

35. Magnetic Field due to a Current

Short Answer

1. Question

An electric current flows in a wire from north to south. What will be the direction of the magnetic field due to this wire at a point

- (a) east of the wire,
- (b) west of the wire,
- (c) vertically above the wire and
- (d) vertically below the wire?

Answer

An electric current flows in a wire from north to south. So the direction of the magnetic field associated with it can be predicted by Maxwell's Right hand Cork-Screw rule.

So we can conclude that,

- (a) The magnetic field to the east of the wire will be directed vertically upwards.
- (b) The magnetic field to the west of the wire will be directed vertically downwards.
- (c) The magnetic field vertically above the wire will be directed towards the west.
- (d) The magnetic field vertically below the wire will be directed towards the east.

2. Question

The magnetic field due to a long straight wire has been derived in terms of μ_0 , i and d . Express this in terms of ϵ_0 , c , i , and d .

Answer

Magnetic field at a point d distance apart from a long straight current carrying wire is given by $B = \frac{\mu_0 i}{2\pi d}$ from Ampere's circuital law. Where μ_0 is the magnetic permeability of free space, i denotes the current through the wire and d denotes the distance from the wire, of the point where magnetic field is to be calculated.

Now we know that, speed of light, $c = \sqrt{\mu_0 \epsilon_0}$ (ϵ_0 denotes the electric permittivity of free space)

$$c^2 = \mu_0 \epsilon_0$$

$$\mu_0 = \frac{c^2}{\epsilon_0}$$

Thus we get magnetic field to be $B = \frac{c^2 i}{2\pi\epsilon_0 d}$

3. Question

You are facing a circular wire carrying an electric current. The current is clockwise as seen by you. Is the field at the centre coming towards you or going away from you?

Answer

We can predict the direction of the magnetic field by Maxwell's Right hand Cork-Screw rule. So we put the thumb of our right hands pointing towards the direction of current and now the direction the other fingers curl toward will be direction of the magnetic field.

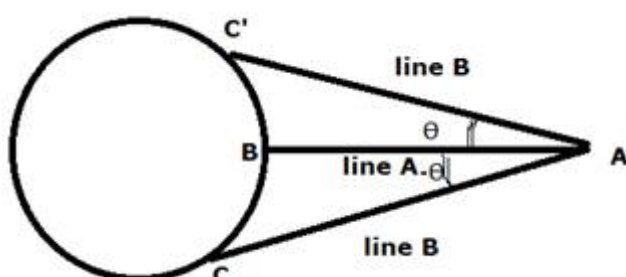
So, we get the field at the center is moving away from me if the current is going in the clockwise direction.

4. Question

In Ampere's law $\oint \vec{B} \cdot d\vec{t} = \mu_0 i$, the current outside the curve is not included on the right hand side. Does it mean that the magnetic field B calculated by using Ampere's law, gives the contribution of only the currents crossing the area bounded by the curve?

Answer

The contribution to the magnetic field B by currents outside the Amperian loop turns out to be zero because if we draw a straight line on any fixed point on the loop from the current vector (the direction vector of current), let line A and draw another line from the current vector to a movable point on the loop, let line B, and move that point in one direction along the loop until the full loop is traversed then we can see the net positive angle travelled by line B with respect to line A is equal to the net negative value travelled by the same. So the total angle travelled is zero. The line integral has a finite value for a current that is within the loop as it traverses an angle of 2π . So, we can say that Ampere's law takes into account every current be it inside the Amperian loop/curve or outside but as for outside currents, the integral is zero, so only the enclosed current is taken into account for calculation but actually takes into consideration the contribution of all the currents.



we consider the circle to be the Amperian loop. Point B is the fixed point C is the moving point. At some point of time C coincides with C' where $\angle BAC = -\angle BAC'$. We will get a point like this for every arbitrary point on the circle, so total angle for all the circle will be zero as stated before.

5. Question

The magnetic field inside a tightly wound, long solenoid is $B = \mu_0 ni$. It suggests that the field does not depend on the total length of the solenoid, and hence if we add more loops at the ends of a solenoid the field should not increase. Explain qualitatively why the extra-added loops do not have a considerable effect on the field inside the solenoid.

Answer

Magnetic field on the axis of a circular current carrying loop is given by,

$$B = \frac{\mu_0 i a^2}{2(a^2 + x^2)^{\frac{3}{2}}}$$

where a denotes the radius of the loop and x is the distance of the point on the axis at which the magnetic field is to be calculated. i is the current flowing in the loop. μ_0 is the magnetic permeability of free space.

And $B = \mu_0 ni$ (where μ_0 is the magnetic permeability of free space, n is the number of turns of the wire per unit length of the solenoid, and i is current in the wire) holds true for a very long solenoid where at the terminal points $x \gg a$. We can say that if we keep adding more loops at the ends, the total number of loops increase but with that length also increases so the turns per unit length n is unvaried. We can also think of it this way,

The value of B for loops added to the ends of the solenoid will have dropped to almost zero as for the condition $x \gg a$,

This can be shown mathematically by taking the limit of B for a circular loop. So, let us take x to be tending to infinity and a to have a finite value.

$$\begin{aligned} \lim_{x \rightarrow \infty} B &= \lim_{x \rightarrow \infty} \frac{\mu_0 i a^2}{2(a^2 + x^2)^{\frac{3}{2}}} \\ &= \lim_{x \rightarrow \infty} \frac{\mu_0 i \frac{a^2}{x^2}}{2(\frac{a^2}{x^2} + 1)^{\frac{3}{2}}} \text{ (Dividing both sides by } x^3) \\ &= 0 \end{aligned}$$

So if we add more loops at the ends of a very long solenoid, the magnetic field due to each one of them will be zero at the centre of the solenoid thus won't have considerable effect.

6. Question

A long, straight wire carries a current. Is Ampere's law valid for a loop that does not enclose the wire, or that encloses the wire but is not circular?

Answer

Ampere's law holds true for any closed loop of magnetic field containing constant electric current. The left hand side of Ampere's circuital law $\oint \vec{B} \cdot d\vec{l} = \mu_0 i$

Where

μ_0 is the magnetic permeability of free space,

i is the current enclosed by the loop and B is the magnetic field

The above equation deals with a closed line integral along the path of the magnetic field. The line integral will not have a finite value if the loop is not closed. So, Ampere's law will not hold for a loop that does not enclose the wire but it will hold for any shape of the loop even if it's not circular.

7. Question

A straight wire carrying an electric current is placed along the axis of a uniformly charged ring. Will there be a magnetic force on the wire if the ring starts rotating about the wire? If yes, in which direction?

Answer

No, there won't be any magnetic force acting on the wire in this case because, if the ring starts rotating, it will create a magnetic field which is parallel or antiparallel to the direction of current or the length vector of the wire at the center of the ring. i.e. angle between the is zero or π respectively. According to $F = ILB\sin\theta$

for $\theta = 0$ or π ,

$F = 0$.

Here I is the current, L is the length of the wire and B is the magnitude of the magnetic field.

So, there will be no force on the wire if the ring starts rotating about the wire.

8. Question

Two wires carrying equal currents i each, are placed perpendicular to each other, just avoiding a contact. If one wire is held fixed and the other is free to move under magnetic forces, what kind of motion will result?

Answer

Using the relation for force acting on a conductor

$$\vec{F} = I(\vec{L} \times \vec{B})$$

Where

I is the current in the wire,

L is the length vector of the wire

B is the external electric field.

Thus, if the wires are kept perpendicular, the angle between them will be 90° and thus the angle between the movable wire and the magnetic field due to the fixed wire is 0° (from Maxwell's Cork-Screw rule). Thus the force is the minimum and it is in an unstable equilibrium. Thus the wire will rotate and make itself parallel to the fixed wire and attain the maximum attractive force and thus a stable equilibrium.

9. Question

Two proton beams going in the same direction repel each other whereas two wires carrying currents in the same direction attract each other. Explain.

Answer

The magnetic field is not as strong as electric field and the magnitude of magnetic field due to a moving charge only equals that of its electric field only when the velocity is equal to the speed of light. So for two protons moving in the same direction, Force due to electric field repulsion will exceed that due to magnetic field attraction. So the proton beams moving in the same direction repel each other.

But two parallel current carrying wires attract due to magnetic attraction as the electric field is inside the conductor and is shielded by it and can't be felt outside. The magnetic attraction can be further reinforced by Fleming's left rule by considering the magnetic field produced by one wire and electric field in the other wire in the direction of current.

10. Question

In order to have a current in a long wire, it should be connected to a battery or some such device. Can we obtain the magnetic field due to a straight, long wire by using Ampere's law without mentioning this other part of the circuit?

Answer

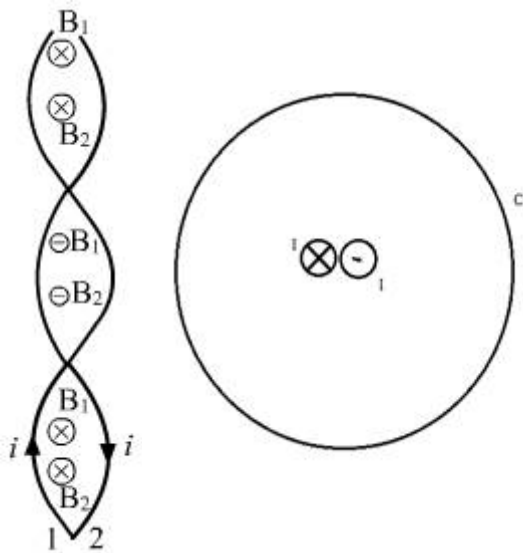
We can always find the magnetic field due to a long, straight current carrying wire along any loop which only contains that wire by using Ampere's circuital law. The law only takes into account the current bound in the loop irrespective of the source. If we take the whole circuit along with the battery within that Amperian loop, then the total current contained in that loop will be zero so the resultant magnetic field will be zero too even if the wire itself produces a non zero magnetic field.

11. Question

Quite often, connecting wires carrying currents in opposite directions are twisted together in using electrical appliances. Explain how it avoids unwanted magnetic fields.

Answer

If we choose an Amperian loop containing both the wires, then the magnetic field along that loop will turn out to be zero as the wires carry currents in the opposite direction and the total current turns out to be lesser than the maximum of that of the two wires. And if the magnitudes of currents are the same then the total current in that loop turns out to be zero. If they are kept very close together, this will hold for even very small Amperian loops. Thus twisting two wires, which carry currents in the opposite directions, minimizes the magnetic field produced and can even cancel it out.



denotes current going perpendicularly inside the screen.



denotes current coming perpendicularly out of the screen.

If this both currents are equal in magnitude then the total current enclosed by the Amperian Loop C will be zero.

12. Question

Two current-carrying wires may attract each other. In absence of other forces, the wires will move towards each other increasing the kinetic energy. Does it contradict the fact that the magnetic force cannot do any work and hence cannot increase the kinetic energy?

Answer

The displacement of the wire due to magnetic forces happens in the direction of the distance vector between the two wires. The magnetic field due to the wires is perpendicular to the wire and the distance vector between the wires so work done by the field is zero as usual. As the wires move towards each other there is a change in magnetic field and according to Lenz law there will be an induced current in the wires which will be directed opposite to the current initially carried by them. As these electrons cannot come out of the wire, they hit the edges of the wire starts moving and gains kinetic energy.

Objective I

1. Question

A vertical wire carries a current in upward direction. An electron beam sent horizontally towards the wire will be deflected.

- A. towards right
- B. towards left
- C. upwards
- D. downwards

Answer

With the right hand cork screw we can get the direction of the magnetic field due to the vertical current carrying wire. And we find it to be directed downwards. Now applying Fleming's Left hand rule, we can find the direction of force as we put the direction of the electron beam as the opposite direction of current. We can find that force is pointed upwards. So the electron beam will be deflected upwards.

2. Question

A current carrying, straight wire is kept along the axis of a circular loop carrying a current. The straight wire.

- A. will exert an inward force on the circular loop
- B. will exert an outward force on the circular loop
- C. will not exert any force on the circular loop.
- D. will exert a force on the circular loop parallel to itself.

Answer

There won't be any magnetic force acting on the wire in this case because, according to the Cork-Screw rule, the magnetic field due to the straight wire will be parallel or anti parallel to the direction of current in the loop, thus creating an angle of 0 or π with the direction of current.

According to $F = ILB\sin\theta$

Where

I is the current,

L is the length of the wire

B is the magnitude of the magnetic field.

for $\theta = 0$ or π , $F = 0$.

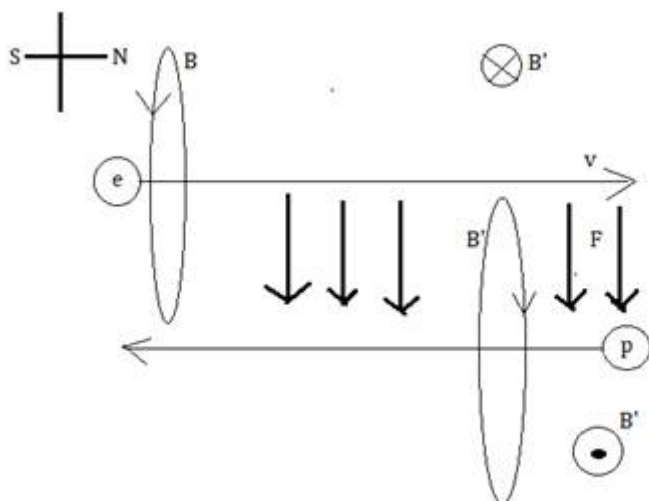
3. Question

A proton beam is going from north to south and an electron beam is going from south to north. Neglecting the earth's magnetic field, the electron beam will be deflected.

- A. towards the proton beam
- B. away from the proton beam
- C. upwards
- D. downwards

Answer

We can consider the beams to be current carrying wires. The proton beam is going from north to south so the current for it will be directed from north to south. And the electron beam is moving from South to north so the current will be directed opposite to the direction of the beam, i.e. from north to south. So we get the case of two current carrying wires carrying current in the same direction and they attract each other. So the electron beam will be deflected towards the proton beam.



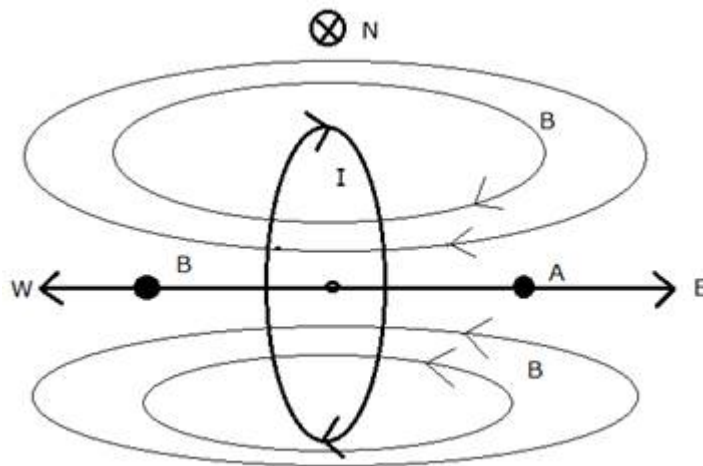
4. Question

A circular loop is kept in that vertical plane which contains the north-south direction. It carries a current that is towards north at the topmost point. Let A be a point on the axis of the circle to the east of it and B a point on this axis to the west of it. The magnetic field due to the loop

- A. is towards east at A and towards west at B
- B. is towards west at A and towards east at B
- C. is towards east at both A and B
- D. is towards west at both A and B.

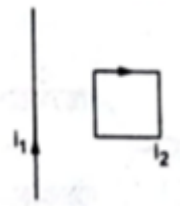
Answer

If the current is directed towards north at the topmost point of that circular loop that means, from point A's perspective, current is in clockwise direction and will cause a magnetic field which is moving inwards i.e. to the west at point A and subsequently to the west at point B.



5. Question

Consider the situation shown in figure. The straight wire is fixed but the loop can move under magnetic force. The loop will



- A. remain stationary
- B. move towards the wire
- C. move away from the wire
- D. rotate about the wire

Answer

The closest side of the loop to the straight wire carries a current in the same direction as the straight wire. So there will be an attractive force. And for the opposite side of the loop the current is moving in the opposite direction to that of the straight wire, so there will be a repulsive force. As the closest side shows attractive force and the farthest side shows repulsive force, the attractive force will be dominant. To explain it mathematically, we introduce the expression for force per unit length on each of the wires.

$$F = \frac{\mu_0 I_1 I_2}{2\pi d}$$

Where

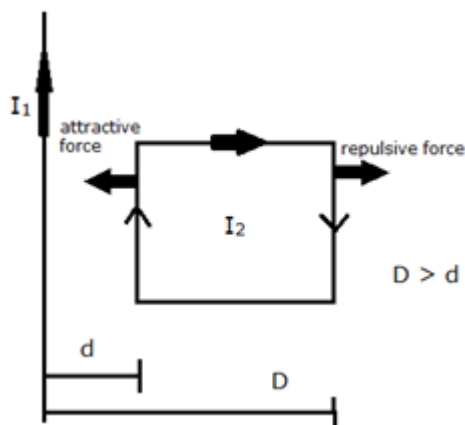
F denote the force per unit length,

I_1, I_2 denote the currents in the two wires,

d is the distance between them

μ_0 is the magnetic permeability of free space.

The current in the loop is of same magnitude everywhere. The parallel sides of the loop to the long straight wire will encounter all the force. As the side closer to the straight wire carries a current in the same direction, $I_1 I_2$ will be positive thus it will encounter attractive force and the opposite side will encounter repulsive force as the, $I_1 I_2$ is negative as the currents are in opposite directions. We can see that the force is inversely proportional to the distance. As the wire carrying current in the same direction is closer than the wire carrying current in the opposite direction, the attractive force will be greater than the repulsive force. The other two sides will experience forces of equal amplitude but opposite directions and thus will cancel each other out. Thus the loop will move towards the wire.



6. Question

A charged particle is moved along a magnetic field line. The magnetic force on the particle is

- A. along its velocity
- B. opposite to its velocity
- C. perpendicular to its velocity
- D. zero

Answer

If a charged particle is moving along a magnetic field line, that means it is moving either parallel or anti parallel to the direction of magnetic field. i.e. the direction of velocity of the particle is either making an angle of 0 or π . According to $F = qvB\sin\theta$ (where q is the charge of the particle, v is the velocity of it and B is the magnitude of the external Magnetic field and θ is the angle between the direction of magnetic field and the velocity) for $\theta = 0$ or π , $F = 0$.

7. Question

A moving charge produces

- A. electric field only
- B. magnetic field only
- C. both of them
- D. none of them

Answer

Under any circumstances, a charge particle will produce an electric field. And if we imagine that the moving charge particle is analogous to an electric current then it is clear to us that it, most certainly, will produce a magnetic field as well.

8. Question

A particle is projected in a plane perpendicular to a uniform magnetic field. The area bounded by the path described by the particle is proportional to

- A. the velocity
- B. the momentum
- C. the kinetic energy
- D. none of these

Answer

If a charged particle is projected with velocity v on a plane perpendicular to a uniform magnetic field B , then the particle will travel in a circle. Whose radius is $R = \frac{mv}{qB}$ where m and q are the mass and the charge of the particle respectively.

So the area enclosed by the path will be $A = \pi R^2 = \pi \frac{m^2 v^2}{q^2 B^2} = \frac{2\pi m}{q^2 B^2} \times \frac{1}{2} mv^2 = \frac{2\pi m}{q^2 B^2} \times \text{kinetic energy}$

Thus, the area will be proportional to the kinetic energy of the particle.

9. Question

Two particles X and Y having equal charge, after being accelerated through the same potential difference enter a region of uniform magnetic field and describe circular paths of radii R_1 and R_2 respectively. The ratio of the mass of X to that of Y is

- A. $(R_1/R_2)^{1/2}$
- B. R_1/R_2
- C. $(R_1/R_2)^2$

D. $R_1 R_2$

Answer

let the charge of both of them be q and mass be m_x and m_y respectively. If they are driven through the same potential V then, $\frac{1}{2}mv^2 = qV$

$$v = \sqrt{\frac{2qV}{m}}$$

Thus, v_x and v_y (their respective velocities after leaving the potential) will be

$$v_x = \sqrt{\frac{2qV}{m_x}} \text{ and } v_y = \sqrt{\frac{2qV}{m_y}}$$

So from $R = \frac{mv}{qB}$ we get, $R \propto \sqrt{m}$ (all the other terms are constant for both the particles)

So, according to the question $\frac{R_1}{R_2} = \sqrt{\frac{m_x}{m_y}}$

$$\text{Thus, } \frac{m_x}{m_y} = \left(\frac{R_1}{R_2}\right)^2$$

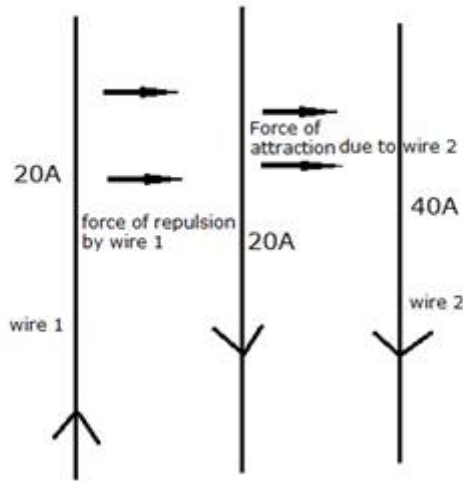
10. Question

Two parallel wires carry currents of 20 A and 40 A in opposite directions. Another wire carrying a current antiparallel to 20 A is placed midway between the two wires. The magnetic force on it will be

- A. towards 20 A
- B. towards 40 A
- C. zero
- D. perpendicular to the plane of the currents.

