SAMPLE PAPER - 4

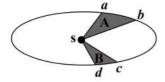
Class 11 - Physics							
Time Al	lowed: 3 hours	Maximum Mark	s: 70				
General	Instructions:						
	1. There are 35 questions in all. All que	stions are compulsory.					
	2. This question paper has five sections: Section A, Section B, Section C, Section D and Section E. All the section						
	are compulsory.						
	3. Section A contains eighteen MCQ of 1 mark each, Section B contains seven questions of two marks each, Se						
	C contains five questions of three marks each, section D contains three long questions of five marks each and						
	Section E contains two case study based questions of 4 marks each.						
	4. There is no overall choice. However, an internal choice has been provided in section B, C, D and E. You						
	attempt only one of the choices in su	ch questions.					
	5. Use of calculators is not allowed.						
		Section A					
1.	If energy (E), velocity (u), and force (F) mass?	be taken as fundamental quantities, then what are the dimensions of	[1]				
	a) [Ev ²]	b) [Fv ⁻²]					
	c) [Ev ⁻²]	d) [Fv ⁻¹]					
2.	2. A force acts on a 2 kg object, so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds?						
	a) 875 J	b) 850 J					
	c) 950 J	d) 900 J					
3.	3. A particle is moving along a straight line parallel to x-axis with constant velocity. Its angular momentum a the origin:		[1]				
	a) decreases with time	b) remains constant					
	c) is zero	d) increases with time					
4.	When a weight of 10 kg is suspended from a copper wire of length 3 m and diameter 0.4 mm, its length increases by 2.4 cm. If the diameter of the wire is doubled, then the extension in its length will be		[1]				
	a) 1.2 cm	b) 4.8 cm					
	c) 7.6 cm	d) 0.6 cm					

The given figure shows the motion of a planet around the sun S in an elliptical orbit with the sun at the focus.

[1]

5.

The shaded areas A and B are also shown in the figure which can be assumed to be equal. If t_1 and t_2 represent the time taken for the planet to move from a to b and c to d respectively, then:



a) $t_1 = t_2$

- b) $t_1 < t_2$
- c) from the given information the relation $% \left(\frac{1}{2}\right) =-\frac{1}{2}\left(\frac{1}{2}\right) =-\frac{1}$
- d) $t_1 > t_2$

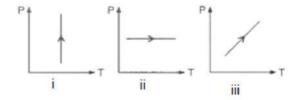
between t₁ and t₂ cannot be determined

6. Pressure versus temperature graphs of an ideal gas are as shown in figure. Choose the wrong statement.

[1]

[1]

[1]



a) None of these

- b) Density of gas is decreasing in graph (ii).
- c) Density of gas is constant in graph (iii).
- d) Density of gas is increasing in graph (i).
- 7. A gas under constant pressure of 4.5×10^5 Pa when subjected to 800 kJ of heat, changes the volume from 0.5 m² to 2.0 m³. The change in internal energy of the gas is:
 - a) $5.25 \times 10^5 J$

b) $1.25 \times 10^5 J$

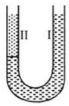
c) $6.75 \times 10^5 J$

- d) $3.25 \times 10^5 J$
- 8. If in an experiment for determination of velocity of sound by resonance tube method using a tuning fork of 512 [1] Hz, first resonance was observed at 30.7 cm and second was obtained at 63.2 cm, then a maximum possible error in velocity of sound is:
 - a) 51.2 cm/sec

b) 102.4 cm/sec

c) 161.3 cm/sec

- d) 204.8 cm/sec
- 9. A U-tube of uniform cross-section (see Fig) is partially filled with a liquid I. Another liquid II which does not mix with the liquid I is poured into one side. It is found that the liquid levels of the two sides of the tube are the same, while the level of liquid I have risen by 2 cm. If the specific gravity of liquid I is 1.1, the specific gravity of liquid II must be



a) 1.1

b) 1.05

c) 1.0

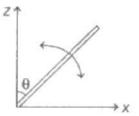
- d) 1.12
- 10. The gravitational potential at the centre of a square of side a and four equal masses (m each) placed at the corners of a square is:
 - a) 0

b) $-4\sqrt{2}\frac{Gm}{a}$

		C 2
6)	1. /2.	Gm-
4	-4V 2	0

d)
$$4\sqrt{2}\frac{Gm}{a}$$

11. A cylinder uniform rod of mass M and length l is pivoted at one end so that it can rotate in a vertical plane (see the figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical, is



a) $\frac{3g}{2l}\cos\theta$

b) $\frac{2g}{3l}\cos\theta$

c) $\frac{3g}{2l}\sin\theta$

- d) $\frac{2g}{3l}\sin\theta$
- 12. The amount of heat energy required to raise the temperature of 1 g of Helium at NTP, from T_1K to T_2K is
 - a) $\frac{3}{2}$ N_ak_B(T₂ T₁)

b) $\frac{3}{8}$ N_ak_B(T₂ - T₁)

[1]

[1]

c) $\frac{3}{4}$ N_ak_B(T₂ - T₁)

- d) $\frac{3}{4}N_ak_B\left(\frac{T_2}{T_1}\right)$
- 13. If in an experimental determination of the velocity of sound using a Kundt's tube, standing waves are set up in the metallic rod as well as in rigid tube containing air, then the sound waves have the same:
 - a) wavelengths

b) amplitudes

c) frequencies

- d) particle velocities
- 14. The work done W during an isothermal process in which the gas expands from an initial volume V_1 to a final volume V_2 is given by: (R is gas constant, T is temperature)
 - a) 2RT $\log_e\left(\frac{V_1}{V_2}\right)$

b) $R(V_2 - V_1)log_e\left(\frac{T_1}{T_2}\right)$

c) R(T₂ - T₁)log_e $\left(\frac{V_2}{V_1}\right)$

- d) RT $\log_e\left(\frac{V_2}{V_1}\right)$
- 15. The escape velocity of a projectile on the earth's surface is 11.2 km s⁻¹. A body is projected out with thrice this speed. The speed of the body far away form the earth will be:
 - a) None of these

b) 33.6 km s⁻¹

c) 31.7 km s⁻¹

- d) 22.4 km s⁻¹
- 16. **Assertion (A):** During a turn, the value of centripetal force should be less than the limiting frictional force. **[1] Reason (R):** The centripetal force is provided by the frictional force between the tyres and the road.
 - a) Both A and R are true and R is the correct explanation of A.
- b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

for assertion.

- d) A is false but R is true.
- 17. **Assertion:** Water is more elastic than air.

