Chapter : 11. APPLICATIONS OF DERIVATIVES

Exercise : 11A

Question: 1

The side of a squ

Solution:

Let the side of the square be a

Rate of change of side= $\frac{da}{dt} = 0.2 \text{ cm/s}$

Perimeter of the square = 4a

Rate of change of perimeter = $4\frac{da}{dt} = 4 \times 0.2$

$\frac{dP}{dt} = 0.8 \text{ cm/s}$

Question: 2

The radius of a c

Solution:

Let the radius of the circle be r

$\frac{dr}{dt} = 0.7 \text{cm/s}$

Circumference of the circle= $2\pi r$

Rate of change of circumference $= 2\pi\frac{dr}{dt}$

$= 2 \times 3.14 \times 0.7$

$\frac{dC}{dt} = 4.4 \text{ cm/s}$

Question: 3

The radius of a c

Solution:

Let the radius of the circle be r

$\frac{dr}{dt} = 0.3 \text{cm/s}$

Area of the circle= πr^2

Rate of change of Area $= 2\pi r \frac{dr}{dt}$

$= 2 \times 3.14 \times 10 \times 0.3$

$\frac{dA}{dt} = 18.84 \text{ cm}^2/\text{s}$

Question: 4

The side of a squ

Solution:

Let the side of the square be a

Rate of change of side $= \dfrac{da}{dt} = 3$ cm/s

Area of the square $= a^2$

Rate of change of Area $= 2a\dfrac{da}{dt} = 2 \times 10 \times 3$

$$\dfrac{dA}{dt} = 60 \text{ cm}^2/\text{s}$$

Question: 5

The radius of a c

Solution:

Soap bubble will be in the shape of a sphere

Let the radius of the soap bubble be r

$$\dfrac{dr}{dt} = 0.2 \text{cm/s}$$

Surface area of the soap bubble $= 4\pi r^2$

Rate of change of Surface area $= 8\pi r\dfrac{dr}{dt}$

$= 8 \times 3.14 \times 7 \times 0.2$

$$\dfrac{dS}{dt} = 35.2 \text{ cm}^2/\text{s}$$

Question: 6

The radius of an

Solution:

Soap bubble will be in the shape of a sphere

Let the radius of the soap bubble be r

$$\dfrac{dr}{dt} = 0.5 \text{cm/s}$$

Volume of the soap bubble $= \dfrac{4}{3}\pi r^3$

Rate of change of Volume $= 4\pi r^2\dfrac{dr}{dt}$

$= 4 \times 3.14 \times 1^2 \times 0.5$

$$\dfrac{dV}{dt} = 6.28 \text{ cm}^3/\text{s}$$

Question: 7

The volume of a s

Solution:

Let the radius of the balloon be r

Let the volume of the spherical balloon be V

$$V = \dfrac{4}{3}\pi r^3$$

$$\dfrac{dV}{dt} = 4\pi r^2\dfrac{dr}{dt}$$

$$25 \text{ cm}^3/\text{s} = 4 \times \pi \times 5^2 \times \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{4\pi}$$

Surface area of the bubble $= 4\pi r^2$

Rate of change of Surface area $= 8\pi r \frac{dr}{dt}$

$$= 8 \times \pi \times 5 \times \frac{1}{4\pi}$$

$$\frac{dS}{dt} = 10 \text{ cm}^2/\text{s}$$

Question: 8

A balloon which a

Solution:

When we pump a balloon its volume changes.

Let the radius of the balloon be r

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$900 \text{ cm}^3/\text{s} = 4 \times \pi \times 15^2 \times \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{900}{4 \times 3.14 \times 225}$$

$$\frac{dr}{dt} = 0.32 \text{ cm/s}$$

Question: 9

The bottom of a r

Solution:

Let the volume of the water tank be V

$$V = l \times b \times h$$

$$V = 25 \times 40 \times h$$

$$\frac{dV}{dt} = 1000 \times \frac{dh}{dt}$$

$$500 = 1000 \times \frac{dh}{dt}$$

$$\frac{dh}{dt} = 0.5 \text{ m/min}$$

Question: 10

A stone is droppe

Solution:

Let the radius of the circle be r

$$\frac{dr}{dt} = 3.5 \text{ cm/s}$$

Area of the circle$=\pi r^2$
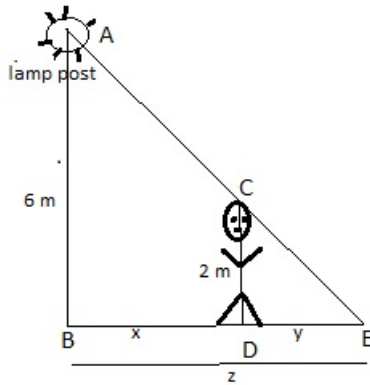
Rate of change of Area $= 2\pi r\dfrac{dr}{dt}$

$= 2 \times 3.14 \times 7.5 \times 3.5$

$= 165 \text{ cm}^2/s$

Question: 11

A 2-m tall man wa

Solution:

ABE and CDE are similar triangles.

So,

$\dfrac{AB}{BE} = \dfrac{CD}{DE}$

$\dfrac{0.006}{x+y} = \dfrac{0.002}{y}$

$6y = 2(x+y)$

$6\dfrac{dy}{dt} = 2(\dfrac{dx}{dt} + \dfrac{dy}{dt})$

$6\dfrac{dy}{dt} = 2(5 + \dfrac{dy}{dt})$

$6\dfrac{dy}{dt} = 10 + 2\dfrac{dy}{dt}$

$4\dfrac{dy}{dt} = 10$

$\dfrac{dy}{dt} = 2.5 \text{km/h}$

Question: 12

An inverted cone

Solution:

Let the volume of the cone be V

$\dfrac{dV}{dt} = 1.5 \text{ cm}^3/s$

$V = \dfrac{1}{3}\pi r^2 h$

$V = \dfrac{1}{3}\pi 5^2 h$

$$V = \frac{25}{3}\pi h$$

$$\frac{dV}{dt} = \frac{25}{3}\pi\frac{dh}{dt}$$

$$\frac{15}{10} = \frac{25}{3}\pi\frac{dh}{dt}$$

Question: 13

Sand is pouring f

Solution:

$$h = \frac{1}{6}r$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi(6h)^2 h$$

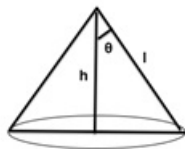$$V = 12\pi h^3$$

$$\frac{dV}{dt} = 36\pi h^2\frac{dh}{dt}$$

$$18 = 36 \times 9 \times \pi \times \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{18\pi}cm/s$$

Question: 14

Water is dripping

Solution:

Let the volume of the cone be V

$$\frac{dV}{dt} = 4cm^3/s$$

$$V = \frac{1}{3}\pi r^2 h$$

$$\cos Q = \frac{h}{1} = \cos 120 = \cos(180 - 60) = -\frac{1}{2}$$

$$\sin Q = \frac{r}{1} = \sin 120 = \sin(180 - 60) = \sin 60 = \frac{\sqrt{3}}{2}$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{\sqrt{3}}{2}1\right)^2 (-\frac{1}{2}1)$$

$$V = -\frac{3}{24}\pi l^3$$

$$\frac{dV}{dt} = -\frac{9}{24}\pi l^2 \frac{dl}{dt}$$

$$4 = -\frac{3}{8}\pi 3^2 \frac{dl}{dt}$$

$$-\frac{32}{27\pi} \, cm/s = \frac{dl}{dt}$$

Question: 15

Oil is leaking at

Solution:

$$\frac{dV}{dt} = 15 \, mL/s$$

$$\frac{d}{dt}(\pi r^2 h) = 15$$
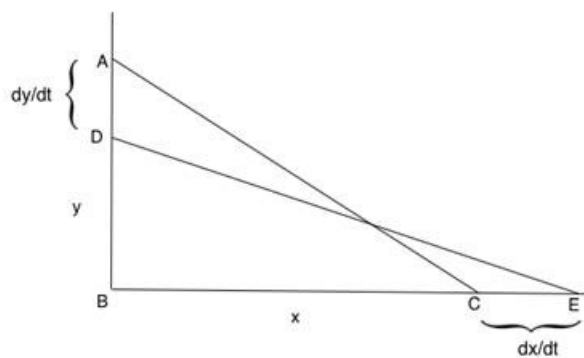
$$\frac{d}{dt}(\pi 7^2 h) = 15$$

$$49\pi \frac{dh}{dt} = 15$$

$$\frac{dh}{dt} = \frac{15}{49\pi}$$

Question: 16

A 13-m long ladde

Solution:

Let the original ladder be AC and the pulled ladder be DE

Let AB=y and BC=x

Applying Pythagoras Theorem in ABC

$$x^2 + y^2 = 13^2 \, ...(1)$$

$$5^2 + y^2 = 13^2$$

$$y = 12cm$$

Differentiating both sides of eqn (1) wrt to t

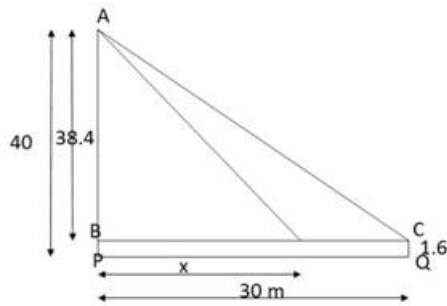$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$5.2 + 12\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{10}{12} = -\frac{5}{6} \text{ cm/s}$$

Question: 17

A man is moving a

Solution:

$$\frac{dx}{dt} = -2 \text{ cm/s}$$

$$\tan Q = \frac{38.4}{x}$$

$$Q = \tan^{-1}\frac{38.4}{x}$$

$$\frac{dQ}{dt} = \frac{1}{1 + \frac{38.4^2}{x^2}}\left(-\frac{1}{x^2}\right).38.4$$

$$\frac{dQ}{dt} = \frac{x^2}{x^2 + 1474.56}\left(-\frac{1}{x^2}\right).38.4$$

$$\frac{dQ}{dt} = -\frac{1}{30^2 + 1474.56}.38.4.\frac{dx}{dt}$$

$$\frac{dQ}{dt} = -\frac{1}{30^2 + 1474.56}.38.4 \times 2$$

$$\frac{dQ}{dt} = -0.032 \text{ radian/second}$$

Question: 18

Find an angle x w

Solution:

ATQ,

$$\frac{dx}{dt} = 2\frac{d}{dt}(\sin x)$$

$$\frac{dx}{dt} = 2\cos x\frac{dx}{dt}$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}$$

Question: 19

The radius of a b

Solution:

$$\frac{dr}{dt} = 10 \text{ m/s}$$

$$S = 4\pi r^2$$

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$$

$$\frac{dS}{dt} = 8\pi . 15.10$$

$$\frac{dS}{dt} = 1200\pi \text{ cm}^2/\text{s}$$

Question: 20

An edge of a vari

Solution:

$$\frac{da}{dt} = 5 \text{cm/s}$$

$$V = a^3$$

$$\frac{dV}{dt} = 3a^2 \frac{da}{dt}$$

$$\frac{dV}{dt} = 3.10^2.5$$

$$\frac{dV}{dt} = 1500 \text{ cm}^3/\text{s}$$

Question: 21

The sides of an e

Solution:

$$\frac{da}{dt} = 2 \text{cm/s}$$

$$A = \frac{\sqrt{3}}{4}a^2$$

$$\frac{dA}{dt} = \frac{\sqrt{3}}{4} 2a \frac{da}{dt}$$

$$\frac{dA}{dt} = \frac{\sqrt{3}}{2} . 10.2$$

$$\frac{dA}{dt} = 10\sqrt{3} \text{ cm}^2/\text{s}$$

Exercise : 11B

Question: 1

Using differentia

Solution:

Let $y = \sqrt{x}$.

Let $x = 36$ and $\Delta x = 1$.

As $y = \sqrt{x}$.

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

We, know

$$\Rightarrow \Delta y = \frac{dy}{dx}\Delta x$$

$$\therefore \Delta y = \frac{1}{2\sqrt{x}}.\Delta x$$

$$\Rightarrow \Delta y = \frac{1}{2\sqrt{36}}.1$$

$$\Rightarrow \Delta y = \frac{1}{12}$$

$$\therefore \Delta y = 0.08$$

Also,

$$\Delta y = f(x+\Delta x)-f(x)$$

$$\therefore 0.08 = \sqrt{36+1} - \sqrt{36}$$

$$\Rightarrow 0.08 = \sqrt{37} - 6$$

$$\Rightarrow \sqrt{37} = 6.08$$

Question: 2

Using diffe

Solution:

Let $y = \sqrt[3]{x}$.

Let x =27 and $\Delta x = 2$.

As $y = \sqrt[3]{x}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{3}x^{\frac{-2}{3}}$$

We, know

$$\Rightarrow \Delta y = \frac{dy}{dx}\Delta x$$

$$\therefore \Delta y = \frac{1}{3}x^{\frac{-2}{3}}.\Delta x$$

$$\Rightarrow \Delta y = \frac{1}{3}27^{\frac{-2}{3}}.2$$

$$\Rightarrow \Delta y = \frac{2}{27}$$

$$\therefore \Delta y = 0.074$$

Also,

$$\Delta y = f(x+\Delta x)-f(x)$$

$$\therefore 0.074 = \sqrt[3]{27+2} - \sqrt[3]{27}$$

$$\Rightarrow 0.074 = \sqrt[3]{29} - 3$$

$$\Rightarrow \sqrt[3]{29} = 3.074$$

Question: 3

Using differentia

Solution:

Let y =√x.

Let x =25 and $\Delta x = 2$.

As y = √x.

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

We, know

$$\Rightarrow \Delta y = \frac{dy}{dx} \Delta x$$

$$\therefore \Delta y = \frac{1}{2\sqrt{x}}. \Delta x$$

$$\Rightarrow \Delta y = \frac{1}{2\sqrt{25}}. 2$$

$$\Rightarrow \Delta y = \frac{1}{5}$$

$$\therefore \Delta y = 0.2$$

Also,

$$\Delta y = f(x+\Delta x)-f(x)$$

$$\therefore 0.2 = \sqrt{25 + 2} - \sqrt{25}$$

$$\Rightarrow 0.2 = \sqrt{27} - 5$$

$$\Rightarrow \sqrt{27} = 5.2$$

Question: 4

Using differentia

Solution:

Let y =√x.

Let x =0.25 and Δx = -0.01.

As y = √x.

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

We, know

$$\Rightarrow \Delta y = \frac{dy}{dx} \Delta x$$

$$\therefore \Delta y = \frac{1}{2\sqrt{x}}. \Delta x$$

$$\Rightarrow \Delta y = \frac{1}{2\sqrt{0.25}}. (-0.01)$$

$$\Rightarrow \Delta y = -0.01$$

$$\therefore \Delta y = -0.01$$

Also,

$$\Delta y = f(x+\Delta x)-f(x)$$

$$\therefore -0.01 = \sqrt{0.25 - 0.01} - \sqrt{0.25}$$

$$\Rightarrow -0.01 = \sqrt{0.24} - 0.5$$

$$\Rightarrow \sqrt{0.24} = 0.49$$

Question: 5

Using diffe

Solution:

Let y =√x.

Let x =19 and Δx = 0.5.

As y = √x.

$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

We, know

$\Rightarrow \Delta y = \frac{dy}{dx} \Delta x$

$\therefore \Delta y = \frac{1}{2\sqrt{x}} . \Delta x$

$\Rightarrow \Delta y = \frac{1}{2\sqrt{49}} . 0.5$

$\Rightarrow \Delta y = \frac{0.5}{14}$

$\therefore \Delta y = 0.0357$

Also,

$\Delta y = f(x+\Delta x)-f(x)$

$\therefore 0.0357 = \sqrt{49+0.5} - \sqrt{49}$

$\Rightarrow 0.0357 = \sqrt{49.5} - 7$

$\Rightarrow \sqrt{49.5} = 7.0357.$

Question: 6

Using diffe

Solution:

Let $y = \sqrt[4]{x}$.

Let x =16 and $\Delta x = 1$.

As $y = \sqrt[4]{x}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{4}x^{\frac{-3}{4}}$

We, know

$\Rightarrow \Delta y = \frac{dy}{dx} \Delta x$

$\therefore \Delta y = \frac{1}{4}x^{\frac{-3}{4}} . \Delta x$

$\Rightarrow \Delta y = \frac{1}{4} 16^{\frac{-3}{4}} . (-1)$

$\Rightarrow \Delta y = \frac{-1}{32}$

$\therefore \Delta y = -0.03125$

Also,

$\Delta y = f(x+\Delta x)-f(x)$

$\therefore -0.03125 = \sqrt[4]{16-1} - \sqrt[4]{16}$

$\Rightarrow -0.03125 = \sqrt[4]{15} - 2$

$\Rightarrow \sqrt[4]{15} = 1.96875$

Question: 7

find the ap

Solution:

Let $y = \frac{1}{x^2}$

Let x =2 and Δx = 0.002.

As $y = \frac{1}{x^2}$

$\Rightarrow \frac{dy}{dx} = \frac{-2}{x^3}$

We, know

$\Rightarrow \Delta y = \frac{dy}{dx} \Delta x$

$\therefore \Delta y = \frac{-2}{x^3} . \Delta x$

$\Rightarrow \Delta y = \frac{-2}{8} . (0.002)$

$\Rightarrow \Delta y = \frac{-0.5}{1000}$

$\therefore \Delta y = -0.0005$

Also,

$\Delta y = f(x+\Delta x) - f(x)$

$\therefore -0.0005 = \frac{1}{(2.002)^2} - \frac{1}{2^2}$

$\Rightarrow -0.0005 = \frac{1}{(2.002)^2} - 0.25$

$\Rightarrow \frac{1}{(2.002)^2} = 0.2495$

Question: 8

find the ap

Solution:

Let $y = \log_e x$

Let x =10 and Δx = 0.02.

As $y = \log_e x$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x}$

We, know

$\Rightarrow \Delta y = \frac{dy}{dx} \Delta x$

$\therefore \Delta y = \frac{1}{x} . \Delta x$

$\Rightarrow \Delta y = \frac{1}{10} . (0.02)$

$\Rightarrow \Delta y = \frac{0.02}{10}$

$\therefore \Delta y = 0.002$

Also,

$\Delta y = f(x+\Delta x) - f(x)$

$\therefore 0.002 = \log_e(10+0.02) - \log_e(10)$

$\Rightarrow 0.002 = \log_e(10.02) - 2.3026$

$\Rightarrow \log_e(10.02) = 2.3046.$

Question: 9

find the approxim

Solution:

Let $y = \log_{10} x$

$\therefore y = \dfrac{\log_e x}{\log_e 10}$

$\therefore y = 0.4343 \log_e x$

Let x =4 and Δx = 0.04.

As $y = 0.4343 \log_e x$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{0.4343}{x}$

We, know

$\Rightarrow \Delta y = \dfrac{dy}{dx} \Delta x$

$\therefore \Delta y = \dfrac{0.4343}{x} . \Delta x$

$\Rightarrow \Delta y = \dfrac{0.4343}{4} . (0.04)$

$\Rightarrow \Delta y = \dfrac{0.017372}{4}$

$\therefore \Delta y = 0.004343$

Also,

Δy = f(x+Δx)-f(x)

$\therefore 0.004343 = \log_e(4+0.04) - \log_e(4)$

$\Rightarrow 0.004343 = \log_e(4.04) - 0.6021$

$\Rightarrow \log_e(4.04) = 0.606443.$

Question: 10

find the approxim

Solution:

Let y = cosx

Let x =60° and Δx = 1° .

As y =cos x

$\Rightarrow \dfrac{dy}{dx} = \sin x$

We, know

$\Rightarrow \Delta y = \dfrac{dy}{dx} \Delta x$

$\therefore \Delta y = \sin x . \Delta x$

$\Rightarrow \Delta y = \sin(60)(0.01745)$

$\Rightarrow \Delta y = (0.86603)(0.01745$

$\therefore \Delta y = 0.01511$

Also,

Δy = f(x+Δx)-f(x)

$\therefore 0.01511 = \cos(60+1) - \cos(60)$

$\Rightarrow 0.01511 = \cos 61° - 0.5$

$\Rightarrow \cos 61° = 0.48489$

Question: 11

If y = sin x and

Solution:

Given x is π /2

Value of π is 22/7

22/14 is π /2

Hence there will be no change.

Question: 12

A circular metal

Solution:

Let the radius of the plate 10cm.

Radius increases by 2% by heating

∴ After increment $= \frac{2}{100} \times 10 = 0.2$

Change in radius dr = 0.2

∴ New radius = 10+0.2 = 10.2cm

Area of circular plate $= A = \pi r^2$

∴ Change in Area $= \frac{dA}{dr}$

$\Rightarrow \frac{dA}{dr} = 2\pi r dr$

$\Rightarrow \frac{dA}{dr} = 2 \times \pi \times 10 \times 0.2$

$\Rightarrow \frac{dA}{dr} = 4\pi \ cm^2$

Question: 13

If the length of

Solution:

The formula for time period -

$\therefore T = 2\pi \sqrt{\frac{l}{g}}$

Here 2,π,g have no dimensions. So we can eliminate them.

Now $\frac{\Delta T}{T} = \frac{1}{2} \times \frac{\Delta L}{L}$

By representing in percentage error

$\Rightarrow \frac{\Delta T}{T} \times 100\% = \frac{1}{2} \times \frac{\Delta L}{L} \times 100\%$

$\Rightarrow \frac{\Delta T}{T} \times 100\% = \frac{1}{2} \times \frac{\Delta L}{L} \times 100\%$

$\Rightarrow \frac{\Delta T}{T}\% = \frac{1}{2} \times 2\%$

$\Rightarrow \frac{\Delta T}{T}\% = 1\%$

Hence the time period becomes 1 %.

Question: 14

The pressure p an

Solution:

Given: $pv^{1/4} = k$

%decrease in the volume = 1/2%s

$\therefore \frac{\Delta V}{V} \times 100 = \frac{-1}{2}$

$pv^{1/4} = k$

taking log on both sides

$\log[pv^{1/4}] = \log a$

$\log P + 1.4\log V = \log a$

Differentiating both the sides we get

$\Rightarrow \frac{1}{P}dP + \frac{1.4}{V}dV = 0$

$\Rightarrow \frac{dP}{P} + 1.4\frac{dV}{V} = 0$

Multiplying both sides by 100.

$\Rightarrow \frac{dP}{P} \times 100 + 1.4 \times \frac{dV}{V} \times 100 = 0$

$\Rightarrow \frac{dP}{P} \times 100 + 1.4\left(\frac{-1}{2}\right) = 0$

$\Rightarrow \frac{dP}{P} \times 100 = 0.7$

%error in P = 0.7%.

Question: 15

The radius of a s

Solution:

Decrease in radius = dr = 10-9.8

$\therefore$ dr = 0.2

Volume of the sphere is given by = $V = \frac{4}{3}\pi r^3$

Change in volume = $dV = 4\pi r^2 dr$

$\therefore dV = 4\pi(10)^2 \times 0.2$

$\Rightarrow dV = 80\pi \text{ cm}^3$

Surface area of the sphere is given by = $A = 4\pi r^2$

Change in volume = $dA = 8\pi r dr$

$\therefore dA = 8\pi \times 10 \times 0.2$

$\therefore dA = 16\pi$.

Question: 16

If there is an er

Solution:

Volume of the sphere is given by = $V = \frac{4}{3}\pi r^2$

Change in volume = $dV = 4\pi r^2 dr$

Given: $\Delta r = 0.1$

$$\Rightarrow \Delta r. \frac{dV}{dr} = 4\pi r^2 \Delta r$$

$$\Rightarrow \Delta V = 4\pi r^2 \Delta r$$

Percentage error

$$\Rightarrow \frac{\Delta V}{V} = \frac{4\pi r^2}{\frac{4\pi r^3}{3}} \times 0.1$$

$$= 0.3\%$$

Question: 17

Show that the rel

Solution:

Let d be the diameter r be the radius and V be the volume of Sphere

Volume of the sphere is given by $= V = \frac{4}{3}\pi r^2$

$$\Rightarrow V = \frac{4}{3}\pi \left(\frac{D}{2}\right)^3 = \frac{1}{6}\pi D^3$$

Let Δd be the error in d and the corresponding error in V be ΔV.

$$\therefore \Delta V = \frac{dV}{dd}\Delta d = \frac{1}{2}\pi d^2 \Delta D$$

$$\therefore \frac{\Delta V}{V} = \frac{\frac{1}{2}\pi d^2 \Delta D}{\frac{1}{6}\pi D^3} = 3\frac{\Delta D}{D}$$

Hence Proved

Exercise : 11C

Question: 1

Verify Rolle's th

Solution:

Condition (1):

Since, $f(x)=x^2$ is a polynomial and we know every polynomial function is continuous for all x∈R.

$\Rightarrow f(x)=x^2$ is continuous on [-1,1].

Condition (2):

Here, f'(x)=2x which exist in [-1,1].

So, $f(x)=x^2$ is differentiable on (-1,1).

Condition (3):

Here, $f(-1)=(-1)^2=1$

And $f(1)=1^1=1$

i.e. f(-1)=f(1)

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one c∈(-1,1) such that f'(c)=0

i.e. 2c=0

i.e. c=0

Value of c=0∈(-1,1)

Thus, Rolle's theorem is satisfied.

Question: 2

Verify Rolle's th

Solution:

Condition (1):

Since, $f(x)=x^2-x-12$ is a polynomial and we know every polynomial function is continuous for all $x \epsilon R$.

$\Rightarrow f(x)= x^2-x-12$ is continuous on [-3,4].

Condition (2):

Here, $f'(x)=2x-1$ which exist in [-3,4].

So, $f(x)= x^2-x-12$ is differentiable on (-3,4).

Condition (3):

Here, $f(-3)=(-3)^2-3-12=0$

And $f(4)=4^2-4-12=0$

i.e. f(-3)=f(4)

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one $c\epsilon(-3,4)$ such that f'(c)=0

i.e. 2c-1=0

i.e. $c = \frac{1}{2}$

Value of $c = \frac{1}{2}\epsilon(-3,4)$

Thus, Rolle's theorem is satisfied.

Question: 3

Verify Rolle's th

Solution:

Condition (1):

Since, $f(x)=x^2-5x+6$ is a polynomial and we know every polynomial function is continuous for all $x \epsilon R$.

$\Rightarrow f(x)= x^2-5x+6$ is continuous on [2,3].

Condition (2):

Here, $f'(x)=2x-5$ which exist in [2,3].

So, $f(x)= x^2-5x+6$ is differentiable on (2,3).

Condition (3):

Here, $f(2)=2^2-5\times2+6=0$

And $f(3)= 3^2-5\times3+6=0$

i.e. f(2)=f(3)

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one $c\epsilon(2,3)$ such that f'(c)=0

i.e. 2c-5=0

i.e. $c = \frac{5}{2}$

Value of $c = \frac{5}{2} \in (2,3)$

Thus, Rolle's theorem is satisfied.

Question: 4

Verify Rolle's th

Solution:

Condition (1):

Since, $f(x) = x^2-5x+6$ is a polynomial and we know every polynomial function is continuous for all $x \in R$.

$\Rightarrow f(x) = x^2-5x+6$ is continuous on [-3,6].

Condition (2):

Here, $f'(x) = 2x-5$ which exist in [-3,6].

So, $f(x) = x^2-5x+6$ is differentiable on (-3,6).

Condition (3):

Here, $f(-3) = (-3)^2-5 \times (-3)+6 = 30$

And $f(6) = 6^2-5 \times 6+6 = 12$

i.e. $f(-3) \neq f(6)$

Conditions (3) of Rolle's theorem is not satisfied.

So, Rolle's theorem is not applicable.

Question: 5

Verify Rolle's th

Solution:

Condition (1):

Since, $f(x) = x^2-4x+3$ is a polynomial and we know every polynomial function is continuous for all $x \in R$.

$\Rightarrow f(x) = x^2-4x+3$ is continuous on [1,3].

Condition (2):

Here, $f'(x) = 2x-4$ which exist in [1,3].

So, $f(x) = x^2-4x+3$ is differentiable on (1,3).

Condition (3):

Here, $f(1) = (1)^2-4(1)+3 = 0$

And $f(3) = (3)^2-4(3)+3 = 0$

i.e. $f(1) = f(3)$

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one $c \in (1,3)$ such that $f'(c) = 0$

i.e. $2c-4 = 0$

i.e. $c = 2$

Value of $c = 2 \in (1,3)$

Thus, Rolle's theorem is satisfied.

Question: 6

Verify Rolle's th

Solution:

Condition (1):

Since, $f(x)=x(x-4)^2$ is a polynomial and we know every polynomial function is continuous for all $x \epsilon R$.

$\Rightarrow f(x) = x(x-4)^2$ is continuous on [0,4].

Condition (2):

Here, $f'(x) = (x-4)^2 + 2x(x-4)$ which exist in [0,4].

So, $f(x) = x(x-4)^2$ is differentiable on (0,4).

Condition (3):

Here, $f(0) = 0(0-4)^2 = 0$

And $f(4) = 4(4-4)^2 = 0$

i.e. $f(0) = f(4)$

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one $c \epsilon (0,4)$ such that $f'(c) = 0$

i.e. $(c-4)^2 + 2c(c-4) = 0$

i.e. $(c-4)(3c-4) = 0$

i.e. $c = 4$ or $c = 3 \div 4$

Value of $c = \frac{3}{4} \epsilon (0,4)$

Thus, Rolle's theorem is satisfied.

Question: 7

Verify Rolle's th

Solution:

Condition (1):

Since, $f(x) = x^3 - 7x^2 + 16x - 12$ is a polynomial and we know every polynomial function is continuous for all $x \epsilon R$.

$\Rightarrow f(x) = x^3 - 7x^2 + 16x - 12$ is continuous on [2,3].

Condition (2):

Here, $f'(x) = 3x^2 - 14x + 16$ which exist in [2,3].

So, $f(x) = x^3 - 7x^2 + 16x - 12$ is differentiable on (2,3).

Condition (3):

Here, $f(2) = 2^3 - 7(2)^2 + 16(2) - 12 = 0$

And $f(3) = 3^3 - 7(3)^2 + 16(3) - 12 = 0$

i.e. $f(2) = f(3)$

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one $c \epsilon (2,3)$ such that $f'(c) = 0$

i.e. $3c^2 - 14c + 16 = 0$

i.e. $(c-2)(3c-7) = 0$

i.e. c=2 or c=7÷3

Value of $c = \frac{7}{3} \in (2,3)$

Thus, Rolle's theorem is satisfied.

Question: 8

Verify Rolle's th

Solution:

Condition (1):

Since, $f(x) = x^3 + 3x^2 - 24x - 80$ is a polynomial and we know every polynomial function is continuous for all x∈R.

⇒ $f(x) = x^3 + 3x^2 - 24x - 80$ is continuous on [-4,5].

Condition (2):

Here, $f'(x) = 3x^2 + 6x - 24$ which exist in [-4,5].

So, $f(x) = x^3 + 3x^2 - 24x - 80$ is differentiable on (-4,5).

Condition (3):

Here, $f(-4) = (-4)^3 + 3(-4)^2 - 24(-4) - 80 = 0$

And $f(5) = (5)^3 + 3(5)^2 - 24(5) - 80 = 0$

i.e. f(-4)=f(5)

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one c∈(-4,5) such that f'(c)=0

i.e. $3c^2 + 6c - 24 = 0$

i.e. c=-4 or c=2

Value of c=2 ∈(-4,5)

Thus, Rolle's theorem is satisfied.

Question: 9

Verify Rolle's th

Solution:

Condition (1):

Since, f(x)=(x-1)(x-2)(x-3) is a polynomial and we know every polynomial function is continuous for all x∈R.

⇒ f(x)= (x-1)(x-2)(x-3) is continuous on [1,3].

Condition (2):

Here, f'(x)= (x-2)(x-3)+ (x-1)(x-3)+ (x-1)(x-2) which exist in [1,3].

So, f(x)= (x-1)(x-2)(x-3) is differentiable on (1,3).

Condition (3):

Here, f(1)= (1-1)(1-2)(1-3) =0

And f(3)= (3-1)(3-2)(3-3) =0

i.e. f(1)=f(3)

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one c∈(1,3) such that f'(c)=0

i.e. (c-2)(c-3)+ (c-1)(c-3)+ (c-1)(c-2)=0

i.e. (c-3)(2c-3)+(c-1)(c-2)=0

i.e. $(2c^2-9c+9)+(c^2-3c+2)=0$

i.e. $3c^2-12c+11=0$

i.e. $c = \frac{12\pm\sqrt{12}}{6}$

i.e. c=2.58 or c=1.42

Value of c=1.42є(1,3) and c=2.58є(1,3)

Thus, Rolle's theorem is satisfied.

Question: 10

Verify Rolle's th

Solution:

Condition (1):

Since, $f(x)=(x-1)(x-2)^2$ is a polynomial and we know every polynomial function is continuous for all xєR.

⇒ $f(x)= (x-1)(x-2)^2$ is continuous on [1,2].

Condition (2):

Here, $f'(x)= (x-2)^2+2(x-1)(x-2)$ which exist in [1,2].

So, $f(x)= (x-1)(x-2)^2$ is differentiable on (1,2).

Condition (3):

Here, $f(1)= (1-1)(1-2)^2=0$

And $f(2)= (2-1)(2-2)^2=0$

i.e. f(1)=f(2)

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one cє(1,2) such that f'(c)=0

i.e. $(c-2)^2+2(c-1)(c-2)=0$

(3c-4)(c-2)=0

i.e. c=2 or c=4÷3

Value of $c = \frac{4}{3} = 1.33$є$(1,2)$

Thus, Rolle's theorem is satisfied.

Question: 11

Verify Rolle's th

Solution:

Condition (1):

Since, $f(x)=(x-2)^4(x-3)^3$ is a polynomial and we know every polynomial function is continuous for all xєR.

⇒ $f(x)= (x-2)^4(x-3)^3$ is continuous on [2,3].

Condition (2):

Here, $f'(x)= 4(x-2)^3(x-3)^3+3(x-2)^4(x-3)^2$ which exist in [2,3].

So, f(x)= (x-2)^4(x-3)^3 is differentiable on (2,3).

Condition (3):

Here, f(2)= (2-2)^4(2-3)^3=0

And f(3)= (3-2)^4(3-3)^3=0

i.e. f(2)=f(3)

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one cϵ(2,3) such that f'(c)=0

i.e. 4(c-2)^3(c-3)^3+3(c-2)^4(c-3)^2=0

(c-2)^3(c-3)^2(7c-18)=0

i.e. c=2 or c=3 or c=18÷7

Value of $c = \frac{18}{7} = 2.57ϵ(2,3)$

Thus, Rolle's theorem is satisfied.

Question: 12

Verify Rolle's th

Solution:

Condition (1):

Since, $f(x) = \sqrt{1-x^2}$ is a polynomial and we know every polynomial function is continuous for all xϵR.

⇒ $f(x) = \sqrt{1-x^2}$ is continuous on [-1,1].

Condition (2):

Here, $f'(x) = -\frac{x}{\sqrt{1-x^2}}$ which exist in [-1,1].

So, $f(x) = \sqrt{1-x^2}$ is differentiable on (-1,1).

Condition (3):

Here, $f(-1) = \sqrt{1-(-1)^2} = 0$

And $f(1) = \sqrt{1-1^2} = 0$

i.e. $f(-1) = f(1)$

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one cϵ(-1,1) such that f'(c)=0

i.e. $-\frac{c}{\sqrt{1-c^2}} = 0$

i.e. c=0

Value of c=0ϵ(-1,1)

Thus, Rolle's theorem is satisfied.