Answer

The third wire is carrying a current 20A antiparallel to the previously set 20A current wire. So these wires will repel each other. As the 40A wire is carrying a current in the opposite direction to the previously set 20A wire, the newly set 20A wire will be having the same current direction to it. So they will attract. So the newly set wire in the midway will move towards the 40A wire.



11. Question

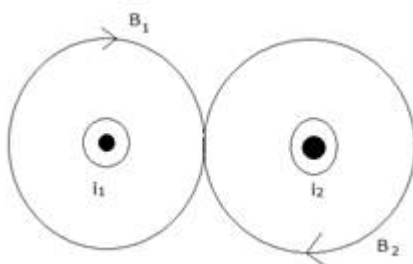
Two parallel, long wires carry currents i_1 and i_2 with $i_1 > i_2$. When the currents are in the same direction, the magnetic field at a point midway between the wires is $10 \mu\text{T}$. If the direction of i_2 is reversed, the field becomes $30 \mu\text{T}$. The ratio i_1/i_2 is

- A. 4
- B. 3
- C. 2
- D. 1

Answer

The magnetic field caused by a long wire is given by $B = \frac{\mu_0 i}{2\pi d}$ where i is the current in the wire, d is the distance at which B is to be calculated, μ_0 is the magnetic permeability of free space. Let the distance between the two wires be $2d$.

Then, when the currents are parallel, one's magnetic field will negate the other's because at the point midway between the two wires, the magnetic fields generated by them will be directed in the opposite directions and it can be verified by the right hand Cork-Screw rule.



So the total B will be

$B = \frac{\mu_0 i_1}{2\pi d} - \frac{\mu_0 i_2}{2\pi d}$ which is given to be $10\mu\text{T}$ and when the direction of i_2 is reversed

then, $B = \frac{\mu_0 i_1}{2\pi d} + \frac{\mu_0 i_2}{2\pi d}$ is given to be $30\mu\text{T}$. So, solving these, we get, $\frac{\frac{\mu_0 i_1}{2\pi d} + \frac{\mu_0 i_2}{2\pi d}}{\frac{\mu_0 i_1}{2\pi d} - \frac{\mu_0 i_2}{2\pi d}} = \frac{30}{10}$

$$\frac{\frac{\mu_0 i_1}{\pi d}}{\frac{\mu_0 i_2}{\pi d}} = \frac{40}{20} \text{ (Componendo- Dividendo rule)}$$

$$\frac{i_1}{i_2} = 2$$

12. Question

Consider a long, straight wire of cross-sectional area A carrying a current i . Let there be n free electrons per unit volume. An observer places himself on a trolley

moving in the direction opposite to the current with a speed $v = \frac{i}{nAe}$ and

separated from the wire by a distance r . The magnetic field seen by the observer is very nearly.

A. $\frac{\mu_0 i}{2\pi r}$

B. zero

C. $\frac{\mu_0 i}{\pi r}$

D. $\frac{2\mu_0 i}{\pi r}$

Answer

We know, drift velocity of electron is

$$v_d = i/(nAe)$$

Where

i is the current in the wire,

n is the electron number density of the wire a ,

A is the area of cross section of the wire,

e is the charge of the electron.

We can see that the trolley is moving with drift velocity but current travels in a conductor with a speed comparable to that of light in free space. So the velocity of the trolley is very low compared to the velocity at which the current travels. So there will be no significant change in the value of B and it will be $\frac{\mu_0 i}{2\pi r}$.

Objective II

1. Question

The magnetic field at the origin due to a current element $\overrightarrow{d\ell}$ placed at a position \vec{r} is

A. $\frac{\mu_0 i}{4\pi} \frac{\overrightarrow{d\ell} \times \vec{r}}{r^3}$

B. $-\frac{\mu_0 i}{4\pi} \frac{\vec{r} \times \overrightarrow{d\ell}}{r^3}$

C. $\frac{\mu_0 i}{4\pi} \frac{\vec{r} \times \overrightarrow{d\ell}}{r^3}$

D. $-\frac{\mu_0 i}{4\pi} \frac{\overrightarrow{d\ell} \times \vec{r}}{r^3}$

Answer

According to Biot-Savart law: The magnetic field due to a current element is given

as $\overrightarrow{dB} = \frac{\mu_0 i}{4\pi} \frac{\overrightarrow{dl} \times \vec{r}}{r^3}$ Where,

i is the current,

\overrightarrow{dl} is the length element vector,

\vec{r} is the vector joining the required point and the length element,

μ_0 is the permeability of vacuum and $\mu_0 = 4\pi \times 10^{-7} \text{ TmA}^{-1}$. Also, by vector

properties: $A \times B = -(B \times A)$ Thus, $\overrightarrow{dB} = -\frac{\mu_0 i}{4\pi} \frac{\vec{r} \times \overrightarrow{dl}}{r^3}$ Thus, options (A) and (B) are the correct options

2. Question

Consider three quantities $x = E/B$, $y = \sqrt{1/\mu_0 \epsilon_0}$ and $z = \frac{\ell}{CR}$. Here, ℓ is the

length of a wire, C is a capacitance and R is a resistance. All other symbols have standard meanings.

A. x, y have the same dimension.

B. y, z have the same dimensions

C. z, x have the same dimensions

D. None of the three pairs have the same dimensions.

Answer

(A) Dimension for electric field E is: $F/q = [MLT^{-3}A^{-1}]$ Force due to magnetic field is given as: $F = qvB$. Where q is the charge, v is the velocity of charge and B is the magnetic field. Hence $B = \frac{F}{qv}$

The dimensions of charge, q is [AT]

The dimension of the velocity, V is $[LT^{-1}]$

Putting the value in the above formula, we get

$$\text{Dimensions of } B = \left(\frac{[MLT^{-3}]}{[AT][LT^{-1}]} \right)$$

$$\therefore \text{Dimensions of } B = [MT^{-2}A^{-1}] \text{ Thus, } \frac{E}{B} = \frac{[MLT^{-3}A^{-1}]}{[MT^{-2}A^{-1}]} = [LT^{-1}]$$

$$(B) y = \sqrt{\frac{1}{\mu_0 \epsilon_0}} \text{ We know that } \frac{1}{\mu_0 \epsilon_0} = c^2 \therefore \sqrt{\frac{1}{\mu_0 \epsilon_0}} = c \text{ Where}$$

μ_0 is permeability of free space,

ϵ_0 is the permittivity of free space and

c is the speed of light. Thus, $y = c$, c is the speed and unit of speed is m/s **Dimensions of y = $[LT^{-1}]$** (C)

$$\text{Reactance, } Z = \frac{l}{CR} \text{ Where}$$

C is the capacitance

R is the resistance

l is the length of the wire.

In a CR circuit the product CR is the time constant and unit is seconds. Thus,

Dimensions of $z = l/CR = m/s = [LT^{-1}]$ Hence x, y and z have same dimensions. Thus, options (A), (B) and (C) are the correct options.

3. Question

A long, straight wire carries a current along the z-axis. One can find two points in the x-y plane such that

A. the magnetic fields are equal

B. the directions of the magnetic fields are the same

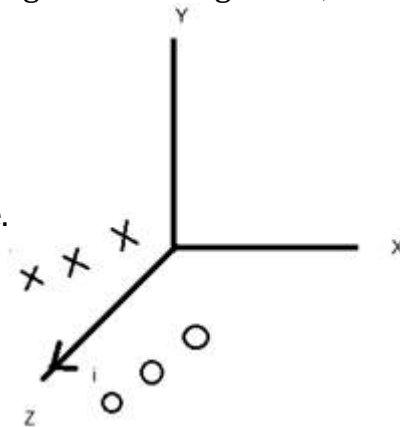
C. the magnitudes of the magnetic fields are equal

D. the field at one point is opposite to that at the other point.

Answer

When a current carrying wire is along z-axis, the magnetic field can be found using

right hand thumb rule.



Here X represents the field is

going in and O represents field is coming out. First case, if two points in the XY plane are above the wire and at same distance from z-axis, then both will lie in the X region (field going in), thus have same direction of magnetic field and magnitudes of magnetic field would be same. Second case, if the two points in XY plane are at same distance from z axis but one point is above the wire and one is below it, then the direction of the electric field would be opposite for both the points. One point having X direction (going in) and other having O direction (coming out). First two cases satisfy options (B), (C) and (D). Thus, options (B), (C) and (D) are the correct options.

4. Question

A long straight wire of radius R carries a current distributed uniformly over its cross section. The magnitude of the magnetic field is

- A. maximum at the axis of the wire.
- B. minimum at the axis of the wire
- C. maximum at the surface of the wire
- D. minimum at the surface of the wire.

Answer

According to Ampere's Law: $\oint B \cdot dl = \mu_0 i$ Where,

dl is the current element,

B is the magnetic field,

μ_0 is the permeability of free space and

i is the current flowing.

Thus, at the cross-section the formula becomes, $B \cdot 2\pi R = \mu_0 i$ $2\pi R$ is the circumference of the wire and R is the radius. We get, $B = \frac{\mu_0 i}{2\pi R}$ Now, at the axis of the wire. $R=0$, and so no area to integrate and hence zero current is enclosed. Thus magnitude of magnetic field is minimum at axis of the wire. At the surface of the wire, $R =$ some minimum value. As R increases, magnitude of magnetic field decreases. Hence, B will be maximum at the surface of the wire. Thus, options (B) and (C) are correct options.

5. Question

A hollow tube is carrying an electric current along its length distributed uniformly over its surface. The magnetic field.

- A. increases linearly from the axis to the surface
- B. is constant inside the tube
- C. is zero at the axis
- D. is zero just outside the tube

Answer

By Ampere's Law, $\oint B \cdot dl = \mu_0 i$ Here, dl is the current element, B is the magnetic field, μ_0 is the permeability of free space and i is the current flowing. We know that, magnetic field due to a circular wire is given $B = \frac{\mu_0 i}{2\pi R}$ R is the radius of the wire. Consider a Hollow tube as the series of circular wires. Inside the tube at a point, $r < R$ $\oint B_{inside} \cdot dl = \mu_0 i_{inside}$ Current inside the hollow tube is zero: $i_{inside} = 0$. $\therefore \oint B \cdot dl = 0 \therefore B_{inside} = \text{Constant}$ Hence, the magnetic field is constant inside the hollow tube. Now, at the axis, $R=0$.

Therefore, zero current is enclosed. Hence, $B=0$. Thus, options (B) and (C) are the correct options.

6. Question

In a coaxial, straight cable, the central conductor and the outer conductor carry equal currents in opposite directions. The magnetic field is zero

- A. outside the cable
- B. inside the inner conductor
- C. inside the outer conductor
- D. in between the two conductors

Answer

According to Ampere's Law: $\oint B \cdot dl = \mu_0 i$ Where,

B is the magnetic field,

dl is the current element,

μ_0 is the permeability of free space

i is the current flowing in the conductor

Magnetic field outside the cable: $B = B_1 + B_0$ B_1 and B_0 are the magnetic field of the inner conductor and outer cable respectively. Due to wire, $B = \frac{\mu_0 i}{2\pi R}$ R is the radius of the wire.

$B = \frac{\mu_0 i}{2\pi R} + \frac{\mu_0 (-i)}{2\pi R} \therefore B = \frac{\mu_0 i}{2\pi R} - \frac{\mu_0 i}{2\pi R} \therefore B = 0$ Thus, option (A) is the correct option.

Magnetic field inside the inner conductor:

$$\oint B \cdot dl = \mu_0 I \quad (I=0 \text{ (inside the wire)})$$

Magnetic field between the two conductors and inside the outer conductor.

$$\oint B \cdot dl = \mu_0 I$$

On integrating the length element over the complete loop, we get $dl = 2\pi r$

$$B \cdot 2\pi r = \mu_0 I$$

On solving the above equation for B, we get

$$B = \frac{\mu_0 I}{2\pi r}$$

Thus, option A and B is the correct option.

7. Question

A steady electric current is flowing through a cylindrical conductor.

- A. The electric field at the axis of the conductor is zero.
- B. The magnetic field at the axis of the conductor is zero
- C. The electric field in the vicinity of the conductor is zero
- D. The magnetic field in the vicinity of the conductor is zero.

Answer

Using Ampere's law, the electric field due to a current carrying conductor of cross section areas is: magnetic field, $B = \frac{\mu_0 i}{2\pi R}$ Where,

dl is the current element,

μ_0 is the permeability of free space,

i is the current flowing

R is the radius of the conductor. At the axis,

R=0 and so no area to enclose current.

Hence, B=0 at the axis of the conductor.

Electric field in the vicinity of the conductor is zero as a minimum distance between the point and the conductor is required to have electric field at that point. Thus, options (B) and (C) are correct options.

Exercises

1. Question

Using the formulae $\vec{F} = q\vec{v} \times \vec{B}$ and $B = \frac{\mu_0 i}{2\pi r}$, show that the S.I. units of the magnetic field B and the permeability constant μ_0 may be written as N mA^{-1} and NA^{-2} respectively.

Answer

Given: $\vec{F} = q\vec{v} \times \vec{B}$

$B = \frac{\mu_0 i}{2\pi r}$ **Formula used:** Lorentz force: $\vec{F} = q\vec{v} \times \vec{B}$ Where, \vec{F} is the magnetic force vector, q is the charge, \vec{v} is the velocity vector and \vec{B} is the magnetic field vector.

Magnetic field can be written as: $B = \frac{F}{qv}$ Unit of force is Newton: N Charge: q =

It Unit of current : I = Amperes = A Unit of time : t = seconds = s Unit of charge : q =

As Unit of velocity : v = m/s Thus, unit of Magnetic field: $B = \frac{N}{As \times \frac{m}{s}} \therefore B = \frac{N}{Am}$

Therefore, unit of Magnetic field : $B = \text{N m}^{-1}\text{A}^{-1}$. Now, Ampere's Law: $B = \frac{\mu_0 i}{2\pi r}$

Where,

μ_0 is the permeability of free space,

I is the current flowing

r is the radius of the current carrying wire. SI Unit of B = N mA^{-1} SI Unit of I = A SI

Unit of r = m $\therefore \mu_0 = \frac{B 2\pi r}{i} \therefore \mu_0 = \frac{N \text{ m}^{-1} \text{A}^{-1} \text{ m}}{A} \therefore \mu_0 = \text{N A}^{-2}$ Thus, unit of

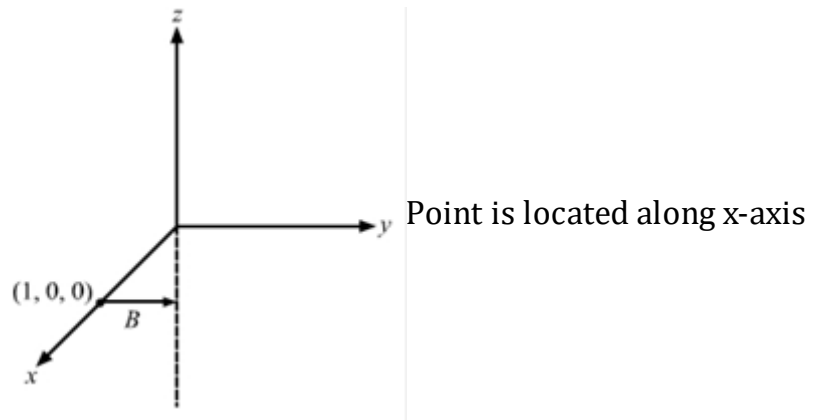
permeability constant μ_0 is N A^{-2} . Hence proved that unit of Magnetic field is $\text{N m}^{-1}\text{A}^{-1}$ and unit of permeability constant μ_0 is N A^{-2} .

2. Question

A current 10 A is established in a long wire along the positive z-axis. Find the magnetic field \vec{B} . At the point (1m, 0, 0).

Answer

Given: Current : $I = 10 \text{ A}$



at 1 m. **Formula used:** Using Ampere's Law for a current carrying straight wire:

$B = \frac{\mu_0 i}{2\pi d}$ Here, B is the magnitude of magnetic field, μ_0 is the permeability of free space and $\mu_0 = 4\pi \times 10^{-7} \text{ T mA}^{-1}$ and d is the distance between the current carrying wire and the required point. Substituting the values, $B = \frac{4\pi \times 10^{-7} \times 10}{2 \times \pi \times 1} \therefore B = 2 \times 10^{-6} \text{ T}$ Hence, magnetic field at a point (1m, 0, 0) is $2 \times 10^{-6} \text{ T}$.

3. Question

A copper wire of diameter 1.6 mm carries a current of 20 A. Find the maximum magnitude of the magnetic field \vec{B} due to this current.

Answer

Given: Diameter of the copper wire: $d = 1.6 \text{ mm} = 1.6 \times 10^{-3} \text{ m}$ Current through the wire: $I = 20 \text{ A}$ **Formula used:** By Ampere's law for current carrying wire of cross section area, $B = \frac{\mu_0 i}{2\pi r}$ Where,

B is the magnitude of magnetic field,

μ_0 is the permeability of free space and $\mu_0 = 4\pi \times 10^{-7} \text{ T mA}^{-1}$ r is the radius of the wire.

Radius of the wire : $r = d/2 = 8 \times 10^{-4} \text{ m}$ Substituting we get,

$B = \frac{4\pi \times 10^{-7} \times 20}{2 \times \pi \times 8 \times 10^{-4}} \therefore B = 5 \times 10^{-3} \text{ T}$ Hence, maximum magnitude of magnetic field due to a current of 20 A is $5 \times 10^{-3} \text{ T}$.

4. Question

A transmission wire carries a current of 100 A. What would be the magnetic field B at a point on the road if the wire is 8 m above the road?

Answer

Given: Current through the transmission wire: $I = 100 \text{ A}$
Distance between wire and the road : $d = 8 \text{ m}$

Formula used: By Ampere's Law for a current carrying wire is $B = \frac{\mu_0 i}{2\pi d}$ Where,

B is the magnitude of magnetic field,

μ_0 is the permeability of free space

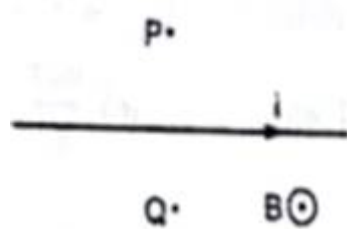
$$\mu_0 = 4\pi \times 10^{-7} \text{ T mA}^{-1}$$

d is the distance between the current carrying wire and the required

point. Substituting we get, $B = \frac{4\pi \times 10^{-7} \times 100}{2\pi \times 8}$ $B = 2.5 \times 10^{-6} \text{ T}$ Hence, magnitude of magnetic field at a point on the road due to current carrying transmission wire is $2.5 \times 10^{-6} \text{ T}$.

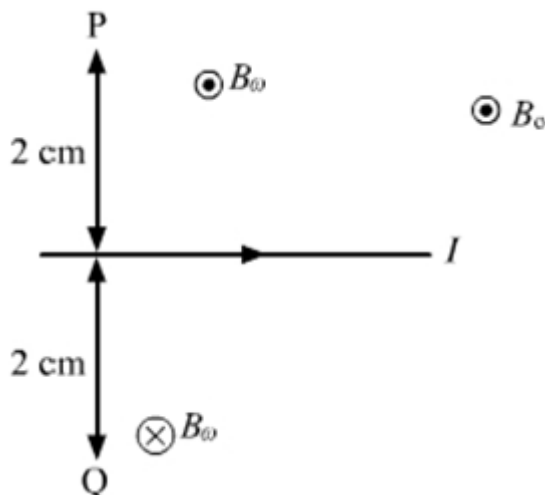
5. Question

A long, straight wire carrying a current of 1.0 A is placed horizontally in a uniform magnetic field $B = 1.0 \times 10^{-8} \text{ T}$ pointing vertically upward figure. Find the magnitude of the resultant magnetic field at the points P and Q , both situated at a distance of 2.0 cm from the wire in the same horizontal plane.



Answer

Given : Current in the wire : $i = 1.0 \text{ A}$
Magnitude of Horizontal magnetic field: $B_0 = 1.0 \times 10^{-5} \text{ T}$
Distance between points P, Q and the wire : $d = 2.0 \text{ cm} = 0.02 \text{ m}$



Right hand rule is used to determine the

direction of the magnetic field due to the current carrying wire. Cross determines the field going into the plane and dot determines the field coming out of the

field. B_w is the magnetic field due to wire. **Formula used:** By Ampere's Law for a current carrying wire is $B = \frac{\mu_0 i}{2\pi d}$ Where,

B is the magnitude of magnetic field,

μ_0 is the permeability of free space, $\mu_0 = 4\pi \times 10^{-7} \text{ T mA}^{-1}$

d is the distance between the current carrying wire and the required point.

