Exponents and Powers

Exponents

Any number of the form a^n, where n is a natural number and "a" is a real number is called the exponents. Here n is called the power of the number a. Power may be positive or negative.

For any rational number $\left(\frac{a}{b}\right)^n$, n is the power of the rational number.

$$\left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^n = \frac{a}{b}\times\frac{a}{b}\times\frac{a}{b}\times\frac{a}{b}\times\frac{a}{b}\times----\times\frac{a}{b} \text{ (n-times)} = \frac{a^n}{b^n}$$

Illustrative EXAMPLE

$$x^n = x\times x\times x\times x\times x\times---\times x$$

$$x^{-n} = \frac{1}{x\times x\times x\times x\times x\times x\times---\times x}$$

Also,

$$x^0 = 1;\ x^1 = x;\ x^{-1} = \frac{1}{x} \text{ and } x^{-n} = \frac{1}{x^n}$$

Illustrative EXAMPLE

$$3^0 = 1$$

$$3^1 = 3$$

$$3^{-1} = \frac{1}{3}$$

Laws of Exponent

There are various laws of exponents. They are laws of addition, laws of multiplication and laws of division.

(i) $a^m \times a^n = a^{m+n}$ (ii) $\frac{a^m}{a^n} = a^{m+n}$

(iii) $a^m \times b^m = (a\times b)^m$ (iv) $\left[\left(\frac{a}{b}\right)^n\right]^m = \left(\frac{a}{b}\right)^{mn}$

(v) $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ (vi) $\left(\frac{a}{b}\right)^0 = 1$

(vii) $(ab)^n = a^n b^n$

Important Points to keep in Mind

- $x^0 = 1$, where $x \neq 0$. x can be anything (except zero), including numbers, variables, or an equation.
- $x^1 = x$
- $x^{-n} = \frac{1}{x^n}$, Where $x \neq 0$

Illustrative EXAMPLE

(i) $2^3 2^5 = 2^{3+5} = 2^8$ (ii) $w^2 w^3 = w^5$ (iii) $xy^2x^3y^3x^4y^4 = x^8y^9$

While working with exponents there are certain rules that we need to remember.

$4^2 \times 4^5 = 47$

It means: $4\times4\times4\times4\times4\times4\times4$ or 4.4.4.4.4.4.4

Add the exponent, if base are same.

Uses of Exponents

The exponents can be used for various purposes such as comparing large and small numbers, expressing large and small numbers in the standard forms. It is used to express the distance between any two celestial bodies which cannot be expressed in the form of normal denotion. It is also useful in writing the numbers in scientific notation. The size of the microorganisms is very-very small and it cannot be written in normal denotion and can easily be expressed in exponential form.

Radicals Expressed with Exponents

Radicals are the fractional exponents of any number. Index of the radical becomes the denominator of the fractional power.

$$\sqrt[n]{a} = \frac{1}{a^n}$$

i.e. $\sqrt{9} = \sqrt[2]{9} = 9^{\frac{1}{2}} = 3$

Express $\sqrt[3]{2}\sqrt[4]{2}$ as a Single Radical Term

Let us convert the radicals to exponential expressions, and then apply laws of exponent to combine the factors:

$$\sqrt[3]{2}\ \sqrt[4]{2} = 2^{\frac{1}{3}}\ 2^{\frac{1}{4}} = 2^{\frac{1}{3}+\frac{1}{4}} = 2^{\frac{7}{12}} = \sqrt[12]{2^7}$$

Illustrative EXAMPLE

Simplify $\frac{\sqrt{5}}{\sqrt[3]{5}}$

Solution:

$$\frac{5^{\frac{1}{2}}}{5^{\frac{1}{3}}} = 5^{\frac{1}{2}-\frac{1}{3}} = 5^{\frac{1}{6}}$$

Illustrative EXAMPLE

$$\left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)$$

Solution:

$$= \frac{2\times2\times2\times2}{3\times3\times3\times3} = \frac{2^4}{3^4} = \frac{16}{81}$$

Expand: $\left(\frac{x}{10}\right)^5$

Solution:

Raising the top and bottom numbers to the power of 5 gives:

$$\left(\frac{x}{10}\right)^5 = \frac{x^5}{10^5} = \frac{x^5}{100000}$$

You Must KNOW

- Zero raised to the power is not always zero.
- The positive quantity raised to the power of negative index is always positive.
- Zero was discovered by the Babylonians in Mesopotamia in around 300 B.C.
- The rule of mathematics given by Brahma gupta is known as Brahmas phutasiddhanta.
- The number which appears maximum number of times in the decimal expansion of pie upto the 6 billion decimal places.

SUMMARY

$a^m \times a^n = a^{m+n}$

$(a^m)^n = a^{mn}$

$(ab)^n = a^n b^n$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$a^0 = 1$

$a^{-n} = \frac{1}{a^n}$

Commonly Asked QUESTIONS

Simplify: $(-5)^3$

(a) -125 (b) -120
(c) -105 (d) -121
(e) None of these

Answer: (a)

Explanation

$(-5)(-5)(-5) = -125$

Number -5 has exponent 3. Therefore, option (a) is correct and rest of the options is incorrect.