- **Reason:** Air is more compressible than water.
 - a) Assertion and reason both are correct statements and reason is correct explanation
- Assertion and reason both are correct statements but reason is not correct explanation for assertion.

	 c) Assertion is correct statement but reason is wrong statement. 	 d) Assertion is wrong statement but reason is correct statement. 			
18.	Assertion (A): Avogadro number is the number of ato		[1]		
10.	Reason (R): Avogadro number is a dimensionless constant.				
	a) Both A and R are true and R is the correct	b) Both A and R are true but R is not the			
	explanation of A.	correct explanation of A.			
	c) A is true but R is false.	d) A is false but R is true.			
	Sec	tion B			
19.	State the number of significant figures in the following:				
	i. 6.032 N m ⁻²				
	ii. 0.0006032 m ²				
20.	A cricket ball of mass 0.15 kg moving with a speed of 20 ms^{-1} is brought to rest by a player in 0.1 s . What is the				
	average force applied by the player?				
21.	Define the period of revolution. Derive an expression of a period of revolution or time period of the satellite. [2] OR				
	Calculate the escape speed of a body from the solar sy	stem from the following data:			
	i. Mass of the sun = 2×10^{30} kg.				
	ii. Separation of the earth from the sun = 1.5 $ imes$ 10^{11}	m.			
22.	Calculate the value of stress in a wire of steel having a radius of 2 mm of 10 kN of force is applied on it.				
23.	Calculate the total number of degrees of freedom possessed by the molecules in $1\ cm^3$ of H_2 gas at temperature				
	273 K and 1 atm pressure?				
		OR			
	On the basis of kinetic theory, obtain a definition of ab	solute zero temperature.			
24.	A bullet travelling with a velocity of 16 ms ⁻¹ penetrates a tree trunk and comes to rest in 0.4 m. Find the time taken during the retardation.				
25.	An aeroplane requires for take off a speed of 80 kmh ⁻¹ , the run on the ground being 100 m. The mass of the				
	aeroplane is 10 ⁴ kg and the coefficient of friction between the plane and the ground is 0.2. Assume that the plane				
	accelerates uniformly during the take off. What is the maximum force required by the engine of the plane for				
	take off?				
	Sec	tion C			
26.	The volume of steam produced by 1 g of water at 100°	°C is 1650 cm ³ . Calculate the change in internal energy	[3]		
	during the change of state given $J = 4.2 \times 10^7$ erg cal				
27.	A pebble of mass 0.05 kg is thrown vertically upwards pebble,	s. Give the direction and magnitude of the net force on the	[3]		
	a. during its upward motion,				
	b. during its downward motion,				
	c. at the highest point where it is momentarily at rest.	Do your answers change if the pebble was thrown at an			
	angle of 45° with the horizontal direction? Ignore	nir resistance.			
28.	The flow rate of water is 0.58 L/mm from a tap of diameter $\frac{1}{2}$	meter of 1.30 cm. After some time, the flow rate is	[3]		

increased to 4 L/min. Determine the nature of the flow for both the flow rates. The coefficient of viscosity of water is 10^{-3} Pa - s and the density of water is 10^{3} kg/m³.

OR

What is equation of continuity? Water plows through a horizontal pipe of radius, 1cm at a speed of 2m/s. What should be the diameter of nozzle if water is to come out at a speed of 10m/s?

29. Explain the terms wavelength, frequency and amplitude for a harmonic wave.

[3]

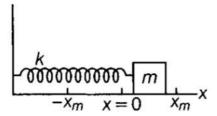
OR

The equation of a wave is given by $y = 6 \sin 10\pi t + 8 \cos 10\pi t$, where y is in centimetre and t in second. Determine the constants involved in the standard equation of the wave.

30. Two rods of the same area of cross-section, but of lengths l_1 and l_2 and conductivities K_1 and K_2 are joined in series. Show that the combination is equivalent of a material of conductivity $K = \frac{l_1 + l_2}{\left(\frac{l_1}{K_1}\right)\left(\frac{l_2}{K_2}\right)}$

Section D

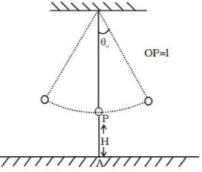
31. Consider a block of mass 700 g is fastened to a spring having spring constant of 70 N/m. Find out the following parameters if block is pulled a distance of 14 cm from its mean position on a frictionless surface and released from rest at t =0.



- i. The angular frequency, the frequency and the period of the resulting motion.
- ii. The amplitude of the oscillation.
- iii. The maximum speed of the oscillating block.
- iv. The maximum acceleration of the block.
- v. The phase constant and hence the displacement function x (t).

OR

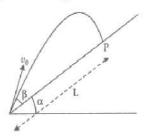
A simple pendulum of time period 1s and length l is hung from fixed support at O, such that the bob is at a distance H vertically above A on the ground (Figure). The amplitude is θ_o . The string snaps at $\theta \frac{\theta_0}{2}$. Find the time taken by the bob to hit the ground. Also, find the distance from A where bob hits the ground. Assume θ_o to be small so that $\sin \theta_o$ and $\cos \theta_o$ 1.



- 32. A particle is projected in air at an angle β to a surface which itself is inclined at an angle α to the horizontal as in [5] figure
 - i. Find an expression of range on the plane surface [distance on the plane from the point of projection at which particle will hit the surface.]

- ii. Time of flight.
- iii. β at Which range will be maximum.

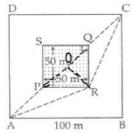
[Hint: This problem can be solved in two different ways:



- a. Point P at which particle hits the plane can be seen as the intersection of its trajectory (parabola) and straight line. Remember particle is projected at an angle $(\alpha + \beta)$ w.r.t. horizontal.
- b. We can take x-direction along the plane and y-direction perpendicular to the plane. In that case resolve g (acceleration due to gravity) in two different components, g_x along the plane and g_y perpendicular to the plane. Now the problem can be solved as two independent motions in x and y directions respectively with time as a common parameter.]

OR

A man wants to reach from A to the opposite corner of the square C (as in figure). The sides of the square are 100 m. A central square of $50m \times 50m$ is filled with sand. Outside this square, he can walk at a speed 1 m/s⁻¹. In the central square, he can walk only at a speed of vm/s (v<1) What is smallest value of v for which he can reach faster via a straight path through the sand than any path in the square outside the sand?



33. From a uniform disk of radius R, a circular hole of radius $\frac{R}{2}$ is cut out. The centre of the hole is at $\frac{R}{2}$ from the centre of the original disc. Locate the centre of gravity of the resulting flat body.

OR

Find the components along the x, y, z axes of the angular momentum l of a particle, whose position vector is r with components x, y, z and momentum is p with components p_x , p_y and p_z . Show that if the particle moves only in the x-y plane the angular momentum has only a z-component.

Section E

34. Read the text carefully and answer the questions:

[4]

2 friends started for a picnic spot, in two different cars. A drove his car at a constant velocity 60 km/h. B drove his car at a constant velocity 50 km/h.

The velocity of B relative to A is v_B- v_A

Similarly, the velocity of object A relative to object B is v_A-v_B

Their friend C was supposed to wait at a point on the road for a lift. Both of them forgot to pick up C. A and B reached the picnic spot within 2 hours and 2 hours 24 minutes respectively.

- (i) What was the velocity of B relative to A?
- (ii) What is the velocity of A relative to B?

(iii) What are the velocities of A and B relative to C?

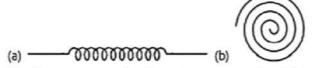
OR

Draw the Velocity vs. time plot for A?

35. Read the text carefully and answer the questions:

[4]

There are many types of spring. Important among these are helical and spiral springs as shown in the figure.



