Chapter 5 Quadratic Functions

Ex 5.5

Answer 1e.

The remainder theorem deals with the remainder obtained when a polynomial is divided by a divisor of the form $x - k$.

By the theorem, when a polynomial $f(x)$ is divided $x - k$, the remainder obtained is $r = f(k)$.

Answer 1gp.

When a polynomial $f(x)$ is divided by a divisor $d(x)$, we obtain a quotient $q(x)$ and a remainder $r(x)$ such that $\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$.

In polynomial long division, include 0 as the coefficient of the missing terms in the dividend. The x^2 term is missing in the given dividend. Use 0 as its coefficient.

$$x^2 + 2x - 1\overline{)2x^4 + x^3 + 0x^2 + x - 1}$$

Then divide the term with highest power in what is left of the dividend at each stage by the first term of the divisor.

Multiply the divisor by $\frac{2x^4}{x^2}$, or $2x^2$.

$$\begin{array}{r} 2x^2 \\ x^2 + 2x - 1\overline{)2x^4 + x^3 + 0x^2 + x - 1} \\ 2x^4 + 4x^3 - 2x^2 \qquad\qquad\quad \end{array}$$

Subtract the corresponding terms and bring down the next term.

$$\begin{array}{r} 2x^2 \\ x^2 + 2x - 1\overline{)2x^4 + x^3 + 0x^2 + x - 1} \\ \underline{2x^4 + 4x^3 - 2x^2} \qquad\qquad\quad \\ -3x^3 + 2x^2 + x \qquad \end{array}$$

Now, multiply the divisor by $\frac{-3x^3}{x^2}$, or $-3x$.

$$\begin{array}{r l}
 & 2x^2 - 3x \\
x^2 + 2x - 1 \big) & 2x^4 + x^3 + 0x^2 + x - 1 \\
 & \underline{2x^4 + 4x^3 - 2x^2} \\
 & \quad -3x^3 + 2x^2 + x \\
 & \quad -3x^3 - 6x^2 + 3x
\end{array}$$

Subtract the terms and bring down the next term.

$$\begin{array}{r l}
 & 2x^2 - 3x + 8 \\
x^2 + 2x - 1 \big) & 2x^4 + x^3 + 0x^2 + x - 1 \\
 & \underline{2x^4 + 4x^3 - 2x^2} \\
 & \quad -3x^3 + 2x^2 + x \\
 & \quad \underline{-3x^3 - 6x^2 + 3x} \\
 & \qquad\qquad 8x^2 - 2x - 1
\end{array}$$

Repeat the process by multiplying the divisor by $\frac{8x^2}{x^2}$, or 8.

$$\begin{array}{r l}
 & 2x^2 - 3x + 8 \\
x^2 + 2x - 1 \big) & 2x^4 + x^3 + 0x^2 + x - 1 \\
 & \underline{2x^4 + 4x^3 - 2x^2} \\
 & \quad -3x^3 + 2x^2 + x \\
 & \quad \underline{-3x^3 - 6x^2 + 3x} \\
 & \qquad\qquad 8x^2 - 2x - 1 \\
 & \qquad\qquad \underline{8x^2 + 16x - 8} \\
 & \qquad\qquad\quad -18x + 7
\end{array}$$

Thus, $\frac{2x^4 + x^3 + x - 1}{x^2 + 2x - 1} = 2x^2 - 3x + 8 + \frac{-18x + 7}{x^2 + 2x - 1}$.

Check the result by multiplying the quotient by the divisor and adding the remainder. The result should be the dividend.

$$\left(2x^2-3x+8\right)\left(x^2+2x-1\right)+\left(-18x+7\right)$$
$$=2x^2\left(x^2+2x-1\right)-3x\left(x^2+2x-1\right)+8\left(x^2+2x-1\right)-18x+7$$
$$=2x^4+4x^3-2x^2-3x^3-6x^2+3x+8x^2+16x-8-18x+7$$
$$=2x^4+x^3+x-1$$

The solution checks.

Answer 1mr.

a. The scientific notation is of the form $c\times10^n$, where $1\le c<10$ and n is an integer.

The given number is positive. Move the decimal point in the given number 11 places to the left so as to obtain a number that is greater than or equal to 1 and less than 10.

Since the decimal point is moved 11 places to the left, the exponent on the base 10 will be 11. Thus, the distance between the sun and Earth in scientific notation is 1.64×10^{11} yards.

b. Let n be the number of football fields required.

The value of n can be determined by dividing the total distance between Earth and the sun by the length of a football field.

$$n=\frac{1.64\times10^{11}}{1.20\times10^2}$$

Rewrite the quotient as a product of two fractions.

$$\frac{1.64\times10^{11}}{1.20\times10^2}=\frac{1.64}{1.20}\times\frac{10^{11}}{10^2}$$

Simplify.

$$\frac{1.64}{1.20}\times\frac{10^{11}}{10^2}\approx1.36\times\frac{10^{11}}{10^2}$$

Apply the quotient of powers property on $\frac{10^{11}}{10^2}$.

Subtract the exponents keeping the base.

$$1.36 \times \frac{10^{11}}{10^2} = 1.36 \times 10^{11-2}$$
$$= 1.36 \times 10^9$$

Therefore, about 1,360,000,000 football fields have to be stretched end-to-end so as to reach from Earth to the sun.

Answer 2e.

Consider the polynomial

$$f(x) = x^4 - 5x^2 + 8x - 2$$

And $f(x) = x^4 - 5x^2 + 8x - 2$ is divided by $x+3$ with the use of syntactic division

That is

$$\begin{array}{r|rrrrr} 3 & 1 & 0 & -5 & 8 & -2 \\ & & -3 & 9 & -12 & 12 \\ \hline & 1 & -3 & 4 & -4 & 10 \end{array}$$

In the above synthetic division, red colored numbers $1, -3, 4$ and -4 represents the coefficients of the quotient which we get after dividing $f(x) = x^4 - 5x^2 + 8x - 2$ by $x+3$ and the blue colored number 10 represents the remainder of the division.

Therefore on dividing $f(x) = x^4 - 5x^2 + 8x - 2$ by $x+3$,

From the syntactic division,

$$\frac{x^4 - 5x^2 + 8x - 2}{x+3} = x^3 - 3x^2 + 4x - 4 + \frac{10}{x+3}$$

Answer 2gp.

Consider the expression

$$x^3 - x^2 + 4x - 10 \div x + 2$$

Divide $x^3 - x^2 + 4x - 10$ by $x+2$ with use of polynomial long division.

Write polynomial division in the same format use when dividing numbers. At each stage, divide the term with the highest power in what is left of the dividend by the first term of the divisor. This gives the next term of the quotient.

Thus,

$$\begin{array}{r l} x^2-3x+10 & \\ x+2\overline{)\,x^3-x^2+4x-10} & \square\ \text{Quotient} \\ \underline{x^3+2x^2} & \text{Multiply divisor by } \frac{x^3}{x}=x^2 \\ -3x^2+4x & \text{Subtract} \\ \underline{-3x^2-6x} & \text{Multiply divisor by } \frac{-3x^2}{x}=-3x \\ 10x-10 & \text{Subtract} \\ \underline{10x+20} & \text{Multiply divisor by } \frac{10x}{x}=10 \\ -30 \leftarrow & \text{Remainder} \end{array}$$

Therefore

$$\frac{x^3-x^2+4x-10}{x+2}=\boxed{x^2-3x+10-\frac{30}{x+2}}$$

Answer 2mr.

Consider the following figure which represents the rectangular picnic cooler.

(a)

Need to write a polynomial function $T(x)$ in standard form for the volume of the rectangular prism formed by the cooler outer surface.

Let x represent the width of the cooler.
Therefore the length of the rectangular cooler is $4x$ and the height is $2x$.
Also the cooler has insulation that is 1 inch thick on each of the four sides and 2 inches thick on the top and bottom.
Therefore volume of the rectangular prism:

$$\text{Volume} = \text{Length} \times \text{Width} \times \text{Height}$$

$$T(x) = (4x)\cdot(x)\cdot(2x)$$

$$= 8x^3$$

Therefore volume of the rectangular prism formed by the outer surface is

$$T(x) = \boxed{8x^3}$$

(b)

Need to write a polynomial function $C(x)$ in standard form for the volume of the inside of the cooler.

Therefore volume inside the rectangular cooler is

$$\text{Volume} = \text{Interior length} \times \text{Interior width} \times \text{Interior height}$$

$$\begin{aligned} C(x) &= (4x-2)\times(x-2)\times(2x-4) \\ &= (4x^2-10x+4)\times(2x-4) && \text{[Using multiplication]} \\ &= 8x^3-36x^2+48x-16 && \text{[Write in the standard form]} \end{aligned}$$

Therefore volume inside the rectangular cooler is

$$C(x) = \boxed{8x^3-36x^2+48x-16}$$

(c)

Let $I(x)$ be the polynomial function for the volume of insulation.

Need to find how $I(x)$ is related to $T(x)$ and $C(x)$.

$T(x)$ is the volume of the rectangular prism formed by the cooler outer surface and $C(x)$ is the volume inside the rectangular cooler.

Therefore,

$$\begin{aligned} I(x) &= T(x)-C(x) \\ &= 8x^3-(8x^3-36x^2+48x-16) \end{aligned}$$

(d)

Need to write $I(x)$ in standard form and then find out the volume of $I(x)$ when $x=8$ inches.

$$\begin{aligned} I(x) &= 8x^3-(8x^3-36x^2+48x-16) \\ &= \cancel{8x^3}-\cancel{8x^3}+36x^2-48x+16 \\ &= 36x^2-48x+16 \end{aligned}$$

Therefore standard form for $I(x)$ is

$$I(x) = \boxed{36x^2-48x+16}$$

Now find the value of $I(x)$ when $x=8$

$$\begin{aligned} I(x) &= 36x^2-48x+16 \\ I(x) &= 36(8)^2-48(8)+16 && \text{Substitute 8 for } x \\ &= 36(64)-48(8)+16 \\ &= 2304-384+16 \\ &= 1936 \end{aligned}$$

So the value for $I(x)$ when $x=8$ is $I(x)=\boxed{1936}$

Answer 3e.

When a polynomial $f(x)$ is divided by a divisor $d(x)$, we obtain a quotient $q(x)$ and a remainder $r(x)$ such that $\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$.

Then divide the term with highest power in what is left of the dividend at each stage by the first term of the divisor.

Multiply the divisor by $\frac{x^2}{x}$, or x.

$$\begin{array}{r} x \\ x-4\overline{)x^2+x-17} \\ x^2-4x \end{array}$$

Subtract the corresponding terms and bring down the next term.

$$\begin{array}{r} x \\ x-4\overline{)x^2+x-17} \\ \underline{x^2-4x} \\ 5x-17 \end{array}$$

Now, multiply the divisor by $\frac{3x}{x}$, or 3.

$$\begin{array}{r} x+5 \\ x-4\overline{)x^2+x-17} \\ \underline{x^2-4x} \\ 5x-17 \\ 5x-20 \end{array}$$

Subtract the terms.

$$\begin{array}{r} x+5 \\ x-4\overline{)x^2+x-17} \\ \underline{x^2-4x} \\ 5x-17 \\ \underline{5x-20} \\ 3 \end{array}$$

Thus, $\frac{x^2+x-17}{x-4} = x+5+\frac{3}{x-4}$.

Check the result by multiplying the quotient by the divisor and adding the remainder. The result should be the dividend.

$$\begin{aligned}(x+5)(x-4)+3 &= x(x-4)+5(x-4)+3 \\ &= x^2-4x+5x-20+3 \\ &= x^2+x-17\end{aligned}$$

The solution checks.

Answer 3gp.

Any polynomial can be divided using synthetic division by a divisor of the form $x - k$. In this case, we evaluate the dividend at $x = k$.

We have $x + 3$ as the divisor. It can be written as $x - (-3)$. Evaluate the given dividend when $x = -3$.

$$\begin{array}{r|rrrr} -3 & 1 & 4 & -1 & -1 \\ & & -3 & -3 & 12 \\ \hline & 1 & 1 & -4 & 11 \end{array}$$

The quotient is $x^2 + x - 4$, and the remainder is 11.