Question: 3

Verify Rolle's th

Solution:

Condition (1):

Since, f(x)=cosx is a trigonometric function and we know every trigonometric function is continuous.

⇒ f(x)=cosx is continuous on $[-\frac{\pi}{2},\frac{\pi}{2}]$.

Condition (2):

Here, f'(x)=-sinx which exist in $[-\frac{\pi}{2},\frac{\pi}{2}]$.

So, f(x)=cosx is differentiable on $(-\frac{\pi}{2},\frac{\pi}{2})$.

Condition (3):

Here, $f\left(-\frac{\pi}{2}\right) = \cos\left(-\frac{\pi}{2}\right) = 0$

And $f\left(\frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) = 0$

i.e. $f\left(-\frac{\pi}{2}\right) = f\left(\frac{\pi}{2}\right)$

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one $c\epsilon(-\frac{\pi}{2},\frac{\pi}{2})$ such that f'(c)=0

i.e. -sinc=0

i.e. c=0

Value of $c = 0\epsilon(-\frac{\pi}{2},\frac{\pi}{2})$

Thus, Rolle's theorem is satisfied.

Question: 14

Verify Rolle's th

Solution:

Condition (1):

Since, f(x)=cos2x is a trigonometric function and we know every trigonometric function is continuous.

⇒ f(x)= cos2x is continuous on [0,π].

Condition (2):

Here, f'(x)= -2sin2x which exist in [0,π].

So, f(x)=cos2x is differentiable on (0,π).

Condition (3):

Here, f(0)=cos0=1

And f(π)=cos2π=1

i.e. f(0)=f(π)

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one c∈(0,π) such that f'(c)=0

i.e. -2sin2c =0

i.e. 2c=π

i.e. $c = \frac{\pi}{2}$

Value of $c = \frac{\pi}{2}\epsilon(0,\pi)$

Thus, Rolle's theorem is satisfied.

Question: 15

Verify Rolle's th

Solution:

Condition (1):

Since, f(x)=sin3x is a trigonometric function and we know every trigonometric function is continuous.

⇒ f(x)= sin3x is continuous on [0,π].

Condition (2):

Here, f'(x)= 3cos3x which exist in [0,π].

So, f(x)= sin3x is differentiable on (0,π).

Condition (3):

Here, f(0)=sin0=0

And f(π)=sin3π=0

i.e. f(0)=f(π)

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one c∈(0,π) such that f'(c)=0

i.e. 3cos3c =0

i.e. $3c = \frac{\pi}{2}$

i.e. $c = \frac{\pi}{6}$

Value of $c = \frac{\pi}{6} \in (0, \pi)$

Thus, Rolle's theorem is satisfied.

Question: 16

Verify Rolle's th

Solution:

Condition (1):

Since, f(x)=sinx+cosx is a trigonometric function and we know every trigonometric function is continuous.

⇒ f(x)= sinx+cosx is continuous on $[0, \frac{\pi}{2}]$.

Condition (2):

Here, f'(x)= cosx-sinx which exist in $[0, \frac{\pi}{2}]$.

So, f(x)= sinx+cosx is differentiable on $(0, \frac{\pi}{2})$

Condition (3):

Here, f(0)=sin0+cos0=1

And $f(\frac{\pi}{2}) = \sin\left(\frac{\pi}{2}\right) + \cos(\frac{\pi}{2}) = 1$

i.e. $f(0) = f(\frac{\pi}{2})$

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one $c \in (0, \frac{\pi}{2})$ such that f'(c)=0

i.e. cosc-sinc =0

i.e. $c = \frac{\pi}{4}$

Value of $c = \frac{\pi}{4} \epsilon (0, \frac{\pi}{2})$

Thus, Rolle's theorem is satisfied.

Question: 17

Verify Rolle's th

Solution:

Condition (1):

Since, $f(x) = e^{-x} \sin x$ is a combination of exponential and trigonometric function which is continuous.

$\Rightarrow f(x) = e^{-x} \sin x$ is continuous on $[0,\pi]$.

Condition (2):

Here, $f'(x) = e^{-x} (\cos x - \sin x)$ which exist in $[0,\pi]$.

So, $f(x) = e^{-x} \sin x$ is differentiable on $(0,\pi)$

Condition (3):

Here, $f(0) = e^{-0} \sin 0 = 0$

And $f(\pi) = e^{-\pi} \sin \pi = 0$

i.e. $f(0) = f(\pi)$

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one $c \epsilon (0,\pi)$ such that $f'(c) = 0$

i.e. $e^{-c} (\cos c - \sin c) = 0$

i.e. $\cos c - \sin c = 0$

i.e. $c = \frac{\pi}{4}$

Value of $c = \frac{\pi}{4} \epsilon (0, \pi)$

Thus, Rolle's theorem is satisfied.

Question: 18

Verify Rolle's th

Solution:

Condition (1):

Since, $f(x) = e^{-x} (\sin x - \cos x)$ is a combination of exponential and trigonometric function which is continuous.

$\Rightarrow f(x) = e^{-x} (\sin x - \cos x)$ is continuous on $[\frac{\pi}{4}, \frac{5\pi}{4}]$.

Condition (2):

Here, $f'(x) = e^{-x} (\sin x + \cos x) - e^{-x} (\sin x - \cos x)$

$= e^{-x} \cos x$ which exist in $[\frac{\pi}{4}, \frac{5\pi}{4}]$.

So, $f(x) = e^{-x} (\sin x - \cos x)$ is differentiable on $(\frac{\pi}{4}, \frac{5\pi}{4})$

Condition (3):

Here, $f(\frac{\pi}{4}) = e^{-\frac{\pi}{4}} (\sin \frac{\pi}{4} - \cos \frac{\pi}{4}) = 0$

And $f(\frac{5\pi}{4}) = e^{-\frac{5\pi}{4}} (\sin \frac{5\pi}{4} - \cos \frac{5\pi}{4}) = 0$

i.e. $f\left(\frac{\pi}{4}\right) = f\left(\frac{5\pi}{4}\right)$

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one $c\in\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ such that f'(c)=0

i.e. $e^{-c} \cos c = 0$

i.e. cos c = 0

i.e. $c = \frac{\pi}{2}$

Value of $c = \frac{\pi}{2}\in\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

Thus, Rolle's theorem is satisfied.

Question: 19

Verify Rolle's th

Solution:

Condition (1):

Since, f(x) = sinx-sin2x is a trigonometric function and we know every trigonometric function is continuous.

⇒ f(x) = sinx-sin2x is continuous on [0,2π].

Condition (2):

Here, f'(x)= cosx-2cos2x which exist in [0,2π].

So, f(x)= sinx-sin2x is differentiable on (0,2π)

Condition (3):

Here, f(0)= sin0-sin0 = 0

And f(2π)=sin(2π)-sin(4π) =0

i.e. f(0)=f(2π)

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one c∈(0,2π) such that f'(c)=0

i.e. cosx-2cos2x =0

i.e. cosx-4cos^{2}x+2=0

i.e. 4cos^{2}x-cosx-2=0

i.e. $\cos x = \frac{1\pm\sqrt{33}}{8}$

i.e. c=32° 32' or c=126°23'

Value of c=32°32'∈(0,2π)

Thus, Rolle's theorem is satisfied.

Question: 20

Verify Rolle's th

Solution:

Condition (1):

Since, f(x)=x(x+2)e^x is a combination of exponential and polynomial function which is continuous for all x∈R.

⇒ f(x)= x(x+2)e^x is continuous on [-2,0].

Condition (2):

Here, f'(x)=(x²+4x+2)eˣ which exist in [-2,0].

So, f(x)=x(x+2)eˣ is differentiable on (-2,0).

Condition (3):

Here, f(-2)= (-2)(-2+2)e⁻² =0

And f(0)= 0(0+2)e⁰=0

i.e. f(-2)=f(0)

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one cϵ(-2,0) such that f'(c)=0

i.e. (c²+4c+2)eᶜ =0

i.e. (c+√2)²=0

i.e. c=-√2

Value of c=-√2 ϵ(-2,0)

Thus, Rolle's theorem is satisfied.

Question: 21

Verify Rolle's th

Solution:

Condition (1):

Since, f(x)=x(x-5)² is a polynomial and we know every polynomial function is continuous for all xϵR.

⇒ f(x)= x(x-5)² is continuous on [0,5].

Condition (2):

Here, f'(x)= (x-5)²+ 2x(x-5) which exist in [0,5].

So, f(x)= x(x-5)² is differentiable on (0,5).

Condition (3):

Here, f(0)= 0(0-5)²=0

And f(5)= 5(5-5)²=0

i.e. f(0)=f(5)

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one cϵ(0,5) such that f'(c)=0

i.e. (c-5)²+ 2c(c-5)=0

i.e.(c-5)(3c-5)=0

i.e. $c = \frac{5}{3}$ or c=5

Value of $c = \frac{5}{3}ϵ(0,5)$

Thus, Rolle's theorem is satisfied.

Question: 22

Discuss the appli

Solution:

Condition (1):

Since, f(x)=(x-1)(2x-3) is a polynomial and we know every polynomial function is continuous for all xϵR.

⇒ f(x)= (x-1)(2x-3) is continuous on [1,3].

Condition (2):

Here, f'(x)= (2x-3)+ 2(x-1) which exist in [1,3].

So, f(x)= (x-1)(2x-3) is differentiable on (1,3).

Condition (3):

Here, f(1)= (1-1)(2(1)-3)=0

And f(5)= (3-1)(2(3)-3)=6

i.e. f(1)≠f(3)

Condition (3) of Rolle's theorem is not satisfied.

So, Rolle's theorem is not applicable.

Question: 23

Discuss the appli

Solution:

Condition (1):

Since, $f(x)=x^{1/2}$ is a polynomial and we know every polynomial function is continuous for all xϵR.

⇒ f(x)= $x^{1/2}$ is continuous on [-1,1].

Condition (2):

Here, $f'(x) = \dfrac{1}{2x^{\frac{1}{2}}}$ which does not exist at x=0 in [-1,1].

f(x)=$x^{1/2}$ is not differentiable on (-1,1).

Condition (2) of Rolle's theorem is not satisfied.

So, Rolle's theorem is not applicable.

Question: 24

Discuss the appli

Solution:

Condition (1):

Since, $f(x)=2+(x-1)^{2/3}$ is a polynomial and we know every polynomial function is continuous for all xϵR.

⇒ f(x)= $2+(x-1)^{2/3}$ is continuous on [0,2].

Condition (2):

Here, $f'(x) = \dfrac{2}{3(x-1)^{\frac{1}{3}}}$ which does not exist at x=1 in [0,2].

f(x)= $2+(x-1)^{2/3}$ is not differentiable on (0,2).

Condition (2) of Rolle's theorem is not satisfied.

So, Rolle's theorem is not applicable.

Question: 25

Discuss the appli

Solution:

Condition (1):

Since, $f(x) = \cos\frac{1}{x}$ which is discontinuous at x=0

$\Rightarrow f(x) = \cos\frac{1}{x}$ is not continuous on [-1,1].

Condition (1) of Rolle's theorem is not satisfied.

So,Rolle's theorem is not applicable.

Question: 26

Discuss the appli

Solution:

Condition (1):

Since, f(x)=[x] which is discontinuous at x=0

$\Rightarrow$ f(x)=[x] is not continuous on [-1,1].

Condition (1) of Rolle's theorem is not satisfied.

So,Rolle's theorem is not applicable.

Question: 27

Using Rolle's the

Solution:

Condition (1):

Since, y=x(x-4) is a polynomial and we know every polynomial function is continuous for all xϵR.

$\Rightarrow$ y= x(x-4) is continuous on [0,4].

Condition (2):

Here, y'= (x-4)+x which exist in [0,4].

So, y= x(x-4) is differentiable on (0,4).

Condition (3):

Here, y(0)=0(0-4)=0

And y(4)= 4(4-4)=0

i.e. y(0)=y(4)

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one cϵ(0,4) such that y'(c)=0

i.e. (c-4)+c=0

i.e. 2c-4=0

i.e. c=2

Value of c=2ϵ(0,4)

So,y(c)=y(2)=2(2-4)=-4

By geometric interpretation, (2,-4) is a point on a curve y=x(x-4),where tangent is parallel to x-axis.

Exercise : 11D

Question: 1

Verify Lagrange's

Solution:

Given:

Since the f(x) is a polynomial function,

It is continuous as well as differentiable in the interval [4,6].

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{(36 + 12 + 3) - (16 + 8 + 3)}{6 - 4}$$

$$= \frac{24}{2}$$

$$= 12$$

$\Rightarrow$ f' (c)=2c+2

$\Rightarrow$ 2c+2=12

$\Rightarrow$ c=5

Question: 2

Verify Lagrange's

Solution:

Given:

Since the f(x) is a polynomial function,

It is continuous as well as differentiable in the interval [0,4].

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{(16 + 4 - 1) - (0 + 0 - 1)}{4 - 0}$$

$$= 5$$

$\Rightarrow$ f'(c)=2c+1

$\Rightarrow$ 2c+1=5

$\Rightarrow$ c=2

Question: 3

Verify Lagrange's

Solution:

Given:

Since the f(x) is a polynomial function,

It is continuous as well as differentiable in the interval [1,3].

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{(18 - 9 + 1) - (2 - 3 + 1)}{3 - 1}$$

$$= 5$$

$\Rightarrow$ f'(c)=4c-3

$\Rightarrow$ 4c-3=5

$\Rightarrow c = 2$

Question: 4

Verify Lagrange's

Solution:

Given:

Since the f(x) is a polynomial function,

It is continuous as well as differentiable in the interval [-1,4].

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{(64 + 16 - 24) - (-1 + 1 + 6)}{4 + 1}$$

$$= \frac{50}{5}$$

$$= 10$$

$f'(c) = 3c^2 + 2c - 6$

$\Rightarrow 3c^2 + 2c - 6 = 10$

$\Rightarrow 3c^2 + 2c - 16 = 0$

$\Rightarrow 3c^2 - 6c + 8c - 16 = 0$

$\Rightarrow 3c(c-2) + 8(c-2) = 0$

$\Rightarrow (3c+8)(c-2) = 0$

$c = 2, \dfrac{-8}{3}$

Question: 5

Verify Lagrange's

Solution:

Given:

$$f(x) = x^3 - 18x^2 + 104x - 192$$

Since the f(x) is a polynomial function,

It is continuous as well as differentiable in the interval [4,6].

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow f'(c) = \frac{(216 - 648 + 624 - 192) - (64 - 288 + 416 - 192)}{6 - 2}$$

$$= 0$$

$\Rightarrow f'(c) = 3c^2 - 36c + 104$

$= 3c^2 - 36c + 10$

$= 0$

$$\Rightarrow c = \frac{36 \pm \sqrt{1296 - 1248}}{6}$$

$$\Rightarrow c = \frac{36 \pm \sqrt{48}}{6}$$
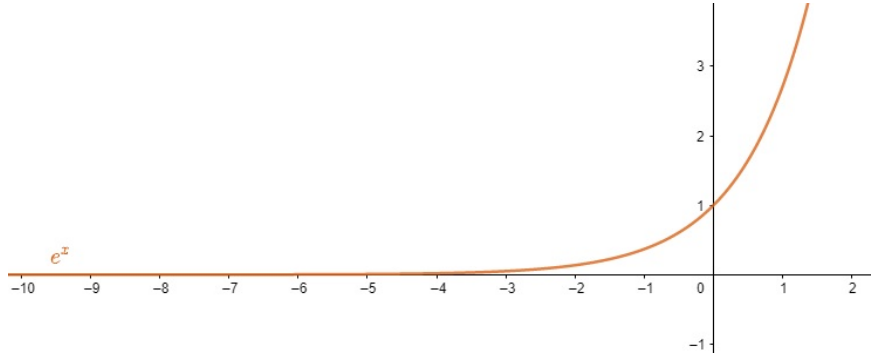
$$\Rightarrow c = 6 \pm \frac{2}{3}\sqrt{3}$$

Question: 6

Verify Lagrange's

Solution:

Given:

Since f(c) is continuous as well as differentiable in the interval [0,1].

e^x

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{e - 1}{1}$$

$$\Rightarrow f'(c) = e^c$$

$$\Rightarrow e^c = e - 1$$

$$\Rightarrow \log_e e^c = \log_e(e - 1)$$

$$\Rightarrow c = \log_e(e - 1)$$

Question: 7

Verify Lagrange's

Solution:

Given:

Since the f(x) is a polynomial function,

It is continuous as well as differentiable in the interval [0,1].

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{1 - 0}{1 - 0}$$

$$= 1$$

$$f'(c) = \frac{2}{3}c^{\frac{1}{3}}$$

$$\Rightarrow \frac{2}{3}c^{\frac{1}{3}} = 1$$

$$\Rightarrow c^{\frac{-1}{3}} = \frac{3}{2}$$

$$\Rightarrow c^{\frac{1}{3}} = \frac{2}{3}$$
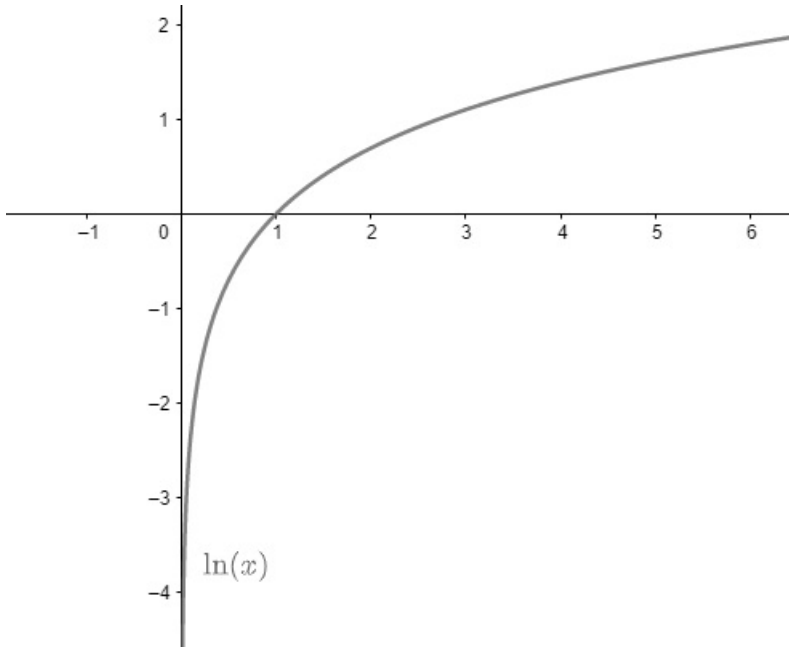
$$\Rightarrow c = \frac{8}{27}$$

Question: 8

Verify Lagrange's

Solution:

Given:

Since log x is a continuous as well as differentiable function in the interval [1,e].

ln(x)

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{\log e - \log 1}{e - 1}$$

$$= \frac{1}{e - 1}$$

$$f'(c) = \frac{1}{c}$$

$$\Rightarrow \frac{1}{e - 1} = \frac{1}{c}$$

c=e-1

Question: 9

Verify Lagrange's

Solution:

Given:

Since $\tan^{-1} x$ is a continuous as well as differentiable function in the interval [0,1].

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{\tan^{-1} 1 - \tan^{-1} 0}{1 - 0}$$

$$= \frac{\pi}{4}$$

$$f'(c) = \frac{1}{1 + c^2}$$

$$\Rightarrow \frac{1}{1+c^2} = \frac{\pi}{4}$$

$$\Rightarrow 1+c^2 = \frac{4}{\pi}$$
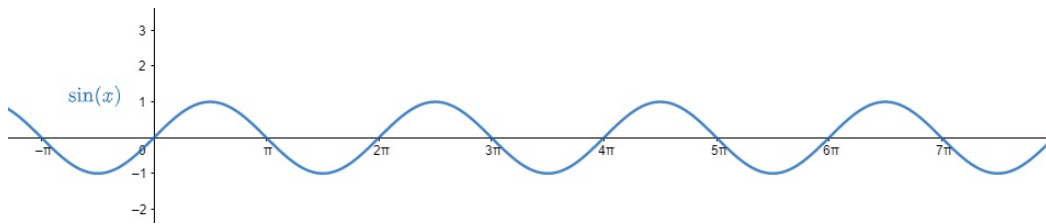
$$\Rightarrow c = \sqrt{\frac{4}{\pi} - 1}$$

Question: 10

Verify Lagrange's

Solution:

Given:

Since sin x is a continuous as well as differentiable function in the interval $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$.

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{\sin \frac{5\pi}{2} - \sin \frac{\pi}{2}}{\frac{5\pi}{2} - \frac{\pi}{2}}$$

=0

f' (c)=cos x

cos x=0
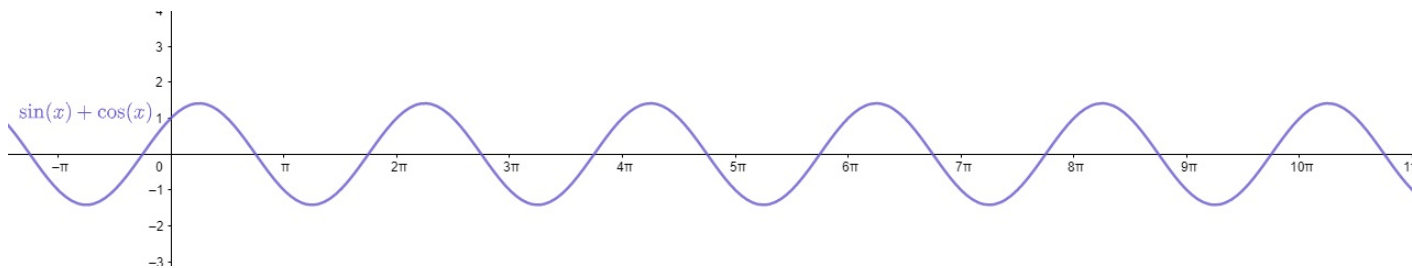
$$x = \frac{n\pi}{2}, n \in \{1,2,3,4,5\}$$

Question: 11

Verify Lagrange's

Solution:

Given:

Since (sin x + cos x) is a continuous as well as differentiable function in the interval $\left[0, \frac{\pi}{2}\right]$.

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{\sin \frac{\pi}{2} + \cos \frac{\pi}{2} - \sin 0 - \cos 0}{\frac{\pi}{2} - 0}$$

=0

f' (c)=cos x-sin x

⇒ cos x-sin x=0

⇒ $\cos x \cos \dfrac{\pi}{4} - \sin x \sin \dfrac{\pi}{4} = 0$

⇒ $\cos\left(x + \dfrac{\pi}{4}\right) = 0$

⇒ $\left(x + \dfrac{\pi}{4}\right) = \cos^{-1} 0$

⇒ $\left(x + \dfrac{\pi}{4}\right) = 1$

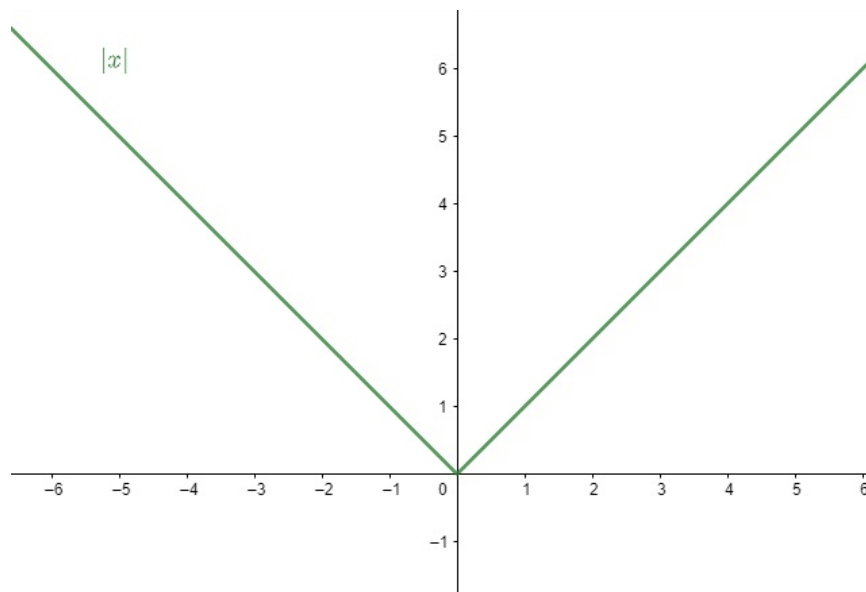⇒ $x = 1 - \dfrac{\pi}{4}$

Question: 12

Show that Lagrang

Solution:

Given:

Since f(x) is continuous in the interval [-1,1].

But is non differentiable at x=0 due to sharp corner.

So LMVT is not applicable to ƒ(x)=|x|

Question: 13

Show that Lagrang

Solution:

Given:

Since the graph is discontinuous at x=0 as shown in the graph.
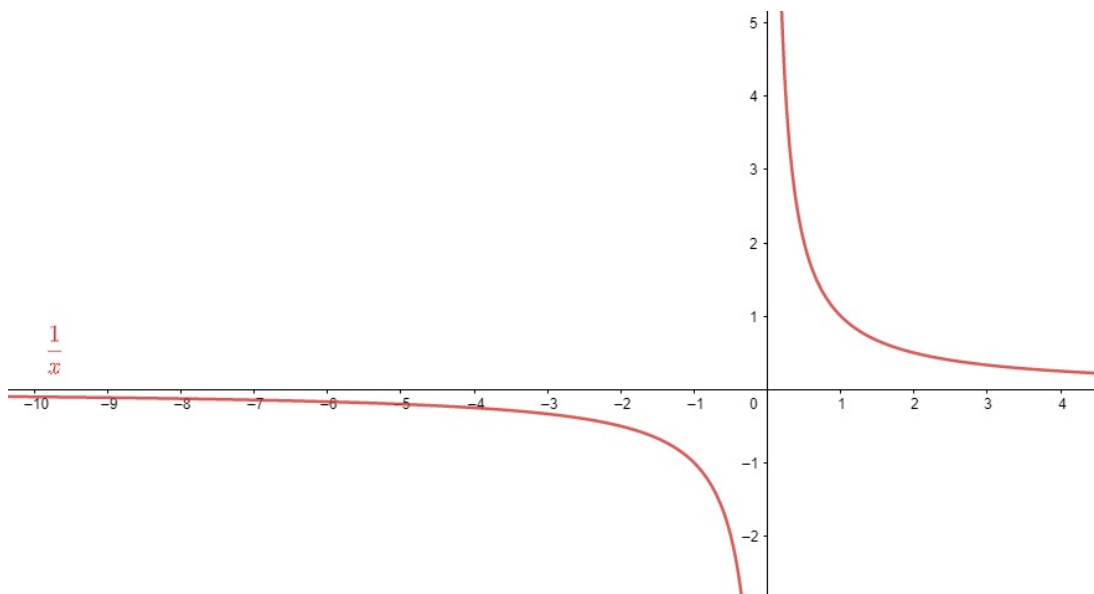
So LMVT is not applicable to the above function.

Question: 14 A

Find 'c' of Lagra

Solution:

Given:

Since the f(x) is a polynomial function,

It is continuous as well as differentiable in the interval $[0, \frac{1}{2}]$.

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{\frac{1}{8} - \frac{3}{4} + 1 - 0}{\frac{1}{2} - 0}$$

$$= \frac{3}{4}$$

f' (c)=3x^2 -6x+2

3 x^2-6x+2=3/4

12 x^2-24x+8=3

12 x^2-24x+5=0

$$x = \frac{24 \pm \sqrt{576 - 240}}{24}$$

$$x = 1 \pm \sqrt{\frac{336}{576}}$$

$$x = 1 \pm \sqrt{\frac{7}{12}}$$

Question: 14 B

Find 'c' of Lagra

Solution:

Given:

Since the f(x) is a polynomial function,

It is continuous as well as differentiable in the interval [1,5].

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{\sqrt{25 - 25} - \sqrt{25 - 1}}{5 - 1}$$

$$= \frac{-\sqrt{24}}{4}$$

$$f'(c) = \frac{1}{2\sqrt{25 - c^2}}(-2c)$$

$$\Rightarrow \frac{-c}{\sqrt{25 - c^2}} = \frac{-\sqrt{24}}{4}$$

$$\Rightarrow 4c = \sqrt{24(25 - c^2)}$$

$$\Rightarrow 16c^2 = 600 - 24c^2$$

$$\Rightarrow 40c^2 = 600$$

$$\Rightarrow c^2 = 15$$

$$\Rightarrow c = \sqrt{15}$$

Question: 14 C

Find 'c' of Lagra

Solution:

Given:

Since the f(x) is a polynomial function,

It is continuous as well as differentiable in the interval [4,6].

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{\sqrt{8} - \sqrt{6}}{6 - 4}$$

$$= \frac{\sqrt{8} - \sqrt{6}}{2}$$

$$f'(c) = \frac{1}{2\sqrt{c + 2}}$$

$$\Rightarrow \frac{1}{2\sqrt{c + 2}} = \frac{\sqrt{8} - \sqrt{6}}{2}$$

$$\Rightarrow \frac{1}{\sqrt{c + 2}} = \frac{\sqrt{8} - \sqrt{6}}{1}$$

$$\Rightarrow \sqrt{c + 2} = \frac{1}{\sqrt{8} - \sqrt{6}} \times \frac{\sqrt{8} + \sqrt{6}}{\sqrt{8} + \sqrt{6}}$$

$$\Rightarrow \sqrt{c + 2} = \frac{\sqrt{8} + \sqrt{6}}{2}$$

$$\Rightarrow c + 2 = \frac{1}{4}(8 + 6 + 2\sqrt{48})$$

$$\Rightarrow c = \frac{3}{2} + 2\sqrt{3}$$

⇒ c=4.964

Question: 15

Using Lagrange's

Solution:

Given:

$y=x^2$

Since y is a polynomial function.

It is continuous and differentiable in [1,2]

So, there exists a c such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{4 - 1}{2 - 1}$$

$$= 3$$

⇒ f' (c)=2c

⇒ 2c=3

$$c = \frac{3}{2}$$

So, the point is $\left(\frac{3}{2}, \frac{9}{4}\right)$

Question: 16

Find a point on t

Solution:

Given:

$y = x^3$

Since y is a polynomial function.

It is continuous and differentiable in [1,3]

So, there exists a c such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{27 - 1}{3 - 1}$$

$$= 13$$

⇒ f' (c)=3c²

⇒ 3c²=13

$$\Rightarrow c = \sqrt{\frac{13}{3}}$$

$$\Rightarrow c = \frac{\sqrt{39}}{3}$$

So the point is $\left(\frac{\sqrt{39}}{3}, \frac{13\sqrt{39}}{9}\right)$

Question: 17

Find the points o

Solution:

Given:

$y = x^3 - 3x$

Since y is a polynomial function.

It is continuous and differentiable in [1,2]

So, there exists a c such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{(8 - 6) - (1 - 3)}{2 - 1}$$

=4

$\Rightarrow f'(c) = 3c^2 - 3$

$\Rightarrow 3c^2 - 3 = 4$

$\Rightarrow 3c^2 = 7$

$\Rightarrow c^2 = \dfrac{7}{3}$

$\Rightarrow c = \pm\sqrt{\dfrac{7}{3}}$

So, the points are $\left(\sqrt{\dfrac{7}{3}}, \dfrac{-2}{3}\sqrt{\dfrac{7}{3}}\right), \left(\dfrac{-7}{3}\sqrt{\dfrac{7}{3}}, \dfrac{2}{3}\sqrt{\dfrac{7}{3}}\right)$

Question: 18

If

Solution:

Given:

$f(x) = x(1 - \log x)$

Since the function is continuous as well as differentiable

So, there exists c such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$\Rightarrow (1 - \log c) - c \times \dfrac{1}{c} = \dfrac{b(1 - \log b) - a(1 - \log a)}{b - a}$

$\Rightarrow \log c = \dfrac{b(1 - \log b) - a(1 - \log a)}{b - a}$

(b-a) log c = b(1-log b)-a(1-log a)

Hence proved.

Exercise : 11E

Question: 1

Find the maximum

Solution:

min. value = 4

Since the square of any no. Is positive, the given function has no maximum value.

The minimum value exists when the quantity $(5x-1)^2=0$

Therefore, minimum value=4

Question: 2

Find the maximum

Solution:

max. value = 9

Since the quantity $(x-3)^2$ has a –ve sign, the max. Value it can have is 9.

Also hence it has no minimum value.

Question: 3

Find the maximum

Solution:

max. value = 6

Since |x+4| is non-negative for all x belonging to R.

Therefore the least value it can have is 0 .

Hence value of function is 6.

It has no minimumvalue as it can have infinitely many.

Question: 4

Find the maximum

Solution:

max. value = 4, min. value = 6

f(x)=sin2x+5

We know that,

$-1 \leq \sin\Theta \leq 1$

$-1 \leq \sin 2x \leq 1$

Adding 5 on both sides,

$-1+5 \leq \sin 2x+5 \leq 1+5$

$4 \leq \sin 2x+5 \leq 6$

Hence

max value of f(x)=2x+5 will be 6

Min value of f(x) =2x+5 will be 4

Question: 5

Find the maximum

Solution:

max. value = 4, min. value = 2

We know that

$-1 \leq \sin\Theta \leq 1$

$-1 \leq \sin 4x \leq 1$

Adding 3 on both sides,

We get

$-1+3 \leq \sin 4x + 3 \leq 1+3$

$2 \leq |\sin 4x + 3| \leq 4$

Hence min.Value is 2 and max value is 4

Question: 6

Find the point of

Solution:

local max. value is 0 at x = 3

$F'(x) = 4(x-3)^3 = 0$

$\Rightarrow X = 3$

∴ local max. Vaue is 0.

Question: 7

Find the point of

Solution:

local min. value is 0 at x = 0

$F'(x) = 2x = 0$

$x = 0$

∴ local min.value is 0

Question: 8

Find the point of

Solution:

local max. value is -3 at x = 1 and local min. value is -128 at x = 6

$F'(x) = 6x^2 - 42x + 36 = 0$

$\Rightarrow 6(x-1)(x-6) = 0$

$\Rightarrow x = 1, 6$

$F''(x) = 12x - 42$

$F''(1) < 0$, 1 is the pont of local max.

$F''(6) > 0$, 6 is the point of localmin.

$F(1) = 2 - 21 + 36 - 20 = -3$

$F(6) = -128$

Question: 9

Find the point of

Solution:

local max. value is 19 at x = 1 and local min. value is 15 at x = 3

$F'(x) = 3x^2 - 12x + 9 = 0$

$\Rightarrow 3(x-3)(x-1) = 0$

$\Rightarrow x = 3, 1$

F''(x)=6x-12

F''(3)=18-12=6>0 , 3 is the of local min.

F''(1)<0, 1 is the point of local max.

F(3)=15

F(1)=19

Question: 10

Find the point of

Solution:

local max. value is 68 at x = 1 and local min. values are −1647 at x = −6 and −316 at x = 5

F'(x)=4x^3-124x+120=0

⇒ 4(x^3-31x+30)=0

For x=1, the given eq is 0

∴x-1 is a factor,

4(x-1)(x+6)(x-5)=0

⇒X=1,-6,5

F''(1)<0, 1is the point of max.

F''(-6) and f''(5)>0, -6 and 5 are point of min.

