# Chapter 6

# Venn Diagrams

**CHAPTER HIGHLIGHTS**

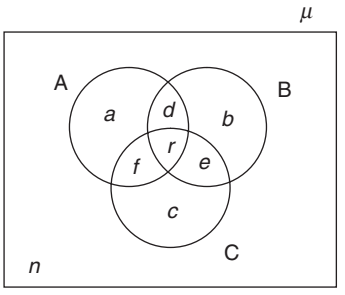
- *Venn Diagrams*
- *Venn Diagram Type I*
- *Venn Diagram Type II*

## Venn Diagrams

Venn Diagrams are diagrammatic representation of sets, using geometrical figures like circles, triangles, and rectangles. Each geometrical figure represents a group as shown in the examples. The area common to two or more figures represent those elements which are common to two or more groups. There are various models in Venn diagrams, which we will discuss with examples.

## Venn Diagram Type I

In this type, two, three, or four different groups could be given with some elements common to two or more groups. Let us observe the diagram given.




Here, A, B, and C are three different groups, and the various regions can be explained as follows:

Only A = $a$
Only B = $b$
Only C = $c$
A and B only = $d$
B and C only = $e$
C and A only = $f$
All the three (A, B, and C) = $r$
Both A and B = $d + r$
Both B and C = $e + r$
Both C and D = $f + r$
Neither A, nor B, nor C = $n$
A, B, or C and none = $\mu$
Also, $\mu = (A \cup B \cup C) + n$

Here, the rectangle represents the sample space, which consists of three groups A, B, and C, and also n, which is the number of people belonging to neither A, nor B, nor C.

Some more formulae are as given under:

1. $A' = (b + e + c) + n$; where $A'$ = A complement (not in A)
2. $B' = (a + f + c) + n$; where $B'$ = B complement (not in B)
3. $C' = (a + d + b) + n$; where $C'$ = C complement (not in C)
4. $A - B = A - (A \cap B)$
5. $A \Delta B = (A - B) \cup (B - A)$
6. Number of people (or things) belonging to at least one out of the three groups = $A \cup B \cup C$
   $= (a + b + c) + (d + e + f) + r$
   ↑ exactly one ↑ exactly two ↑ exactly three
7. $A + B + C = (A \cup B \cup C) + (d + e + f) + 2r$
   $= (a + b + c) + 2(d + e + f) + 3r$

## Solved Examples

***Direction for questions 1 to 3:*** These questions are based on the data given.

In a class of 165 students, 45 students are passed in Maths a well as in English, whereas 60 students are failed in Maths and 65 students are failed in English.

**Example 1:** How many students are passed in exactly one subject?

(A) 160 (B) 100
(C) 115 (D) 165

**Example 2:** How many students are failed in both the subjects?

(A) 25 (B) 20 (C) 45 (D) 5

**Example 3:** How many students are failed only in Maths?

(A) 55 (B) 60 (C) 65 (D) 70

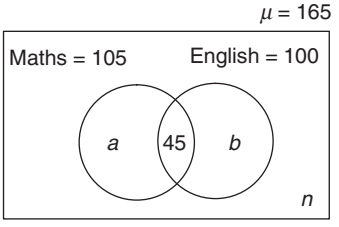
***Solutions for questions 1 to 3:***

Since, 60 students are failed in Maths.

$\therefore$ 165 – 60 = 105 students passed in Maths similarly, 65 students failed in English.

$\therefore$ 165 – 65 = 100 students passed in English.

The respective Venn diagrams is as follows, which shows the number of students who passed the subject.




45 students passed in Maths as well as English.

$\therefore$ $a$ = 105 – 45 = 60 students passed only in Maths and $b$ = 100 – 45 = 55 students passed only English.

Number of students passed in atleast one subject = 60 + 55 + 45 = 160.

Hence, $n$ = 165 – 160 = 5 students failed in both.

**Solution 1:** $a + b$ = 60 + 55 = 115 students passed exactly in one subject.

Hence, the correct option is (C).

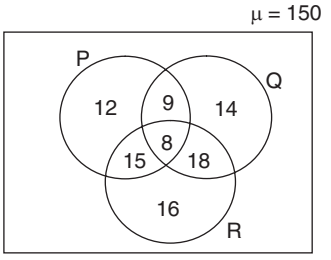
**Solution 2:** 5 students failed in both the subjects.