Magnetic field at point P due to wire, $B_w = \frac{4\pi \times 10^{-7} \times 1}{2\pi \times 0.02} \therefore B_w = 1 \times 10^{-5} \text{ T}$

Resultant magnetic field at P $B_P = B_0 + B_w \therefore B_P = 1 \times 10^{-5} + 1 \times 10^{-5} \therefore B_P = 2 \times 10^{-5}$

Magnetic field at point Q due to wire, $B_w = - \frac{4\pi \times 10^{-7} \times 1}{2\pi \times 0.02} \therefore B_w = - 1 \times 10^{-5}$

The negative sign is because the field at Q is opposite to the direction of horizontal field and at P. Resultant magnetic field at P $B_Q = B_0 + B_w \therefore B_Q = 1 \times 10^{-5} - 1 \times 10^{-5} \therefore B_Q = 0$ Hence, resultant magnetic field at P is $2 \times 10^{-5} \text{ T}$ and at Q is 0.

6. Question

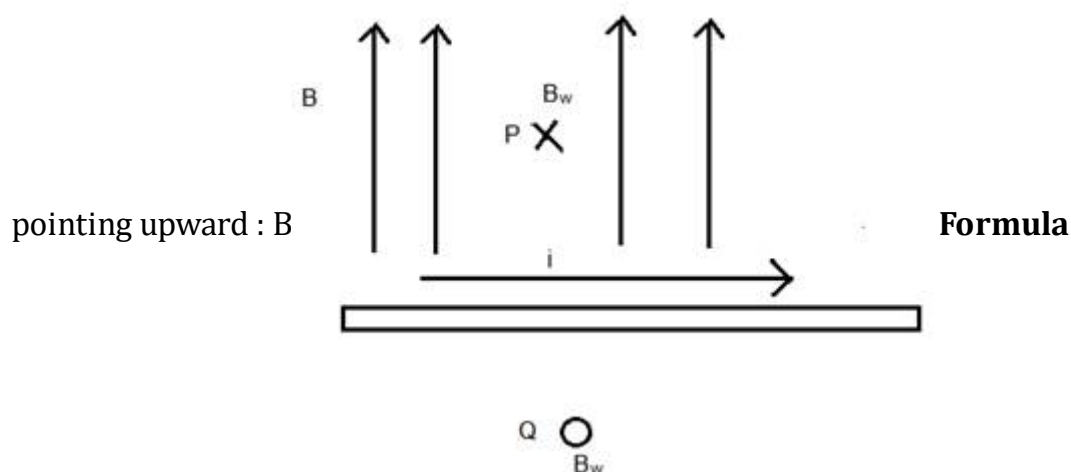
A long, straight wire of radius r carries a current i and is placed horizontally in a uniform magnetic field B pointing vertically upward. The current is uniformly distributed over its cross section.

(a) At what points will the resultant magnetic field have maximum magnitude?

(b) What will be the minimum magnitude of the resultant magnetic field?

Answer

Given: Radius of the wire : r Current through the wire : i Uniform magnetic field



used: Magnetic field due to a current carrying wire is

Magnitude of magnetic field due to wire, $B_w = \frac{\mu_0 i}{2\pi r}$

Where, μ_0 is the permeability of free space and $\mu_0 = 4\pi \times 10^{-7} \text{ T mA}^{-1}$.

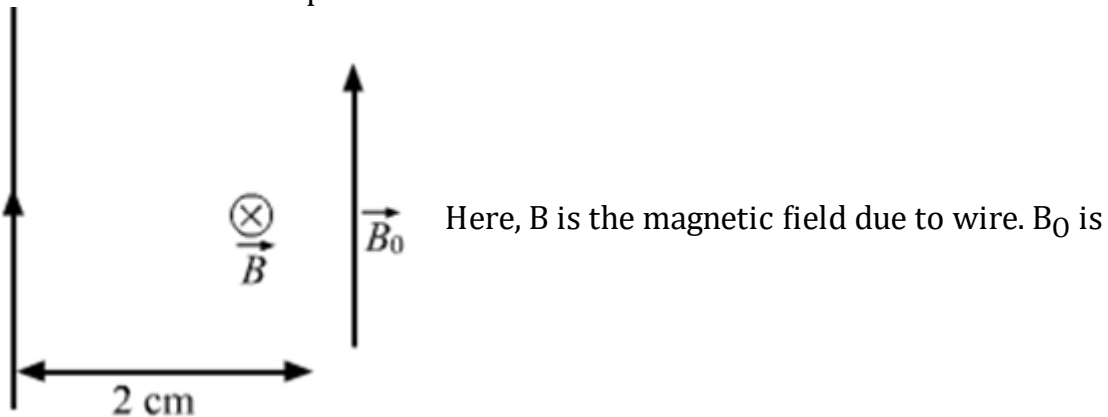
r is the radius of the wire. (a) Maximum net magnetic field would be above the wire at P. Considering direction of the current as shown in the diagram, right hand rule will give us direction of the magnetic field due to wire. thus, $B_{\text{net}} = B + B_w \therefore B_{\text{net}} = B + \frac{\mu_0 i}{2\pi r}$ (b) Similarly, the field at Q would be in opposite direction to the P. Minimum magnitude of the magnetic field will be at Q $B_{\text{net}} = B - B_w \therefore B_{\text{net}} = B - \frac{\mu_0 i}{2\pi r}$ Hence, magnitude of magnetic field will be maximum at points above at wire and minimum at points below the wire only if the direction of the current is as shown in the figure.

7. Question

A long, straight wire carrying a current of 30 A is placed in an external, uniform magnetic field of 4.0×10^{-4} T parallel to the current. Find the magnitude of the resultant magnetic field at a point 2.0 cm away from the wire.

Answer

Given: Current in the wire : $I = 30$ A External Uniform magnetic field: $B_0 = 4.0 \times 10^{-4}$ T Distance between the point and the wire : $d = 2 \text{ cm} = 0.02 \text{ m}$



parallel to the current in the wire. Using Right hand rule we can see that direction of magnetic field at the desired point is going into the plane (cross). This direction due to wire is perpendicular to the External magnetic field. $\vec{B} \perp \vec{B_0}$ **Formula**

used: By Ampere's Law for a current carrying wire is $B = \frac{\mu_0 i}{2\pi d}$ Here, B is the magnitude of magnetic field, μ_0 is the permeability of free space and $\mu_0 = 4\pi \times 10^{-7} \text{ T mA}^{-1}$ and d is the distance between the current carrying wire and the required point. Substituting we get, $B = \frac{4\pi \times 10^{-7} \times 30}{2\pi \times 0.02} \therefore B = 3 \times 10^{-4} \text{ T}$ As Magnetic field due to wire is perpendicular to the external magnetic field, the resultant net magnetic field would be: $B_{\text{net}} = \sqrt{B^2 + B_0^2}$
 $\therefore B_{\text{net}} = \sqrt{(3 \times 10^{-4})^2 + (4.0 \times 10^{-4})^2} \therefore B_{\text{net}} = 5 \times 10^{-4} \text{ T}$ Hence, the magnitude of the resultant magnetic field at a point 2.0 cm away from the wire is $5 \times 10^{-4} \text{ T}$.

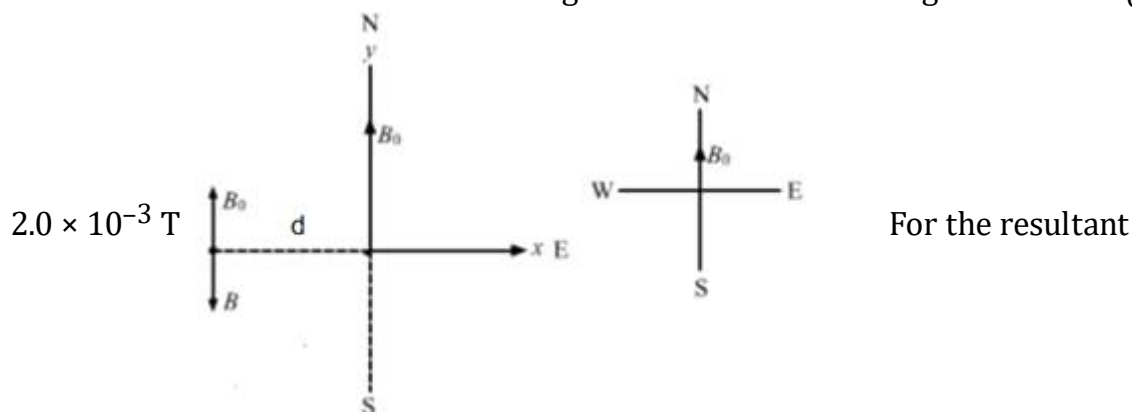
8. Question

A long, vertical wire carrying a current of 10 A in the upward direction is placed in a region where a horizontal magnetic field of magnitude 2.0×10^{-3} T exists from

south to north. Find the point where the resultant magnetic field is zero.

Answer

Given: Current in the wire : $I = 10 \text{ A}$ Magnitude of Horizontal magnetic field : $B_0 =$



magnetic field to be zero, we need to have a field in opposite direction to that of the existing field. Hence, magnetic field B due to wire in the direction from north to south as shown in the diagram. **Formula used:** By Ampere's Law for a current

carrying wire is $B = \frac{\mu_0 i}{2\pi d}$ Where,

B is the magnitude of magnetic field,

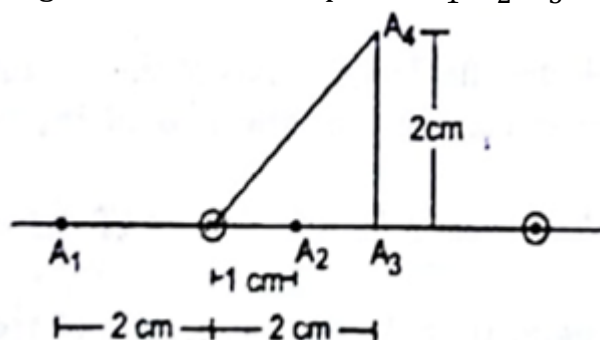
μ_0 is the permeability of free space and $\mu_0 = 4\pi \times 10^{-7} \text{ T mA}^{-1}$ d is the distance between the current carrying wire and the required point. To get zero

resultant, $B = B_0$. $\frac{\mu_0 i}{2\pi d} = 2.0 \times 10^{-3} \therefore d = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 2.0 \times 10^{-3}} \therefore d = 0.001 \text{ m} = 1$

mm. Hence, the point should be placed at a distance of 0.001 m from the wire in west direction

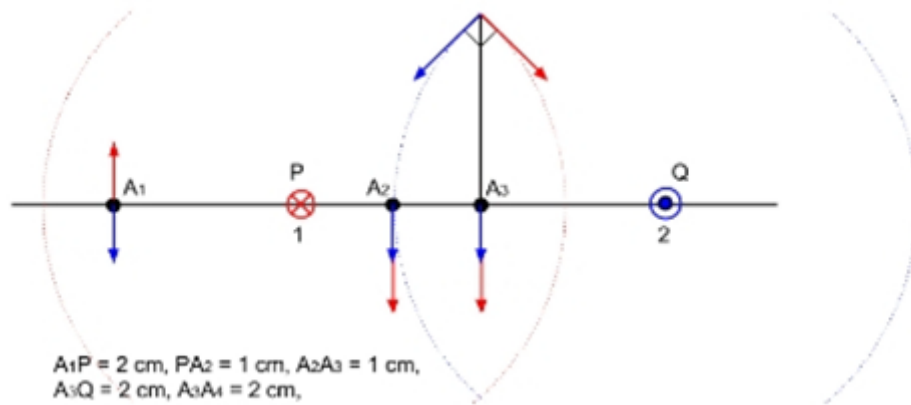
9. Question

Figure shows two parallel wires separated by a distance of 4.0 cm and carrying equal currents of 10 A along opposite directions. Find the magnitude of the magnetic field B at the points A_1 , A_2 , A_3 and A_4 .



Answer

Given: Magnitude of current in both the wires: $i = 10 \text{ A}$ Distance between 2 wires : $d = 4 \text{ cm} = 0.04 \text{ m}$



P and Q are

the two wires having opposite direction of currents. From the diagram, P has current into the plane and Q has current coming out of the plane. By right hand rule, P will have field coming up at A_1 and going down at A_2 and A_3 and tangent to A_4 as shown by the red arrow. Similarly, Q will have field going down at A_1, A_2 and A_3 and tangent to A_4 as shown by blue arrow. Distance between A_1, A_2 and A_3 is 2 cm each.

Formula used: By Ampere's Law for a current carrying wire is $B = \frac{\mu_0 i}{2\pi d}$ Here,

B is the magnitude of magnetic field, μ_0 is the permeability of free space and $\mu_0 = 4\pi \times 10^{-7} \text{ T mA}^{-1}$ and d is the distance between the current carrying wire and the required point. At A_1 : Net magnetic field due to P and Q would be subtraction of individual fields at A_1 . $PA_1: d_1 = 0.02 \text{ m}$ $QA_1: d_2 = 0.06 \text{ m}$ $B_{\text{net}} = B_P - B_Q$ Where, B_P is the magnetic field due to wire P and B_Q is the magnetic field due to wire Q.

$$\therefore B_{\text{net}} = \frac{\mu_0 i}{2\pi d_1} - \frac{\mu_0 i}{2\pi d_2} \therefore B_{\text{net}} = \frac{\mu_0 i}{2\pi} \left(\frac{1}{0.02} - \frac{1}{0.06} \right)$$

$$\therefore B_{\text{net}} = \frac{4\pi \times 10^{-7} \times 10}{2\pi} \times 33.33 \therefore B_{\text{net}} = 6.66 \times 10^{-5} \text{ T}$$
 Hence, magnetic field at

A_1 is $6.66 \times 10^{-5} \text{ T}$. At A_2 Effect of magnetic field at A_2 will be sum of magnetic field due to P and Q as both are in same direction as shown. $PA_2: d_1 = 0.01 \text{ m}$ $QA_3: d_2 = 0.03 \text{ m}$ $B_{\text{net}} = B_P + B_Q$ Where, B_P is the magnetic field due to wire P and B_Q is the

$$\text{magnetic field due to wire Q.} \therefore B_{\text{net}} = \frac{\mu_0 i}{2\pi d_1} + \frac{\mu_0 i}{2\pi d_2}$$

$$\therefore B_{\text{net}} = \frac{\mu_0 i}{2\pi} \left(\frac{1}{0.01} + \frac{1}{0.03} \right) \therefore B_{\text{net}} = \frac{4\pi \times 10^{-7} \times 10}{2\pi} \times 133.33 \therefore B_{\text{net}} = 2.66 \times$$

10^{-4} T Hence, magnetic field at A_2 is $2.66 \times 10^{-4} \text{ T}$. At A_3 Effect of magnetic field at A_3 will be sum of magnetic field due to P and Q as both are in same direction as shown. $PA_3: d_1 = 0.02 \text{ m}$ $QA_3: d_2 = 0.02 \text{ m}$ $B_{\text{net}} = B_P + B_Q$ Where, B_P is the magnetic field due to wire P and B_Q is the magnetic field due to wire Q.

$$\therefore B_{\text{net}} = \frac{\mu_0 i}{2\pi d_1} + \frac{\mu_0 i}{2\pi d_2} \therefore B_{\text{net}} = \frac{\mu_0 i}{2\pi} \left(\frac{1}{0.02} + \frac{1}{0.02} \right)$$

$$\therefore B_{\text{net}} = \frac{4\pi \times 10^{-7} \times 10}{2\pi} \times 100 \therefore B_{\text{net}} = 2 \times 10^{-4} \text{ T}$$
 Hence, magnetic field at A_3 is

$2 \times 10^{-4} \text{ T}$. At A_4 : The resultant magnetic field at A_4 would be $B_{\text{net}} = \sqrt{B_P^2 + B_Q^2}$ From

Pythagoras theorem, $(PA_4)^2 = (PA_3)^2 + (A_3A_4)^2 \therefore (PA_4)^2 = 0.02^2 + 0.02^2 \therefore PA_4 =$

$$0.028 \text{ m} = d_1 \text{ Now, } B_P = \frac{\mu_0 i}{2\pi d_1} \therefore B_P = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.028} \therefore B_P = 7.14 \times 10^{-5}$$

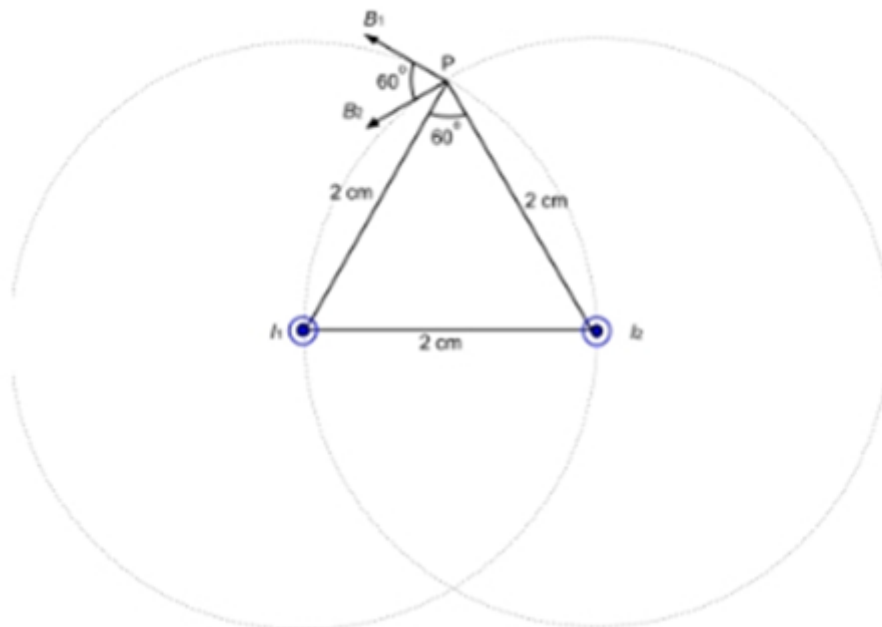
TSimilarly,QA₄ = 0.028 m = d₂Thus, $B_Q = \frac{\mu_0 i}{2\pi d_2} \therefore B_Q = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.028} \therefore B_Q = 7.14 \times 10^{-5} \text{ T}$ Substituting to get net magnetic field at A₄,
 $B_{net} = \sqrt{(7.14 \times 10^{-5})^2 + (7.14 \times 10^{-5})^2} \therefore B_{net} = 1 \times 10^{-4} \text{ T}$ Hence magnetic field at A₄ is $1 \times 10^{-4} \text{ T}$.

10. Question

Two parallel wires carry equal currents of 10 A along the same direction and are separated by a distance of 2.0 cm. Find the magnetic field at a point which is 2.0 cm away from each of these wires.

Answer

Given: Current in the equal wires: $i = 10 \text{ A}$
 Distance between the two wires: $a = 2 \text{ cm} = 0.02 \text{ m}$
 Distance between the wire and the point : $d = 2 \text{ cm} = 0.02 \text{ m}$



From the

diagram l_1 and l_2 are the wires coming out of the plane. These two wires and point P form an equilateral triangle of side 0.02 m. By right hand rule, Magnetic field at P due to l_1 is shown by arrow B_1 tangent to the field's path and Magnetic field at P due to l_2 is shown by B_2 tangent to the field's path. By geometry we can calculate the angle between B_1 and B_2 which turns out to be 60° . **Formula used:** By

Ampere's Law for a current carrying wire is $B = \frac{\mu_0 i}{2\pi d}$ Where,

B is the magnitude of magnetic field,

μ_0 is the permeability of free space, $\mu_0 = 4\pi \times 10^{-7} \text{ T mA}^{-1}$

d is the distance between the current carrying wire and the required

point. Magnetic field due to wire l_1 : $B_1 = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.02} \therefore B_1 = 1 \times 10^{-4}$

Magnetic Field due to l_2 : $B_2 = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.02} \therefore B_2 = 1 \times 10^{-4} \text{ T}$ $B_1 = B_2$ as same magnitude of current is flowing through both the wires and point P is located at

same distance from each wire. Now, angle between \vec{B}_1 and \vec{B}_2 : $\theta = 60^\circ$ Resultant of two vectors is given as $B_R = \sqrt{B_1^2 + B_2^2 + 2B_1B_2\cos\theta}$ B_R is the resultant magnetic field at P. $\therefore B_R = \sqrt{(1 \times 10^{-4})^2 + (1 \times 10^{-4})^2 + 2(1 \times 10^{-4})^2\cos(60^\circ)} \therefore B_R = 1.73 \times 10^{-4} \text{ T}$ Hence, resultant magnetic field at a point 2 cm away from due to two current carrying wires is $1.73 \times 10^{-4} \text{ T}$.