Usually, we assume that the springs are massless. Therefore, work done is stored in the spring in the form of the elastic potential energy of the spring. Thus, the potential energy of a spring is the energy associated with the state of compression or expansion of an elastic spring.

- (i) When the potential energy of a spring may be considered as zero?
- (ii) The ratio of spring constants of two springs is 2 : 3. What is the ratio of their potential energy, if they are stretched by the same force?
- (iii) The potential energy of a spring increases by 15 J when stretched by 3 cm. If it is stretched by 4 cm, What will be the increase in potential energy?

OR

The potential energy of a spring when stretched through a distance x is 10 J. What is the amount of work done on the same spring to stretch it through an additional distance x?

Solution

SAMPLE PAPER - 4

Class 11 - Physics

Section A

1. **(c)** $[Ev^{-2}]$

Explanation: According to Einstein mass-energy relation, energy $(E) = (mass) \times (velocity of light)$

or [mass] =
$$\left[\frac{\text{energy}}{(\text{velocity of light})^2}\right] = \frac{E}{v^2} = [\text{Ev}^{-2}]$$

2. **(d)** 900 J

Explanation: Here, the displacement of an object is given by $x = (3t^2 + 5)m$

Therefore, velocity (v) =
$$\frac{dx}{dt} = \frac{d(3t^2+5)}{dt}$$

or
$$v = 6t \text{ m/s ...(i)}$$

The work done in moving the object from t = 0 to t = 5 s

$$W = \int\limits_{x_0}^{x_5} F \cdot dx \,$$
 ...(ii)

The force acting on this object is given by $F = ma = m \times \frac{dv}{dt}$

= m
$$\times \frac{d(6t)}{dt}$$
 [.: using (i)]

$$F = m \times 6 = 6m = 12N$$

Also,
$$x_0 = 3t^2 + 5 = 3 \times (0)^2 + 5 = 5$$
 m and at $t = 5$ s,

$$x_5 = 3 \times (5)^2 + 5 = 80 \text{ m}$$

Put the values in Eq. (ii),

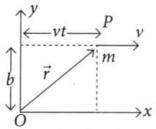
W =
$$12 \times \int_{x_0}^{x_5} dx = 12 [80 - 5]$$

$$W = 12 \times 75 = 900 J$$

3. (b) remains constant

Explanation:

Suppose the particle of mass m is moving with speed v parallel to x-axis as shown in figure, then at any time t coordinates of P will be



$$x = vt$$
, $y = b$ and $z = 0$

While components of velocity will be $v_x = v$, $v_y = 0$ and $v_z = 0$

(As it is moving parallel to x-axis)

So,
$$\vec{L} = \vec{r} \times \vec{p}$$

$$= m \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ vt & b & 0 \\ v & 0 & 0 \end{vmatrix}$$

$$= \hat{k}m[vt \times 0 - vb]$$

$$= -\text{mvbk}$$

4. **(d)** 0.6 cm

Explanation: For constant F, l, Y,

$$\Delta l \propto rac{1}{r^2}$$

$$\therefore rac{\Delta l_2}{\Delta l_1} = \left(rac{r_1}{r_2}
ight)^2 = \left(rac{1}{2}
ight)^2 = rac{1}{4}$$
 $\Delta l_2 = rac{1}{4} imes 2.4 ext{ cm} = 0.6 ext{ cm}$

$$\Delta l_2 = \frac{1}{4} imes 2.4 ext{ cm} = 0.6 ext{ cm}$$

5. **(a)**
$$t_1 = t_2$$

Explanation: Second law of Kepler states that the radius vector from sun to the planet swaps equal area in equal time.

(c) Density of gas is constant in graph (iii).

Explanation:
$$\rho = \frac{PM}{RT}$$

Density ρ remains constant when P/T or volume remains constant. In graph (i) volume is decreasing, hence density is increasing; while in graphs (ii) and (iii) volume is increasing, hence, density is decreasing.

[Note: That volume would have been constant in case the straight line in graph (iii) had passed through origin.]

7. **(b)**
$$1.25 \times 10^5 \text{J}$$

Explanation:
$$P = 4.5 \times 10^5 \text{ Pa}$$
; $dQ = 800 \text{ kJ}$

$$V_1 = 0.5 \text{ m}^3$$
; $V_2 = 2\text{m}^3$

$$dW = P(V_2 - V_1) = 4.5 \times 10^5 (2 - 0.5) = 6.75 \times 10^5 J$$

Change in internal energy,

$$dU = dQ - dW$$

=
$$800 \times 10^3$$
 - 6.75×10^5 = 1.25×10^5 J

(a) 51.2 cm/sec

Explanation: For first resonance:

$$30.7 = \frac{\lambda}{4} + e$$
(i)

$$63.2 = \frac{3\lambda}{4} + e$$
(ii)

$$63.2 - 30.7 = \frac{\lambda}{2}$$

or
$$\lambda$$
 = (65.0 ± 0.1) cm

(Because maximum error in measurement of length using metric scale would be 1 mm)

$$v = \lambda v = (65.0 \pm 0.1) \times 512 \text{ cm/sec}$$

Hence, maximum error in velocity will be 51.2 cm/sec

9. (a) 1.1

Explanation:



Pressure in limb I at B = Pressure in limb II at A

$$h\rho_1g = h\rho_2g$$

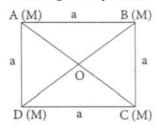
$$\Rightarrow \rho_1 = \rho_2$$

Hence specific gravity of liquid II = sp gravity of liquid I = 1.1

10. **(b)**
$$-4\sqrt{2}\frac{Gm}{a}$$

Explanation:

According to the question,



From the above figure,

$$OA = OB = OC = OD$$

$$= \frac{a\sqrt{2}}{2}$$

$$= \frac{a}{2}$$

Total gravitational potential at the centre of the square

$$= \frac{-Gm \times 4}{OA}$$

$$= \frac{-4Gm}{a/\sqrt{2}}$$

$$-4\sqrt{2}Gm$$

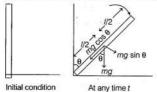
$$= \frac{-4\sqrt{2}Gm}{a}$$

$$(c) \frac{3g}{2l} \sin \theta$$

11.

Explanation:

As the rod rotates in the vertical plane so torque is acting on it, which is due to the vertical component of the weight of the rod.



Now, Torque au = force imes perpendicular distance of line of action of force from axis of rotation

$$= mg\sin\theta imes rac{l}{2}$$

Again, Torque,
$$au = I lpha$$

Where, 7 = moment of inertia =
$$\frac{ml^2}{3}$$

[Force and Torque frequency along axis of rotation passing through in end]

$$\alpha$$
 = angular acceleration

$$\therefore mg\sin\theta imes rac{l}{2} = rac{ml^2}{3}lpha \ \therefore lpha = rac{3g\sin heta}{2l}$$

12. **(b)**
$$\frac{3}{8}$$
 N_ak_B(T₂ - T₁)

Explanation:
$$Q = \frac{f}{2}nR\Delta T$$

= $\frac{3}{2} \times \frac{1}{4} \times k_B N_a (T_2 - T_1)$
= $\frac{3}{8} N_a k_B (T_2 - T_1)$

13. (c) frequencies

Explanation: Speed, wavelength and amplitude change as it is travelling through different material on the other side frequency must remain constant to conserve energy (which is dependent solely on frequency).

