CBSE Class 10 Mathematics Standard

Sample Paper - 02 (2020-21)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. This question paper contains two parts A and B.
ii. Both Part A and Part B have internal choices.

Part – A consists 20 questions

i. Questions 1-16 carry 1 mark each. Internal choice is provided in 5 questions.
ii. Questions 17-20 are based on the case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B consists 16 questions

i. Question No 21 to 26 are Very short answer type questions of 2 mark each,
ii. Question No 27 to 33 are Short Answer Type questions of 3 marks each
iii. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
iv. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

Part-A

1. The HCF of 45 and 105 is 15. Write their LCM.

OR

$\sqrt{7}$ is an irrational number.

2. Without factorization, find the nature of the roots of the quadratic equation.

$4x^2 - 12x + 9 = 0$.

3. Find the value of k so that the following system of equations has no solution:

$3x - y - 5 = 0, 6x - 2y + k = 0$

4. How many common tangents can be drawn to two circles touching externally?
5. Write the first four terms of the AP, when the first term a = -1 and the common difference $d = \frac{1}{2}$

OR

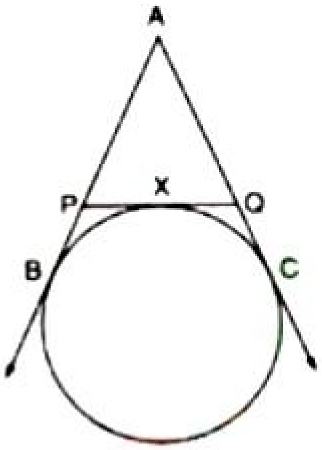
Find the 6^{th} term from the end of the A.P. 17,14,11,..., - 40

6. For the AP 0.6, 1.7, 2.8, 3.9 ...., write the first term and the common difference.
7. Solve: $4x^2 + 5x = 0$.

OR

If p, q and r are rational numbers and $p \neq q \neq r$, then find the roots of the equation $(p^2 - q^2)x^2 - (q^2 - r^2)x + r^2 - p^2 = 0$.

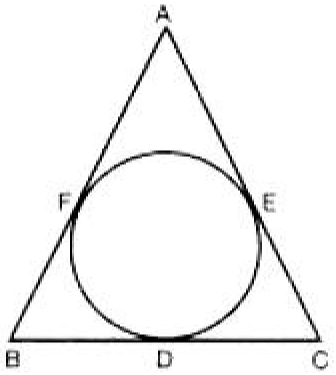
8. In the given figure,AB, AC and PQ are tangents. If AB = 5 cm, then find the perimeter of $\triangle APQ$.



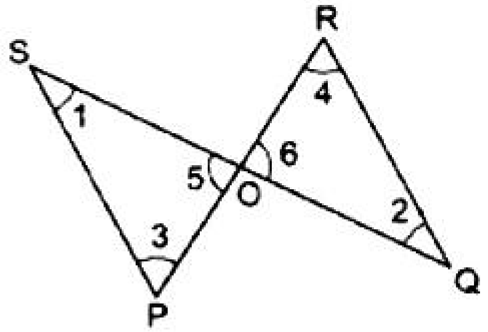
9. What is the maximum number of parallel tangents a circle can have on a diameter?

OR

A triangle ABC is drawn to circumscribe a circle. If AB = 13 cm, BC = 14 cm and AE = 7 cm, then find AC.



10. In Fig. if ΔPOS ~ ΔROQ, prove that $PS \| QR$.



11. Which term of the sequence 4,9,14,19, ... is 124?
12. Prove that: $\frac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}+\sin\theta\cos\theta=1$
13. If cot A + $\frac{1}{cot A}$ = 1, then find the value of $\cot^2$ A + $\frac{1}{cot^2 A}$.
14. A metallic cone of radius 12 cm and height 24 cm is melted and made into spheres of radius 2 cm each. How many spheres are formed?
15. What is the sum of first n terms of the AP a, 3a, 5a,.....
16. A card is drawn at random from a well-shuffled pack of 52 cards. Find the probability that the card was drawn is neither a red card nor a queen.
17. **2-DIMENSINAL PLANE/ CARTESIAN PLANE**

Using Cartesian Coordinates we mark a point on a graph by **how far along** and **how far up** it is.

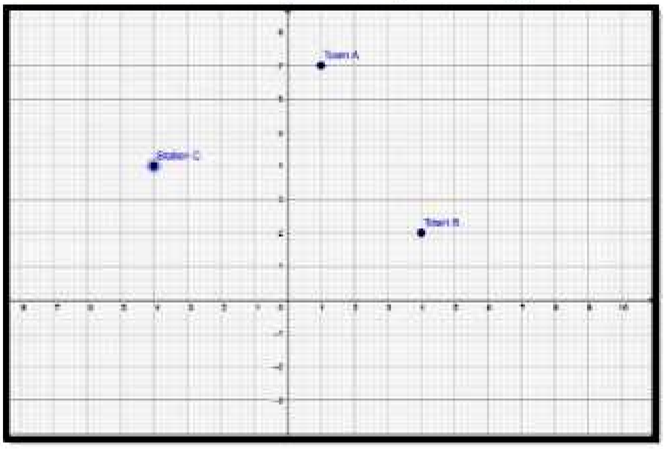
The left-right (**horizontal)** direction is commonly called X-axis.

The up-down (**vertical)** direction is commonly called Y-axis.

When we include negative values, the x and y axes divide the space up into 4 pieces.