Hence, the correct option is (D).

**Solution 3:** As 55 students passed only in English which implies that 55 students failed only in Maths.

Hence, the correct option is (A).

***Direction for questions 4 to 7:*** These questions are based on the given diagram.




**Example 4:** How many elements are there in Q′ (complement of Q)?

(A) 100 (B) 49 (C) 101 (D) 50

**Example 5:** How many elements are there in P′ ∩ Q′ ∩ R′?

(A) 35 (B) 8 (C) 58 (D) 48

**Example 6:** How many elements are there in R?

(A) 16 (B) 57 (C) 41 (D) 8

**Example 7:** How many elements are there in P ∩ (Q ∪ R)?

(A) 32 (B) 48 (C) 54 (D) 44

***Solutions for questions 4 to 7:***

**Solution 4:** Number of elements in Q′ = ($\mu$) – (number of elements in Q)

= 150 – (14 + 18 + 8 + 9)

$\Rightarrow$ 150 – 49 = 101.

Hence, the correct option is (C).

**Solution 5:** Number of elements in P′ ∩ Q′ ∩ R′

$= \mu - (P \cup Q \cup R)$

= 150 – (12 + 15 + 9 + 8 + 18 + 16 + 14)

= 150 – 92 = 58.

Hence, the correct option is (C).

**Solution 6:** Number of elements in R = 16 + 15 + 8 + 18 = 57

Hence, the correct option is (B).

**Solution 7:** Number of elements in Q ∪ R =

(14 + 9 + 8 + 15 + 18 + 16)

Number of elements in P = (12 + 9 + 15 + 8)

P ∩ (Q ∪ R) is the region common to P and Q ∪ R

Number of elements in P ∩ (Q ∪ R)

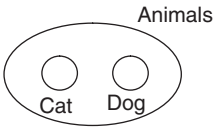
= 9 + 8 +15 = 32.

Hence, the correct option is (A).

## Venn Diagram Type II

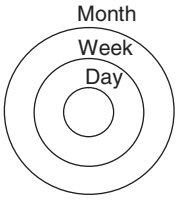
In this type, Venn diagrams are used to establish relationship between the given groups. In other words, two or more groups are given, and the Venn diagram, which most correctly establishes a relation between them, has to be chosen out of the various Venn diagrams given in the choices. Let us look at some of the examples given.

**Examples:** Animals, Cat, Dog



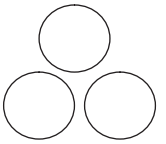

Here, in animals we have many species of which cat and dog are two different kinds of species, having nothing in common. So the above diagram is the most appropriate representation of the given groups.

**Examples:** Month, week, day




We know that day is a part of the week and week is a part of the month. So, the given diagram is the most appropriate representation of the given groups.

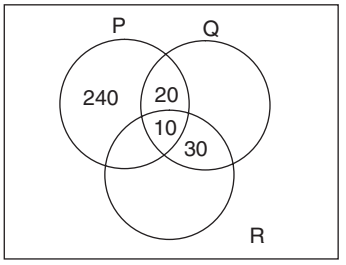
**Examples:** Mars, Earth, Jupiter



We know that Mars, Earth, and Jupiter are three independent entities having nothing in common. So the above-given diagram is the most appropriate representation of the given groups.

## Exercises

***Direction for questions 1 to 5:*** There are 1500 students in a college. Each student can be a member of three student communities namely P, Q, and R. Now, using the data mentioned and the diagram given, answer the questions that follow.




- Total members in community P is 300.
- Total members in community Q is 420.
- Total members in community R is 490.

**1.** How many students are part of only community R?
(A) 360 (B) 420 (C) 210 (D) 350

**2.** How many students is not part of any community?
(A) 390 (B) 420 (C) 410 (D) 490

**3.** How many students are part of at least two communities?
(A) 10 (B) 30
(C) 80 (D) 90

**4.** How many students are part of at least one community?
(A) 1000 (B) 1090
(C) 1110 (D) 1100

**5.** How many students are part of exactly two communities?
(A) 90 (B) 80
(C) 100 (D) 120

***Direction for questions 6 to 10:*** These questions are based on the data given.

In a class of 95 students, 40 play cricket, 50 play football, and 10 play both cricket and football.