F(1)=68

F(-6)=-1647

F(5)=-316

Question: 11

Find the point of

Solution:

local max. value is 251 at x = 8 and local min. value is −5 at x = 0

'}(x)=-3x^2+24x=0

⇒ -3x(x-8)=0

⇒ x=0,8

F''(x)=-6x+24

F''(0)>0, 0 is the point of local min.

F''(8)<0, 8 is the point of local max.

F(8)=251 and f(0)=-5

Question: 12

Find the point of

Solution:

local max. value is 0 at x = −2 and local min. value is −4 at x = 0

f'(x)=(x-1)2(x+2)+(x+2)^2=0

x=0,-2

f''(0)>0, 0 is the point of local min.

f''(-2)<0, -2 is the point of local max.

f(0)=-4

f(-2)=0

Question: 13

Find the point of

Solution:

local max. value is 0 at each of the points x = 1 and x = −1 and local min. value is $\dfrac{-3456}{3125}$ at

$x = -\dfrac{1}{5}$

$F'(x) = -(x-1)^3 2(x+1) - 3(x-1)^2 (x+1)^2 = 0$

$\Rightarrow x = 1, -1, -\dfrac{1}{5}$

Since, $f^{||}(1)$ and $f^{||}(-1) < 0$, 1 and -1 are the points of local max.

$F^{||}\left(\dfrac{1}{5}\right) > 0$, $\dfrac{1}{5}$ is the point of local min.

$F(1) = f(-1) = 0$

Also, $f\left(-\dfrac{1}{5}\right) = -\dfrac{3456}{3125}$

Question: 14

Find the point of

Solution:

local min. value is 2 at x = 2

$F'(x) = \dfrac{1}{2} - \dfrac{2}{x^2} = 0$

$\Rightarrow x^2 - 4 = 0$

$\Rightarrow x = \pm 2$

But since x>0, x=2

$F''(2) = \dfrac{2}{x^3}$

$= \dfrac{2}{8} < 0$

∴point of local mini. is 2

$F(2) = \dfrac{2}{2} + \dfrac{2}{2} = 2$

Question: 15

Find the ma

Solution:

max. value is 139 at x = −2 and min. value is 89 at x = 3

$F'(x) = 6x^2 - 24 = 0$

$6(x^2 - 4) = 0$

$6(x^2 - 2^2) = 0$

$6(x-2)(x+2) = 0$

X=2,-2

Now, we shall evaluate the value of f at these points and the end points

F(2)=2(2)³-24(2)+107=75

F(-2)=2(-2)³-24(-2)+107=139

F(-3)=2(-3)³-24(-3)+107=125

F(3)=2(3)³-24(3)+107=89

Question: 16

Find the ma

Solution:

max. value is 257 at x = 4 and min. value is −63 at x = 2

Fˡ(x)=12x³-24x²+24x-48=0

12(x³-2x²+2x-4)=0

Since for x=2, x³-2x²+2x-4=0, x-2 is a factor

On dividing x³-2x²+2x-4 by x-2, we get,

12(x-2)(x²+2)=0

X=2,4

Now, we shall evaluate the value of f at these points and the end points

F(1)=3(1)⁴-8(1)³+12(1)²-48(1)+1=-40

F(2)= 3(2)⁴-8(2)³+12(2)²-48(2)+1=-63

F(4)= 3(4)⁴-8(4)³+12(4)²-48(4)+1=257

Question: 17

Find the ma

Solution:

max. value is $\dfrac{3}{4}$ at $x = \dfrac{\pi}{6}$ and min. value is $\dfrac{1}{2}$ at $x = \dfrac{\pi}{2}$

Fˡ(x)=cos x$-\dfrac{1}{2}$sin x=0

δ 2 cos x=sin x

$\Rightarrow \dfrac{\pi}{6} = \dfrac{\pi}{3}$

$f\left(\dfrac{\pi}{2}\right) = \sin\dfrac{\pi}{2} + \dfrac{1}{2}\cos\dfrac{\pi}{2} = \dfrac{1}{2}$

$f\left(\dfrac{\pi}{6}\right) = \sin\dfrac{\pi}{6} + \dfrac{1}{2}\cos\dfrac{\pi}{6} = \dfrac{1}{2} + \dfrac{\sqrt{3}}{4}$

$f\left(\dfrac{\pi}{3}\right) = \sin\dfrac{\pi}{3} + \dfrac{1}{2}\cos\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2} + \dfrac{1}{4}$

Question: 18

Show that t

Solution:

The given function is

$Y = x^{\frac{1}{x}}$

Now, taking logarithm from both sides, we get..

$\log y = \dfrac{1}{x} \log x$

Differentiating both sides w.r.t x....

$\dfrac{1}{y} y' = -\dfrac{1}{x^2} \ln(x) + \dfrac{1}{x^2}$

$\Rightarrow y' = \dfrac{y}{x^2}(1 - \ln(x))$

$(1 - \ln(x)) = 0$

$\ln(x) = 1$

$x = e$

hence the max. point is $x = e$

max value is $e^{\frac{1}{e}}$.

Question: 19

Show that

Solution:

$F(x) = x + \dfrac{1}{x}$

Taking first derivative and equating it to zero to find extreme points.

$F'(x) = 1 - \dfrac{1}{x^2} = 0$

$X^2 = 1$

$x = 1, x = -1$

now to determine which of these is min. And max. We use second derivative.

$f''(x) = \dfrac{2}{x^3}$

$f''(1) = 2$ and $f''(-1) = -2$

since $f''(1)$ is +ve it is minimum point while $f''(-1)$ is –ve it is maximum point

max value-> $f(-1) = -1 + \dfrac{1}{-1} = -2$

min vaue-> $f(1) = 1 + \dfrac{1}{1} = 2$

hence maximum value is less than minimum value

Question: 20

Find the ma

Solution:

49

$\dfrac{dp}{dx} = -24 - 36x$

$= 0$

$\Rightarrow x = -23$

Step 2

$\dfrac{d^2p}{dx^2} = -36$ is negative

Step 3

maximum profit $= p\left(-\dfrac{2}{3}\right)$

$= 49$

Question: 21

An enemy je

Solution:

(1, 3)

Let P(x,y) be the position of the jet and the soldier is placed at A(3,2)

$AP = \sqrt{(x-3)^2 + (y-2)^2}$

As $y = x^2 + 2$ or $y - 2 = x^2$

∴ $AP^2 = (x-3)^2 + x^4 = z$ (say)

$\dfrac{dz}{dx} = 2(x-3) + 4x^3$

$\dfrac{dz}{dx} = 0$

$2 \cdot x - 6 + 4x^3 = 0$

Put x=1

$2 - 6 + 4 = 0$

∴ x-1 is a factor

And $\dfrac{d^2z}{dx^2} = 12x^2 + 2$

$\dfrac{dz}{dx} = 0$ or x=1

and $\dfrac{d^2z}{dx^2}$(at x=1)>0

∴ z is minimum when x=1,y=1+2=3

Point is (1,3)

Question: 22

Find the ma

Solution:

max. value is $\left(-\dfrac{\pi}{3} + \sqrt{3}\right)$ at $x = \dfrac{\pi}{3}$ and min. value is $\left(\dfrac{5\pi}{3} + \sqrt{3}\right)$ at $x = \dfrac{5\pi}{3}$

$f'(x) = -1 + 2\cos x = 0$

$\Rightarrow \cos x = -\dfrac{1}{2}$

$x = \dfrac{2\pi}{3}$

By finding the general solution, we get $x = \frac{\pi}{3}$ and $x = \frac{5\pi}{3}$

Now, by finding the second derivative, we get that $f''\left(\frac{\pi}{3}\right) < 0$ and $f''\left(\frac{5\pi}{3}\right) > 0$

Therefore, max. value is $\left(-\frac{\pi}{3} + \sqrt{3}\right)$ at $x = \frac{\pi}{3}$ and min. value is $\left(\frac{5\pi}{3} + \sqrt{3}\right)$ at $x = \frac{5\pi}{3}$

Exercise : 11F

Question: 1

Find two positive

Solution:

Given,

• The two numbers are positive.

• the product of two numbers is 49.

• the sum of the two numbers is minimum.

Let us consider,

• x and y are the two numbers, such that x > 0 and y > 0

• Product of the numbers : x × y = 49

• Sum of the numbers : S = x + y

Now as,

x × y = 49

$y = \frac{49}{x}$ ------ (1)

Consider,

S = x + y

By substituting (1), we have

$S = x + \frac{49}{x}$ ------ (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. <u>This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.</u>

Differentiating the equation (2) with x

$\frac{dS}{dx} = \frac{d}{dx}\left(x + \frac{49}{x}\right)$

$\frac{dS}{dx} = \frac{d}{dx}(x) + \frac{d}{dx}\left(\frac{49}{x}\right)$

$\frac{dS}{dx} = 1 + 49\left(\frac{-1}{x^2}\right)$ ----- (3)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}\left(\frac{1}{x^n}\right) = \frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

Now equating the first derivative to zero will give the critical point c.

So,

$\frac{dS}{dx} = 1 + 49\left(\frac{-1}{x^2}\right) = 0$

$= 1 - \left(\frac{49}{x^2}\right) = 0$

$$= 1 = \left(\frac{49}{x^2}\right)$$

$$= x^2 = 49$$

$$= x = \pm\sqrt{49}$$

As x > 0, then x = 7

Now, for checking if the value of S is maximum or minimum at x=7, <u>we will perform the second differentiation and check the value of $\frac{d^2S}{dx^2}$ at the critical value x = 7.</u>

Performing the second differentiation on the equation (3) with respect to x.

$$\frac{d^2S}{dx^2} = \frac{d}{dx}\left[1 + 49\left(\frac{-1}{x^2}\right)\right]$$

$$\frac{d^2S}{dx^2} = \frac{d}{dx}[1] + \frac{d}{dx}\left[49\left(\frac{-1}{x^2}\right)\right]$$

$$\frac{d^2S}{dx^2} = 0 + \left[49\left(\frac{-1 \times -2}{x^3}\right)\right]$$

[Since $\frac{d}{dx}$ (constant) = 0 and $\frac{d}{dx}\left(\frac{1}{x^n}\right) = \frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

$$\frac{d^2S}{dx^2} = 49\left(\frac{2}{x^3}\right) = \frac{98}{x^3}$$

Now when x = 7,

$$\left[\frac{d^2S}{dx^2}\right]_{x=7} = \frac{98}{7^3} = \frac{98}{343} > 0$$

As second differential is positive, hence the critical point x = 7 will be the minimum point of the function S.

Therefore, the function S = sum of the two numbers is minimum at x = 7.

From Equation (1), if x= 7

$$y = \frac{49}{7} = 7$$

Therefore, x = 7 and y = 7 are the two positive numbers whose product is 49 and the sum is minimum.

Question: 2

Find two positive

Solution:

Given,

• The two numbers are positive.

• the sum of two numbers is 16.

• the sum of the squares of two numbers is minimum.

Let us consider,

• x and y are the two numbers, such that x > 0 and y > 0

• Sum of the numbers : x + y = 16

• Sum of squares of the numbers : S = $x^2 + y^2$

Now as,

x + y = 16

$y = (16-x)$ ------ (1)

Consider,

$S = x^2 + y^2$

By substituting (1), we have

$S = x^2 + (16-y)^2$ ------ (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. <u>This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.</u>

Differentiating the equation (2) with x

$\dfrac{dS}{dx} = \dfrac{d}{dx}[x^2 + (16-x)^2]$

$\dfrac{dS}{dx} = \dfrac{d}{dx}(x^2) + \dfrac{d}{dx}[(16-x)^2]$

$\dfrac{dS}{dx} = 2x + 2(16-x)(-1)$ ----- (3)

[Since $\dfrac{d}{dx}(x^n) = nx^{n-1}$]

Now equating the first derivative to zero will give the critical point c.

So,

$\dfrac{dS}{dx} = 2x + 2(16-x)(-1) = 0$

$\Rightarrow 2x - 2(16 - x) = 0$

$\Rightarrow 2x - 32 + 2x = 0$

$= 4x = 32$

$\Rightarrow x = \dfrac{32}{4}$

$\Rightarrow x = 8$

As x > 0, x = 8

Now, for checking if the value of S is maximum or minimum at x=8, <u>we will perform the second differentiation and check the value of $\dfrac{d^2S}{dx^2}$ at the critical value x = 8.</u>

Performing the second differentiation on the equation (3) with respect to x.

$\dfrac{d^2S}{dx^2} = \dfrac{d}{dx}[2x + 2(16-x)(-1)]$

$\dfrac{d^2S}{dx^2} = \dfrac{d}{dx}[2x] - 2\dfrac{d}{dx}[16-x]$

$\dfrac{d^2S}{dx^2} = 2 - 2[0-1]$

[Since $\dfrac{d}{dx}(\text{constant}) = 0$ and $\dfrac{d}{dx}(x^n) = nx^{n-1}$]

$\dfrac{d^2S}{dx^2} = 2 - 0 + 2 = 4$

Now when x = 8,

$\left[\dfrac{d^2S}{dx^2}\right]_{x=8} = 4 > 0$

As second differential is positive, hence the critical point x = 8 will be the minimum point of the

function S.

Therefore, the function S = sum of the squares of the two numbers is minimum at x = 8.

From Equation (1), if x= 8

y = 16 – 8 = 8

Therefore, x = 8 and y = 8 are the two positive numbers whose su is 16 and the sum of the squares is minimum.

Question: 3

Divide 15 into tw

Solution:

Given,

• the number 15 is divided into two numbers.

• the product of the square of one number and cube of another number is maximum.

Let us consider,

• x and y are the two numbers

• Sum of the numbers : x + y = 15

• Product of square of the one number and cube of anther number : $P = x^3 y^2$

Now as,

x + y = 15

y = (15-x) ------ (1)

Consider,

$P = x^3y^2$

By substituting (1), we have

$P = x^3 \times (15-x)^2$ ------ (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. <u>This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.</u>

Differentiating the equation (2) with x

$$\frac{dP}{dx} = \frac{d}{dx} [x^3 \times (15-x)^2]$$

$$\frac{dP}{dx} = (15-x)^2 \frac{d}{dx} (x^3) + x^3 \frac{d}{dx} [(15-x)^2]$$

$$\frac{dP}{dx} = (15-x)^2 (3x^2) + x^3 [2(15-x)(-1)]$$

[Since $\frac{d}{dx} (x^n) = nx^{n-1}$ and if u and v are two functions of x, then

$\frac{d}{dx} (u \times v) = v \times \frac{d}{dx} (u) + u \times \frac{d}{dx} (v)$]

$$\frac{dP}{dx} = (15-x)^2 (3x^2) + x^3 [-30 + 2x]$$

$= 3 \times [15^2 - 2 \times (15) \times (x) + x^2] x^2 + x^3(2x-30)$

$= x^2[3 \times (225 - 30x + x^2) + x (2x - 30)]$

$= x^2[675 - 90x + 3x^2 + 2x^2 - 60x]$

$= x^2[5x^2 - 120x + 675]$

$= 5x^2 [x^2 - 24x + 135]$ ----- (3)

Now equating the first derivative to zero will give the critical point c.

So,

$$\frac{dP}{dx} = 5x^2[x^2 - 24x + 135] = 0$$

Hence $5x^2 = 0$ (or) $x^2 - 24x + 135 = 0$

$x = 0$ (or) $x = \frac{-(-24)\pm\sqrt{(-24)^2 - 4(1)(135)}}{2 \times 1}$

$x = 0$ (or) $x = \frac{24\pm\sqrt{576 - 540}}{2}$

$x = 0$ (or) $x = \frac{24\pm\sqrt{36}}{2}$

$x = 0$ (or) $x = \frac{24\pm 6}{2}$

$x = 0$ (or) $x = \frac{24+6}{2}$ (or) $x = \frac{24-6}{2}$

$x = 0$ (or) $x = \frac{30}{2}$ (or) $x = \frac{18}{2}$

$x = 0$ (or) $x = 15$ (or) $x = 9$

Now considering the critical values of $x = 0,9,15$

Now, for checking if the value of P is maximum or minimum at x=0,9,15, we will perform the second differentiation and check the value of $\frac{d^2P}{dx^2}$ at the critical value x = 0,9,15.

Performing the second differentiation on the equation (3) with respect to x.

$$\frac{d^2P}{dx^2} = \frac{d}{dx}[5x^2(x^2 - 24x + 135)]$$

$$\frac{d^2P}{dx^2} = (x^2 - 24x + 135)\frac{d}{dx}[5x^2] + 5x^2\frac{d}{dx}[x^2 - 24x + 135]$$

$= (x^2 - 24x + 135)(5 \times 2x) + 5x^2(2x - 24 + 0)$

[Since $\frac{d}{dx}$ (constant) $= 0$ and $\frac{d}{dx}(x^n) = nx^{n-1}$ and if u and v are two functions of x, then $\frac{d}{dx}(u \times v) = v \times \frac{d}{dx}(u) + u \times \frac{d}{dx}(v)$]

$= (x^2 - 24x + 135)(10x) + 5x^2(2x - 24)$

$= 10x^3 - 240x^2 + 1350x + 10x^3 - 120x^2$

$= 20x^3 - 360x^2 + 1350x$

$= 5x(4x^2 - 72x + 270)$

$$\frac{d^2P}{dx^2} = 5x(4x^2 - 72x + 270)$$

Now when x = 0,

$$\left[\frac{d^2P}{dx^2}\right]_{x=0} = 5 \times 0[4(0)^2 - 72(0) + 270]$$

$= 0$

So, we reject x = 0

Now when x = 15,

$$\left[\frac{d^2P}{dx^2}\right]_{x=15} = 5 \times 15 \, [4(15)^2 - 72(15) + 270]$$

$= 65 \, [(4 \times 225) - 1080 + 270]$

$= 65 \, [900 - 1080 + 270]$

$= 65 \, [1170 - 1080]$

$= 65 \times (90) > 0$

Hence $\left[\frac{d^2P}{dx^2}\right]_{x=15} > 0$, so at x = 15, the function P is minimum

Now when x = 9,

$$\left[\frac{d^2P}{dx^2}\right]_{x=9} = 5 \times 9 \, [4(9)^2 - 72(9) + 270]$$

$= 45 \, [(4 \times 81) - 648 + 270]$

$= 45 \, [324 - 648 + 270]$

$= 45 \, [594 - 648]$

$= 45 \times (-54)$

$= -2430 < 0$

As second differential is negative, hence at the critical point x = 9 will be the maximum point of the function P.

Therefore, the function P is maximum at x = 9.

From Equation (1), if x = 9

y = 15 – 9 = 6

Therefore, x = 9 and y = 6 are the two positive numbers whose sum is 15 and the product of the square of one number and cube of another number is maximum.

Question: 4

Divide 8 into two

Solution:

Given,

• the number 8 is divided into two numbers.

• the product of the square of one number and cube of another number is minimum.

Let us consider,

• x and y are the two numbers

• Sum of the numbers : x + y = 8

• Product of square of the one number and cube of anther number : $S = x^3 + y^2$

Now as,

x + y = 8

y = (8-x) ------ (1)

Consider,

$S = x^3 + y^2$

By substituting (1), we have

$S = x^3 + (8-x)^2$ ------ (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.

Differentiating the equation (2) with x

$$\frac{dS}{dx} = \frac{d}{dx}[x^3 + (8-x)^2]$$

$$\frac{dS}{dx} = \frac{d}{dx}(x^3) + \frac{d}{dx}[(8-x)^2]$$

$$\frac{dS}{dx} = (3x^2) + 2(8-x)(-1)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$$\frac{dS}{dx} = 3x^2 - 16 + 2x$$

$$= 3x^2 + 2x - 16 \text{ ------ (3)}$$

Now equating the first derivative to zero will give the critical point c.

So,

$$\frac{dS}{dx} = 3x^2 + 2x - 16 = 0$$

Hence $3x^2 + 2x - 16 = 0$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(3)(-16)}}{2 \times 3}$$

$$= \frac{-2 \pm \sqrt{4 + 192}}{6}$$

$$= \frac{-2 \pm \sqrt{196}}{6}$$

$$x = \frac{-2 \pm 14}{6}$$

$$x = \frac{-2+14}{6} \text{ (or) } x = \frac{-2-14}{6}$$

$$x = \frac{12}{6} \text{ (or) } x = \frac{-16}{6}$$

x = 2 (or) x = -2.67

Now considering the critical values of x = 2,-2.67

Now, for checking if the value of P is maximum or minimum at x=2,-2.67, we will perform the second differentiation and check the value of $\frac{d^2S}{dx^2}$ at the critical value x = 2,-2.67.

Performing the second differentiation on the equation (3) with respect to x.

$$\frac{d^2S}{dx^2} = \frac{d}{dx}[3x^2 + 2x - 16]$$

$$\frac{d^2S}{dx^2} = \frac{d}{dx}[3x^2] + \frac{d}{dx}[2x] - \frac{d}{dx}[16]$$

$$= 3(2x) + 2(1) - 0$$

[Since $\frac{d}{dx}(\text{constant}) = 0$ and $\frac{d}{dx}(x^n) = nx^{n-1}$]

$$= 6x + 2$$

$$\frac{d^2S}{dx^2} = 6x + 2$$

Now when x = -2.67,

$$\left[\frac{d^2S}{dx^2}\right]_{x=-2.67} = 6\,(-2.67) + 2$$

= - 16.02 + 2 = -14.02

At x = -2.67 $\frac{d^2S}{dx^2} = -14.02 < 0$ hence, the function S will be maximum at this point.

Now consider x = 2,

$$\left[\frac{d^2S}{dx^2}\right]_{x=2} = 6\,(2) + 2$$

= 12 + 2 = 14

Hence $\left[\frac{d^2S}{dx^2}\right]_{x=2} = 14 > 0$, so at x = 2, the function S is minimum

As second differential is positive, hence at the critical point x = 2 will be the maximum point of the function S.

Therefore, the function S is maximum at x = 2.

From Equation (1), if x = 2

y = 8 – 2 = 6

Therefore, x = 2 and y = 6 are the two positive numbers whose sum is 8 and the sum of the square of one number and cube of another number is maximum.

Question: 5

Divide a into two

Solution:

Given,

• the number 'a' is divided into two numbers.

• the product of the pth power of one number and qth power of another number is maximum.

Let us consider,

• x and y are the two numbers

• Sum of the numbers : x + y = a

• Product of square of the one number and cube of anther number : $P = x^p\,y^q$

Now as,

x + y = a

y = (a-x) ------ (1)

Consider,

$P = x^p y^q$

By substituting (1), we have

$P = x^p \times (a-x)^q$ ------ (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. <u>This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.</u>

Differentiating the equation (2) with x

$$\frac{dP}{dx} = \frac{d}{dx}[x^P \times (a-x)^q]$$

$$\frac{dP}{dx} = (a-x)^q \frac{d}{dx}(x^P) + x^P \frac{d}{dx}[(a-x)^q]$$

$$\frac{dP}{dx} = (a-x)^q (px^{P-1}) + x^P [q(a-x)^{q-1}(-1)]$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and if u and v are two functions of x, then

$\frac{d}{dx}(u \times v) = v \times \frac{d}{dx}(u) + u \times \frac{d}{dx}(v)$]

$$\frac{dP}{dx} = x^{P-1}(a-x)^{q-1}[(a-x)p - xq]$$

$= x^{p-1}(a-x)^{q-1}[ap-xp-xq]$

$= x^{p-1}(a-x)^{q-1}[ap - x(p+q)]$ ----- (3)

Now equating the first derivative to zero will give the critical point c.

So,

$$\frac{dP}{dx} = x^{P-1}(a-x)^{q-1}[ap - x(p+q)] = 0$$

Hence $x^{p-1} = 0$ (or) $(a-x)^{q-1}$ (or) $ap - x(p+q) = 0$

$x = 0$ (or) $x = a$ (or) $x = \frac{ap}{p+q}$

Now considering the critical values of $x = 0, a$ and $x = \frac{ap}{p+q}$

Now, using the First Derivative test,

<u>For f, a continuous function which has a critical point c, then, function has the local maximum at c, if f'(x) changes the sign from positive to negative as x increases through c, i.e. f'(x)>0 at every point close to the left of c and f'(x)<0 at every point close to the right of c.</u>

Now when $x = 0$,

$$\left[\frac{dP}{dx}\right]_{x=0} = 0$$

So, we reject $x = 0$

Now when $x = a$,

$$\left[\frac{dP}{dx}\right]_{x=a} = 0$$

Hence we reject $x = a$

Now when $x < \frac{ap}{p+q}$,

$$\left[\frac{dP}{dx}\right]_{x<\frac{ap}{p+q}} = \left(\frac{ap}{p+q}\right)^{P-1}\left(a - \frac{ap}{p+q}\right)^{q-1}[ap - \frac{ap}{p+q}(p+q)] > 0 \text{ ---- (4)}$$

Now when $x > \frac{ap}{p+q}$,

$$\left[\frac{dP}{dx}\right]_{x>\frac{ap}{p+q}} = \left(\frac{ap}{p+q}\right)^{P-1}\left(a - \frac{ap}{p+q}\right)^{q-1}[ap - \frac{ap}{p+q}(p+q)] < 0 \text{ ---- (5)}$$

By using first derivative test, from (4) and (5), we can conclude that, the function P has local maximum at $x = \frac{ap}{p+q}$

From Equation (1), if $x = \frac{ap}{p+q}$

$$y = a - \frac{ap}{p+q} = \frac{a(p+q) - ap}{p+q} = \frac{ap}{p+q}$$

Therefore, $x = \frac{ap}{p+q}$ and $y = \frac{aq}{p+q}$ are the two positive numbers whose sum together to give the number 'a' and whose product of the pth power of one number and qth power of the other number is maximum.

Question: 6

The rate of worki

Solution:

Given:

Rate of working of an engine R, v is the speed of the engine:

$R = 15v + \frac{6000}{v}$, where $0 < v < 30$

For finding the maximum/ minimum of given function, we can find it by differentiating it with v and then equating it to zero. This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.

Now, differentiating the function R with respect to v.

$$\frac{dR}{dv} = \frac{d}{dv}[15v + \frac{6000}{v}]$$

$$\frac{dR}{dv} = \frac{d}{dv}[15v] + \frac{d}{dv}\left[\frac{6000}{v}\right]$$

$$\frac{dR}{dv} = 15 + \left[\frac{6000}{v^2}\right](-1) = 15 - \frac{6000}{v^2} \text{ ----- (1)}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}\left(\frac{1}{x^n}\right) = \frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

Equating equation (1) to zero to find the critical value.

$$\frac{dR}{dv} = 15 - \frac{6000}{v^2} = 0$$

$$15 = \frac{6000}{v^2}$$

$$v^2 = \frac{6000}{15} = 400$$

$$v^2 = 400$$

$$v = \pm\sqrt{400}$$

v = 20 (or) v = -20

As given in the question $0 < v < 30$, v = 20

Now, for checking if the value of R is maximum or minimum at v=20, we will perform the second differentiation and check the value of $\frac{d^2R}{dv^2}$ at the critical value v = 20.

Differentiating Equation (1) with respect to v again:

$$\frac{d^2R}{dv^2} = \frac{d}{dx}[15 - \frac{6000}{v^2}]$$

$$= \frac{d}{dx}[15] - \frac{d}{dx}\left[\frac{6000}{v^2}\right]$$

$$= 0 - (-2)\left[\frac{6000}{v^3}\right]$$

[Since $\frac{d}{dx}(\text{constant}) = 0$ and $\frac{d}{dx}\left(\frac{1}{x^n}\right) = \frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

$$= 2\left[\frac{6000}{v^3}\right]$$

$$\frac{d^2R}{dv^2} = \left[\frac{12000}{v^3}\right] \text{ ----- (2)}$$

Now find the value of $\left(\frac{d^2R}{dv^2}\right)_{v=20}$

$$\left(\frac{d^2R}{dv^2}\right)_{v=20} = \left[\frac{12000}{(20)^3}\right] = \frac{12000}{20 \times 20 \times 20} = \frac{3}{2} > 0$$

So, at critical point v = 20. The function R is at its minimum.

Hence, the function R is at its minimum at v = 20.

Question: 7
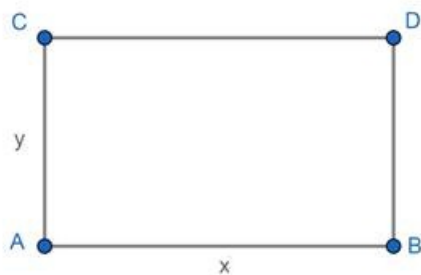
Find the dimensio

Solution:

Given,

• Area of the rectangle is 93 cm^2.

• The perimeter of the rectangle is also fixed.

Let us consider,

• x and y be the lengths of the base and height of the rectangle.

• Area of the rectangle = A = x × y = 96 cm^2

• Perimeter of the rectangle = P = 2 (x + y)

As,

x × y = 96

$$y = \frac{96}{x} \text{ ------ (1)}$$

Consider the perimeter function,

P = 2 (x + y)

Now substituting (1) in P,

$$P = 2\left(x + \frac{96}{x}\right) \text{ ----- (2)}$$

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.

Differentiating the equation (2) with respect to x:

$$\frac{dP}{dx} = \frac{d}{dx}\left[2\left(x + \frac{96}{x}\right)\right]$$

$$\frac{dP}{dx} = \frac{d}{dx}(2x) + 2\frac{d}{dx}\left(\frac{96}{x}\right)$$

$$\frac{dP}{dx} = 2(1) + 2\left(\frac{96}{x^2}\right)(-1)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}\left(\frac{1}{x^n}\right) = \frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

$$\frac{dP}{dx} = 2 - \left(\frac{192}{x^2}\right) \text{ ----- (3)}$$

To find the critical point, we need to equate equation (3) to zero.

$$\frac{dP}{dx} = 2 - \left(\frac{192}{x^2}\right) = 0$$

$$2 = \left(\frac{192}{x^2}\right)$$

$$x^2 = \left(\frac{192}{2}\right) = 96$$

$$x = \sqrt{96}$$

$$x = \pm 4\sqrt{6}$$

As the length and breadth of a rectangle cannot be negative, hence $x = 4\sqrt{6}$

Now to check if this critical point will determine the least perimeter, we need to check with second differential which needs to be positive.

Consider differentiating the equation (3) with x:

$$\frac{d^2P}{dx^2} = \frac{d}{dx}\left[2 - \left(\frac{192}{x^2}\right)\right]$$

$$\frac{d^2P}{dx^2} = \frac{d}{dx}(2) - \frac{d}{dx}\left(\frac{192}{x^2}\right)$$

$$\frac{d^2P}{dx^2} = 0 - (-2)\left(\frac{192}{x^3}\right)$$

[Since $\frac{d}{dx}(\text{constant}) = 0$ and $\frac{d}{dx}\left(\frac{1}{x^n}\right) = \frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

$$\frac{d^2P}{dx^2} = \left(\frac{2 \times 192}{x^3}\right) \text{ ------ (4)}$$

Now, consider the value of $\left(\frac{d^2P}{dx^2}\right)_{x=4\sqrt{6}}$

$$\frac{d^2P}{dx^2} = \left(\frac{2 \times 192}{(4\sqrt{6})^3}\right)$$

$$= \left(\frac{2 \times 192}{4\sqrt{6} \times 4\sqrt{6} \times 4\sqrt{6}}\right)$$

$$= \left(\frac{2 \times 192}{4\sqrt{6} \times 4\sqrt{6} \times 4\sqrt{6}}\right) = \frac{1}{\sqrt{6}}$$

As $\left(\frac{d^2P}{dx^2}\right)_{x=4\sqrt{6}} = \frac{1}{\sqrt{6}} > 0$, so the function P is minimum at $x = 4\sqrt{6}$.

Now substituting $x = 4\sqrt{6}$ in equation (1):

$$y = \frac{96}{4\sqrt{6}}$$

$$y = \frac{96\sqrt{6}}{4 \times 6}$$

[By rationalizing he numerator and denominator with $\sqrt{6}$]

$$\therefore y = 4\sqrt{6}$$

Hence, area of the rectangle with sides of a rectangle with $x = 4\sqrt{6}$ and $y = 4\sqrt{6}$ is 96cm² and has the least perimeter.

Now the perimeter of the rectangle is

$$P = 2(4\sqrt{6} + 4\sqrt{6}) = 2(8\sqrt{6}) = 16\sqrt{6} \text{ cms}$$

The least perimeter is $16\sqrt{6}$ cms.

Question: 8

Prove that the la

Solution:

Given,

• Rectangle with given perimeter.

Let us consider,

• 'p' as the fixed perimeter of the rectangle.

• 'x' and 'y' be the sides of the given rectangle.

• Area of the rectangle, A = x × y.

Now as consider the perimeter of the rectangle,

p = 2(x +y)

p = 2x + 2y

$y = \frac{p - 2x}{2}$ ----- (1)

Consider the area of the rectangle,

A = x × y

Substituting (1) in the area of the rectangle,

$$A = x \times \left(\frac{p - 2x}{2}\right)$$

$A = \frac{1}{2} \times (px - 2x^2)$ ----- (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. <u>This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.</u>

Differentiating the equation (2) with respect to x:

$$\frac{dA}{dx} = \frac{d}{dx}\left[\frac{1}{2}(px - 2x^2)\right]$$

$$\frac{dA}{dx} = \frac{1}{2}\frac{d}{dx}(px) - \frac{1}{2}\frac{d}{dx}(2x^2)$$

$$\frac{dA}{dx} = \frac{1}{2}(p) - \frac{2}{2}(2x)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$\frac{dA}{dx} = \frac{p}{2} - (2x)$ ----- (3)

To find the critical point, we need to equate equation (3) to zero.

$$\frac{dA}{dx} = \frac{p}{2} - (2x) = 0$$

$$2x = \frac{p}{2}$$

$$x = \frac{p}{4}$$

Now to check if this critical point will determine the largest rectangle, we need to check with second differential which needs to be negative.

Consider differentiating the equation (3) with x:

$$\frac{d^2A}{dx^2} = \frac{d}{dx}\left[\frac{p}{2} - (2x)\right]$$

$$\frac{d^2A}{dx^2} = \frac{d}{dx}\left(\frac{p}{2}\right) - \frac{d}{dx}(2x)$$

$$\frac{d^2A}{dx^2} = 0 - 2 = -2$$

[Since $\frac{d}{dx}$ (constant) $= 0$ and $\frac{d}{dx}(x^n) = nx^{n-1}$]

$$\frac{d^2A}{dx^2} = -2 \ \text{------ (4)}$$

Now, consider the value of $\left(\frac{d^2A}{dx^2}\right)_{x=\frac{p}{4}}$

$$\frac{d^2A}{dx^2} = -2 < 0$$

As $\left(\frac{d^2P}{dx^2}\right)_{x=\frac{p}{4}} = -2 < 0$, so the function P is maximum at $x = \frac{p}{4}$.

Now substituting $x = \frac{p}{4}$ in equation (1):

$$y = \frac{p - 2\left(\frac{p}{4}\right)}{2}$$

$$y = \frac{p - \frac{p}{2}}{2} = \frac{p}{4}$$

$$\therefore y = \frac{p}{4}$$

As $= y = \frac{p}{4}$ the sides of the taken rectangle are equal, we can clearly say that a largest rectangle which has a given perimeter is a square.
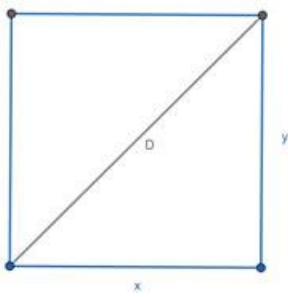
Question: 9

Given the perimet

Solution:

Given,

• Rectangle with given perimeter.

