Chapter 12 Coordinate Geometry

From the CAT point of view, Coordinate Geometry by itself is not a very significant chapter. Basically, applied questions are asked in the form of tabular representation or regarding the shape of the structure formed. However, it is advised to go through the basics and important formulae to have a feel-good effect as also to be prepared for surprises, if any, in the examination. Logical questions might be asked based on the formulae and concepts contained in this chapter. Besides, the student will have an improved understanding of the graphical representation of functions if he/she has gone through coordinate geometry.

The students who face any problems in this chapter can stop after solving LOD II instead of getting frustrated.

CARTESIAN COORDINATE SYSTEM

Rectangular Coordinate Axes

Let *X*¢*OX* and *Y'OY* be two mutually perpendicular lines through any point *O* in the plane of the paper. Point *O* is known as the origin. The line *X*¢*OX* is called the *x*-axis or axis of *x*; the line *Y*¢*OY* is known as the *y*-axis or axis of *y*; and the two lines taken together are called the coordinate axes or the axes of coordinates.

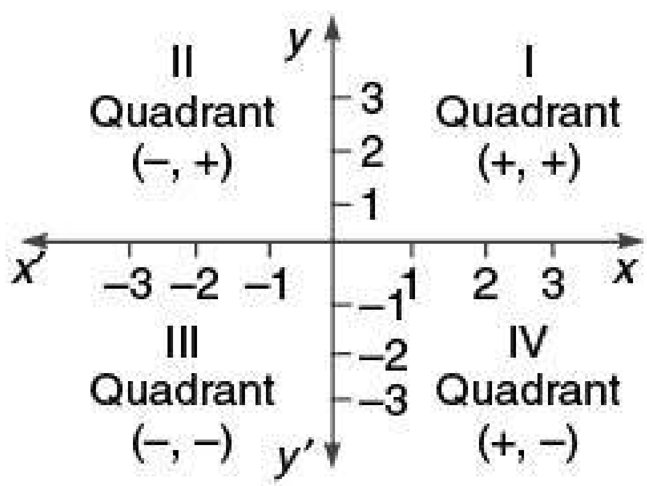


Fig. 12.1

Any point can be represented on the plane described by the coordinate axes by specifying its x and y coordinates. The x coordinate of the point is also known as the abscissa while the y coordinate is also known as the ordinate.

1. Distance Formula If two points P and Q are such that they are represented by the points (x_1, y_1) and (x_2, y_2) on the x-y plane (cartesian plane), then the distance between the points P and Q = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Illustration

Question 1: Find the distance between the points (5, 2) and (3, 4).

Answer: Distance = $\sqrt{(5-3)^2 + (2-4)^2}$

$\left[\text{Using the formula } \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}\right]$

$= 2\sqrt{2}$ units

2. Section Formula If any point (x, y) divides the line segment joining the points (x_1, y_1) and (x_2, y_2) in the ratio $m : n$ internally,

then $x = (mx_2 + nx_1)/(m + n)$

$y = (my_2 + ny_1)/(m + n)$ (See figure)

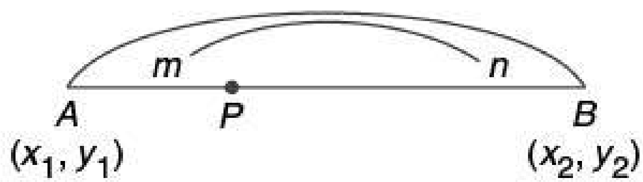


Fig. 12.2

If any point (x, y) divides the line segment joining the points (x_1, y_1) and (x_2, y_2) in the ratio $m : n$ externally,

then $x = (mx_2 - nx_1)/(m - n)$

$y = (my_2 - ny_1)/(m - n)$

Illustration

Question 2: Find the point which divides the line segment joining (2, 5) and (1, 2) in the ratio 2 : 1 internally.

Answers: $X = (2.1 + 1.2)/(1 + 2) = 4/3$

$Y = (2.2 + 1.5)/(2 + 1) = 9/3 = 3$

3. Area of a Triangle The area of a triangle whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is given by

$$\left[\frac{\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}}{2}\right]$$

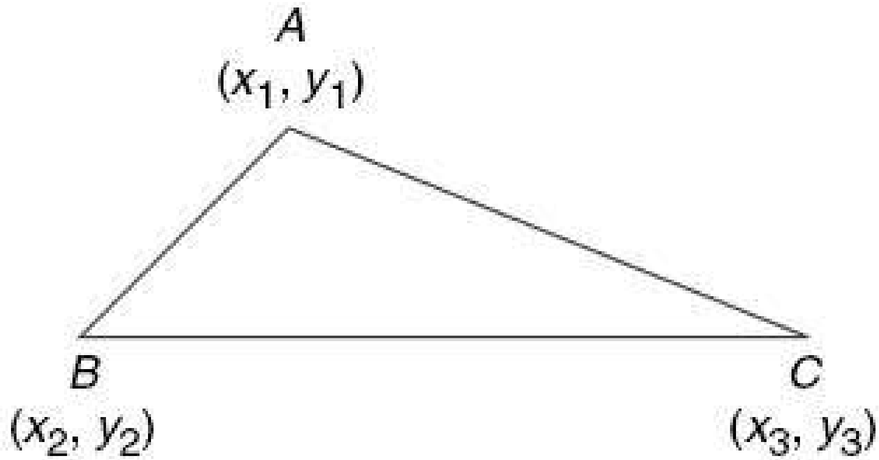


Fig. 12.3

[Note: Since the area cannot be negative, we have to take the modulus value given by the above equation.]

Corollary: If one of the vertices of the triangle is at the origin and the other two vertices are $A\,(x_1, y_1)$, $B\,(x_2, y_2)$, then the area of triangle is $\left|\dfrac{(x_1 y_2 - x_2 y_1)}{2}\right|$.

Illustration

Question 3: Find the area of the triangle (0, 4), (3, 6) and (–8, –2).

Answer: Area of triangle = |1/2 {0 (6– (–2)) + 3 ((–2) –4) + (–8) (4–6)}|
= |1/2 {(0) + 3 (–6) + (–8) (–2)}|
= |1/2 (–2)| = |–1| = 1 square unit.

4. Centre of gravity or centroid of a triangle The centroid of a triangle is the point of intersection of its medians (the line joining the vertex to the middle point of the opposite side). Centroid divides the medians in the ratio 2 : 1. In other words, the *CG* or the centroid can be viewed as a point at which the whole weight of the triangle is concentrated.

Formula: If $A\,(x_1, y_1)$, $B\,(x_2, y_2)$ and $C\,(x_3, y_3)$ are the coordinates of the vertices of a triangle, then the coordinates of the centroid G of that triangle are
$x = (x_1 + x_2 + x_3)/3$ and $y = (y_1 + y_2 + y_3)/3$

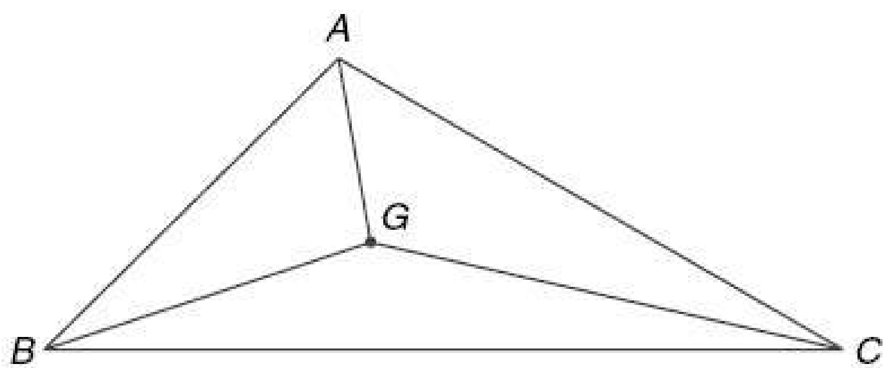


Fig. 12.4

Illustration

Question 4: Find the centroid of the triangle whose vertices are (5, 3), (4, 6) and (8, 2).