14. **(d)** RT
$$\log_{e} \left(\frac{V_2}{V_1} \right)$$

Explanation: Work done W, is given by

$$W = -\int_{W_1}^{W_2} pdV$$

Since the expansion is isothermal:

$$\begin{aligned} & pV = RT \\ & \text{or } p = \frac{RT}{V} \\ & \therefore W = -RT \int\limits_{V_1}^{V_2} \frac{dV}{V} \\ & = -RT[\log_e V]_{V_1}^{V_2} \\ & W = -RT \log_e \frac{V_2}{V_1}, = RT \log_e \left(\frac{V_2}{V_1}\right) \end{aligned}$$

Explanation: Escape velocity of a projectile from the Earth, $v_{esc} = 11.2 \text{ km/s}$

Projection velocity of the projectile, $v_p = 3v_{esc}$

Let velocity of the projectile far away from the Earth = v_f

Total energy of the projectile on the Earth = $rac{1}{2}mv_p^2-rac{1}{2}mv_{esc}^2$

Gravitational potential energy of the projectile far away from the Earth is zero.

Total energy of the projectile far away from the Earth = $\frac{1}{2}mv_f^2$

From the law of conservation of energy, we have $\frac{1}{2}mv_p^2 - \frac{1}{2}mv_{esc}^2 = \frac{1}{2}mv_f^2$

$$v_{\mathrm{f}}$$
 = $\sqrt{v_{p}^{2}-v_{esc}^{2}}$

$$=\sqrt{8}v_{esc}$$

$$=\sqrt{8}\times11.2$$

$$= 31.68 \text{ km/s} = 31.7 \text{ km/s}$$

16. (a) Both A and R are true and R is the correct explanation of A.

Explanation: The body is able to move on a circular path due to centripetal force. The centripetal force in case of vehicle is provided by frictional force. Thus, if the value of frictional force, μ mg is less than centripetal force, then it is not possible for a vehicle to take a turn and the bicycle would overturn. Thus, condition for no overturning of vehicle is,

$$\mu mg \geq \frac{mv^2}{r}$$

17. (a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

Explanation: Assertion and reason both are correct statements and reason is correct explanation for assertion.

18. (c) A is true but R is false.

Explanation: Avogadro number (N) represents the number of atoms in 1 gram mole of an element, i.e. it has the dimensions of mole⁻¹.

Section B

19. i.4

Explanation: Significant figure- 6, 0, 3, 2. 0's between 2 non-zero digits are significant.

ii. 4

Explanation: Significant figure- 6, 0, 3, 2. Since, **the number is less than 1**, the zeros on the right to the decimal before the first non-zero integer is insignificant.

20. Here m = 0.15 kg, u = 20 ms⁻¹, v = 0 and t = 0.1 s

Average force applied by the player, F = ma =
$$\frac{m(v-u)}{t} = \frac{0.15 \times (0-20)}{(0.1)}$$
 = -30 N

The negative sign suggests that the force is a retarding force.

21. Period of revolution of a satellite, T is the time taken by the satellite to complete one revolution around the earth.

$$\begin{split} \therefore T &= \frac{\text{Circumference of circular orbit}}{\text{Orbital velocity}} \\ \text{or } T &= \frac{2\pi r}{v_o} \\ \text{or } T &= \frac{2\pi (R+h)}{v_o} \text{ [$\cdot\cdot:$ r = R + h$]} \\ \text{or } T &= 2\pi (R+h) \sqrt{\frac{R+h}{GM}} \text{ [$\cdot:$ v_o$ } = \sqrt{\frac{GM}{R+h}}$]} \\ \text{or } T &= 2\pi \sqrt{\frac{(R+h)^3}{GM}} \\ \text{Also, } T &= 2\pi \sqrt{\frac{(R+h)^3}{GM}} \\ \text{or } T &= 2\pi \sqrt{\frac{(R+h)^3}{gR^2}} \\ \therefore gR^3 &= GM \\ \therefore T &= 2\pi \sqrt{\frac{(R+h)^3}{gR^2}} \end{split}$$

Suppose M be the mass of the sun and R be the distance of the earth from the sun, then escape velocity,

$$v_e = \sqrt{rac{2GM}{R}} = \sqrt{rac{2 imes 6.67 imes 10^{-11} imes 2 imes 10^{30}}{1.5 imes 10^{11}}} ext{ ms}^{-1} = \sqrt{rac{4 imes 6.67}{1.5}} imes 10^4 ms^{-1} = 4.217 imes 10^4 ms^{-1}
onumber \ v_e = 42.17 \ kms^{-1}$$

 \therefore The escape speed for the solar system is $42.17 \ km s^{-1}$

22. Force,
$$F = 10kN = 1 \times 10^4 N$$

Radius,
$$r = 2mm = 2 \times 10^{-3}m$$

Area,
$$A = \pi r^2 = \pi \times (2 \times 10^{-3})^2$$

$$=12.56 imes 10^{-6} m^2$$

$$= 12.56 \times 10^{-8} m^{-8}$$
Stress = $\frac{\text{Force}}{\text{Area}} = \frac{1 \times 10^{4} N}{12.56 \times 10^{-6} m^{2}} = 7.96 \times 10^{8} N/m^{2}$

23. At 273 K temperature and 1 atm pressure means STP condition.

 \therefore Number of H₂ molecules in volume of 22400 cm³ at STP = 6.02 \times 10²³(Avogadro's number)

Hence, number of H₂ molecules in 1 cm³ of volume at STP

$$=rac{6.02 imes10^{23}}{22400}=2.6875 imes10^{19}$$

Now, number of degrees of freedom associated with each H_2 (diatomic) molecule = 5

Total number of degrees of freedom associated with 1cm^3 of gas = total number of molecules × total degrees of freedom = $2.6875 imes 10^{19} imes 5 = 1.34375 imes 10^{20}$

OR

According to the kinetic interpretation of temperature, mean kinetic energy of a gas molecule is directly proportional to its absolute temperature i.e., $\frac{1}{2}m\overline{v}^2 \propto T$ or $\overline{v} \propto \sqrt{T}$.

If gas temperature T = 0 K, the rms speed of gas molecules and hence kinetic energy of each molecule will be zero. Thus, the absolute zero temperature is the temperature at which all molecular motions stop altogether.

