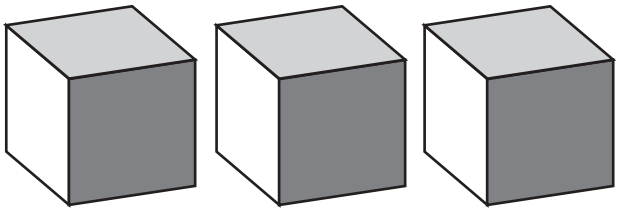
Activity 24



Algebraic identity

Objective

To verify the identity $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ using cuboids and unit cubes.

Pre–requisite knowledge

1. Express the volume of an object as the number of unit cubes in it.
2. Knowledge of the identity $(a + b)^3 = a^3 + a^2b + a^2b + a^2b + ab^2 + ab^2 + ab^2 + b^3$

Material Required

64 unit cubes made of wood (dimension is 1 unit × 1 unit × 1 unit).

Procedure

1. Take a = 3 and make a cube of dimensions a^3 i.e. 3 × 3 × 3 using 27 unit cubes as shown in Fig 24 (a).
2. Take b = 1 and make a cuboid of dimensions a^2b i.e. 3 × 3 × 1. [Fig 24 (b)] Add this three times in Fig 24 (a) as shown in Fig 24 (c).
3. Make a cuboid of dimensions ab^2 i.e. 3 × 1 × 1. [Fig 24 (d)] Add this cuboid three times in Fig 24 (c) as shown in Fig 24 (e).
4. Make a cuboid of dimensions b^3 i.e. 1 × 1 × 1. [Fig 24 (f)] Add this cube in Fig 24 (e) as shown in Fig 24 (g).
5. The total number of cubes will be $64 = 4^3$ i.e. $(a + b)^3$ as shown in Fig 24 (g).

Observations

1. Number of unit cube in $a^3 = 3^3 = 27$
2. Number of unit cube in $a^2b = 9$
3. Number of unit cube in $a^2b = 9$
4. Number of unit cube in $a^2b = 9$
5. Number of unit cube in $ab^2 = 3$
6. Number of unit cube in $ab^2 = 3$
7. Number of unit cube in $ab^2 = 3$
8. Number of unit cube in $b^3 = 1$
9. Total cubes = 64
10. $64 = 4^3$
11. It is verified that

$$(a + b)^3 = a^3 + a^2b + a^2b + a^2b + ab^2 + ab^2 + ab^2 + b^3$$
$$= a^3 + 3a^2b + 3ab^2 + b^3$$

Learning Outcomes

1. The students obtain the skill of making cuboids using unit cubes.
2. The students obtain the skill of adding and subtracting the volume of cuboids.
3. Showing the volume of a cube as the sum of cuboids helps them to get a geometric feeling of volume.

Remark

1. Teachers can take any value of a and b and verify the result.
2. The dimensions of cuboid added and removed should be calculated by students.

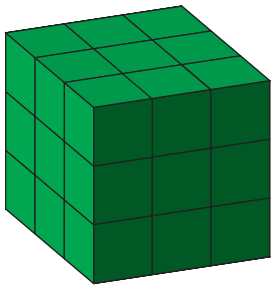


Fig 24 (a)

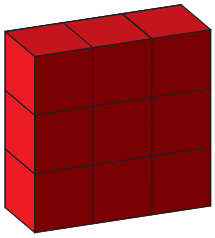


Fig 24 (b)

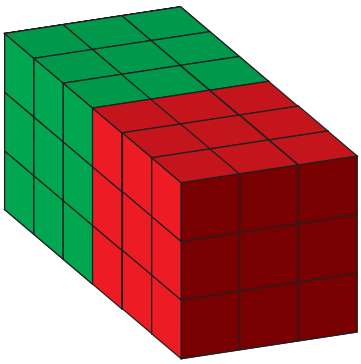


Fig 24 (c)

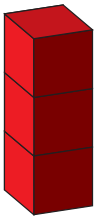


Fig 24 (d)

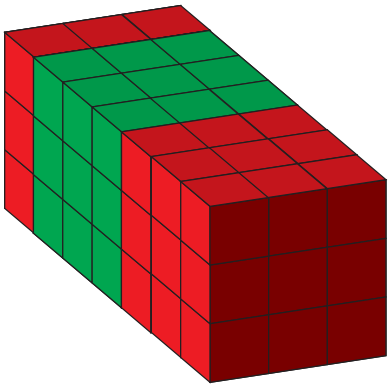


Fig 24 (e)

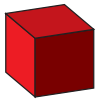


Fig 24 (f)

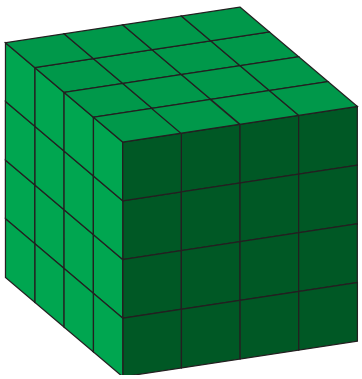


Fig 24 (g)