Answer: X coordinate = (5 + 4 + 8)/3 = 17/3

Y coordinate = (3 + 6 + 2)/3 = 11/3

5. In-centre of a triangle The centre of the circle that touches the sides of a triangle is called its In-centre. In other words, if the three sides of the triangle are tangential to the circle then the centre of that circle represents the in-centre of the triangle.

The in-centre is also the point of intersection of the internal bisectors of the angles of the triangle. The distance of the in-centre from the sides of the triangle is the same and this distance is called the in-radius of the triangle.

If A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are the coordinates of the vertices of a triangle, then the coordinates of its in-centre are

$x = \dfrac{(ax_1 + bx_2 + cx_3)}{(a + b + c)}$ and $y = \dfrac{(ay_1 + by_2 + cy_3)}{(a + b + c)}$

where $BC = a$, $AB = c$ and $AC = b$.

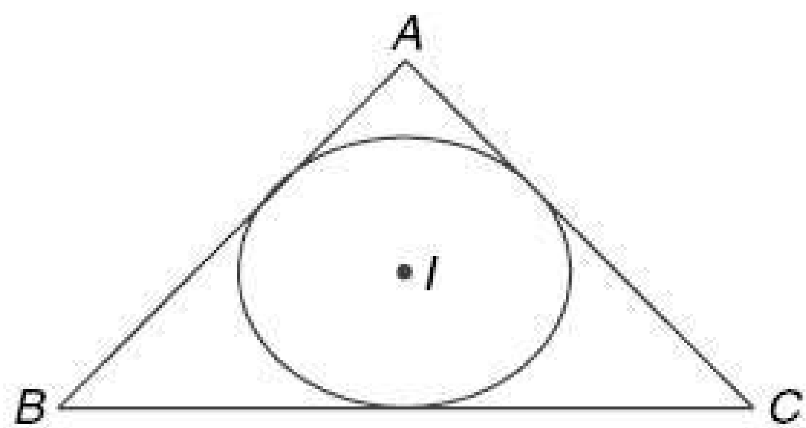


Fig. 12.5

Illustration

Question 5: Find the in-centre of the right angled isosceles triangle having one vertex at the origin and having the other two vertices at (6, 0) and (0, 6).

Answer: Obviously, the length of the two sides AB and BC of the triangle is 6 units and the length of the third side is $(6^2 + 6^2)^{1/2}$.

Hence $a = c = 6$, $b = 6\sqrt{2}$

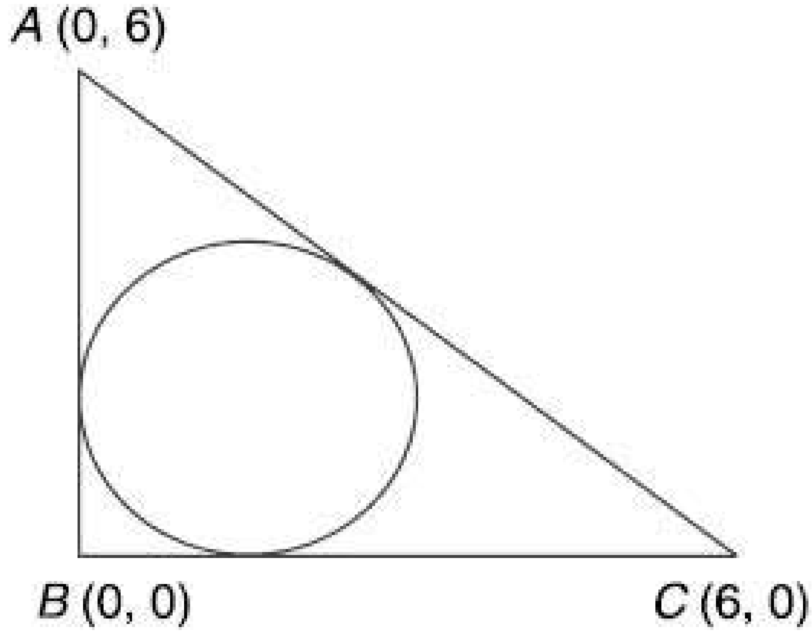


Fig. 12.6

In-centre will be at

$$\frac{(6.0 + 6\sqrt{2}.0 + 6.6)}{(6 + 6 + \sqrt{2})}, \frac{(6.6 + 6\sqrt{2}.0 + 6.0)}{(6 + 6 + \sqrt{2})}$$

$$= \frac{36}{12 + 6\sqrt{2}}, \frac{36}{12 + 6\sqrt{2}}$$

6. Circumcentre of a triangle The point of intersection of the perpendicular bisectors of the sides of a triangle is called its circumcentre. It is equidistant from the vertices of the triangle. It is also known as the centre of the circle which passes through the three vertices of a triangle (or the centre of the circle that circumscribes the triangle.)

Let *ABC* be a triangle. If *O* is the circumcentre of the triangle *ABC*, then *OA* = *OB* = *OC* and each of these three represent the circum radius.

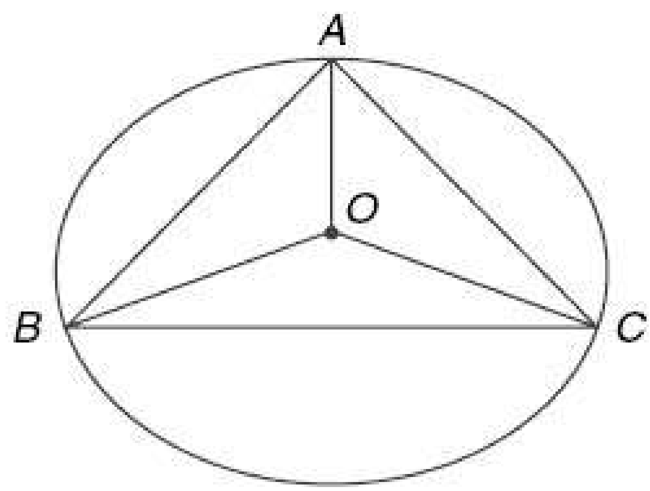


Fig. 12.7

Illustration

Question 6: What will be the circumcentre of a triangle whose sides are $3x - y + 3 = 0$, $3x + 4y + 3 = 0$

and $x + 3y + 11 = 0$?

Answer: Let ABC be the triangle whose sides AB, BC and CA have the equations $3x – y + 3 = 0$, $3x + 4y + 3 = 0$ and $x + 3y + 11 = 0$ respectively.Solving the equations, we get the points A, B and C as (–2, –3), (–1, 0) and (7, – 6) respectively.

The equation of a line perpendicular to BC is $4x – 3y + k = 0$.

[For students unaware of this formula, read the section on straight lines later in the chapter.]

This will pass through (3, –3), the mid-point of BC, if $12 + 9 + k = 0$ fi $k = –21$

Putting $k_1 = –21$ in $4x – 3y + k = 0$, we get $4x – 3y – 21 = 0$ (i)

as the equation of the perpendicular bisector of BC.