Thus, $\dfrac{x^3 + 4x^2 - x - 1}{x + 3} = x^2 + x - 4 + \dfrac{11}{x + 3}$.

Answer 3mr.

The surface area of a cube of side length x is $6x^2$, and its volume is x^3. The ratio of surface area and volume is $\dfrac{6x^2}{x^3}$.

Rewrite the quotient as a product of fractions such that terms with same base appear together.

$$\frac{6x^2}{x^3} = 6 \cdot \frac{x^2}{x^3}$$

Apply the quotient of powers property which states that $\dfrac{a^m}{a^n} = a^{m-n}$.

$$6 \cdot \frac{x^2}{x^3} = 6 \cdot x^{2-3}$$

$$= 6 \cdot x^{-1}$$

Use the negative exponent property.

$$6 \cdot x^{-1} = 6 \cdot \frac{1}{x}$$

$$= \frac{6}{x}$$

The surface area-to-volume ratio of a cubic cell is $\dfrac{6}{x}$.

Similarly, find the respective ratio for a spherical cell.

For a sphere of diameter x, the radius is $\frac{x}{2}$ and so the surface area is $4\pi\left(\frac{x}{2}\right)^2$, or πx^2.

The volume is $\frac{4}{3}\pi\left(\frac{x}{2}\right)^3$, or $\frac{\pi x^3}{6}$.

The surface area-to-volume ratio is $\frac{\pi x^2}{\frac{\pi x^3}{6}}$, or $\frac{6}{x}$.

Since the surface area-to-volume ratio is the same for cubic cell and spherical cell, the exchange of materials with their environment will be at the same rate.

Answer 4e.

Consider the expression

$$3x^2-11x-26 \div x-5$$

Divide $3x^2-11x-26$ by $x-5$ with the use of polynomial long division.

Write polynomial division in the same format use when dividing numbers.
At each stage, divide the term with the highest power in what is left of the dividend by the first term of the divisor.
This gives the next term of the quotient.

Thus,

$$\begin{array}{r|l} 3x+4 & \leftarrow \text{Quotient} \\ x-5\overline{)3x^2-11x-26} & \\ \underline{3x^2-15x} & \text{Multiply divisor by } \frac{3x^2}{x}=3x \\ 4x-26 & \text{Subtract} \\ \underline{4x-20} & \text{Multiply divisor by } \frac{4x}{x}=4 \\ -6 & \leftarrow \text{Remainder} \end{array}$$

Therefore

$$\frac{3x^2-11x-26}{x-5}=\boxed{3x+4-\frac{6}{x-5}}$$

Answer 4gp.

Consider the expression

$$4x^3 + x^2 - 3x + 7 \div x - 1$$

Divide $4x^3 + x^2 - 3x + 7$ by $x - 1$ with the use of synthetic division.

Since the divisor is $x - 1$, evaluate the dividend when $x = 1$

$$\begin{aligned} f(1) &= 4 \cdot 1^3 + 1^2 - 3 \cdot 1 + 7 \\ &= 4 + 1 - 3 + 7 \\ &= 9 \end{aligned}$$

When $4x^3 + x^2 - 3x + 7$ is divided by $x - 1$, see that $f(1)$ equals the remainder.

Also the other values below the line match the coefficients of the quotient.

Now

$$\begin{array}{r|rrrr} 1 & 4 & 1 & -3 & 7 \\ & & 3 & 8 & 2 \\ \hline \text{coefficients of quotient} \rightarrow & 4 & 5 & 2 & 9 \leftarrow \text{reminder} \end{array}$$

Here $4, 5$ and 2 are the coefficients of the quotient and 9 is the remainder.

Therefore,

$$4x^3 + x^2 - 3x + 7 \div x - 1 = \boxed{4x^2 + 5x + 2 + \frac{9}{x - 1}}$$

Answer 4mr.

Consider the polynomial function has degree 4, i.e. even and end behavior given by $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$ and $f(x) \rightarrow -\infty$ as $x \rightarrow \infty$.

Therefore, the leading coefficient is negative.

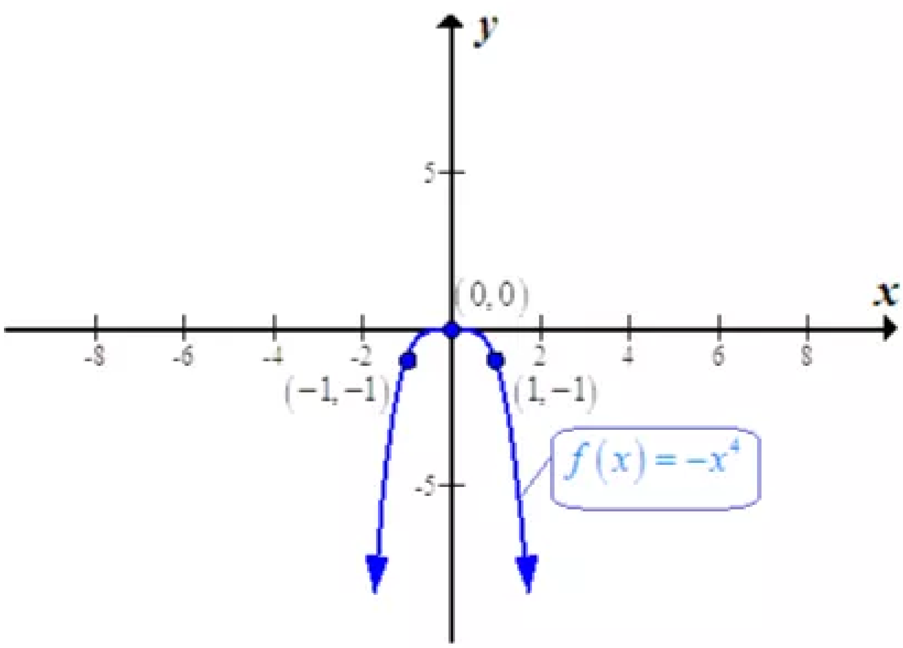
Thus, a polynomial function satisfying the above criterion is,

$$f(x) = -x^4$$

To graph the function make a table of values and plot the corresponding points. Connect the points with a smooth curve and check end behavior.

X	*-2*	*-1*	*0*	*1*	*2*
y	*-16*	*-1*	*0*	*-1*	*-16*

The graph of the function is shown below:



Answer 5e.

When a polynomial $f(x)$ is divided by a divisor $d(x)$, we obtain a quotient $q(x)$ and a remainder $r(x)$ such that $\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$.

Then divide the term with highest power in what is left of the dividend at each stage by the first term of the divisor.

Multiply the divisor by $\frac{x^3}{x}$, or x^2.

$$\begin{array}{r|l} & x^2 \\ x-1 & x^3+3x^2+3x+2 \\ & x^3-x^2 \end{array}$$

Subtract the corresponding terms and bring down the next term.

$$\begin{array}{r|l} & x^2 \\ x-1 & x^3+3x^2+3x+2 \\ & \underline{x^3-x^2} \\ & \quad 4x^2+3x \end{array}$$

Now, multiply the divisor by $\frac{4x^2}{x}$, or $4x$.

$$\begin{array}{r} x^2 + 4x \\ x - 1\overline{)x^3 + 3x^2 + 3x + 2} \\ \underline{x^3 - x^2} \\ 4x^2 + 3x \\ 4x^2 - 4x \end{array}$$

Subtract the terms and bring down the next term.

$$\begin{array}{r} x^2 + 4x \\ x - 1\overline{)x^3 + 3x^2 + 3x + 2} \\ \underline{x^3 - x^2} \\ 4x^2 + 3x \\ \underline{4x^2 - 4x} \\ 7x + 2 \end{array}$$

Repeat the process by multiplying the divisor by $\frac{7x}{7}$, or 7.

$$\begin{array}{r} x^2 + 4x + 7 \\ x - 1\overline{)x^3 + 3x^2 + 3x + 2} \\ \underline{x^3 - x^2} \\ 4x^2 + 3x \\ \underline{4x^2 - 4x} \\ 7x + 2 \\ \underline{7x - 7} \\ 9 \end{array}$$

Thus, $\frac{x^3 + 3x^2 + 3x + 2}{x - 1} = x^2 + 4x + 7 + \frac{9}{x - 1}$.

Check the result by multiplying the quotient by the divisor and adding the remainder. The result should be the dividend.

$$\begin{aligned} (x^2 + 4x + 7)(x - 1) + 9 &= x^2(x - 1) + 4x(x - 1) + 7(x - 1) + 9 \\ &= x^3 - x^2 + 4x^2 - 4x + 7x - 7 + 9 \\ &= x^3 + 3x^2 + 3x + 2 \end{aligned}$$

The solution checks.

Answer 5gp.

By the factor theorem, $x - k$ is a factor of a polynomial if and only if $f(k)$ is 0.

Since $x - 4$ is a factor of the given polynomial, $f(4)$ must be 0. Use synthetic division to find the other factors by dividing $x^3 - 6x^2 + 5x + 12$ by $x - 4$.

$$\begin{array}{r|rrrr} 4 & 1 & -6 & 5 & 12 \\ & & 4 & -8 & -12 \\ \hline & 1 & -2 & -3 & 0 \end{array}$$

The function $f(x)$ can be thus written as $f(x) = (x - 4)(x^2 - 2x - 3)$.

Factor the trinomial $x^2 - 2x - 3$.
$(x - 4)(x^2 - 2x - 3) = (x - 4)(x - 3)(x + 1)$

Therefore, the given polynomial can be completely factored as $f(x) = (x - 4)(x - 3)(x + 1)$.

Answer 5mr.

a. The given function is of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ (where $a_n \neq 0$), having leading coefficient as –0.027, and the degree as 4.

We know that a polynomial function which has a degree of 4 is called a quartic function. Thus, the given function is quartic.

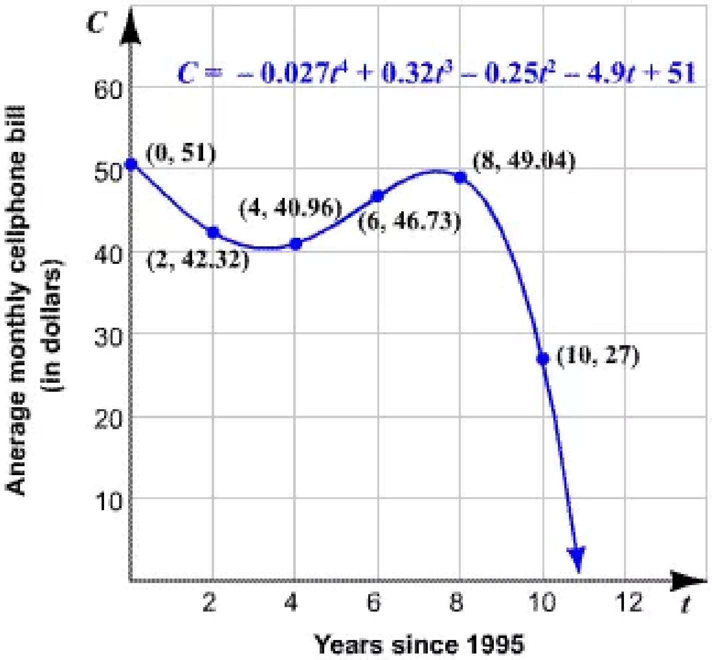
b. Let us create a table of values for the given model.

Since t represents the number of years, the model deals with only positive values of t. Also, as t is the number of years since 1995, $t = 0$ corresponds to the year 1995.

t	0	2	4	6	8	10
C	51	42.32	40.96	46.73	49.04	27

c. Plot the points and connect them with a smooth curve.

Since the degree is even and the leading coefficient is negative, the graph falls to the right.



From the graph, it is clear that the average cell phone bill falls for years after 2003, which corresponds to $t = 8$. For the year 2005, the average bill is about \$27, which is same as the calculated value.

Therefore, the model accurately predicts the bill for years beyond 2003 also.

Answer 6e.