Read the information given above and below and answer the questions that follow:

Two friends Seema and Aditya work in the same office in Delhi. In the Christmas vacations, both decided to go their hometowns represented by Town A and Town B respectively in the figure given below. Town A and Town B are connected by trains from the same station C (in the given figure) in Delhi.



i. Who will travel more distance to reach to their hometown?
   a. Seema
   b. Aditya
   c. Both travel the same distance
   d. None of these

ii. Seema and Aditya planned to meet at a location D situated at a point D represented by the mid-point of the line joining the point represented by Town A and Town B. Then the coordinates of the point represented by the point D are:
   a. $\left(\frac{2}{5}, \frac{9}{2}\right)$
   b. $\left(\frac{5}{2}, \frac{2}{9}\right)$
   c. $\left(\frac{9}{2}, \frac{5}{2}\right)$
   d. $\left(\frac{5}{2}, \frac{9}{2}\right)$

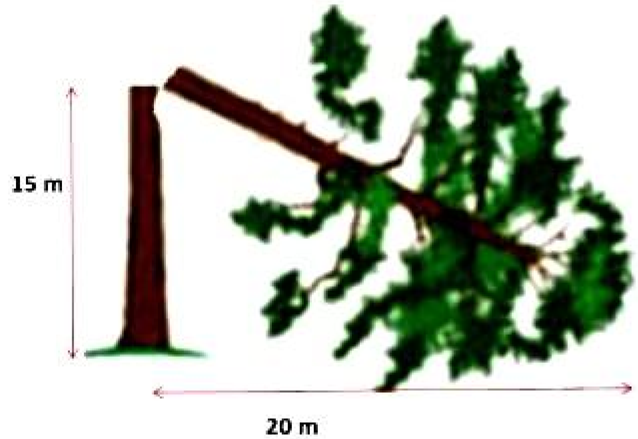
iii. The area of the triangle formed by joining the points represented by A, B and C is:
   a. 17 sq. units
   b. 27 sq. units
   c. 7 sq. units
   d. 15 sq. units

iv. The location of the station is given by:
   a. (4, -4)
   b. (-4, 4)
   c. (-2, 4)
   d. (4, 2)

v. The location of the Town B is given by:

a. (4, -4)
b. (1, 7)
c. (2, 4)
d. (4, 2)

18. Suresh is having a garden near Delhi. In the garden, there are different types of trees and flower plants. One day due to heavy rain and storm one of the trees got broken as shown in the figure.



The height of the unbroken part is 15 m and the broken part of the tree has fallen at 20 m away from the base of the tree.

Using the Pythagoras answer the following questions:

a. What is the length of the broken part?
   a. 15 m
   b. 20 m
   c. 25 m
   d. 30 m

b. What was the height of the full tree?
   a. 40 m
   b. 50 m
   c. 35 m
   d. 30 m

c. In the formed right-angle triangle what is the length of the hypotenuse?
   a. 15 m
   b. 20 m
   c. 25 m

d. 30m

d. What is the area of the formed right angle triangle?

a. 100 m^2

b. 200 m^2

c. 60 m^2

d. 150 m^2

e. What is the perimeter of the formed triangle?

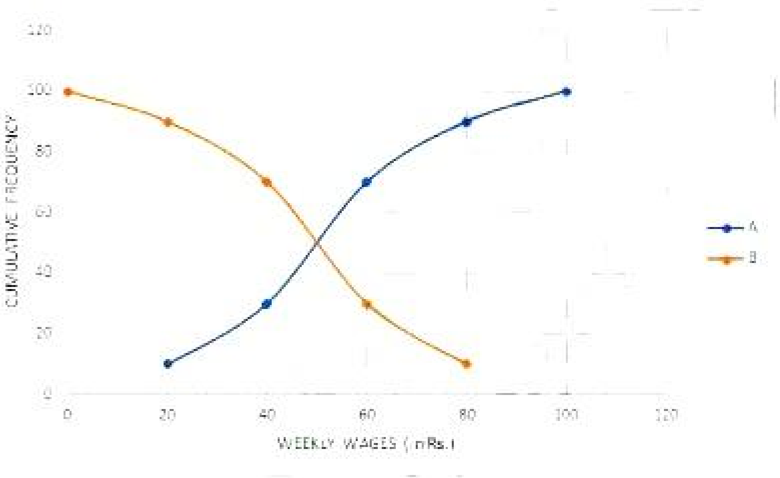
a. 60 m

b. 50 m

c. 45 m

d. 100 m

19. A Mall is constructing in the city, Jaipur. 100 workers are working in the Mall. The data of the distribution of weekly wages of 100 workers are recorded and the following graph is made:



Based on the above graph, answer the following questions:

i. Identify less than type ogive from the given graph.

a. A

b. point of intersection of A and B

c. B

d. none of these

ii. Find the Median Wages.

a. Rs.60

b. Rs.150

c. Rs.50

d. Rs.55

iii. Find Mode of the data if Mean Wages is Rs. 50

a. Rs.52

b. Rs.60

c. Rs.55

d. Rs.50

iv. The construction of the cumulative frequency table is useful in determining the:

a. Median

b. Mean

c. Mode

d. All of the above

v. The intersection of the Ogive graph(abscissa) represents which of the following:

a. Mean

b. Median

c. Mode

d. All of these

20. To make the teaching, learning process easier, creative, and innovative, A teacher brings clay in the classroom to teach the topic mensuration. She thought this method of teaching is more interesting, leave a long-lasting impact She forms a cylinder of radius 6 cm and height 8 cm with the clay, then she moulds the cylinder into a sphere and asks some question to students [use π = 3.14]

i. The radius of the sphere so form:

a. 6 cm

b. 7 cm

c. 4 cm

d. 8 cm

ii. The volume of the sphere so formed:

a. 902.32 cm^3

b. 899.34 cm^3

c. 904.32 cm^3

d. 999.33 cm^3

iii. What is the ratio of the volume of a sphere to the volume of a cylinder?

a. 1:2

b. 2:1

c. 1:1

d. 3:1

iv. The total surface area of the cylinder is:

a. 525.57 cm^2

b. 557.55 cm^2

c. 534.32 cm^2

d. 527.52 cm^2

v. During the conversion of a solid from one shape to another the volume of the new shape will:

a. increase

b. decrease

c. remain unaltered

d. be double

Part-B

21. Express the following in the form p/q, where p and q are integers and q ≠0.

$0.\overline{2341}$

22. The mid-points of the sides of a triangle are (3, 4), (4,1) and (2, 0). Find the coordinates of the vertices of the triangle.