Again, the equation of a line perpendicular to CA is $3x – y + k_1 = 0$.

This will pass through (5/2, –9/2), the mid-point of AC if

$15/2 + 9/2 + k_1 = 0$ fi $k_1 = –12$

Putting $k_1 = –12$ in $3x – y + k_1 = 0$, we get $3x – y – 12 = 0$ (ii)

as the perpendicular bisector of AC.

Solving (i) and (ii), we get $x = 3$, $y = –3$.

Hence, the coordinates of the circumcentre of DABC are (3, –3).

7. Orthocentre of a triangle The orthocentre of a triangle is the point of intersection of the perpendiculars drawn from the vertices to the opposite sides of the triangle.

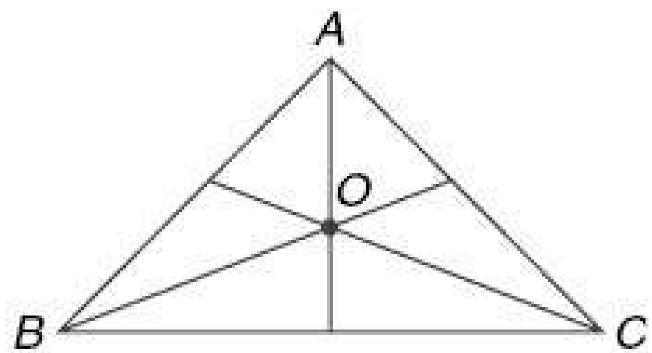


Fig. 12.8

Illustration

Question 7: Find the orthocentre of the triangle whose sides have the equations $y = 15$, $3x = 4y$, and $5x + 12y = 0$.

Answer: Let ABC be the triangle whose sides BC, CA and AB have the equations $y = 15$, $3x = 4y$, and $5x + 12y = 0$ respectively.

Solving these equations pairwise, we get coordinates of A, B and C as (0, 0), (–36, 15) and (20, 15) respectively.

AD is a line passing through A (0, 0) and perpendicular to $y = 15$.

So, equation of AD is $x = 0$.

The equation of any line perpendicular to $3x – 4y = 0$ is represented by $4x + 3y + k = 0$.

This line will pass through (–36, 15) if $–144 + 45 + k = 0$ fi $k = 99$.

So the equation of BE is $4x + 3y + 99 = 0$.

Solving the equations of AD and BE we get $x = 0$, $y = -33$.

Hence the coordinates of the orthocentre are $(0, -33)$.

8. Collinearity of three points: Three given points A, B and C are said to be collinear, that is, lie on the same straight line, if any of the following conditions occur:

(i) Area of triangle formed by these three points is zero.

(ii) Slope of AB = Slope of AC.

(iii) Any one of the three points (say C) lies on the straight line joining the other two points (here A and B).

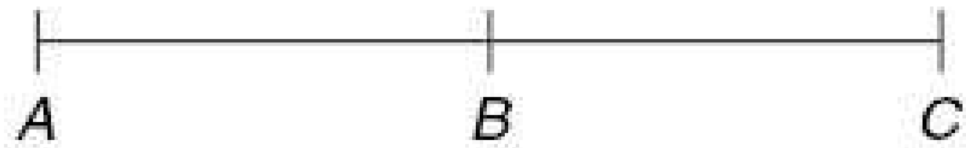


Fig. 12.9

Illustration

Question 8: Select the right option the points $(-a, -b)$, $(0, 0)$ and (a, b) are

(a) Collinear
(b) Vertices of square
(c) Vertices of a rectangle
(d) None of these

Answer: We can use either of the three methods to check whether the points are collinear.

But the most convenient one is (ii) in this case.

Let A, B, C are the points whose coordinates are $(-a, -b)$, $(0, 0)$ and (a, b)

Slope of $BC = b/a$

Slope of $AB = b/a$

So, the straight line made by points A, B and C is collinear.

Hence, (a) is the answer.

[If you have not understood this here, you are requested to read the following section on straight lines and their slopes and then re-read this solution]

Alternative: Draw the points on paper assuming the paper to be a graph paper. This will give you an indication regarding the nature of points. In the above question, point (a, b) is in first quadrant for $a > 0$, $b > 0$ and point $(-a, -b)$ is directly opposite to the point (a, b) in the third quadrant with the third point $(0, 0)$ in the middle of the straight line joining the points A and B.

You can check this by assuming any value for 'a' and 'b'.

Also, you can use this method for solving any problem involving points and diagrams made by those points. However you should be fast enough to trace the points on paper. A little practice of tracing points might help you.

9. Slope of a line The slope of a line joining two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is denoted by m and is given by $m = (y_2 - y_1)/(x_2 - x_1) = \tan q$, where q is the angle that the line makes with the positive direction of x-axis. This angle q is taken positive when it is measured in the anti-clockwise direction from the positive direction of the axis of x.

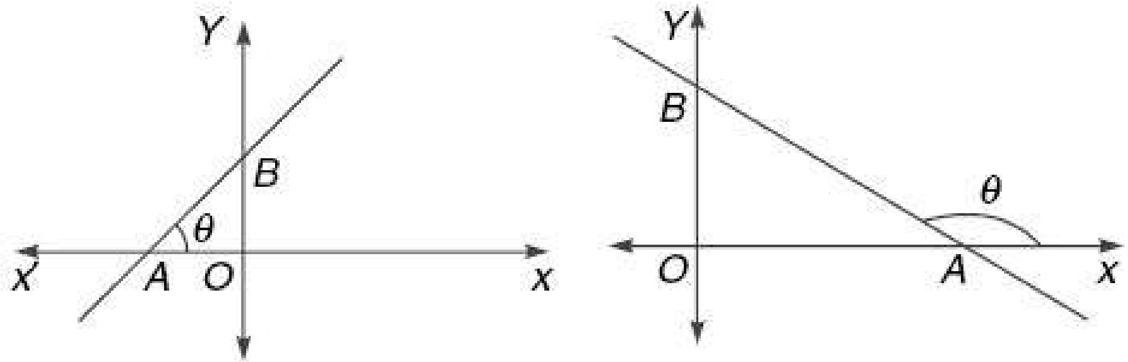


Fig. 12.10

Illustration

Question 9: Find the equation of a straight line passing through (2, –3) and having a slope of 1 unit.

Answer: Here slope = 1

And point given is (2, –3).

So, we will use point-slope formula for finding the equation of straight line. This formula is given by:

$(y - y_1) = m (x - x_1)$

So, equation of the line will be $y - (-3) = 1 (x - 2)$

fi $y + 3 = x - 2$

fi $y - x + 5 = 0$

10. Different Forms of the Equations of a Straight Line

(a) General Form The general form of the equation of a straight line is $ax + by + c = 0$. (First degree equation in x and y). Where a, b and c are real constants and a, b are not simultaneously equal to zero.

In this equation, slope of the line is given by $\frac{-a}{b}$.

The general form is also given by $y = mx + c$; where m is the slope and c is the intercept on y-axis.

In this equation, slope of the line is given by m.

(b) Line Parallel to the X-axis The equation of a straight line *parallel to the x-axis* and at a distance b from it, is given by $y = b$.

Obviously, the **equation of the *x*-axis is *y* = 0**

(c) Line Parallel to Y-axis The equation of a straight line *parallel to the y-axis* and at a distance a from it, is given by $x = a$.