Consider the expression

$$8x^2 + 34x - 1 \div 4x - 1$$

Divide $8x^2 + 34x - 1$ by $4x - 1$ with the use of polynomial long division.

Write polynomial division in the same format use when dividing numbers.
At each stage, divide the term with the highest power in what is left of the dividend by the first term of the divisor.
This gives the next term of the quotient.

Thus,

$$\begin{array}{r} 2x+9 \\ 4x-1\overline{)8x^2+34x-1} \end{array} \quad \square \quad \text{Quotient}$$

$$\underline{8x^2-2x} \qquad \text{Multiply divisor by } \frac{8x^2}{4x}=2x$$

$$36x-1 \qquad \text{Subtract}$$

$$\underline{36x-9} \qquad \text{Multiply divisor by } \frac{36x}{4x}=9$$

$$8 \leftarrow \quad \text{Remainder}$$

Therefore

$$\frac{8x^2+34x-1}{4x-1}=\boxed{2x+9+\frac{8}{4x-1}}$$

Answer 6gp.

Consider the polynomial

$$f(x)=x^3-x^2-22x+40$$

Factorize completely.

From the data, $x-4$ is a factor of $f(x)$

Therefore

$$f(4)=4^3-4^2-22\cdot 4+40$$
$$=64-16-88+40$$
$$=0$$

Therefore, use synthetic division to find the other factors.
Since the divisor is $x-4$, evaluate the dividend when $x=4$
Now

$$\begin{array}{c|cccc} 4 & 1 & -1 & -22 & 40 \\ & & 4 & 12 & 40 \\ \hline & 1 & 3 & -10 & 0 \end{array}$$

Now using the result to write $f(x)$ as a product of two factors and then factor completely.

Therefore,

$$f(x)=x^3-x^2-22x+40$$ Writhe the original equation

$$=(x-4)(x^2+3x-10)$$ Write as a product of two factors

$$=(x-4)(x^2+5x-2x-10)$$ Replace $3x$by $5x-2x$ so that product of Coefficients is -10

$$=(x-4)(x(x+5)-2(x+5))$$ Take x common of x^2+5x ; -2 common of $-2x-10$

$$=\boxed{(x-4)(x+5)(x-2)}$$ Factor trinomial

Answer 6mr.

Consider the price p (in dollars) that a camera manufacturer is able to charge for a camera is given by $p=100-10x^2$ where x (in millions) is number of cameras produced. It cost the company \$30 to make a camera.

(a)

Need to write a function that gives the total revenue R in terms of x.
To find the total revenue, we multiply x with the price of one camera.
Therefore,

$$\text{Total revenue}(R)=x\times\text{price of one camera}$$

$$R=x\times(100-10x^2)$$

So, the total revenue *(R)* is $\boxed{R=x\times(100-10x^2)}$.

(b)

Need to write a function that gives the company's profit (P) in terms of x.
For this first we compare cost of one camera with the price that a camera manufacturer is able to charge for a camera.
If we subtract cost of one camera from price then we get the profit.
Therefore,

$$\text{Profit}(P)=\text{Price}-\text{Cost}$$

$$P=(100-10x^2)-30x \qquad \ldots\ldots (1)$$

So, the company's profit is $\boxed{P=(100-10x^2)-30x}$.

(c)

Need to solve equation (1) to find other values of x that yield a profit of \$60,000,000.
First we need to convert the value of profit from dollars to millions. So the profit is 60 million.
If we substitute the value of profit ($P=60$) in equation (1), then we have

$$P=(100-10x^2)-30x$$

$$60=(100-10x^2)-30x$$

$$60=100-10x^2-30x \qquad [\text{Write in the standard form}]$$

$$10x^2+30x-40=0$$

$$10(x^2+3x-4)=0$$

$$x(x+4)-1(x+4)=0 \qquad [\text{Factor by grouping}]$$

$$(x+4)(x-1)=0$$

$$x=-4 \text{ or } x=1$$

The only real value is $x=1$.

So the company will profit \$60,000,000 for $\boxed{x=1}$.

(d)

Need to explain does all the solution in part (C) make any sense in this situation or not.

The company produce 2 million cameras and make a profit of \$60,000,000.
But by solving the equation in part (C) we find that the company make a profit of \$60,000,000 for producing 1 million cameras.
So in this situation all the solution in part(C) make no sense.

Answer 7e.

When a polynomial $f(x)$ is divided by a divisor $d(x)$, we obtain a quotient $q(x)$ and a remainder $r(x)$ such that $\dfrac{f(x)}{d(x)} = q(x) + \dfrac{r(x)}{d(x)}$.

Then divide the term with highest power in what is left of the dividend at each stage by the first term of the divisor.

Multiply the divisor by $\dfrac{3x^3}{x^2}$, or $3x$.

$$\begin{array}{r} 3x \\ x^2+x\overline{\big)3x^3+11x^2+4x+1} \\ 3x^3+3x^2 \end{array}$$

Subtract the corresponding terms and bring down the next term.

$$\begin{array}{r} 3x \\ x^2+x\overline{)3x^3+11x^2+4x+1} \\ \underline{3x^3+3x^2} \\ 8x^2+4x \end{array}$$

Now, multiply the divisor by $\frac{8x^2}{x^2}$, or 8.

$$\begin{array}{r} 3x+8 \\ x^2+x\overline{)3x^3+11x^2+4x+1} \\ \underline{3x^3+3x^2} \\ 8x^2+4x \\ 8x^2+8x \end{array}$$

Subtract the terms and bring down the next term.

$$\begin{array}{r} 3x+8 \\ x^2+x\overline{)3x^3+11x^2+4x+1} \\ \underline{3x^3+3x^2} \\ 8x^2+4x \\ \underline{8x^2+8x} \\ -4x+1 \end{array}$$

Thus, $\dfrac{3x^3+11x^2+4x+1}{x^2+x} = 3x+8+\dfrac{-4x+1}{x^2+x}$.

Check the result by multiplying the quotient by the divisor and adding the remainder. The result should be the dividend.

$$\begin{aligned} (3x+8)\left(x^2+x\right)+(-4x+1) &= 3x\left(x^2+x\right)+8\left(x^2+x\right)-4x+1 \\ &= 3x^3+3x^2+8x^2+8x-4x+1 \\ &= 3x^3+11x^2+4x+1 \end{aligned}$$

The solution checks.

Answer 7gp.

By the factor theorem, $x - k$ is a factor of a polynomial if and only if $f(k)$ is 0.

Since $f(-2)$ is 0, $x + 2$ is a factor of the given polynomial. Use synthetic division to find the other factors by dividing $x^3 + 2x^2 - 9x - 18$ by $x + 2$.

$$\begin{array}{r|rrrr} -2 & 1 & 2 & -9 & -18 \\ & & -2 & 0 & 18 \\ \hline & 1 & 0 & -9 & 0 \end{array}$$

The function $f(x)$ can be thus written as $f(x) = (x + 2)(x^2 - 9)$.

Factor $x^2 - 9$.
$(x + 2)(x^2 - 9) = (x + 2)(x + 3)(x - 3)$

Therefore, the other zeros are -3 and 3.

Answer 7mr.

It is given that the base of the pyramid is $3x - 6$ feet, and the height is x feet.

We know that the volume of a pyramid is $\frac{1}{3}$ times the product of the area of its base and height. We have the volume as 48 cubic feet.

$$\begin{array}{ccccccc} \text{Volume (cubic feet)} & = & \frac{1}{3} & \cdot & \text{area of the base (square feet)} & \cdot & \text{height (feet)} \\ \Downarrow & & \Downarrow & & \Downarrow & & \Downarrow \\ 48 & = & \frac{1}{3} & \cdot & (3x-6)^2 & \cdot & x \end{array}$$

We get the equation $48 = \frac{1}{3}(3x - 6)^2 x$.

Multiply both sides of the equation by 3 to clear the fraction.
$144 = (3x - 6)^2 x$

Expand $(3x - 6)^2$, and simplify.
$144 = 9x^3 - 36x^2 + 36x$

Rewrite the equation in standard form.
$0 = 9x^3 - 36x^2 + 36x - 144$

Now, factor $9x^3 - 36x^2 + 36x - 144$ by grouping.
$0 = 9x^2(x-4) + 36(x-4)$

Apply the distributive property.
$0 = (9x^2 + 36)(x-4)$

Factor the expression completely.
$0 = 9(x^2 + 4)(x-4)$

Use the zero product property and solve for x.

$$x - 4 = 0 \quad \text{or} \quad x^2 + 4 = 0$$
$$x = 4 \qquad\qquad x^2 = -4$$

The only real solution being 4, we get the height of the sculpture as 4 feet.

Answer 8e.

Consider the expression

$$7x^3 + 11x^2 + 7x + 5 \div x^2 + 1$$

Divide $7x^3 + 11x^2 + 7x + 5$ by $x^2 + 1$ with the use of polynomial long division.

Write polynomial division in the same format use when dividing numbers.
At each stage, divide the term with the highest power in what is left of the dividend by the first term of the divisor.
This gives the next term of the quotient.

Thus,

$$\begin{array}{r l}
 7x + 11 & \leftarrow \text{Quotient} \\
x^2 + 1 \overline{)\, 7x^3 + 11x^2 + 7x + 5} & \\
7x^3 \qquad\quad + 7x & \text{Multiply divisor by } \dfrac{7x^3}{x^2} = 7x \\
\hline
11x^2 \qquad + 5 & \text{Subtract} \\
11x^2 \qquad + 11 & \text{Multiply divisor by } \dfrac{11x^2}{x^2} = 11 \\
\hline
-6 & \leftarrow \text{Remainder}
\end{array}$$

Therefore,

$$\frac{7x^3 + 11x^2 + 7x + 5}{x^2 + 1} = \boxed{7x + 11 - \frac{6}{x^2 + 1}}$$

Answer 8gp.

Consider the function

$$f(x)=x^3+8x^2+5x-14$$

And $f(-2)=0$.

Find the other zeros of $f(x)$.

Since $f(-2)=0$, $x+2$ is a factor of $f(x)$.

Now use synthetic division,

$$\begin{array}{r|rrrr} -2 & 1 & 8 & 5 & -14 \\ & & -2 & -12 & 14 \\ \hline & 1 & 6 & -7 & 0 \end{array}$$

Use the result to write $f(x)$ as a product of two factors.

Then factor completely.

$f(x)=x^3+8x^2+5x-14$	Writhe the original equation
$=(x+2)(x^2+6x-7)$	Write as a product of two factors
$=(x+2)(x^2+7x-x-7)$	Replace $6x$ by $7x-x$ so that product of Coefficients is -7
$=(x+2)(x(x+7)-1(x+7))$	Take x common of x^2+7x; -1 common of $-x-7$
$=(x+2)(x+7)(x-1)$	Factor trinomial

Therefore other zeros are $\boxed{-7 \text{ and } 1}$

Answer 9e.

When a polynomial $f(x)$ is divided by a divisor $d(x)$, we obtain a quotient $q(x)$ and a remainder $r(x)$ such that $\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$.

In polynomial long division, include 0 as the coefficient of the missing terms in the dividend. The x term is missing in the given dividend. Use 0 as its coefficient.

$$x^2+2x-4\overline{)5x^4-2x^3-7x^2+0x-39}$$

Then divide the term with highest power in what is left of the dividend at each stage by the first term of the divisor.