OR

Find the values of k so that the area of the triangle with vertices (1, -1), (- 4, 2k) and (- k, - 5) is 24 sq. units.

23. Find a cubic polynomial whose zeros are $\frac{1}{2}$, 1 and -3.
24. Draw a circle of radius 4cm from a point P, 7cm from the centre of the circle, draw a pair of tangents to the circle measure the length of each tangent segment.
25. Prove the following identity, where the angles involved are acute angles for which the expressions are defined. $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}=1+\sec A \cos ec A$
[Hint : Write the expression in terms of sin θ and cos θ]

OR

If $\cot \theta=\frac{7}{8}$, evaluate $\frac{(1+\sin \theta)(1-\sin \theta)}{(1+\cos \theta)(1-\cos \theta)}$.

26. From a point P, the length of the tangent to a circle is 15 cm and distance of P from the centre of the circle is 17 cm. Then what is the radius of the circle?
27. Show that 3 + 5$\sqrt{2}$ is an irrational number.
28. The hypotenuse of a right triangle is 25 cm. The difference between the lengths of the other two sides of the triangle is 5 cm. Find the lengths of these sides.

OR

Solve the quadratic equation by factorization:
$\frac{1}{2a+b+2x}=\frac{1}{2a}+\frac{1}{b}+\frac{1}{2x}$

29. Find the zeros of the polynomial f(x) = $x^3 - 12x^2 + 39x - 28$, if it is given that the zeros are in A.P.
30. D is the mid-point of side BC of $\triangle$ABC and E is the mid-point of AD. BE produced meets AC at the point M. Prove that BE = 3EM.

OR

In a $\triangle ABC$, AB = AC and D is a point on AC such that $BC^2 = AC \times DC$. Prove that BD = BC.

31. A number is selected at random from first 50 natural numbers. Find the probability that it is a multiple of 3 and 4.

32. If a 1.5-m-tall girl stands at a distance of 3m from a lamp-post and casts a shadow of length 4.5m on the ground then find the height of the lamp-post.

33. Compute the mode of the following data:

Class Interval	1 - 5	6 - 10	11 - 15	16 - 20	21 - 25	26 - 31	31 - 35	36 - 40	41 - 45	46 - 50
Frequency	3	8	13	18	28	20	13	8	6	4

34. A semicircular region and a square region have equal perimeters. The area of the square region exceeds that of the semicircular region by 4 cm^2. Find the perimeters and areas of the two regions.

35. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. find the dimensions of the garden.

36. From the top of a 7 m high building, the angle of elevation of the top of a tower is 60^o and the angle of depression of its foot is 45^o. Find the height of the tower.

CBSE Class 10 Mathematics Standard

Sample Paper - 02 (2020-21)

Solution

Part-A

1. HCF(45, 105) = 15

   $\therefore \text{LCM} = \frac{45\times105}{15}$ = 315

OR

since we know that 7, is a prime number

as it has only 2, factors i,e 1 and itself

also, we know that the square root of every prime number is an irrational number.

therefore $\sqrt{7}$ is an irrational number.

2. $4x^2 - 12x + 9 = 0$

   Here a = 4, b = -12, c = 9

   $D = (-12)^2 - 4 \times 4 \times 9 = 144 - 144 = 0$

   $\therefore$ Equation has real and equal roots.

3. Given system of equations is, 3x - y - 5 = 0 and 6x - 2y + k = 0

   Here $a_1 = 3$, $b_1 = -1$, $c_1 = -5$ and $a_2 = 6$, $b_2 = -2$, $c_2 = k$

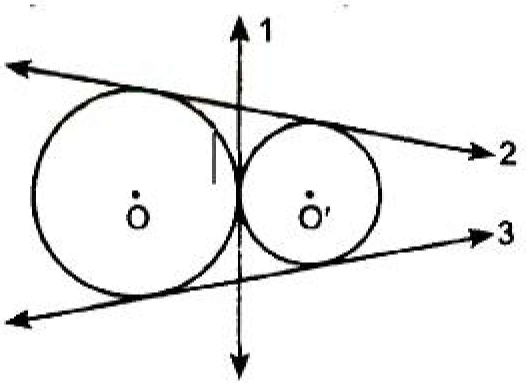
   For no solution $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

   $\implies \frac{3}{6} = \frac{1}{2} \neq \frac{-5}{k}$

   $\Rightarrow \frac{1}{2} \neq \frac{-5}{k}$

   $\implies k \neq -10$

4. 3 common tangents can be drawn when circles touch externally as shown in the figure.



5. a = -1, $d = \frac{1}{2}$

First term = a = -1

Second term = -1 + d $= -1 + \frac{1}{2} = -\frac{1}{2}$

Third term $= -\frac{1}{2} + d = -\frac{1}{2} + \frac{1}{2} = 0$

Fourth term = 0 + d $= 0 + \frac{1}{2} = \frac{1}{2}$

Hence, the first four terms of the given AP are -1, $-\frac{1}{2}, 0, \frac{1}{2}$.

OR

A.P. is 17,14,11,..., - 40

We have,

l = Last term = -40 , a = 17 and, d = Common difference= 14 - 17 = - 3

∴ 6th term from the end = l - (n -1)d

= l - (6-1) d

= -40 - 5 × (-3)

= -40 + 15

= -25

So, 6th term of given A.P. is -25.