24. Here u = 16 ms, v = 0, s = 0.4 m, t = ?

As
$$v^2 - u^2 = 2$$
 as

$$\therefore 0^2 - 16^2 = 2a \times 0.4$$

or a =
$$-\frac{16 \times 16}{2 \times 0.4}$$
 = -320 ms⁻²

or a =
$$-\frac{16 \times 16}{2 \times 0.4}$$
 = -320 ms⁻²
Times, t = $\frac{v, -u}{a}$ = $\frac{0 - 16}{-320}$ = 0.05 s

25. Here u = 0, s = 100m

$$v = 80 \text{ kmh}^{-1} = 80 \times \frac{5}{18} = \frac{200}{9} \text{ ms}^{-1}$$

As
$$v^2 - u^2 = 2as$$

$$\therefore \left(\frac{200}{9}\right)^2 - 0 = 2a \times 100$$
or $a = \frac{40000}{81 \times 200} = \frac{200}{81} \text{ ms}^{-2}$

or
$$a = \frac{40000}{81 \times 200} = \frac{200}{81} \text{ ms}^{-2}$$

Force required to produce acceleration a,

$$F_1 = ma = 10^4 \times \frac{200}{81} = 2.47 \times 10^4 \text{ N}$$

Force required to overcome friction,

$$F_2 = \mu R = \mu mg = 0.2 \times 10^4 \times 9.8 = 1.96 \times 10^4 \text{ N}$$

Maximum force required by the engine for take off,

$$F = F_1 + F_2 = 2.47 \times 10^4 + 1.96 \times 10^4 = 4.43 \times 10^4 \text{ N}.$$

Section C

26. Here J =
$$4.2 \times 10^7$$
 erg cal⁻¹

Latent heat of steam, L = 540 cal g^{-1}

Mass of water =
$$1 g$$

Temperature of water = 100°C

Initial volume,
$$V_1 = 1 \text{ cm}^3$$

Final volume, $V_2 = 1650 \text{ cm}^3$

.: Change in volume,

$$dV = V_2 - V_1 = 1650 - 1 = 1649 \text{ cm}^3$$

When 1 g of water at 100°C is changed to steam at 100°C, temperature remains constant. so the heat supplied is

$$dQ = mL = 1 \times 540 = 540 \text{ cal} = 540 \times 4.2 \times 10^7 \text{ erg}$$

Pressure, P = 1 atm =
$$76 \times 13.6 \times 981$$
 dyne cm⁻²

From first law of thermodynamics,

$$dU = dQ - PdV$$

$$= 540 \times 4.2 \times 10^7 - 76 \times 13.6 \times 981 \times 1649$$

=
$$22.68 \times 10^9$$
 - 1.67×10^9

$$= 21.01 \times 10^9 = 2.1 \times 10^{10} \text{ erg}$$

27. When an object is thrown vertically upward or it falls vertically downward under gravity then an acceleration $g = 10m/s^{-2}$ acts downward due to the earth's gravitational pull.

Mass of pebble (m) = 0.05 kg

i. During upward motion

Net force acting on pebble (F) = ma = $0.05 \times 10 \text{ N}$

- = 0.50N (vertically downward)
- ii. During downward motion

Net force acting on pebble (F) = ma = $0.05 \times 10 \text{ N}$

- = 0.50N (vertically downward)
- iii. At the highest point

Net force acting on pebble

(F) = ma =
$$0.05 \times 10N$$

- = 0.50 N (vertically downward) If pebble was thrown at an angle of 45° with the horizontal direction then acceleration acting on it and therefore force acting on it will remain unchanged, i.e., 0.50 N (vertically downward). In case , at the highest point the vertical component of velocity will be zero but horizontal component of velocity will not be zero.
- 28. Given, diameter, $D = 1.30cm = 1.3 \times 10^{-2}m$

Coefficient of viscosity of water, $\eta = 10^{-3} Pa - s$

Density of water, $ho = 10^3 kg/m^3$

The volume of the water flowing out per second is

$$V = vA = v \times \pi r^2 = v\pi \frac{D^2}{4}$$

Reynold's number, $R_e = \frac{\rho v \overset{7}{D}}{\eta} = \frac{4pv}{\eta \pi D}$

Case I When
$$V=0.58L/min=rac{0.58 imes10^{-3} ext{m}^3}{1 imes60 ext{s}}$$

$$=9.67\times 10^{-6}m^3s^{-1}$$

$$R_{e} = \tfrac{4 \times 10^{3} \times 9.67 \times 10^{-6}}{10^{-3} \times 3.14 \times 1.3 \times 10^{-2}} = 948$$

 $\therefore R_e < 1000$, so the flow is steady or streamline

Case II When
$$V = 4L/min$$

$$=rac{4 imes 10^{-3}}{60}~m^3s^{-1}=6.67~ imes 10^{-5}m^3s^{-1}$$

$$R_e = \frac{\frac{4 \times 10^3 \times 6.67 \times 10^{-5}}{10^{-3} \times 3.14 \times 1.3 \times 10^{-2}} = 6536$$

 \therefore R_e > 3000, so the flow will be turbulent.

OR

The Navier–Stokes equations form a vector continuity equation describing the conservation of linear momentum. If the fluid is an incompressible flow (p is constant), the mass continuity equation simplifies to a volume continuity equation: which means that the divergence of velocity field is zero everywhere. Consider a non-viscous liquid in streamline flow through a tube A B of varying cross-section



Let a_1 , a_2 = area of cross-section at A and B

 V_1 , V_2 = Velocity of flow of liquid at A and B

 S_1 , S_2 = Density of liquid at A and B

Volume of liquid entering per second at $A = a_1 v_1$

Mass of liquid entering per second at $A = a_1 v_1 s_1$

Moss of liquid entering per second at $B = a_1 v_1 s_2$.

If there is no loss of liquid in tube and flow is steady, then

Mass of liquid entering per second at A = Mass of liquid leaving per second at B

$$a_1 v_1 s_1 = a_2 v_2 s_2$$

If the liquid is incompressible,

$$s_1 = s_2 = s$$

$$\mathbf{a}_1 \, \mathbf{v}_1 \, \mathbf{s} = \mathbf{a}_2 \mathbf{v}_2 \mathbf{s}$$

$$a_1 v_1 = a_2 v_2$$

or a
$$v = constant$$

i. e V
$$\propto \frac{1}{a}$$

It means the larger the area of cross-section, the smaller will be the flow of liquid.

Here 1
$$D_1 = 2r_1 = 2 \times 1 = 2cm$$

$$D_2 = ?$$

$$V_1 = 2m|s$$

$$V_2 = 10 \text{m/s}$$

$$D_1 = Diameter$$

$$R_1 = Radius$$

$$V_1$$
 = velocity

$$a=\pi r^2$$
 , D=2r

$$=\pi \frac{d^2}{4}, \frac{D}{2}=r$$

From equation of continuity

$$a_1 v_1 = a_2 v_2$$

$$\Rightarrow \left(\pirac{D_1^2}{4}
ight)^2 \quad x
u_1 = \left(\pirac{D_2^2}{4}
ight)
u_2$$

$$D_2 = D_1 \left(\frac{v_1}{v_2}\right)^{\frac{1}{2}}$$

$$=2\left(rac{2}{10}
ight)^{rac{1}{2}} =2 imesrac{1}{\sqrt{5}}$$

$$=2\times\frac{1}{\sqrt{2}}$$

$$=2 imes rac{1}{2.236}$$

$$D_2 = 0.894 \text{ cm}$$

Hence, diameter of the outer opening is 0.894 cm as flow rate through this area is high as compared to initial one.