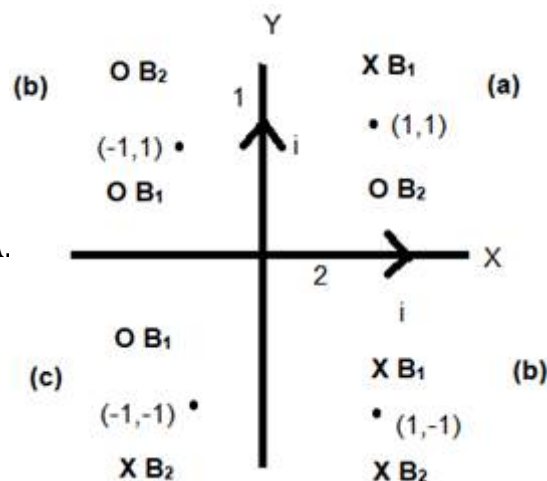
11. Question

Two long, straight wires, each carrying a current of 5A, are placed along the x-and y-axis respectively. The currents point along the positive directions of the axes.

Find the magnetic fields at the points (a) (1m, -1m), (b) (-1m, 1m) (c) (-1m, -1m) and (d) (1m, -1m).

Answer

Given: Current in both the wires : $I = 5 \text{ A}$.



Wires 1 and 2 generate magnetic fields B_1 and B_2 respectively. By right hand rule, direction of magnetic field in each quadrant due to wires 1 and 2 is shown by X and O. X : Field is going into the plane O : Field is coming out of the plane. **Formula**

used: By Ampere's Law for a current carrying wire is $B = \frac{\mu_0 i}{2\pi d}$ Where,

B is the magnitude of magnetic field,

μ_0 is the permeability of free space and $\mu_0 = 4\pi \times 10^{-7} \text{ T mA}^{-1}$ d is the distance between the current carrying wire and the required point. (a) At (1m,1m) As we can see from the diagram at (1,1) the magnetic fields due to wire 1 and 2 are in opposite direction. (X and O) Hence, net magnetic field would be zero. (b) At (-1m, 1m) Here, magnetic field due to wire 1 and 2 would add up as direction of magnetic field is same. (O and O) Thus $B = B_X + B_Y$ B_X is the magnetic field due to wire on x axis B_Y is the magnetic field due to wire on y axis

$$B = \frac{4\pi \times 10^{-7} \times 5}{2\pi \times 1} + \frac{4\pi \times 10^{-7} \times 5}{2\pi \times 1} \therefore B = 2 \times 10^{-6} \text{ T}$$

This B would be along z-Axis.

(c) At (-1m,-1m) As we can see from the diagram at (-1,-1) the magnetic fields due to wire 1 and 2 are in opposite direction. (O and X) Hence, net magnetic field would be zero. (d) At (1m,-1m) Here, magnetic field due to wire 1 and 2 would add up as direction of magnetic field is same. (X and X) Thus $B = B_X + B_Y$ B_X is the

magnetic field due to wire on x axis B_y is the magnetic field due to wire on y axis

$$B = \frac{4\pi \times 10^{-7} \times 5}{2\pi \times 1} + \frac{4\pi \times 10^{-7} \times 5}{2\pi \times 1} \therefore B = 2 \times 10^{-6} \text{ T}$$

This B would be along negative z-axis.

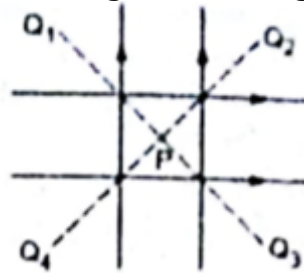
12. Question

Four long, straight wires, each carrying a current of 5.0 A, are placed in a plane as shown in figure. The points of intersection from a square of side 5.0 cm.

(a) Find the magnetic field at the centre P of the square

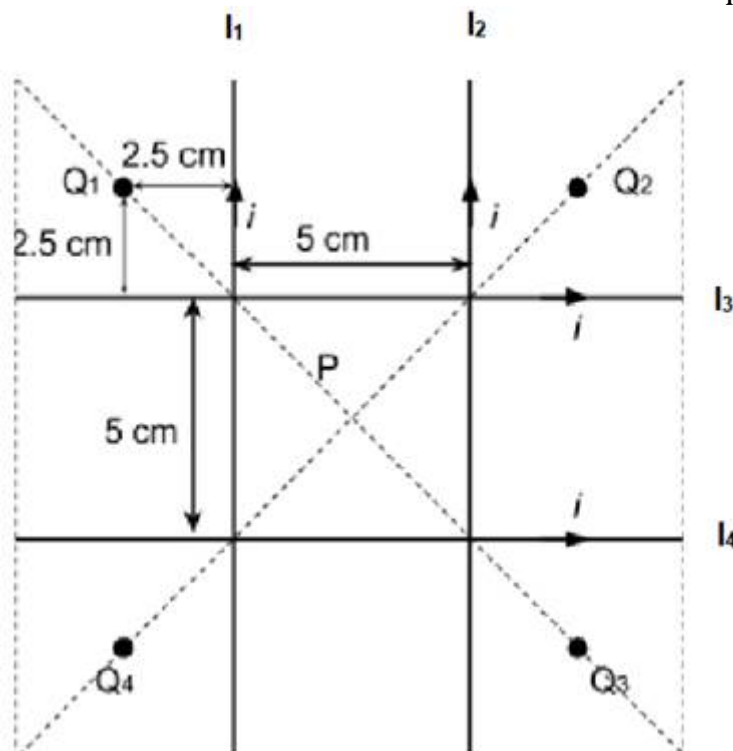
(b) Q_1 , Q_2 , Q_3 and Q_4 are points situated on the diagonal of the square and at a distance from P that to equal to the length of the diagonal of the square. Find the

magnetic fields at these points.



Answer

Given: Current in the 4 wires: $i = 5.0 \text{ A}$ Side of the square : $d = 5 \text{ cm} = 0.05 \text{ m}$



In the diagram above, l_1 and l_2

represents vertical wires. l_3 and l_4 represent horizontal wires. Arrows represent the direction of current in the wire. We can determine the direction of magnetic field due to any of the wire using Right hand rule. **Formula used:** By Ampere's Law for a

current carrying wire is $B = \frac{\mu_0 i}{2\pi d}$ Here, B is the magnitude of magnetic field, μ_0 is

the permeability of free space and $\mu_0 = 4\pi \times 10^{-7} \text{ T mA}^{-1}$ and d is the distance

between the current carrying wire and the required point. (a) At point P, if we use

Right hand rule for every wire. Field due to l_1 would be into the plane and field due to l_2 would come out of the plane. Thus canceling out each other. Similarly, field due to l_3 would go into the plane and field due to l_4 would come out, canceling each other. Thus, net field at P is zero. (b) At Q_1 , By geometry: Distance between wire l_1 and Q_1 : $d_1 = 0.025$ m Distance between wire l_2 and Q_1 : $d_2 = 0.075$ m Distance between wire l_3 and Q_1 : $d_3 = 0.025$ m Distance between wire l_4 and Q_1 : $d_4 = 0.075$ m Net magnetic field at Q_1 due to wires l_1, l_2, l_3, l_4 is $B_{Q_1} = B_{l_1} + B_{l_2} + B_{l_3} + B_{l_4}$

$$\therefore B_{Q_1} = \frac{\mu_0 i}{2\pi d_1} + \frac{\mu_0 i}{2\pi d_2} + \frac{\mu_0 i}{2\pi d_3} + \frac{\mu_0 i}{2\pi d_4}$$

$$\therefore B_{Q_1} = \frac{4\pi \times 10^{-7} \times 5}{2\pi} \left(\frac{1}{0.025} + \frac{1}{0.075} + \frac{1}{0.025} + \frac{1}{0.075} \right) \therefore B_{Q_1} = 1.06 \times 10^{-4}$$

THence, magnetic field at Q_1 is 1.06×10^{-4} T and in upward direction. At Q_2 ,

Similarly by geometry Distance between wire l_1 and Q_2 : $d_1 = 0.075$ m Distance between wire l_2 and Q_2 : $d_2 = 0.025$ m Distance between wire l_3 and Q_2 : $d_3 = 0.025$ m Distance between wire l_4 and Q_2 : $d_4 = 0.075$ m Now, Net magnetic field at Q_2 due to wires l_1, l_2, l_3, l_4 is $B_{Q_2} = B_{l_1} + B_{l_2} + B_{l_3} + B_{l_4}$

$$\therefore B_{Q_2} = \frac{\mu_0 i}{2\pi d_1} + \frac{\mu_0 i}{2\pi d_2} - \frac{\mu_0 i}{2\pi d_3} - \frac{\mu_0 i}{2\pi d_4}$$

$$\therefore B_{Q_2} = \frac{4\pi \times 10^{-7} \times 5}{2\pi} \left(\frac{1}{0.075} + \frac{1}{0.025} - \frac{1}{0.025} - \frac{1}{0.075} \right) \therefore B_{Q_2} = 0$$

Hence, magnetic field at Q_2 is zero. Negative sign for B_{l_3} and B_{l_4} as direction of magnetic field due to l_1 (going in) is opposite to l_4 (coming out) and l_2 (going in) is opposite to l_3 (coming out). At Q_3 , By geometry Distance between wire l_1 and Q_3 : $d_1 = 0.075$ m Distance between wire l_2 and Q_3 : $d_2 = 0.025$ m Distance between wire l_3 and Q_3 : $d_3 = 0.075$ m Distance between wire l_4 and Q_3 : $d_4 = 0.025$ m Net magnetic field at Q_3 due to wires l_1, l_2, l_3, l_4 is $B_{Q_3} = B_{l_1} + B_{l_2} + B_{l_3} + B_{l_4}$

$$\therefore B_{Q_3} = \frac{\mu_0 i}{2\pi d_1} + \frac{\mu_0 i}{2\pi d_2} + \frac{\mu_0 i}{2\pi d_3} + \frac{\mu_0 i}{2\pi d_4}$$

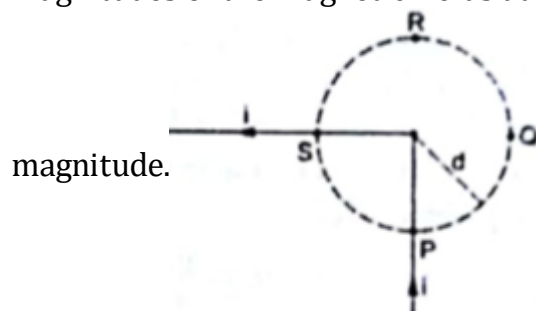
$$\therefore B_{Q_3} = \frac{4\pi \times 10^{-7} \times 5}{2\pi} \left(\frac{1}{0.075} + \frac{1}{0.025} + \frac{1}{0.075} + \frac{1}{0.025} \right) \therefore B_{Q_3} = 1.06 \times 10^{-4}$$

THence, magnetic field at Q_3 is 1.06×10^{-4} T and in downward direction. At

Q_4 , Magnetic field at Q_4 would be same as that of Q_2 because at Q_4 , l_1 and l_4 have opposite fields, l_2 and l_3 have opposite fields. Thus canceling out each other and B_{Q_4} is zero. Hence, Magnetic field at Q_1 and Q_3 have same magnitude of 1.06×10^{-4} T but in opposite direction and magnetic field at Q_2 and Q_4 is zero.

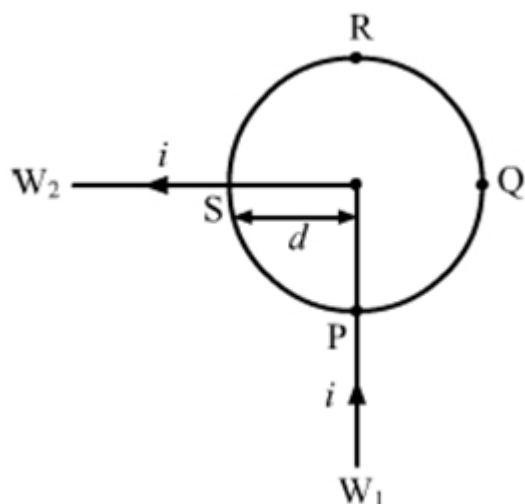
13. Question

Figure shows a long wire bent at the middle to form a right angle. Show that the magnitudes of the magnetic fields at the points, P, Q, R and S are equal and find this



Answer

Given: Current through the wire : i radius of the circle formed: d



Let W_1 and W_2 be the terms for wire 1

and wire 2. Arrow shows the direction of the current in the wire. By right hand rule we can determine the direction of the magnetic field due to any of the two wire at any point. **Formula used:** By Ampere's Law for a current carrying wire is $B = \frac{\mu_0 i}{2\pi d}$

Here, B is the magnitude of magnetic field, μ_0 is the permeability of free space and $\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$ and d is the distance between the current carrying wire and the required point. In this case, d is the radius of the circle. (1) At point P, Magnetic field at P due to W_1 is zero as P is on the axis of W_1 . Magnetic field at P due to W_2

is $B_{Pw_2} = \frac{\mu_0 i}{2\pi d}$ Hence, net magnetic field at P due to both the wires is $B_P = \frac{\mu_0 i}{2\pi d}$

Direction of B_P is perpendicular to the plane and in outward direction. (2) At point

Q, Magnetic field at Q due to W_1 , $B_{Qw_1} = \frac{\mu_0 i}{2\pi d}$ Magnetic field at Q due to W_2 is zero as Q is not in the effective zone of W_2 . Hence, net magnetic field at Q due to both

the wires is $B_Q = \frac{\mu_0 i}{2\pi d}$ Direction of B_Q is perpendicular to the plane and inward direction (3) At point R, Magnetic field at R due to W_1 is zero as R is not in the

effective zone of W_1 . Magnetic field at R due to W_2 is $B_{Rw_2} = \frac{\mu_0 i}{2\pi d}$ Hence, net

magnetic field at R due to both the wires is $B_R = \frac{\mu_0 i}{2\pi d}$ Direction of B_R is

perpendicular to the plane and in inward direction. (4) At point S, Magnetic field at S due to W_2 is zero as S is on the axis of W_2 . Magnetic field at S due to W_1 is

$B_{Sw_1} = \frac{\mu_0 i}{2\pi d}$ Hence, net magnetic field at S due to both the wires is $B_S = \frac{\mu_0 i}{2\pi d}$

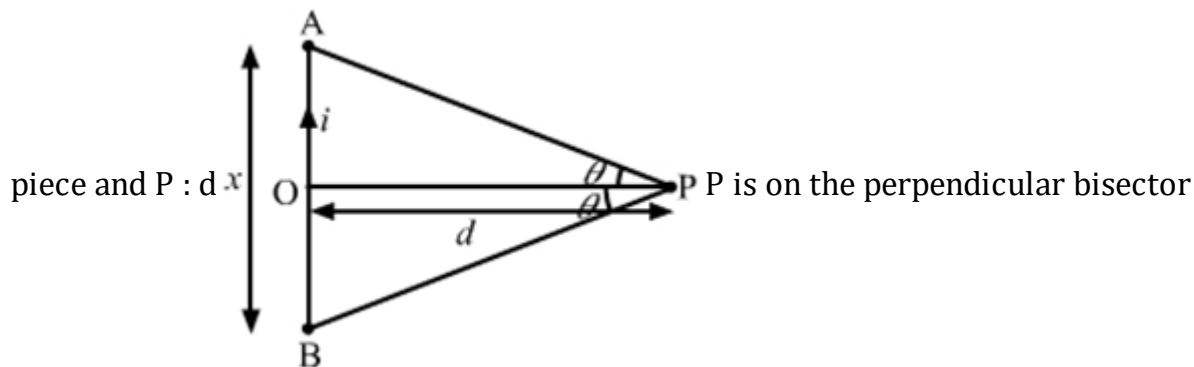
Direction of B_P is perpendicular to the plane and in outward direction.

14. Question

Consider a straight piece of length x of a wire carrying a current i . Let P be a point on the perpendicular bisector of the piece, situated at a distance d from its middle point. Show that for $d \gg x$, the magnetic field at P varies at $1/d^2$ whereas for $d \ll x$, it varies as $1/d$.

Answer

Given: Length of the piece: x Current in the piece: i Distance between midpoint of



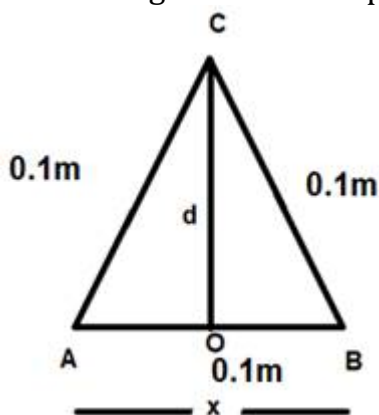
of AB meeting at O. **Formula used:** By Biot-Savart Law: $dB = \frac{\mu_0 i dl \times \bar{r}}{4\pi r^3}$ Here, dB is the magnitude of magnetic field element, μ_0 is the permeability of free space and $\mu_0 = 4\pi \times 10^{-7} \text{ T mA}^{-1}$, dl is the length element, r is the distance between the current carrying wire and the required point. We know that, magnetic field at a point on the perpendicular bisector is $B = \frac{\mu_0 i}{2\pi d} \left(\frac{a}{\sqrt{a^2 + 4d^2}} \right)$ Here, a is the length of the wire. In this case $a = x$ d is the distance between point P and the midpoint of AB. For $d \gg x$ x^2 can be neglected. $B = \frac{\mu_0 i}{2\pi d} \left(\frac{x}{\sqrt{4d^2}} \right) \therefore B = \frac{\mu_0 i}{2\pi d} \frac{x}{2d} \therefore B = \frac{\mu_0 i x}{4\pi d^2} \therefore B \propto 1/d^2$ Hence for $d \gg x$ magnetic field is inversely proportional to the square of the distance d . For $d \ll x$ d^2 can be neglected. $B = \frac{\mu_0 i}{2\pi d} \left(\frac{x}{\sqrt{x^2}} \right) \therefore B = \frac{\mu_0 i}{2\pi d} \therefore B \propto 1/d$ Hence, when $d \ll x$ the magnetic field is inversely proportional to the distance d .

15. Question

Consider a 10 cm long piece of a wire which carries a current of 10A. Find the magnitude of the magnetic field due to the piece at a point which makes an equilateral triangle with the ends of the piece.

Answer

Given: Length of the wire piece: $x = 10 \text{ cm} = 0.1 \text{ m}$ Current in the wire piece: $i = 10 \text{ A}$



Formula used: By Biot-Savart Law: $dB = \frac{\mu_0 i dl \times \bar{r}}{4\pi r^3}$

Here, dB is the magnitude of magnetic field element, μ_0 is the permeability of free space and $\mu_0 = 4\pi \times 10^{-7} \text{ T mA}^{-1}$, dl is the length element, r is the distance between the current carrying wire and the required point. We know that, magnetic field at a point on the perpendicular bisector is $B = \frac{\mu_0 i}{2\pi d} \left(\frac{x}{\sqrt{x^2 + 4d^2}} \right)$ Here, x is the length

of the wire. d is the distance between point C and the midpoint O. From Pythagoras theorem, $(BC)^2 = (OB)^2 + (OC)^2 \therefore (OC)^2 = (BC)^2 - (OB)^2 \therefore (OC)^2 = (0.1)^2 - (0.05)^2 \therefore OC = (7.5 \times 10^{-3})^{1/2} \therefore OC = 0.086 \text{ m} = d$ Substituting for B we get,

$$B = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.086} \times \frac{0.01}{\sqrt{0.01^2 + 4 \times (0.086)^2}} \therefore B = 2.32 \times 10^{-5} \times 0.502 \therefore B =$$

$1.16 \times 10^{-5} \text{ T}$ Hence the magnitude of the magnetic field due to the piece at a point which makes an equilateral triangle with the ends of the piece is $1.16 \times 10^{-5} \text{ T}$.

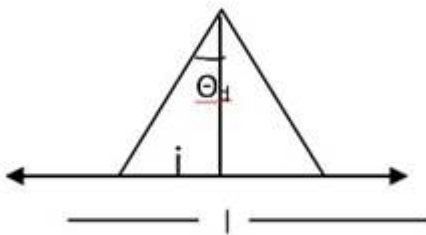
16. Question

A long, straight wire carries a current i . Let B_1 be the magnetic field at a point P at a distance d from the wire. Consider a section of length ℓ of this wire such that the point P lies on the perpendicular bisector of the section. Let B_2 be the magnetic field at this point due to this section only. Find the value of d/ℓ so that B_2 differs from B_1 by 1%.

