CHAPTER-8

BINOMIAL THEOREM

THREE MARKS QUESTIONS:

1. Using Binomial Theorem evaluate$i)\ (102)^5\ ii)\ (99)^5$. (K)
2. Show that $9^{n+1} - 8n - 9$ is divisible by 64, whenever n is a positive integer. (A)
3. Using Binomial theorem, prove that $6^n - 5n$ always leaves remainder 1 when divided by 25. (S)
4. Find $aifthe17^{th}and18^{th}termsoftheexpansion(2+a)^{50}areequal.$ (U)
5. Find the coefficient of $a^5b^7in(a-2b)^{12.}$ (U)
6. Find a positive value of m for which the coefficient of x^2 in the expansion $(1+x)^m$ is 6. (U)
7. In the expansion of $(1+a)^{m+n}$, prove that coefficients of $a^m and a^n$ are equal. (A)
8. Prove that the coefficient of x^n in the expansion of $(1+x)^{2n}$ is twice the coefficient of x^n in the expansion of $(1+x)^{2n-1}$. (U)
9. Which is larger $(1.01)^{100000}$ or 10,000? (K)
10. Prove that$\sum_{r=0}^{n} 3^r\ .\ n_{C_r} = 4^n$. (K)
11. Find the coefficient of $x^5in(x+3)^8$. (U)
12. Find the 4^{th}term in the expansion of $(x-2y)^{12}$. (U)
13. Find the 13^{th}term in the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$. (U)
14. Find the middle term in the expansion of $\left(\frac{x}{3} + 9y\right)^{10}$. (U)
15. Show that the middle term in the expansion of$(1+\mathrm{x})^{2\mathrm{n}}$ is $\frac{1.3.5\ldots\ldots(2\mathrm{n}-1)}{\mathrm{n}!}\ 2\mathrm{n}.\mathrm{x}^{\mathrm{n}}$. (A)
16. If the coefficients of $(r-5)^{th}$and $(2r-1)^{th}$ terms of the expansion $(1+x)^{34}$ are equal, find r. (A)
17. Find the r^{th} term from the end in the expansion of $(x+a)^n$. (A)
18. Find the value of $\left(a^2+\sqrt{a^2-1}\right)^4 + \left(a^2-\sqrt{a^2-1}\right)^4$. (A)
19. If a and b are distinct integers, prove that a – b is a factor of $a^n - b^n$, when n is a positive integer. (A)
20. Find the coefficient of x^5 in the product $(1+2x)^6(1-x)^7$. (A)
21. Expand each of the following and find the sum of the binomial coefficients in each case:

i) $\left(\frac{2}{x} - \frac{x}{2}\right)^5$ ii) $\left(\frac{x}{3} + \frac{1}{x}\right)^6$ iii) $(2x-3)^7$. (U)

FIVE MARKS QUESTIONS:

1. State and prove Binomial Theorem for any positive integer n. (K)
2. Find $(a+b)^4 - (a-b)^4$. Hence, evaluate$\left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4$. (S)
3. The second, third and fourth terms in the binomial expansion $(x+a)^n$ are 24, 720 and 1080 respectively. Find x , a and n. (A)
4. The coefficients of the $(r-1)^{th}$, $r^{th}$and$(r+1)^{th}$terms in the expansion of $(x+1)^n$ are in the ratio $1:3:5$. Find nandr. (A)
5. Find the middle terms in the expansion of i) $\left(3 - \frac{x^3}{6}\right)^7$ ii) $\left(\frac{x}{3} + 9y\right)^{11}$. (U)

6. Find the term independent of x in the expansionof i) $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^6$ ii) $\left(\sqrt[3]{x} + \frac{1}{2\sqrt[3]{x}}\right)^{18}, x > 0$. (U)

7. The coefficients of three consecutive terms in the expansion of $(1 + a)^n$are in the ratio of 1:7:42. Find n. (A)

8. The sum of the coefficients of the first three terms in the expansion of$\left(x - \frac{3}{x^2}\right)^m, m \neq 0$, m being natural number, is 559. Find the term of the expansion containing x^3. (A)

9. Show that the coefficient of the middle term in the expansion of $(x + 1)^{2n}$ is equal to the sum of the coefficients of two middle terms in the expansion of $(x + 1)^{2n-1}$. (A)

10. Find n, if the ratio of fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}: 1$. (S)

11. If the coefficients of a^{r-1}, a^r, and a^{r+1} in the expansion of $(1 + x)^n$ are in arithmetic progression, prove that $n^2 - n(4r+1) + 4r^2 - 2 = 0$. (A)

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