- 29. i. The wavelength of a harmonic wave is the distance covered by the wave motion during the time in which a medium particle completes one vibration to and fro about its mean position. Alternately, it is the distance (parallel to the direction of wave propagation) between the consecutive repetitions of the shape of a wave. It is the minimum distance between two consecutive points in the same phase.
 - ii. Frequency of a harmonic wave is the number of vibrations per unit time by a medium element as the wave passes through it. The frequency of a wave is defined as reciprocal of its time period and is related to angular frequency ω by the relation, Frequency $\nu = \frac{1}{T} = \frac{\omega}{2\pi}$

SI unit of frequency is s⁻¹ or Hz.

iii. The amplitude of a harmonic wave is the magnitude of maximum displacement of a medium particle (or element) from its equilibrium position as the wave passes through it. The amplitude of a wave is a positive quantity and its SI unit is 1 metre.

OR

Given, $y = 6 \sin 10\pi t + 8 \cos \pi \text{ cm...(i)}$

Now the general equation of this type of wave is

$$y = A \sin(\omega t + \phi)$$

- $=A\sin\omega t\cos\phi+A\cos\omega t\sin\phi$
- $= (A\cos\phi)\sin\omega t + (A\sin\phi)\cos\omega t$ (ii)

Comparing Eqs.(i) and (ii), we get

$$A\cos\phi=6$$
.....(iii)

and A sin
$$\phi = 8$$
(iv)

Time period,
$$T=rac{2\pi}{\omega}=rac{2\pi}{10\pi}=0.2 ext{s}$$

Squaring and adding Eqs.(iii) and (iv), we get

$$A^2 (\cos^2 \phi + \sin^2 \phi) = 6^2 + 8^2$$

$$= 36 + 64 = 100$$

or
$$A^2 = 100$$

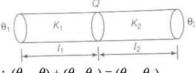
 \therefore A = 10 cm, the amplitude of the given wave.

Dividing Eq.(iv) by (iii), we get

$$\tan \phi = \frac{8}{6} = 1.3333$$

 $\therefore \phi = \tan^{-1}(1.3333) = 53^{\circ}8'$, the value of phase angle of the given wave in the question

30. It is given conductivities K_1 and K_2 are in series, Therefore rate of flow of heat energy is same.



$$\therefore (\theta_1 - \theta) + (\theta - \theta_2) = (\theta_1 - \theta_2)$$

i,e.
$$\frac{\theta}{t} \frac{l_1}{K_1 A} + \frac{\theta}{t} \frac{l_2}{K_2 A} = \frac{\theta}{t} \cdot \frac{(l_1 + l_2)}{K_e q A}$$

$$\Rightarrow \frac{l_1}{K_1} + \frac{l_2}{K_2} = \frac{l_1 + l_2}{K_{eq}}$$

$$\therefore K_{eq} = \frac{l_1 + l_2}{\left(\frac{l_1}{K_1} + \frac{l_2}{K_2}\right)}$$

$$\Rightarrow \frac{l_1}{K_1} + \frac{l_2}{K_2} = \frac{l_1 + l_2}{K_{eq}}$$

$$\therefore K_{eq} = rac{ extstyle l_1 + extstyle l_2}{\left(rac{ extstyle l_1}{ extstyle K_1} + rac{ extstyle l_2}{ extstyle K_2}
ight)}$$

Section D

31. i. The angular frequency is given by

$$\omega = \sqrt{rac{k}{m}} = \sqrt{rac{70 \mathrm{N/m}}{0.700 \mathrm{kg}}}$$
 = 10 rad/s

Frequency,
$$f = \frac{\omega}{2\pi} = \frac{10}{2\pi} \simeq 1.59 \mathrm{Hz}$$

Frequency,
$$f = \frac{\omega}{2\pi} = \frac{10}{2\pi} \simeq 1.59 \mathrm{Hz}$$

The time period, $T = \frac{1}{f} = \frac{1}{1.59} = 0.63 = 630 \mathrm{\ ms}$

ii. The maximum amplitude of the oscillation = maximum displacement

$$\therefore x_{\rm m} = 14 \text{ cm} = 0.14 \text{ m}$$

iii. The maximum speed of the oscillation v_{m} is given by

$$v_{\rm m}$$
 = $\omega x_{\rm m}$ =10 $imes$ 0.14 =1.4 m/s

iv. The magnitude of maximum acceleration of the block is given by

$$a_{\rm m} = \omega^2 x_{\rm m} = 100 \times 0.14 = 14 \text{ m/s}^2$$

At time t = 0, the block is located at position, $x = x_m$

v. Then, from general equation of oscillation, x(t) = ${
m x_m}{
m cos}\left(\omega t + \phi
ight)$

$$\Rightarrow x_m = x_m \cos(0 \times \omega + \phi)$$

$$\therefore \cos \phi = 1 \Rightarrow \phi = 0$$

The required displacement function of the given oscillation with all the above values becomes,

$$x(t) = x_m \cos(\omega t + \phi)$$

$$\Rightarrow$$
 x(t) = 0.14 \times cos(10t + 0)

$$\Rightarrow$$
x(t) = 0.14 cos 10t

OR

Assume that t = 0 when $\theta = \theta_0$. Then,

$$\theta = \theta_0 \cos \omega t$$

Given a seconds pendulum $\omega=2\pi$

At time
$$t_1$$
, let $\theta = \frac{\theta_0}{2}$

$$\therefore \quad \cos 2\pi t_1 = 1/2 \Rightarrow t_1 = \frac{1}{6}$$

$$heta = - heta_0 2\pi \sin 2\pi t \quad \left[heta = rac{\mathrm{d} heta}{\mathrm{d}t}
ight]$$

At
$$t_1 = \frac{1}{6}$$

$$\theta = -\theta_0 2\pi \sin \frac{2\pi}{6} = -\sqrt{3}\pi\theta_0$$

Thus the linear velocity is

 $u = -\sqrt{3}\pi\theta_0 l$ perpendicular to the string.

The vertical component is

$$u_y = -\sqrt{3}\pi\theta_0 t \sin\theta_0$$

and the horizontal component is

$$u_x = -\sqrt{3}\pi\theta_0 l\cos\theta_0$$

At the time it snaps, the vertical height is

$$H' = H + l \left(1 - \cos\left(rac{ heta_0}{2}
ight)
ight)$$

Let the time required for fall be t, then

$$H' = u_g t + (1/2)gt^2$$

(notice g is also in the negative direction)

or,
$$\frac{1}{2}gt^2 + \sqrt{3}\pi\theta_0 l\sin\theta_0 t - H' = 0$$

$$\therefore t = \frac{-\sqrt{3}\pi\theta_0 l \sin\theta_0 \pm \sqrt{3\pi^2\theta_0^2} e^2 \sin^2\theta_0 + 2gH'}{-\sqrt{3}\pi l\theta_0^2 \pm \sqrt{3\pi^2\theta_0^4} l^2 + 2gH'}$$

$$-\sqrt{3}\pi l\theta_{0}^{2}\pm\sqrt{3\pi^{2}\theta_{0}^{4}l^{2}+2gH'}$$

Neglecting terms of order θ_0^2 and heigher.

$$\sqrt{\frac{2H'}{g}}$$

Now H' H + l(1 - 1) = H :
$$t\sqrt{\frac{2H}{g}}$$

The distance travelled in the x-direction is u_x t to the left of where it snapped.

$$X = \sqrt{3}\pi heta_0 l\cos heta_0\sqrt{rac{2H}{g}}$$

To order of $heta_0$

$$X = \sqrt{3}\pi heta_0 l \sqrt{rac{2H}{g}} = \sqrt{rac{6H}{g}} heta_0 l$$

At the time of snapping, the bob was

 $l\sin\theta_0$ $l\theta_0$ distance from A.