Let us consider,

• 'p' as the fixed perimeter of the rectangle.

• 'x' and 'y' be the sides of the given rectangle.

• Diagonal of the rectangle, $D = \sqrt{x^2 + y^2}$. (using the hypotenuse formula)

Now as consider the perimeter of the rectangle,

p = 2(x +y)

p = 2x + 2y

$y = \frac{p-2x}{2}$ ----- (1)

Consider the diagonal of the rectangle,

$D = \sqrt{x^2 + y^2}$

Substituting (1) in the diagonal of the rectangle,

$D = \sqrt{x^2 + \left(\frac{p-2x}{2}\right)^2}$

[squaring both sides]

$Z = D^2 = x^2 + \left(\frac{p-2x}{2}\right)^2$ ----- (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.

Differentiating the equation (2) with respect to x:

$\frac{dZ}{dx} = \frac{d}{dx}\left[x^2 + \left(\frac{p-2x}{2}\right)^2\right]$

$\frac{dZ}{dx} = \frac{d}{dx}(x^2) + \frac{1}{4}\frac{d}{dx}[(p-2x)^2]$

$\frac{dZ}{dx} = 2x + \frac{1}{4}[2(p-2x)(-2)]$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

= 2x - p + 2x

$\frac{dZ}{dx} = 4x - p$ ------ (3)

To find the critical point, we need to equate equation (3) to zero.

$\frac{dZ}{dx} = 4x - p = 0$

4x -p = 0

4x = p

$x = \frac{p}{4}$

Now to check if this critical point will determine the minimum diagonal, we need to check with second differential which needs to be positive.

Consider differentiating the equation (3) with x:

$$\frac{d^2Z}{dx^2} = \frac{d}{dx}[4x - p]$$

$$\frac{d^2Z}{dx^2} = \frac{d}{dx}(4x) - \frac{d}{dx}(p)$$

$$= 4 + 0$$

[Since $\frac{d}{dx}$ (constant) = 0 and $\frac{d}{dx}(x^n) = nx^{n-1}$]

$$\frac{d^2Z}{dx^2} = 4 \text{ ------ (4)}$$

Now, consider the value of $\left(\frac{d^2Z}{dx^2}\right)_{x=\frac{p}{4}}$

$$\frac{d^2Z}{dx^2} = 4 > 0$$

As $\left(\frac{d^2Z}{dx^2}\right)_{x=\frac{p}{4}} = 4 > 0$, so the function Z is minimum at $x = \frac{p}{4}$.

Now substituting $x = \frac{p}{4}$ in equation (1):

$$y = \frac{p - 2\left(\frac{p}{4}\right)}{2}$$

$$y = \frac{p - \frac{p}{2}}{2} = \frac{p}{4}$$

$$\therefore y = \frac{p}{4}$$

As $x = y = \frac{p}{4}$ the sides of the taken rectangle are equal, we can clearly say that a rectangle with minimum diagonal which has a given perimeter is a square.
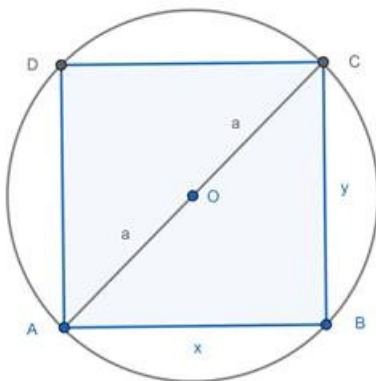
Question: 10

Show that a recta

Solution:

Given,

• Rectangle is of maximum perimeter.

• The rectangle is inscribed inside a circle.

• The radius of the circle is 'a'.

Let us consider,

• 'x' and 'y' be the length and breadth of the given rectangle.

• Diagonal $AC^2 = AB^2 + BC^2$ is given by $4a^2 = x^2 + y^2$ (as AC = 2a)

• Perimeter of the rectangle, P = 2(x+y)

Consider the diagonal,

$4a^2 = x^2 + y^2$

$y^2 = 4a^2 - x^2$

$y = \sqrt{4a^2 - x^2}$ ---- (1)

Now, perimeter of the rectangle, P

P = 2x + 2y

Substituting (1) in the perimeter of the rectangle.

$P = 2x + 2\sqrt{4a^2 - x^2}$ ------ (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.

Differentiating the equation (2) with respect to x:

$\dfrac{dP}{dx} = \dfrac{d}{dx}\left[2x + 2\sqrt{4a^2 - x^2}\right]$

$\dfrac{dP}{dx} = \dfrac{d}{dx}(2x) + 2\dfrac{d}{dx}[\sqrt{4a^2 - x^2}]$

$\dfrac{dP}{dx} = 2 + 2[\dfrac{1}{2}(4a^2 - x^2)^{-\frac{1}{2}}(-2x)]$

[Since $\dfrac{d}{dx}(x^n) = nx^{n-1}$]

$\dfrac{dP}{dx} = 2 - \dfrac{2x}{\sqrt{4a^2 - x^2}}$ ------ (3)

To find the critical point, we need to equate equation (3) to zero.

$\dfrac{dP}{dx} = 2 - \dfrac{2x}{\sqrt{4a^2 - x^2}} = 0$

$2 = \dfrac{2x}{\sqrt{4a^2 - x^2}}$

$\sqrt{4a^2 - x^2} = x$

[squaring on both sides]

$4a^2 - x^2 = x^2$

$2x^2 = 4a^2$

$x^2 = 2a^2$

$x = \pm a\sqrt{2}$

$x = a\sqrt{2}$

[as x cannot be negative]

Now to check if this critical point will determine the maximum diagonal, we need to check with second differential which needs to be negative.

Consider differentiating the equation (3) with x:

$\dfrac{d^2P}{dx^2} = \dfrac{d}{dx}\left[2 - \dfrac{2x}{\sqrt{4a^2 - x^2}}\right]$

$\dfrac{d^2P}{dx^2} = \dfrac{d}{dx}(2) - \dfrac{d}{dx}\left(\dfrac{2x}{\sqrt{4a^2 - x^2}}\right)$

$$\frac{d^2P}{dx^2} = 0 - \left[\frac{\sqrt{4a^2 - x^2}\frac{d}{dx}(2x) - (2x)\frac{d}{dx}(\sqrt{4a^2 - x^2})}{(\sqrt{4a^2 - x^2})^2}\right]$$

[Since $\frac{d}{dx}$ (constant) $= 0$ and $\frac{d}{dx}(x^n) = nx^{n-1}$ and if u and v are two functions of x, then

$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$]

$$\frac{d^2P}{dx^2} = -\left[\frac{\sqrt{4a^2 - x^2}\,(2) - (2x)\frac{1}{2}(4a^2 - x^2)^{-\frac{1}{2}}(-2x)}{4a^2 - x^2}\right]$$

$$\frac{d^2P}{dx^2} = -\left[\frac{\sqrt{4a^2 - x^2}\,(2) + (2x^2)(4a^2 - x^2)^{-\frac{1}{2}}}{4a^2 - x^2}\right]$$

$$\frac{d^2P}{dx^2} = -\left[\frac{2\sqrt{4a^2 - x^2} + \frac{2x^2}{\sqrt{4a^2 - x^2}}}{4a^2 - x^2}\right]$$

$$\frac{d^2P}{dx^2} = -\left[\frac{2(4a^2 - x^2) + 2x^2}{(4a^2 - x^2)^{\frac{3}{2}}}\right]$$

$$\frac{d^2P}{dx^2} = -\left[\frac{8a^2}{(4a^2 - x^2)^{\frac{3}{2}}}\right] \text{ ----- (4)}$$

Now, consider the value of $\left(\frac{d^2P}{dx^2}\right)_{x=a\sqrt{2}}$

$$\left(\frac{d^2P}{dx^2}\right)_{x=a\sqrt{2}} = -\left[\frac{8a^2}{(4a^2 - (a\sqrt{2})^2)^{\frac{3}{2}}}\right]$$

$$\left(\frac{d^2P}{dx^2}\right)_{x=a\sqrt{2}} = -\left[\frac{8a^2}{(4a^2 - 2a^2)^{\frac{3}{2}}}\right] = -\frac{8a^2}{(2a^2)^{\frac{3}{2}}} = -\frac{8a^2}{2\sqrt{2}\,a^3} = -\frac{2\sqrt{2}}{a}$$

As $\left(\frac{d^2P}{dx^2}\right)_{x=a\sqrt{2}} = -\frac{2\sqrt{2}}{a} < 0$, so the function P is maximum at $x = a\sqrt{2}$.

Now substituting $x = a\sqrt{2}$ in equation (1):

$$y = \sqrt{4a^2 - (a\sqrt{2})^2}$$

$$y = \sqrt{4a^2 - 2a^2} = \sqrt{2a^2}$$

$$\therefore y = a\sqrt{2}$$

As $x = y = a\sqrt{2}$ the sides of the taken rectangle are equal, we can clearly say that a rectangle with maximum perimeter which is inscribed inside a circle of radius 'a' is a square.

Question: 11

The sum of the pe

Solution:

Given,

• Sum of perimeter of square and circle.

Let us consider,

• 'x' be the side of the square.

- 'r' be the radius of the circle.

- Let 'p' be the sum of perimeters of square and circle.

$p = 4x + 2\pi r$

Consider the sum of the perimeters of square and circle.

$p = 4x + 2\pi r$

$4x = p - 2\pi r$

$x = \dfrac{p - 2\pi r}{4}$ ---- (1)

Sum of the area of the circle and square is

$A = x^2 + \pi r^2$

Substituting (1) in the sum of the areas,

$A = \left(\dfrac{p - 2\pi r}{4}\right)^2 + \pi r^2$

$A = \dfrac{1}{16}[p^2 + 4\pi^2 r^2 - 4\pi p r] + \pi r^2$ ------ (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with r and then equating it to zero. This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.

Differentiating the equation (2) with respect to r:

$\dfrac{dA}{dr} = \dfrac{d}{dr}\left[\dfrac{1}{16}[p^2 + 4\pi^2 r^2 - 4\pi p r] + \pi r^2\right]$

$\dfrac{dA}{dr} = \dfrac{1}{16}\dfrac{d}{dr}(p^2 + 4\pi^2 r^2 - 4\pi p r) + \dfrac{d}{dr}[\pi r^2]$

$\dfrac{dA}{dr} = \dfrac{1}{16}(0 + 8\pi^2 r - 4\pi p) + 2\pi r$

[Since $\dfrac{d}{dx}(x^n) = nx^{n-1}$ and $\dfrac{d}{dx}(\text{constant}) = 0$]

$\dfrac{dA}{dr} = \dfrac{\pi^2 r}{2} - \dfrac{\pi p}{4} + 2\pi r$ ------ (3)

To find the critical point, we need to equate equation (3) to zero.

$\dfrac{dA}{dr} = \dfrac{\pi^2 r}{2} - \dfrac{\pi p}{4} + 2\pi r = 0$

$\left(\dfrac{\pi^2}{2} + 2\pi\right)r - \dfrac{\pi p}{4} = 0$

$r = \dfrac{\dfrac{\pi p}{4}}{\dfrac{\pi^2}{2} + 2\pi} = \dfrac{2\pi p}{4(\pi^2 + 4\pi)} = \dfrac{\pi p}{2(\pi^2 + 4\pi)}$

$r = \dfrac{\pi p}{2\pi(\pi + 4)} = \dfrac{p}{2(\pi + 4)}$

$r = \dfrac{p}{2(\pi + 4)}$

Now to check if this critical point will determine the least of the sum of the areas of square and circle, we need to check with second differential which needs to be positive.

Consider differentiating the equation (3) with r:

$\dfrac{d^2 A}{dr^2} = \dfrac{d}{dx}\left[\dfrac{\pi^2 r}{2} - \dfrac{\pi p}{4} + 2\pi r\right]$

$$\frac{d^2A}{dr^2} = \frac{d}{dr}\left(\frac{\pi^2 r}{2}\right) - \frac{d}{dr}\left(\frac{\pi p}{4}\right) + \frac{d}{dr}(2\pi r)$$

$$\frac{d^2A}{dr^2} = \frac{\pi^2}{2} - 0 + 2\pi$$

[Since $\frac{d}{dx}$ (constant) $= 0$ and $\frac{d}{dx}(x^n) = nx^{n-1}$]

$$\frac{d^2A}{dr^2} = \frac{\pi^2}{2} + 2\pi \text{ ----- (4)}$$

Now, consider the value of $\left(\frac{d^2A}{dr^2}\right)_{r=\frac{p}{2(\pi+4)}}$

$$\left(\frac{d^2A}{dr^2}\right)_{r=\frac{p}{2(\pi+4)}} = \frac{\pi^2}{2} + 2\pi$$

As $\left(\frac{d^2A}{dr^2}\right)_{r=\frac{p}{2(\pi+4)}} = \frac{\pi^2}{2} + 2\pi > 0$, so the function A is minimum at $r = \frac{p}{2(\pi+4)}$.

Now substituting $r = \frac{p}{2(\pi+4)}$ in equation (1):

$$x = \frac{p - 2\pi\left(\frac{p}{2(\pi+4)}\right)}{4}$$

$$x = \frac{p(\pi+4) - \pi p}{4 \times (\pi+4)} = \frac{\pi p + 4p - \pi p}{4\pi + 16} = \frac{4p}{4(\pi+4)}$$

$$x = \frac{p}{\pi+4}$$

As the side of the square,

$$x = \frac{p}{\pi+4}$$

$$x = 2\left[\frac{p}{2(\pi+4)}\right] = 2r$$

[as $r = \frac{p}{2(\pi+4)}$]

Therefore, side of the square, $x = 2r$ = diameter of the circle.

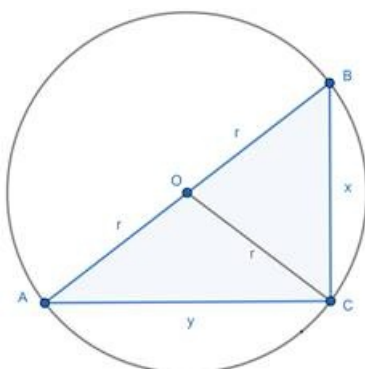**Question: 12**

Show that the rig

Solution:

Given,

• A right angle triangle is inscribed inside the circle.

• The radius of the circle is given.

Let us consider,

- 'r' is the radius of the circle.

- 'x' and 'y' be the base and height of the right angle triangle.

- The hypotenuse of the $\triangle ABC = AB^2 = AC^2 + BC^2$

$AB = 2r$, $AC = y$ and $BC = x$

Hence,

$4r^2 = x^2 + y^2$

$y^2 = 4r^2 - x^2$

$y = \sqrt{4r^2 - x^2}$ --- (1)

Now, Area of the $\triangle ABC$ is

$A = \dfrac{1}{2} \times \text{base} \times \text{height}$

$A = \dfrac{1}{2} \times x \times y$

Now substituting (1) in the area of the triangle,

$A = \dfrac{1}{2} x \left(\sqrt{4r^2 - x^2} \right)$

[Squaring both sides]

$Z = A^2 = \dfrac{1}{4} x^2 (4r^2 - x^2)$ ------ (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.

Differentiating the equation (2) with respect to x:

$\dfrac{dZ}{dx} = \dfrac{d}{dx} \left[\dfrac{1}{4} x^2 (4r^2 - x^2) \right]$

$\dfrac{dZ}{dx} = \dfrac{1}{4} \left[(4r^2 - x^2) \dfrac{d}{dx} (x^2) + x^2 \dfrac{d}{dx} (4r^2 - x^2) \right]$

$\dfrac{dZ}{dx} = \dfrac{1}{4} [(4r^2 - x^2) \times (2x) + x^2 (0 - 2x)]$

[Since $\dfrac{d}{dx} (x^n) = nx^{n-1}$ and if u and v are two functions of x, then $\dfrac{d}{dx} (u.v) = v \dfrac{du}{dx} + u \dfrac{dv}{dx}$]

$\dfrac{dZ}{dx} = \dfrac{1}{4} [8r^2 x - 2x^3 - 2x^3]$

$\dfrac{dZ}{dx} = \dfrac{1}{4} [8r^2 x - 4x^3] = \dfrac{4x}{4} [2r^2 - x^2]$

$\dfrac{dZ}{dx} = 2r^2 x - x^3$ ------ (3)

To find the critical point, we need to equate equation (3) to zero.

$\dfrac{dZ}{dx} = 2r^2 x - x^3 = 0$

$2r^2 x = x^3$

$x^2 = 2r^2$

$x = \pm \sqrt{2r^2}$

$x = r\sqrt{2}$

[as the base of the triangle cannot be negative.]

Now to check if this critical point will determine the maximum area of the triangle, we need to check with second differential which needs to be negative.

Consider differentiating the equation (3) with x:

$$\frac{d^2Z}{dx^2} = \frac{d}{dx}[2r^2x - x^3]$$

$$\frac{d^2Z}{dx^2} = \frac{d}{dx}(2r^2x) - \frac{d}{dx}(x^3)$$

$$\frac{d^2Z}{dx^2} = 2r^2 - 3x^2 \text{ ----- (4)}$$

$$[\text{Since } \frac{d}{dx}(x^n) = nx^{n-1}]$$

Now, consider the value of $\left(\frac{d^2Z}{dx^2}\right)_{x=r\sqrt{2}}$

$$\left(\frac{d^2Z}{dx^2}\right)_{x=r\sqrt{2}} = 2r^2 - 3(r\sqrt{2})^2 = 2r^2 - 6r^2 = -4r^2$$

As $\left(\frac{d^2Z}{dx^2}\right)_{x=r\sqrt{2}} = -4r^2 < 0$, so the function A is maximum at $x = r\sqrt{2}$.

Now substituting $x = r\sqrt{2}$ in equation (1):

$$y = \sqrt{4r^2 - (r\sqrt{2})^2}$$

$$y = \sqrt{4r^2 - 2r^2} = \sqrt{2r^2} = r\sqrt{2}$$

As $x = y = r\sqrt{2}$, the base and height of the triangle are equal, which means that two sides of a right angled triangle are equal,

Hence the given triangle, which is inscribed in a circle, is an isosceles triangle with sides AC and BC equal.
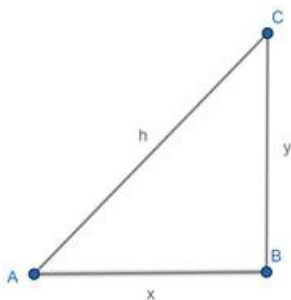
Question: 13

Prove that the pe

Solution:

Given,

• A right angle triangle.

• Hypotenuse of the given triangle is given.

Let us consider,

• 'h' is the hypotenuse of the given triangle.

• 'x' and 'y' be the base and height of the right angle triangle.

• The hypotenuse of the ΔABC = $AC^2 = AB^2 + BC^2$

AC = h, AB = x and BC = y

Hence,

$h^2 = x^2 + y^2$

$y^2 = h^2 - x^2$

$y = \sqrt{h^2 - x^2}$ --- (1)

Now, perimeter of the $\triangle ABC$ is

$P = h + x + y$

Now substituting (1) in the area of the triangle,

$P = h + x + \sqrt{h^2 - x^2}$ ----- (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.

Differentiating the equation (2) with respect to x:

$$\frac{dP}{dx} = \frac{d}{dx}\left[h + x + \sqrt{h^2 - x^2}\right]$$

$$\frac{dP}{dx} = \left[\frac{d}{dx}(h) + \frac{d}{dx}(x) + \frac{d}{dx}(\sqrt{h^2 - x^2})\right]$$

$$\frac{dP}{dx} = 0 + 1 + \frac{1}{2}\left(\frac{-2x}{\sqrt{h^2 - x^2}}\right)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$$\frac{dP}{dx} = 1 - \frac{x}{\sqrt{h^2 - x^2}}$$ ------ (3)

To find the critical point, we need to equate equation (3) to zero.

$$\frac{dP}{dx} = 1 - \frac{x}{\sqrt{h^2 - x^2}} = 0$$

$$\frac{x}{\sqrt{h^2 - x^2}} = 1$$

$$x = \sqrt{h^2 - x^2}$$

[squaring on both sides]

$x^2 = h^2 - x^2$

$$x^2 = \frac{h^2}{2}$$

$$x = \pm\sqrt{\frac{h^2}{2}}$$

$$x = \frac{h}{\sqrt{2}}$$

[as the base of the triangle cannot be negative.]

Now to check if this critical point will determine the maximum perimeter of the triangle, we need to check with second differential which needs to be negative.

Consider differentiating the equation (3) with x:

$$\frac{d^2P}{dx^2} = \frac{d}{dx}\left[1 - \frac{x}{\sqrt{h^2 - x^2}}\right]$$

$$\frac{d^2P}{dx^2} = \frac{d}{dx}(1) - \frac{d}{dx}\left(\frac{x}{\sqrt{h^2 - x^2}}\right)$$

$$\frac{d^2P}{dx^2} = 0 - \left[\frac{\sqrt{h^2 - x^2}\,\frac{d}{dx}(x) - x\,\frac{d}{dx}\left(\sqrt{h^2 - x^2}\right)}{\left(\sqrt{h^2 - x^2}\right)^2}\right]$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ if u and v are two functions of x, then $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$]

$$\frac{d^2P}{dx^2} = -\left[\frac{\sqrt{h^2 - x^2}\,(1) - x\left(\frac{-2x}{2\sqrt{h^2 - x^2}}\right)}{h^2 - x^2}\right]$$

$$\frac{d^2P}{dx^2} = -\left[\frac{\left(\sqrt{h^2 - x^2}\right)^2 + x^2}{h^2 - x^2\,\sqrt{h^2 - x^2}}\right] = -\left[\frac{h^2}{(h^2 - x^2)\,\sqrt{h^2 - x^2}}\right]$$

$$\frac{d^2P}{dx^2} = -\left[\frac{h^2}{(h^2 - x^2)^{\frac{3}{2}}}\right]$$

Now, consider the value of $\left(\frac{d^2P}{dx^2}\right)_{x = \frac{h}{\sqrt{2}}}$

$$\left(\frac{d^2P}{dx^2}\right)_{x = \frac{h}{\sqrt{2}}} = -\left[\frac{h^2}{\left(h^2 - \left(\frac{h}{\sqrt{2}}\right)^2\right)^{\frac{3}{2}}}\right] = -\left[\frac{h^2}{\left(\frac{h^2}{2}\right)^{\frac{3}{2}}}\right] = -\left[\frac{h^2}{\left(\frac{h^2}{2}\right)^{\frac{3}{2}}}\right] = -\frac{2^{\frac{3}{2}}}{h}$$

As $\left(\frac{d^2P}{dx^2}\right)_{x = \frac{h}{\sqrt{2}}} = -\frac{2^{\frac{3}{2}}}{h} < 0$, so the function A is maximum at $x = \frac{h}{\sqrt{2}}$.

Now substituting $x = \frac{h}{\sqrt{2}}$ in equation (1):

$$y = \sqrt{h^2 - \left(\frac{h}{\sqrt{2}}\right)^2}$$

$$y = \sqrt{\frac{h^2}{2}} = \frac{h}{\sqrt{2}}$$

As $x = y = \frac{h}{\sqrt{2}}$, the base and height of the triangle are equal, which means that two sides of a right angled triangle are equal,

Hence the given triangle is an isosceles triangle with sides AB and BC equal.

Question: 14

The perimeter of

Solution:

Given,

• Perimeter of a triangle is 8 cm.

• One of the sides of the triangle is 3 cm.

• The area of the triangle is maximum.

Let us consider,

• 'x' and 'y' be the other two sides of the triangle.

Now, perimeter of the ∆ABC is

$8 = 3 + x + y$

$y = 8\text{-}3\text{-}x = 5\text{-}x$

$y = 5\text{-}x$ --- (1)

Consider the Heron's area of the triangle,

$A = \sqrt{s(s-a)(s-b)(s-c)}$

Where $s = \frac{a+b+c}{2}$

As perimeter $= a + b + c = 8$

$s = \frac{8}{2} = 4$

Now Area of the triangle is given by

$A = \sqrt{8\,(8-3)(8-x)(8-y)}$

Now substituting (1) in the area of the triangle,

$A = \sqrt{4(4-3)(4-x)(4-(5-x))}$

$A = \sqrt{4(4-x)(x-1)}$

$A = \sqrt{4(\,4x-4-x^2+x)} = \sqrt{4(\,5x-x^2-4)}$

$A = \sqrt{4(\,5x-x^2-4)}$

[squaring on both sides]

$Z = A^2 = 4(5x -x^2-4)$ ----- (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.

Differentiating the equation (2) with respect to x:

$\frac{dZ}{dx} = \frac{d}{dx}\,[4(5x -x^2-4)]$

$\frac{dZ}{dx} = 4\frac{d}{dx}(5x) - 4\frac{d}{dx}(x^2) - 4\frac{d}{dx}(4)$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$\frac{dZ}{dx} = 4\,(5) - 4(2x) - 0$

$\frac{dZ}{dx} = 20 - 8x$ ------ (3)

To find the critical point, we need to equate equation (3) to zero.

$\frac{dZ}{dx} = 20 - 8x = 0$

$20 - 8x = 0$

$8x = 20$

$x = \frac{5}{2}$

Now to check if this critical point will determine the maximum area of the triangle, we need to check with second differential which needs to be negative.

Consider differentiating the equation (3) with x:

$$\frac{d^2Z}{dx^2} = \frac{d}{dx}[20 - 8x]$$

$$\frac{d^2Z}{dx^2} = -8 \ \text{----- (4)}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

As $\left(\frac{d^2Z}{dx^2}\right)_{x=\frac{5}{2}} = -8 < 0$, so the function A is maximum at $x = \frac{5}{2}$.

Now substituting $x = \frac{5}{2}$ in equation (1):

y = 5 – 2.5

y = 2.5

As x = y = 2.5, two sides of the triangle are equal,

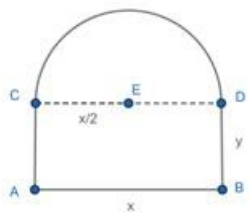Hence the given triangle is an isosceles triangle with two sides equal to 2.5 cm and the third side equal to 3cm.

Question: 15

A window is in th

Solution:

Given,

• Window is in the form of a rectangle which has a semicircle mounted on it.

• Total Perimeter of the window is 10 metres.

• The total area of the window is maximum.

Let us consider,

• The breadth and height of the rectangle be 'x' and 'y'.

• The radius of the semicircle will be half of the base of the rectangle.

Given Perimeter of the window is 10 meters:

$$10 = (x + 2y) + \frac{1}{2}\left[2\pi\left(\frac{x}{2}\right)\right]$$

[as the perimeter of the window will be equal to one side (x) less to the perimeter of rectangle and the perimeter of the semicircle.]

$$10 = (x + 2y) + \left(\frac{\pi x}{2}\right)$$

From here,

$$2y = 10 - x - \left(\frac{\pi x}{2}\right) = \frac{20 - 2x - \pi x}{2}$$

$$y = \frac{20 - 2x - \pi x}{4} \ \text{----- (1)}$$

Now consider the area of the window,

Area of the window = area of the semicircle + area of the rectangle

$$A = \frac{1}{2}\left[\pi\left(\frac{x}{2}\right)^2\right] + xy$$

Substituting (1) in the area equation:

$$A = \frac{1}{2}\left[\pi\left(\frac{x}{2}\right)^2\right] + x\left(\frac{20 - 2x - \pi x}{4}\right)$$

$$A = \frac{1}{8}\left[\pi x^2\right] + \left(\frac{20x - 2x^2 - \pi x^2}{4}\right)$$

$$A = \frac{\pi x^2 - 2\pi x^2 + 40x - 4x^2}{8}$$

$$A = \frac{1}{8}\left[x^2(\pi - 2\pi - 4) + 40x\right] \text{------ (2)}$$

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.

Differentiating the equation (2) with respect to x:

$$\frac{dA}{dx} = \frac{d}{dx}\left[\frac{1}{8}\left[x^2(\pi - 2\pi - 4) + 40x\right]\right]$$

$$\frac{dA}{dx} = \frac{1}{8}\frac{d}{dx}\left(x^2(\pi - 2\pi - 4)\right) + \frac{1}{8}\frac{d}{dx}(40x)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$$\frac{dA}{dx} = \frac{1}{8}\left[2x(-\pi - 4)\right] + \frac{1}{8}(40)$$

$$\frac{dA}{dx} = \frac{1}{4}\left[x(-\pi - 4)\right] + 5 \text{------ (3)}$$

To find the critical point, we need to equate equation (3) to zero.

$$\frac{dA}{dx} = \frac{1}{4}\left[x(-\pi - 4)\right] + 5 = 0$$

$$\frac{1}{4}\left[x(-\pi - 4)\right] + 5 = 0$$

$$\frac{1}{4}\left[x(4 + \pi)\right] = 5$$

$$x(4 + \pi) = 20$$

$$x = \frac{20}{(4 + \pi)}$$

Now to check if this critical point will determine the maximum area of the window, we need to check with second differential which needs to be negative.

Consider differentiating the equation (3) with x:

$$\frac{d^2A}{dx^2} = \frac{d}{dx}\left[\frac{1}{4}\left[x(-\pi - 4)\right] + 5\right]$$

$$\frac{d^2A}{dx^2} = \frac{d}{dx}\left[x(-\pi - 4)\right] + \frac{d}{dx}(5)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$$\frac{d^2A}{dx^2} = (-\pi - 4)(1) + 0 = -(\pi + 4) \text{------ (4)}$$

As $\left(\frac{d^2A}{dx^2}\right)_{x = \frac{20}{(4+\pi)}} = -(\pi + 4) < 0$, so the function A is maximum at $x = \frac{20}{(4+\pi)}$.

Now substituting $x = \frac{20}{(4+\pi)}$ in equation (1):

$$y = \frac{20 - \left(\frac{20}{(4+\pi)}\right)(\pi + 2)}{4}$$

$$y = \frac{20(4+\pi) - (20)(\pi+2)}{4(4+\pi)} = \frac{20[4+\pi-\pi-2]}{4(4+\pi)} = \frac{20 \times 2}{4(4+\pi)}$$

$$y = \frac{5 \times 2}{(4+\pi)} = \frac{10}{(4+\pi)}$$

Hence the given window with maximum area has breadth, $x = \frac{20}{(4+\pi)}$ and height, $y = \frac{10}{(4+\pi)}$.
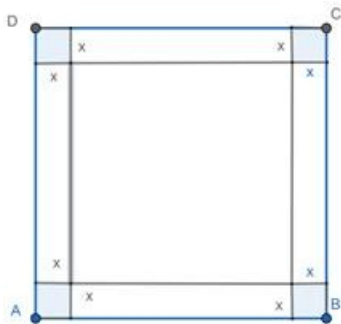
Question: 16

A square piece of

Solution:

Given,

• Side of the square piece is 12 cms.

• the volume of the formed box is maximum.

Let us consider,

• 'x' be the length and breadth of the piece cut from each vertex of the piece.

• Side of the box now will be (12-2x)

• The height of the new formed box will also be 'x'.

Let the volume of the newly formed box is :

V = (12-2x)^2 × (x)

V = (144 + 4x^2 – 48x) x

V = 4x^3 -48x^2 +144x ------ (1)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.

Differentiating the equation (1) with respect to x:

$$\frac{dV}{dx} = \frac{d}{dx}[4x^3 - 48x^2 + 144x]$$

$$\frac{dV}{dx} = 12x^2 - 96x + 144 \text{-------- (2)}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

To find the critical point, we need to equate equation (2) to zero.

$$\frac{dV}{dx} = 12x^2 - 96x + 144 = 0$$

$x^2 - 8x + 12 = 0$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(12)}}{2(1)} = \frac{8 \pm \sqrt{64 - 48}}{2} = \frac{8 \pm \sqrt{16}}{2}$$

$$x = = \frac{8 \pm 4}{2}$$

x = 6 or x = 2

x = 2

[as x = 6 is not a possibility, because 12-2x = 12-12= 0]

<u>Now to check if this critical point will determine the maximum area of the box, we need to check with second differential which needs to be negative.</u>

Consider differentiating the equation (3) with x:

$$\frac{d^2V}{dx^2} = \frac{d}{dx}[12x^2 - 96x + 144]$$

$$\frac{d^2V}{dx^2} = 24x - 96 \text{ ----- (4)}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now let us find the value of

$$\left(\frac{d^2V}{dx^2}\right)_{x=2} = 24\,(2) - 96 = 48 - 96 = -48$$

As $\left(\frac{d^2V}{dx^2}\right)_{x=2} = -48 < 0$, so the function A is maximum at x = 2

Now substituting x = 2 in 12 – 2x, the side of the considered box:

Side = 12-2x = 12 - 2(2) = 12-4= 8cms

Therefore side of wanted box is 8cms and height of the box is 2cms.

Now, the volume of the box is

V = $(8)^2 \times 2 = 64 \times 2 = 128cm^3$

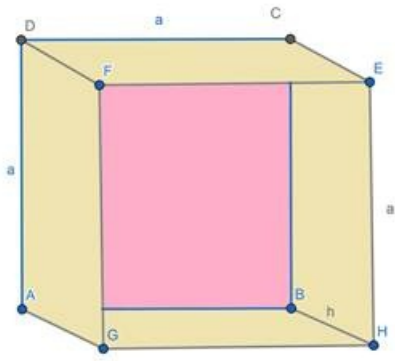Hence maximum volume of the box formed by cutting the given 12cms sheet is $128cm^3$ with 8cms side and 2cms height.

Question: 17

An open box with

Solution:

Given,

• The open box has a square base

• The area of the box is c^2 square units.

• The volume of the box is maximum.

Let us consider,

• The side of the square base of the box be 'a' units. (pink coloured in the figure)

• The breadth of the 4 sides of the box will also be 'a'units (skin coloured part).

• The depth of the box or the length of the sides be 'h' units (skin coloured part).

Now, the area of the box =

(area of the base) + 4 (area of each side of the box)

So as area of the box is given c^2,

$c^2 = a^2 + 4ah$

$h = \dfrac{c^2 - a^2}{4a}$ ---- (1)

Let the volume of the newly formed box is :

$V = (a)^2 \times (h)$

[substituting (1) in the volume formula]

$V = a^2 \times \left(\dfrac{c^2 - a^2}{4a} \right)$

$V = \left(\dfrac{ac^2 - a^3}{4} \right)$ ------ (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with a and then equating it to zero. <u>This is because if the function f(a) has a maximum/minimum at a point c then f'(c) = 0.</u>

Differentiating the equation (2) with respect to a:

$\dfrac{dV}{da} = \dfrac{d}{da} \left[\left(\dfrac{ac^2 - a^3}{4} \right) \right]$

$\dfrac{dV}{da} = \dfrac{c^2}{4} - \dfrac{3a^2}{4}$ ------- (3)

[Since $\dfrac{d}{dx}(x^n) = nx^{n-1}$]

To find the critical point, we need to equate equation (3) to zero.

$\dfrac{dV}{da} = \dfrac{c^2}{4} - \dfrac{3a^2}{4} = 0$

$c^2 - 3a^2 = 0$

$a^2 = \dfrac{c^2}{3}$

$a = \pm \sqrt{\dfrac{c^2}{3}}$

$$a = \frac{c}{\sqrt{3}}$$

[as 'a' cannot be negative]

Now to check if this critical point will determine the maximum Volume of the box, we need to check with second differential which needs to be negative.