Answer

Magnitude of current = i

Distance of point P from wire = d



The magnetic field induced in the wire is defined as

$$B_1 = \frac{\mu_0 i}{2\pi d}$$

Also, the magnetic field due to a section of length l on the perpendicular bisector is given by

$$B_2 = \frac{\mu_0 i}{4\pi d} \times \frac{2l}{\sqrt{l^2 + 4d^2}}$$

$$\frac{\mu_0 i}{4\pi d} \times \frac{2l}{d \sqrt{\frac{l^2}{d^2} + 4}}$$

Neglecting $\frac{l^2}{d^2}$ (as it is very small), we get

$$B_2 = \frac{\mu_0 i l}{4\pi d^2} \times \frac{2}{\sqrt{2}}$$

$$B_2 = \frac{\sqrt{2}\mu_0 il}{4\pi d^2}$$

now, $B_1 > B_2$

We are given that, $\frac{B_1 - B_2}{B_1} = \frac{1}{100}$

$$\Rightarrow B_2 = 0.99B_1$$

substituted the value of B_1 and B_2 in the above equation we get,

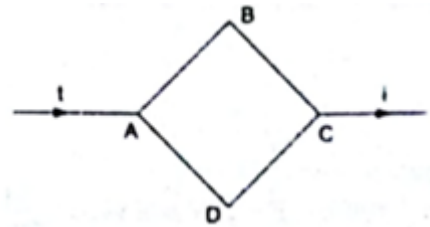
$$\Rightarrow \frac{\sqrt{2}\mu_0 il}{4\pi d^2} = 0.99 \times \frac{\mu_0 i}{2\pi d}$$

$$\Rightarrow \frac{d}{l} = \frac{1.414}{1.98}$$

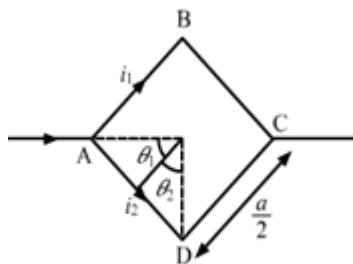
$$\Rightarrow \frac{d}{l} = 0.71$$

17. Question

Figure shows a square loop ABCD with edge length a . The resistance of the wire ABC is r and that of ADC is $2r$. Find the magnetic field B at the centre of the loop assuming uniform wires.



Answer



Resistance of wire ADC is given twice that of wire ABC. hence current flow through ADC is half of ABC

i.e.,

$$\frac{i_2}{i_1} = \frac{1}{2} \text{-----(1)}$$

$$\text{also } i_1 + i_2 = i \text{-----(2)}$$

Equating equations (1) and (2), we get

$$i_1 = \frac{2i}{3} \text{ and } i_2 = \frac{i}{3}$$

Hence, magnetic field at centre O due to wire AB and BC

$$B_1 = B_2 = \frac{\mu_0}{4\pi} \times \frac{2i_1 \sin 45}{\frac{a}{2}}$$

$$B_1 = B_2 = \frac{\mu_0}{4\pi} \times \frac{2\sqrt{2}i_1}{a}$$

and magnetic field at centre o due to wire AD and DC

$$B_3 = B_4 = \frac{\mu_0}{4\pi} \times \frac{2i_2 \sin 45}{\frac{a}{2}}$$

$$B_3 = B_4 = \frac{\mu_0}{4\pi} \times \frac{2\sqrt{2}i_2}{a}$$

Also $i_1 = 2i_2$, so $(B_1 = B_2) > (B_3 = B_4)$

Hence net magnetic field at centre O

$$B_{net} = (B_1 + B_2) - (B_3 + B_4)$$

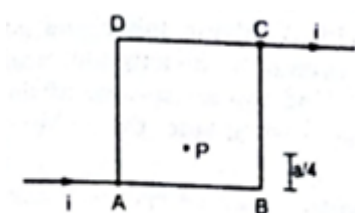
$$B_{net} = \left(2 \times \frac{\mu_0}{4\pi} \times \frac{2\sqrt{3} \times \frac{2}{3}(i)}{a} \right) - \left(\frac{\mu_0}{4\pi} \times \frac{2\sqrt{3} \times \frac{1}{3}(2)}{a} \right)$$

$$B_{net} = \frac{\mu_0}{4\pi} \times \frac{4\sqrt{2}i}{3a} (2 - 1)$$

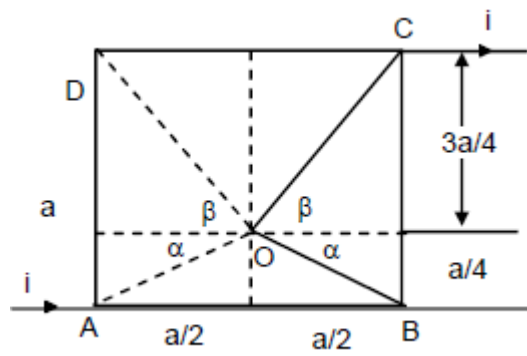
$$B_{net} = \frac{\sqrt{2}\mu_0 i}{3\pi a}$$

18. Question

Figure shows a square loop of edge a made of a uniform wire. A current i enters the loop at the point A and leaves it at the point C. Find the magnetic field at the point P which is not the perpendicular bisector of AB at a distance $a/4$ from it.



Answer



$$A_0 = \sqrt{\frac{a^2}{16} + \frac{a^2}{4}} = \sqrt{\frac{5a^2}{16}} = \frac{a\sqrt{5}}{4}$$

$$D_0 = \sqrt{\left(\frac{3a}{4}\right)^2 + \left(\frac{a}{2}\right)^2} = \sqrt{\frac{9a^2}{16} + \frac{a^2}{4}} = \sqrt{\frac{13a^2}{16}} = \frac{a\sqrt{13}}{4}$$

magnetic field due to AB

$$B_{AB} = \frac{\mu_0}{4\pi} \times \frac{i}{2\left(\frac{a}{4}\right)} [\sin \sin (90^\circ - i) + \sin \sin (90^\circ - \alpha)]$$

$$B_{AB} = \frac{\mu_0 \times 2i}{4\pi a} \times 2\cos\alpha$$

$$B_{AB} = \frac{\mu_0 \times 2i}{4\pi a} \times 2 \frac{\frac{a}{2}}{a\left(\frac{\sqrt{5}}{4}\right)}$$

$$B_{AB} = \frac{2i\mu_0}{\pi\sqrt{5}}$$

magnetic field due to DC

$$B_{DC} = \frac{\mu_0}{4\pi} \times \frac{i}{2\left(\frac{3a}{4}\right)} (90^\circ - \alpha)$$

$$B_{DC} = \frac{\mu_0 i \times 4 \times 2}{4\pi \times 3a} \times \cos\beta$$

$$B_{DC} = \frac{\mu_0 \times i}{3\pi a} \times \frac{\frac{a}{2}}{\left(\frac{\sqrt{13}a}{4}\right)}$$

$$B_{DC} = \frac{2i\mu_0}{a^3\pi\sqrt{13}}$$

The magnetic field due to AD and BC are equal and opposite hence nullify each other

Hence, net magnetic field

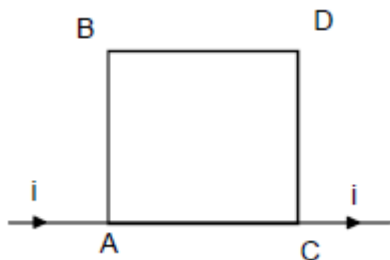
$$B_{net} = \frac{2i\mu_0}{\pi\sqrt{5}} - \frac{2i\mu_0}{a^3\pi\sqrt{13}}$$

$$B_{net} = \frac{2i\mu_0}{\pi a} \times \left[\frac{1}{\sqrt{5}} - \frac{1}{3\sqrt{13}} \right]$$

19. Question

Consider the situation described in the previous problem. Suppose the current i enters the loop at the point A and leaves it at the point B. Find the magnetic field at the centre of the loop.

Answer



Magnetic vector B due to BC and due to AD at PT equal and opposite.

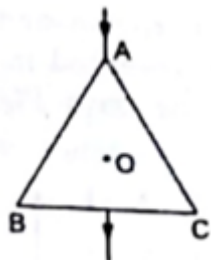
Hence net vector $B = 0$

Similarly, due to AB and CD at P = 0

\therefore The net vector B at the centre of the square loop = 0

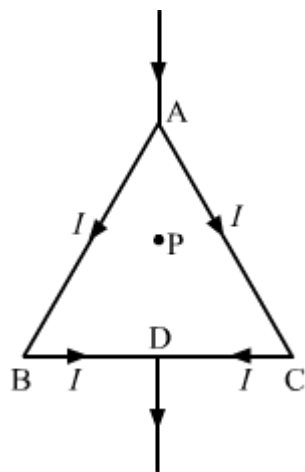
20. Question

The wire ABC shown in figure forms an equilateral triangle. Find the magnetic field B at the centre O of the triangle assuming the wire to be uniform.



Answer

Let current $2I$ flow through the circuit. Since the wire is uniform, the current will enter the loop at point A and divide equally between the two arms AB and AC (as shown in the figure)



Now, Magnetic field induced at point P due to wire AB and wire AC = B (say)
(Perpendicular to the plane in outward direction)

$$B = \frac{\mu_0 i}{4\pi r} (\sin \sin 60^\circ + \sin \sin 60^\circ)$$

Magnetic field at P due to wire BD and DC = B' (say) (Perpendicular to the plane in outward direction)

$$B' = \frac{\mu_0 i}{4\pi r} (\sin \sin 60^\circ)$$

$$\therefore \text{Net magnetic field at } P = B + B' - B - B' = 0$$

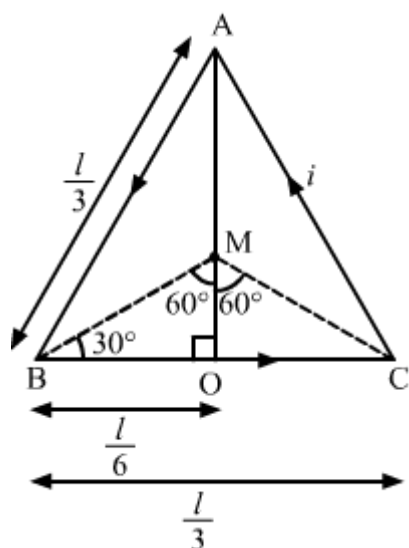
21. Question

A wire of length ℓ is bent in the form of an equilateral triangle and carries an electric current i . (a) Find the magnetic field B at the centre.

(b) If the wire is bent in the form of a square, what would be the value of B at the centre?

Answer

(a)



Let M be the midpoint of equilateral ΔABC

$$\therefore AB = BC = CA = \frac{l}{2}$$

current is given to be i

Hence, in ΔAOB

$$AO = \sqrt{\left(\frac{l}{3}\right)^2 - \left(\frac{l}{6}\right)^2}$$

$$AO = l \sqrt{\frac{1}{9} - \frac{1}{36}}$$

$$AO = l \sqrt{\frac{9-1}{36}}$$

$$AO = l \sqrt{\frac{1}{12}}$$

$$MO = \frac{1}{3} \times l \sqrt{\frac{1}{12}}$$

$$MO = \frac{l}{6\sqrt{3}}$$

The angles made by points B and C with centre M are $\theta_1 = \theta_2 = 60^\circ$ respectively,

Hence separation of the point from the wire is given by, $d = MO = \frac{l}{6\sqrt{3}}$

Thus, the magnetic field induced due to current in wire BC is-

$$B = \frac{\mu_0 i}{4\pi d} (\sin \theta_1 + \sin \theta_2)$$

$$B = \frac{\mu_0 i}{4\pi \frac{l}{6\sqrt{3}}} (\sin 60^\circ + \sin 60^\circ)$$

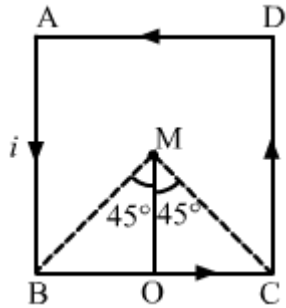
$$B = \frac{\mu_0 i}{4\pi l} \times 6\sqrt{3} \times \sqrt{3}$$

Now, Net magnetic field at point M is given as Magnetic field due to wire BC + Magnetic field due to wire CA + Magnetic field due to wire AB. Since all wires are equal,

$$B_{net} = 3B = \frac{27\mu_0 i}{\pi l}$$

It is perpendicular to the plane in outward direction if current is anticlockwise and in an inward direction if the current is clockwise.

(b)



The angles made by B and C with centre M are $\theta_1 = 45^\circ, \theta_2 = 60^\circ$

Distance between point from the wire, $d = \frac{l}{8}$

Thus, the induced magnetic field due to electric current in wire BC is given by

$$B = \frac{\mu_0 i}{4\pi d} (\sin \theta_1 + \sin \theta_2)$$

$$B = \frac{\mu_0 i}{4\pi \frac{l}{8}} (\sin 45^\circ + \sin 45^\circ)$$

$$B = \frac{2\sqrt{2}\mu_0 i}{\pi l}$$

Since all wires are equal, Net magnetic field at point M = 4 × Magnetic field due to wire BC

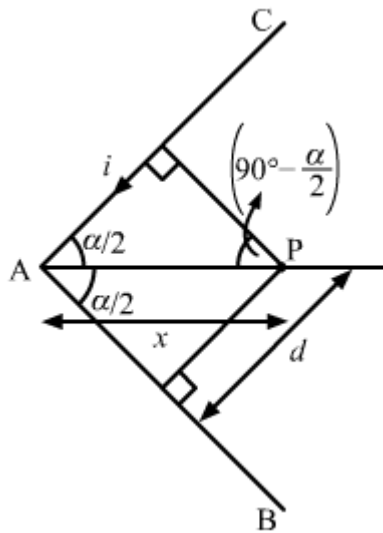
$$B_{net} = 3B = \frac{8\sqrt{2}\mu_0 i}{\pi l}$$

22. Question

A very long wire carrying a current i is bent to form a plane at an angle α . Find the magnetic field B at a point P on the bisector of this angle situated at a distance x from the vertex A .

Answer

Let CAB be the wire making an angle α , P be a point on the bisector of this angle situated at a separation x from the vertex A and d be the perpendicular distance of AC and AB from P .



From the diagram, $\sin\left(\frac{\alpha}{2}\right) = \frac{r}{x}$

$$\Rightarrow r = x \sin\left(\frac{\alpha}{2}\right)$$

The induced magnetic field B due to wire AB

$$\frac{\mu_0 i}{4\pi r} \left[\sin \sin \left(180 - \left(90 - \left(\frac{\alpha}{2} \right) \right) \right) + 1 \right]$$

$$\Rightarrow \frac{\mu_0 i \left[\sin \sin \left(180 - \left(90 - \left(\frac{\alpha}{2} \right) \right) \right) + 1 \right]}{4\pi \times \sin\left(\frac{\alpha}{2}\right)}$$

$$\Rightarrow \frac{\mu_0 i \left[\cos \cos \left(\frac{\alpha}{2} \right) + 1 \right]}{4\pi \times \sin\left(\frac{\alpha}{2}\right)}$$

$$= \frac{\mu_0 i 2 \cos^4\left(\frac{\alpha}{4}\right)}{4\pi \times \sin\left(\frac{\alpha}{4}\right) \cos \cos \left(\frac{\alpha}{4} \right)}$$

$$= \frac{\mu_0 i}{4\pi x} \cot\left(\frac{\alpha}{4}\right)$$

The magnetic field due to current in both the wires

$$= \frac{2\mu_0 i}{4\pi x} \cot\left(\frac{\alpha}{4}\right)$$

$$= \frac{\mu_0 i}{2\pi x} \cot\left(\frac{\alpha}{4}\right)$$

23. Question

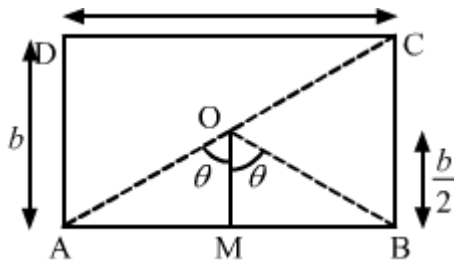
Find the magnetic field B at the centre of a rectangular loop of length ℓ and width b, carrying a current i.

Answer

Let us consider two angles made by points A and B with point O be θ_1 and θ_2 respectively.

$$\theta_1 = \theta_2 = \theta$$

Let distance of the point from the wire $d = \frac{b}{2}$



In triangle ΔAOM

$$AO = \sqrt{\left(\frac{b}{2}\right)^2 + \left(\frac{l}{2}\right)^2} = \frac{1}{2}\sqrt{b^2 + l^2}$$

$$\sin\theta = \frac{\frac{l}{2}}{\frac{1}{2}\sqrt{b^2 + l^2}} = \frac{l}{\sqrt{b^2 + l^2}}$$

Thus induced magnetic field due to AB is given by,

$$B = \frac{\mu_0 i}{4\pi d} (\sin\theta_1 + \sin\theta_2)$$

$$\Rightarrow B = \frac{\mu_0 i}{4\pi d} \times 2\sin\theta$$

$$\Rightarrow B = \frac{\mu_0 i}{4\pi \frac{b}{2}} \times \frac{2l}{\sqrt{l^2 + b^2}}$$

$$\Rightarrow B = \frac{\mu_0 i}{\pi b} \times \frac{l}{\sqrt{l^2 + b^2}}$$

Similarly the induced magnetic field due to current in BC is given as:

$$B' = \frac{\mu_0 i}{\pi l} \times \frac{b}{\sqrt{l^2 + b^2}}$$

Now, magnetic field due to CD = magnetic field due to current in wire AB = B

Also, magnetic field due to current in wire DA = magnetic field due to current in wire BC = B'

$$\therefore B_{\text{net}} = l(B + B')$$

$$B_{nut} = 2 \left[\frac{\mu_0 i}{\pi b} \times \frac{l}{\sqrt{l^2 + b^2}} + \frac{\mu_0 i}{\pi l} \times \frac{b}{\sqrt{l^2 + b^2}} \right]$$

$$B_{nut} = \frac{2\mu_0 i}{\pi \sqrt{l^2 + b^2}} \left[\frac{l}{b} + \frac{b}{l} \right]$$

$$B_{nut} = \frac{2\mu_0 i \sqrt{b^2 + l^2}}{\pi b l}$$

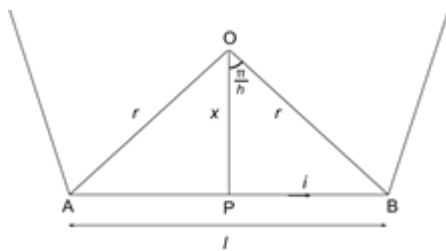
24. Question

A polygon of n equal sides is formed by bending a current carrying wire to total length $2\pi r$ which carries a current i.

(a) Find the magnetic field B at the center of the n-sided polygon.

(b) By letting $n \rightarrow \infty$, derive the expression for the induced magnetic field at the center of a circular current carrying wire.

Answer



a) Using the diagram,

For a polygon of n equal sides, the angle at the centre is $\frac{2\pi}{n}$.

$$\tan \theta = \frac{l}{2x}$$

$$\Rightarrow x = \frac{l}{2 \tan \theta}$$

Considering angle to be very small

$$\frac{l}{2} = \frac{\pi r}{n}$$

Using Biot-Savart's law for one side of the n-sided polygon,

$$B = \frac{\mu_0 i}{4\pi} \frac{dl \sin \theta}{x^2}$$

$$\Rightarrow B = \frac{\mu_0 i}{4\pi} \frac{(\sin \theta + \sin \theta)}{x^2}$$

$$B = \frac{\mu_0 i 2(\tan \theta)(2 \sin \theta)}{4\pi l} \text{ (substituting the value of r)}$$

$$B = \frac{\mu_0 i 2n \left[\tan\left(\frac{\pi}{n}\right) \right] \left[2 \sin\left(\frac{\pi}{n}\right) \right]}{4\pi(2\pi r)} \text{ (substituting the value of } l \text{)}$$

For n-sided polygon

$$B' = nB$$

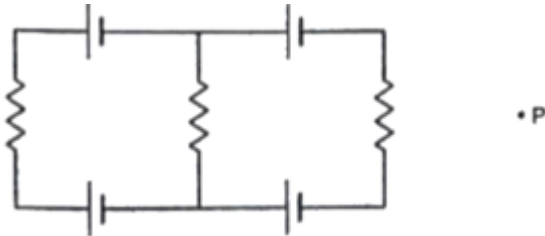
$$\Rightarrow B = \frac{\mu_0 i n^2 \tan\left(\frac{\pi}{n}\right) \sin\left(\frac{\pi}{n}\right)}{2\pi^2 r}$$

b) When $n \rightarrow \infty$, polygon tends to a circle with radius r and magnetic field will tend toward

$$B = \frac{\mu_0 i}{2r}$$

25. Question

Each of the batteries shown in figure has an emf 5V. Prove that the magnetic field B at the point P is zero for any set of values of the resistances.



Answer

By applying Kirchhoff's Voltage Law, we can observe that the current in the circuit is zero

Now, since net current in the circuit = 0

Induced magnetic field is always proportional to the current flowing in the circuit. Hence,

Net magnetic field \vec{A} at the point $P=0$

Induced magnetic field at point P is independent of the values of the various resistances in the electric circuit.

26. Question

A straight, long wire carries an electric current of 20 A. Another wire carrying same current is kept parallel to it. If force acting on a length of 10 cm of the second wire is 2.0×10^{-5} N, what is the distance between them?