Multiply the divisor by $\dfrac{5x^4}{x^2}$, or $5x^2$.

$$\begin{array}{r} 5x^2 \\ x^2+2x-4\overline{)5x^4-2x^3-7x^2+0x-39} \\ 5x^4+10x^3-20x^2 \qquad\qquad\quad \end{array}$$

Subtract the corresponding terms and bring down the next term.

$$\begin{array}{r} 5x^2 \\ x^2+2x-4\overline{)5x^4-2x^3-7x^2+0x-39} \\ \underline{5x^4+10x^3-20x^2} \qquad\qquad\quad \\ -12x^3+13x^2+0x \qquad\quad \end{array}$$

Now, multiply the divisor by $\dfrac{-12x^3}{x^2}$, or $-12x$.

$$\begin{array}{r} 5x^2-12x \\ x^2+2x-4\overline{)5x^4-2x^3-7x^2+0x-39} \\ \underline{5x^4+10x^3-20x^2} \qquad\qquad\quad \\ -12x^3+13x^2+0x \qquad\quad \\ -12x^3-24x^2+48x \qquad\quad \end{array}$$

Subtract the terms and bring down the next term.

$$\begin{array}{r} 5x^2-12x \\ x^2+2x-4\overline{)5x^4-2x^3-7x^2+0x-39} \\ \underline{5x^4+10x^3-20x^2} \qquad\qquad\quad \\ -12x^3+13x^2+0x \qquad\quad \\ \underline{-12x^3-24x^2+48x} \qquad\quad \\ 37x^2-48x-39 \end{array}$$

Repeat the process by multiplying the divisor by $\frac{37x^2}{x^2}$, or 37.

$$
\begin{array}{r}
5x^2 - 12x + 37 \\
x^2+2x-4\,\overline{)\,5x^4 - 2x^3 - 7x^2 + 0x - 39} \\
\underline{5x^4 + 10x^3 - 20x^2} \qquad\qquad\quad \\
-12x^3 + 13x^2 + 0x \qquad \\
\underline{-12x^3 - 24x^2 + 48x} \qquad \\
37x^2 - 48x - 39 \\
\underline{37x^2 + 74x - 148} \\
-122x + 109
\end{array}
$$

Thus, $\dfrac{5x^4 - 2x^3 - 7x^2 - 39}{x^2 + 2x - 4} = 5x^2 - 12x + 37 + \dfrac{-122x + 109}{x^2 + 2x - 4}$.

Check the result by multiplying the quotient by the divisor and adding the remainder. The result should be the dividend.

$$\left(5x^2 - 12x + 37\right)\left(x^2 + 2x - 4\right) + \left(-122x + 109\right)$$
$$= 5x^2\left(x^2 + 2x - 4\right) - 12x\left(x^2 + 2x - 4\right) + 37\left(x^2 + 2x - 4\right) - 122x + 109$$
$$= 5x^4 + 10x^3 - 20x^2 - 12x^3 - 24x^2 + 48x + 37x^2 + 74x - 148 - 122x + 109$$
$$= 5x^4 - 2x^3 - 7x^2 - 39$$

The solution checks.

Answer 9gp.

We seek the number of shoes that must be produced to make the same profit of \$25,000,000, where the profit is modeled by the function $P = -15x^3 + 40x$.

Replace P with 25 and write the resulting equation in standard form.
$25 = -15x^3 + 40x$
or $15x^3 - 40x + 25 = 0$

Since $x = 1$ is a solution of the equation, $x - 1$ is factor. Use synthetic division to find the other factor.

$$
\begin{array}{r|rrrr}
1 & 15 & 0 & -40 & 25 \\
 & & 15 & 15 & -25 \\
\hline
 & 15 & 15 & -25 & 0
\end{array}
$$

The equation thus becomes $(x - 1)(15x^2 + 15x - 25) = 0$

Solve $15x^2 + 15x - 25 = 0$ using the quadratic formula to find the positive value of x.

$$x = \frac{-15 + \sqrt{225 + 1725}}{30}$$

$$\approx 0.97$$

The company could make the same profit by producing about 970,000 shoes.

Answer 10e.

Consider the expression

$$4x^4 + 5x - 4 \div x^2 - 3x - 2$$

Divide $4x^4 + 5x - 4$ by $x^2 - 3x - 2$ with the use of polynomial long division.

Write polynomial division in the same format use when dividing numbers, include a "0" as the coefficient of x^3 and x^2 in the dividend.

At each stage, divide the term with the highest power in what is left of the dividend by the first term of the divisor.

This gives the next term of the quotient.

Thus,

$$\begin{array}{r|l}
 & 4x^2 + 12x + 44 \quad \leftarrow \text{Quotient} \\
x^2 - 3x - 2 & 4x^4 + 0 \cdot x^3 + 0 \cdot x^2 + 5x - 4 \\
 & \underline{4x^4 - 12x^3 - 8x^2} \quad \text{Multiply divisor by } \frac{4x^4}{x^2} = 4x^2 \\
 & 12x^3 + 8x^2 + 5x \quad \text{Subtract} \\
 & \underline{12x^3 - 36x^2 - 24x} \quad \text{Multiply divisor by } \frac{12x^3}{x^2} = 12x \\
 & 44x^2 + 29x - 4 \\
 & \underline{44x^2 - 132x - 88} \quad \text{Multiply divisor by } \frac{44x^2}{x^2} = 44 \\
 & 161x + 84 \leftarrow \text{Remainder}
\end{array}$$

Therefore

$$\frac{4x^4 + 5x - 4}{x^2 - 3x - 2} = \boxed{4x^2 + 12x + 44 + \frac{161x + 84}{x^2 - 3x - 2}}$$

Answer 11e.

Any polynomial can be divided using synthetic division by a divisor of the form $x - k$. In this case, we evaluate the dividend at $x = k$.

We have $x - 5$ as the divisor. Evaluate the given dividend when $x = 5$.

$$\begin{array}{r|rrr} 5 & 2 & -7 & 10 \\ & & 10 & 15 \\ \hline & 2 & 3 & 25 \end{array}$$

The quotient is $2x + 3$, and the remainder is 25.

Thus, $\dfrac{2x^2 - 7x + 10}{x - 5} = 2x + 3 + \dfrac{25}{x - 5}$.

Answer 12e.

Consider the expression

$$4x^2 - 13x - 5 \div x - 2$$

Divide $4x^2 - 13x - 5$ by $x - 2$ with the use synthetic division.

Since the divisor is $x - 2$, evaluate the dividend when $x = 2$

$$\begin{aligned} f(2) &= 4 \cdot 2^2 - 13 \cdot 2 - 5 \\ &= 16 - 26 - 5 \\ &= -15 \end{aligned}$$

When $4x^2 - 13x - 5$ is divided by $x - 2$, see that $f(2)$ equals the remainder.

Also the other values below the line match the coefficients of the quotient.

Thus,

$$\begin{array}{r|rrr} 2 & 4 & -13 & -5 \\ & & 8 & -10 \\ \hline \text{coefficients of quotient} \rightarrow & 4 & -5 & -15 \leftarrow \text{reminder} \end{array}$$

Here 4 and -5 are the coefficients of the quotient and -15 is the remainder.

Therefore,

$$\frac{4x^2 - 13x - 5}{x - 2} = \boxed{4x - 5 - \frac{15}{x - 2}}$$

Answer 13e.

Any polynomial can be divided using synthetic division by a divisor of the form $x-k$. In this case, we evaluate the dividend at $x=k$.

We have $x+4$ as the divisor. It can be written as $x-(-4)$. Evaluate the given dividend when $x=-4$.

$$\begin{array}{r|rrr} -4 & 1 & 8 & 1 \\ & & -4 & -16 \\ \hline & 1 & 4 & -15 \end{array}$$

The quotient is $x+4$, and the remainder is -15.

Thus, $\dfrac{x^2+8x+1}{x+4} = x+4+\dfrac{-15}{x+4}$.

Answer 14e.

We have to divide $f(x)=x^2+9$ by $x-3$ using synthetic division.

Because the divisor is $x-3$, evaluate the dividend when $x=3$

From the Remainder theorem, When $f(x)=x^2+9$ is divided by $x-3$, $f(3)$ is the remainder.

Now

$$\begin{array}{r|rrr} 3 & 1 & 0 & 9 \\ & & 3 & 9 \\ \hline & 1 & 3 & 18 \end{array}$$

Here 1 and 3 are the coefficients of the quotient and 18 is the remainder.

Therefore

$$\frac{x^2+9}{x-3} = \boxed{x+3+\frac{18}{x-3}}$$

Answer 15e.

Any polynomial can be divided using synthetic division by a divisor of the form $x - k$. In this case, we evaluate the dividend at $x = k$.

We have $x - 4$ as the divisor. Evaluate the given dividend when $x = 4$.

$$\begin{array}{r|rrrr} 4 & 1 & -5 & 0 & -2 \\ & & 4 & -4 & -16 \\ \hline & 1 & -1 & -4 & -18 \end{array}$$

The quotient is $x^2 - x - 4$, and the remainder is -18.

Thus, $\dfrac{x^3 - 5x^2 - 2}{x - 4} = x^2 - x - 4 + \dfrac{-18}{x - 4}$.

Answer 16e.

We have to divide $f(x) = x^3 - 4x + 6$ by $x + 3$ using synthetic division.

Since the divisor is $x + 3 = x - (-3)$, evaluate the dividend when $x = -3$

From the Remainder Theorem When $x^3 - 4x + 6$ is divided by $x + 3$, $f(-3)$ is the remainder.

Also the other values below the line match the coefficients of the quotient.

Now

$$\begin{array}{r|rrrr} -3 & 1 & 0 & -4 & 6 \\ & & -3 & 9 & -15 \\ \hline & 1 & -3 & 5 & -9 \end{array}$$

Here $1, -3$ and 5 are the coefficients of the quotient and -9 is the remainder.

Therefore

$$\frac{x^3 - 4x + 6}{x + 3} = \boxed{x^2 - 3x + 5 - \frac{9}{x + 3}}$$

Answer 17e.

Any polynomial can be divided using synthetic division by a divisor of the form $x - k$. In this case, we evaluate the dividend at $x = k$.

We have $x - 6$ as the divisor. Evaluate the given dividend when $x = 6$.

$$\begin{array}{r|rrrrr} 6 & 1 & -5 & -8 & 13 & -12 \\ & & 6 & 6 & -12 & 6 \\ \hline & 1 & 1 & -2 & 1 & -6 \end{array}$$

The quotient is $x^3 + x^2 - 2x + 1$, and the remainder is -6.

Thus, $\dfrac{x^4 - 5x^3 - 8x^2 + 13x - 12}{x - 6} = x^3 + x^2 - 2x + 1 + \dfrac{-6}{x - 6}$.

Answer 19e.

When a polynomial $f(x)$ is divided by a divisor $d(x)$, we obtain a quotient $q(x)$ and a remainder $r(x)$ such that $\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$.

The quotient will always be one degree less than the divisor. In the synthetic division shown, the quotient is written incorrectly as its degree is the same as the degree of the divisor.

The first three terms in the bottom row of the synthetic division are the coefficients of the quotient and the last term is the remainder.

Thus, the correct way of writing the result is $\frac{x^3 - 5x + 3}{x - 2} = x^2 + 2x - 1 + \frac{1}{x - 2}$.

Answer 20e.

We are given that $f(x) = x^3 - 5x + 3$ is divided by $x - 2$ using synthetic division.

Since the divisor is $x - 2$, evaluate the dividend when $x = 2$

From the Remainder Theorem When $x^3 - 5x + 3$ is divided by $x - 2$, $f(2)$ is the remainder.

Also the other values below the line match the coefficients of the quotient.

Now we will use "0" as a coefficient of x^2 since the expression $x^3 - 5x + 3$ doesn't have term containing x^2.

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -5 & 3 \\ & & 2 & 4 & -2 \\ \hline & 1 & 2 & -1 & 1 \end{array}$$

Here $1, 2$ and -1 are the coefficients of the quotient and 1 is the remainder.

Therefore

$$\frac{x^3 - 5x + 3}{x - 2} = x^2 + 2x - 1 + \frac{1}{x - 2}$$

Answer 21e.

By the factor theorem, $x - k$ is a factor of a polynomial if and only if $f(k)$ is 0.

Since $x - 6$ is a factor of the given polynomial, $f(6)$ must be 0. Use synthetic division to find the other factors by dividing $x^3 - 10x^2 + 19x + 30$ by $x - 6$.

$$\begin{array}{r|rrrr} 6 & 1 & -10 & 19 & 30 \\ & & 6 & -24 & -30 \\ \hline & 1 & -4 & -5 & 0 \end{array}$$

The function $f(x)$ can be thus written as $f(x) = (x - 6)(x^2 - 4x - 5)$.