6. 0.6, 1.7, 2.8, 3.9...

First term = a= 0.6

Common difference (d) = Second term - First term

= Third term - Second term and so on

Therefore, Common difference (d) = 1.7−0.6=1.1

7. $4x^2 + 5x = 0$

$\Rightarrow x(4x+5) = 0$

$\Rightarrow x = 0 \; or \; 4x + 5 = 0$

⇒ x = 0 or x = $-\frac{5}{4}$

OR

In quadratic equation $ax^2 + bx + c = 0$, if $a + c = b$, then roots are -1 and $\frac{-c}{a}$.

Here $p^2 - q^2 + r^2 - p^2 = -(q^2 - r^2)$

∴ roots are -1 and $\frac{-(r^2-p^2)}{p^2-q^2}$.

8. Let PQ touch the circle at point R.

We know that tangents drawn from a external point to a circle are equal in length.

$\therefore$ AB = AC = 5 cm

$\Rightarrow$ AP + BP = AQ + QC = 5 cm

$\Rightarrow$ AP + PR = AQ + QR = 5 cm ...(i) [$\because$ BP = PR and QC = QR]

Now,Perimeter of $\triangle APQ$ = Addition of all three sides = AP + PQ + AQ

= AP + RP + QR + AQ [$\because$ from (i)]

= 5 + 5

= 10 cm

The perimeter of $\triangle APQ$ is 10 cm.

9. Since Tangent touches A circle on a distinct point. On the diameter of a circle, only two parallel tangents can be drawn.

OR

$AF = AE = 7cm$ (tangents from same external point are equal)

$\therefore BF = AB - AF = 13 - 7 = 6cm$

$BD = BF = 6cm$ (tangents from same external point)

$\therefore CD = BC - BD = 14 - 6 = 8cm$

$CE = CD = 8cm$

$\therefore AC = AE + EC$

$= 7 + 8 = 15cm.$

10. We have,

ΔPOS ~ ΔROQ

$\Rightarrow \angle 3 = \angle 4$ and $\angle 1 = \angle 2$

Thus, PS and QR are two lines and the transversal PR cuts them in such a way that $\angle 3$ = $\angle 4$ i.e., alternate angles are equal. Hence, $PS \| QR$.

11. Sequence is 4,9,14,19, ...

Clearly, the given sequence is an A.P. with first term a = 4 and common difference d = 5

Let 124 be the nth term of the given sequence.

Then, $a_n = 124$

$\Rightarrow$ a+(n-1)d = 124

$\Rightarrow$ 4+(n-1) $\times$ 5 = 124

$\Rightarrow 4 + 5n - 5 = 124$

$\Rightarrow 5n-1 = 124$

$\Rightarrow 5n = 124 + 1$

$\Rightarrow 5n = 125$

$\Rightarrow n = 125/5$

$\Rightarrow n=25$

Hence, 25th term of the given sequence 4,9,14,19, ... is 124.

12. We have,

$$\text{LHS} = \frac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta} + \sin\theta\cos\theta$$

$$\Rightarrow \quad \text{LHS} = \frac{(\sin\theta+\cos\theta)\left(\sin^2\theta+\cos^2\theta-\sin\theta\cos\theta\right)}{\sin\theta+\cos\theta} + \sin\theta\cos\theta$$

$\Rightarrow$ LHS = 1 - $\sin\theta\cos\theta$ + $\sin\theta\cos\theta$ = 1 = RHS

13. Given $\frac{\cot A+1}{\cot A} = 1$

squaring both sides we get

$$\cot^2 A + \left(\frac{1}{\cot A}\right)^2 + 2 = 1$$

$\cot^2 A + \frac{1}{cot^2 A} = 1 - 2 = -1$

14. Number of spheres $= \frac{\text{Volume of the cone}}{\text{Volume of each sphere}}$

$$= \frac{\frac{1}{3}\pi^2 h}{\frac{4}{3}\pi R^3}$$

$$= \frac{(12)^2(24)}{4\times(2)^3}$$

= 108

15. First-term = a

the common difference, d = 3a - a = 2a

Sum of n terms, $S_n = \frac{n}{2}$ [2a+(n-1)d]

Putting the values

$S_n = \frac{n}{2}[2a + (n-1)(2a)]$

$S_n = \frac{n}{2}[2a + 2an - 2a]$

$S_n = \frac{n}{2}[2an] = (n)(an)$

$S_n = an^2$

So, the sum of n terms is an^2.

16. Total number of all possible outcomes = 52

There are 26 red cards (including 2 queens) and apart from these, there are 2 more queens.

Number of cards, each one of which is either a red card or a queen = 26 + 2 = 28

Let E be the event that the card drawn is neither a red card nor a queen.

Then, the number of favorable outcomes = (52 – 28) = 24

Therefore, P(getting neither a red card nor a queen) = P(E) = $\frac{24}{52} = \frac{6}{13}$

17. i. (b) A(1, 7), B(4, 2), C(-4, 4)

Distance travelled by Seema, AC = $\sqrt{[-4-1]^2+[4-7]^2} = \sqrt{34}$ units

Distance travelled by Aditya, BC = $\sqrt{[-4-4]^2+[4-2]^2} = \sqrt{68}$ units

∴ Aditya travels more distance

ii. (d) By using mid-point formula,

Coordinates of D are $\left(\frac{1+4}{2}, \frac{7+2}{2}\right) = \left(\frac{5}{2}, \frac{9}{2}\right)$

iii. $ar(\Delta ABC) = \frac{1}{2}$ [1(2 - 4) + 4(4 - 7) - 4(7 - 2)]

= 17 sq. units

iv. (b) (-4, 4)

v. (d) (4, 2)

18. i. (c) 25 m

ii. (a) 40 m

iii. (c) 25 m

iv. (d) 150 m^2

v. (a) 60 m

19. i. (a) Curve A - Less than type ogive and Curve B - More than type ogive

ii. (c) Median Wages = 50 Rs.

iii. (d) Mode = 3 Median - 2 Mean = 3(50) - 2(50) = 50 Rs.