Obviously, the **equation of *y*-axis is *x* = 0**

(d) Slope Intercept Form The equation of a straight line passing through the point $A (x_1, y_1)$ and having a slope m is given by

$(y - y_1) = m (x - x_1)$

(e) Two Points Form The equation of a straight line passing through two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$(y - y_1) = \frac{(y_2 - y_1)(x - x_1)}{(x_2 - x_1)}$$

Its slope $= \left(\dfrac{y_2 - y_1}{x_2 - x_1}\right)$

(f) Intercept Form The equation of a straight line making intercepts a and b on the axes of x and y respectively is given by

$x/a + y/b = 1$

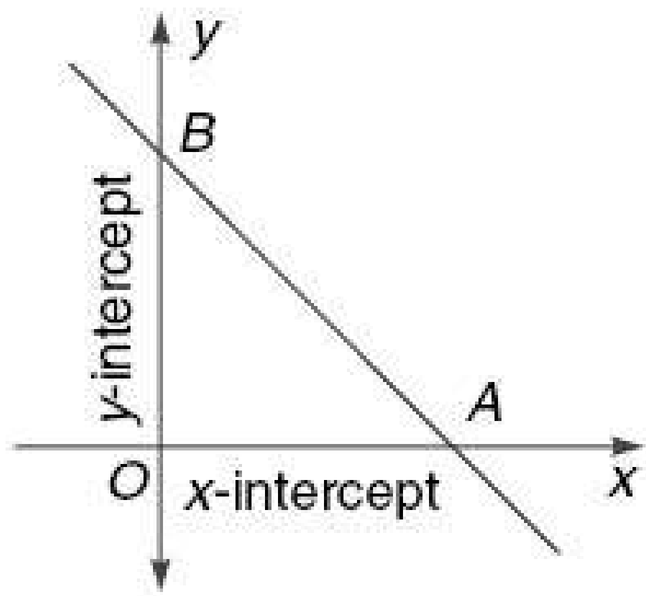


Fig. 12.11

If a straight line cuts x-axis at A and the y-axis at B then OA and OB are known as the intercepts of the line on x-axis and y-axis respectively.

11. Perpendicularity and Parallelism

Condition for two lines to be parallel: Two lines are said to be parallel if their slopes are equal.

For this to happen, ratio of coefficient of x and y in both the lines should be equal.

In a general form, this can be stated as: line parallel to $ax + by + c = 0$ is $ax + by + k = 0$ or $dx + ey + k = 0$ if $a/d = b/e$ where k is a constant.

Illustration

Question 10: Which of the lines represented by the following equations are parallel to each other?

1. $x + 2y = 5$
2. $2x - 4y = 6$
3. $x - 2y = 4$
4. $2x + 6y = 8$

(a) 1 and 2 (b) 2 and 4 (c) 2 and 3 (d) 1 and 4

Answer: Go through the options and check which of the two lines given will satisfy the criteria for two lines to be parallel. It will be obvious that option c is correct, that is, the line $2x - 4y = 6$ is parallel to the line $x - 2y = 4$.

Question 11: Find the equation of a straight line parallel to the straight line $3x + 4y = 7$ and passing through the point $(3, -3)$.

Answer: Equation of the line parallel to $3x + 4y = 7$ will be of the form $3x + 4y = k$.

This line passes through $(3, -3)$, so this point will satisfy the equation of straight line $3x + 4y = k$. So, $3.3 + 4. (-3) = k$ fi $k = -3$.

Hence, equation of the required straight line will be $3x + 4y + 3 = 0$.

Condition for two lines to be perpendicular: Two lines are said to be perpendicular if product of the slopes of the lines is equal to –1.

For this to happen, the product of the coefficients of x + the product of the coefficients of y should be equal to zero.

Illustration

Question 12: Which of the following two lines are perpendicular?

1. $x + 2y = 5$	2. $2x - 4y = 6$
3. $2x + 3y = 4$	4. $2x - y = 4$

(a) 1 and 2 (b) 2 and 4 (c) 2 and 3 (d) 1 and 4

Check the equations to get option 4 as the correct answer.

Question 13: Find the equation of a straight line perpendicular to the straight line $3x + 4y = 7$ and passing through the point $(3, -3)$.

Answer: Equation of the line perpendicular to $3x + 4y = 7$ will be of the form $4x - 3y = K$.

This line passes through $(3, -3)$, so this point will satisfy the equation of straight line $4x - 3y = K$. So, $4.3 - 3.-3$ fi $K = 21$.

Hence, equation of required straight line will be $4x - 3y = 21$.

12. Length of perpendicular or Distance of a point from a line The length of perpendicular from a given point (x_1, y_1) to a line $ax + by + c = 0$ is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Corollary:

(a) Distance between two parallel lines.

If two lines are parallel, the distance between them will always be the same.

When two straight lines are parallel whose equations are $ax + by + c = 0$ and $ax + by + c_1 = 0$, then the distance between them is given by $\dfrac{|c - c_1|}{\sqrt{a^2 + b^2}}$.

(b) The length of the perpendicular from the origin to the line $ax + by + c = 0$ is given by $\frac{|c|}{\sqrt{a^2 + b^2}}$

Illustration

Question 14: Two sides of a square lie on the lines $x + y = 2$ and $x + y = -2$. Find the area of the square formed in this way.

Answer: Obviously, the difference between the parallel lines will be the side of the square.

To convert it into the form of finding the distance of a point from a line, we will have to find out a point at which any one of these two lines cut the axes and then we will draw a perpendicular from that point to the other line, and this distance will be the side of the square.

To find the point at which the equation of the line $x + y = 2$ cut the axes, we will put once $x = 0$ and then again $y = 0$.

When $x = 0$, $y = 2$, so the coordinates of the point where it cuts y-axis is (0, 2).

Now the point is (0, 2), and the equation of line on which perpendicular is to be drawn is $x + y = -2$.

So, distance = $\frac{|1.0 + 1.2 + 2|}{\sqrt{1^2 + 1^2}} = \frac{4}{\sqrt{2}}$.

\ Area = $\frac{16}{2} = 8$

Alternatively: Draw the points on the paper and you will get the length of diagonal as 4 units; so, length of side will be $2\sqrt{2}$ and, therefore, the area will be 8 sq units.

Alternatively: You can also use the formula for the distance between two parallel lines as

$$\frac{|2 + 2|}{\sqrt{1^2 + 1^2}} = \frac{4}{\sqrt{2}}$$

13. Change of axes If origin (0, 0) is shifted to (h, k) then the coordinates of the point (x, y) referred to the old axes and (X, Y) referred to the new axes can be related with the relation $x = X + h$ and $y = Y + k$.

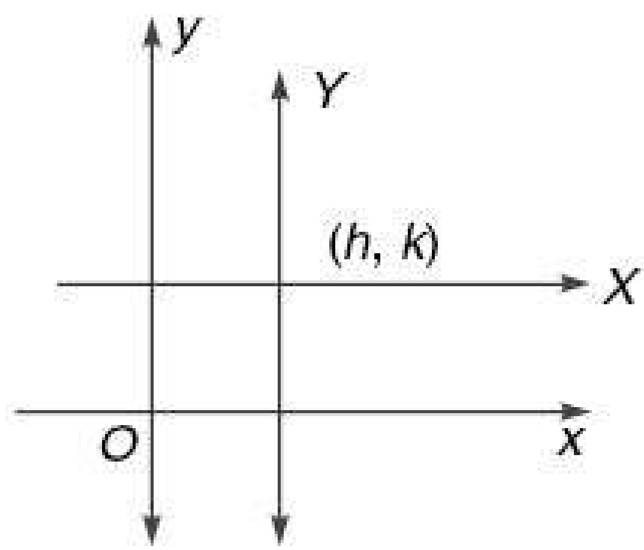


Fig. 12.12

Illustration

Question 15: If origin (0, 0) is shifted to (5, 2), what will be the coordinates of the point in the new axis which was represented by (1, 2) in the old axis?