Factor the trinomial $x^2 - 4x - 5$.
$(x-6)(x^2-4x-5) = (x-6)(x-5)(x+1)$

Therefore, the given polynomial can be completely factored as $f(x) = (x-6)(x-5)(x+1)$.

Answer 22e.

conisder $f(x) = x^3 + 6x^2 + 5x - 12$.

We have $x+4$ is a factor of $f(x)$

Because $x+4$ is a factor of $f(x)$, by the Remainder Theorem $f(-4)$ is the remainder of $f(x)$.

consider

$$\begin{aligned} f(-4) &= (-4)^3 + 6(-4)^2 + 5(-4) - 12 \\ &= -64 + 96 - 20 - 12 \\ &= 0 \end{aligned}$$

Therefore, we will use synthetic division to find the other factors.

Since the divisor is $x+4 = x-(-4)$, evaluate the dividend when $x = -4$

Now

$$\begin{array}{r|rrrr} -4 & 1 & 6 & 5 & -12 \\ & & 4 & -8 & 12 \\ \hline & 1 & 2 & -3 & 0 \end{array}$$

Now using the result to write $f(x)$ as a product of two factors and then factor completely.

Therefore

$$\begin{aligned} f(x) &= x^3 + 6x^2 + 5x - 12 && [\text{Write original polynomial}] \\ &= (x+4)(x^2 + 2x - 3) && [\text{Write as a product of two factors}] \\ &= (x+4)(x^2 + 3x - x - 3) && \begin{bmatrix}\text{Replace } 2x \text{ by } 3x - x \text{ so that} \\ \text{product of the coefficients is } -3\end{bmatrix} \\ &= (x+4)(x(x+3) - 1(x+3)) && \begin{bmatrix}\text{Taking } x \text{ common of } x^2 + 3x; \\ -1 \text{ common of } -x - 3\end{bmatrix} \\ &= \boxed{(x+4)(x+3)(x-1)} && [\text{Factor trinomial}] \end{aligned}$$

Answer 23e.

By the factor theorem, $x - k$ is a factor of a polynomial if and only if $f(k)$ is 0.

Since $x - 8$ is a factor of the given polynomial, $f(8)$ must be 0. Use synthetic division to find the other factors by dividing $x^3 - 2x^2 - 40x - 64$ by $x - 8$.

$$\begin{array}{r|rrrr} 8 & 1 & -2 & -40 & -64 \\ & & 8 & 48 & 64 \\ \hline & 1 & 6 & 8 & 0 \end{array}$$

The function $f(x)$ can be thus written as $f(x) = (x - 8)(x^2 + 6x + 8)$.

Factor the trinomial $x^2 + 6x + 8$.
$(x - 8)(x^2 + 6x + 8) = (x - 8)(x + 4)(x + 2)$

Therefore, the given polynomial can be completely factored as $f(x) = (x - 8)(x + 4)(x + 2)$.

Answer 24e.

We have to factorize completely $f(x) = x^3 + 18x^2 + 95x + 150$.

We are given that $x + 10$ is a factor of $f(x)$

Because $x + 10$ is a factor of $f(x)$, By the Remainder Theorem $f(-10)$ is the remainder of $f(x)$

Consider,

$$\begin{aligned} f(-10) &= (-10)^3 + 18(-10)^2 + 95(-10) + 150 \\ &= -1000 + 1800 - 950 + 150 \\ &= 0 \end{aligned}$$

Therefore, we will use synthetic division to find the other factors.

Since the divisor is $x + 10 = x - (-10)$, evaluate the dividend when $x = -10$

Now

$$\begin{array}{r|rrrr} -10 & 1 & 18 & 95 & 150 \\ & & -10 & -80 & -150 \\ \hline & 1 & 8 & 15 & 0 \end{array}$$

Now using the result to write $f(x)$ as a product of two factors and then factor completely.

Therefore

$$\begin{aligned}
f(x) &= x^3+18x^2+95x+150 && \text{[Write original polynomial]}\\
&= (x+10)(x^2+8x+15) && \text{[Write as a product of two factors]}\\
&= (x+10)(x^2+5x+3x+15) && \left[\begin{array}{l}\text{Replace } 8x \text{ by } 5x+3x \text{ so that}\\ \text{product of the coefficients is } 15\end{array}\right]\\
&= (x+10)(x(x+5)+3(x+5)) && \left[\begin{array}{l}\text{Taking } x \text{ common of } x^2+5x;\\ 3 \text{ common of } 3x+5\end{array}\right]\\
&= \boxed{(x+10)(x+5)(x+3)} && \text{[Factor trinomial]}
\end{aligned}$$

Answer 25e.

By the factor theorem, $x-k$ is a factor of a polynomial if and only if $f(k)$ is 0.

Since $x+9$ is a factor of the given polynomial, $f(-9)$ must be 0. Use synthetic division to find the other factors by dividing $x^3+2x^2-51x+108$ by $x+9$.

$$\begin{array}{r|rrrr}
-9 & 1 & 2 & -51 & 108\\
 & & -9 & 63 & -108\\
\hline
 & 1 & -7 & 12 & 0
\end{array}$$

The function $f(x)$ can be thus written as $f(x)=(x+9)(x^2-7x+12)$.

Factor the trinomial $x^2-7x+12$.

$(x+9)(x^2-7x+12)=(x+9)(x-4)(x-3)$

Therefore, the given polynomial can be completely factored as $f(x)=(x+9)(x-4)(x-3)$.

Answer 26e.

Consider, $f(x)=x^3-9x^2+8x+60$.

We have $x+2$ is a factor of $f(x)$

Because $x+2$ is a factor of $f(x)$, From Remainder Theorem $f(-2)$ is a remainder of $f(x)$

Consider,

$$f(-2)=(-2)^3-9(-2)^2+8(-2)+60$$
$$=-8-36-16+60$$
$$=0$$

Therefore, we will use synthetic division to find the other factors.

Since the divisor is $x+2=x-(-2)$, evaluate the dividend when $x=-2$

Now

$$\begin{array}{r|rrrr} -2 & 1 & -9 & 8 & 60 \\ & & -2 & 22 & -60 \\ \hline & 1 & -11 & 30 & 0 \end{array}$$

Now using the result to write $f(x)$ as a product of two factors and then factor completely.

Therefore

$$f(x)=x^3-9x^2+8x+60 \quad \text{[Write original polynomial]}$$
$$=(x+2)(x^2-11x+30) \quad \text{[Write as a product of two factors]}$$
$$=(x+2)(x^2-5x-6x+30) \quad \left[\begin{array}{l}\text{Replace } -11x \text{ by } -5x-6x \text{ so that} \\ \text{product of the coefficients is } 30\end{array}\right]$$
$$=(x+2)(x(x-5)-6(x-5)) \quad \left[\begin{array}{l}\text{Taking } x \text{ common of } x^2-5x; \\ -6 \text{ common of } -6x+30\end{array}\right]$$
$$=\boxed{(x+2)(x-5)(x-6)} \quad \text{[Factor trinomial]}$$

Answer 27e.

By the factor theorem, $x-k$ is a factor of a polynomial if and only if $f(k)$ is 0.

Since $x-1$ is a factor of the given polynomial, $f(1)$ must be 0. Use synthetic division to find the other factors by dividing $2x^3-15x^2+34x-21$ by $x-1$.

$$\begin{array}{r|rrrr} 1 & 2 & -15 & 34 & -21 \\ & & 2 & -13 & 21 \\ \hline & 2 & -13 & 21 & 0 \end{array}$$

The function $f(x)$ can be thus written as $f(x)=(x-1)(2x^2-13x+21)$.

Factor the trinomial $2x^2 - 13x + 21$.
$(x - 1)(2x^2 - 13x + 21) = (x - 1)(2x - 7)(x - 3)$

Therefore, the given polynomial can be completely factored as
$f(x) = (x - 1)(2x - 7)(x - 3)$.

Answer 28e.

Consider, $f(x) = 3x^3 - 2x^2 - 61x - 20$.

We have $x - 5$ is a factor of $f(x)$

Because $x - 5$ is a factor of $f(x)$, from Remainder Theorem $f(5)$ is a remainder of $f(x)$

Consider,

$$
\begin{aligned}
f(5) &= 3(5)^3 - 2(5)^2 - 61(5) - 20 \\
&= 375 - 50 - 305 - 20 \\
&= 0
\end{aligned}
$$

Therefore, we will use synthetic division to find the other factors.
Since the divisor is $x - 5$, evaluate the dividend when $x = 5$
Now

$$
\begin{array}{r|rrrr}
5 & 3 & -2 & -61 & -20 \\
 & & 15 & 65 & 20 \\
\hline
 & 3 & 13 & 4 & 0
\end{array}
$$

Now using the result to write $f(x)$ as a product of two factors and then factor completely.
Therefore

$$
\begin{aligned}
f(x) &= 3x^3 - 2x^2 - 61x - 20 && [\text{Write original polynomial}] \\
&= (x-5)(3x^2 + 13x + 4) && [\text{Write as a product of two factors}] \\
&= (x-5)(3x^2 + 12x + x + 4) && \begin{bmatrix}\text{Replace } 13x \text{ by } 12x + x \text{ so that} \\ \text{product of the coefficients is } 12\end{bmatrix} \\
&= (x-5)(3x(x+4) + 1(x+4)) && \begin{bmatrix}\text{Taking } x \text{ common of } 3x^2 + 12x; \\ 1 \text{ common of } x+4\end{bmatrix} \\
&= \boxed{(x-5)(x+4)(3x+1)} && [\text{Factor trinomial}]
\end{aligned}
$$

Answer 29e.

By the factor theorem, $x - k$ is a factor of a polynomial if and only if $f(k)$ is 0.

Since $f(-3)$ is 0, $x + 3$ is a factor of the given polynomial. Use synthetic division to find the other factors by dividing $x^3 - 2x^2 - 21x - 18$ by $x + 3$.

$$\begin{array}{r|rrrr} -3 & 1 & -2 & -21 & -18 \\ & & -3 & 15 & 18 \\ \hline & 1 & -5 & -6 & 0 \end{array}$$

The function $f(x)$ can be thus written as $f(x) = (x + 3)(x^2 - 5x - 6)$.

Factor the trinomial $x^2 - 5x - 6$.
$(x + 3)(x^2 - 5x - 6) = (x + 3)(x + 1)(x - 6)$

Therefore, the other zeros are -1 and 6.

Answer 30e.

consider $f(x) = 4x^3 - 25x^2 - 154x + 40$

and we have $f(10) = 0$

Since $f(10) = 0$, $x - 10$ is a factor of $f(x)$, from the Factor Theorem.

Now using synthetic division, we will get

$$\begin{array}{r|rrrr} 10 & 4 & -25 & -154 & 40 \\ & & 40 & 150 & 40 \\ \hline & 4 & 15 & -4 & 0 \end{array}$$

Now we will use the result to write $f(x)$ as a product of two factors. Then factor completely.

$$\begin{aligned} f(x) &= 4x^3 - 25x^2 - 154x + 40 \\ &= (x-10)(4x^2 + 15x - 4) \\ &= (x-10)(4x^2 + 16x - x - 4) && \left[\begin{array}{l}\text{Replace } 15x \text{ by } 16x - x \\ \text{so that product of coefficient} \\ \text{of } 16x - x \text{ is } -16\end{array}\right] \\ &= (x-10)(4x(x+4) - 1(x+4)) && \left[\begin{array}{l}\text{Taking } 4x \text{ as a common in } 4x^2 + 16x; \\ -1 \text{ common in } -x - 4\end{array}\right] \\ &= (x-10)(x+4)(4x-1) && [\text{Factor trinomial}] \end{aligned}$$

Now equating $f(x)$ to zero, we will get,

$x+4=0$ and $4x-1=0$

Therefore

$$x+4=0$$
$$x=-4$$

And

$$4x-1=0$$
$$4x=1$$
$$x=\frac{1}{4}$$

Therefore other zeros are $\boxed{-4 \text{ and } \frac{1}{4}}$

Answer 31e.

By the factor theorem, $x-k$ is a factor of a polynomial if and only if $f(k)$ is 0.

