_{h \to 0} \frac{|\sin(0 + h)|}{h} = \lim_{h \to 0} \frac{\sin h}{h} = 1$$

and

$$f(0 - 0) = \lim_{h \to 0} \frac{|\sin(0 - h)|}{-h} = \lim_{h \to 0} \frac{\sin h}{-h} = -1$$

$$\because \quad f(0 + 0) \neq f(0 - 0)$$

$\therefore$ The limit does not exist.

63. We have,

$$\lim_{h \to 0} f(1 - h) = \lim_{h \to 0} a(1 - h)^2 + b = a + b$$

$$\Rightarrow \lim_{h \to 0} f(1 + h) = \lim_{h \to 0} (1 + h) + 3 = 4$$

and $f(1) = 4$

$\therefore$ f(x) will not be continuous at $x = 1$, if $a + b \neq 4$

64. We have, $f(x) = \log_x(\log x) = \frac{\log(\log x)}{\log x}$

$$\Rightarrow \quad f'(x) = \frac{\log x \cdot \frac{1}{\log x} \cdot \frac{1}{x} - \log(\log x) \cdot \frac{1}{x}}{(\log x)^2}$$

$$= \frac{1 - \log(\log x)}{x(\log x)^2}$$

$$\therefore \quad f'(e) = \frac{1 - \log(\log e)}{e(\log e)^2} = \frac{1}{e}$$

65. Given, circle is intersection of sphere.

$$x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0 \qquad \ldots\text{(i)}$$

and plane $x - 2y + 2z + 8 = 0 \qquad \ldots$(ii)

Centre of sphere is $(-1, 1, 2)$.

P = length of the perpendicular from $(-1, 1, 2)$ upon Eq. (ii)

$$= \frac{-1 - 2 + 4 + 8}{\sqrt{1 + 4 + 4}} = \frac{9}{3} = 3$$

R = Radius of the sphere $= \sqrt{1 + 1 + 4 + 19} = 5$

Radius of the circle $= \sqrt{R^2 - P^2} = \sqrt{25 - 9}$

$$= \sqrt{16} = 4$$

66. We have, $x^y = e^{x - y}$

$$\Rightarrow \quad y \log x = (x - y)\log e = x - y$$

$$\Rightarrow \quad y = \frac{x}{1 + \log x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1 + \log x) \cdot 1 - x \cdot \frac{1}{x}}{(1 + \log x)^2} = \frac{\log x}{(1 + \log x)^2}$$

67. We have, $y = \sin(m\sin^{-1} x)$

$$\frac{dy}{dx} = \cos(m\sin^{-1} x) \cdot \frac{m}{\sqrt{1 - x^2}}$$

$$\Rightarrow (1 - x^2)\left(\frac{dy}{dx}\right)^2 = m^2\cos^2(m\sin^{-1} x)$$

$$= m^2(1 - y^2)$$

On differentiating w.r.t. x, again, we get

$$(1-x^2)2\cdot\frac{dy}{dx}\cdot\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\cdot(-2x)$$

$$=-2m^2y\frac{dy}{dx}$$

$$\Rightarrow \quad (1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=-m^2y$$

[$\because$ Dividing by 2dy/dx]

68. We have, $f(x)=(ax+b)\sin x+(cx+d)\cos x$

$f'(x)=a\sin x+(ax+b)\cos x+c\cos x$
$-(cx+d)\sin x$

But $f'(x)=x\cos x$ for all x (given)

$\therefore\ x\cos x=(a-d)$
$\sin x+(b+c)\cos x+ax\cos x-cx\sin x$

Equating the coefficients of sin x, cos x, x cos x and x sin x, we get

$$a-d=0,\ b+c=0,\ a=1,\ c=0$$

$\therefore b=c=0$ and $a=d=1$

69. Let v be the velocity of the particle when the distance covered is s. Then,

$$v\propto\sqrt{s} \qquad \text{(given)}$$

$$\Rightarrow \quad v=\lambda\sqrt{s}$$

$$\Rightarrow \quad \frac{dv}{dx}=\frac{\lambda}{2\sqrt{s}}$$

$$\Rightarrow \quad v\frac{dv}{ds}=\frac{\lambda v}{2\sqrt{s}}=\frac{\lambda^2}{2}=\text{constant}$$

$\therefore$ The acceleration is constant

70. We have, $f(x)=e^x\sin x$

$$\Rightarrow \quad f'(x)=e^x\cos x+\sin x\cdot e^x$$

and $f''(x)=-\sin x\cdot e^x+\cos x\cdot e^x+\cos x\cdot e^x$
$+\sin x\cdot e^x$

Now, for maximum or minimum slope put $(f'(x))'=0\Rightarrow f''(x)=0$

$$\Rightarrow \quad 2\cos x\cdot e^x=0$$

$$\Rightarrow \quad \cos x=0\Rightarrow x=\frac{\pi}{2}$$

Also, $f'''(x)=-2\sin x\cdot e^x+2\cos x\cdot e^x=$ negative

$\therefore$ Slope is maximum at $x=\pi/2$.

71. Clearly, $f(2)=-1$

$$\Rightarrow \quad -1=\frac{2a+b}{(2-1)(2-4)} \Rightarrow 2a+b=2$$

Now, $\quad f'(x)=\dfrac{4a+5b-2bx-ax^2}{(x-1)^2(x-4)^2}$,

$$f'(2)=0$$

$$\Rightarrow \quad b=0 \Rightarrow a=1$$

$$\Rightarrow \quad f'(x)=-\frac{(x-2)(x+2)}{(x-1)^2(x-4)^2}$$

Clearly, for $x>2$, $f'(x)<0$ and for $x<2$, $f'(x)>0$.