Answer: Let (X, Y) be the coordinates of the point in the new axis.

Then, $1 = X + 5 \qquad \backslash \quad X = –4$

$2 = Y + 2 \qquad \backslash \quad Y = 0$

So, the new coordinates of the point will be (–4, 2).

14. Point of intersection of two lines Point of intersection of two lines can be obtained by solving the equations as simultaneous equations.

An Important Result
If all the three vertices of a triangle have integral coordinates, then that triangle cannot be an Equilateral triangle.

LEVEL OF DIFFICULTY (I)

1. Find the distance between the points (3, 4) and (8, –6).

   (a) $\sqrt{5}$ (b) $5\sqrt{5}$

   (c) $2\sqrt{5}$ (d) $4\sqrt{5}$

2. Find the distance between the points (5, 2) and (0, 0).

   (a) $\sqrt{27}$ (b) $\sqrt{21}$

   (c) $\sqrt{29}$ (d) $\sqrt{31}$

3. Find the value of p if the distance between the points (8, p) and (4, 3) is 5.

   (a) 6 (b) 0

   (c) Both (a) and (b) (d) None of these

4. Find the value of c if the distance between the point (c, 4) and the origin is 5 units.

   (a) 3 (b) –3

   (c) Both a and b (d) None of these

5. Find the mid-point of the line segment made by joining the points (3, 2) and (6, 4).

   (a) $\left(\frac{9}{2}, 3\right)$ (b) $\left(\frac{-3}{2}, -1\right)$

   (c) $\left(\frac{9}{2}, -\frac{3}{2}\right)$ (d) $\left(\frac{3}{-1}\right)$

6. If the origin is the mid-point of the line segment joined by the points (2, 3) and (x, y), find the value of (x, y).

   (a) (2, 3) (b) (–2, 3)

   (c) (–2, –3) (d) (2, –3)

7. Find the points that divide the line segment joining (2, 5) and (–1, 2) in the ratio 2 : 1 internally.

   (a) (1, 2) (b) (–3, 2)

   (c) (3, 1) (d) (0, 3)

8. In what ratio does the x-axis divide the line segment joining the points (2, –3) and (5, 6)?

   (a) 2 : 1 (b) 1 : 2

   (c) 3 : 4 (d) 2 : 3

9. How many squares are possible if two of the vertices of a quadrilateral are (1, 0) and (2, 0)?

   (a) 1 (b) 2

   (c) 3 (d) 4

10. If the point R (1, –2) divides externally the line segment joining P (2, 5) and Q in the ratio 3 : 4,

what will be the coordinates of Q?

(a) (–3, 6)
(b) (2, –4)
(c) (3, 6)
(d) (1, 2)

11. Find the coordinates of the points that trisect the line segment joining (1, –2) and (–3, 4).

(a) $\left(\frac{-1}{3}, 0\right)$
(b) $\left(\frac{-5}{3}, 2\right)$
(c) Both (a) and (b)
(d) None of these

12. Find the coordinates of the point that divides the line segment joining the points (6, 3) and (–4, 5) in the ratio 3 : 2 internally.

(a) $\left(0, \frac{-21}{5}\right)$
(b) $\left(0, \frac{21}{5}\right)$
(c) $\left(\frac{11}{2}, \frac{14}{3}\right)$
(d) $\left(\frac{-11}{2}, \frac{-14}{3}\right)$

13. In Question 12 question, find the coordinates of the point if it divides the points externally.

(a) (24, –9)
(b) (3, –5)
(c) (–24, 9)
(d) (5, –3)

14. In what ratio is the line segment joining (–1, 3) and (4, –7) divided at the point (2, –3)?

(a) 3 : 2
(b) 2 : 3
(c) 3 : 5
(d) 5 : 3

15. In question 14, find the nature of division?

(a) Internal
(b) External
(c) Cannot be said

16. In what ratio is the line segment made by the points (7, 3) and (–4, 5) divided by the y-axis?

(a) 2 : 3
(b) 4 : 7
(c) 3 : 5
(d) 7 : 4

17. What is the nature of the division in the above question?

(a) External
(b) Internal
(c) Cannot be said

18. If the coordinates of the mid-point of the line segment joining the points (2, 1) and (1, –3) is (x, y) then the relation between x and y can be best described by

(a) $3x + 2y = 5$
(b) $6x + y = 8$
(c) $5x – 2y = 4$
(d) $2x – 5y = 4$

19. Points (6, 8), (3, 7), (–2, –2) and (1, –1) are joined to form a quadrilateral. What will be this structure?

(a) Rhombus

(b) Parallelogram

(c) Square

(d) Rectangle

20. Points (4, –1), (6, 0), (7, 2) and (5, 1) are joined to be a vertex of a quadrilateral. What will be the structure?

(a) Rhombus

(b) Parallelogram

(c) Square

(d) Rectangle

21. What will be the centroid of a triangle whose vertices are (2, 4), (6, 4) and (2, 0)?

(a) $\left(\frac{7}{2}, \frac{5}{2}\right)$

(b) (3, 5)

(c) $\left(\frac{10}{3}, \frac{8}{3}\right)$

(d) (1, 4)

22. The distance between the lines $4x + 3y = 11$ and $8x + 6y = 15$ is

(a) 4

(b) $\frac{7}{10}$

(c) $\frac{5}{7}$

(d) 26

23. If the mid-point of the line joining (3, 4) and $(p, 7)$ is (x, y) and $2x + 2y + 1 = 0$, then what will be the value of p?

(a)15

(b) $\frac{-17}{2}$

(c) –15

(d) $\frac{17}{2}$

24. Find the third vertex of the triangle whose two vertices are (–3, 1) and (0, –2) and the centroid is the origin.

(a) (2, 3)

(b) $\left(\frac{-4}{3}, \frac{14}{3}\right)$

(c) (3, 1)

(d) (6, 4)

25. Find the area of the triangle whose vertices are (1, 3), (–7, 6) and (5, –1).

(a)20

(b) 10

(c)18

(d) 24

26. Find the area of the triangle whose vertices are $(a, b + c)$, $(a, b - c)$ and $(-a, c)$.

(a) $2ac$

(b) $2bc$

(c) $b(a + c)$

(d) $c(a - b)$

27. The number of lines that are parallel to $2x + 6y + 7 = 0$ and have an intercept of length 10 between

the coordinate axes is

(a) 0 (b) 1

(c) 2 (d) Infinite

28. Which of the following three points represent a straight line?

(a) $\left(\frac{-1}{2}, 3\right)$, (–5, 6) and (– 8, 8)

(b) $\left(\frac{-1}{2}, 3\right)$, (5, 6) and (– 8, 8)

(c) $\left(\frac{1}{2}, 3\right)$, (–5, 6) and (– 8, 8)

(d) $\left(\frac{-1}{2}, 3\right)$, $\left(\frac{5}{6}\right)$ and (8, 8)

29. Which of the following will be the equation of a straight line that is parallel to the y-axis at a distance 11 units from it?