Consider differentiating the equation (3) with x:

$$\frac{d^2V}{da^2} = \frac{d}{dx}\left[\frac{c^2}{4} - \frac{3a^2}{4}\right]$$

$$\frac{d^2V}{da^2} = 0 - \frac{3 \times 2 \times a}{4} = -\frac{3a}{2} \quad \text{----- (4)}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now let us find the value of

$$\left(\frac{d^2V}{da^2}\right)_{a=\frac{c}{\sqrt{3}}} = -\frac{3\left(\frac{c}{\sqrt{3}}\right)}{2} = -\frac{c\sqrt{3}}{2}$$

As $\left(\frac{d^2V}{da^2}\right)_{a=\frac{c}{\sqrt{3}}} = -48 - \frac{c\sqrt{3}}{2} < 0$, so the function V is maximum at $a = \frac{c}{\sqrt{3}}$

Now substituting a in equation (1)

$$h = \frac{c^2 - \left(\frac{c}{\sqrt{3}}\right)^2}{4\left(\frac{c}{\sqrt{3}}\right)} = \frac{\frac{2c^2}{3}}{\frac{4c}{\sqrt{3}}} = \frac{c\sqrt{3}}{6} = \frac{c}{2\sqrt{3}}$$

$$\therefore h = \frac{c}{2\sqrt{3}}$$

Therefore side of wanted box has a base side, $a = \frac{c}{\sqrt{3}}$ is and height of the box, $h = \frac{c}{2\sqrt{3}}$.

Now, the volume of the box is

$$V = \left(\frac{c}{\sqrt{3}}\right)^2 \times \left(\frac{c}{2\sqrt{3}}\right)$$

$$V = \frac{c^2}{3} \times \left(\frac{c}{2\sqrt{3}}\right) = \frac{c^3}{6\sqrt{3}}$$
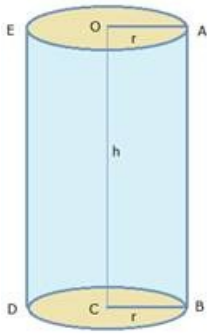
$$\therefore V = \frac{c^3}{6\sqrt{3}}$$

Question: 18

A cylindrical can

Solution:

Given,

• The can is cylindrical with a circular base

• The volume of the cylinder is 1 litre = 1000 cm^2.

• The surface area of the box is minimum as we need to find the minimum dimensions.

Let us consider,

• The radius base and top of the cylinder be 'r' units. (skin coloured in the figure)

• The height of the cylinder be 'h'units.

• As the Volume of cylinder is given, V = 1000cm^3

The Volume of the cylinder= $\pi r^2 h$

$1000 = \pi r^2 h$

$h = \frac{1000}{\pi r^2}$ ---- (1)

The Surface area cylinder is = area of the circular base + area of the circular top + area of the cylinder

$S = \pi r^2 + \pi r^2 + 2\pi rh$

$S = 2\pi r^2 + 2\pi rh$

[substituting (1) in the volume formula]

$S = 2\pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$

$S = 2\left[\pi r^2 + \left(\frac{1000}{r}\right)\right]$ ------ (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with r and then equating it to zero. This is because if the function f(r) has a maximum/minimum at a point c then f'(c) = 0.

Differentiating the equation (2) with respect to r:

$\frac{dS}{dr} = \frac{d}{dr}\left[2\left[\pi r^2 + \left(\frac{1000}{r}\right)\right]\right]$

$\frac{dS}{dr} = 2(2\pi r) + \left(\frac{1000}{r^2}\right)(-1)$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

$\frac{dS}{dr} = 2(2\pi r) - 2\left(\frac{1000}{r^2}\right)$ ------- (3)

To find the critical point, we need to equate equation (3) to zero.

$\frac{dS}{dr} = 2(2\pi r) - 2\left(\frac{1000}{r^2}\right) = 0$

$2(2\pi r) - 2\left(\frac{1000}{r^2}\right) = 0$

$2\pi r = \frac{1000}{r^2}$

$r^3 = \frac{500}{\pi}$

$$r = \sqrt[3]{\frac{500}{\pi}}$$

<u>Now to check if this critical point will determine the minimum surface area of the box, we need to check with second differential which needs to be positive.</u>

Consider differentiating the equation (3) with r:

$$\frac{d^2S}{dr^2} = \frac{d}{dr}\left[2(2\pi r) - 2\left(\frac{1000}{r^2}\right)\right]$$

$$\frac{d^2S}{dr^2} = 4\pi - \frac{2 \times 1000 \times (-2)}{r^3} = 4\pi + \frac{4000}{r^3} \quad \text{----- (4)}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

Now let us find the value of

$$\left(\frac{d^2S}{dr^2}\right)_{r=\sqrt[3]{\frac{500}{\pi}}} = 4\pi + \frac{4000}{\left(\sqrt[3]{\frac{500}{\pi}}\right)^3} = 4\pi + \frac{4000 \times \pi}{500} = 4\pi + 8\pi = 12\pi$$

As $\left(\frac{d^2S}{dr^2}\right)_{r=\sqrt[3]{\frac{500}{\pi}}} = 12\pi > 0$, so the function S is minimum at $r = \sqrt[3]{\frac{500}{\pi}}$

Now substituting r in equation (1)

$$h = \frac{1000}{\pi r^2} = \frac{1000}{\pi\left(\sqrt[3]{\frac{500}{\pi}}\right)^2} = \frac{1000}{\pi^{\frac{1}{3}}(500)^{\frac{2}{3}}}$$

$$\therefore h = \frac{1000}{\pi^{\frac{1}{3}}(500)^{\frac{2}{3}}}$$

Therefore the radius of base of the cylinder, $r = \sqrt[3]{\frac{500}{\pi}}$ and height of the cylinder, $h = \frac{1000}{\pi^{\frac{1}{3}}(500)^{\frac{2}{3}}}$ where the surface area of the cylinder is minimum.
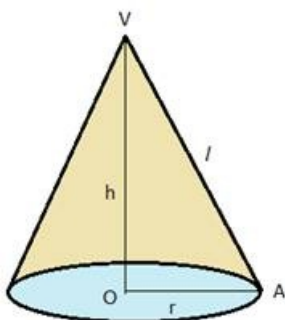
Question: 19

Show that the rig

Solution:

Given,

• The volume of the cone.

• The cone is right circular cone.

• The cone has least curved surface.

Let us consider,

• The radius of the circular base be 'r' cms.

• The height of the cone be 'h' cms.

- The slope of the cone be 'l' cms.

Given the Volume of the cone = $\pi r^2 l$

$$V = \frac{\pi r^2 h}{3}$$

$$h = \frac{3v}{\pi r^2} \quad \text{----- (1)}$$

The Surface area cylinder is = $\pi r l$

$$S = \pi r l$$

$$S = \pi r (\sqrt{h^2 + r^2})$$

[substituting (1) in the Surface area formula]

$$S = \pi r \left[\sqrt{\left(\frac{3V}{\pi r^2}\right)^2 + r^2} \right]$$

[squaring on both sides]

$$Z = S^2 = \pi^2 r^2 \left(\frac{9V^2}{\pi^2 r^4} + r^2\right)$$

$$Z = \pi^2 \left(\frac{9V^2}{\pi^2 r^2} + r^4\right) \quad \text{----- (2)}$$

For finding the maximum/ minimum of given function, we can find it by differentiating it with r and then equating it to zero. This is because if the function Z has a maximum/minimum at a point c then Z'(c) = 0.

Differentiating the equation (2) with respect to r:

$$\frac{dZ}{dr} = \frac{d}{dr}\left[\pi^2\left(\frac{9V^2}{\pi^2 r^2} + r^4\right)\right]$$

$$\frac{dZ}{dr} = \pi^2\left(\frac{9V^2}{\pi^2}\right)\frac{d}{dr}\left(\frac{1}{r^2}\right) + \pi^2 \frac{d}{dr}(r^4)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

$$\frac{dZ}{dr} = \left(\frac{-18V^2}{r^3}\right) + \pi^2(4r^3) \quad \text{------- (3)}$$

To find the critical point, we need to equate equation (3) to zero.

$$\frac{dZ}{dr} = \left(\frac{-18V^2}{r^3}\right) + \pi^2(4r^3) = 0$$

$$\pi^2(4r^3) = \frac{18V^2}{r^3}$$

$$2\pi^2 r^6 = 9V^2 \quad \text{---- (4)}$$

Now to check if this critical point will determine the minimum surface area of the cone, we need to check with second differential which needs to be positive.

Consider differentiating the equation (3) with r:

$$\frac{d^2Z}{dr^2} = \frac{d}{dr}\left[\left(\frac{-18V^2}{r^3}\right) + \pi^2(4r^3)\right]$$

$$\frac{d^2Z}{dr^2} = \frac{-18V^2(-3)}{r^4} + \pi^2(4 \times 3 r^2)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

$$\frac{d^2Z}{dr^2} = \frac{54V^2}{r^4} + \pi^2 (12\,r^2)$$

Now let us find the value of

$$\left(\frac{d^2Z}{dr^2}\right) = \frac{54V^2}{r^4} + \pi^2 (12\,r^2) > 0$$

As $\left(\frac{d^2Z}{dr^2}\right) > 0$, so the function $Z = S^2$ is minimum

Now consider, the equation (4),

$$9V^2 = 2\pi^2 r^6$$

Now substitute the volume of the cone formula in the above equation.

$$9\left(\frac{\pi r^2 h}{3}\right)^2 = 2\pi^2 r^6$$

$$\pi^2 r^4 h^2 = 2\,\pi^2 r^6$$

$$2r^2 = h^2$$

$$h = r\sqrt{2}$$

Hence, the relation between h and r of the cone is proved when S is the minimum.

Question: 20

Find the radius o

Solution:

Given,

• The closed is cylindrical can with a circular base and top.

• The volume of the cylinder is 1 litre = 100 cm^3.

• The surface area of the box is minimum.
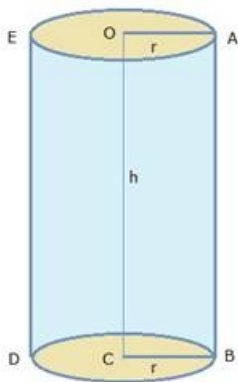
Let us consider,

• The radius base and top of the cylinder be 'r' units. (skin coloured in the figure)

• The height of the cylinder be 'h'units.

• As the Volume of cylinder is given, V = 100cm^3

The Volume of the cylinder= $\pi r^2 h$

$$100 = \pi r^2 h$$

$$h = \frac{100}{\pi r^2} \text{ ---- (1)}$$

The Surface area cylinder is = area of the circular base + area of the circular top + area of the cylinder

$$S = \pi r^2 + \pi r^2 + 2\pi r h$$

$$S = 2\,\pi r^2 + 2\pi r h$$

[substituting (1) in the volume formula]

$$S = 2\pi r^2 + 2\pi r\left(\frac{100}{\pi r^2}\right)$$

$$S = 2\left[\pi r^2 + \left(\frac{100}{r}\right)\right] \text{------ (2)}$$

For finding the maximum/ minimum of given function, we can find it by differentiating it with r and then equating it to zero. <u>This is because if the function f(r) has a maximum/minimum at a point c then f'(c) = 0.</u>

Differentiating the equation (2) with respect to r:

$$\frac{dS}{dr} = \frac{d}{dr}\left[2\left[\pi r^2 + \left(\frac{100}{r}\right)\right]\right]$$

$$\frac{dS}{dr} = 2\,(2\pi r) + \left(\frac{100}{r^2}\right)(-1)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

$$\frac{dS}{dr} = 2\,(2\pi r) - 2\left(\frac{100}{r^2}\right) \text{------- (3)}$$

To find the critical point, we need to equate equation (3) to zero.

$$\frac{dS}{dr} = 2\,(2\pi r) - 2\left(\frac{100}{r^2}\right) = 0$$

$$2\,(2\pi r) - 2\left(\frac{100}{r^2}\right) = 0$$

$$2\pi r = \frac{100}{r^2} \text{---- (4)}$$

<u>Now to check if this critical point will determine the minimum surface area of the box, we need to check with second differential which needs to be positive.</u>

Consider differentiating the equation (3) with r:

$$\frac{d^2S}{dr^2} = \frac{d}{dr}\left[2\,(2\pi r) - 2\left(\frac{100}{r^2}\right)\right]$$

$$\frac{d^2S}{dr^2} = 4\pi - \frac{2 \times 100 \times (-2)}{r^3} = 4\pi + \frac{400}{r^3} \text{----- (5)}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

Now let us find the value of

$$\left(\frac{d^2S}{dr^2}\right)_{r=\sqrt[3]{\frac{50}{\pi}}} = 4\pi + \frac{400}{\left(\sqrt[3]{\frac{50}{\pi}}\right)^3} = 4\pi + \frac{400 \times \pi}{50} = 4\pi + 8\pi = 12\pi$$

As $\left(\frac{d^2S}{dr^2}\right)_{r=\sqrt[3]{\frac{50}{\pi}}} = 12\pi > 0$, so the function S is minimum at $r = \sqrt[3]{\frac{50}{\pi}}$

As S is minimum from equation (4)

$$2\pi r = \frac{100}{r^2}$$

$$2\pi r = \frac{V}{r^2}$$

$$V = 2\pi r^3$$

Now in equation (1) we have,

$$h = \frac{V}{\pi r^2} = \frac{2\pi r^3}{\pi r^2}$$

h = 2r = diameter

Therefore when the total surface area of a cone is minimum, then height of the cone is equal to twice the radius or equal to its diameter.

Question: 21

Show that the hei

Solution:

Let r be the radius of the base and h the height of a cylinder.

The surface area is given by,

S = 2 π r² + 2 π rh

$$h = \frac{S - 2\pi r^2}{2\pi r} \ldots\ldots\ldots(1)$$

Let V be the volume of the cylinder.

Therefore, V = πr²h

$$V = \pi r^2 \left(\frac{S - 2\pi r^3}{2\pi r} \right) \ldots\ldots\text{Using equation 1}$$

$$V = \frac{Sr - 2\pi r^3}{2}$$

Differentiating both sides w.r.t r, we get,

$$\frac{dV}{dr} = \frac{S}{2} - 3\pi r^2 \ldots\ldots\ldots(2)$$

For maximum or minimum, we have,

$$\frac{dV}{dr} = 0$$

$$\Rightarrow \frac{S}{2} - 3\pi r^2 = 0$$

$$\Rightarrow S = 6\pi r^2$$

2πr² + 2πrh = 6πr²

h = 2r

Differentiating equation 2, with respect to r to check for maxima and minima, we get,

$$\frac{d^2V}{dr^2} = -6\pi r < 0$$

Hence, V is maximum when h = 2r or h = diameter

Question: 22

Prove that the vo

Solution:

Given,

• Volume of the sphere.

• Volume of the cone.

• Cone is inscribed in the sphere.

• Volume of cone is maximum.
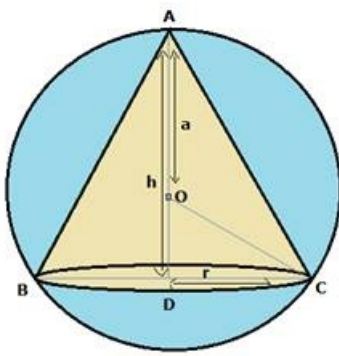
Let us consider,

• The radius of the sphere be 'a' units.

• Volume of the inscribed cone be 'V'.

• Height of the inscribed cone be 'h'.

• Radius of the base of the cone is 'r'.

Given volume of the inscribed cone is,

$$V = \frac{\pi r^2 h}{3}$$

Consider OD = (AD-OA) =(h-a)

Now let $OC^2 = OD^2 + DC^2$, here OC = a, OD = (h-a), DC = r,

So $a^2 = (h-a)^2 + r^2$

$r^2 = a^2 - (h^2 + a^2 - 2ah)$

$r^2 = h\,(2a - h)$ ----- (1)

Let us consider the volume of the cone:

$$V = \frac{1}{3}\,(\pi r^2 h)$$

Now substituting (1) in the volume formula,

$$V = \frac{1}{3}\,(\pi h(2a - h)h)$$

$$V = \frac{1}{3}\,(2\pi h^2 a - \pi h^3)\ ---- (2)$$

For finding the maximum/ minimum of given function, we can find it by differentiating it with h and then equating it to zero. <u>This is because if the function V(r) has a maximum/minimum at a point c then V'(c) = 0.</u>

Differentiating the equation (2) with respect to h:

$$\frac{dV}{dh} = \frac{d}{dh}\left[\frac{1}{3}\,(2\pi h^2 a - \pi h^3)\right]$$

$$\frac{dV}{dh} = \frac{1}{3}\,(2\pi a)(2h) - \frac{1}{3}\,(\pi)(3h^2)$$

[Since $\frac{d}{dx}\,(x^n) = nx^{n-1}$]

$$\frac{dV}{dh} = \frac{1}{3}[\,4\pi ah - 3\pi h^2] \ ------- (3)$$

To find the critical point, we need to equate equation (3) to zero.

$$\frac{dV}{dh} = \frac{1}{3}[\,4\pi ah - 3\pi h^2] \ = 0$$

$4\pi ah - 3\pi h^2 = 0$

h(4πa-3πh)= 0

$h = 0$ (or) $h = \dfrac{4\pi a}{3\pi} = \dfrac{4a}{3}$

$h = \dfrac{4a}{3}$

[as h cannot be zero]

<u>Now to check if this critical point will determine the maximum volume of the inscribed cone, we need to check with second differential which needs to be negative.</u>

Consider differentiating the equation (3) with h:

$$\dfrac{d^2V}{dh^2} = \dfrac{d}{dh}\left[\dfrac{1}{3}[4\pi a h - 3\pi h^2]\right]$$

$$\dfrac{d^2V}{dh^2} = \dfrac{1}{3}[4\pi a - (3\pi)(2h)] = \dfrac{\pi}{3}[4a - 6h] \text{ ----- (4)}$$

[Since $\dfrac{d}{dx}(x^n) = nx^{n-1}$]

Now let us find the value of

$$\left(\dfrac{d^2V}{dh^2}\right)_{h=\frac{4a}{3}} = \dfrac{\pi}{3}\left[4a - 6\left(\dfrac{4a}{3}\right)\right] = \dfrac{4a\pi}{3}[1-2] = -\dfrac{4a\pi}{3}$$

As $\left(\dfrac{d^2V}{dh^2}\right)_{h=\frac{4a}{3}} = -\dfrac{4a\pi}{3} < 0$, so the function V is maximum at $h = \dfrac{4a}{3}$

Substituting h in equation (1)

$$r^2 = \left(\dfrac{4a}{3}\right)\left(2a - \dfrac{4a}{3}\right)$$

$$r^2 = \left(\dfrac{4a}{3}\right)\left(2a - \dfrac{4a}{3}\right)$$

$$r^2 = \dfrac{8a^2}{9}$$

As V is maximum, substituting h and r in the volume formula:

$$V = \dfrac{1}{3}\pi\left(\dfrac{8a^2}{9}\right)\left(\dfrac{4a}{3}\right)$$

$$V = \dfrac{8}{27}\left(\dfrac{4}{3}\pi a^3\right)$$

$$V = \dfrac{8}{27} \text{ (volume of the sphere)}$$

Therefore when the volume of a inscribed cone is maximum, then it is equal to $\dfrac{8}{27}$ times of the volume of the sphere in which it is inscribed.

Question: 23

Which fraction ex

Solution:

Given,

The pth power of a number exceeds by a fraction to be the greatest.

Let us consider,

• 'x' be the required fraction.

• The greatest number will be $y = x - x^p$ ------ (1)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function y(x)has a maximum/minimum at a point c then y'(c) = 0.

Differentiating the equation (1) with respect to x:

$$\frac{dy}{dx} = \frac{d}{dx}(x - x^p)$$

$$\frac{dy}{dx} = 1 - px^{p-1} \text{ ---- (2)}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

To find the critical point, we need to equate equation (2) to zero.

$$\frac{dy}{dx} = 1 - px^{p-1} = 0$$

$$1 = px^{p-1}$$

$$x = \left(\frac{1}{p}\right)^{\frac{1}{p-1}}$$

Now to check if this critical point will determine the if the number is the greatest, we need to check with second differential which needs to be negative.

Consider differentiating the equation (2) with x:

$$\frac{d^2y}{dx^2} = \frac{d}{dx}[1 - px^{p-1}]$$

$$\frac{d^2y}{dx^2} = -p(p-1)x^{p-2} \text{ ----- (3)}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now let us find the value of

$$\left(\frac{d^2y}{dx^2}\right)_{x=\left(\frac{1}{p}\right)^{\frac{1}{p-1}}} = -p(p-1)\left(\left(\frac{1}{p}\right)^{\frac{1}{p-1}}\right)^{p-2}$$

As $\left(\frac{d^2y}{dx^2}\right)_{x=\left(\frac{1}{p}\right)^{\frac{1}{p-1}}} = -p(p-1)\left(\left(\frac{1}{p}\right)^{\frac{1}{p-1}}\right)^{p-2} < 0$, so the number y is greatest at $x = \left(\frac{1}{p}\right)^{\frac{1}{p-1}}$

Hence, the y is the greatest number and exceeds by a fraction $x = \left(\frac{1}{p}\right)^{\frac{1}{p-1}}$

Question: 24

Find the point on

Solution:

Given,

- A point is present on a curve $y^2 = 4x$

- The point is near to the point (2,-8)

Let us consider,

- The co-ordinates of the point be P(x,y)

- As the point P is on the curve, then $y^2 = 4x$

$$x = \frac{y^2}{4}$$

- The distance between the points is given by,

$D^2 = (x-2)^2 + (y+8)^2$

$D^2 = x^2 - 4x + 4 + y^2 + 64 + 16y$

Substituting x in the distance equation

$$D^2 = \left(\frac{y^2}{4}\right)^2 - 4\left(\frac{y^2}{4}\right) + y^2 + 16y + 68$$

$Z = D^2 = \frac{y^4}{16} + 16y + 68$ ---- (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with y and then equating it to zero. <u>This is because if the function Z(x) has a maximum/minimum at a point c then Z'(c) = 0.</u>

Differentiating the equation (2) with respect to y:

$$\frac{dZ}{dy} = \frac{d}{dy}\left(\frac{y^4}{16} + 16y + 68\right)$$

$\frac{dZ}{dy} = \frac{4y^3}{16} + 16 = \frac{y^3}{4} + 16$ ---- (2)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

To find the critical point, we need to equate equation (2) to zero.

$\frac{dZ}{dy} = \frac{y^3}{4} + 16 = 0$

$y^3 + 64 = 0$

$(y + 4)(y^2 - 4y + 16) = 0$

$(y+4) = 0$ (or) $y^2 - 4y + 16 = 0$

$y = -4$

(as the roots of the $y^2 - 4y + 16$ are imaginary)

<u>Now to check if this critical point will determine the distance is mimimum, we need to check with second differential which needs to be positive.</u>

Consider differentiating the equation (2) with y:

$$\frac{d^2Z}{dy^2} = \frac{d}{dy}\left[\frac{y^3}{4} + 16\right]$$

$\frac{d^2Z}{dy^2} = \frac{3y^2}{4}$ ----- (3)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now let us find the value of

$\left(\frac{d^2Z}{dy^2}\right)_{y=-4} = \frac{3}{4}(-4)^2 = 12$

As $\left(\frac{d^2Z}{dy^2}\right)_{y=-4} = 12 > 0$, so the Distance D^2 is minimum at y = -4

Now substituting y in x, we have

$x = \frac{(-4)^2}{4} = 4$

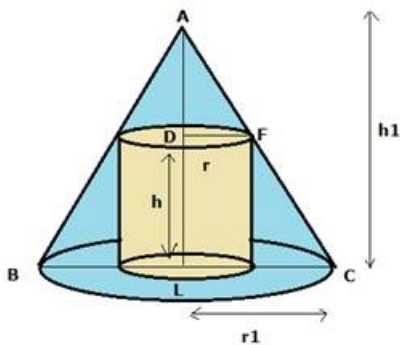So, the point P on the curve $y^2 = 4x$ is (4,-4) which is at nearest from the (2,-8)

Question: 25

A right circular

Solution:

Given,

- A right circular cylinder is inscribed inside a cone.
- The curved surface area is maximum.

Let us consider,

- 'r_1' be the radius of the cone.
- 'h_1' be the height of the cone.
- 'r' be the radius of the inscribed cylinder.
- 'h' be the height of the inscribed cylinder.

DF = r, and AD = AL – DL = h_1 – h

Now, here ΔADF and ΔALC are similar,

Then

$$\frac{AD}{AL} = \frac{DF}{LC} \Rightarrow \frac{h_1 - h}{h_1} = \frac{r}{r_1}$$

$$h_1 - h = \frac{rh_1}{r_1}$$

$$h = h_1 - \frac{rh_1}{r_1} = h_1\left(1 - \frac{r}{r_1}\right)$$

$$h = h_1\left(1 - \frac{r}{r_1}\right) \text{----- (1)}$$

Now let us consider the curved surface area of the cylinder,

S = 2πrh

Substituting h in the formula,

$$S = 2\pi r \left[h_1\left(1 - \frac{r}{r_1}\right)\right]$$

$$S = 2\pi rh_1 - \frac{2\pi h_1 r^2}{r_1} \text{---- (2)}$$

For finding the maximum/ minimum of given function, we can find it by differentiating it with r and then equating it to zero. This is because if the function S(r) has a maximum/minimum at a point c then S'(c) = 0.

Differentiating the equation (2) with respect to r:

$$\frac{dS}{dr} = \frac{d}{dr}\left[2\pi rh_1 - \frac{2\pi h_1 r^2}{r_1}\right]$$

$$\frac{dS}{dr} = 2\pi h_1 - \frac{2\pi h_1(2r)}{r_1}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$$\frac{dS}{dr} = 2\pi h_1 - \frac{4\pi h_1 r}{r_1} \text{ ------- (3)}$$

To find the critical point, we need to equate equation (3) to zero.

$$\frac{dS}{dr} = 2\pi h_1 - \frac{4\pi h_1 r}{r_1} = 0$$

$$\frac{4\pi h_1 r}{r_1} = 2\pi h_1$$

$$r = \frac{2\pi h_1 r_1}{4\pi h_1}$$

$$r = \frac{r_1}{2}$$

<u>Now to check if this critical point will determine the maximum volume of the inscribed cylinder, we need to check with second differential which needs to be negative.</u>

Consider differentiating the equation (3) with r:

$$\frac{d^2 S}{dr^2} = \frac{d}{dr}\left[2\pi h_1 - \frac{4\pi h_1 r}{r_1}\right]$$

$$\frac{d^2 S}{dr^2} = 0 - \frac{4\pi h_1}{r_1} = -\frac{4\pi h_1}{r_1} \text{ ----- (4)}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now let us find the value of

$$\frac{d^2 S}{dr^2}_{r=\frac{r_1}{2}} = -\frac{4\pi h_1}{r_1}$$

As $\frac{d^2 S}{dr^2}_{r=\frac{r_1}{2}} = -\frac{4\pi h_1}{r_1} < 0$, so the function S is maximum at $r = \frac{r_1}{2}$

Substituting r in equation (1)

$$h = h_1\left(1 - \frac{\frac{r_1}{2}}{r_1}\right)$$

$$h = h_1\left(1 - \frac{1}{2}\right) = \frac{h_1}{2} \text{ --- (5)}$$

As S is maximum, from (5) we can clearly say that $h_1 = 2h$ and

$$r_1 = 2r$$

this means the radius of the cone is twice the radius of the cylinder or equal to diameter of the cylinder.

Question: 26

Show that the sur

Solution:

Given,

• Closed cuboid has square base.

• The volume of the cuboid is given.

- Surface area is minimum.

Let us consider,

- The side of the square base be 'x'.

- The height of the cuboid be 'h'.

- The given volume, $V = x^2h$

$h = \frac{V}{x^2}$ ----- (1)

Consider the surface area of the cuboid,

Surface Area =

2(Area of the square base) + 4(areas of the rectangular sides)

$S = 2x^2 + 4xh$

Now substitute (1) in the Surface Area formula

$S = 2x^2 + 4x\left(\frac{V}{x^2}\right)$

$S = 2x^2 + \left(\frac{4V}{x}\right)$ ----- (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function S(x) has a maximum/minimum at a point c then S'(c) = 0.

Differentiating the equation (2) with respect to x:

$\frac{dS}{dx} = \frac{d}{dx}\left[2x^2 + \left(\frac{4V}{x}\right)\right]$

$\frac{dS}{dx} = 2(2x) + 4V\left(\frac{-1}{x^2}\right)$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

$\frac{dS}{dx} = 4x - \frac{4V}{x^2}$ ------- (3)

To find the critical point, we need to equate equation (3) to zero.

$\frac{dS}{dx} = 4x - \frac{4V}{x^2} = 0$

$4x = \frac{4V}{x^2}$

$x^3 = V$

Now to check if this critical point will determine the minimum surface area, we need to check with second differential which needs to be positive.

Consider differentiating the equation (3) with x:

$\frac{d^2S}{dx^2} = \frac{d}{dx}\left[4x - \frac{4V}{x^2}\right]$

$\frac{d^2S}{dx^2} = 4 + \frac{8V}{x^3}$ ----- (4)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

Now let us find the value of

$\frac{d^2S}{dx^2}_{x=V^{\frac{1}{3}}} = 4 + \frac{8V}{V} = 12$

As $\frac{d^2S}{dx^2}_{x=V^{\frac{1}{3}}} = 12 > 0$, so the function S is minimum at $x = \sqrt[3]{V}$

Substituting x in equation (1)

$$h = \frac{V}{x^2} = \frac{x^3}{x^2} = x$$

h = x

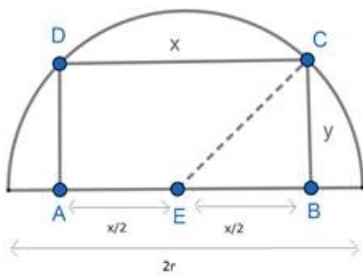As S is minimum and h = x, this means that the cuboid is a cube.

Question: 27

A rectangle is in

Solution:

Given,

• Radius of the semicircle is 'r'.

• Area of the rectangle is maximum.

Let us consider,

• The base of the rectangle be 'x' and the height be 'y'.

Consider the ΔCEB,

$$CE^2 = EB^2 + BC^2$$

As CE = r, $EB = \frac{x}{2}$ and CB = y

$$r^2 = \left(\frac{x}{2}\right)^2 + y^2$$

$$y^2 = r^2 - \left(\frac{x}{2}\right)^2 \quad \text{---- (1)}$$

Now the area of the rectangle is

A = x × y

Squaring on both sides

$$A^2 = x^2 y^2$$

Substituting (1) in the above Area equation

$$A^2 = x^2 \left[r^2 - \left(\frac{x}{2}\right)^2\right]$$

$$Z = A^2 = x^2 r^2 - x^2 \frac{x^2}{4} = x^2 r^2 - \frac{x^4}{4} \quad \text{----- (2)}$$

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function Z(x) has a maximum/minimum at a point c then Z'(c) = 0.

Differentiating the equation (2) with respect to x:

$$\frac{dZ}{dx} = \frac{d}{dx}\left[x^2 r^2 - \frac{x^4}{4}\right]$$

$$\frac{dZ}{dx} = r^2 (2x) - \frac{4x^3}{4}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$$\frac{dZ}{dx} = 2xr^2 - x^3 \text{ ------- (3)}$$

To find the critical point, we need to equate equation (3) to zero.

$$\frac{dZ}{dx} = 2xr^2 - x^3 = 0$$

$$x(2r^2 - x^2) = 0$$

x = 0 (or) $x^2 = 2r^2$

x = 0 (or) $x = r\sqrt{2}$

$x = r\sqrt{2}$

[as x cannot be zero]

<u>Now to check if this critical point will determine the maximum area, we need to check with second differential which needs to be negative.</u>

Consider differentiating the equation (3) with x:

$$\frac{d^2Z}{dx^2} = \frac{d}{dx}[2xr^2 - x^3]$$

$$\frac{d^2Z}{dx^2} = 2r^2 - 3x^2 \text{ ----- (4)}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now let us find the value of

$$\frac{d^2Z}{dx^2}_{x=r\sqrt{2}} = 2r^2 - 3(r\sqrt{2})^2 = 2r^2 - 6r^2 = -4r^2$$

As $\frac{d^2Z}{dx^2}_{x=r\sqrt{2}} = -4r^2 < 0$, so the function Z is maximum at $x = r\sqrt{2}$

Substituting x in equation (1)

$$y^2 = r^2 - \left(\frac{r\sqrt{2}}{2}\right)^2 = r^2 - \frac{r^2}{2} = \frac{r^2}{2}$$

$$y = \sqrt{\frac{r^2}{2}} = \frac{r}{\sqrt{2}} = \frac{r\sqrt{2}}{2}$$

As the area of the rectangle is maximum, and $x = r\sqrt{2}$ and $y = \frac{r\sqrt{2}}{2}$

So area of the rectangle is

$$A = r\sqrt{2} \times \frac{r\sqrt{2}}{2}$$

$$A = r^2$$

Hence the maximum area of the rectangle inscribed inside a semicircle is r^2 square units.

Question: 28

Two sides of a tr

Solution:

Given,

- The length two sides of a triangle are 'a' and 'b'
- Angle between the sides 'a' and 'b' is θ.
- The area of the triangle is maximum.
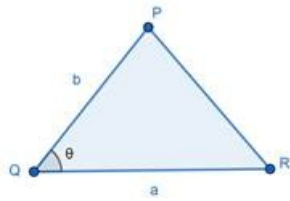
Let us consider,

The area of the ΔPQR is given be

$A = \frac{1}{2} ab \sin\theta$ ---- (1)

For finding the maximum/ minimum of given function, we can find it by differentiating it with θ and then equating it to zero. This is because if the function A (θ) has a maximum/minimum at a point c then A'(c) = 0.

Differentiating the equation (1) with respect to θ:

$\frac{dA}{d\theta} = \frac{d}{d\theta} \left[\frac{1}{2} ab \sin\theta \right]$

$\frac{dA}{d\theta} = \frac{1}{2} ab \cos\theta$ ---- (2)

[Since $\frac{d}{dx} (\sin\theta) = \cos\theta$]

To find the critical point, we need to equate equation (2) to zero.

$\frac{dA}{d\theta} = \frac{1}{2} ab \cos\theta = 0$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$

Now to check if this critical point will determine the maximum area, we need to check with second differential which needs to be negative.

Consider differentiating the equation (2) with θ :

$\frac{d^2A}{d\theta^2} = \frac{d}{d\theta} \left[\frac{1}{2} ab \cos\theta \right]$

$\frac{d^2A}{d\theta^2} = -\frac{1}{2} ab \sin\theta$ ----- (2)

[Since $\frac{d}{dx} (\cos\theta) = -\sin\theta$]

Now let us find the value of

$\frac{d^2A}{d\theta^2}\Big|_{\theta=\frac{\pi}{2}} = -\frac{1}{2} ab \sin\left(\frac{\pi}{2}\right) = -\frac{1}{2} ab$

As $\frac{d^2A}{d\theta^2}\Big|_{\theta=\frac{\pi}{2}} = -\frac{1}{2} ab < 0$, so the function A is maximum at $\theta = \frac{\pi}{2}$

As the area of the triangle is maximum when $\theta = \frac{\pi}{2}$
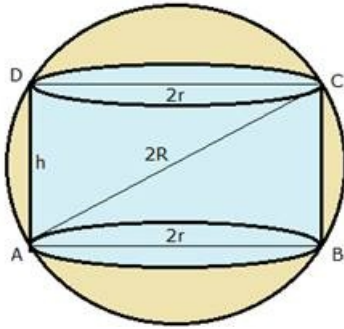
Question: 29

Show that the max

Solution:

Given,

- Radius of the sphere is $5\sqrt{3}$.