As, Mean = Median = Mode, so it is a symmetrical distribution

iv. (a) Median

v. (b) Median

20. i. (a) 6 cm

ii. (c) 904.32 cm^3

iii. (c) 1:1

iv. (d) 527.52 cm^2

v. (c) Remain unaltered

Part-B

21. Let $x = 0.\overline{2341} = 0.2341341 \quad 341 \ldots (1)$

Multiplying both sides of (1) by 10, we get

10x = 2. 341 341 341 ....(2)

Multiplying both sides of (2) by 1000, we get

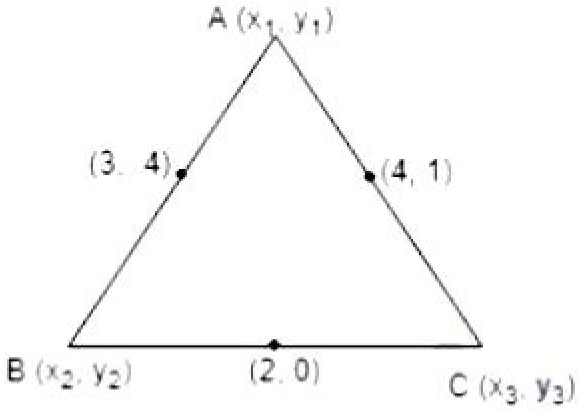
10000x = 2341. 341 341 31 ....(3)

Subtracting (2) from (3), we get

$9990x = 2339 \Rightarrow x = \frac{2339}{9990}$

Here, $p = 2339, q = 9990$

22. Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of a given triangle as shown in the figure.



Now, (3,4) is the mid-point of AB, therefore,

$3 = \frac{x_1+x_2}{2}$ and $4 = \frac{y_1+y_2}{2}$

$x_1 + x_2 = 6$ and $y_1 + y_2 = 8$ ..... (i)

(2,0) is the mid-point of BC, then,

$2 = \frac{x_2+x_3}{2}$ and $0 = \frac{y_2+y_3}{2}$

$x_2 + x_3 = 4$ and $y_2 + y_3 = 0$ ....(ii)

(4,1) is the mid-point of AC, then,

$4 = \frac{x_1+x_3}{2}$ and $1 = \frac{y_1+y_3}{2}$

$x_1 + x_3 = 8$ and $y_1 + y_3 = 2$ ....(iii)

Subtracting (ii) from (iii), we get,

$x_1 - x_2 = 4$ and $y_1 - y_2 = 2$ .... (iv)

Adding (i) and (iv), we get,

$2x_1 = 10$ and $2y_1 = 10$

$x_1 = 5$ and $y_1 = 5$

From (i), we have,

$x_2 = 6 - 5 = 1$ and $y_2 = 8 - 5 = 3$

From (ii), we have,

$x_3 = 4 - 1 = 3$ and $y_3 = 0 - y_2 = 0 - 3 = -3$

Thus (5, 5), (1, 3) and (3, -3) are the vertices of triangle.

OR

Area of triangle

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

or, $24 = \frac{1}{2}$ [1(2k + 5) - 4(- 5 + 1) - k(-1 - 2k)]

or, $48 = 2k + 5 + 16 + k + 2k^2$

or, $2k^2 + 3k - 27 = 0$

or, (k - 3) (2k + 9) = 0

or, $k = 3, k = -\frac{9}{2}$

23. The zeros of the said cubic polynomial are $\frac{1}{2}$, 1 and -3

Let $\alpha = \frac{1}{2}$, $\beta = 1$ and $\gamma = -3$

Now,

$\alpha + \beta + \gamma = \frac{1}{2} + 1 - 3 = \frac{1+2-6}{2} = \frac{-3}{2}$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{1}{2}(1) + (1)(-3) + (-3)(\frac{1}{2}) = \frac{1}{2} - 3 - \frac{3}{2} = \frac{1-6-3}{2} = \frac{-8}{2} =$

$\alpha\beta\gamma = \frac{1}{2}(1)(-3) = -\frac{3}{2}$

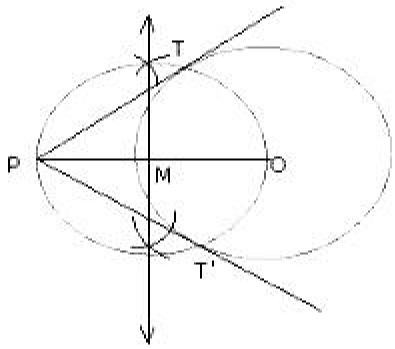
Now, a cubic polynomial whose zeros are $\alpha,\ \beta\ and\ \gamma$ is given by

$p(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$

Hence

$$p(x) = x^3 - \frac{-3}{2}x^2 + (-4)x + \frac{3}{2}$$

$$\text{or}\ \ p(x) = 2x^3 + 3x^2 - 8x + 3$$

24. 

Steps of construction:

i. Take a point O in the plane of a paper and draw a circle of the radius 4 cm.

ii. Make a point P at a distance of 7cm from the centre O and Join OP.

iii. Bisect the line segment OP. Let M be the mid-point of OP.

iv. Taking M as a centre and OM as radius draw a circle to intersect the given circle at the points, T and T'.

v. Join PT and PT', then PT and PT' are required tangents.