(a) $x = +11, x = -11$ (b) $y = 11, y = -11$

(c) $y = 0$ (d) None of these

30. Which of the following will be the equation of a straight line parallel to the y-axis at a distance of 9 units to the left?

(a) $x = -9$ (b) $x = 9$

(c) $y = 9$ (d) $y = -9$

31. What can be said about the equation of the straight line $x = 7$?

(a) It is the equation of a straight line at a distance of 7 units towards the right of the y-axis.

(b) It is the equation of a straight line at a distance of 7 units towards the left of the y-axis.

(c) It is the equation of a straight line at a distance of 7 units below the x-axis.

(d) It is the equation of a straight line at a distance of 7 units above the x-axis.

32. What can be said about the equation of the straight line $y = -8$?

(a) It is the equation of a straight line at a distance of 8 units below the x-axis.

(b) It is the equation of a straight line at a distance of 8 units above the x-axis.

(c) It is the equation of a straight line at a distance of 8 units towards the right of the y-axis.

(d) It is the equation of a straight line at a distance of 8 units towards the left of the y-axis.

33. Which of the following straight lines passes through the origin?

(a) $x + y = 4$ (b) $x^2 + y^2 = -6$

(c) $x + y = 5$ (d) $x = 4y$

34. What will be the point of intersection of the equation of lines $2x + 5y = 6$ and $3x + 4y = 7$?

(a) $\left(\frac{11}{7}, \frac{4}{7}\right)$ (b) $\left(\frac{-11}{7}, 4\right)$

(c) $\left(3, \frac{-2}{7}\right)$ (d) $\left(4, \frac{-2}{5}\right)$

35. If P (6, 7), Q (2, 3) and R (4, –2) be the vertices of a triangle, then which of the following is not a point contained in this triangle?

(a) (4, 3) (b) (3, 3)

(c) (4, 2) (d) (6, 1)

36. What will be the reflection of the point (4, 5) in the second quadrant?

(a) (– 4, – 5) (b) (– 4, 5)

(c) (4, – 5) (d) None of these

37. What will be the reflection of the point (4, 5) in the third quadrant?

(a) (– 4, – 5) (b) (– 4, 5)

(c) (4, – 5) (d) None of these

38. What will be the reflection of the point (4, 5) in the second quadrant?

(a) (– 4, – 5) (b) (– 4, 5)

(c) (4, –5) (d) None of these

39. If the origin gets shifted to (2, 2), then what will be the new coordinates of the point (4, –2)?

(a) (– 2, 4) (b) (2, 4)

(c) (4, 2) (d) (2, – 4)

40. What will be the length of the perpendicular drawn from the point (4, 5) upon the straight line $3x + 4y = 10$?

(a) $\frac{12}{5}$ (b) $\frac{32}{5}$

(c) $\frac{22}{5}$ (d) $\frac{42}{5}$

LEVEL OF DIFFICULTY (II)

1. Find the area of the quadrilateral the coordinates of whose angular points taken in order are (1, 1), (3, 4), (5, –2) and (4, –7).

   (a) 20.5 (b) 41

   (c)82 (d) 61.5

2. Find the area of the quadrilateral the coordinates of whose angular points taken in order are (–1, 6), (–3, –9), (5, –8) and (3, 9).

   (a)48 (b) 96

   (c) 192 (d) 72

3. Two vertices of a triangle are (5, –1) and (–2, 3). If the orthocenter of the triangle is the origin, what will be the coordinates of the third point?

   (a) (4, 7) (b) (– 4, 7)

   (c) (– 4, –7) (d) (4, –7)

4. Find the equation of the straight line passing through the origin and the point of intersection of the lines $x/a + y/b = 1$ and $x/b + y/a = 1$.

   (a) $y = x$ (b) $y = -x$

   (c) $y = 2x$ (d) $y = -2x$

5. One side of a rectangle lies along the line $4x + 7y + 5 = 0$. Two of its vertices are (–3, 1) and (1, 1). Which of the following may be an equation which represents any of the other three straight lines?

   (a) $7x - 4y = 3$ (b) $7x - 4y + 3 = 0$

   (c) $y + 1 = 0$ (d) $4x + 7y = 3$

6. The points $(p–1, p + 2)$, $(p, p + 1)$, $(p + 1, p)$ are collinear for

   (a) $p = 0$ (b) $p = 1$

   (c) $p = -1/2$ (d) Any value of p

7. The straight line joining (1, 2) and (2, –2) is perpendicular to the line joining (8, 2) and (4, p). What will be the value of p?

   (a) –1 (b) 1

   (c) 3 (d) None of these

8. What will be the length of the perpendicular drawn from the point (–3, –4) to the straight line $12(x + 6) = 5(y - 2)$?

   (a) $5\left(\frac{4}{13}\right)$ (b) $5\left(\frac{1}{13}\right)$

   (c) $3\left(\frac{2}{13}\right)$ (d) $3\left(\frac{1}{13}\right)$

9. The area of the triangle with vertices at $(a, b + c)$, $(b, c + a)$ and $(c, a + b)$ is

(a) 0 (b) a + b + c

(c) $a^2 + b^2 + c^2$ (d) 1

10. Find the distance between the two parallel straight lines $y = mx + c$ and $y = mx + d$? [Assume $c > d$]

(a) $\left(\frac{(c-d)}{(1+m^2)^{\frac{1}{2}}}\right)$ (b) $\left(\frac{(d-c)}{(1+m^2)^{\frac{1}{2}}}\right)$

(c) $\left(\frac{d}{(1+m^2)^{\frac{1}{2}}}\right)$ (d) $\left(\frac{-d}{(1+m)^{\frac{1}{2}}}\right)$

11. What will be the equation of the straight line that passes through the intersection of the straight lines $2x - 3y + 4 = 0$ and $3x + 4y - 5 = 0$ and is perpendicular to the straight line $3x - 4y = 5$?

(a) $8x + 6y = \frac{32}{7}$ (b) $4x + 3y = \frac{84}{17}$

(c) $4x + 3y = \frac{62}{17}$ (d) $8x + 6y = \frac{58}{17}$

12. In question 11, find the equation of the straight line if it is parallel to the straight line $3x + 4y = 5$?

(a) $12x + 16y = \frac{58}{17}$ (b) $3x + 4y = \frac{58}{17}$

(c) $6x + 8y = \frac{58}{17}$ (d) None of these

13. The orthocenter of the triangle formed by the points (0, 0), (8, 0) and (4, 6) is

(a) $\left(4, \frac{8}{3}\right)$ (b) (3, 4)

(c) (4, 3) (d) $\left(3, \frac{5}{2}\right)$

14. The area of a triangle is 5 square units, two of its vertices are (2, 1) and (3, –2). The third vertex lies on $y = x + 3$. What will be the third vertex?

(a) $\left(\frac{5}{3}, \frac{13}{3}\right)$ (b) $\left(\frac{7}{2}, \frac{13}{2}\right)$

(c) (3, 4) (d) (1, 2)

15. The equations of two equal sides AB and AC of an isosceles triangle ABC are $x + y = 5$ and $7x - y = 3$ respectively. What will be the equation of the side BC if area of triangle ABC is 5 square units.

(a) $x + 3y - 1 = 0$ (b) $x - 3y + 1 = 0$

(c) $2x - y = 5$ (d) $x + 2y = 5$

16. Three vertices of a rhombus, taken in order are (2, –1), (3, 4) and (–2, 3). Find the fourth vertex.

(a) (3, 2) (b) (–3, –2)

(c) (–3, 2) (d) (3, –2)

17. Four vertices of a parallelogram taken in order are (–3, –1), (a, b), (3, 3) and (4, 3). What will be the ratio of a to b?