- Volume of cylinder is maximum.

Let us consider,

- The radius of the sphere be 'R' units.

- Volume of the inscribed cylinder be 'V'.

- Height of the inscribed cylinder be 'h'.

- Radius of the cylinder is 'r'.

Now let $AC^2 = AB^2 + BC^2$, here AC = 2R, AB = 2r, BC = h,

So $4R^2 = 4r^2 + h^2$

$r^2 = \frac{1}{4}[4R^2 - h^2]$ ----- (1)

Let us consider, the volume of the cylinder:

$V = \pi r^2 h$

Now substituting (1) in the volume formula,

$V = \pi h \left(\frac{1}{4}[4R^2 - h^2]\right)$

$V = \frac{\pi}{4}(4R^2 h - h^3)$ ---- (2)

For finding the maximum/ minimum of given function, we can find it by differentiating it with h and then equating it to zero. This is because if the function V(h) has a maximum/minimum at a point c then V'(c) = 0.

Differentiating the equation (2) with respect to h:

$\frac{dV}{dh} = \frac{d}{dh}\left[\frac{\pi}{4}(4R^2 h - h^3)\right]$

$\frac{dV}{dh} = \frac{4R^2\pi}{4} - \frac{\pi}{4}(3h^2)$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$\frac{dV}{dh} = R^2\pi - \frac{3h^2\pi}{4}$ ------- (3)

To find the critical point, we need to equate equation (3) to zero.

$\frac{dV}{dh} = R^2\pi - \frac{3h^2\pi}{4} = 0$

$3h^2\pi = 4R^2\pi$

$h^2 = \frac{4}{3}R^2 = \frac{4}{3}(5\sqrt{3})^2 = \frac{4}{3}(25 \times 3) = 100$

h = 10

[as h cannot be negative]

Now to check if this critical point will determine the maximum volume of the inscribed cone, we need to check with second differential which needs to be negative.

Consider differentiating the equation (3) with h:

$$\frac{d^2V}{dh^2} = \frac{d}{dh}\left[R^2\pi - \frac{3h^2\pi}{4} \right]$$

$$\frac{d^2V}{dh^2} = 0 - \frac{3\,(2h)\pi}{3} = -2h\pi \text{ ----- (4)}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now let us find the value of

$$\left(\frac{d^2V}{dh^2}\right)_{h=10} = -2h\pi = -2(10)\pi = -20\pi$$

As $\left(\frac{d^2V}{dh^2}\right)_{h=10} = -20\pi < 0$, so the function V is maximum at h=10

Substituting h in equation (1)

$$r^2 = \frac{1}{4}[4(5\sqrt{3})^2 - (10)^2]$$

$$r^2 = \frac{1}{4}[4\,(25 \times 3) - 100]$$

$$r^2 = \frac{300 - 100}{4} = \frac{200}{4} = 50$$

As V is maximum, substituting h and r in the volume formula:

$$V = \pi\,(50)\,(10)$$

$$V = 500\pi \text{ cm}^3$$

Therefore when the volume of a inscribed cylinder is maximum and is equal 500π cm³

Question: 30

A square tank of

Solution:

Given,

• Capacity of the square tank is 250 cubic metres.

• Cost of the land per square meter Rs.50.

• Cost of digging the whole tank is Rs. $(400 \times h^2)$.

• Where h is the depth of the tank.

Let us consider,

• Side of the tank is x metres.

• Cost of the digging is; $C = 50x^2 + 400h^2$ ---- (1)

• Volume of the tank is; $V = x^2h$; $250 = x^2h$

$$h = \frac{250}{x^2} \text{ ----- (2)}$$

Substituting (2) in (1),

$$C = 50x^2 + 400\left(\frac{250}{x^2}\right)^2$$

$$C = 50x^2 + \frac{400 \times 62500}{x^4} \quad \text{----- (3)}$$

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. <u>This is because if the function C(x) has a maximum/minimum at a point c then C'(c) = 0.</u>

Differentiating the equation (3) with respect to x:

$$\frac{dC}{dx} = \frac{d}{dx}\left[50x^2 + \frac{400 \times 62500}{x^4}\right]$$

$$\frac{dC}{dx} = 50\,(2x) + \frac{25000000\,(-4)}{x^5}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$$\frac{dC}{dx} = 100x - \frac{10^8}{x^5} \quad \text{------- (4)}$$

To find the critical point, we need to equate equation (4) to zero.

$$\frac{dC}{dx} = 100x - \frac{10^8}{x^5} = 0$$

$$x^6 = 10^6$$

$$x = 10$$

<u>Now to check if this critical point will determine the minimum volume of the tank, we need to check with second differential which needs to be positive.</u>

Consider differentiating the equation (4) with x:

$$\frac{d^2C}{dx^2} = \frac{d}{dx}\left[100x - \frac{10^8}{x^5}\right]$$

$$\frac{d^2C}{dx^2} = 100 - \frac{10^8\,(-5)}{x^6} = 100 + \frac{10^8\,(5)}{x^6} \quad \text{----- (5)}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

Now let us find the value of

$$\left(\frac{d^2C}{dx^2}\right)_{x=10} = 100 + \frac{10^8\,(5)}{(10)^6} = 100 + 500 = 600$$

As $\left(\frac{d^2C}{dx^2}\right)_{x=10} = 600 > 0$, so the function C is minimum at x=10

Substituting x in equation (2)

$$h = \frac{250}{(10)^2} = \frac{250}{100} = \frac{5}{2}$$

$$h = 2.5 \text{ m}$$

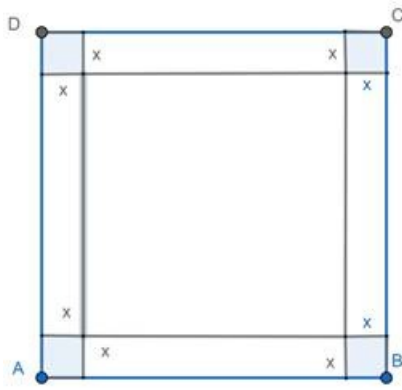Therefore when the cost for the digging is minimum, when x = 10m and h = 2.5m

Question: 31

A square piece of

Solution:

Given,

• Side of the square piece is 18 cms.

• the volume of the formed box is maximum.

Let us consider,

- 'x' be the length and breadth of the piece cut from each vertex of the piece.

- Side of the box now will be (18-2x)

- The height of the new formed box will also be 'x'.

Let the volume of the newly formed box is :

V = (18-2x)^2 × (x)

V = (324+ 4x^2 – 72x) x

V = 4x^3 -72x^2 +324x ------ (1)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function V(x) has a maximum/minimum at a point c then V'(c) = 0.

Differentiating the equation (1) with respect to x:

$\frac{dV}{dx} = \frac{d}{dx}[4x^3 - 72x^2 + 324x]$

$\frac{dV}{dx} = 12x^2 - 144x + 324$ -------- (2)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

To find the critical point, we need to equate equation (2) to zero.

$\frac{dV}{dx} = 12x^2 - 144x + 324 = 0$

x^2 – 12x + 27 = 0

$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(27)}}{2(1)} = \frac{12 \pm \sqrt{144 - 108}}{2} = \frac{12 \pm \sqrt{36}}{2}$

$x == \frac{12 \pm 6}{2}$

x = 9 or x =3

x= 2

[as x = 9 is not a possibility, because 18-2x = 18-18= 0]

Now to check if this critical point will determine the maximum area of the box, we need to check with second differential which needs to be negative.

Consider differentiating the equation (3) with x:

$\frac{d^2V}{dx^2} = \frac{d}{dx}[12x^2 - 144x + 324]$

$\frac{d^2V}{dx^2} = 24x - 144$ ----- (4)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now let us find the value of

$$\left(\frac{d^2V}{dx^2}\right)_{x=3} = 24\,(3) - 144 = 72 - 144 = -72$$

As $\left(\frac{d^2V}{dx^2}\right)_{x=3} = -72 < 0$, so the function V is maximum at x = 3cm

Now substituting x = 3 in 18 – 2x, the side of the considered box:

Side = 18-2x = 18 - 2(3) = 18-6= 12cm

Therefore side of wanted box is 12cms and height of the box is 3cms.

Now, the volume of the box is

V = $(12)^2 \times 3$ = 144 × 3 = 432cm^3

Hence maximum volume of the box formed by cutting the given 18cms sheet is 432cm^3 with 12cms side and 3cms height.

Question: 32

An open tank with

Solution:

Given,

• The tank is square base open tank.

• The cost of the construction to be least.

Let us consider,

• Side of the tank is x metres.

• Height of the tank be 'h' metres.

• Volume of the tank is; V = x^2h

• Surface Area of the tank is S = x^2 +4xh

• Let Rs.P is the price per square.

Volume of the tank,

$h = \frac{V}{x^2}$ ---- (1)

Cost of the construction be:

C = $(x^2$ +4xh)P ---- (2)

Substituting (1) in (2),

$$C = \left[x^2 + 4x\,\frac{V}{x^2}\right]P$$

$C = [x^2 + \frac{4V}{x}]\,P$ ----- (3)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. This is because if the function C(x) has a maximum/minimum at a point c then C'(c) = 0.

Differentiating the equation (3) with respect to x:

$$\frac{dC}{dx} = \frac{d}{dx}\left[x^2 + \frac{4V}{x}\right]P$$

$$\frac{dC}{dx} = \left[(2x) + \frac{4V(-1)}{x^2}\right]P$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

$$\frac{dC}{dx} = \left[2x - \frac{4V}{x^2}\right]P \text{ ------- (4)}$$

To find the critical point, we need to equate equation (4) to zero.

$$\frac{dC}{dx} = \left[2x - \frac{4V}{x^2}\right]P = 0$$

$$x^3 = 2V$$

Now to check if this critical point will determine the minimum volume of the tank, we need to check with second differential which needs to be positive.

Consider differentiating the equation (4) with x:

$$\frac{d^2C}{dx^2} = P\frac{d}{dx}\left[2x - \frac{4V}{x^2}\right]$$

$$\frac{d^2C}{dx^2} = \left[2 - \frac{4V(-2)}{x^3}\right]P = \left[2 + \frac{8V}{x^3}\right]P \text{ ----- (5)}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

Now let us find the value of

$$\left(\frac{d^2C}{dx^2}\right)_{x=(2V)^{\frac{1}{3}}} = \left[2 + \frac{8V}{2V}\right]P = [2 + 4]P = 6P$$

As $\left(\frac{d^2C}{dx^2}\right)_{x=(2V)^{\frac{1}{3}}} = 6P > 0$, so the function C is minimum at $x = \sqrt[3]{2V}$

Substituting x in equation (2)

$$h = \frac{V}{(2V)^{\frac{2}{3}}} = \frac{V\sqrt[3]{(2V)}}{2V} = \frac{1}{2}\sqrt[3]{2V}$$

$$h = \frac{1}{2}\sqrt[3]{2V}$$

Therefore when the cost for the digging is minimum, when $x = \sqrt[3]{2V}$ and $h = \frac{1}{2}\sqrt[3]{2V}$

Question: 33

A wire of length

Solution:

Given,

• Length of the wire is 36 cm.

• The wire is cut into 2 pieces.

• One piece is made to a square.

• Another piece made into a equilateral triangle.

Let us consider,

• The perimeter of the square is x.

• The perimeter of the equilateral triangle is (36-x).

• Side of the square is $\frac{x}{4}$

- Side of the triangle is $\frac{(36-x)}{3}$

Let the Sum of the Area of the square and triangle is

$$A = \left(\frac{x}{4}\right)^2 + \frac{\sqrt{3}}{4}\left(\frac{36-x}{3}\right)^2$$

$$A = \left(\frac{x}{4}\right)^2 + \frac{\sqrt{3}}{4}\left(12 - \frac{x}{3}\right)^2 = \frac{x^2}{16} + \frac{\sqrt{3}}{4}\left(144 + \frac{x^2}{9} - 8x\right)$$

$$A = \frac{x^2}{16} + \frac{\sqrt{3}}{4}\left(144 + \frac{x^2}{9} - 8x\right) \text{ --- (1)}$$

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero. <u>This is because if the function A(x) has a maximum/minimum at a point c then A'(c) = 0.</u>

Differentiating the equation (1) with respect to x:

$$\frac{dA}{dx} = \frac{d}{dx}\left[\frac{x^2}{16} + \frac{\sqrt{3}}{4}\left(144 + \frac{x^2}{9} - 8x\right)\right]$$

$$\frac{dA}{dx} = \frac{2x}{16} + \frac{\sqrt{3}}{4}\left(0 + \frac{2x}{9} - 8\right)$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$$\frac{dA}{dx} = \frac{2x}{16} + \frac{\sqrt{3}}{4}\left(\frac{2x}{9} - 8\right) \text{ ----- (2)}$$

To find the critical point, we need to equate equation (2) to zero.

$$\frac{dA}{dx} = \frac{2x}{16} + \frac{\sqrt{3}}{4}\left(\frac{2x}{9} - 8\right) = 0$$

$$\frac{2x}{16} = \frac{\sqrt{3}}{4}\left(8 - \frac{2x}{9}\right)$$

$$\frac{2x}{16} = 2\sqrt{3} - \frac{\sqrt{3}x}{18}$$

$$\frac{2x}{16} + \frac{\sqrt{3}x}{18} = 2\sqrt{3}$$

$$x\left(\frac{2(9) + \sqrt{3}(8)}{144}\right) = 2\sqrt{3}$$

$$x\left(\frac{18 + 8\sqrt{3}}{144}\right) = 2\sqrt{3}$$

$$x = 2\sqrt{3}\left(\frac{144}{18 + 8\sqrt{3}}\right) = \frac{144\sqrt{3}}{(9 + 4\sqrt{3})}$$

<u>Now to check if this critical point will determine the minimum area, we need to check with second differential which needs to be positive.</u>

Consider differentiating the equation (3) with x:

$$\frac{d^2A}{dx^2} = \frac{d}{dx}\left[\frac{2x}{16} + \frac{\sqrt{3}}{4}\left(\frac{2x}{9} - 8\right)\right]$$

$$\frac{d^2A}{dx^2} = \frac{1}{8} + \frac{\sqrt{3}}{4}\left(\frac{2}{9}\right) = \frac{9 + 4\sqrt{3}}{72} \text{ ----- (4)}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now let us find the value of

$$\left(\frac{d^2A}{dx^2}\right)_{x=\frac{144\sqrt{3}}{(9+4\sqrt{3})}} = \frac{9+4\sqrt{3}}{72}$$

As $\left(\frac{d^2A}{dx^2}\right)_{x=\frac{144\sqrt{3}}{(9+4\sqrt{3})}} = \frac{9+4\sqrt{3}}{72} > 0$, so the function A is minimum at

$$x = \frac{144\sqrt{3}}{(9+4\sqrt{3})}$$

Now, the length of each piece is $x = \frac{144\sqrt{3}}{(9+4\sqrt{3})}$ cm and $36-x = 36 - \frac{144\sqrt{3}}{(9+4\sqrt{3})} = \frac{324}{(9+4\sqrt{3})}$ cm
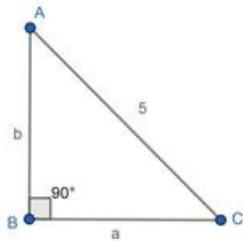
Question: 34

Find the largest

Solution:

Given,

• The triangle is right angled triangle.

• Hypotenuse is 5cm.

Let us consider,

• The base of the triangle is 'a'.

• The adjacent side is 'b'.

Now $AC^2 = AB^2 + BC^2$

As $AC = 5$, $AB = b$ and $BC = a$

$25 = a^2 + b^2$

$b^2 = 25 - a^2$ ---- (1)

Now, the area of the triangle is

$$A = \frac{1}{2}ab$$

Squaring on both sides

$$A^2 = \frac{1}{4}a^2b^2$$

Substituting (1) in the area formula

$$Z = A^2 = \frac{1}{4}a^2(25-a^2) \text{ ----- (2)}$$

For finding the maximum/ minimum of given function, we can find it by differentiating it with a and then equating it to zero. <u>This is because if the function Z (x) has a maximum/minimum at a point c then Z'(c) = 0.</u>

Differentiating the equation (2) with respect to a:

$$\frac{dZ}{da} = \frac{d}{da}\left[\frac{1}{4}a^2(25-a^2)\right]$$

$$\frac{dZ}{da} = \frac{1}{4}\left[25\,(2a) - 4a^3\right]$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

$$\frac{dZ}{da} = \frac{25a}{2} - a^3 \text{ ----- (3)}$$

To find the critical point, we need to equate equation (3) to zero.

$$\frac{dZ}{da} = \frac{25a}{2} - a^3 = 0$$

$$a\left(\frac{25}{2} - a^2\right) = 0$$

$$a = 0 \text{ (or) } a = \frac{5}{\sqrt{2}}$$

$$a = \frac{5}{\sqrt{2}}$$

[as a cannot be zero]

<u>Now to check if this critical point will determine the maximum area, we need to check with second differential which needs to be negative.</u>

Consider differentiating the equation (3) with a:

$$\frac{d^2Z}{da^2} = \frac{d}{da}\left[\frac{25a}{2} - a^3\right]$$

$$\frac{d^2Z}{da^2} = \frac{25}{2} - 3a^2 \text{ ----- (4)}$$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

Now let us find the value of

$$\left(\frac{d^2Z}{da^2}\right)_{a=\frac{5}{\sqrt{2}}} = \frac{25}{2} - 3\left(\frac{5}{\sqrt{2}}\right)^2 = \frac{25}{2} - \frac{(3)25}{2} = -25$$

As $\left(\frac{d^2Z}{da^2}\right)_{a=\frac{5}{\sqrt{2}}} = -25 < 0$, so the function A is maximum at $a = \frac{5}{\sqrt{2}}$

Substituting value of A in (1)

$$b^2 = 25 - \frac{25}{2} = \frac{25}{2}$$

$$b = \frac{5}{\sqrt{2}}$$

Now the maximum area is

$$A = \frac{1}{2}\left(\frac{5}{\sqrt{2}}\right)\left(\frac{5}{\sqrt{2}}\right) = \frac{25}{4}$$

$$\therefore A = \frac{25}{4} \text{ cm}^2$$

Exercise : 11G

Question: 1

Show that t

Solution:

Domain of the function is R

Finding derivative f′(x)=5

Which is greater than 0

Mean strictly increasing in its domain i.e R

Question: 2

Show the fu

Solution:

Domain of the function is R

Finding derivative f′(x)=-2

Which is less than 0

Means strictly decreasing in its domain i.e R

Question: 3

Prove that

Solution:

Domain of the function is R

Finding derivative i.e f′(x)=a

As given in question it is given that a>0

Derivative>0

Means strictly increasing in its domain i.e R

Question: 4

Prove that

Solution:

Domain of the function is R

finding derivative i.e f′(x)=2e^x

As we know e^x is strictly increasing its domain

f′(x)>0

hence f(x) is strictly increasing in its domain

Question: 5

Show that t

Solution:

Domain of function is **R.**

f′(x)=2x

for x>0 f′(x)>0 i.e. increasing

for x<0 f′(x)<0 i.e. decreasing

hence it is neither increasing nor decreasing in R

Question: 6

Show that t

Solution:

For x>0

Modulus will open with + sign

f(x)=+x

$\Rightarrow$ f'(x)=+1 which is <0

for x<0

Modulus will open with -ve sign

f(x)=-x => f'(x)=-1 which is >0

hence f(x) is increasing in x>0 and decreasing in x<0

Question: 7

Prove that

Solution:

f(x)=ln(x)

$$f'(x) = \frac{1}{x}$$

for x<0

f'(x)=-ve →increasing

for x>0

f'(x)=+ve →decreasing

f(x) in increasing when x>0 i.e x∈(0,∞)

Question: 8

Prove that the fu

Solution:

Consider $f(x)=\log_a x$

domain of f(x) is x>0

$$f'(x) = \frac{1}{x}\ln(a)$$

$\Rightarrow$ for a>1, ln(a)>0,

hence f'(x) >0 which means strictly increasing.

$\Rightarrow$ for 0<a<1, ln(a)<0,

hence f'(x)<0 which means strictly decreasing.

Question: 9

Prove that

Solution:

Consider $f(x)=3^x$

The domain of f(x) is R.

f'(x)=3^xln(3)

3^x is always greater than 0 and ln(3) is also + ve.

Overall f'(x) is >0 means strictly increasing in its domain i.e. R.

Question: 10

Show that <

Solution:

Consider $f(x)=x^3-15x^2+75x-50$

Domain of the function is R.

$f'(x)=3x^2-30x+75$

$=3(x^2-10+25)$

$=3(x-5)(x-5)$

$=3(x-5)^2$

$f'(x)=0$ for $x=5$

for $x<5$

$f'(x)>5$

and

for $x>5$

$f'(x)>5$

we can see throughout R the derivative is +ve but at x=5 it is 0 so it is increasing.

Question: 11

Show that <

Solution:

$$f(x) = \left(x - \frac{1}{x}\right)$$

domain of function is R-{0}

$$f'(x) = 1 + \frac{1}{x^2}$$

$f'(x) \forall x \in R$ is greater than 0.

Question: 12

Show that <

Solution:

$$f(x) = \frac{1}{x} + 5$$

domain of function is R-{0}

$$f'(x) = -\frac{1}{x^2}$$

for all x, $f'(x)<0$

Hence function is decreasing.

Question: 13

Show that <

Solution:

Consider $f(x) = \frac{1}{(1+x^2)}$,

$$f'(x) = -\frac{2x}{(1+x^2)^2}$$

for $x \geq 0$,

$f'(x)$ is -ve.

hence function is decreasing for x ≤ 0

Question: 14

Show that <

Solution:

$f(x)=x^3+x^{-3}$

$f'(x)=3x^2-3x^{-4}$

$=3(x^2-1/x^4)$

$= 3(\dfrac{x^3-1}{x^2} \cdot \dfrac{x^3+1}{x^2})$

$= \dfrac{3(x-1)(x^2+x+1)(x+1)(x^2-x+1)}{x^4}$

Root of f'(x)=1and-1

-1   1
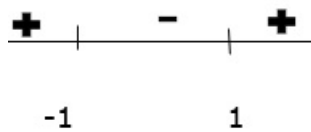
Here we can clearly see that f'(x) is decreasing in [-1,1]

So, f(x) is decreasing in interval [-1,1]

Question: 15

Show that <

Solution:

Consider $f(x) = \dfrac{x}{\sin x}$,

$f'(x) = \dfrac{\sin x - x.\cos x}{\sin^2 x}$

$f'(x) = \dfrac{\cos x(\tan x - x)}{\sin^2 x}$

in $\left]0, \dfrac{\pi}{2}\right[$ cos>0,

tan x-x>0,

$\sin^2 x>0$

hence f'(x)>0,

so, function is increasing in the given interval.

Question: 16

Prove that the fu

Solution:

Consider $f(x) = \log(1+x) - \dfrac{2x}{(x+2)}$,

$f'(x) = \dfrac{1}{1+x} - \dfrac{4}{(x+2)^2}$

$= \dfrac{(x+2)^2-4(x+1)}{(x+1)(x+2)^2}$

$= \dfrac{x^2}{(x+1)(x+2)^2}$
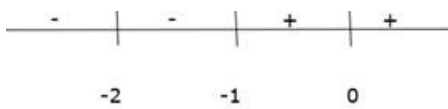
Clearly we can see that f'(x)>o for x>-1.

Hence function is increasing for all x>-1

Question: 17

Let I be an inter

Solution:

Consider $f(x) = \left(x + \dfrac{1}{x}\right)$

$f'(x) = 1 - \dfrac{1}{x^2}$

$f'(x) = \dfrac{x^2 - 1}{x^2}$

$= \dfrac{x - 1 . x + 1}{x^2}$
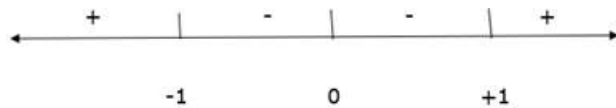
We can see f'(x) <0 in [-1,1]

i.e. f(x) is decreasing in this interval.

We can see f'(x) >0 in (-∞, -1) ∪(1, ∞)

i.e. f(x) is increasing in this interval.

Question: 18

Show that <

Solution:

Consider $f(x) = \dfrac{(x-2)}{(x+1)}$,

$f'(x) = \dfrac{3}{(x + 1)^2}$

f'(x) at x=-1 is not defined

and for all x ∈ R- {-1}

f'(x)>0

hence f(x) is increasing.

Question: 19

Find the in

Solution:

f(x)=(2x²-3x)

f'(x)=4x-3

f'(x)=0 at x=3/4
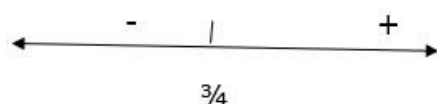
Clearly we can see that function is increasing for x∈[3/4, ∞) and is decreasing for x∈(-∞,3/4)

Question: 20

Find the in

Solution:

f(x)=$2x^3$-$3x^2$-36x+7

f'(x)=$6x^2$-6x-36

f'(x)=6(x^2-x-6)

f'(x)=6(x-3)(x+2)

f'(x) is 0 at x=3 and x=-2
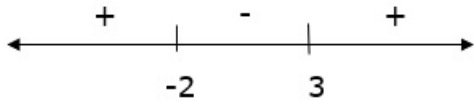
F'(x)>0 for x ∈ (-∞, -2] ∪ [3, ∞)

hence in this interval function is increasing.

F'(x)<0 for x ∈ (-2 ,3)
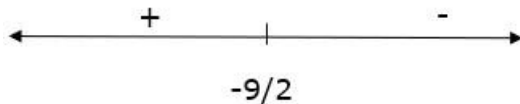
hence in this interval function is decreasing.

Question: 21

Find the in

Solution:

f(x)=6-9x-x^2

f'(x)=-(2x+9)

We can see that f(x) is increasing for $x \in \left(-\infty, -\frac{9}{2}\right]$ and decreasing in $x \in \left(-\frac{9}{2}, \infty\right)$

Question: 22

Find the interval

Solution:

Consider $f(x) = \left(x^4 - \frac{x^3}{3}\right)$

f'(x)=$4x^3$-x^2

=x^2(4x-1)

F'(x)=0 for x=0 and x=1/4
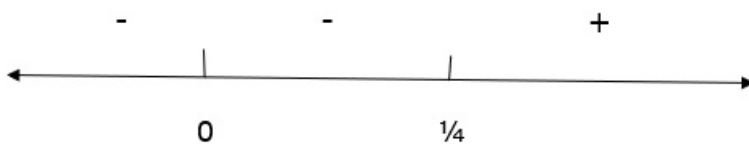
Function f(x) is decreasing for x ∈ (-∞,1/4] and increasing in x ∈ (1/4 , ∞)

Question: 23

Find the interval

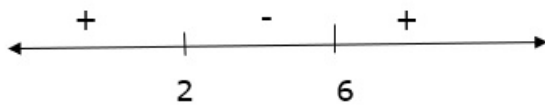**Solution:**

$f(x)=x^3-12x^2+36x+17$

$f'(x)=3x^2-24x+36$

$f'(x)=3(x^2-8x+12)$

$=3(x-6)(x-2)$

Function f(x) is decreasing for x ∈ [2,6] and increasing in x ∈ (-∞,2) ∪ (6, ∞)

Question: 24

Find the interval

Solution:

$f(x)=x^3-6x^2+9x+10$

$f'(x)=3x^2-12x+9$

$f'(x)=3(x^2-4x+3)$

$=3(x-3)(x-1)$

Function f(x) is decreasing for x ∈ [1,3] and increasing in x ∈ (-∞,1) ∪ (3, ∞)
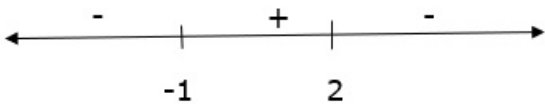
Question: 25

Find the interval

Solution:

$f(x)=-2x^3+3x^2+12x+6$

$f'(x)=-6x^2+6x+12$

$f'(x)=-6(x^2-x-2)$

$=-6(x-2)(x+1)$

Function f(x) is increasing for x ∈ [-1,2] and decreasing in x ∈ (-∞,-1) ∪ (2, ∞)

Question: 26

Find the interval

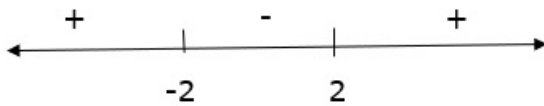**Solution:**

$f(x)=2x^3-24x+5$

$f'(x)=6x^2-24$

$f'(x)=6(x^2-4)$

$=6(x-2)(x+2)$

Function f(x) is decreasing for x ∈ [-2,2] and increasing in x ∈ (-∞,-2) ∪ (2, ∞)

Question: 27

Find the interval

Solution:

f(x)=(x-1)(x-2)^2=x2-4x+4 * x-1= x3-4x2+4x-x2+4x-4

f(x)=x^3-5x^2+8x-4

f'(x)=3x^2-10x+8

f'(x)=3x^2-6x-4x+8

=3x(x-2)-4(x-2)

=(3x-4)(x-2)
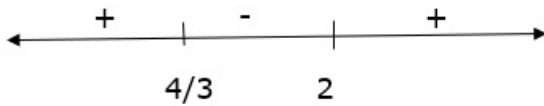
Function f(x) is decreasing for x ∈ [4/3,2] and increasing in x ∈ (-∞,4/3) ∪ (2, ∞)
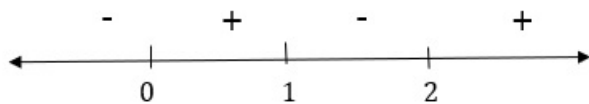
Question: 28

Find the interval

Solution:

f(x)=x^4-4x^3+4x^2+15

f'(x)=4x^3-12x^2+8x

= 4x(x^2-3x+2)

=4x(x-1)(x-2)

Function f(x) is decreasing for x ∈ (-∞,0] ∪ [1, 2] and increasing in x ∈ (0,1) ∪ (2, ∞)

Question: 29

Find the interval

Solution:

f(x)=2x^3+9x^2+12x+15

f'(x)=6x^2+18x+12

f'(x)=6(x^2+3x+2)

=6(x+2)(x+1)
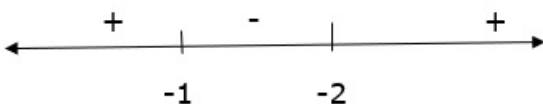
Function f(x) is decreasing for x ∈ [-1,-2] and increasing in x ∈ (-∞,-1) ∪ (-2, ∞)
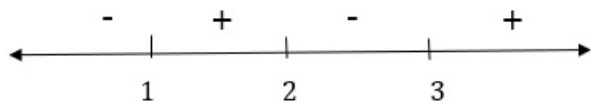
Question: 30

Find the interval

Solution:

$f(x)=x^4-8x^3+22x^2-24x+21$

$f'(x)=4x^3-24x^2+44x-24$

$= 4(x^3 -6x^2+11x-6)$

$=4(x-3)(x-1)(x-2)$

Function f(x) is decreasing for x ∈ (-∞,1] ∪ [2, 3] and increasing in x ∈ (1,2) ∪ (3, ∞)

Question: 31

Find the interval

Solution:

$f(x)=3x^4-4x^3-12x^2+5$

$f'(x)=12x^3-12x^2-24x$

$=12x(x^2 -x-2)$

$=12(x)(x+1)(x-2)$

Function f(x) is decreasing for x ∈ (-∞,-1] ∪ [0, 2] and increasing in x ∈ (-1,0) ∪ (2, ∞)

Question: 32

Find the interval

Solution:

$f'(x) = \frac{12x^3}{10} - \frac{12x^2}{5} - 6x + \frac{36}{5}$

$f'(x)=(12x^3-24x^2-60x+72)/10$

$=1.2(x^3-2x^2-5x+6)$

$=1.2(x-1)(x-3)(x+2)$
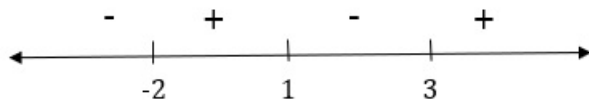
Function f(x) is decreasing for x ∈ (-∞,-2] ∪ [1, 3] and increasing in x ∈ (-2,1) ∪ (3, ∞)

Exercise : 11H

Question: 1

Find the slope of

Solution:

i. $\frac{dy}{dx} = 3x^2 - 1$

$\frac{dy}{dx}$ at $(x = 2) = 11$

ii. $\frac{dy}{dx} = 4x + 3\cos x$

$\frac{dy}{dx}$ at $(x = 0) = 3$

iii. $\frac{dy}{dx} = 2(\sin 2x + \cot x + 2)(2\cos 2x - \mathrm{cosec}^2 x)$

$\frac{dy}{dx}$ at $\left(x = \frac{\pi}{2}\right) = 2(0 + 0 + 2)(-2 - 1) = -12$

Question: 2

Find the equation

Solution:

$m : \frac{dy}{dx} = 3x^2 - 2$

m at (1, 6) = 1

Tangent : y – b = m(x – a)

y – 6 = 1(x – 1)

x – y + 5 = 0

Normal : $y - b = \frac{-1}{m}(x - a)$

y – 6 = -1(x – 1)

x + y – 7 = 0

Question: 3

Find the equation

Solution:

$m : 2y\frac{dy}{dx} = 4a$

m at $\left(\frac{a}{m^2}, \frac{2a}{m}\right) = m$

Tangent : y – b = m(x – a)

$y - \frac{2a}{m} = m\left(x - \frac{a}{m^2}\right)$

m²x – my + a = 0

Normal : $y - b = \frac{-1}{m}(x - a)$

$y - \frac{2a}{m} = \frac{-1}{m}\left(x - \frac{a}{m^2}\right)$

m²x + m³y – 2am² – a = 0

Question: 4

Find the equation

Solution:

$m : \frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$

m at $(a\cos\theta, b\sin\theta) = \frac{-b\cos\theta}{a\sin\theta}$

Tangent : y – b = m(x – a)

$y - b\sin\theta = \frac{-b\cos\theta}{a\sin\theta}(x - a\cos\theta)$

$bx \cos\theta + ay \sin\theta = ab$

Normal : $y - b = \dfrac{-1}{m}(x - a)$

$y - b\sin\theta = \dfrac{a\sin\theta}{b\cos\theta}(x - a\cos\theta)$

$ax \sec\theta - by \csc\theta = a^2 - b^2$

Question: 5

Find the equation

Solution:

$m : \dfrac{2x}{a^2} - \dfrac{2y}{b^2}\dfrac{dy}{dx} = 0$

$m \text{ at } (a\sec\theta, b\tan\theta) = \dfrac{b\sec\theta}{a\tan\theta}$

Tangent : $y - b = m(x - a)$

$y - b\tan\theta = \dfrac{b\sec\theta}{a\tan\theta}(x - a\sec\theta)$

$bx \sec\theta - ay \tan\theta = ab$

Normal : $y - b = \dfrac{-1}{m}(x - a)$

$y - b\sin\theta = \dfrac{-a\sin\theta}{b\cos\theta}(x - a\cos\theta)$

$by \csc\theta + ax \sec\theta = (a^2 + b^2)$

Question: 6

Find the equation

Solution:

$m : \dfrac{dy}{dx} = 3x^2$

$m \text{ at } (1, 1) = 3$

Tangent : $y - b = m(x - a)$

$y - 1 = 3(x - 1)$

$y = 3x - 2$

Normal : $y - b = \dfrac{-1}{m}(x - a)$

$y - 1 = \dfrac{-1}{3}(x - 1)$

$x + 3y = 4$

Question: 7

Find the equation

Solution:

$m : 2y\dfrac{dy}{dx} = 4a$

$m \text{ at } (at^2, 2at) = 1/t$

Tangent : $y - b = m(x - a)$

$y - 2at = \dfrac{1}{t}(x - at^2)$

x − ty + at² = 0

Normal : $y - b = \frac{-1}{m}(x - a)$

y − 2at = -t(x − at²)

tx + y = at³ + 2at

Question: 8

Find the equation

Solution:

$m : \frac{dy}{dx} = 2\cot x(-\csc^2 x) + 2\csc^2 x$

m at (x = π/4) = 2(-2) + 2(2) = 0

Tangent : y − b = m(x − a)

y − 1 = 0(x − π/4)

y = 1

Normal : $y - b = \frac{-1}{m}(x - a)$

$y - 1 = \frac{-1}{0}\left(x - \frac{\pi}{4}\right)$

x = π/4

Question: 9

Find the equation

Solution:

$m : 32x + 18y\frac{dy}{dx} = 0$

$m \text{ at } (2, y_1) = \frac{-32}{9y_1}$

16(2)² + 9(y₁)² = 144

$y_1 = \frac{4\sqrt{5}}{3}$

Tangent : y − b = m(x − a)

$y - \frac{4\sqrt{5}}{3} = \frac{-32}{9\frac{4\sqrt{5}}{3}}(x - 2)$

$8x + 3\sqrt{5}y - 36 = 0$

Normal : $y - b = \frac{-1}{m}(x - a)$

$y - \frac{4\sqrt{5}}{3} = \frac{9\frac{4\sqrt{5}}{3}}{32}(x - 2)$

$9\sqrt{5}x - 24y + 14\sqrt{5} = 0$

Question: 10

Find the equation

Solution:

$m : \frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$

m at $(x = 1) = 2$

y at $(x = 1) = (1)^4 - 6(1)^3 + 13(1)^2 - 10(1) + 5 = 3$

Tangent : $y - b = m(x - a)$

$y - 3 = 2(x - 1)$

$2x - y + 1 = 0$

Normal : $y - b = \frac{-1}{m}(x - a)$

$y - 3 = \frac{-1}{2}(x - 1)$

$x + 2y - 7 = 0$

Question: 11

Find the equation

Solution:

$m : \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$

m at $\left(\frac{a^2}{4}, \frac{a^2}{4}\right) = -1$

$y - b = m(x - a)$

$y - \frac{a^2}{4} = -1\left(x - \frac{a^2}{4}\right)$

$2(x + y) = a^2$

Question: 12

Show that the equ

Solution:

m at $(x_1, y_1) = \frac{b^2 x_1}{a^2 y_1}$

At $(x_1, y_1) : \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} = 1 \Rightarrow b^2 x_1^2 - a^2 y_1^2 = a^2 b^2$

$y - b = m(x - a)$

$y - y_1 = \frac{b^2 x_1}{a^2 y_1}(x - x_1)$

$a^2 y_1 y - a^2 y_1^2 = b^2 x_1 x - b^2 x_1^2$

$b^2 x_1 x - a^2 y_1 y = a^2 b^2$

$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$

Question: 13

Find the equation

Solution:

$m : \frac{dy}{dx} = 4\sec^3 x(\tan x \sec x) - 4\tan^3 x(\sec^2 x)$

m at $\left(x = \frac{\pi}{3}\right) = 4(2)^3(\sqrt{3} \times 2) - 4(\sqrt{3})^3(2)^2 = 16\sqrt{3}$

At $x = \pi/3$, $y = 7$

$$y - b = m(x - a)$$

$$y - 7 = 16\sqrt{3}\left(x - \frac{\pi}{3}\right)$$

$$3y - 48\sqrt{3}x + 16\sqrt{3}\pi - 21 = 0$$

Question: 14

Find the equation

Solution:

$$m : \frac{dy}{dx} = 2(\sin 2x + \cot x + 2)(2\cos 2x - \operatorname{cosec}^2 x)$$

$$\frac{dy}{dx} \text{ at } \left(x = \frac{\pi}{2}\right) = 2(0 + 0 + 2)(-2 - 1) = -12$$

At $x = \pi/2$, $y = 4$

$$y - b = \frac{-1}{m}(x - a)$$

$$y - 4 = \frac{1}{12}\left(x - \frac{\pi}{2}\right)$$

$$24y - 2x + \pi - 96 = 0$$

Question: 15

Show that the tan

Solution:

$$m : \frac{dy}{dx} = 6x^2$$

m at $(x = 2) = 24$

m at $(x = -2) = 24$

We know that if the slope of curve at two different point is

equal then straight lines are parallel at that points.