Answer

Current on both wires $i_1 = i_2 = 20\text{A}$

Force acting on 10 cm of the second wire, $F = 20 \times 10^{-5}$

$$\therefore \text{force per unit length} = \frac{2 \times 10^{-5}}{0.1} = 20 \times 10^{-4}$$

Now, let the distance between the two wires be d . Thus, the force per unit length is given as:

$$\frac{F}{l} = \frac{\mu_0 i_1 i_2}{2\pi d}$$

$$\Rightarrow 2 \times 10^{-4} = \frac{2 \times 10^{-7} \times 20 \times 20}{d}$$

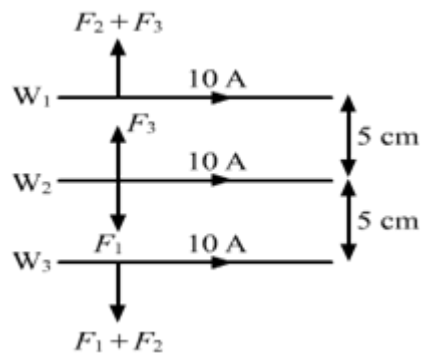
$$\Rightarrow \text{distance } d = 0.4\text{m} = 40\text{cm}$$

27. Question

Three coplanar parallel wires, each carrying a current of 10 A along the same direction, are placed with a separation 5.0 cm between the consecutive ones. Find the magnitude of the magnetic force per unit length acting on the wires.

Answer

-



Given: - Current flowing in each wire is $i_1 = i_2 = i_3 = 10\text{A}$

The magnetic force acting per unit length on wire due to the flow of the current is

$$\frac{F}{l} = \frac{\mu_0 i_1 i_2}{2\pi d}$$

So, for wire w_1 ,

$$\frac{F}{l} = \frac{F}{l} \text{ by wire } w_2 + \frac{F}{l} \text{ by wire } w_3$$

$$\frac{F}{l} = \frac{\mu_0 \times 10 \times 10}{2\pi \times 5 \times 10^{-2}} + \frac{\mu_0 \times 10 \times 10}{2\pi \times 10 \times 10^{-2}}$$

$$\frac{F}{l} = 6.4 \times 10^{-4} \text{ N}$$

And for wire w_2

$$\frac{F}{l} = \frac{F}{l} \text{ by wire } w_1 - \frac{F}{l} \text{ by wire } w_3$$

$$\frac{F}{l} = \frac{\mu_0 \times 10 \times 10}{2\pi \times 5 \times 10^{-2}} - \frac{\mu_0 \times 10 \times 10}{2\pi \times 5 \times 10^{-2}}$$

$$\frac{F}{l} = 0$$

Similarly for wire w_3

$$\frac{F}{l} = \frac{F}{l} \text{ by wire } w_1 + \frac{F}{l} \text{ by wire } w_2$$

$$\frac{F}{l} = \frac{\mu_0 \times 10 \times 10}{2\pi \times 5 \times 10^{-2}} + \frac{\mu_0 \times 10 \times 10}{2\pi \times 5 \times 10^{-2}}$$

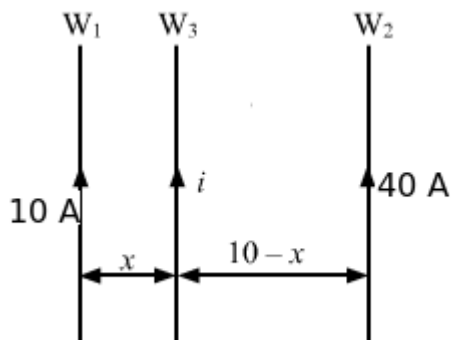
$$\frac{F}{l} = 6.4 \times 10^{-4} \text{ N}$$

28. Question

Two parallel wires separated by a distance of 10 cm carry currents of 10 A and 40 A along the same direction. Where should a third current be placed so that it experiences no magnetic force?

Answer

Let x be the distance of the wire W_3 from the wire carrying a current of 10A.



Magnetic force acting per unit length on a wire due to a parallel current-carrying wire is given by

$$\frac{F}{l} = \frac{\mu_0 i_1 i_2}{2\pi d}$$

In this case as the wire W_3 experiences no magnetic force.

$$\therefore \frac{F}{l} \text{ by wire } w_1 = \frac{F}{l} \text{ by wire } w_2$$

$$\Rightarrow \frac{\mu_0 10i}{2\pi x} = \frac{\mu_0 40i}{2\pi(10-x)}$$

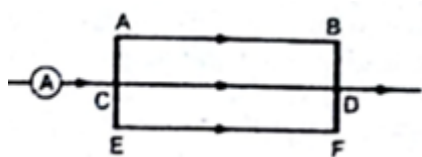
$$\Rightarrow 10 - x = 4x$$

$$\Rightarrow x = 2 \text{ cm}$$

Thus, wire W_3 is placed 2 cm from the 10 A current wire.

29. Question

Figure shows a part of an electric circuit. The wires AB, CD and EF are long and have identical resistances. The separation between the neighboring wires is 1.0 cm. The wires AE and BF have negligible resistance and the ammeter reads 30 A. Calculate the magnetic force per unit length of AB and CD.



Answer

Current will equally divide into three parts since wires AB, CD and EF have identical resistance, 10 A each.

So, the Magnetic force on wire AB due to wire CD is,

$$\frac{F_1}{l} = \frac{\mu_0 i_1 i_2}{2\pi d} = \frac{\mu_0 \times 10 \times 10}{2\pi \times 1 \times 10^{-2}} = \frac{\mu_0}{2\pi} \times 10^4 \text{ N/m}$$

and Magnetic force on wire AB due to wire EF is,

$$\frac{F_2}{l} = \frac{\mu_0 i_1 i_2}{2\pi d'} = \frac{\mu_0 \times 10 \times 10}{2\pi \times 2 \times 10^{-2}} = \frac{\mu_0}{4\pi} \times 10^4 \text{ N/m}$$

Total force per unit length,

$$\frac{F}{l} = \frac{F_1}{l} + \frac{F_2}{l} = \frac{3\mu_0}{4\pi} \times 10^4 = 3 \times 10^{-3} \frac{\text{N}}{\text{m}}$$

Now, magnetic force on wire CD due to wire AB,

$$\frac{F_1}{l} = \frac{\mu_0 i_1 i_2}{2\pi d} = \frac{\mu_0 \times 10 \times 10}{2\pi \times 1 \times 10^{-2}} = \frac{\mu_0}{2\pi} \times 10^4 \text{ N/m}$$

and magnetic force on wire CD due to wire EF is

$$\frac{F_2}{l} = \frac{\mu_0 i_1 i_2}{2\pi d} = \frac{\mu_0 \times 10 \times 10}{2\pi \times 1 \times 10^{-2}} = \frac{\mu_0}{2\pi} \times 10^4 \text{ N/m}$$

$$\frac{F}{l} = \frac{F_1}{l} + \frac{F_2}{l} = \frac{4\mu_0}{4\pi} \times 10^4 = 4 \times 10^{-3} \frac{\text{N}}{\text{m}}$$

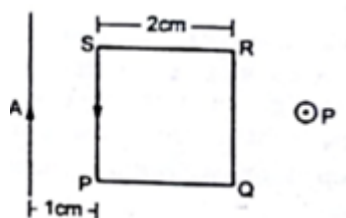
30. Question

A long, straight wire is fixed horizontally and carries a current of 50.0 A. A second wire having linear mass density $1.0 \times 10^{-4} \text{ kg m}^{-1}$ is placed parallel to and directly above this wire at a separation of 5.0 mm. What current should this second wire carry such that the magnetic repulsion can balance its weight?

. A square loop PQRS carrying a current of 6.0 A is placed near a long wire carrying 10 A as shown in the figure.

(a) Show that the magnetic force acting on the part PQ is equal and opposite to that on the part Rs.

(b) Find the magnetic force on the square loop.



Answer

The current in the coil is 10A

There will be induction of magnetic force due to two current carrying wire placed in each other's vicinity.

The force is given by

$$F_m = \frac{\mu_0 i_1 i_2}{2\pi d}$$

Where l is the length of the wire and $i_1 i_2$ are the currents flowing through the wires.

To balance the weight, the gravitational force should be equal to the magnetic force that is exerted.

Hence

$$F_g = \lambda \times g \times l$$

Where λ is the linear mass density of the wire.

Hence, equating these forces, we get,

$$\frac{\mu_0 i_1 i_2}{2\pi d} = \lambda \times g \times l$$

$$i_2 = \frac{\lambda \times g \times l \times 2\pi d}{\mu_0 i_1}$$

Substituting the numerical values, we get

$$i_2 = \frac{2 \times 10^{-7} \times 50}{5 \times 10^{-8}}$$

$$i_2 = \frac{9.8 \times 10^{-7}}{20 \times 10^{-7}} = 0.49A$$

31. Question

A square loop PQRS carrying a current of 6.0 A is placed near long wire carrying 10 A as shown in figure (35-E10). A Show that the magnetic force acting on the part

PQ is equal and opposite to that on the part RS. B Find the magnetic force on the square loop.

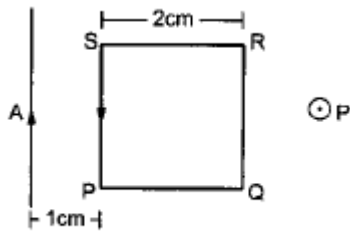


Figure 35-E10

Answer

Here, current in the wire is $I_1 = 10A$

Current in the loop, $I_2 = 6A$

Force on an element dx in the arm PQ a distance x away is

$$dF_{PQ} = \frac{\mu_0 I_1 I_2}{2\pi x} dx$$

$$\Rightarrow F_{PQ} = \int_1^3 \frac{\mu_0 I_1 I_2}{2\pi x} dx = \frac{\mu_0 I_1 I_2}{2\pi} \ln x \Big|_1^3 = 120 \times 10^{-7} [\log 3 - \log 1]$$

similarly, force on RS is,

$$\Rightarrow F_{RS} = \int_1^3 \frac{\mu_0 I_1 I_2}{2\pi x} dx = \frac{\mu_0 I_1 I_2}{2\pi} \ln x \Big|_1^3 = 120 \times 10^{-7} [\log 3 - \log 1]$$

Hence, $F_{PQ} = F_{RS}$

B distance of PS from wire, $x = 1cm$

distance of PS from RQ, $x' = 2cm$

Force on element PS is

$$F_{PS} = \frac{\mu_0 I_1 I_2}{2\pi x} - \frac{\mu_0 I_1 I_2}{2\pi x'}$$

$$F_{PS} = \frac{4\pi \times 10^{-7} \times 10 \times 6}{2\pi \times 10^{-2}} - \frac{4\pi \times 10^{-7} \times 6 \times 6}{2\pi \times 2 \times 10^{-2}} = 8.4 \times 10^{-6} N \text{ towards right}$$

Now distance of RQ from wire, $x = 3cm$

distance of RQ from PS, $x' = 2cm$

similarly, force on RQ is

$$F_{RQ} = \frac{\mu_0 I_1 I_2}{2\pi x} - \frac{\mu_0 I_1 I_2}{2\pi x'}$$

$$F_{RQ} = \frac{4\pi \times 10^{-7} \times 6 \times 10}{2\pi \times 3 \times 10^{-2}} - \frac{4\pi \times 10^{-7} \times 6 \times 6}{2\pi \times 2 \times 10^{-2}} = 7.6 \times 10^{-6} \text{ N towards left}$$

$$\text{Net force, } F = F_{RQ} + F_{PS} = (8.4 + 7.6) \times 10^{-6} = 16 \times 10^{-6} \text{ N}$$

32. Question

A circular loop of one turn carries a current of 5.00 A. If the magnetic field B at the centre is 0.200 mT, find the radius of the loop.

Answer

Given: No. of turns, $n = 1$ Magnitude of current, $i = 5.00 \text{ A}$ the magnetic field at the center due to the current in the loop is given by

We know that

$$B = \frac{\mu_0 n i}{2r}$$

Now,

Where $B = \text{Magnetic Field} = 0.2 \text{ mT}$

$r = \text{radius of loop}$

$$r = \frac{\mu_0 n i}{2B}$$

$$r = \frac{4\pi \times 10^{-7} \times 5}{2 \times 0.2 \times 10^{-3}}$$

$$r = 1.57 \times 10^{-2} \text{ m} = 1.57 \text{ cm}$$

33. Question

A current carrying circular coil of 100 turns and radius 5.0 cm produces a magnetic field of $6.0 \times 10^{-3} \text{ T}$ at its centre. Find the value of the current.

Answer

number of turns $n = 100$ turns

Radius $r = 5 \text{ cm} = 0.05 \text{ m}$

Magnetic field $B = 6.0 \times 10^{-3} \text{ T}$

Now,

$$B = \frac{n\mu_0 i}{2r}$$

$$6.0 \times 10^{-3} = \frac{100 \times 4\pi \times 10^{-7} \times i}{2 \times 0.05}$$

$$i = 4.7740 \text{ A}$$

34. Question

An electron makes 3×10^5 revolutions per second in a circle of radius 0.5 angstrom. Find the magnetic field B at the centre of the circle.

Answer

$$\text{no of revolutions} = 3 \times 10^5$$

$$\text{radius} = 0.5 \text{ angstrom} = 5 \times 10^{-11} \text{ m}$$

Thus, the magnetic field at the centre due to the current in the loop is given by

We know that,

$$B = \frac{\mu_0 n I}{2r}$$

Here $n=1$,

$$I = \frac{q}{t} = 1.6 \times 10^{-19}$$

$$\text{and } r = 5 \times 10^{-11} \text{ m}$$

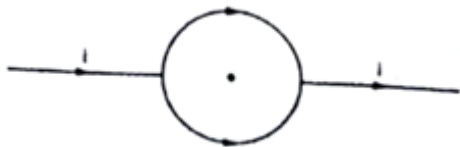
Hence,

$$B = \frac{4\pi \times 10^{-7} \times 1.6 \times 10^{-19}}{2 \times 0.5 \times 10^{-10} \times 3 \times 10^5}$$

$$B = 6 \times 10^{-10} \text{ T}$$

35. Question

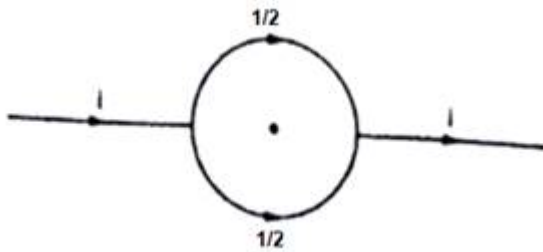
A conducting circular loop of radius a is connected to two long, straight wires. The straight wires carry a current i as shown in figure. Find the magnetic field B at the centre of the loop.



Answer

The current flows into the circular coil and using the KCL, splits equally between the two arms because both the arms are equal in length and are made up of the same material.

Hence the current in the upper section as well as the lower section will be $i/2$.



The current leaves the loop again and the resultant current leaving the coil will again be i .

The magnitude of the current is same and the direction is opposite at every point of time. Magnetic fields produced will hence also be zero as both the arms produce equivalent magnetic field which neutralize each other. Hence net magnetic field at the center will be 0

36. Question

Two circular coils of radii 5.0 cm and 10 cm carry equal currents of 2.0 A. The coils have 50 and 100 turns respectively and are placed in such a way that their planes as well as the centers coincide. Find the magnitude of the magnetic field B at the common centre of the coils if the currents in the coils are (a) in the same sense (b) in the opposite sense.

Answer

$$n_1 = 100$$

$$n_2 = 50$$

$$r_1 = 10 \text{ cm}$$

$$r_2 = 5 \text{ cm}$$



Both the coils have a common center and hence the effect due to the current flowing will depend on the direction of flow. We can calculate the individual results and then do the vector addition of both the forces to get the resultant magnetic fields

1) In the same sense

Field due to larger coil

$$B_1 = \frac{\mu_0 n_1 I}{2r_1}$$

Field due to smaller coil

$$B_l = \frac{\mu_0 n_s I}{2r_s}$$

As the direction is same and both the fields are acting in the same direction, that outside the plane of the paper, we can simply perform a scalar addition

$$B_t = \frac{\mu_0 n_l I}{2r_l} + \frac{\mu_0 n_s I}{2r_s}$$

$$B_t = \frac{\mu_0 I}{2} \left(\frac{n_s}{r_s} + \frac{n_l}{r_l} \right)$$

$$B_t = \frac{4 \times \pi \times 10^{-7}}{2} \left(\frac{100}{10 \times 10^{-2}} + \frac{50}{5 \times 10^{-2}} \right)$$

$$B = 25.12 \text{ mT}$$

Similarly for

2) Opposite sense

As the direction is opposite, we can simply perform scalar subtraction to get the resultant

$$B_t = \frac{\mu_0 n_l I}{2r_l} - \frac{\mu_0 n_s I}{2r_s}$$

$$B_t = \frac{\mu_0 I}{2} \left(\frac{n_s}{r_s} - \frac{n_l}{r_l} \right)$$

$$B_t = \frac{4 \times \pi \times 10^{-7}}{2} \left(\frac{100}{10 \times 10^{-2}} - \frac{50}{5 \times 10^{-2}} \right)$$

$$\therefore B_t = 0$$

37. Question

If the outer coil of the previous problem is rotated through 90° about a diameter, what would be the magnitude of the magnetic field B at the center?

Answer

Given, for outer circle,

Number of turns $n = 100$,

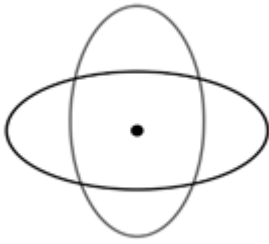
current, $I_o = 2A$

Radius, $r = 0.1m$

therefore, magnetic field is pointing west,

$$\vec{B}_o = \frac{\mu_0 n I}{2r} = \frac{4\pi \times 10^{-7} \times 2 \times 100}{2 \times 0.1} = 4\pi \times 10^{-4} \text{ T}$$

for inner circle,



Number of turns $n = 100$,

current, $I_o = 2A$

Radius, $r = 0.1m$

therefore, magnetic field is pointing downwards,

$$\vec{B}_i = \frac{\mu_0 n I}{2r} = \frac{4\pi \times 10^{-7} \times 2 \times 50}{2 \times 0.05} = 4\pi \times 10^{-4} \text{ T}$$

Therefore, total magnetic field is

$$B = \sqrt{B_o^2 + B_i^2} = \sqrt{2 \times (4 \times 10^{-4})^2} \approx 1.8 \times 10^{-3} \text{ T} \approx 1.8 \text{ mT}$$

38. Question

A circular loop of radius 20 cm carries a current of 10 A. An electron crosses the plane of the loop with a speed of $2.0 \times 10^6 \text{ m s}^{-1}$. The direction of motion makes an angle of 30° with the axis of the circle and passes through its center. Find the magnitude of the magnetic force on the electron at the instant it crosses the plane.

Answer

Given,

Radius, $r = 20 \text{ cm} = 0.2 \text{ m}$

Velocity of electron, $v = 2 \times 10^6 \text{ m/s}$

direction of motion, $\theta = 30^\circ$

Current in the loop, $I = 10 \text{ A}$

Therefore, force, $F = e(v \times B) = evB \sin \theta = ev \sin \theta \times \frac{\mu_0 I}{2r}$

$$F = 1.6 \times 10^{-19} \times 2 \times 10^6 \times \frac{4\pi \times 10^{-7}}{2 \times 0.2} \times \frac{1}{2} = 16\pi \times 10^{-19} N$$

39. Question

A circular loop of radius R carries a current I . Another circular loop of radius r ($r \ll R$) carries a current i and is placed at the center of the larger loop. The planes of the two circles are at right angle to each other. Find the torque acting on the smaller loop.

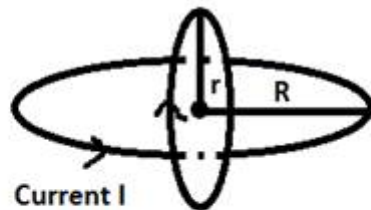
Answer

Given:

Current carried by circular loop of radius $R = I$

Current carried by circular loop of radius r ($r \ll R$) = i

Diagram:



Two loops of radius R and r carrying currents I and i respectively, passing through a common centre with their planes perpendicular to each other

Formula used:

Magnetic field at the center of a circular loop = $\mu_0 I / 2r$,

where I = current through the loop, r = radius of the circle,

$$\mu_0 = 4\pi \times 10^{-7} \text{ kgm s}^{-2} \text{ A}^{-2}$$

Magnetic torque acting on a loop of radius $r = I (A \times B)$, where I = current carried by the loop, A = area of loop, B = magnetic field

Therefore, magnetic field acting at the center of the loops due to the bigger loop of radius R (B) = $\mu_0 I / 2R$.

$$\text{Area of the smaller loop (A)} = \pi r^2$$

Hence, torque acting on the smaller loop = $I (A \times B)$, since current carried by smaller loop is i . Here A = area of smaller loop = πr^2 and B = magnetic field due to bigger loop.

Now, we know that $A \times B = |A||B|\sin\theta$, where θ is the angle between the area vector of the smaller loop and the magnetic field of the larger loop. $|A|$ = modulus value of area of smaller loop, $|B|$ = modulus value of magnetic field due to larger loop.

Since the planes of the two loops are perpendicular to each other, $\theta = 90^\circ$.