(a) 4 : 1 (b) 1 : 2

(c) 1 : 3 (d) 3 : 1

18. What will be the new equation of straight line $3x + 4y = 6$ if the origin gets shifted to (3, – 4)?

(a) $3x + 4y = 5$ (b) $4x – 3y = 4$

(c) $3x + 4y + 1 = 0$ (d) $3x + 4y – 13 = 0$

19. What will be the value of p if the equation of straight line $2x + 5y = 4$ gets changed to $2x + 5y = p$ after shifting the origin at (3, 3)?

(a)16 (b) –17

(c)12 (d) 10

20. A line passing through the points (a, $2a$) and (–2, 3) is perpendicular to the line $4x + 3y + 5 = 0$. Find the value of a?

(a) –14/3 (b) 18/5

(c) 14/3 (d) –18/5

LEVEL OF DIFFICULTY (III)

1. The area of a triangle is 5 square units. Two of its vertices are (2, 1) and (3, –2). The third vertex lies on $y = x + 3$. What will be the third vertex?

   (a) (4, –7) (b) (4, 7)

   (c) (– 4, –7) (d) (– 4, 7)

2. One side of a rectangle lies along the line $4x + 7y + 5 = 0$. Two of its vertices are (–3, 1) and (1, 1). Which of the following is not an equation of the other three straight lines?

   (a) $14x – 8y = 6$ (b) $7x – 4y = –25$

   (c) $4x + 7y = 11$ (d) $14x – 8y = 20$

3. The area of triangle formed by the points (p, 2–2p), (1–p, 2p) and (–4–p, 6–2p) is 70 units. How many integral values of p are possible?

   (a) 2 (b) 3

   (c) 4 (d) None of these

4. What are the points on the axis of x whose perpendicular distance from the straight line $x/p + y/q = 1$ is p?

   (a) $\frac{p}{q}\left[q+\sqrt{(p^2+q^2)}\right], 0$

   (b) $\frac{p}{q}\left[q-\sqrt{(p^2+q^2)}\right], 0$

   (c) Both (a) and (b)

   (d) None of these

5. If the medians PT and RS of a triangle with vertices P (0, b), Q (0, 0) and R (a, 0) are perpendicular to each other, which of the following satisfies the relationship between a and b?

   (a) $4b^2 = a^2$ (b) $2b^2 = a^2$

   (c) $a = –2b$ (d) $a^2 + b^2 = 0$

6. The point of intersection of the lines $x/a + y/b = 1$ and $x/b + y/b = 1$ lies on the line

   (a) $x + y = 1$ (b) $x + y = 0$

   (c) $x – y = 1$ (d) $x – y = 0$

7. PQR is an isosceles triangle. If the coordinates of the base are Q (1, 3) and R (–2, 7), then the coordinates of the vertex P can be

   (a) $\left(4, \frac{7}{2}\right)$ (b) (2, 5)

   (c) $\left(\frac{5}{6}, 6\right)$ (d) $\left(\frac{1}{3}, 2\right)$

8. The extremities of a diagonal of a parallelogram are the points (3, – 4) and (– 6, 5). If the third vertex is the point (–2, 1), the coordinate of the fourth vertex is

(a) (1, 0)
(b) (–1, 0)
(c) (–1, 1)
(d) (1, –1)

9. If the points (a, 0), (0, b) and (1, 1) are collinear then which of the following is true?

(a) $\frac{1}{a} + \frac{1}{b} = 2$
(b) $\frac{1}{a} - \frac{1}{b} = 1$
(c) $\frac{1}{a} - \frac{1}{b} = 2$
(d) $\frac{1}{a} + \frac{1}{b} = 1$

10. If P and Q are two points on the line $3x + 4y = -15$, such that $OP = OQ = 9$ units, the area of the triangle POQ will be

(a) $18\sqrt{2}$ sq units
(b) $3\sqrt{2}$ sq units
(c) $6\sqrt{2}$ sq units
(d) $15\sqrt{2}$ sq units

11. If the coordinates of the points A, B, C and D are (6, 3), (–3, –5), (4, –2) and (a, $3a$) respectively and if the ratio of the area of triangles ABC and DBC is 2 : 1, then the value of a is

(a) $\frac{-9}{2}$
(b) $\frac{9}{2}$
(c) $\frac{-23}{36}$
(d) $\frac{23}{18}$

12. The equations of two equal sides AB and AC of an isosceles triangle ABC are $x + y = 5$ and $7x - y = 3$ respectively. What will be the length of the intercept cut by the side BC on the y-axis?

(a) $\frac{9}{5}$
(b) 8
(c) 1.5
(d) No unique solution

13. A line is represented by the equation $4x + 5y = 6$ in the coordinate system with the origin (0, 0). You are required to find the equation of the straight line perpendicular to this line that passes through the point (1, –2) [which is in the coordinate system where origin is at (–2, –2)].

(a) $5x - 4y = 11$
(b) $5x - 4y = 13$
(c) $5x - 4y = -3$
(d) $5x - 4y = 7$

14. P (3, 1), Q (6, 5) and R (x, y) are three points such that the angle PRQ is a right angle and the area of D PRQ is 7. The number of such points R that are possible is

(a) 1
(b) 2
(c) 3
(d) 4

15. Two sides of a square lie on the lines $x + y = 1$ and $x + y + 2 = 0$. What is its area?

(a) $\frac{11}{2}$ (b) $\frac{9}{2}$

(c) 5 (d) 4

16. Find the value of k if the straight line $2x + 3y + 4 + k(6x - y + 12) = 0$ is perpendicular to the line $7x + 5y = 4$.

(a) $\frac{-33}{37}$ (b) $\frac{-29}{37}$

(c) $\frac{19}{37}$ (d) None of these

17. If p is the length of the perpendicular from the origin to the line $\frac{x}{a} + \frac{y}{b} = 1$, then which of the following is true?

(a) $\frac{1}{p^2} = \frac{1}{b^2} - \frac{1}{a^2}$ (b) $\frac{1}{p^2} = \frac{1}{a^2} - \frac{1}{b^2}$

(c) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$ (d) None of these

18. How many points on $x + y = 4$ are there that lie at a unit distance from the line $4x + 3y = 10$?

(a) 1 (b) 2

(c) 3 (d) None of these

19. What will be the area of the rhombus $ax \pm by \pm c = 0$?

(a) $\frac{3c^2}{ab}$ (b) $\frac{4c^2}{ab}$

(c) $\frac{2c^2}{ab}$ (d) $\frac{c^2}{ab}$

20. The coordinates of the mid-points of the sides of a triangle are (4, 2), (3, 3) and (2, 2). What will be the coordinates of the centroid of the triangle?