PT = PT' = 5.75 cm

25. LHS-

$\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$

$=\frac{\tan A}{1-\frac{1}{\tan A}}+\frac{\frac{1}{\tan A}}{1-\tan A}$

$=\frac{\tan A}{\frac{\tan A-1}{\tan A}}+\frac{1}{\tan A(1-\tan A)}$

$=\frac{\tan^2 A}{\tan A-1}+\frac{1}{\tan A(1-\tan A)}$

$=\frac{\tan^3 A-1}{\tan A(\tan A-1)}$

$=\frac{(\tan A-1)(\tan^2 A+\tan A+1)}{\tan A(\tan A-1)}$ $[a^3-b^3=(a-b)(a^2+ab+b^2)]$

$=\frac{\tan^2 A+\tan A+1}{\tan A}$

$=\tan A+1+\cot A$

$=\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}+1$

$=\frac{\sin^2 A+\cos^2 A}{\sin A\cos A}+1$

$=\frac{1}{\sin A\cos A}+1$

= sec A cosec A + 1

= R.H.S

OR

Given: $\cot\theta = \frac{7}{8}$

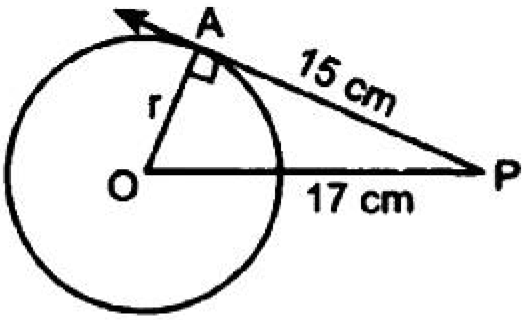
To Evaluate: $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta}$

= $\cot^2\theta$

$= \left(\frac{7}{8}\right)^2$

$= \frac{49}{64}$

26. 

$\angle$OAP = 90°

in $\triangle OAP$,

By applying Pythagoras theorem , we get

$\Rightarrow 17^2 = r^2 + 15^2$

$\Rightarrow r^2 = 17^2 - 15^2 = (17-15)(17+15)$

= 2×32

$\Rightarrow r^2 = 64 \Rightarrow r = \pm 8\text{cm}$

we should not take negative value because length cannot be negative.

$\Rightarrow$ r = 8 cm

27. Let $3 + 5\sqrt{2}$ be rational and have only common factor 1.

Let, $3 + 5\sqrt{2} = \frac{a}{b}$

$5\sqrt{2} = \frac{a}{b} - 3$

$\sqrt{2} = \frac{a-3b}{5b}$

If $\frac{a-3b}{5b}$ is rational so, $\sqrt{2}$ is also rational number but it is not true as $\sqrt{2}$ is an irrational number.

So it is a contradiction to our assumption,

Therefore, $3 + 5\sqrt{2}$ is an irrational number.

28. Let the length of the sides of the right triangle be x cm and (x + 5) cm. Given, length of hypotenuse = 25 cm.

According to Pythagoras theorem;

$p^2 + b^2 = h^2$ (where, p, b & h are respectively perpendicular, base & hypotenuse of right angled triangle)

$\therefore x^2 + (x + 5)^2 = (25)^2$ [Using pythagoras theorem]

$\Rightarrow x^2 + x^2 + 25 + 10x = 625$

$\Rightarrow 2x^2 + 10x + 25 - 625 = 0$

$\Rightarrow 2x^2 + 10x - 600 = 0$

$\Rightarrow 2(x^2 + 5x - 300) = 0$

$\Rightarrow x^2 + 5x - 300 = 0$

$\Rightarrow x^2 + 20x - 15x - 300 = 0$

$\Rightarrow x(x + 20) - 15(x + 20) = 0$

$\Rightarrow (x + 20)(x - 15) = 0$

$\Rightarrow x - 15 = 0$ [$\because$ Length can never be negative $\therefore x + 20 \neq 0$]

$\Rightarrow x = 15$ cm

$\therefore x + 5 = 15 + 5 = 20$ cm

Hence, the lengths of required sides are 15 cm and 20 cm.

OR

Consider $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$

$\Rightarrow \frac{1}{2a+b+2x} - \frac{1}{2x} = \frac{1}{2a} + \frac{1}{b}$

$\Rightarrow 2ab\ (2x - 2a - b - 2x) = (2a + b)2x\ (2a + b + 2x)$

$\Rightarrow 2ab\ (-2a - b) = 2(2a + b)\ (2ax + bx + 2x^2)$

$\Rightarrow -ab = 2ax + bx + 2x^2$

$\Rightarrow 2x^2 + 2ax + bx + ab = 0$

$\Rightarrow 2x\ (x + a) + b\ (x + a) = 0$

$\Rightarrow (2x + b)\ (x + a) = 0$

$\Rightarrow x = -a, -\frac{b}{2}$

Hence the roots are $-a, -\frac{b}{2}$.

29. Let α, β and γ be the zeroes of $f(x)$ and the zeroes are in AP.

Suppose $\alpha = a - d, \beta = a$ and $\gamma = a + d$

$f(x) = x^3 - 12x^2 + 39x - 28$

$\therefore \quad \alpha+\beta+\gamma=-\left(\frac{-12}{1}\right) = 12$ ...(1)

and, $\alpha\beta\gamma=-\left(\frac{-28}{1}\right) = 28$ ...(2)

From equation (1) , we have

$a - d + a + a + d = 12$

$3a = 12 \Rightarrow a = 4.$

Now from equation (2), we have

$(a-d)a(a+d) = 28$

$a(a^2-d^2) = 28$

$4(16-d^2) = 28$

$16-d^2 = 7$

$d^2 = 9$

$d = \pm 3$

Case I: When a = 4 and d = 3 : In this case,

$\alpha = a - d = 4 - 3 = 1, \beta = a = 4$ and $\gamma = a + d = 7$

CASE II: When a = 4 and d = -3: In this case,

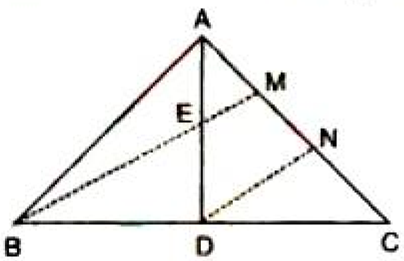
$\alpha = a - d = 4 - (-3) = 7, \beta = a = 4$ and $\gamma = a + d = 4 - 3 = 1$

Hence, in either case the zeroes of the given polynomial are 1,4 and 7.