Question: 16

Find the equation

Solution:

We know that if two straight lines are parallel then their slope

are equal. So, slope of required tangent is also equal to 4.

$$m : \frac{dy}{dx} = \frac{-2x}{3} = 4$$

x = -6 and y = -11

$$y - b = m(x - a)$$

$$y - (-11) = 4(x - (-6))$$

$$4x - y + 13 = 0$$

Question: 17

At what point on

Solution:

If the tangent is parallel to y-axis it means that it's slope is

not defined or 1/0.

$$m : 2x + 2y\frac{dy}{dx} - 2 - 4\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-(2x-2)}{(2y-4)} = \frac{1}{0}$$

$2y - 4 = 0 \Rightarrow y = 2$

$x^2 + (2)^2 - 2x - 4(2) + 1 = 0$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow x = 3$ and $x = -1$

So, the requied points are (-1, 2) and (3, 2).

Question: 18

Find the point on

Solution:

If the tangent is parallel to x-axis it means that it's slope is 0

$$m : 2x + 2y\frac{dy}{dx} - 2 = 0$$

$2x + 2y(0) - 2 = 0$

$x = 1$

$(1)^2 + y^2 - 2(1) - 3 = 0$

$\Rightarrow y^2 = 4 \Rightarrow y = 2$ and $y = -2$

So, the requied points are (1, 2) and (1, -2).

Question: 19

Prove the tangent

Solution:

We know that if the slope of two tangent of a curve are satisfies a relation $m_1 m_2 = -1$, then tangents are at right angles

$$m : \frac{dy}{dx} = 2x - 5$$

m_1 at (2, 0) = -1

m_2 at (3, 0) = 1

$m_1 m_2 = (-1)(1) = -1$

So, we can say that tangent at (2, 0) and (3, 0) are at right angles.

Question: 20

Find the point on

Solution:

If tangent is pass through origin it means that equation of tangent is $y = mx$

Let us suppose that tangent is made at point (x_1, y_1)

$y_1 = x_1^2 + 3x_1 + 4$ ...(1)

$$m : \frac{dy}{dx} = 2x + 3$$

m at $(x_1, y_1) = 2x_1 + 3$

Equation of tangent : $y_1 = (2x_1 + 3)x_1$ ...(2)

On compairing eq(1) and eq(2)

$x_1^2 + 3x_1 + 4 = (2x_1 + 3)x_1$

$x_1^2 - 4 = 0 \Rightarrow x_1 = 2$ and -2

At $x_1 = 2$, $y_1 = 14$

At $x_1 = -2$, $y_1 = 2$

So, required points are (2, 14) and (-2, 2)

Question: 21

Find the point on

Solution:

Slope of $y = x - 11$ is equal to 1

$m : \dfrac{dy}{dx} = 3x^2 - 11$

$3x^2 - 11 = 1 \Rightarrow x = 2$ and -2

At x = 2

From the equation of curve, $y = (2)^3 - 11(2) + 5 = -9$

From the equation of tangent, $y = 2 - 11 = -9$

At x = -2

From the equation of curve, $y = (-2)^3 - 11(-2) + 5 = 19$

From the equation of tangent, $y = -2 - 11 = -13$

So, the final answer is (2, -9) because at x = -2, y is come different from the equation of curve and tangent which is not possible.

Question: 22

Find the equation

Solution:

If tangent is parallel to the line x + 3y = 4 then it's slope is -1/3.

$m : 4x + 6y\dfrac{dy}{dx} = 0$

$m = \dfrac{-2x}{3y} = \dfrac{-2x}{3\sqrt{\dfrac{14 - 2x^2}{3}}} = \dfrac{-1}{3}$

$2x = \sqrt{\dfrac{14 - 2x^2}{3}}$

$4x^2 = \dfrac{14 - 2x^2}{3}$

x = 1 and -1

At x = 1, y = 2 and y = -2 (not possible)

At x = -1, y = -2 and y = 2 (not possible)

y – b = m(x – a)

At (1, 2)

$$y - 2 = \frac{-1}{3}(x - 1)$$

$3y + x = 7$

At (-1, -2)

$$y - (-2) = \frac{-1}{3}(x - (-1))$$

$3y + x = -7$

Question: 23

Find the equation

Solution:

.: If tangent is perpendicular to the line x – 2y + 1 = 0 then it's -1/m is -2.

$$m : 2x + 2\frac{dy}{dx} = 0$$

$m = -x = 1/2$

$x = -1/2$

At x = -1/2, y = 31/8

$$y - b = \frac{-1}{m}(x - a)$$

At (-1/2, 31/8)

$$y - \frac{31}{8} = \frac{-1}{\frac{1}{2}}\left(x - \left(-\frac{1}{2}\right)\right)$$

$16x + 8y - 23 = 0$

Question: 24

Find the point on

Solution:

We know that if tangent is parallel to x-axis then it's slope is equal to 0.

$$m : \frac{dy}{dx} = 4x - 6$$

$4x - 6 = 0 \Rightarrow x = 3/2$

At x = 3/2 , y = -17/2

So, the required points are $\left(\frac{3}{2}, \frac{-17}{2}\right)$.

Question: 25

Find the point on

Solution:

If the tangent is parallel to chord joining the points (3, 0) and (4, 1) then slope of tangent is equal to slope of chord.

$$m = \frac{1 - 0}{4 - 3} = 1$$

$$m : \frac{dy}{dx} = 2(x - 3)$$

$2(x - 3) = 1 \Rightarrow x = 7/2$

At x = 7/2, y = 1/4

So, the required points are $\left(\frac{7}{2}, \frac{1}{4}\right)$.

Question: 26

Show that the cur

Solution:

If curves cut at right angle if $8k^2 = 1$ then vice versa also true. So, we have to prove that $8k^2 = 1$ if curve cut at right angles.

If curve cut at right angle then the slope of tangent at their intersecting point satisfies the relation $m_1 m_2 = -1$

We have to find intersecting point of two curves.

$x = y^2$ and $xy = k$ then $y = k^{\frac{1}{3}}$ and $x = k^{\frac{2}{3}}$

$m_1 : \dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x}}$

m_1 at $\left(k^{\frac{2}{3}}, k^{\frac{1}{3}}\right) = \dfrac{1}{2k^{\frac{1}{3}}}$

$m_2 : \dfrac{dy}{dx} = \dfrac{-k}{x^2}$

m_2 at $\left(k^{\frac{2}{3}}, k^{\frac{1}{3}}\right) = \dfrac{-k}{k^{\frac{4}{3}}} = -\dfrac{1}{k^{\frac{1}{3}}}$

$m_1 m_2 = -1$

$\left(\dfrac{1}{2k^{\frac{1}{3}}}\right)\left(-\dfrac{1}{k^{\frac{1}{3}}}\right) = -1$

$k^{\frac{2}{3}} = \dfrac{1}{2} \Rightarrow k^2 = \dfrac{1}{8} \Rightarrow 8k^2 = 1$

Question: 27

Show that the cur

Solution:

If the two curve touch each other then the tangent at their intersecting point formed a angle of 0.

We have to find the intersecting point of these two curves.

$xy = a^2$ and $x^2 + y^2 = 2a^2$

$\Rightarrow x^2 + \left(\dfrac{a^2}{x}\right)^2 = 2a^2$

$\Rightarrow x^4 - 2a^2x^2 + a^4 = 0$

$\Rightarrow (x^2 - a^2) = 0$

$\Rightarrow x = +a$ and $-a$

At $x = a$, $y = a$

At $x = -a$, $y = -a$

$m_1 : \dfrac{dy}{dx} = \dfrac{-a^2}{x^2}$

m_1 at $(a, a) = -1$

m_1 at $(-a, -a) = -1$

$m_2 : 2x + 2y\dfrac{dy}{dx} = 0$

m_2 at (a, a) = -1

m_2 at (-a, -a) = -1

At (a, a)

$\tan\theta = \dfrac{m_1 - m_2}{1 + m_1 m_2}$

$\tan\theta = \dfrac{(-1)-(-1)}{1+(-1)(-1)} = 0 \Rightarrow \theta = 0$

At (-a, -a)

$\tan\theta = \dfrac{m_1 - m_2}{1 + m_1 m_2}$

$\tan\theta = \dfrac{(-1)-(-1)}{1+(-1)(-1)} = 0 \Rightarrow \theta = 0$

So, we can say that two curves touch each other because the angle between two tangent at their intersecting point is equal to 0.

Question: 28

Show that the cur

Solution:

If the two curve cut orthogonally then angle between their tangent at intersecting point is equal to 90⁰.

We have to find their intersecting point.

$x^3 - 3xy^2 + 2 = 0$ ...(1) and $3x^2y - y^3 - 2 = 0$ ...(2)

On adding eq (1) and eq (2)

$x^3 - 3xy^2 + 2 + 3x^2y - y^3 - 2 = 0$

$x^3 - y^3 - 3xy^2 + 3x^2y = 0$

$(x - y)^3 = 0 \Rightarrow x = y$

Put x = y in eq (1)

$y^3 - 3y^3 + 2 = 0 \Rightarrow y = 1$

At y = 1, x = 1

$m_1 : 3x^2 - 3\left(x \times 2y\dfrac{dy}{dx} + y^2\right) = 0$

m_1 at (1, 1) = 0

$m_2 : 3\left(x^2\dfrac{dy}{dx} + 2xy\right) - 3y^2\dfrac{dy}{dx} = 0$

m_2 at (1, 1) = -2/0

At (1, 1)

$\tan\theta = \dfrac{m_1 - m_2}{1 + m_1 m_2}$

$\tan\theta = \dfrac{m_2\left(1 - \dfrac{m_1}{m_2}\right)}{m_2\left(\dfrac{1}{m_2} + m_1\right)}$

$$\tan \theta = \frac{(1-0)}{(0+0)} = \text{not defined} \Rightarrow \theta = \frac{\pi}{2}$$

So, we can say that two curve cut each other orthogonally because angle between two tangent at their intersecting point is equal to 90⁰.

Question: 29

Find the equation

Solution:

$$m : \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-\sin\theta}{1+\cos\theta}$$

$$m \text{ at } \left(\theta = \frac{\pi}{4}\right) = \frac{-1}{1+\sqrt{2}} = 1-\sqrt{2}$$

At $\theta = \frac{\pi}{4}, x = \left(\frac{\pi}{4} + \frac{1}{\sqrt{2}}\right)$ and $y = \left(1 + \frac{1}{\sqrt{2}}\right)$

$$y - b = m(x - a)$$

$$y - \left(1 + \frac{1}{\sqrt{2}}\right) = (1 - \sqrt{2})\left(x - \left(\frac{\pi}{4} + \frac{1}{\sqrt{2}}\right)\right)$$

$$y = (1 - \sqrt{2})x + \frac{(\sqrt{2}-1)\pi}{4} + 2$$

Question: 30

Find the equation

Solution:

$$m : \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2\sin 2t}{3\cos 3t}$$

$$m \text{ at } \left(t = \frac{\pi}{4}\right) = \frac{2\sqrt{2}}{3}$$

At $t = \frac{\pi}{4}, x = \frac{1}{\sqrt{2}}$ and $y = 0$

$$y - b = m(x - a)$$

$$y - 0 = \frac{2\sqrt{2}}{3}\left(x - \frac{1}{\sqrt{2}}\right)$$

$$4x - 3\sqrt{2}y - 2\sqrt{2} = 0$$

Exercise : OBJECTIVE QUESTIONS

Question: 1

Mark (√) against

Solution:

Given that $y = 2^x$

Taking log both sides, we get

$\log_e y = x \log_e 2$ (Since $\log_a b^c = c \log_a b$)

Differentiating with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx} = \log_e 2 \text{ or } \frac{dy}{dx} = \log_e 2 \times y$$

Hence $\frac{dy}{dx} = 2^x \log_e 2$

Question: 2

Mark (√) against

Solution:

Given that $y = \log_{10} x$

Using the property that $\log_a b = \frac{\log_e b}{\log_e a}$, we get

$$y = \frac{\log_e x}{\log_e 10}$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{x \log_e 10}$$

Question: 3

Mark (√) against

Solution:

Given that $y = e^{\frac{1}{x}}$

Taking log both sides, we get

$$\log_e y = \frac{1}{x} \text{ (Since } \log_a b^c = c \log_a b)$$

Differentiating with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx} = -\frac{1}{x^2} \text{ or} \frac{dy}{dx} = -\frac{1}{x^2} \times y$$

Hence $\frac{dy}{dx} = -\frac{1}{x^2} \times e^{\frac{1}{x}}$

Question: 4

Mark (√) against

Solution:

Let y=f(x)=x^x

Taking log both sides, we get

$$\log_e y = x \times \log_e x \text{-(1) (Since } \log_a b^c = c \log_a b)$$

Differentiating (1) with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{x} + \log_e x \times 1$$

$$\Rightarrow \frac{dy}{dx} = y \times (1 + \log_e x)$$

$$\Rightarrow \frac{dy}{dx} = f'(x) = x^x(1 + \log_e x)$$

Question: 5

Mark (√) against

Solution:

Let y=f(x)=x^{sinx}

Taking log both sides, we get

$\log_e y = \sin x \times \log_e x$-(1) (Since $\log_a b^c = c \log_a b$)

Differentiating (1) with respect to x, we get

$\dfrac{1}{y}\dfrac{dy}{dx} = \sin x \times \dfrac{1}{x} + \log_e x \times \cos x$

$\Rightarrow \dfrac{dy}{dx} = y \times \left(\dfrac{\sin x}{x} + \log_e x \cos x\right)$

$\Rightarrow \dfrac{dy}{dx} = f'(x) = x^x \left(\dfrac{\sin x + x \log_e x \cos x}{x}\right)$

Question: 6

Mark (√) against

Solution:

Let $y = f(x) = x^{\sqrt{x}}$

Taking log both sides, we get

$\log_e y = \sqrt{x} \times \log_e x$ -(1)

(Since $\log_a b^c = c \log_a b$)

Differentiating (1) with respect to x, we get

$\dfrac{1}{y}\dfrac{dy}{dx} = \sqrt{x} \times \dfrac{1}{x} + \log_e x \times \dfrac{1}{2\sqrt{x}}$

$\Rightarrow \dfrac{dy}{dx} = y \times \left(\dfrac{2 + \log_e x}{2\sqrt{x}}\right)$

$\Rightarrow \dfrac{dy}{dx} = f'(x)$

$= x^{\sqrt{x}} \left(\dfrac{2 + \log_e x}{2\sqrt{x}}\right)$

Question: 7

Mark (√) against

Solution:

Given that $y = e^{\sin\sqrt{x}}$

Taking log both sides, we get

$\log_e y = \sin\sqrt{x}$

(Since $\log_a b^c = c \log_a b$)

Differentiating with respect to x, we get

$\dfrac{1}{y}\dfrac{dy}{dx} = \cos\sqrt{x} \times \dfrac{1}{2\sqrt{x}}$

Or

$\dfrac{dy}{dx} = \cos\sqrt{x} \times \dfrac{1}{2\sqrt{x}} \times y$

Hence $\dfrac{dy}{dx} = \dfrac{e^{\sin\sqrt{x}}\cos\sqrt{x}}{2\sqrt{x}}$

Question: 8

Mark (√) against

Solution:

Given that $y = (\tan x)^{\cot x}$

Taking log both sides, we get

$\log_e y = \cot x \times \log_e \tan x$ (Since $\log_a b^c = c \log_a b$)

Differentiating with respect to x, we get

$\frac{1}{y}\frac{dy}{dx} = \cot x \times \frac{1}{\tan x} \times \sec^2 x - \log_e \tan x \times \csc^2 x = \csc^2 x(1 - \log_e \tan x)$

Hence, $\frac{dy}{dx} = \csc^2 x(1 - \log_e \tan x \times y = \csc^2 x(1 - \log_e \tan x)(\tan x)^{\cot x}$

Question: 9

Mark (√) against

Solution:

Given that $y = (\sin x)^{\log_e x}$

Taking log both sides, we get

$\log_e y = \log_e x \times \log_e \sin x$ (Since $\log_a b^c = c \log_a b$)

Differentiating with respect to x, we get

$\frac{1}{y}\frac{dy}{dx} = \log_e x \times \frac{1}{\sin x} \times \cos x + \log_e \sin x \times \frac{1}{x}$

$= \frac{x \cot x \log_e x + \log_e \sin x}{x}$

Hence, $\frac{dy}{dx} = \frac{x \cot x \log_e x + \log_e \sin x}{x} \times y$

$= \frac{x \cot x \log_e x + \log_e \sin x}{x}(\sin x)^{\log_e x}$

Question: 10

Mark (√) against

Solution:

Given that y=sin(x^x)

Let x^x=u, then y=sin u

Differentiating with respect to x, we get

$\frac{dy}{dx} = \cos u \times \frac{du}{dx} = \cos(x^x)\frac{du}{dx}$ –(1)

Also, u=x^x

Taking log both sides, we get

$\log_e u = x \times \log_e x$

(Since $\log_a b^c = c \log_a b$)

Differentiating with respect to x, we get

$\frac{1}{u}\frac{du}{dx} = x \times \frac{1}{x} + \log_e x \times 1$

$$\Rightarrow \frac{du}{dx} = u \times (1 + \log_e x)$$

$$\Rightarrow \frac{du}{dx} = x^x(1 + \log_e x) \text{ -(2)}$$

From (1) and (2), we get

$$\frac{dy}{dx} = \cos(x^x) x^x (1 + \log_e x)$$

Question: 11

Mark (√) against

Solution:

Given that $y = \sqrt{x\sin x}$

Squaring both sides, we get

$$y^2 = x\sin x$$

Differentiating with respect to x, we get

$$2y \frac{dy}{dx} = x\cos x + \sin x \text{ or } \frac{dy}{dx} = \frac{x\cos x + \sin x}{2y}$$

Hence, $\frac{dy}{dx} = \frac{x\cos x + \sin x}{2\sqrt{x\sin x}}$

Question: 12

Mark (√) against

Solution:

Given that $xy = e^{x+y}$

Taking log both sides, we get

$\log_e xy = x + y$ (Since $\log_a b^c = c\log_a b$)

Since $\log_a bc = \log_a b + \log_a c$, we get

$$\log_e x + \log_e y = x + y$$

Differentiating with respect to x, we get

$$\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

Or

$$\frac{dy}{dx}\left(\frac{y-1}{y}\right) = \frac{1-x}{x}$$

Hence, $\frac{dy}{dx} = \frac{y(1-x)}{x(y-1)}$

Question: 13

Mark (√) against

Solution:

Given that $x+y = \sin(x+y)$

Differentiating with respect to x, we get

$$1 + \frac{dy}{dx} = \cos(x+y)\left(1 + \frac{dy}{dx}\right) \text{ or } (\cos(x+y) - 1)\left(1 + \frac{dy}{dx}\right) = 0$$

Hence, $\cos(x+y) = 1$ or $\frac{dy}{dx} = -1$

If cos(x+y)=1 then, x+y=2nπ, n∈ℤ

Hence x+y=sin(2nπ)=0 or y=-x

Differentiating with respect to x, we get

$$\frac{dy}{dx} = -1$$

Hence, $\frac{dy}{dx} = -1$

Question: 14

Mark (√) against

Solution:

Given that $\sqrt{x} + \sqrt{y} = \sqrt{a}$

Differentiating with respect to x, we get

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$$

Or

$$\frac{dy}{dx} = -\sqrt{\frac{y}{x}}$$

Question: 15

Mark (√) against

Solution:

Given that $x^y = y^x$

Taking log both sides, we get

$$y \log_e x = x \log_e y$$

(Since $\log_a b^c = c \log_a b$)

Differentiating with respect to x, we get

$$\frac{y}{x} + \log_e x \frac{dy}{dx} = \frac{x}{y}\frac{dy}{dx} + \log_e y$$

$$\Rightarrow \frac{x - y \log_e x}{y}\frac{dy}{dx} = \frac{y - x \log_e y}{x}$$

Hence $\frac{dy}{dx} = \frac{y(y - x \log_e y)}{x(x - y \log_e x)}$

Question: 16

Mark (√) against

Solution:

Given that $x^p y^q = (x+y)^{p+q}$

Taking log both sides, we get

$$\log_e x^p y^q = (p + q) \log_e (x + y)$$

(Since $\log_a b^c = c \log_a b$)

Since $\log_a bc = \log_a b + \log_a c$, we get

$$\log_e x^p + \log_e y^q = (p + q) \log_e (x + y)$$

$$p \log_e x + q \log_e y = (p + q) \log_e(x + y)$$

Differentiating with respect to x, we get

$$\frac{p}{x} + \frac{q}{y} \frac{dy}{dx} = \frac{p + q}{x + y}\left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{xq - yp}{y(x + y)}\right) = \frac{xq - yp}{x(x + y)}$$

Hence, $\frac{dy}{dx} = \frac{y}{x}$

Question: 17

Mark (√) against

Solution:

Given that $y = x^2 \sin\frac{1}{x}$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = x^2 \cos\frac{1}{x} \times -\frac{1}{x^2} + 2x\sin\frac{1}{x} = 2x\sin\frac{1}{x} - \cos\frac{1}{x}$$

Question: 18

Mark (√) against

Solution:

$y = \cos^2 x^3 = (\cos(x^3))^2$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = 2\cos(x^3) \times -\sin(x^3) \times 3x^2$$

Using 2sinAcosA=sin2A

$$\frac{dy}{dx} = -3x^2 \sin(2x^3)$$

Question: 19

Mark (√) against

Solution:

Given that $y = \log_e\left(x + \sqrt{x^2 + a^2}\right)$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 + a^2}}\left(1 + \frac{1}{2\sqrt{x^2 + a^2}} \times 2x\right)$$

Hence, $\frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 + a^2}} \times \frac{x + \sqrt{x^2 + a^2}}{\sqrt{x^2 + a^2}} = \frac{1}{\sqrt{x^2 + a^2}}$

Question: 20

Mark (√) against

Solution:

Given that $y = \log_e\frac{1 + \sqrt{x}}{1 - \sqrt{x}}$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{1-\sqrt{x}}{1+\sqrt{x}} \times \frac{(1-\sqrt{x}) \times \frac{1}{2\sqrt{x}} - (1+\sqrt{x}) \times -\frac{1}{2\sqrt{x}}}{(1-\sqrt{x})^2} = \frac{1}{(1-x)\sqrt{x}}$$

Question: 21

Mark (√) against

Solution:

Given that $y = \log_e\left(\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x}\right)$

Differentiating with respect to x, we get

$$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{1+x^2}-x}{\sqrt{1+x^2}+x} \times \frac{\left(\sqrt{1+x^2}-x\right)\times\left(\frac{1}{2\sqrt{1+x^2}}\times2x+1\right)-\left(\sqrt{1+x^2}+x\right)\times\left(\frac{1}{2\sqrt{1+x^2}}\times2x-1\right)}{\left(\sqrt{1+x^2}-x\right)^2}$$

Hence, $\frac{dy}{dx} = \frac{2}{\sqrt{1+x^2}}$

Question: 22

Mark (√) against

Solution:

Given that $y = \sqrt{\frac{1+\sin x}{1-\sin x}}$

Using, $\cos^2\theta + \sin^2\theta = 1$ and $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$

$$y = \sqrt{\frac{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2}}}$$

$$= \frac{\cos\frac{x}{2} + \sin\frac{x}{2}}{\cos\frac{x}{2} - \sin\frac{x}{2}}$$

Dividing by $\sin\frac{x}{2}$ in numerator and denominator, we get

$$y = \frac{\cot\frac{x}{2} + 1}{\cot\frac{x}{2} - 1} = \cot\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$\left(\text{Using } \cot\left(\frac{\pi}{4} - A\right) = \frac{\cot A + 1}{\cot A - 1}\right)$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = -\csc^2\left(\frac{\pi}{4} - \frac{x}{2}\right) \times -\frac{1}{2}$$

Hence, $\frac{dy}{dx} = \frac{1}{2}\csc^2\left(\frac{\pi}{4} - \frac{x}{2}\right)$

Question: 23

Mark (√) against

Solution:

Given that $y = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$

Multiplying by cos x in numerator and denominator, we get

$$y = \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

Using $1 - \cos x = 2\sin^2\frac{x}{2}$ and $1 + \cos x = 2\cos^2\frac{x}{2}$, we get

$$y = \sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}$$

$$= \tan\left(\frac{x}{2}\right)$$

Differentiating with respect to x, we get

$$y = \sec^2\frac{x}{2} \times \frac{1}{2}$$

$$= \frac{1}{2}\sec^2\frac{x}{2}$$

Question: 24

Mark (√) against

Solution:

Given that $y = \sqrt{\frac{1 + \tan x}{1 - \tan x}}$

Using $\tan\left(\frac{\pi}{4} + x\right) = \frac{1 + \tan x}{1 - \tan x}$, we get

$$y = \sqrt{\tan\left(\frac{\pi}{4} + x\right)}$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\tan\left(\frac{\pi}{4} + x\right)}} \times \sec^2\left(\frac{\pi}{4} + x\right) \times 1$$

Hence, $\dfrac{dy}{dx} = \dfrac{\sec^2\left(\frac{\pi}{4} + x\right)}{2\sqrt{\tan\left(\frac{\pi}{4} + x\right)}}$

Question: 25

Mark (√) against

Solution:

Given that $y = \tan^{-1}\left(\frac{1 - \cos x}{\sin x}\right)$

Using $1 - \cos x = 2\sin^2\frac{x}{2}$ and Using $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$, we get

$$y = \tan^{-1}\left(\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right) \text{ or } y = \tan^{-1}\tan\frac{x}{2}$$

$$y = \frac{x}{2}$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{2}$$

Question: 26

Mark (√) against

Solution:

Given that $y = \tan^{-1}\left(\frac{\cos x + \sin x}{\cos x - \sin x}\right)$

Dividing numerator and denominator with cosx, we get

$y = \tan^{-1}\left(\frac{1 + \tan x}{1 - \tan x}\right)$

Using $\tan\left(\frac{\pi}{4} + x\right) = \frac{1 + \tan x}{1 - \tan x}$, we get

$y = \tan^{-1}\tan\left(\frac{\pi}{4} + x\right) = \frac{\pi}{4} + x$

Differentiating with respect to x, we get

$\frac{dy}{dx} = 1$

Question: 27

Mark (√) against

Solution:

Given that $y = \tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right)$

Using $\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$, $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$ and $\cos^2\theta + \sin^2\theta = 1$

Hence, $y = \tan^{-1}\left(\frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\cos^2\frac{x}{2} + \sin^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}\right) = \tan^{-1}\left(\frac{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}\right)$

$\Rightarrow y = \tan^{-1}\frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}$

Dividing by $\cos\frac{x}{2}$ in numerator and denominator, we get

$y = \tan^{-1}\frac{1 - \tan\frac{x}{2}}{1 + \tan\frac{x}{2}}$

Using $\tan\left(\frac{\pi}{4} - x\right) = \frac{1 - \tan x}{1 + \tan x}$, we get

$y = \tan^{-1}\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$

$= \frac{\pi}{4} - \frac{x}{2}$

Differentiating with respect to x, we get

$\frac{dy}{dx} = -\frac{1}{2}$

Question: 28

Mark (√) against

Solution:

Given that $y = \tan^{-1}\sqrt{\frac{1 - \cos x}{1 + \cos x}}$

Using $1 - \cos x = 2\sin^2\frac{x}{2}$ and $1 + \cos x = 2\cos^2\frac{x}{2}$, we get

$$y = \tan^{-1}\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}} = \tan^{-1}\tan\left(\frac{x}{2}\right) = \frac{x}{2}$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{2}$$

Question: 29

Mark (√) against

Solution:

Given that $y = \tan^{-1}\left(\dfrac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$

Dividing by bcosx in numerator and denominator, we get

$$y = \tan^{-1}\left(\frac{\dfrac{a}{b} - \tan x}{1 + \dfrac{a}{b}\tan x}\right)$$

Let $\dfrac{a}{b} = \tan\alpha \Rightarrow \alpha = \tan^{-1}\dfrac{a}{b}$

Then $y = \tan^{-1}\left(\dfrac{\tan\alpha - \tan x}{1 + \tan\alpha\tan x}\right)$

Using $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$, we get

$$y = \tan^{-1}\tan(\alpha - x) = \alpha - x = \tan^{-1}\frac{a}{b} - x$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = -1$$

Question: 30

Mark (√) against

Solution:

Given that y=sin⁻¹(3x-4x³)

Let x=sin θ

⇒ θ=sin⁻¹x

Then, y= sin⁻¹(3sinθ -4sin³θ)

Using sin3θ=3sinθ -4sin³θ, we get

y=sin⁻¹(sin3θ)=3θ=3sin⁻¹x

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{3}{\sqrt{1 - x^2}}$$

Question: 31

Mark (√) against

Solution:

Given that y=cos⁻¹(4x³-3x)

Let x=cos θ

$\Rightarrow \theta = \cos^{-1}x$

Then, $y = \cos^{-1}(4\cos^3\theta - 3\cos\theta)$

Using $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$, we get

$y = \cos^{-1}(\cos 3\theta) = 3 = 3\cos^{-1}x$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{-3}{\sqrt{1-x^2}}$$

Question: 32

Mark (√) against

Solution:

Given that $y = \tan^{-1}\frac{\sqrt{a}+\sqrt{x}}{1-\sqrt{ax}}$

Let $\sqrt{a} = \tan A$ and $\sqrt{x} = \tan B$, then $A = \tan^{-1}\sqrt{a}$ and $B = \tan^{-1}\sqrt{x}$

Hence, $y = \tan^{-1}\frac{\tan A + \tan B}{1 - \tan A \tan B}$

Using $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$, we get

$y = \tan^{-1}\tan(A+B) = A+B$

$= \tan^{-1}\sqrt{a} + \tan^{-1}\sqrt{x}$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = 0 + \frac{1}{1 + \left(\sqrt{x}\right)^2} \times \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}(1+x)}$$

Question: 33

Mark (√) against

Solution:

Given that $y = \cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$

$\Rightarrow \cos y = \frac{x^2-1}{x^2+1}$ or $\sec y = \frac{x^2+1}{x^2-1}$

Since $\tan^2 x = \sec^2 x - 1$, therefore

$$\tan^2 y = \left(\frac{x^2+1}{x^2-1}\right)^2 - 1$$

$$= \frac{4x^2}{(x^2-1)^2}$$

Hence, $\tan y = -\frac{2x}{1-x^2}$ or $y = \tan^{-1}\left(-\frac{2x}{1-x^2}\right)$

Let $x = \tan\theta$

$\Rightarrow \theta = \tan^{-1}x$

Hence, $y = \tan^{-1}\left(-\frac{2\tan\theta}{1-\tan^2\theta}\right)$

Using $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$, we get

$y = \tan^{-1}(-\tan 2\theta)$

Using -tan x = tan(-x), we get

$$y = \tan^{-1}(\tan(-2\theta))$$

$$= -2\theta$$

$$= -2\tan^{-1} x$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{-2}{1+x^2}$$

Question: 34

Mark (√) against

Solution:

Given that $y = \tan^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

Let $x^2 = \tan\theta$

$\Rightarrow \theta = \tan^{-1} x^2$

Hence, $y = \tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$

Using $\tan\left(\frac{\pi}{4} + x\right) = \frac{1+\tan x}{1-\tan x}$, we get

$$y = \tan^{-1}\tan\left(\frac{\pi}{4} + \theta\right) = \frac{\pi}{4} + \theta = \frac{\pi}{4} + \tan^{-1}(x^2)$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{1+x^4} \times 2x = \frac{2x}{1+x^4}$$

Question: 35

Mark (√) against

Solution:

Given that $y = \tan^{-1}(-\sqrt{x})$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{1+\left(-\sqrt{x}\right)^2} \times \frac{-1}{2\sqrt{x}} = \frac{-1}{2\sqrt{x}(1+x)}$$

Question: 36

Mark (√) against

Solution:

Given that $y = \cos^{-1} x^3$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-(x^3)^2}} \times 3x^2 = \frac{-3x^2}{\sqrt{1-x^6}}$$

Question: 37

Mark (√) against

Solution:

Given that $y = \tan^{-1}(\sec x + \tan x)$

Hence, $y = \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)$

Using $\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$, $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$ and $\cos^2\theta + \sin^2\theta = 1$

Hence, $y = \tan^{-1}\left(\frac{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}\right) = \tan^{-1}\left(\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\right)$

Dividing by $\cos\frac{x}{2}$ in numerator and denominator, we get

$y = \tan^{-1}\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}$

Using $\tan\left(\frac{\pi}{4}+x\right) = \frac{1+\tan x}{1-\tan x}$, we get

$y = \tan^{-1}\tan\left(\frac{\pi}{4}+\frac{x}{2}\right) = \frac{\pi}{4}+\frac{x}{2}$

Differentiating with respect to x, we get

$\frac{dy}{dx} = \frac{1}{2}$

Question: 38

Mark (√) against

Solution:

Given that $y = \cot^{-1}\left(\frac{1-x}{1+x}\right)$

Let $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$ and using $\cot^{-1}x = \frac{\pi}{2} - \tan^{-1}x$

Hence, $y = \frac{\pi}{2} - \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$

Using $\tan\left(\frac{\pi}{4}-x\right) = \frac{1-\tan x}{1+\tan x}$, we get

$y = \frac{\pi}{2} - \tan^{-1}\tan\left(\frac{\pi}{4}-\theta\right) = \frac{\pi}{2} - \left(\frac{\pi}{4}-\theta\right) = \frac{\pi}{4}+\theta = \frac{\pi}{4}+\tan^{-1}x$

Differentiating with respect to x, we get

$\frac{dy}{dx} = \frac{1}{1+x^2}$

Question: 39

Mark (√) against

Solution:

Given that $y = \sqrt{\frac{1+x}{1-x}}$

Let $x = -\cos\theta \Rightarrow \theta = \cos^{-1}(-x)$.