Simplify: $-(3)^3 - (-3)^2 + (-2)^2$

(a) -31 (b) -21
(c) -32 (d) -33
(e) None of these

Answer: (c)

Explanation

$-(3\times3\times3) - (-3\times-3) + (-2\times-2) = -27 - 9 + 4 = -32$

Find the value of x such that $\left(\frac{64}{125}\right)^2\left(\frac{4}{5}\right)^4\left(\frac{16}{25}\right)^{2x+1}=\left(\frac{256}{625}\right)^{3x}$

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$

(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

(e) None of these

Answer: (a)

By what number $\left(-\frac{4}{3}\right)^{-5}$ **must be multiplied so that the result is** $\frac{16}{9}$**.**

(a) $\left(\frac{4}{3}\right)^5$ (b) $\left(\frac{4}{3}\right)^7$

(c) $\left(\frac{4}{3}\right)^{-5}$ (d) $\left(\frac{4}{3}\right)^{-7}$

(e) None of these

Answer: (b)

The scientific notation of 165000000000000 is given by:

(a) 16.5×10^{13} (b) 165×10^{12}

(c) 1650×10^{11} (d) 1.65×10^{14}

(e) None of these

Answer: (d)

Self Evaluation TEST

Duration 10 Minutes

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1. **Find the value of the given expression** $\left(\frac{5}{9}\right)^{(-2)} \times \left(\frac{3}{5}\right)^{(-2)} \times \left(\frac{3}{5}\right)^{(0)}$ **:**

(a) 10 (b) 9
(c) 20 (d) 30
(e) None of these

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2. **Simplify:** $x^2y^2x^5y^3$ **.**

(a) x^7y^5 (b) x^7y^7
(c) x^5y^5 (d) x^5y^7
(e) None of these

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3. **The value of the expression** $\frac{4^{-5}\times 10^{-6}\times 625}{40^{-8}\times 5^2\times 4^2}$ **is given by:**

(a) 2.5×10^4 (b) 3.5×10^5
(c) 4.5×10^5 (d) 5.5×10^5
(e) None of these

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4. **Find the value of m such that** $\frac{(16)^{2m+1}(64)}{(256)^2\times 4} = (256)^{3m}$ **.**

(a) 1 (b) 0
(c) 4 (d) 5
(e) None of these

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5. **The value of** $\frac{1}{2}\left[\frac{-7}{4}+\frac{5}{3}+\frac{-5}{6}+\frac{1}{3}+\frac{-1}{2}\right]+\frac{1}{3}\left[\frac{-12}{5}+\frac{-7}{20}+\frac{3}{14}+\frac{1}{7}+\frac{-1}{10}\right]$ **:**

(a) $\frac{-1153}{840}$
(b) $\frac{128}{105}$
(c) $\frac{128}{105}$
(d) $\frac{-125}{256}$ (e) None of these

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6. **The value of** $1 \div \left[\left\{ \left(\frac{2}{3} \right)^2 \right\}^3 \times \left(\frac{1}{3} \right)^{-4} \times 3^{-1} \times 6^{-1} \right] + \left[\left(\frac{1}{3} \right)^{-3} - \left(\frac{1}{2} \right)^{-3} \right] \div \left(\frac{1}{4} \right)^3$

(a) $\frac{181}{64}$ (b) $\frac{151}{64}$

(c) $\frac{172}{21}$ (d) $\frac{147}{32}$

(e) None of these

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7. **Evaluate** $\left(\frac{-1728}{2744} \right)^{\frac{2}{3}} \left(\frac{4096}{9216} \right)^{\frac{1}{2}} - \left(\frac{729}{343} \right)^{\frac{2}{3}}$

(a) 0 (b) $\frac{4208}{2152}$

(c) $-\frac{57}{49}$ (d) $\frac{2152}{4208}$

(e) None of these

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8. **Evaluate** $\left[\left(\sqrt{\frac{2}{3}} \right)^2 - \sqrt[3]{\frac{8}{27}} \right]^{1000}$

(a) 0 (b) 1

(c) 2 (d) -1 (e) None of these

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9. **Simplify:** $(5an^{-2})^{-1}$.

(a) $\frac{n^2}{5a}$ (b) $\frac{n}{5a}$

(c) $\frac{n^3}{5a}$ (d) $\frac{n^4}{5a}$ (e) None of these

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10. **Simplify** $\left(\frac{a^{-2}}{b^2} \right)^{-3} \left(\frac{a^{-3}}{b^5} \right)^2 =$

(a) $\frac{1}{b^2}$ (b) $\frac{1}{b^4}$

(c) $\frac{a}{b}$ (d) $\frac{a^2}{b^3}$ (e) None of these

Answers – Self Evaluation Test

1.	B	**2.**	A	**3.**	A	**4.**	A	**5.**	A	**6.**	A	**7.**	C	**8.**	A	**9.**	A	**10.**	B

Self Evaluation Test
SOLUTIONS

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1. $\frac{81}{25}\times\frac{25}{9}\times 1 = 9$

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2. $= x^2x^5y^2y^3$

$= x^7y^5$

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3. $\frac{4^{-5}\times 10^{-6}\times 625}{4^{-8}\times 10^{-8}\times 4^2} = \frac{4^3\times 10^2\times 625}{4^2}$

$= 2.5\times 10^5$

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4. The above expression get reduced to the form

$4^{4m+1}\times 4^{15} = 4^{2m+8}$

$4^{4m+16} = 4^{12m+8}$

$4m+16 = 12m+8$

$8m = 8$

$m = 1$

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9. $(5an^{-2})^{-1} = \frac{n^2}{5a}$