30. Given: D is the mid-point of side BC, E is mid-point of AD, BE produced meets AC at M.

To prove: BE = 3 EM

Construction: Draw DN $\parallel$ BM



Proof: In $\triangle$ADN, EM $\parallel$ DN (construction)

$\therefore \quad \triangle AEM \sim \triangle ADN$ (AA similarity)

$\therefore \quad \frac{EM}{DN} = \frac{AE}{AD}$ (By BPT)

But AD = 2AE

$\therefore \quad \frac{EM}{DN} = \frac{AE}{2AE}$

$\therefore$ DN = 2EM ..(i)

In $\triangle$BCM, DN $\|$ BM (construction)

$\therefore \quad \triangle CDN \sim \triangle CBM$ (AA similarity)

$\therefore \quad \frac{DN}{BM} = \frac{CD}{CB}$ (by BPT)

But CB = 2CD

$\therefore \quad \frac{DN}{BM} = \frac{CD}{2CD}$

$\therefore$ BM = 2DN ..(ii)

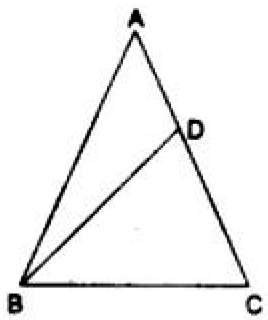
from (i) and (ii),

BM = 2(2EM)

$\therefore$ BM = 4EM

$\therefore$ BE = BM - EM = 4EM - EM = 3EM. Hence proved

OR



Given A $\triangle ABC$ in which AB = AC and D is a point on AC such that $BC^2 = AC \times DC$.

To Prove BD = BC

Proof $BC^2 = AC \times DC$ (given)

$\Rightarrow \frac{BC}{DC} = \frac{AC}{BC}$

Thus, in $\triangle ABC$ and $\triangle BDC$, we have

$\frac{BC}{DC} = \frac{AC}{BC}$ and $\angle C = \angle C$ (common).

$\therefore \quad \triangle ABC \sim \triangle BDC$ [by SAS-similarity].

$\Rightarrow \quad \frac{AC}{BC} = \frac{AB}{BD}$

$\Rightarrow \quad \frac{AC}{BC} = \frac{AC}{BD}$ [$\because$ AB = AC (given)]

$\Rightarrow$ BD = BC

Hence BD = BC.

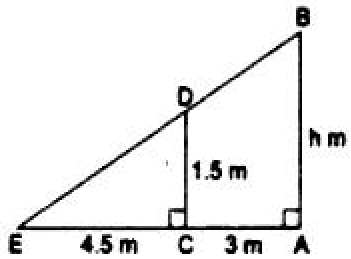
31. Number of possible outcomes = 50

Numbers which are multiple of 3 and 4 from 1 to 50 are = 12, 24, 36 and 48

No. of favorable outcomes = 4

P(the selected number is multiple of 3 and 4) = $\frac{4}{50}=\frac{2}{25}$

32. Let AB be the lamp-post and CD be the girl.



Let CE be the shadow of CD. Then,

CD = 1.5m, CE = 4.5m and AC = 3m.

Let AB = h m.

Now, ΔAEB and ΔCED are similar.

$\therefore \quad \frac{AB}{AE}=\frac{CD}{CE} \Rightarrow \frac{h}{(3+4.5)}=\frac{1.5}{4.5}=\frac{1}{3}$

$\Rightarrow h=\frac{1}{3}\times 7.5=2.5$

33. The given series is converted from inclusive to exclusive form and on preparing the frequency table, we get

Class	Frequency
0.5 - 5.5	3
5.5 - 10.5	8
10.5 - 15.5	13
15.5 - 20.5	18
20.5 - 25.5	28
25.5 - 30.5	20
30.5 - 35.5	13
35. 5- 40.5	8
40.5 - 45.5	6
45.5 - 50.5	4

Clearly, the modal class is 20.5 - 25.5, as it has the maximum frequency.

Now, x_k (lower limit of modal class) = 20.5, h (length of interval of modal class) = 5, f_k (

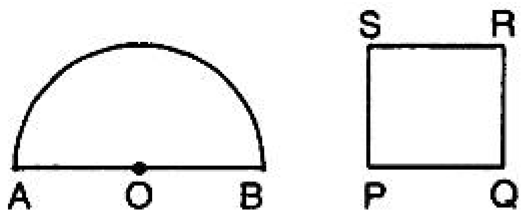
frequency of modal class) = 28, f_{k-1} (frequency of the class just preceding the modal class) = 18, f_{k+1} (frequency of the class just exceeding the modal class) = 20

Mode (M_o) is given by the formula ,

$$M_o = x_k + \left\{h \times \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$$
$$= 20.5 + \left[5 \times \frac{(28-18)}{(56-18-20)}\right]$$
$$= 20.5 + \left[\frac{5\times10}{18}\right]$$
$= 20.5 + 2.78$

$= 23.28$

Hence, mode = 23.28

34. 

Let radius of semicircular region be r units.

Perimeter = $2r + \pi r$

Let side of square be x units

Perimeter = 4x units.