Thus, $x=2$ is indeed the point of local maxima for $y=f(x)$

72. We have, $z=\tan(y+ax)+\sqrt{y-ax}$

$$\Rightarrow z_x=a\sec^2(y+ax)+\frac{(-a)}{2\sqrt{y-ax}}$$

$$\Rightarrow z_{xx}=2a^2\sec^2(y+ax)\tan(y+ax)$$
$$-\frac{a^2}{4}(y-ax)^{-3/2}$$

and $\quad z_y=\sec^2(y+ax)+\dfrac{1}{2\sqrt{y-ax}}$

$$\Rightarrow z_{yy}=2\sec^2(y+ax)\tan(y+ax)$$
$$-\frac{1}{4}(y-ax)^{-3/2}$$

$$a^2z_{yy}=2a^2\sec^2(y+ax)\cdot\tan(y+ax)$$
$$-\frac{a^2}{4}(y-ax)^{-3/2}$$

Now, $\quad z_{xx}-a^2z_{yy}=0$

73. We have, $\int f(x)\,dx=F(x)$

$$\therefore\ \int x^3 f(x^2)\,dx=\frac{1}{2}\int x^2f(x^2)\,d(x^2)$$

$$=\frac{1}{2}[x^2F(x^2)-\int F(x^2)d(x^2)]$$

74. $\displaystyle\int\frac{x^2+4}{x^4+16}\,dx$

$$=\int\frac{1+\frac{4}{x^2}}{x^2+\frac{16}{x^2}}\,dx=\int\frac{d\left(x-\frac{4}{x}\right)}{\left(x-\frac{4}{x}\right)^2+8}$$

$$=\int\frac{dt}{t^2+(2\sqrt{2})^2},\ \text{where } t=x-\frac{4}{x}$$

$$=\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2-4}{2\sqrt{2}\,x}\right)+c$$

75. Let $I=\displaystyle\int\frac{1}{(x+1)\sqrt{x^2-1}}\,dx$

Put $\quad x+1=\dfrac{1}{t}$

$$\Rightarrow \quad dx=-\frac{1}{t^2}\,dt,\ \text{then}$$

$$I = \int \frac{1}{\frac{1}{t}\sqrt{\left(\frac{1}{t}-1\right)^2 - 1}}\left(\frac{-1}{t^2}\right)dt$$

$$\Rightarrow \quad I = -\int \frac{dt}{\sqrt{1-2t}} = -\int (1-2t)^{-1/2}dt$$

$$= -\frac{(1-2t)^{1/2}}{(-2)\left(\frac{1}{2}\right)} + c$$

$$\Rightarrow \quad I = \sqrt{1-2t} + c$$

$$\Rightarrow \quad I = \sqrt{1 - \frac{2}{x+1}} + c$$

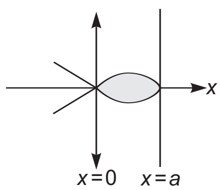
$$\Rightarrow \quad I = \sqrt{\frac{x-1}{x+1}} + c$$

76. $\int_{\pi/2}^{3\pi/2} [\sin x]dx = \int_{\pi/2}^{\pi} [\sin x]dx + \int_{\pi}^{3\pi/2} [\sin x]\, dx$

$$= \int_{\pi/2}^{\pi} 0\, dx + \int_{\pi}^{3\pi/2} (-1)dx = -[x]_{x}^{3\pi/2}$$

$$= -\left(\frac{3\pi}{2} - \pi\right) = -\frac{\pi}{2}$$

77. The area of the loop of the curve $ay^2 = x^2(a-x)$ is given by



$$2\int_0^a y\, dx = 2\int_0^a x\sqrt{\frac{a-x}{a}}dx$$

$$= 4a^2\int_0^{\pi/2} \sin^3\theta \cdot \cos^2\theta \cdot d\theta$$

[Let $x = a\sin^2\theta$

$\Rightarrow \quad dx = 2a\sin\theta \cdot \cos\theta \cdot d\theta$]

$$= 4a^2 \cdot \frac{2 \cdot 1}{5 \cdot 3 \cdot 1} = \frac{8a^2}{15} \text{ sq unit.}$$

78. The given differential equation can be written as

$$x = e^{xy\frac{dy}{dx}}$$

$$\Rightarrow \quad \log x = xy\frac{dy}{dx} \Rightarrow ydy = \frac{\log x}{x}dx$$

$$\Rightarrow \quad ydy = \log x\, d(\log x)$$

On integrating, we get

$$\frac{y^2}{2} = \frac{(\log x)^2}{2} + c$$

$$\Rightarrow \quad y^2 = (\log_e x)^2 + 2c$$

$$\Rightarrow \quad y = \pm\sqrt{(\log_e x)^2 + 2c}$$

79. We have, $\frac{dy}{dx} = \frac{ax+3}{2y+f}$

$$\Rightarrow \quad (ax+3)\,dx = (2y+f)dy$$

On integrating, we obtain

$$a\frac{x^2}{2} + 3x = y^2 + fy + c$$

$$\Rightarrow \quad -\frac{a}{2}x^2 + y^2 - 3x + fy + c = 0$$

This will represent a circle, if

$$-\frac{a}{2} = 1$$

$$(\because \text{coefficient of } x^2 = \text{coefficient of } y^2)$$

$$\Rightarrow \quad a = -2$$

80. $\int_1^5 x^2dx$

$$= \frac{4}{4}\left[\frac{1}{2}f(1) + f(2) + f(3) + f(4) + \frac{1}{2}f(5)\right]$$

$$= \left[\frac{1}{2} + 4 + 9 + 16 + \frac{25}{2}\right] = 42 \text{ [using } f(x) = x^2]$$