Using $1 - \cos\theta = 2\sin^2\frac{\theta}{2}$ and $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$, we get

$y = \sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}} = \tan\left(\frac{\theta}{2}\right)$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \sec^2\left(\frac{\theta}{2}\right) \times \frac{1}{2}\frac{d\theta}{dx} \quad -(1)$$

Since, $x = -\cos\theta \Rightarrow 2\cos^2\frac{\theta}{2} = 1 + \cos\theta = 1 - x$ or $\sec^2\left(\frac{\theta}{2}\right) = \frac{2}{1-x}$ -(2)

Also, since $\theta = \cos^{-1}(-x)$, therefore $\frac{d\theta}{dx} = \frac{1}{\sqrt{1-x^2}}$ -(3)e

Substituting (2) and (3) in (1), we get

$$\frac{dy}{dx} = \frac{2}{1-x} \times \frac{1}{2} \times \frac{1}{\sqrt{1-x^2}} = \frac{1}{(1-x)\sqrt{1-x^2}} = \frac{1}{(1-x)^{\frac{3}{2}}(1+x)^{\frac{1}{2}}}$$

Question: 40

Mark (√) against

Solution:

Given that $y = \sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$

$$\Rightarrow \sec y = \frac{x^2+1}{x^2-1}$$

Since $\tan^2 x = \sec^2 x - 1$, therefore

$$\tan^2 y = \left(\frac{x^2+1}{x^2-1}\right)^2 - 1 = \frac{4x^2}{(x^2-1)^2}$$

Hence, $\tan y = -\frac{2x}{1-x^2}$ or $y = \tan^{-1}\left(-\frac{2x}{1-x^2}\right)$

Let $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

Hence, $y = \tan^{-1}\left(-\frac{2\tan\theta}{1-\tan^2\theta}\right)$

Using $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$, we get

$$y = \tan^{-1}(-\tan 2\theta)$$

Using $-\tan x = \tan(-x)$, we get

$$y = \tan^{-1}(\tan(-2\theta)) = -2\theta = -2\tan^{-1}x$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{-2}{1+x^2}$$

Question: 41

Mark (√) against

Solution:

$$\Rightarrow y = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$$

$$\Rightarrow \sec y = \frac{1}{2x^2-1}$$

$$\Rightarrow \cos y = 2x^2 - 1$$

$$\Rightarrow y = \cos^{-1}(2x^2 - 1)$$

Put $x = \cos\theta$

$$\Rightarrow y = \cos^{-1}(2\cos^2\theta - 1)$$

$\Rightarrow y = \cos^{-1}(\cos 2\theta)$

$\Rightarrow y = 2\theta$

But $\theta = \cos^{-1}x$.

$\Rightarrow \dfrac{dy}{dx} = \dfrac{d(\cos^{-1}x)}{dx}$

$\Rightarrow \dfrac{dy}{dx} = 2.\dfrac{d(\cos^{-1}x)}{dx}$

$\Rightarrow \dfrac{dy}{dx} = 2.\left(\dfrac{-1}{\sqrt{1-x^2}}\right)$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{-2}{\sqrt{1-x^2}}$

Question: 42

Mark (√) against

Solution:

Put $x = \tan\theta$

$\Rightarrow y = \tan^{-1}\left(\dfrac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$

$\Rightarrow y = \tan^{-1}\left(\dfrac{\sec\theta - 1}{\tan\theta}\right)$

$\Rightarrow y = \tan^{-1}\left(\dfrac{1-\cos\theta}{\sin\theta}\right)$

$\Rightarrow y = \tan^{-1}\left(\dfrac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}\right)$

$\Rightarrow y = \tan^{-1}\left(\tan\dfrac{\theta}{2}\right)$

$\Rightarrow y = \dfrac{\theta}{2}$

$\theta = \tan^{-1}x$

$\Rightarrow y = \dfrac{\tan^{-1}x}{2}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2(1+x^2)}$

Question: 43

Mark (√) against

Solution:

Put $x = \cos 2\theta$

$\Rightarrow y = \sin^{-1}\left(\dfrac{\sqrt{1+\cos 2\theta}}{2} + \dfrac{\sqrt{1-\cos 2\theta}}{2}\right)$

$\Rightarrow y = \sin^{-1}\left(\dfrac{\sqrt{2\cos^2 2\theta}}{2} + \dfrac{\sqrt{2.\sin^2\theta}}{2}\right)$

$\Rightarrow y = \sin^{-1}\left(\dfrac{\cos 2\theta}{\sqrt{2}} + \dfrac{\sin 2\theta}{\sqrt{2}}\right)$

$\Rightarrow y = \sin^{-1}\left(\sin\left(\dfrac{\pi}{4} + 2\theta\right)\right)$

$\Rightarrow y = \dfrac{\pi}{4} + 2\theta.$

$\Rightarrow \dfrac{dy}{d\theta} = 2$

Put $\theta = \dfrac{\cos^{-1}x}{2}$

$\Rightarrow \dfrac{d\theta}{dx} = \dfrac{-1}{4\sqrt{1-x^2}}$

$\therefore \dfrac{dy}{dx} = \dfrac{-1}{2\sqrt{1-x^2}}$

Question: 44

Mark (√) against

Solution:

$x = at^2$

$\therefore \dfrac{dx}{dt} = 2at$

$\therefore \dfrac{dt}{dx} = \dfrac{1}{2at}$

$Y = 2at$

$\therefore \dfrac{dy}{dt} = 2a$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx}$

$\Rightarrow \dfrac{dy}{dx} = 2a \times \dfrac{1}{2at}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{t}$

Question: 45

Mark (√) against

Solution:

$x = a \sec\theta$

$\therefore \dfrac{dx}{d\theta} = a\sec\theta.\tan\theta$

$\therefore \dfrac{d\theta}{dx} = \dfrac{1}{a\sec\theta.\tan\theta}$

$y = b \tan\theta$

$\therefore \dfrac{dy}{d\theta} = b.\sec^2\theta$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{dy}{d\theta} \times \dfrac{d\theta}{dx}$

$$\Rightarrow \frac{dy}{dx} = b.\sec^2\theta \times \frac{1}{a\sec\theta.\tan\theta}$$

$$\Rightarrow \frac{dy}{dx} = \frac{b\sec\theta}{a\tan\theta}$$

$$\Rightarrow \frac{dy}{dx} = \frac{b.\dfrac{1}{\cos\theta}}{a.\dfrac{\sin\theta}{\cos\theta}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{b}{a}\csc\theta$$

Question: 46

Mark (√) against

Solution:

$$x = a.\cos^2\theta$$

$$\therefore \frac{dx}{d\theta} = -2a\cos\theta.\sin\theta$$

$$\Rightarrow \frac{d\theta}{dx} = \frac{-1}{2a.\cos\theta.\sin\theta}$$

$$y = b.\sin^2\theta$$

$$\therefore \frac{dy}{d\theta} = 2b\sin\theta.\cos\theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$

$$\Rightarrow \frac{dy}{dx} = 2b\sin\theta.\cos\theta \times \frac{-1}{2a\cos\theta.\sin\theta}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-b}{a}$$

Question: 47

Mark (√) against

Solution:

$$x = a(\cos\theta + \theta\sin\theta)$$

$$\therefore \frac{dx}{d\theta} = a(-\sin\theta + \sin\theta + \theta\cos\theta)$$

$$\Rightarrow \frac{d\theta}{dx} = \frac{1}{a\theta\cos\theta}$$

$$y = a(\sin\theta - \theta\cos\theta)$$

$$\therefore \frac{dy}{d\theta} = a(\cos\theta - (\cos\theta + \theta(-\sin\theta)))$$

$$\Rightarrow \frac{dy}{d\theta} = a\cos\theta - a\cos\theta + \theta a\sin\theta$$

$$\Rightarrow \frac{dy}{d\theta} = a\theta\sin\theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$

$$\Rightarrow \frac{dy}{dx} = a\theta\sin\theta \times \frac{1}{a\theta\cos\theta}$$

$$\Rightarrow \frac{dy}{dx} = \tan\theta$$

Question: 48

Mark (√) against

Solution:

Given:

$$\Rightarrow y = x^{x^{x^{x\dots\infty}}}$$

We can write it as

$$\Rightarrow y = x^y$$

Taking log of both sides we get

log y = y log x

Differentiating

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\log x + y.\frac{1}{x}$$

$$\Rightarrow \left(\frac{1}{y} - \log x\right)\frac{dy}{dx} = \frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left(\frac{y}{1 - \log x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2}{x(1 - \log x)}$$

Question: 49

Mark (√) against

Solution:

Given:

$$\Rightarrow y = \sqrt{x + \sqrt{x + \sqrt{x +}\dots\infty}}$$

We can write it as

$$\Rightarrow y = \sqrt{x + y}$$

Squaring we get

$$\Rightarrow y^2 = x + y$$

Differentiating

$$\Rightarrow 2y\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{(2y - 1)}$$

Question: 50

Mark (√) against

Solution:

Given:

$$\Rightarrow y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots \infty}}}$$

We can write it as

$$\Rightarrow y = \sqrt{\sin x + y}$$

Squaring we get

$$\Rightarrow y^2 = \sin x + y$$

Differentiating

$$\Rightarrow 2y\frac{dy}{dx} = \cos x + \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos x}{(2y - 1)}$$

Question: 51

Mark (√) against

Solution:

We can write it as

$$\Rightarrow y = e^{x+y}$$

$$\log y = (x + y) \log e$$

Differentiating

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\Rightarrow \left(\frac{1}{y} - 1\right)\frac{dy}{dx} = 1$$

$$\Rightarrow \frac{dy}{dx} = 1\left(\frac{y}{1 - y}\right)$$

Question: 52

Mark (√) against

Solution:

Since f(x) is continuous on 0.

$$\Rightarrow \lim_{x \to 0}\frac{\sin 5x}{3x} = f(0)$$

$$\Rightarrow \lim_{x \to 0}\frac{\sin 5x}{3x} \times \frac{5x}{5x} = f(0)$$

$$\Rightarrow \lim_{x \to 0}\frac{\sin 5x}{5x} \times \frac{5x}{3x} = f(0)$$

$$\Rightarrow f(0) = \frac{5}{3}$$

$$\Rightarrow k = \frac{5}{3}$$

Question: 53

Mark (√) against

Solution:

Left hand limit =

$\Rightarrow \lim\limits_{x\to 0^-} f(x)$

$\Rightarrow \lim\limits_{h\to 0} f(0-h)$

$\Rightarrow \lim\limits_{h\to 0} h.\sin\left(\dfrac{-1}{h}\right)$

$\Rightarrow \lim\limits_{h\to 0} -h.\dfrac{\sin\left(\dfrac{-1}{h}\right)}{-\dfrac{1}{h}}\times\dfrac{-1}{h} = 1$

Right hand limit =

$\Rightarrow \lim\limits_{x\to 0^+} f(x)$

$\Rightarrow \lim\limits_{h\to 0} f(0+h)$

$\Rightarrow \lim\limits_{h\to 0} h.\sin\left(\dfrac{1}{h}\right)$

$\Rightarrow \lim\limits_{h\to 0} h.\dfrac{\sin\left(\dfrac{1}{h}\right)}{\dfrac{1}{h}}\times\dfrac{1}{h}$

$= 1$

As L.H.L = R.H.L

F(x) is continuous.

Question: 54

Mark (√) against

Solution:

$\Rightarrow f(x) = \dfrac{3x+4\tan x}{x}$ is continuous at x = 0.

$\Rightarrow f(x) = \lim\limits_{x\to 0} \dfrac{3x+4\tan x}{X}$

$\Rightarrow f(x) = \lim\limits_{x\to 0} \dfrac{3x}{X} + \dfrac{4\tan x}{X}$

$\Rightarrow f(x) = 3 + 4\lim\limits_{x\to 0} \dfrac{\tan x}{X}$

$\Rightarrow f(x) = 3+4$

$\therefore$ K = 7.

Question: 55

Mark (√) against

Solution:

$f(x) = x^{3/2}$

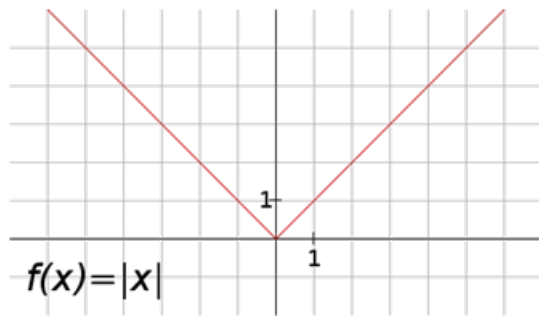$\Rightarrow f'(x) = \dfrac{3}{2\sqrt{x}}$

As x→0, f'(x)→∞

$\therefore$ f'(x) does not exist.

Question: 56

Mark (√) against

Solution:

$f(x)=|x|$

(Sometimes it's easier to get the answer by graphs)

Now in the above graph

We can see f(x) is Continuous on 0.

But it has sharp curve on x = 0 which implies it is not differentiable.

Question: 57

Mark (√) against

Solution:

For continuity left hand limit must be equal to right hand limit and value at the point.

Continuity at x =2.

For continuity at x=2,

L.H.L = $\lim\limits_{x\to2^-}(1+x) = 3$

R.H.L = $\lim\limits_{x\to2^+}(5-x) = 3$

f(2) = 1+2 = 3

∴ f(x) is continuous at x = 2

Now for differentiability.

$\Rightarrow f'(2^-) = \lim\limits_{x\to2^-} \dfrac{f(x)-f(2)}{x-2}$

$\Rightarrow f'(2^-) = \lim\limits_{h\to0} \dfrac{f(2-h)-f(2)}{2-h-2}$

$\Rightarrow f'(2^-) = \lim\limits_{h\to0} \dfrac{1+2-h-3}{2-h-2} = \lim\limits_{h\to0} \dfrac{-h}{-h} = 1.$

$\Rightarrow f'(2^+) = \lim\limits_{x\to2^-} \dfrac{f(x)-f(2)}{x-2}$

$\Rightarrow f'(2^+) = \lim\limits_{h\to0} \dfrac{f(2+h)-f(2)}{2+h-2}$

$\Rightarrow f'(2^-) = \lim\limits_{h\to0} \dfrac{5-(2-h)-3}{2+h-2}$

$= \lim\limits_{h\to0} \dfrac{h}{-h}$

=-1

As, f'(2⁻) is not equal to f(2⁺)

∴ f(x) is not differentiable.

Question: 58

Mark (√) against

Solution:

For continuity left hand limit must be equal to right hand limit and value at the point.

Continuous at x =2.

L.H.L $= \lim_{x \to 2^-}(kx + 5)$

$\Rightarrow \lim_{h \to 0}(k(2 - h) + 5)$

$\Rightarrow$ k(2-0)+5 = 2k+5

R.H.L $= \lim_{x \to 2^+}(x + 1)$

$\Rightarrow \lim_{h \to 0}(2 + h + 1)$

$\Rightarrow$ 2+0+1

=3

As f(x) is continuous

$\therefore$ 2k+5 = 3

K = -1.

Question: 59

Mark (√) against

Solution:

Given:

$\Rightarrow$ f(x) $= \frac{1-\cos 4x}{8x^2}$ is continuous at x = 0.

$\Rightarrow$ 1-cos4x = 2sin²2x

$\Rightarrow$ f(x) $= \lim_{x \to 0} \frac{2 \sin^2 2x}{8x^2}$

$\Rightarrow$ f(x) $= \lim_{x \to 0} \frac{2 \sin^2 2x}{2 \times 4x^2}$

$\Rightarrow$ f(x) $= \lim_{x \to 0} \left(\frac{\sin 2x}{2x}\right)^2$

$\Rightarrow$ f(x) = 1

$\therefore$ Kk= 1

Question: 60

Mark (√) against

Solution:

F(x) is continuous at x = 0.

$\Rightarrow$ f(x) $= \lim_{x \to 0} \frac{\sin^2 ax}{x^2}$

$\Rightarrow$ f(x) $= \lim_{x \to 0} \frac{\sin^2 ax}{x^2} \times \frac{a^2}{a^2}$

$\Rightarrow$ f(x) $= \lim_{x \to 0} \left(\frac{\sin ax}{ax}\right)^2 \times a^2$

$\Rightarrow$ f(x) = a²

$\therefore \text{k} = \text{a}^2$

Question: 61

Mark (√) against

Solution:

Given: f(x) is continuous at x= π/2.

$\therefore \text{L. H. L} = \lim\limits_{x \to \frac{\pi}{2}} f(x)$

$= \lim\limits_{x \to \frac{\pi}{2}} \dfrac{k\cos x}{\pi - 2x}$

Putting $x = \dfrac{\pi}{2} - h$;

As $x \to \dfrac{\pi^-}{2}$ then $h \to 0$.

$\therefore \lim\limits_{x \to \frac{\pi}{2}} \dfrac{k\cos\left(\frac{\pi}{2} - h\right)}{\pi - 2\left(\frac{\pi}{2} - h\right)} = k.\lim\limits_{h \to 0} \dfrac{\sin h}{h}$

$\therefore \text{L.H.L} = k$

As it is continuous which implies right hand limit equals left hand limit equals the value at that point.
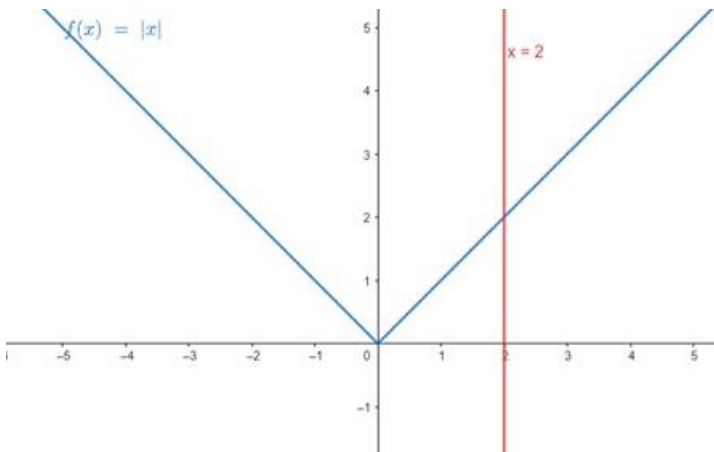
$\therefore \text{k} = 3.$

Question: 62

Mark (√) against

Solution:

Given:

Let us see that graph of the modulus function.

We can see that f(x) = |x| is neither continuous and nor differentiable at x = 2. Hence, D is the correct answer.

Question: 63

Mark (√) against

Solution:

$\Rightarrow f(x) = \dfrac{x^2 - 2x - 3}{x+1}$ is continuous at x = 0.

$\Rightarrow f(x) = \lim\limits_{x \to -1} \dfrac{(x+1)(x-3)}{x+1}$

$\Rightarrow f(x) = \lim_{x \to -1} x - 3$

$\Rightarrow f(x) = -4$

$\therefore K = 1.$

Question: 64

Mark (√) against

Solution:

Given:

$f(x) = x^3+6x^2+15x-12.$

$f'(x) = 3x^2+12x+15$

$f'(x) = 3x^2+12x+12+3$

$f'(x) = 3(x^2+4x+4)+3$

$f'(x) = 3(x+2)^2+3$

As square is a positive number

$\therefore f'(x)$ will be always positive for every real number

Hence $f'(x) > 0$ for all x ϵ R

$\therefore f(x)$ is strictly increasing.

Question: 65

Mark (√) against

Solution:

$f(x) = -x^3+3x^2-3x+4.$

$f'(x) = -3x^2+6x-3$

$f'(x) = -3(x^2-2x+1)$

$f'(x) = -3(x-1)^2$

As $f'(x)$ has –ve sign before 3

$\Rightarrow f'(x)$ is decreasing over R.

Question: 66

Mark (√) against

Solution:

Given:

$f(x) = 3x+\cos 3x$

$f'(x) = 3-3\sin 3x$

$f'(x) = 3(1-\sin 3x)$

$\sin 3x$ varies from[-1,1]

when $\sin 3x$ is 1 $f'(x) = 0$ and $\sin 3x$ is -1 $f'(x) = 6$

As the function is increasing in 0 to 6.

$\therefore$ The function is increasing on R.

Question: 67

Mark (√) against

Solution:

Given:

$f(x) = x^3+6x^2+9x+3$.

$f'(x) = 3x^2+12x+9 = 0$

$f'(x) = 3(x^2+4x+3) =0$

$f'(x) = 3(x+1)(x+3) = 0$

x = -1 or x =-3

for x>-1 f(x) is increasing

for x<-3 f(x) is increasing

But for -1<x<-3 it is decreasing.

Question: 68

Mark (√) against

Solution:

Given:

$f(x) = x^3-27x+8$.

$f'(x) = 3x^2-27x = 0$

$f'(x) = 3(x^2-9) =0$

$f'(x) = 3(x-3)(x+3) = 0$

x = 3 or x =-3

for x>3 f(x) is increasing

for x<-3 f(x) is increasing

∴ for |x|>3 f(x) is increasing.

Question: 69

Mark (√) against

Solution:

Given: f(x) is sin x

∴ f'(x) = cos x

⇒ f'(x) =cos x

= 0

⇒ for $x \epsilon \left(\dfrac{-\pi}{2},\dfrac{\pi}{2}\right)$

f'(x) is increasing

∴ f(x) is increasing in $\left(\dfrac{-\pi}{2},\dfrac{\pi}{2}\right)$.

Question: 70

Mark (√) against

Solution:

⇒ $f(x) = \dfrac{2x}{\log x}$

$\Rightarrow f'(x) = \dfrac{2.\log x - 2}{\log^2 x}$

Put f'(x) = 0

We get

$\Rightarrow \dfrac{2.\log x - 2}{\log^2 x} = 0$

$\Rightarrow 2.\log x = 2$

log x = 1

$\Rightarrow$ x = e

We only have one critical point

So, we can directly say x>e f(x) would be increasing

∴ f(x) will be increasing in (e, ∞)

Question: 71

Mark (√) against

Solution:

Given:

f(x) = sin x – cos x

f'(x) = cos x + sin x

Multiply and divide by √2.

$\Rightarrow \sqrt{2}\left(\dfrac{1}{\sqrt{2}}\cos x + \dfrac{1}{\sqrt{2}}\sin x\right)$

$\Rightarrow \sqrt{2}\left(\sin\dfrac{\pi}{4}.\cos x + \cos\dfrac{\pi}{4}.\sin x\right)$

$\Rightarrow \sqrt{2}(\sin\left(\dfrac{\pi}{4}+x\right))$

$\Rightarrow f'(x) = \sqrt{2}\sin\left(\dfrac{\pi}{4}+x\right)$

For f(x) to be decreasing .f'(x)<0

$\Rightarrow f'(x) = \sqrt{2}\sin\left(\dfrac{\pi}{4}+x\right) < 0$

$\Rightarrow \pi < x + \dfrac{\pi}{4} < 2\pi$

(∵ sin θ <0 for π <θ <2π)

$\Rightarrow \pi - \dfrac{\pi}{4} < x < 2\pi - \dfrac{\pi}{4}$

$\Rightarrow \dfrac{3\pi}{4} < x < \dfrac{7\pi}{4}$

∴ f(x) decreases in the interval.

$\Rightarrow \left(\dfrac{3\pi}{4}, \dfrac{7\pi}{4}\right)$

Question: 72

Mark (√) against

Solution:

$$\Rightarrow f(x) = \frac{x}{\sin x}$$

$$\Rightarrow f'(x) = \frac{\sin x - x\cos x}{\sin x}$$

Now see

In (0,1) sin x is increasing and cos x is decreasing

sin x – x cos x will be increasing

∴ f(x) is increasing in (0,1)

Question: 73

Mark (√) against

Solution:

Given: f(x) = x^x.

$\Rightarrow$ f'(x)=(log x+1) x^x

$\Rightarrow$ keeping f'(x) = 0

We get

$\Rightarrow x = 0 \text{ or } x = \frac{1}{e}$

Now

When x>1/e the function is increasing

x<0 function is increasing.

But in the interval (0,1/e) the function is decreasing.

Question: 74

Mark (√) against

Solution:

Given f(x) = $x^2.e^{-x}$

$\Rightarrow$ f'(x) = 2x. e^{-x} – x^2 e^{-x}

$\Rightarrow$ Put f'(x) = 0

$\Rightarrow$ - (x^2– 2x)e^{-x} = 0

$\Rightarrow$ x = 0 or x =2.

Now as there is a -ve sign before f'(x)

When x>2 the function is decreasing

x<0 function is decreasing

But in the interval (0,2) the function is increasing.

Question: 75

Mark (√) against

Solution:

f(x) =sin x - kx

f'(x) = cos x –k

∴ f decreases, if f'(x)≤ 0

$\Rightarrow$ cos x – k ≤ 0

⇒ cos x≤ k

So, for decreasing k≥ 1.

Question: 76

Mark (√) against

Solution:

Given:

⇒ $f(x) = (x+1)^3.(x-3)^3$

⇒ $f'(x) = 3(x+1)^2(x-3)^3 + 3(x-3)^3 (x+1)^3$

Put f'(x) = 0

⇒ $3(x+1)^2(x-3)^3 = -3(x-3)^2(x+1)^3$

⇒ x-3 = -(x+1)

⇒ 2x = 2

⇒ x =1

When x>1 the function is increasing.

x<1 function is decreasing.

So, f(x) is increasing in (1, ∞).

Question: 77

Mark (√) against

Solution:

⇒ $f(x)=[x(x-3)]^2$

⇒ $f'(x)=2[x(x-3)] =0$

⇒ $x = 3 \text{ and } x = \dfrac{3}{2}$

When x> 3/2 the function is increasing

X<3 function is increasing.

⇒ $\left(0,\dfrac{3}{2}\right) \cup (3,\infty)$ Function is increasing.

Question: 78

Mark (√) against

Solution:

Given $f(x) = kx^3 -9x^2+ 9x+3$

⇒ $f'(x) = 3kx^2-18x+9$

⇒ $f'(x) = 3(kx^2 – 6x + 3)>0$

⇒ $kx^2 – 6x + 3 >0$

For quadratic equation to be greater than 0. a>0 and D<0.

⇒ k>0 and $(-6)^2- 4(k)(3)<0$

⇒ 36 – 12k<0

⇒ 12k>36

⇒ k>3

∴ k>3.

Question: 79

Mark (√) against

Solution:

$$\Rightarrow f(x) = \frac{x}{x^2 + 1}$$

$$\Rightarrow f'(x) = \frac{x^2 - 2x^2 + 1}{x^2 + 1}$$

$$\Rightarrow f'(x) = -\frac{x^2 - 1}{x^2 + 1}$$

⇒ For critical points f'(x) = 0

When f'(x) = 0

We get x = 1 or x = -1

When we plot them on number line as f'(x) is multiplied by –ve sign we get

For x>1 function is decreasing

For x<-1 function is decreasing

But between -1 to 1 function is increasing.

∴ Function is increasing in(-1,1).

Question: 80

Mark (√) against

Solution:

f(x) =x^2+kx+1

For increasing

f'(x) = 2x+k

k≥ -2x

thus,

k≥ -2.

Least value of -2.

Question: 81

Mark (√) against

Solution:

f(x) = |x|

Now to check the maxima and minima at x =0.

It can be easily seen through the option.

See |x| is x for x>0 and –x for x<0

That is no matter if you put a number greater than zero or number less than zero you will get positive answer.

∴ for x = 0 we will get minima.

Question: 82

Mark (√) against

Solution:

Given: $f(x) = x^x$.

$\Rightarrow f'(x) = (\log x + 1) \, x^x$

$\Rightarrow$ keeping $f'(x) = 0$

We get

$\Rightarrow x = 0 \text{ or } x = \dfrac{1}{e}$

$\Rightarrow f''(x) = x^x(1 + \log x)\left[1 + \log x + \dfrac{1}{x(1 + \log x)}\right]$

When x is greater than zero,

We get a maximum value as the function will be negative.

Therefore,

$F(x) = x^x$

$F(e) = \left(\dfrac{1}{e}\right)^{1/e} = e^{-\frac{1}{e}}$

Hence, C is the correct answer.

Question: 83

Mark (√) against

Solution:

$\Rightarrow f(x) = \dfrac{\log x}{x}$

$\therefore f'(x) = \dfrac{\log x - x \cdot \frac{1}{x}}{x^2}$

$\Rightarrow f'(x) = \log x - 1$

$\Rightarrow$ Put $f'(x) = 0$

We get $x = e$

$F''(x) = 1/x$

Put $x = e$ in $f''(X)$

$1/e$ is point of maxima

$\therefore$ The max value is $1/e$.

Question: 84

Mark (√) against

Solution:

We can go through options for this question

Option a is wrong because 0 is not included in (-π,0)

At $x = -π/4$ value of $f(x)$ is $-\sqrt{2} = -1.41$

At $x = -π/3$ value of $f(x)$ is -2.

At $x = -π/2$ value of $f(x)$ is -1.

$\therefore f(x)$ has max value at $x = -π/2$.

Which is -1.

Question: 85

Mark (√) against

Solution:

Given: x>0 and xy = 1

We need to find the minimum value of (x + y).

$\Rightarrow y = \dfrac{1}{x}$

$\Rightarrow f(x) = x + \dfrac{1}{x}$

$\Rightarrow f(x) = \dfrac{x^2 + 1}{x}$

$\Rightarrow f'(x) = \dfrac{x.2x - (x^2 + 1).1}{x^2}$

$\Rightarrow f'(x) = \dfrac{x^2 - 1}{x^2}$

$\Rightarrow f''(x) = \dfrac{x^2(2x) - (x^2 - 1).2x}{x^4}$

$\Rightarrow f''(x) = \dfrac{2x}{x^4}$

$\Rightarrow f''(x) = \dfrac{2}{x^3}$

For maximum or minimum value f'(x) = 0.

$\therefore \dfrac{x^2 - 1}{x^2} = 0$

$\therefore$ x = 1 or x =-1

f''(x) at x = 1.

$\therefore$ f''(x) = 2.

F''(x)>0 it is decreasing and has minimum value at x = 1

At x = -1

f''(x) = -2

f''(x)<0 it is increasing and has maximum value at x = -1.

$\therefore$ Substituting x = 1 in f(x) we get

f(x) = 2.

$\therefore$ The minimum value of given function is 2.

Question: 86

Mark (√) against

Solution:

$\Rightarrow f(x) = x^2 + \dfrac{250}{x}$

$\Rightarrow f'(x) = 2x - \dfrac{250}{x^2} = 0$

$\Rightarrow 2x^3 = 250$

$\Rightarrow x^3 = 125$

⇒ x = 5

Substituting x = 5 in f(x) we get

f(x) = 25+50

f(x) = 75.

Question: 87

Mark (√) against

Solution:

Given:

f(x) = $3x^4$-$8x^3$-48x+25.

F'(x) = $12x^3$-$24x^2$-48 = 0

F'(x) = $12(x^3$-$2x^2$-4) = 0

Differentiating again, we get,

F''(x) = $3x^2$ – 4x = 0

x(3x – 4) = 0

x = 0 or x = 4/3

Putting the value in equation, we get,

f(x) = -39

Hence, C is the correct answer.

Question: 88

Mark (√) against

Solution:

f(x) = $(x-2)(x-3)^2$

f(x) = $(x-2)(x^2$-6x+9)

f(x) = x^3-$8x^2$+21x-18.

f'(x) = $3x^2$-16x+21

f''(x) = 6x-16

For maximum or minimum value f'(x) = 0.

∴ $3x^2$-9x-7x+21 = 0

⇒ 3x(x-3)-7(x-3)=0

⇒ x = 3 or x =7/3.

f''(x) at x = 3.

∴ f''(x) = 2

f''(x)>0 it is decreasing and has minimum value at x = 3

At x = 7/3

F''(x) = -2

F''(x)<0 it is increasing and has maximum value at x = 7/3.

Substituting x = 7/3 in f(x) we get

$$\Rightarrow \left(\frac{7}{3}-2\right)\left(\frac{7}{3}-3\right)^2$$

$$\Rightarrow \left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)^2$$

$$\Rightarrow \frac{4}{27}$$

Question: 89

Mark (√) against

Solution:

f(x) = e^x + e^{-x}

$$\Rightarrow f(x) = e^x + \frac{1}{e^x}$$

$$\Rightarrow f(x) = \frac{e^{2x} + 1}{e^x}$$

f(x) is always increasing at x = 0it has the least value

$$\Rightarrow f(x) = \frac{1+1}{1} = 2$$

∴ The least value is 2.