A.T.Q, $4x = 2r + \pi r \Rightarrow x = \frac{2r+\pi r}{4}$

Area of semicircle = $\frac{1}{2}\pi r^2$

Area of square = x^2

A.T.Q, $x^2 = \frac{1}{2}\pi r^2 + 4$

$$\Rightarrow \left(\frac{2r+\pi r}{4}\right)^2 = \frac{1}{2}\pi r^2 + 4$$
$$\Rightarrow \frac{1}{16}\left(4r^2 + \pi^2 r^2 + 4\pi r^2\right) = \frac{1}{2}\pi r^2 + 4$$
$$\Rightarrow 4r^2 + \pi^2 r^2 + 4\pi r^2 = 8\pi r^2 + 64$$
$$\Rightarrow 4r^2 + \pi^2 r^2 - 4\pi r^2 = 64$$
$$\Rightarrow r^2\left(4 + \pi^2 - 4\pi\right) = 64$$
$$\Rightarrow r^2(\pi - 2)^2 = 64$$
$$\Rightarrow r = \sqrt{\frac{64}{(\pi-2)^2}}$$

$\Rightarrow r = \frac{8}{\pi - 2} = \frac{8}{\frac{22}{7} - 2}$ = 7 cm

Perimeter of semicircle =$2 \times 7 + \frac{22}{7} \times 7$= 36 cm

Perimeter of square = 36 cm

Side of square = $\frac{36}{4}$ = 9 cm

Area of square = 9$\times$ 9 = 81 cm^2

Area of semicircle = $\frac{\pi r^2}{2} = \frac{22}{2 \times 7} \times 7 \times 7$= 77 cm^2

35. Let the dimensions (i.e., length and width) of the garden be x and y m respectively.

Then, x = y + 4 and $\frac{1}{2}$(2x + 2y) = 36

$\Rightarrow$ x - y = 4 ...(1)

x + y = 36 ...(2)

Let us draw the graphs of equations (1) and (2) by finding two solutions for each of the equations. These two solutions of the equations (1) and (2) are given below in table 1 and table 2 respectively.

For equation (1)

x - y = 4

$\Rightarrow$ y = x - 4

Table 1 of the solutions

x	4	2
y	0	-2

For equation (2) x + y = 36

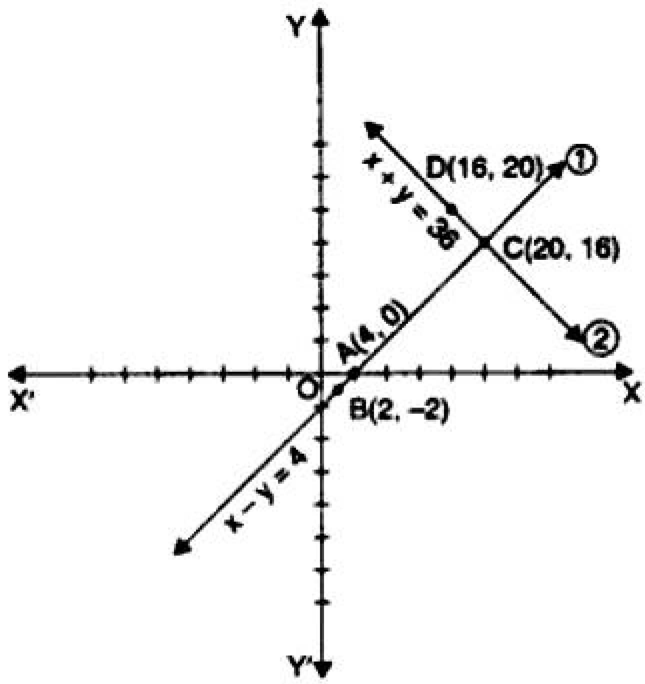
$\Rightarrow$ y = 36 - x

Table 2 of the solutions

x	20	16
y	16	20

We plot the points A(4, 0) and B(2, -2) on a graph paper and join these points to form the line AB representing. The equation (1) as shown in the figure.

Also, we plot the points C(20, 16) and D(16, 20) on the same graph paper and join these points to form the line CD representing the equation (2) as shown in the same figure.



In the figure, we observe that the two lines intersect at the point C(20, 16) So x = 20, y = 16 is the required solution of the pair of linear equations formed. i.e., the dimensions of the garden are 20 m and 16 m.

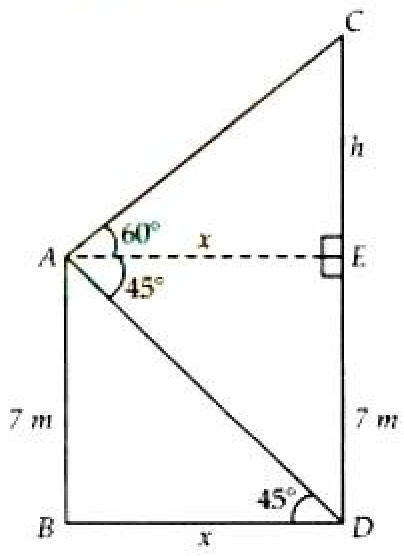
Verification : substituting x = 20 and y = 16 in (1) and (2), we find that both the equations are satisfied as shown below:

20 - 16 = 4

20 + 16 = 36

This verifies the solution.

36. Given: The height of the building is 7m and the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°.



In $\triangle ABD$, $\angle ADB = \angle EAD = 45^\circ$

(alternate angles)

$\therefore \quad \frac{AB}{BD} = \tan 45^\circ$

x = 7

In $\triangle AEC$, $\frac{CE}{AE} = \tan 60^\circ$

$\Rightarrow \quad \frac{h-7}{x} = \sqrt{3}$

$\Rightarrow \quad h - 7 = x\sqrt{3}$

$\Rightarrow \quad h - 7 = 7\sqrt{3}$ (using x = 7)

$\Rightarrow \quad h = 7\sqrt{3} + 7$

$= 7(\sqrt{3} + 1)$

= 7(1.732 + 1)

Hence, the height of the tower = 19.124 m