Hence, torque acting on the smaller loop = $i (|A||B|\sin 90^\circ) = i(|A||B|)$

We know, $|A| = \pi r^2$, $|B| = \frac{\mu_0 I}{2R}$

Hence, torque on smaller loop = $i \left(\pi r^2 \times \frac{\mu_0 I}{2R} \right) = \frac{\mu_0 I i \pi r^2}{2R}$ (Answer)

40. Question

A circular loop of radius r carrying a current i is held at the center of another circular loop of radius R ($\gg r$) carrying a current I . The plane of the smaller loop makes an angle of 30° with that of the larger loop. If the smaller loop is held fixed in this position by applying a single force at a point on its periphery, what would be the minimum magnitude of the force?

Answer

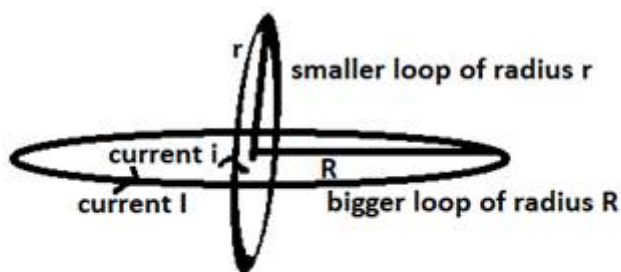
Given:

Current carried by circular loop of radius $r = i$

Current carried by circular loop of radius R ($\gg r$) = I

Angle between the planes of the two loops = 30°

Diagram:



Two loops of radius R and r , carrying currents I and i , with their planes making an angle of 30° with each other

Formula used:

Magnetic field at the center of a circular loop of radius r

$$M = \frac{\mu_0 I}{2r},$$

where

I = current through the loop,

r = radius of the circle,

$$\mu_0 = 4\pi \times 10^{-7} \text{ kgm s}^{-2} \text{ A}^{-2}$$

Magnetic torque acting on a loop of radius r

$$\boldsymbol{\tau} = I(\mathbf{A} \times \mathbf{B}) = \mathbf{F} \times \mathbf{r},$$

where

I = current carried by the loop,

A = area of loop,

B = magnetic field,

F = force on periphery

Therefore, magnetic field at the center of the loops due to the bigger loop(B) = $\frac{\mu_0 I}{2R}$

Area of the smaller loop(A) = πr^2

Therefore, torque on the smaller loop = i (A X B), since the smaller loop carries current i.

A = area of smaller loop, B = magnetic field due to bigger loop.

Now, we know that A X B = |A||B|sinθ,

Where

θ is the angle between the area vector of the smaller loop and the magnetic field of the larger loop.

|A| = magnitude of area of smaller loop,

|B| = magnitude of magnetic field due to larger loop.

Given:

The angle between the area vector of the smaller loop and the magnetic field of the larger loop is θ = 30°.

Hence, torque acting on the smaller loop = i(|A||B|sin 30°) = $\frac{i(|A||B|)}{2}$

We know, |A| = πr^2 , |B| = $\frac{\mu_0 I}{2R}$

Hence,

Torque on smaller loop due to magnetic field,

$$\frac{i \left(\pi r^2 \times \frac{\mu_0 I}{2R} \right)}{2} = \frac{\mu_0 I i \pi r^2}{4R}$$

Torque due to external force on its periphery,

$$\frac{\mu_0 I i \pi r^2}{4R}$$

Therefore

$$\frac{\mu_0 I i \pi r^2}{4R} = F \times r$$

Where

F = external force,

r = perpendicular distance between the line of action of force and the point at which the torque is acting.

Here, F and r are perpendicular to each other, since the force is being applied to the periphery.

Here r = radius of smaller loop.

$$\text{Hence, } \frac{\mu_0 I i \pi r^2}{4R} = F \times r$$

which gives us

$$F = \frac{\mu_0 I i \pi r}{4R} \text{ (Answer)}$$

41. Question

Find the magnetic field B due to a semicircular wire of radius 10.0 cm carrying a current of 5.0 A at its center of curvature.

Answer

Given:

Radius of semicircular wire(r) = 10 cm = 0.1 m

Current carried by it(i) = 5 A

Formula used:

Magnetic field at the center of a semicircular loop =

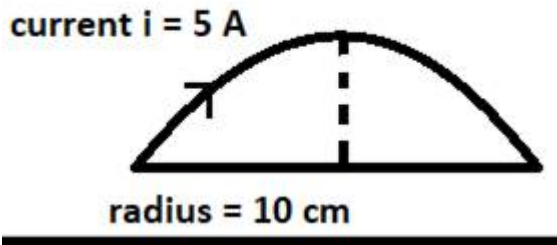
$$\frac{1}{2} \times \frac{\mu_0 i}{2r} = \frac{\mu_0 i}{4r}$$

Where

i = current carried by loop,

r = radius of loop,

μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7} \text{ kg m s}^{-2} \text{ A}^{-2}$



Hence, required magnetic field(B) = $\frac{\mu_0 \times 5}{4 \times 0.1}$ Tesla(T) ,

where $\mu_0 = 4\pi \times 10^{-7} \text{ kg m s}^{-2} \text{ A}^{-2}$

$$B = \frac{4\pi \times 10^{-7} \times 5}{4 \times 0.1} \text{ T} = 1.57 \times 10^{-5} \text{ T}$$

42. Question

A piece of wire carrying a current of 6.00 A is bent in the form of a circular arc of radius 10.0 cm, and it subtends an angle of 120° at the center. Find the magnetic field B due to this piece of wire at the center.

Answer

Given:

Radius of circular arc(r) = 10 cm = 0.1 m

Current carried by it(i) = 6 A

Angle subtended at the center = 120°

Formula used:

Magnetic field at the center of a circular arc,

$$\frac{\mu_0 i}{2r} \times \frac{\theta}{2\pi}$$

Where

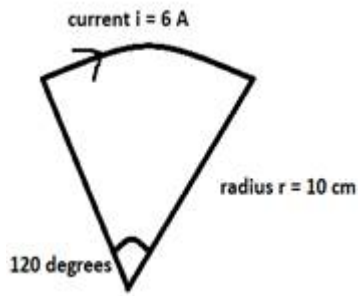
μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7} \text{ kg m s}^{-2} \text{ A}^{-2}$,

i = current carried by the wire,

r = radius of the wire,

θ = angle subtended at the center of the arc

Diagram:



Hence, required magnetic field

$$B = \frac{\mu_0 i}{4\pi r} (\theta)$$

where, $i = 6 \text{ A}$, $r = 0.1 \text{ m}$, $\theta = 120^\circ = 2\pi/3 \text{ radian}$, $\mu_0 = 4\pi \times 10^{-7} \text{ kg m s}^{-2} \text{ A}^{-2}$

$$\text{Hence, } B = \frac{4\pi \times 10^{-7} \times 6}{4\pi \times 0.1} \times \frac{2\pi}{3} = 1.256 \times 10^{-5} \text{ T} = 12.56 \text{ } \mu\text{T (Ans)}$$

43. Question

A circular loop of radius r carries a current i . How should a long, straight wire carrying a current $4i$ be placed in the plane of the circle so that the magnetic field at the center becomes zero?

Answer

For the magnetic field at the center to be zero, the magnetic field due to the circular loop has to be exactly equal and opposite to the long straight wire at the center.

Given:

Current carried by circular loop of radius $r = i$

Current carried by long straight wire = $4i$

Formula used:

Magnetic field at the center of a circular loop due to current in the loop (B_l) = $\frac{\mu_0 i}{2r}$, which points into the plane of the paper.

Here

μ_0 = magnetic permeability of vacuum,

i = current carried by the loop,

r = radius of the loop

Magnetic field due to an infinitely long wire at a distance x from it (B_w) = $\frac{\mu_0 i}{2\pi x}$, which points out of the plane of the paper.

Here

μ_0 = magnetic permeability of vacuum,

i = current carried by the wire,

x = distance of the center of the circle from the wire

Hence, from given information,

$$B_l = \frac{\mu_0 i}{2r}, \text{ and } B_w = \frac{\mu_0 \times 4i}{2\pi x}$$

where B_l = magnetic field due to loop, B_w = magnetic field due to wire, r = radius of loop, x = distance of the center of the circle from the wire

Hence, for the magnetic field at the center to be 0,

$$\mu_0 \frac{i}{2r} = \mu_0 \frac{4i}{2\pi x}$$

$$\Rightarrow \frac{1}{r} = \frac{4}{\pi x} \Rightarrow x = \frac{4r}{\pi}$$

Hence, for the magnetic field to be 0, the wire has to be placed at a distance of $4r/\pi$ from the center of the circular loop. (Answer)

44. Question

A circular coil of 200 turns has a radius of 10 cm and carries a current of 2.0 A.

(a) Find the magnitude of the magnetic field vector B at the center of the coil.

(b) At what distance from the center along the axis of the coil will the field B drop to half its value at the center?

$$(3\sqrt{4} = 1.5874)$$

Answer

Given:

Number of turns(n) = 200

Radius of circular coil(r) = 10 cm = 0.1 m

Current carried by the coil(i) = 2 A

Formula used:

(a) Magnetic field at the center of a circular coil of n turns(B) =

$$B = \frac{n\mu_0 i}{2r},$$

where

μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7} \text{ T m A}^{-1}$,

i = current carried by coil,

r = radius of coil

Hence, from the given information,

Magnetic field at the center of the given coil

$$= \frac{200 \times 4\pi \times 10^{-7} \times 2}{2 \times 0.1} \text{ T} = 2.51 \times 10^{-3} \text{ T} = 2.51 \text{ mT}$$

(b) The magnetic field at a distance point in the coil is given by B_p ,

$$\frac{n\mu_0 i r^2}{2(x^2 + r^2)^{\frac{3}{2}}}$$

The magnetic field at the center of the coil is $\frac{n\mu_0 i}{2r}$

Given: the magnetic field at the center is 1/2 of the initial

The magnetic field at the center of the coil is $\frac{n\mu_0 i}{4r}$

On equation the magnetic field at the center and the magnetic field at the any distant point from the coil, we get

$$\frac{n\mu_0 i}{4r} = \frac{n\mu_0 i r^2}{2(x^2 + r^2)^{\frac{3}{2}}}$$

On equating the above equation, we get

$$(r^2 + x^2)^3 = 4r^6$$

We get

$$x^2 + r^2 = 4^{1/3} r^2$$

$$x^2 = 0.58 r^2$$

$$x^2 = 0.5 \times 100 = 50$$

$$x = \pm 7.66 \text{ cm}$$

Magnetic field will drop to half of its value at the center if the distance of that point from the center of the coil along the axis of coil is equal to 7.66 cm.

45. Question

A circular loop of radius 4.0 cm is placed in a horizontal plane and carries an electric current of 5.0 A in the clockwise direction as seen from above. Find the

magnetic field.

(a) at a point 3.0 cm above the center of the loop

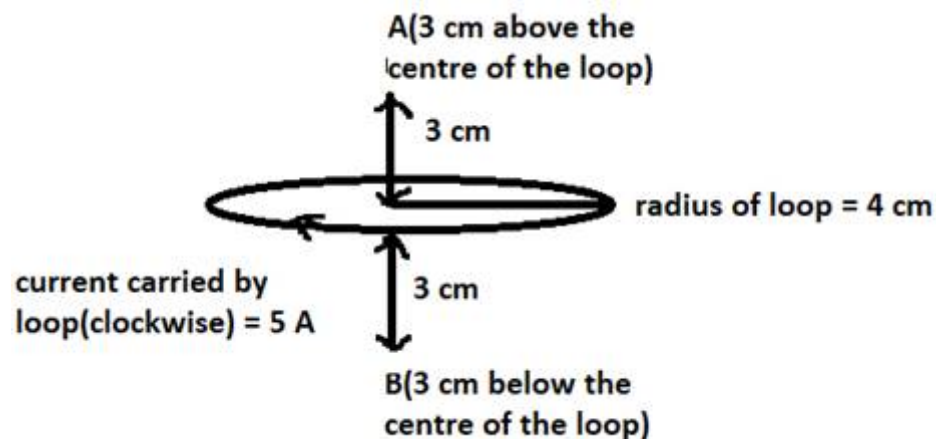
(b) at a point 3.0 cm below the center of the loop.

Answer

Given:

Radius of circular loop(r) = 4 cm = 0.04 m

Current carried by it(i) = 5 A



Formula used:

Magnetic field at a distance x from the center of a current carrying circular loop(B),

$$\frac{\mu_0 i r^2}{2(x^2 + r^2)^{3/2}}$$

Where

μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7} \text{ T m A}^{-1}$,

i = current carried by coil,

r = radius of coil

(a) To find the magnetic field 3 cm above the loop:

Magnetic field,

$$B = \frac{\mu_0 i r^2}{2(x^2 + r^2)^{3/2}}$$

Here, $x = 3 \text{ cm} = 0.03 \text{ m}$

Hence, substituting the given values:

$$B = \frac{(4\pi \times 10^{-7} \times 5 \times 0.042)}{\left(2 \times \frac{(0.032 + 0.042)3}{2}\right)} T$$

= **4.019 x 10⁻⁵ T** (in the downward direction, by right hand thumb rule) (Ans)

(b) To find the magnetic field 3 cm below the loop:

Here, all the values are same as the previous part. Hence,

B = 4.019 x 10⁻⁵ T (in the upward direction, by right hand thumb rule)

46. Question

A charge of 3.14×10^{-6} C is distributed uniformly over a circular ring of radius 20.0 cm. The ring rotates about its axis with an angular velocity of 60.0 rad s^{-1} . Find the ratio of the electric field to the magnetic field at a point on the axis at a distance of 5.00 cm from the center.

Answer

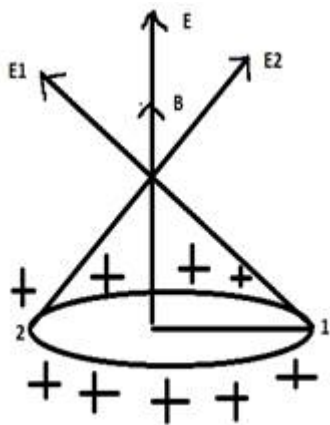
Given:

Charge of the ring(q) = 3.14×10^{-6} C

Radius of circular ring(r) = 20 cm = 0.2 m

Angular velocity with which the ring rotates(ω) = 60 rad s^{-1}

Diagram:



E_1 and E_2 denote the electric field due to two point elements 1 and 2 on the circumference of the circular loop. The resultant electric field due to them is E . Similarly, the resultant magnetic field due to them is B .

Formula used:

Electric field due to a circular loop(E),

$$\frac{qx}{4\pi\epsilon_0(x^2 + r^2)^{\frac{3}{2}}}$$

where q = charge carried by the ring, x = distance from the center, r = radius of the ring, ϵ_0 = electric permittivity of vacuum = $8.85 \times 10^{-12} \text{ N m}^2 \text{ C}^{-2}$

Magnetic field due to a current carrying loop(B),

$$\frac{\mu_0 i r^2}{2(x^2 + r^2)^{\frac{3}{2}}}$$

where μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7} \text{ T m A}^{-1}$, i = current carried by the loop, r = radius of the loop, x = distance from the center

$$\frac{E}{B} = \frac{\frac{qx}{4\pi\epsilon_0(x^2 + r^2)^{\frac{3}{2}}}}{\frac{\mu_0 i r^2}{2(x^2 + r^2)^{\frac{3}{2}}}} = \frac{2qx}{4\pi\epsilon_0\mu_0 i r^2}$$

Now, $i = q/t$, where i = current, q = charge, t = time taken.

Angular velocity(ω) = 60 rad/s (given)

Hence, time period of one revolution(T) = $2\pi/\omega = 2\pi/60 \text{ s}$

Hence, current(i) = charge per unit time = $q/T = 60q/2\pi = 30q/\pi \text{ A}$, where q = charge, T = time period of revolution

Therefore, $E/B = (2 \times 3.14 \times 10^{-6} \times 0.05 \times 9 \times 10^9 \times 3.14) / (4 \times 3.14 \times 10^{-7} \times 30 \times 3.14 \times 10^{-6} \times 0.2^2) = \mathbf{1.88 \times 10^{15} \text{ ms}^{-1}}$ (Ans)

47. Question

A thin but long, hollow, cylindrical tube of radius r carries a current i along its length. Find the magnitude of the magnetic field at a distance $r/2$ from the surface

(a) inside the tube

(b) outside the tube.

Answer

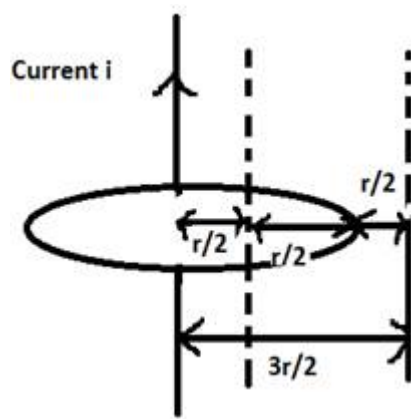
Formula used:

Ampere's circuital law states that the line integral of the magnetic field for a closed surface is μ_0 times the current enclosed by the surface.

$$\oint B \cdot dl = \mu_0 I$$

where B = magnetic field, dl = line element, μ_0 = magnetic permeability of vacuum, I = current enclosed.

Diagram:



(a) Now, since inside a conductor, the current enclosed is 0, in Ampere's circuital law, we put $I = 0$.

Hence, the magnetic field at a distance $r/2$ from the surface inside the tube is 0.
(Ans).

(b) Now, for any point outside the tube, the current enclosed will be given by Ampere's circuital law:

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

Where

B = magnetic field,

$d\mathbf{l}$ = line element,

μ_0 = magnetic permeability of vacuum,

I = current enclosed.

Now, since we have to find the magnetic field at a distance $r/2$ from the surface,

we consider an Amperian loop of radius $\left(r + \frac{r}{2}\right) = \frac{3r}{2}$ from the centre of the loop.

The total current enclosed will be i , since we are considering a loop outside the tube.

Hence,

$$\oint \mathbf{B} \cdot d\mathbf{l} = B \oint d\mathbf{l} = B \times 2\pi \times \frac{3r}{2} = \mu_0 \times i,$$

since $\oint d\mathbf{l} = 2\pi \times \frac{3r}{2}$ = circumference of the Amperian loop.

Hence, we get, B (at a distance $r/2$ from the surface outside the tube) = $\frac{\mu_0 i}{3\pi r}$ (Ans)

48. Question

A long, cylindrical tube of inner and outer radii a and b carries a current i distributed uniformly over its cross section. Find the magnitude of the magnetic field at a point

(a) just inside the tube

(b) just outside the tube

Answer

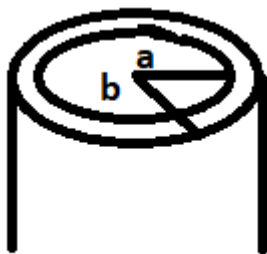
Given:

Inner radii of tube = a

Outer radii = b

Current distributed over its cross section = i

Diagram:



Formula used:

Ampere's circuital law states that the line integral of the magnetic field for a closed surface is μ_0 times the current enclosed by the surface.

$$\oint B \cdot dl = \mu_0 I$$

Where

B = magnetic field, dl = line element, μ_0 = magnetic permeability of vacuum, I = current enclosed.

(a) Inside a conducting tube, I (current enclosed) is 0.

Hence, by Ampere's circuital law, $\oint B \cdot dl = 0 \Rightarrow B = 0$.

Hence, the magnetic field just inside the tube is 0.

(b) Just outside the tube, the distance from the center is b .

Hence, we consider an Amperian loop of radius b from the center.

Hence, from Ampere's circuital law:

$$\oint B \cdot dl = \mu_0 I$$

Where B = magnetic field,

dl = line element, μ_0 = magnetic permeability of vacuum, I = current enclosed.

In this case, $\oint dl = 2\pi b$ (circumference of the loop of radius b)

Therefore, $B \times 2\pi b = \mu_0 i$ (since the total current enclosed is i).

Hence, the magnetic field at a point just outside the tube = $\mu_0 i / 2\pi b$ (Ans)

49. Question

A long, cylindrical wire of radius b carries a current i distributed uniformly over its cross section. Find the magnitude of the magnetic field at a point inside the wire at a distance a from the axis.

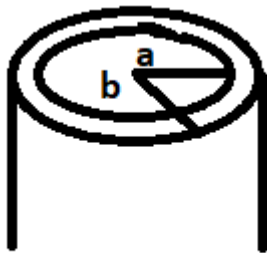
Answer

Given:

Radius of wire = b

Current distributed throughout the cross section = i

Diagram:



Formula used:

Ampere's circuital law states that the line integral of the magnetic field for a closed surface is μ_0 times the current enclosed by the surface.

$\oint B \cdot dl = \mu_0 I$, where B = magnetic field, dl = line element, μ_0 = magnetic permeability of vacuum, I = current enclosed.

Since the current i is uniformly distributed throughout the cross section of the wire,

For an amperian loop of radius a ($a < b$) from the center of the wire,

I (current enclosed)

$$\left(\frac{i}{\pi b^2} \right) \times \pi a^2 = \frac{ia^2}{b^2}$$

Where

i = total current distributed throughout the wire,

b = radius of the wire,

a = distance from the axis at which the magnetic field is to be found.