Since $f(7)$ is 0, $x-7$ is a factor of the given polynomial. Use synthetic division to find the other factors by dividing $10x^3-81x^2+71x+42$ by $x-7$.

$$\begin{array}{r|rrrr} 7 & 10 & -81 & 71 & 42 \\ & & 70 & -77 & -42 \\ \hline & 10 & -11 & -6 & 0 \end{array}$$

The function $f(x)$ can be thus written as $f(x)=(x-7)(10x^2-11x-6)$.

Factor the trinomial $10x^2-11x-6$.

$(x-7)(10x^2-11x-6)=(x-7)(5x+2)(2x-3)$

Therefore, the other zeros are $-\frac{2}{5}$ and $\frac{3}{2}$.

Answer 32e.

Consider, $f(x)=3x^3+34x^2+72x-64$

And we have $f(-4)=0$

Since $f(-4)=0$, $x+4$ is a factor of $f(x)$ from the Factor Theorem

Now using synthetic division, we will get

$$\begin{array}{r|rrrr} -4 & 3 & 34 & 72 & -64 \\ & & -12 & -88 & 64 \\ \hline & 3 & 22 & -16 & 0 \end{array}$$

Now we will use the result to write $f(x)$ as a product of two factors. Then factor completely.

$$f(x)=3x^3+34x^2+72x-64$$

$$=(x+4)(3x^2+22x-16)$$

$$=(x+4)(3x^2+24x-2x-16) \quad \left[\begin{array}{l}\text{Replace } 22x \text{ by } 24x-2x \\ \text{so that product of coefficient} \\ \text{of } 24x-2x \text{ is } -48\end{array}\right]$$

$$=(x+4)(3x(x+8)-2(x+8)) \quad \left[\begin{array}{l}\text{Taking } 3x \text{ as a common in } 3x^2+24x; \\ -2 \text{ common in } -2x-16\end{array}\right]$$

$$=(x+4)(x+8)(3x-2) \quad [\text{Factor trinomial}]$$

Now equating $f(x)$ to zero, we will get,

$x+8=0$ and $3x-2=0$

Therefore

$$x+8=0$$
$$x=-8$$

And

$$3x-2=0$$
$$3x=2$$
$$x=\frac{2}{3}$$

Therefore other zeros are $\boxed{-8 \text{ and } \frac{2}{3}}$

Answer 33e.

By the factor theorem, $x-k$ is a factor of a polynomial if and only if $f(k)$ is 0.

Since $f(9)$ is 0, $x-9$ is a factor of the given polynomial. Use synthetic division to find the other factors by dividing $2x^3-10x^2-71x-9$ by $x-9$.

$$\begin{array}{r|rrrr} 9 & 2 & -10 & -71 & -9 \\ & & 18 & 72 & 9 \\ \hline & 2 & 8 & 1 & 0 \end{array}$$

The function $f(x)$ can be thus written as $f(x)=(x-9)(2x^2+8x+1)$.

Since the trinomial $2x^2 + 8x + 1$ cannot be factored, use the quadratic formula to solve for x.

$$x = \frac{-8 \pm \sqrt{56}}{4}$$
$$= \frac{-8 \pm 2\sqrt{14}}{4}$$
$$= \frac{-4 \pm \sqrt{14}}{2}$$

Therefore, the other zeros are $\frac{-4 - \sqrt{14}}{2}$ and $\frac{-4 + \sqrt{14}}{2}$.

Answer 34e.

Consider the polynomial function

$$f(x) = 5x^3 - x^2 - 18x + 8$$

And a zero of the polynomial function is $f(-2) = 0$

Since $f(-2) = 0$, $x+2$ is a factor of $f(x)$ from the Factor Theorem

Now using synthetic division,

-2	5	-1	-18	8
		-10	22	-8
	5	-11	4	0

Now we will use the result to write $f(x)$ as a product of two factors. Then factor completely.

$$f(x) = 5x^3 - x^2 - 18x + 8$$
$$= (x+2)(5x^2 - 11x + 4)$$

Now for the quadratic equation $5x^2 - 11x + 4 = 0$, we will find the solutions by the quadratic formula $ax^2 + bx + c = 0, a \neq 0 \Rightarrow \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here $a = 5, b = -11, c = 4$

So,

$$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4 \cdot 5 \cdot 4}}{2 \cdot 5}$$
$$= \frac{11 \pm \sqrt{121 - 80}}{10}$$
$$= \frac{11 \pm \sqrt{41}}{10}$$

Thus, the two solutions of the equation $5x^2 - 11x + 4 = 0$ are $\frac{11 + \sqrt{41}}{10}$ and $\frac{11 - \sqrt{41}}{10}$

Therefore, other zeros of the polynomial function $f(x) = 5x^3 - x^2 - 18x + 8$ are

$$\boxed{\frac{11 + \sqrt{41}}{10} \text{ and } \frac{11 - \sqrt{41}}{10}}.$$

Answer 35e.

Since $x = -6$ is a zero of $f(x)$, $x + 6$ is a factor and $f(-6) = 0$. To find the other zeros, divide $4x^3 + 15x^2 - 63x - 54$ by $x + 6$ using synthetic division.

−6	4	15	−63	−54
		−24	54	54
	4	−9	−9	0

The function $f(x)$ can be thus written as $f(x) = (x + 6)(4x^2 - 9x - 9)$.

Factor the trinomial $4x^2 - 9x - 9$.

$(x + 6)(4x^2 - 9x - 9) = (x + 6)(4x + 3)(x - 3)$

The other zeros are $-\frac{3}{4}$ and 3.

Among the given choices, the value in choice **D** is 3, which is a zero of the given function.

Answer 36e.

Consider the volume of a rectangular prism is

$$v = 2x^3 + 17x^2 + 46x + 40$$

Recall that the volume of a rectangular prism is the product of length, width and height.

$$\text{volume} = \text{length} \times \text{width} \times \text{height}$$

We are also given that height and width are $x + 2$ and $x + 4$ respectively.

To find the length of the prism, to divide the volume of the prism by the product of $x + 2$ and $x + 4$.

Product of $x+2$ and $x+4$ is given by

$$(x+2)(x+4)=x^2+2x+4x+8$$
$$=x^2+6x+8$$

Now we will divide $v=2x^3+17x^2+46x+40$ by x^2+6x+8

Write polynomial division in the same format we use when dividing numbers. At each stage, divide the term with the highest power in what is left of the dividend by the first term of the divisor. This gives the next term of the quotient.

Now

$$\begin{array}{r|l} & 2x+5 \quad \leftarrow \text{Quotient} \\ x^2+6x+8 & 2x^3+17x^2+46x+40 \\ & \underline{2x^3+12x^2+16x} \quad \left[\text{Multiply divisor by } \frac{2x^3}{x^2}=2x\right] \\ & 5x^2+30x+40 \quad [\text{Subtract}] \\ & \underline{5x^2+30x+40} \quad \left[\text{Multiply divisor by } \frac{5x^2}{x^2}=5\right] \\ & 0 \quad \leftarrow \text{Remainder} \end{array}$$

Thus,

$$\frac{2x^3+17x^2+46x+40}{x^2+6x+8}=2x+5$$

Therefore, expression for the missing dimension is $\boxed{2x+5}$.

Answer 37e.

We know that the volume of a rectangular prism is the product of its length, breadth, and height. This means that the dimensions of the prism are all factors of the polynomial that represents the volume.

In the figure given, we can see that $x-1$ and $x+6$ are two dimensions and therefore the factors of V. On dividing V by any one of its factors, we can find the other factors.

Let us divide $x^3+13x^2+34x-48$ by $x-1$ using synthetic division.

$$\begin{array}{r|rrrr} 1 & 1 & 13 & 34 & -48 \\ & & 1 & 14 & 48 \\ \hline & 1 & 14 & 48 & 0 \end{array}$$

The function V can be thus written as $V=(x-1)(x^2+14x+48)$.

Factor the trinomial $x^2 + 14x + 48$.
$(x - 1)(x^2 + 14x + 48) = (x - 1)(x + 6)(x + 8)$

Therefore, the third dimension is $x + 8$.

Answer 38e.

Consider the polynomial function

$$f(x) = x^3 - 5x^2 - 12x + 36$$

And a zero of the polynomial function is $f(2) = 0$

(a)

Since $f(2) = 0$, $x - 2$ is a factor of $f(x)$ from the Factor Theorem
Now using synthetic division,

$$\begin{array}{r|rrrr} 2 & 1 & -5 & -12 & 36 \\ & & 2 & -6 & -36 \\ \hline & 1 & -3 & -18 & 0 \end{array}$$

Now we will use the result to write $f(x)$ as a product of two factors. Then factor completely.

$$\begin{aligned} f(x) &= x^3 - 5x^2 - 12x + 36 \\ &= (x-2)(x^2 - 3x - 18) \\ &= (x-2)(x^2 - 6x + 3x - 18) && \left[\begin{array}{l}\text{Replace } -3x \text{ by } -6x + 3x \\ \text{so that product of coefficient} \\ \text{of } -6x + 3x \text{ is } -18\end{array}\right] \\ &= (x-2)(x(x-6) + 3(x-6)) && \left[\begin{array}{l}\text{Taking } x \text{ as a common in } x^2 - 6x; \\ 3 \text{ common in } 3x - 18\end{array}\right] \\ &= (x-2)(x-6)(x+3) && [\text{Factor trinomial}] \end{aligned}$$

Now equating $f(x)$ to zero,

$$x - 6 = 0 \text{ and } x + 3 = 0$$

Therefore

$$x - 6 = 0$$
$$x = 6$$

And

$$x + 3 = 0$$
$$x = -3$$

Therefore, other zeros of the polynomial function $f(x) = x^3 - 5x^2 - 12x + 36$ are

$\boxed{6 \text{ and } -3}$.

(b)

From part (a), the factors of the polynomial function $f(x)=x^3-5x^2-12x+36$ are

$\boxed{x-2, x-6 \text{ and } x+3}$

(c)

Since the factors of the polynomial equation $x^3-5x^2-12x+36=0$ are
$x-2, x-6 \text{ and } x+3$.

So,

$$x^3-5x^2-12x+36=0$$
$$(x-2)(x-6)(x+3)=0$$

Thus,

$$x-2=0$$
$$x=2$$
$$x-6=0$$
$$x=6$$

And

$$x+3=0$$
$$x=-3$$

Therefore, the solutions of the polynomial equation $x^3-5x^2-12x+36=0$ are
$\boxed{2, 6 \text{ and } -3}$.

Answer 39e.

By the factor theorem, if $x-k$ is a factor of the polynomial $f(x)$, then $f(k)=0$.

It is given that $x-5$ is a factor of $x^3-x^2+kx-30$. On substituting 5 for x, the polynomial should evaluate to 0.

$$5^3-5^2+k(5)-30=0$$

Simplify.

$$125-25+5k-30 = 0$$
$$70+5k = 0$$

Subtract 70 from both the sides.

$$70+5k-70 = 0-70$$
$$5k = -70$$

Divide each side by 5 to solve for k.

$$\frac{5k}{5} = \frac{-70}{5}$$
$$k = -14$$

Therefore, the correct answer is choice **A**.

Answer 40e.