Thus, the distance from A is

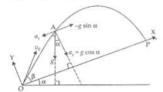
$$l heta_o - \sqrt{rac{6H}{g}}l heta_o = l heta_o(1-\sqrt{6H/g})$$

32. Consider new Cartesian coordinates in which X-axis is along inclined plane OP and OY-axis perpendicular to it as shown in the

Consider the motion of the projectile from OAP.

$$a_y = -g\cos\alpha$$

$$a_x = g \sin \alpha$$



AT O and P,
$$y = 0$$

$$u_y = v_o \sin(\beta)$$
 , t = T where T is time of flight

We calculate the time Of flight part (b) before part (a):

The motion of projectile along new OY axis.

Using the equation: $s = ut + \frac{1}{2}gt^2$

$$s=0,\quad u=u_y=v_0\sineta\;,\;g=g_y=-g\coslpha\;,\;t=T$$

$$0 = v_0 \sin \beta(T) + \frac{1}{2}(-g\cos\alpha)T^2$$

$$0 = v_0 \sin \beta(T) - \frac{g}{2} \cos \alpha(T)^2$$

$$T\left[v_0\sineta-Trac{g}{2}\coslpha
ight]=0$$

Either
$$T=0$$
 or $v_0\sin eta - rac{gt}{2}\cos lpha = 0$

$$\frac{gT}{2}\cos\alpha = v_o\sin\beta$$

$$\therefore$$
 Time of flight from O to P is, $T = \frac{2v_0 \sin \beta}{g \cos \alpha}$

∴ Time of flight from O to P is,
$$T = \frac{2v_0 \sin \beta}{g \cos \alpha}$$

At T = 0, projectile is at O and at T = $\frac{2v_0 \sin \beta}{g \cos \alpha}$, it is at P.

a. Consider motion along OX axis x = L, $u_x = v_0 \cos(\beta)$, $a_x = -g \sin(\alpha)$

$$t = T = \frac{2v_0 \sin \beta}{g \cos \alpha}$$

$$s = u_x t + \frac{1}{2} a_x t^2$$

$$L = v_0 \cos eta(T) + rac{1}{2}(-g \sin lpha)T^2 = T\left[v_0 \cos eta - rac{1}{2}g \sin lpha.T
ight] \ L = rac{2v_0 \sin eta}{g \cos lpha}\left[v_o \cos eta - rac{1}{2}g \sin lpha \cdot rac{2v_0 \sin eta}{g \cos lpha}
ight]$$

$$L = rac{2v_0\sineta}{g\coslpha}igg[v_o\coseta - rac{1}{2}g\sinlpha\cdotrac{2v_0\sineta}{g\coslpha}$$

$$L = rac{2v_0^2\sineta}{g\cos^2lpha}[\coseta\cdot\coslpha-\sineta\sinlpha] \ \Rightarrow L = rac{2v_0^2\sineta}{g\cos^2lpha}\cos(lpha+eta) \,\,\, [{
m Range~On~the~Plane~Surface}]$$

b. Time of flight done above.

c. L will be maximum or maximum range along new OX axis.

From above relation of L, it will be maximum when $\sin(\beta)\cos(\alpha+\beta)$ is maximum as ' α ' is a constant angle of inclination of the plane.

so, $\cos^2(\alpha)$ is constant.

$$Z = \sin(\beta) \cos(\alpha + \beta)$$

$$Z = \sin \beta [\cos \alpha \cos \beta - \sin \alpha \sin \beta]$$

$$Z = \frac{1}{2} \left[\cos \alpha 2 \sin \beta \cos \beta - \sin \alpha 2 \sin^2 \beta \right]$$

$$Z = \frac{1}{2} [\cos \alpha \sin 2\beta - \sin \alpha (1 - \cos 2\beta)]$$

$$Z = \frac{1}{2} [\cos \alpha \sin 2\beta - \sin \alpha + \sin \alpha \cos 2\beta]$$

$$Z = \frac{1}{2}[\cos \alpha \sin 2\beta + \sin \alpha \cos 2\beta - \sin \alpha]$$

$$Z = \frac{1}{2}[\sin(2\beta + \alpha) - \sin \alpha]$$

For Z to be maximum

$$\sin(2\beta + \alpha) = 1$$

$$\sin(2\beta + \alpha) = \sin 90^{\circ}$$

$$2\beta + \alpha = 90^{\circ}$$

$$2\beta = 90^{\circ} - \alpha$$

$$\Rightarrow eta = rac{90^\circ}{2} - rac{lpha}{2} = 45^\circ - rac{lpha}{2}$$

$$\therefore \beta = \left(\frac{\pi}{4} - \frac{\alpha}{2}\right) \text{ radian}$$

OR

Let us first calculate the lengths of PQ and AC , $PQ = \sqrt{50^2 + 50^2} = 50\sqrt{2}$

$$AC = \sqrt{100^2 + 100^2} = 100\sqrt{2}$$

Time(T1) taken through path $A \to P \to Q \to C$

$$T_1 = rac{(AP+QC)}{1m/s} + rac{PQ}{v}$$

$$T_1 = rac{AC - PQ}{4C - PQ} + rac{PQ}{
u} = 100\sqrt{2} - 50\sqrt{2} + rac{50\sqrt{2}}{v}$$

$$T_1 = 50\sqrt{2} + rac{50\sqrt{2}}{2} = 50\sqrt{2}(1 + rac{1}{v})$$

Time taken along the path $A o R o C = rac{(AR+RC)}{1} = 2AR = T_2$,

Using Pythagoras theorem, we get

$$AR^2 = AO^2 + OR^2 = (\frac{100\sqrt{2}}{2})^2 + (\frac{50\sqrt{2}}{2})^2 = 5000 + 1250 = 6250$$

$$AR = \sqrt{6250} = 25\sqrt{10}$$
 s

$$T_2 = 2 \times 25\sqrt{10}s = 50\sqrt{10}s$$

For $T_{sand} < T_{outside}$, we have

$$50\sqrt{2}[1+\frac{1}{3}]<50\sqrt{10}$$

$$[1+\frac{1}{2}]<\sqrt{5}$$

$$\frac{1}{v} < \sqrt{5} - 1$$

$$v < rac{1}{(\sqrt{5}-1)} = rac{3.3}{4} = 0.83 m/s$$

$$v < 0.82 \text{ m} / \text{s}$$

33. The centre of mass of an object is the point at which the object can be balanced. Mathematically, it is the point at which the torques from the mass elements of an object sum to zero. The centre of mass is useful because problems can often be simplified by treating a collection of masses as one mass at their common centre of mass. The weight of the object then acts through this point.