Hence, from Ampere's circuital law,

$$\oint \mathbf{B} \cdot d\mathbf{l} = B \times 2\pi a = \mu_0 I = \frac{\mu_0 i a^2}{b^2}$$

since $2\pi a$ = circumference of Amperian loop of radius a, and I = enclosed current = $i a^2 / b^2$.

Therefore, magnetic field at a distance a from the axis inside the wire, $\frac{\mu_0 i a}{2\pi b^2}$

50. Question

A solid wire of radius 10 cm carries a current of 5.0 A distributed uniformly over its cross section. Find the magnetic field B at a point at a distance.

(a) 2 cm

(b) 10 cm and

(c) 20 cm away from the axis. Sketch a graph of B versus x for $0 < x < 20$ cm.

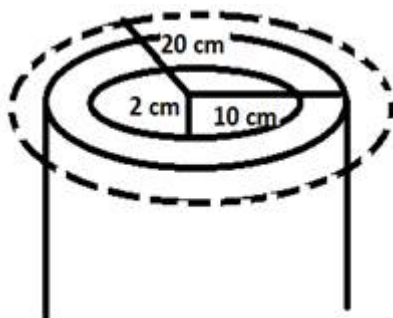
Answer

Given:

Radius of wire (r) = 10 cm = 0.1 m

Current carried by it (i) = 5 A

Diagram:



Formula used:

Ampere's circuital law states that the line integral of the magnetic field for a closed surface is μ_0 times the current enclosed by the surface.

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

Where B = magnetic field, dl = line element, μ_0 = magnetic permeability of vacuum, I = current enclosed.

(a) Since the current is uniformly distributed over the cross section of the wire, at a distance of 2 cm (0.02 m) from the axis (inside the wire), I (current enclosed)

$$= \left(\frac{5}{\pi(0.1)^2} \right) \times (\pi(0.02)^2)$$

Where,

$A = 0.2$ A, where 0.1 m is the radius of the wire.

Hence, from Ampere's circuital law,

$$\oint B \cdot dl = \mu_0 I$$

where μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7}$ T m A⁻¹, B = magnetic field, dl = line element, I = current enclosed

Substituting the values, we get

$$B \times 2\pi \times 0.02 = 4\pi \times 10^{-7} \times 0.2 \text{ (since } 2\pi \times 0.02 = \text{circumference of the loop of radius 2 cm)}$$

=> Magnetic field at a distance of 2 cm from the axis

$$= 2 \times 10^{-6} \text{ T} = 2\mu\text{T (Ans)}$$

(b) At a distance of 10 cm from the axis, we are basically on the surface of the wire. Hence the whole current of 5 A is enclosed by it.

$$\text{Hence, from Ampere's circuital law, } \oint B \cdot dl = \mu_0 I$$

where μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7}$ T m A⁻¹, B = magnetic field, dl = line element, I = current enclosed

Here, $I = 5$ A.

Substituting the values, we get

$$B \times 2\pi \times 0.1 = 4\pi \times 10^{-7} \times 5 \text{ (since } 2\pi \times 0.1 \text{ is the circumference of the loop of radius 10 cm)}$$

=> Magnetic field at a distance of 10 cm from the axis

$$= 10^{-5} \text{ T} = 10\mu\text{T (Ans)}$$

(c) At a distance of 20 cm from the axis, we are outside the wire. Hence the whole current of 5 A is enclosed by it.

$$\text{Hence, from Ampere's circuital law, } \oint B \cdot dl = \mu_0 I$$

where μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7} \text{ T m A}^{-1}$, B = magnetic field, dl = line element, I = current enclosed

Here, $I = 5 \text{ A}$.

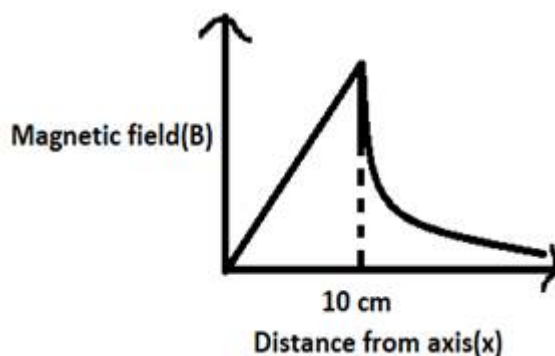
Substituting the values, we get

$$B \times 2\pi \times 0.2 = 4\pi \times 10^{-7} \times 5 \text{ (since } 2\pi \times 0.2 \text{ is the circumference of the loop of radius 20 cm)}$$

=> Magnetic field at a distance of 20 cm from the axis

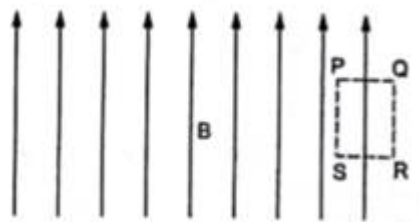
$$= 5 \times 10^{-6} \text{ T} = 5 \mu\text{T (Ans)}$$

Graph for B vs x for $0 < x < 20 \text{ cm}$:



51. Question

Sometimes we show an idealized magnetic field which is uniform in a given region and falls to zero abruptly. One such field is represented in figure. Using Ampere's law over the path PQRS, show that such a field is not possible.



Answer

Half of the given loop is in a region with zero magnetic field, while half of it is in a region with magnetic field B .

Now, we take the Amperian loop PQRS.

Ampere's circuital law states that the line integral of the magnetic field for a closed surface is μ_0 times the current enclosed by the surface.

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

where

B = magnetic field,

dl = line element,

μ_0 = magnetic permeability of vacuum,

I = current enclosed.

Hence, I (current enclosed) is not 0, but is finite (since half of it lies in a region of magnetic field B). Hence, this states the magnetic field inside the loop PQRS is not zero.

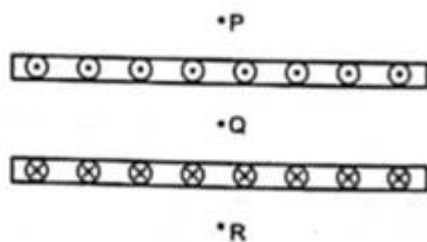
However, it is obvious that the magnetic field at any point in the region outside the magnetic field lines = 0.

These are two contradictory statements.

Hence, such a field is not possible. (proved)

52. Question

Two large metal sheets carry surface currents as shown in figure. The current through a strip of width dl is Kdl where K is a constant. Find the magnetic field at the points P, Q and R.



Answer

Ampere's circuital law states that the line integral of the magnetic field for a closed surface is μ_0 times the current enclosed by the surface.

$\oint B \cdot dl = \mu_0 I$, where B = magnetic field, dl = line element, μ_0 = magnetic permeability of vacuum, I = current enclosed.

(i) At point P, the current enclosed(I) = 0.

Hence, from Ampere's circuital law, $\oint B \cdot dl = 0 \Rightarrow B$ (magnetic field at P) = 0. (Ans)

(ii) Point Q is in between the two strips carrying current.

Hence, from Ampere's circuital law, we consider only $B \cdot dl$ for the strip of width dl carrying surface current K . Hence, I (current enclosed by the strip) = Kdl .

Therefore, $B \cdot dl = \mu_0 Kdl \Rightarrow B$ (magnetic field at Q) = $\mu_0 K$ (Ans)

(iii) At point Q, again, current enclosed(I) = 0.

Hence, from Ampere's circuital law, $\oint B \cdot dl = 0 \Rightarrow B$ (magnetic field at Q) = 0. (Ans)

53. Question

Consider the situation of the previous problem. A particle having charge q and mass m is projected from the point Q in a direction going into the plane of the diagram. It is found to describe a circle of radius r between the two plates. Find the speed of the charged particle.

Answer

Given:

Charge of the particle = q

Mass of the particle = m

Radius of circle described by the particle between the two plates = r

Formula used:

Ampere's circuital law states that the line integral of the magnetic field for a closed surface is μ_0 times the current enclosed by the surface.

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

where B = magnetic field, dl = line element, μ_0 = magnetic permeability of vacuum, I = current enclosed.

Now, from the previous problem, we found out that the magnetic field(B) at point Q is $\mu_0 K$, where μ_0 = magnetic permeability of vacuum, K = surface current.

Also, we know that the velocity of a particle in a magnetic field is given as

$$v = \frac{qBr}{m}$$

Where

q = charge of particle, B = magnetic field, r = radius of circle described by particle, m = mass of particle.

Substituting the value of B , we get

Velocity of the particle is $q\mu_0 K \frac{r}{m}$

54. Question

The magnetic field B inside a long solenoid, carrying a current of 5.00 A, is 3.14×10^{-2} T. Find the number of turns per unit length of the solenoid.

Answer

Given:

Current(i) = 5 A

Magnetic field(B) = 3.14×10^{-2} T

Formula used:

Magnetic field inside a long solenoid $B = \mu_0 ni$,

Where

μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7} \text{ T m A}^{-1}$, n = number of turns per unit length, i = current carried by the wire

Hence, from the given information:

$$3.14 \times 10^{-2} = 4\pi \times 10^{-7} \times n \times 5$$

$$\Rightarrow n = 5000 \text{ turns/m}$$

Therefore, number of turns per unit length of the solenoid = **5000(Ans)**

55. Question

A long solenoid is fabricated by closely winding a wire of radius 0.5 mm over a cylindrical non-magnetic frame so that the successive turns nearly touch each other. What would be the magnetic field B at the center of the solenoid if it carries a current of 5A?

Answer

Given:

Radius of cylinder(r) = 0.5 mm = $5 \times 10^{-4} \text{ m}$

Current(i) = 5 A

Formula used:

Magnetic field inside a long solenoid $B = \mu_0 ni$,

where

μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7} \text{ T m A}^{-1}$, n = number of turns per unit length, i = current carried by the wire

Now, width of each turn = diameter of the wire = $(0.5 \times 2) \text{ mm} = 1 \text{ mm} = 10^{-3} \text{ m}$.

Hence, number of turns per unit length(n) (per m) = $10^3 = 1000$

Therefore, substituting the values:

B (magnetic field at the center) = $4\pi \times 10^{-7} \times 1000 \times 5 \text{ T}$

= **$6.28 \times 10^{-3} \text{ T}$ (Ans)**

56. Question

A copper wire having resistance 0.01 ohm in each meter is used to wind a 400-turn solenoid of radius 1.0 cm and length 20 cm. Find the emf of a battery which when connected across the solenoid will cause a magnetic field of 1.0×10^{-2} T near the center of the solenoid.

Answer

Given:

Resistance per unit length(R_0) = 0.01 ohm

Total number of turns(N) = 400

Radius of the wire(r) = 1 cm = 0.01 m

Length of the wire(l) = 20 cm = 0.2 m

Magnetic field near the center(B) = 1.0×10^{-2} T

Formula used:

Magnetic field at the center of a solenoid, $B = \mu_0 ni$,

Where

μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7}$ T m A⁻¹,

n = number of turns per unit length,

i = current carried by the wire

Emf(E) = iR ,

where i = current, R = resistance

Now, n (number of turns per unit length) = Total number of turns(N)/Length of wire(l) = $\left(\frac{400}{0.2}\right) = 2000$

Total resistance (R of all turns) = $R_0 \times 2\pi r \times 400 = (0.01 \times 2\pi \times 0.01 \times 400) = 0.25$ ohm

Now, substituting the given values:

$$1.0 \times 10^{-2} = 4\pi \times 10^{-7} \times 2000 \times i \Rightarrow i = 3.98 \text{ A}$$

Therefore, Emf(E) = $iR = (3.98 \times 0.25) = 0.995$ V which is almost equal to 1 V.

Emf of the battery = **1 V** (Ans)

57. Question

A tightly-wound solenoid of radius a and length l has n turns per unit length. It carries an electric current i . Consider a length dx of the solenoid at a distance x

from one end. This contains $n \, dx$ turns and may be approximated as a circular current $i \, n \, dx$.

(a) Write the magnetic field at the center of the solenoid due to this circular current. Integrate this expression under proper limits to find the magnetic field at the center of the solenoid.

(b) Verify that if $l \gg a$, the field tends to $B = \mu_0 n i$ and if $a \gg l$, the field tends to

$B = \frac{\mu_0 n i l}{2a}$. Interpret these results

Answer

Given

Radius of solenoid is a

Length of the solenoid is l

Number of turns per unit length is n

The current in the circular loop is $i \, n \, dx$

The magnetic field due to the circular ring at the any distance from the ring is

$$B = \frac{\mu_0 I r^2}{4\pi(x^2 + r^2)^{\frac{3}{2}}}$$

Where

μ_0 is the permeability of the free space,

I is the current in the ring

r is the radius of the loop

x is the distance from the loop at which the magnetic field to be found.

Integrating the above equation for $x=0$ and r

$$B = \int dB = \int_0^l \frac{\mu_0 n i a^2 dx}{4\pi \left(\left(\frac{l}{2} - x \right)^2 + a^2 \right)^{\frac{3}{2}}}$$

$$B = \int_0^l \frac{\mu_0 n i a^2 dx}{4\pi \left(\left(\frac{l}{2} - x \right)^2 + a^2 \right)^{\frac{3}{2}}} = \frac{\mu_0 n i}{4\pi} \int_0^l \frac{a^2 dx}{a^3 \left(\left(\frac{l-2x}{2a} \right)^2 + 1 \right)^{\frac{3}{2}}}$$

On solving the above equation, we get

$$B = \frac{\mu_0 nI}{4\pi a} \int_0^1 \frac{dx}{\left(\left(\frac{1-2x}{2a}\right)^2 + 1\right)^{\frac{3}{2}}} = \frac{\mu_0 nI}{4\pi a} \times \frac{4\pi a}{\left(1 + \left(\frac{2a}{l}\right)^2\right)^{1/2}}$$

The magnetic field at the center of the solenoid due to this circular current

$$B = \frac{\mu_0 nI}{\left(1 + \left(\frac{2a}{l}\right)^2\right)^{1/2}} \quad (i)$$

(b) When $l \gg a$

Putting in the equation (i), we get

$$\frac{2a}{l} \ll 1 \text{ or } \left(1 + \left(\frac{2a}{l}\right)^2\right)^{\frac{1}{2}} = 1$$

Putting the values in the above formula, we get

$$B = \mu_0 n I$$

When $a \gg l$

Putting in the equation (i), we get

$$\frac{2a}{l} \gg 1 \text{ or } \left(1 + \left(\frac{2a}{l}\right)^2\right)^{\frac{1}{2}} = \frac{2a}{l}$$

Putting the above value in the equation (i), we get

$$B = \frac{\mu_0 nIl}{2a}$$

Hence, proved if $l \gg a$, the field tends to $B = \mu_0 ni$ and

if $a \gg l$, the field tends to $B, \frac{\mu_0 nil}{2a}$.

58. Question

A tightly-wound, long solenoid carries a current of 2.00 A. An electron is found to execute a uniform circular motion inside the solenoid with a frequency of $1.00 \times 10^8 \text{ rev s}^{-1}$. Find the number of turns per meter in the solenoid.

Answer

Given:

$$\text{Current}(i) = 2 \text{ A}$$

Frequency(f) = $1.00 \times 10^8 \text{ rev s}^{-1}$.

Formula used:

Magnetic field inside a solenoid(B) = $\mu_0 ni$,

Where

μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7} \text{ T m A}^{-1}$,

n = number of turns per unit length,

i = current carried by the wire

Now, charge of electron(q) = $1.6 \times 10^{-19} \text{ C}$

Mass of electron(m) = $9.1 \times 10^{-31} \text{ kg}$

Now, frequency of the particle in uniform circular motion in a magnetic field is given by

$$f = \frac{qB}{2\pi m},$$

Where

q is the charge,

B is the magnetic field,

m is the mass of particle

Hence, $B = \frac{2\pi mf}{q}$

Substituting the given values, we obtain B as,

$$\frac{2 \times 3.14 \times 9.1 \times 10^{-31} \times 1.00 \times 10^8}{1.6} \times 10^{-19} = 0.0036 \text{ T}$$

Hence, $B = \mu_0 ni$,

Where

μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7} \text{ T m A}^{-1}$,

n = number of turns per unit length,

i = current carried by the wire

=> Number of turns per meter(n) = $\frac{B}{\mu_0 i} = 0.0036 / (4\pi \times 10^{-7} \times 2) = \mathbf{1420 \text{ turns/meter. (Ans)}}$

59. Question

A tightly-wound, long solenoid has n turns per unit length, a radius r and carries a current i . A particle having charge q and mass m is projected from a point on the axis in a direction perpendicular to the axis. What can be the maximum speed for which the particle does not strike the solenoid?

Answer

Given:

Number of turns per unit length = n

Current = i

Radius = r

Charge of particle = q

Mass = m

Formula used:

Magnetic field inside a solenoid(B) = $\mu_0 ni$,

where

μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7} \text{ T m A}^{-1}$,

n = number of turns per unit length,

i = current carried by the wire

Now, when a particle is projected perpendicular to a magnetic field, it describes a circle. Now, for the particle to not strike to solenoid, the required radius is $r/2$.

Since it is moving in a circular path, centripetal acceleration = mv^2/r ,

Where

m = mass of particle,

v = velocity,

r = radius

Force due to the magnetic field = qvB ,

Where

q = charge of particle,

v = velocity,

B = magnetic field.

Here, $r \Rightarrow r/2$.

The centripetal and magnetic forces balance each other.

Hence, using the given data:

$$\frac{mv^2}{r} = qv\mu_0 ni \Rightarrow v = q\mu_0 n i r / 2m \text{ (Ans)}$$

60. Question

A tightly-wound, long solenoid is kept with its axis parallel to a large metal sheet carrying a surface current. The surface current through a width dl of the sheet is Kdl and the number of turns per unit length of the solenoid is n . The magnetic field near the center of the solenoid is found to be zero.

(a) Find the current in the solenoid.

(b) If the solenoid is rotated to make its axis perpendicular to the metal sheet, what would be the magnitude of the magnetic field near its center?

Answer

Given:

Number of turns per unit length = n

Surface current of sheet = K

Formula used:

Ampere's circuital law states that the line integral of the magnetic field for a closed surface is μ_0 times the current enclosed by the surface.

$$\oint B \cdot dl = \mu_0 I$$

Where

B = magnetic field,

dl = line element,

μ_0 = magnetic permeability of vacuum,

I = current in the circuit.

(a) Since the magnetic field near the center of the solenoid is 0, we infer that the magnetic field due to the solenoid = magnetic field due to the sheet

For the sheet, using Ampere's circuital law, we get:

$B \times 2l = \mu_0 \times Kl$, where Kl = current enclosed by sheet and we take $2l$ because of the two surfaces of the sheet.

$$\Rightarrow B_{\text{solenoid}} = \mu_0 K / 2$$

Now, magnetic field due to a solenoid $B_{\text{solenoid}} = \mu_0 ni$, where μ_0 = magnetic permeability of vacuum, i = current, n = number of turns per unit length

Hence, $\mu_0 K/2 = \mu_0 ni \Rightarrow i$ (current in solenoid) = **$K/2n$** (Ans)

(b) Now, if the axes of the solenoid and the plate are perpendicular to each other, their magnetic fields are also perpendicular to each other.

Hence, net magnetic field at center,

$$\sqrt{\left(\frac{\mu_0 K}{2}\right)^2 + \left(\frac{\mu_0 K}{2}\right)^2} = \frac{\mu_0 K}{\sqrt{2}}$$

61. Question

A capacitor of capacitance $100 \mu\text{F}$ is connected to a battery of 20 volts for a long time and then disconnected from it. It is now connected across a long solenoid having 4000 turns per meter. It is found that the potential difference across the capacitor drops to 90% of its maximum value in 2.0 seconds. Estimate the average magnetic field produced at the center of the solenoid during this period.

Answer

Given:

Capacitance(C) = $100 \mu\text{F} = 10^{-4} \text{ F}$

Initial Voltage of battery(V) = 20 V

Number of turns per meter of solenoid(n) = 4000

Final potential difference(V') = 90% of 20 V = 18 V

Time taken(t) = 2 s

Formula used:

Initial charge stored in capacitor(Q) = CV ,

where C = capacitance, V = potential difference

Therefore $Q = (10^{-4} \times 20) \text{ C} = 2 \times 10^{-3} \text{ C}$

New potential difference = $V' = 18 \text{ V}$

Therefore, New charge $Q' = CV' = (10^{-4} \times 18) = 1.8 \times 10^{-3} \text{ C}$

Hence, current flowing in conductor(i) = $(Q - Q')/t$, where Q = initial charge, Q' = final charge, t = time taken

Therefore, $i = (2.0 - 1.8) \times 10^{-3} / 2 \text{ A} = 10^{-4} \text{ A}$

Thus, average magnetic field at the center of solenoid = $B = \mu_0 ni$,

where μ_0 = magnetic permeability of vacuum = $4\pi \times 10^{-7} \text{ T m A}^{-1}$, n = number of turns per unit length, i = current carried by the wire

Hence, from given data, $B = (4\pi \times 10^{-7} \times 4000 \times 10^{-4}) \text{ T}$

= **$0.5 \times 10^{-6} \text{ T}$** (Ans)