Consider the polynomial function

$$f(x) = 30x^3 + 7x^2 - 39x + 14$$

And a factor of the polynomial function is $2x - 1$

(a)

Since $2x - 1$ is a factor of $f(x) = 30x^3 + 7x^2 - 39x + 14$ therefore equating $f(x)$ to 0.

$$2x - 1 = 0$$
$$2x = 1$$
$$x = \frac{1}{2}$$

Therefore, zero of the polynomial function $f(x) = 30x^3 + 7x^2 - 39x + 14$ is at $\boxed{x = \frac{1}{2}}$

(b)

Now using synthetic division,

$$\begin{array}{r|rrrr} \frac{1}{2} & 30 & 7 & -39 & 14 \\ & -15 & 4 & 25 & -14 \\ \hline & 15 & 11 & -14 & 0 \end{array}$$

Now we will use the result to write $f(x)$ as a product of two factors.

$$f(x) = 30x^3 + 7x^2 - 39x + 14$$
$$= (2x - 1)(15x^2 + 11x - 14)$$

Therefore $f(x)$ can be written in the form of $(x-k)\cdot q(x)$ as below:

$$f(x)=(2x-1)(15x^2+11x-14)$$

$$=2\left(x-\frac{1}{2}\right)(15x^2+11x-14) \quad [\text{Taking 2 common from } 2x-1]$$

$$=\boxed{\left(x-\frac{1}{2}\right)(30x^2+22x-28)} \quad [\text{Multiply } 15x^2+11x-14 \text{ by } 2]$$

Where $k=\frac{1}{2}$ and $q(x)=30x^2+22x-28$

(c)

Also, $f(x)=(2x-1)(15x^2+11x-14)$

$$=(2x-1)(15x^2+21x-10x-14) \quad \left[\begin{array}{l}\text{Replace } 11x \text{ by } 21x-10x \\ \text{so that product of coefficient} \\ \text{of } 21x-10x \text{ is } -210\end{array}\right]$$

$$=(2x-1)(3x(5x+7)-2(5x+7)) \quad \left[\begin{array}{l}\text{Taking } 3x \text{ as a common in } 15x^2+21x; \\ -2 \text{ common in } -10x-14\end{array}\right]$$

$$=\boxed{(2x-1)(5x+7)(3x-2)} \quad [\text{Linear factors with integer coefficients}]$$

Answer 41e.

We seek the number of T-shirts that must be produced to make the same profit of \$4,000,000.

Replace P with 4 in the given model and write the resulting equation in standard form.
$4=-x^3+4x^2+x$
or $x^3-4x^2-x+4=0$

Since $x=4$ is a solution of the equation, $x-4$ is factor. Use synthetic division to find the other factor.

4	1	−4	−1	4
		4	0	−4
	1	0	−1	0

The equation thus becomes $(x-4)(x^2-1)=0$

Factor x^2-1.
$(x-4)(x+1)(x-1)=0$

The other solutions are 1 and −1. Since the number of T-shirts cannot be negative, we consider the solution 1 only.

Therefore, the company could make the same profit by producing about 1 million T-shirts.

Answer 42e.

Consider the polynomial function

$$P = -4x^3 + 12x^2 + 16x$$

Where x is the number of MP3 players.

Now,

$$48 = -4x^3 + 12x^2 + 16x \quad [\text{Substitute 48 for } P = -4x^3 + 12x^2 + 16x]$$

$$0 = 4x^3 - 12x^2 - 16x + 48 \quad [\text{Write in standard form}]$$

Recall that $x = 3$ is one solution of the equation. This implies that $x - 3$ is a factor of $4x^3 - 12x^2 - 16x + 48$

Now we will use synthetic division to find the other factors.

$$\begin{array}{r|rrrr} 3 & 4 & -12 & -16 & 48 \\ & & 12 & 0 & -48 \\ \hline & 4 & 0 & -16 & 0 \end{array}$$

Now we will use the result to write $f(x)$ as a product of two factors. Then factor completely.

$$(x-3)(4x^2 - 16) = 0$$

$$(x-3)((2x)^2 - 4^2) = 0$$

$$(x-3)(2x+4)(2x-4) = 0 \quad [\text{Since } a^2 - b^2 = (a+b)(a-b)]$$

Now, $2x + 4 = 0$ and $2x - 4 = 0$

$$2x + 4 = 0$$
$$2x = -4$$
$$x = \frac{-4}{2}$$
$$x = -2$$

And

$$2x - 4 = 0$$
$$2x = 4$$
$$x = \frac{4}{2}$$
$$x = 2$$

Thus, $x = 2$ is the other positive solution.

Therefore, the company could still make the same profit producing about $\boxed{2,000,000}$ MP3 players.

Answer 43e.

The average attendance per team can be determined by dividing the function A by the function T. Let $f(x) = \frac{A}{T} = \frac{-1.95x^3 + 70.1x^2 - 188x + 2150}{14.8x + 725}$.

Use polynomial long division method.

$$\begin{array}{r} -0.132x^2 + 11.2x - 561.4 \\ 14.8x + 725 \overline{) -1.95x^3 + 70.1x^2 - 188x + 2150} \\ \underline{-1.95x^3 - 95.7x^2} \qquad\qquad\qquad \\ 165.8x^2 - 188x \qquad\quad \\ \underline{165.8x^2 + 8120x} \qquad\quad \\ -8308x + 2150 \\ \underline{-8308x - 407{,}015} \\ 409{,}165 \end{array}$$

Thus, the function for the average attendance per team from 1985 to 2003 is

$$f(x) = -0.132x^2 + 11.2x - 561.4 + \frac{409{,}165}{14.8x + 725}.$$

Answer 44e.

Consider the price p (in dollars) that a radio manufacturer able to charge for a radio is given by $p = 40 - 4x^2$ where x in number (in millions) of radios produced.

It cost the company \$15 to make a radio.

(a)

Need to write an expression for the company total revenue R in terms of x. We can get the total revenue if we multiply x with the price of radio.

Therefore,

$$\text{Total revenue}(R) = x \times \text{price of one radio}$$

$$R = x \times (40 - 4x^2)$$

So, the total revenue (R) is $\boxed{R = x \times (40 - 4x^2)}$.

(b)

Need to write a function for company's profit p by subtracting the total cost to make x radios from the expression in part (a).
Company required \$15 to make one radio. So if we multiply x with \$15 then we get total cost to make x radios.
Therefore total cost to make x radios is $= 15x$.
So we can get company's profit p by subtracting $15x$ from the expression in part (a).
Therefore,

$$\text{Profit}(p) = x(40 - 4x^2) - 15x$$

$$= 40x - 4x^3 - 15x \qquad \text{[Using distributive property]}$$

$$= -4x^3 + 25x \qquad \text{[Write in standard form]} \ldots\ldots (1)$$

So, the company's profit is $\boxed{P = -4x^3 + 25x}$.

(c)

Need to write and solve equation (1) to find a lesser number of radios that the company could produce and still make a profit of \$24,000,000.
Currently the company produce 1.5 million radios and makes a profit of \$24,000,000. First we need to convert the value of profit from dollars to millions. So the profit is 24 million.
If we substitute the value of profit ($P = 24$) in equation (1), then we have

$$24 = -4x^3 + 25x$$

$$4x^3 - 25x + 24 = 0 \qquad \text{[Write in standard form]}$$

We know that $x = 1.5$ is one solution for this equation.
This means that $x - 1.5$ is the solution for the equation $4x^3 - 25x + 24 = 0$. We use synthetic division to find other factors.

$$\begin{array}{r|rrrr} 1.5 & 4 & 0 & -25 & 24 \\ & & 6 & 9 & -24 \\ \hline & 4 & 6 & -16 & 0 \end{array}$$

So, $(x - 1.5)(4x^2 + 6x - 16) = 0$

We use quadratic equation to find that $x = 1$ is other positive solution for the equation $4x^2 + 6x - 16 = 0$.
The company could make the same profit by producing 1 million radios.

(d)

Need to explain does all the solution in part (C) make any sense in this situation or not.

The company produce 1.5 million radios and make a profit of \$24,000,000.
But by solving the equation in part (C) we find that the company make a profit of \$24,000,000 for producing 1 million cameras.
So in this situation all the solution in part(C) makes no sense.

Answer 45e.

Let $f(x)$ be the function for the percent of visits to national park. The function can be modeled by dividing the function for the overnight stays S by function for the total visits V.

$$f(x) = \frac{S}{V} = \frac{-0.00722x^4 + 0.176x^3 - 1.40x^2 + 3.39x + 17.6}{3.10x + 256}$$

Use polynomial long division method.

$$\begin{array}{r}
-0.00233x^3 + 0.249x^2 - 21.0x + 1735.3 \\
3.10x + 256 \overline{\smash{\big)}\,-0.00722x^4 + 0.176x^3 - 1.40x^2 + 3.39x + 17.6} \\
\underline{-0.00722x^4 - 0.596x^3} \qquad\qquad\qquad\qquad\qquad \\
0.772x^3 - 1.40x^2 \qquad\qquad\qquad \\
\underline{0.772x^3 + 63.74x^2} \qquad\qquad\qquad \\
-65.14x^2 + 3.39x \qquad\qquad \\
\underline{-65.14x^2 - 5376x} \qquad\qquad \\
5379.39x + 17.6 \\
\underline{5379.39x + 444,236.8} \\
-444,219.2
\end{array}$$

Thus, the function for the percent visits to national parks is

$$f(x) = -0.00233x^3 + 0.249x^2 - 21.0x - 1735.3 + \frac{-444,219.2}{3.10x + 256}$$

Answer 46e.

Consider the polynomial function

$$P = -6x^3 + 72x$$

Where x is the number of DVDs

Now,

$$96 = -6x^3 + 72x \quad \left[\text{Substitute 96 for } P = -6x^3 + 72x\right]$$

$$0 = 6x^3 - 72x + 96 \quad \left[\text{Write in standard form}\right]$$

Recall that $x = 2$ is one solution of the equation. This implies that $x - 2$ is a factor of $6x^3 - 72x + 96$

Use synthetic division to find the other factors.

$$\begin{array}{c|cccc}
2 & 6 & 0 & -72 & 96 \\
 & & 12 & 24 & -96 \\
\hline
 & 6 & 12 & -48 & 0
\end{array}$$

Now, we will use the result to write $f(x)$ as a product of two factors.

$$(x-2)(6x^2+12x-48)=0$$

$$(x-2)(x^2+2x-8)=0 \qquad [\text{Dividing both sides by } 6]$$

$$(x-2)(x^2+4x-2x-8)=0 \qquad \begin{bmatrix}\text{Replace } 2x \text{ by } 4x-2x \text{ so that} \\ \text{product of coefficients of } 4x-2x \text{ is } -8\end{bmatrix}$$

$$(x-2)(x(x+4)-2(x+4))=0$$

$$(x-2)(x+4)(x-2)=0$$

Now, $x+4=0$ and $x-2=0$

$$x+4=0$$
$$x=-4$$

And

$$x-2=0$$
$$x=2$$

Neglecting the negative value, $x=2$. Thus, $x=2$ is the other positive solution.
Therefore, we have got $x=2$ again. Hence, 2 million DVDs is the only production level for the company that yields a profit of $\boxed{\$96,000,000}$.

Answer 47e.

We know that an ordered pair (x, y) is a solution of a linear inequality in two variables, if the inequality is true when the values of x and y are substituted into it.

In order determine whether (1, 4) is a solution of the given inequality, substitute 1 for x, and 4 for y.

$$1-4(4) \stackrel{?}{<} 5$$

$$-15 \stackrel{?}{<} 5$$

The inequality is true and thus (1, 4) is a solution.

Now, substitute 4 for x, and -1 for y to check whether $(4, -1)$ is a solution.

$$4-4(-1) \stackrel{?}{<} 5$$

$$8 \stackrel{?}{<} 5$$

Since the inequality statement is not true, $(4, -1)$ is not a solution.

Answer 48e.

Consider the inequality

$3x+2y \geq 1$ ...... (1)

And the ordered pairs $(-2,4)$ and $(1,-3)$

Now we will check whether the ordered pairs $(-2,4)$ and $(1,-3)$ are the solutions of the given inequality. For that we will put the ordered pairs in the given equation (1) and check.

$$\begin{aligned} 3x+2y &\geq 1 \\ 3\cdot(-2)+2\cdot(4) &\geq 1 \\ -6+8 &\geq 1 \\ 2 &\geq 1 \quad \text{true.} \end{aligned}$$

Because on putting the ordered pair $(-2,4)$ in the inequality, 2 which is greater than 1 and hence satisfy the condition of the inequality. Therefore $(-2,4)$ is the solution of the given inequality $3x+2y \geq 1$.

Similarly,

$$\begin{aligned} 3x+2y &\geq 1 \\ 3\cdot(1)+2\cdot(-3) &\geq 1 \\ 3-6 &\geq 1 \\ -3 &\geq 1 \quad \text{false.} \end{aligned}$$

Because on putting the ordered pair $(1,-3)$ in the inequality, -3 which is not greater than or equal to 1. Therefore, the ordered pair $(1,-3)$ is not a solution of the given inequality $3x+2y \geq 1$. Hence, $(-2,4)$ is a solution of the inequality and $(1,-3)$ is not a solution of the given inequality $3x+2y \geq 1$.