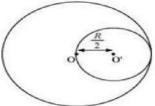
To solve this problem, first we assume that the whole disc was present whose centre of mass lies at the origin from which a small disc was cut out. So CM of remaining portion and cut out disc will lie exactly at the origin i.e Centre of Mass of the original disc at x = 0

Mass per unit area of the original disc = σ

Radius of the original disc = R

Mass of the original disc, $M = \pi R^2 \sigma$

The disc with the cut portion is shown in the following figure:



Radius of the smaller disc = $\frac{R}{2}$

Mass of the smaller disc, $M'=\pi\Big(\frac{R}{2}\Big)^2\sigma=\frac{1}{4}\pi R^2\sigma=\frac{M}{4}$

Let O and O' be the respective centers of the original disc and the disc cut off from the original. As per the definition of the centre of mass, the centre of mass of the original disc is supposed to be concentrated at O, while that of the smaller disc is supposed to be concentrated at O'.

It is given that:

$$OO' = \frac{R}{2}$$

After the smaller disc has been cut from the original, the remaining portion is considered to be a system of two masses. The two masses are:

M (concentrated at O), and

$$\left(-M' = \frac{M}{4}\right)$$
 concentrated at O'

(The negative sign indicates that this portion has been removed from the original disc.)

Let x be the distance through which the centre of mass of the remaining portion shifts from point O.

The relation between the centers of masses of two masses is given as:

$$x=rac{m_{1}r_{1}+m_{2}r_{2}}{m_{1}+m_{2}}$$

For the given system

$$x = \frac{M \times 0 - M \times \left(\frac{R}{2}\right)}{M + (-M')} \text{ (here M' is M/4)}$$
$$= \frac{\frac{-M}{4} \times \frac{R}{2}}{M - \frac{M}{4}} = \frac{-MR}{8} \times \frac{4}{3M} = \frac{-R}{6}$$

Note that shift in Centre of Mass is very less(only 0.16 R or $\frac{R}{6}$) as removed portion has very less mass as compared to the remaining portion.

(The negative sign indicates that the centre of mass gets shifted toward the left of point O and lies at $\frac{R}{6}$ left towards origin.)

OR

$$l_{x} = yp_{z} - zp_{y}$$
$$l_{y} = zp_{x} - xp_{z}$$

$$l_z = xp_y - yp_x$$

The linear momentum of the particle in cartesian coordinate, $ec{p}=p_x\hat{i}+p_y\hat{j}+p_z\hat{k}$

Position vector of the particle in cartesian coordiantes , $\vec{r}=x\hat{i}+\hat{y}+z\hat{k}$

As we know the angular momentum of a moving particle about a point is given as, $\vec{l}=\vec{r}\times\vec{p}$ where p and r are linear momentum and position vector respectively,

$$egin{aligned} &= \left(x \hat{i} + \hat{y}' + z \hat{k}
ight) imes \left(p_x \hat{i} + p_y \hat{j} + p_z \hat{k}
ight) \ &= \left| egin{aligned} \hat{i} & \hat{j} & \hat{k} \ x & y & z \ p_x & p_y & p_z \end{aligned}
ight| \ &l_x \hat{i} + l_y \hat{j} + l_z \hat{k} = \hat{i} \left(y p_z - z p_y
ight) - \hat{j} \left(x p_z - z p_x
ight) + \hat{k} \left(x p_y - y p_x
ight) \ &= = \hat{i} \left(y p_z - z p_y
ight) + \hat{j} \left(-x p_z + z p_x
ight) + \hat{k} \left(x p_y - y p_x
ight) \end{aligned}$$

Comparing the coefficients of $\hat{i},\hat{j},$ and \hat{k} we get the components of angular momentum as :

$$l_x = yp_z - zp_y$$

$$l_y = xp_z - zp_x \dots (i)$$

$$l_z = xp_y - yp_x$$

b) If the particle moves in the x-y plane only. Hence, the z-component of the position vector and z component of linear momentum vector become zero, i.e.,

$$z = p_z = 0$$

Thus, equation (i) reduces to:

$$l_x = 0$$

$$l_v = 0$$

$$l_z = xp_v - yp_x$$

Therefore, when the particle is confined to move in the x-y plane, the x and y components of linear momentum are zero and hence the direction of angular momentum is along the z-direction.

Section E

34. Read the text carefully and answer the questions:

2 friends started for a picnic spot, in two different cars. A drove his car at a constant velocity 60 km/h. B drove his car at a constant velocity 50 km/h.

The velocity of B relative to A is v_B- v_A

Similarly, the velocity of object A relative to object B is v_A-v_B

Their friend C was supposed to wait at a point on the road for a lift. Both of them forgot to pick up C. A and B reached the picnic spot within 2 hours and 2 hours 24 minutes respectively.

(i) The velocity of B relative to A is

$$v_B - v_A = 50 - 60$$

= -10 km/h

(ii) The velocity of A relative to B is

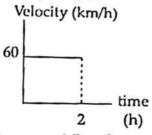
$$v_A - v_B = 60 - 50 = 10 \text{ km/h}$$

(iii)Since C is in stationary position, his velocity was 0.

Hence the velocity of A relative to C is 60 - 0 = 60 km/h and the velocity of B relative to C is 50 - 0 = 50 km/h.

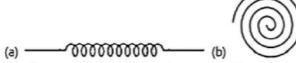
OR

Velocity time graph is as shown below



35. Read the text carefully and answer the questions:

There are many types of spring. Important among these are helical and spiral springs as shown in the figure.



Usually, we assume that the springs are massless. Therefore, work done is stored in the spring in the form of the elastic potential energy of the spring. Thus, the potential energy of a spring is the energy associated with the state of compression or expansion of an elastic spring.

(i) It may be considered as zero, when the spring is in normal position.

(ii)
$$\frac{E_1}{E_2} = \frac{k_1 x_1^2}{k_2 x_2^2}$$

here
$$x_1 = x_2$$

so
$$\frac{E_1}{E_2} = \frac{k_1}{k_2} = \frac{2}{3}$$

$$\begin{array}{c} \text{so } \frac{E_1}{E_2} = \frac{k_1}{k_2} = \frac{2}{3} \\ \text{(iii)} \frac{E_1}{E_2} = \frac{k \ x_1^2}{k \ x_2^2} \end{array}$$

here $x_1 = 3$ cm and $x_2 = 4$ cm

so
$$\frac{E_1}{E_2} = \frac{x_1^2}{x_2^2} = \frac{9}{16}$$

here
$$E_1 = 15 J$$

so E
$$_2$$
 = $\frac{16}{9}$ $imes$ E_1 = $\frac{16}{9}$ $imes$ 15 = 26.7 J

$$egin{aligned} rac{E_1}{E_2} &= rac{k x_1^2}{k x_2^2} \ E &= rac{1}{2} k x^2 \end{aligned}$$

if distance = x,
$$E_1 = \frac{1}{2} kx^2 = 10 J$$

on increasing the distance x more

$$E_2 = \frac{1}{2}k(2x)^2 = 2kx^2$$

so increase in potential energy = E_2 - E1 = (2 - $\frac{1}{2}$) KX^2

$$=rac{3}{2}Kx^2=3 imesrac{1}{2}kx^2=3 imes10$$
 = 30 Joule

so work required = 30 - 10 = 20 Joule