**6.** How many students play only football?
(A) 45 (B) 30 (C) 40 (D) 28

**7.** How many students play at least one game?
(A) 80 (B) 70 (C) 60 (D) 50

**8.** How many students play only cricket?
(A) 30 (B) 35 (C) 40 (D) 25

**9.** How many students play exactly one game?
(A) 85 (B) 80 (C) 70 (D) 75

**10.** How may students play neither cricket nor football?
(A) 12 (B) 15 (C) 18 (D) 20

***Direction for questions 11 to 15:*** Study the following data and answer the questions given.

In a certain college, 37% of the students write EAMCET exam, 47% of the students write IIT-JEE exam, and 50% of the students write AIEEE exam. Also known that, 11% of the students write both EAMCET and IIT-JEE, 11% of the students write both EAMCET and AIEEE, 15% of the students write both IIT-JEE and AIEEE, while 15 students write all the three exams. Each student in the college writes at least one of the three exams.

**11.** How many students appear for the exams from the college?
(A) 400 (B) 200
(C) 500 (D) 600

**12.** How many students write exactly two exams?
(A) 120 (B) 110 (C) 140 (D) 150

**13.** The number of students who write only EAMCET as a percentage of the number of students who write only AIEEE is

(A) $33\frac{1}{3}\%$ (B) $66\frac{2}{3}\%$

(C) $33\frac{2}{3}\%$ (D) $66\frac{1}{3}\%$

**14.** How many students write exactly one exam?

(A) 345 (B) 395 (C) 198 (D) 398

**15.** What is the ratio of the number of students who write only AIEEE to that of those who write only IIT JEE?

(A) 3 : 2 (B) 2 : 3 (C) 8 : 9 (D) 9 : 8

***Direction for questions 16 to 20:*** These questions are based on the data given.

In a library maintained by a student, there are books on different subjects. It was found that 35 books are on sports, 45 books are on business, and 15 books are on current affairs;14 books are on at least two subjects among sports, business, and current affairs; 3 books have sports, business as well as current affairs in them. Every book in the library is assumed to contain at least one of sports, business, or current affairs in them.

**16.** How many books are there, which contain information regarding only one subject?

(A) 58 (B) 64 (C) 60 (D) 62

**17.** What are the total number of books in his library?

(A) 78 (B) 72 (C) 68 (D) 80

**18.** How many books contained information regarding exactly two subjects?

(A) 11 (B) 10 (C) 9 (D) 14

**19.** How many books are there, which contain information regarding at most two subjects?

(A) 11 (B) 64 (C) 72 (D) 75

**20.** If the number of books on only sports is equal to 26, then how many books are there in the library, which are on both business and current affairs but not sports?

(A) 5 (B) 3 (C) 2 (D) 8

***Direction for questions 21 to 25:*** These questions are based on the data given.

In a colony, it is known that three brands of mobile phones are used, namely Nokia, Sony Ericsson, and Motorola. 70 families use only one brand, 47 families use exactly two brands, and 8 use all the three brands. It is assumed that each family uses at least one of these three brands.

**21.** How many families are there in the colony?

(A) 75 (B) 100 (C) 105 (D) 125

**22.** How many families use at least two brands?

(A) 117 (B) 55 (C) 47 (D) 125

**23.** If 10 families stop using Nokia and start using Motorola, then what is the maximum number of families who use exactly two brands?

(A) 57 (B) 37 (C) 47 (D) 67

**24.** What is the ratio of the number of families which use exactly one brand to that which use at least one brand?

(A) 14 : 25 (B) 14 : 11

(C) 11 : 25 (D) 11 : 14

**25.** How many families do not use all the three brands?

(A) 125 (B) 117 (C) 0 (D) 8

## Answer Keys

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|---|---|---|---|---|---|---|---|---|---|
| **1.** B | **2.** A | **3.** D | **4.** C | **5.** B | **6.** C | **7.** A | **8.** A | **9.** C | **10.** B |
| **11.** C | **12.** C | **13.** B | **14.** A | **15.** D | **16.** B | **17.** A | **18.** A | **19.** D | **20.** A |
| **21.** D | **22.** B | **23.** A | **24.** A | **25.** B | | | | | |