Answer 49e.

We know that an ordered pair (x, y) is a solution of a linear inequality in two variables, if the inequality is true when the values of x and y are substituted into it.

In order determine whether (4, 6) is a solution of the given inequality, substitute 4 for x, and 6 for y.

$$5(4) - 2(6) \overset{?}{>} 10$$

$$8 \overset{?}{>} 10$$

Since the inequality statement is not true, (4, 6) is not a solution.

Now, substitute 8 for x, and 10 for y to check whether (8, 10) is a solution.

$$5(8) - 2(10) \stackrel{?}{>} 10$$

$$20 \stackrel{?}{>} 10$$

The inequality is true and thus (8, 10) is a solution.

Answer 50e.

Consider the inequality

$$6x + 5y \le 15 \qquad \ldots\ldots (1)$$

And the ordered pairs $(-5,10)$ and $(-1,4)$

Now we will check whether the ordered pairs $(-5,10)$ and $(-1,4)$ are the solutions of the given inequality. For that we will put the ordered pairs in the given equation (1) and check.

$$6x + 5y \le 15$$
$$6\cdot(-5) + 5\cdot(10) \le 15$$
$$-30 + 50 \le 15$$
$$20 \le 15 \quad \text{false.}$$

Because on putting the ordered pair $(-5,10)$ in the inequality, 20 which is not less than or equal to 15 .Therefore $(-5,10)$ is not a solution of the inequality $6x + 5y \le 15$.

Similarly,

$$6x + 5y \le 15$$
$$6\cdot(-1) + 5\cdot(4) \le 15$$
$$-6 + 20 \le 15$$
$$14 \le 15 \quad \text{true.}$$

Because on putting the ordered pair $(-1,4)$ in the inequality, 14 which is less than 15 and hence satisfy the condition of inequality. Therefore the ordered pair $(-1,4)$ is a solution of the given inequality. Hence, $(-5,10)$ is not a solution of the inequality and $(-1,4)$ is a solution of the inequality $6x + 5y \le 15$.

Answer 51e.

The given equation is of the form $ax^2 + bx + c = 0$. This equation can be solved using the zero product property only if the left side can be factored.

The trinomial on the left side of the given equation is of the form $x^2 + bx + c$, which when factored will be of the form $(x + m)(x + n)$, where the product of m and n gives c, and their sum gives b.

Compare the trinomial with $x^2 + bx + c = 0$. The value of b is 3 and of c is –40. We need to find m and n such that their product gives –40 and sum gives 3. Since the sum is positive, the larger factor must be positive.

List the factors of –40 and find their sums.

Factors of $-40: m, n$	$-1, 40$	$-2, 20$	$-4, 10$	$-5, 8$
Sum of factors: $m + n$	39	18	6	3

From the table, it is clear that $m = -2$ and $n = 8$ gives the product –40 and sum 3.

Thus, the equation becomes $(x - 5)(x + 8) = 0$.

Apply the zero product property. This property states that if the product of two expressions is zero, then one or both of the expressions equal zero.

$x - 5 = 0$ or $x + 8 = 0$.

Solve the equations.

$$x - 5 + 5 = 0 + 5 \quad \text{or} \quad x + 8 - 8 = 0 - 8$$
$$x = 5 \quad \text{or} \quad x = -8$$

Thus, the solutions to the given equation are –8 and 5.

Answer 52e.

Consider the equation

$$5x^2 + 13x + 6 = 0 \qquad \ldots\ldots (1)$$

The given equation is a quadratic equation of the form $ax^2 + bx + c = 0, a \neq 0$.
Therefore, use the following quadratic formula to solve the equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \ldots\ldots (2)$$

Now putting the values of $a=5, b=13$ and $c=6$ in the equation (2),

$$x=\frac{-13\pm\sqrt{(13)^2-4\cdot5\cdot6}}{2\cdot5} \quad \text{[Putting values of } a, b \text{ and } c\text{]}$$

$$=\frac{-13\pm\sqrt{169-120}}{10}$$

$$=\frac{-13\pm\sqrt{49}}{10}$$

$$=\frac{-13\pm7}{10} \quad \text{[Square root of 49 is 7]}$$

Therefore,

$$x=\frac{-13+7}{10} = \frac{-6}{10} = -\frac{3}{5}$$

And

$$x=\frac{-13-7}{10} = \frac{-20}{10} = -2$$

Hence, the solutions of the given equation $5x^2+13x+6=0$ are $\boxed{-\frac{3}{5} \text{ and } -2}$.

Answer 53e.

The given equation is in standard form. Since it cannot be factored, use the quadratic formula to solve for x.

The solutions of a quadratic equation of the form $ax^2+bx+c=0$ are

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$, where a, b, c are real numbers and $a \neq 0$.

Substitute 1 for a, 7 for b, and 2 for c in the formula.

$$x = \frac{-7\pm\sqrt{7^2-4(1)(2)}}{2(1)}$$

Evaluate.

$$x = \frac{-7\pm\sqrt{49-8}}{2}$$

$$= \frac{-7\pm\sqrt{39}}{2}$$

Therefore, the solutions are $\frac{-7-\sqrt{39}}{2}$ and $\frac{-7+\sqrt{39}}{2}$.

Answer 54e.

Consider the equation

$$4x^2+15x+10=0 \quad \text{...... (1)}$$

The given equation is a quadratic equation of the form $ax^2+bx+c=0, a\neq 0$,
Therefore, use the following quadratic formula to solve the equation.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \quad \text{...... (2)}$$

Now putting the values of $a=4, b=15$ and $c=10$ in the equation (2),

$$x=\frac{-15\pm\sqrt{(15)^2-4\cdot 4\cdot 10}}{2\cdot 4} \quad \text{[Putting values of } a, b \text{ and } c\text{]}$$

$$=\frac{-15\pm\sqrt{225-160}}{8}$$

$$=\frac{-15\pm\sqrt{65}}{8}$$

Therefore,

$$x=\frac{-15+\sqrt{65}}{8} \quad \text{And} \quad x=\frac{-15-\sqrt{65}}{8}$$

Hence, the solutions of the given equation $4x^2+15x+10=0$ are

$$\boxed{\frac{-15+\sqrt{65}}{8} \text{ and } \frac{-15-\sqrt{65}}{8}}.$$

Answer 55e.

The given equation is in standard form. Since it cannot be factored, use the quadratic formula to solve for x.

The solutions of a quadratic equation of the form $ax^2+bx+c=0$ are

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$, where a, b, c are real numbers and $a\neq 0$.

Substitute 2 for a, 15 for b, and 31 for c in the formula.

$$x=\frac{-15\pm\sqrt{15^2-4(2)(31)}}{2(2)}$$

Evaluate.

$$x = \frac{-15 \pm \sqrt{225-248}}{4}$$
$$= \frac{-15 \pm \sqrt{-23}}{4}$$
$$= \frac{-15 \pm \sqrt{23}\,i}{4}$$
$$= \frac{-15 - \sqrt{23}\,i}{4} \text{ or } \frac{-15 + \sqrt{23}\,i}{4}$$

Therefore, the solutions are $\frac{-15 - \sqrt{23}i}{4}$ and $\frac{-15 + \sqrt{23}i}{4}$.

Answer 56e.

Consider the equation,

$$x^2 + 2x + 10 = 0 \quad \ldots\ldots (1)$$

The equation is a quadratic equation of the form $ax^2 + bx + c = 0$, where

$a = 1,\ b = 2$ and $c = 10$

Therefore, use the following quadratic formula to solve the equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Now put the values of a, b and c in the above formula.

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4\cdot1\cdot10}}{2\cdot1}$$
$$= \frac{-2 \pm \sqrt{4-40}}{2}$$
$$= \frac{-2 \pm \sqrt{-36}}{2}$$
$$= \frac{-2 \pm \sqrt{(-1)36}}{2}$$
$$= \frac{-2 \pm \sqrt{i^2 36}}{2}$$
$$= \frac{-2 \pm \sqrt{(6i)^2}}{2}$$
$$= \frac{-2 \pm 6i}{2}$$
$$= -1 \pm 3i$$

That is,

$x = -1 + 3i$ And $x = -1 - 3i$

Therefore the solutions of the equation are $\boxed{= -1 + 3i \text{ and } -1 - 3i}$.

Answer 57e.

Clear the parentheses in the given expression.

$\left(x^2 - 4x + 15\right) + \left(-3x^2 + 6x - 12\right) \Rightarrow x^2 - 4x + 15 - 3x^2 + 6x - 12$

Rewrite the expression such that the like terms appear together.

$x^2 - 4x + 15 - 3x^2 + 6x - 12 \Rightarrow x^2 - 3x^2 - 4x + 6x + 15 - 12$

Add the like terms.

$x^2 - 3x^2 - 4x + 6x + 15 - 12 = -2x^2 + 2x + 3$

Therefore, the sum is $-2x^2 + 2x + 3$.

Answer 58e.

Consider:

$\left(5x^2 - 7x - 7\right) - \left(2x^2 - 5x + 8\right)$.

Simplify by subtracting the polynomials.

Now,

$$\left(2x^2 - 5x + 8\right) - \left(5x^2 - 7x - 7\right)$$

$$= 2x^2 - 5x + 8 - 5x^2 + 7x + 7 \quad \left[\begin{array}{l}\text{Change the sign of}\\ \text{each terms of } 5x^2 - 7x - 7\\ \text{and drop the parentheses}\end{array}\right]$$

$$= \left(2x^2 - 5x^2\right) + \left(-5x + 7x\right) + \left(8 + 7\right) \quad \left[\text{Combine like terms}\right]$$

$$= -3x^2 + 2x + 15 \quad \left[\begin{array}{l}\text{Add the coefficients of like}\\ \text{terms}\end{array}\right]$$

Therefore, $\left(5x^2 - 7x - 7\right) - \left(2x^2 - 5x + 8\right) = \boxed{-3x^2 + 2x + 15}$.

Answer 59e.

In order to multiply two polynomials, multiply the first polynomial by each term in the second polynomial.

$(3x - 4)(3x^3 + 2x^2 - 8) = (3x - 4)3x^3 + (3x - 4)2x^2 + (3x - 4)(-8)$

Use the distributive property to remove the parentheses.

$(3x - 4)3x^3 + (3x - 4)2x^2 + (3x - 4)(-8) = 9x^4 - 12x^3 + 6x^3 - 8x^2 - 24x + 32$

Combine the like terms.

$9x^4 - 12x^3 + 6x^3 - 8x^2 - 24x + 32 = 9x^4 - 6x^3 - 8x^2 - 24x + 32$

Therefore, the product is $9x^4 - 6x^3 - 8x^2 - 24x + 32$.

Answer 60e.

Consider the expression:

$(3x-5)^3$

Simplify the expression.

Now,

$$(3x-5)^3 = (3x-5)(3x-5)^2 \quad \left[\text{Making } (3x-5)^3 \text{ as a product of } (3x-5) \text{ and } (3x-5)^2\right]$$

$$= (3x-5)\left((3x)^2 - 2\cdot 3x\cdot 5 + 5^2\right) \quad \left[\text{Since } (a-b)^2 = a^2 - 2ab + b^2\right]$$

$$= (3x-5)\left(9x^2 - 30x + 25\right) \quad [\text{Simplify and evaluate powers}]$$

$$= 3x\left(9x^2 - 30x + 25\right) - 5\left(9x^2 - 30x + 25\right) \quad [\text{Distributive property}]$$

$$= 27x^3 - 90x^2 + 75x - 45x^2 + 150x - 125$$

$$= 27x^3 - 90x^2 - 45x^2 + 75x + 150x - 125 \quad [\text{Combine like terms}]$$

$$= 27x^3 - 135x^2 + 225x - 125$$

Therefore, $(3x-5)^3 = \boxed{27x^3 - 135x^2 + 225x - 125}$.