(a) $\left(3, \frac{7}{3}\right)$ (b) $\left(-3, \frac{-7}{3}\right)$

(c) $\left(3, \frac{-7}{3}\right)$ (d) $\left(-3, \frac{7}{3}\right)$

ANSWER KEY

Level of Difficulty (I)

1. (b)	2. (c)	3. (c)	4. (c)
5. (a)	6. (c)	7. (d)	8. (b)
9. (c)	10. (c)	11. (c)	12. (b)
13. (c)	14. (a)	15. (a)	16. (d)
17. (b)	18. (b)	19. (b)	20. (a)
21. (c)	22. (b)	23. (c)	24. (c)
25. (b)	26. (a)	27. (c)	28. (a)
29. (a)	30. (a)	31. (a)	32. (a)
33. (d)	34. (a)	35. (d)	36. (b)
37. (a)	38. (b)	39. (d)	40. (c)

Level of Difficulty (II)

1. (a)	2. (b)	3. (c)	4. (a)
5. (a)	6. (d)	7. (b)	8. (b)
9. (a)	10. (a)	11. (c)	12. (d)
13. (a)	14. (b)	15. (d)	16. (b)
17. (a)	18. (c)	19. (b)	20. (b)

Level of Difficulty (III)

1. (c)	2. (d)	3. (d)	4. (c)
5. (b)	6. (d)	7. (c)	8. (b)
9. (d)	10. (a)	11. (c)	12. (b)
13. (a)	14. (b)	15. (b)	16. (b)
17. (c)	18. (b)	19. (c)	20. (a)

Hints

Level of Difficulty (II)

1. Use the area of a triangle formula for the two parts of the quadrilateral separately and then add them.
4. Find the point of intersection of the lines by solving the simultaneous equations and then use the two-point formula of a straight line.

   Alternative: After finding out the point of intersection, use options to check.
6. For 3 points to be collinear,

(i) Either the slope of any two of the 3 points should be equal to the slope of any other two points. OR

(ii) The area of the triangle formed by the three points should be equal to zero.

Solve using options.

7. Form the equation of the straight lines and then use the options.
10. Point of intersection of $y = mx + c$ with x-axis is $(–c/m, 0)$.

    Now use the formula for the distance of a point to a straight line.
11. Find the point of intersection of the lines and then put the coordinate of this point into the equation $4x + 3y = K$, which is perpendicular to the equation of straight line $3x – 4y = 5$, to find out K.

13. Orthocenter is the point of intersection of altitudes of a triangle and centroid divides the straight line formed by joining circumcenter and the orthocenter in the ratio 2 : 1.

    Let the vertices of the triangle be $O(0, 0)$, $A(8, 0)$ and $B(4, 6)$.

    The equation of an altitude through O and perpendicular to AB is $y = 2/3x$ and similarly the equation of an altitude through A and perpendicular to OB is $2x + 3y = 16$. Now find the point of intersection of these two straight lines.

14. Use the options.

    Alternative: Draw the points in the cartesian co-ordinate system and then use the simple geometry formula to calculate the point using the options.

15. Draw the points and then check with the options.

    Alternative: Find out the point of intersection with the help of options and then use the formula for area of ?.

17. Sum of x and y co-ordinates of opposite vertices in a parallelogram are same.

19. If the origin gets changed to (h, k) from $(0, 0)$ then

    Old x co-ordinate = New x co-ordinate + h

    Old y co-ordinate = New y co-ordinate + k

20. Equation of any straight line perpendicular to the line $4x + 3y + 5 = 0$ will be of the form of $3x - 4y = k$, where k is any constant.

    Now form the equation of the straight line with the given two points and then equate.

Level of Difficulty (III)

1. First check the options to see that which of the points lie on the equation of straight line $y = x + 3$.

   And then again check the options, if needed, to confirm the second constraint regarding area of triangle.

2. Use the options.

3 Use the formula of area of a quadrilateral which will lead to a quadratic equation. Now solve the quadratic equation to see the number of integral solutions it can have.

4. Use the formula of distance of a point from the straight line using the options.

7. Use the options to find the length using the distance formula.

9. Make the slope of any two points equal to the slope of any other two points.

   Slope = Difference of Y coordinates/Difference of X coordinates.

14. Draw the points on cartesian coordinate system.

15. Length of the square can be find out using the method of finding out the distance between two parallel lines.

17. Use the formula (perpendicular distance of a point from a straight line.)

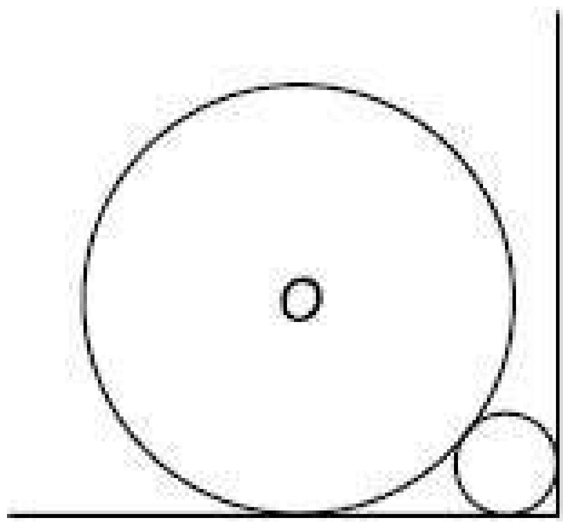
TRAINING GROUND FOR BLOCK IV

HOW TO THINK IN PROBLEMS ON BLOCK IV

In the back to school section of this block, I have already mentioned that there is very little use of complex and obscure formulae and results while solving questions on this block.

The following is a list of questions (with solutions) of what has been asked in previous years' CAT questions from this chapter. Hopefully you will realise through this exercise, what I am talking about when I say this. For each of the questions given below, try to solve on your own first, before looking at the solution provided.

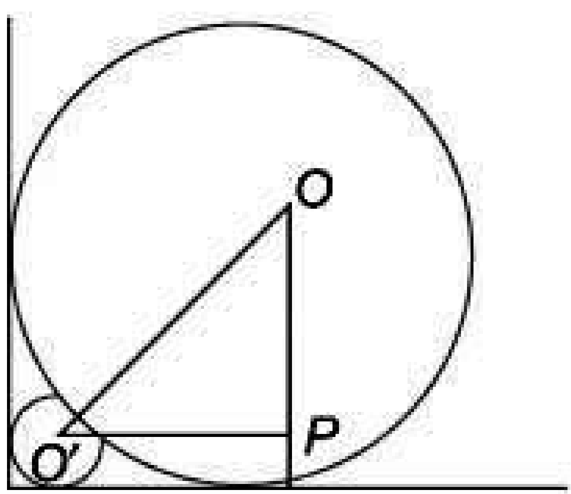
1. A circle with radius 2 is placed against a right angle. As shown in the figure below, another smaller circle is placed in the gap between the circle and the right angle. What is the radius of the smaller circle?



(a) $3 - 2\sqrt{2}$ (b) $4 - 2\sqrt{2}$

(c) $7 - 4\sqrt{2}$ (d) $6 - 4\sqrt{2}$

Solution: The solution of the above question is based on the following construction.



In the right triangle *OO¢P*,

OP = (2 – *r*), *O¢P* = (2 – *r*) and *OO¢* = 2 + *r*

where *r* is the radius of the smaller circle.

Using Pythagoras theorem:

$(2 + r)^2 = (2 - r)^2 + (2 - r)^2$

Solving, we get $r = 6 \pm 4\sqrt{2}$

$6 + 4\sqrt{2}$ cannot be correct since the value of r should be less than 2.

Note: The key to solving this question is in the visualisation of the construction. If you try to use complex formulae while solving, your mind unnecessarily gets cluttered. The key to your thinking in this question is:

(1) Realise that you only have to use length measuring formulae. Hence, put all angle measurement formulae into the back seat.

(2) A quick mental search of the length measuring formulae available for this situation will narrow down your mind to the Pythagoras theorem.

(3) The key then becomes the construction of a triangle (right angled of course) where the only unknown is r.

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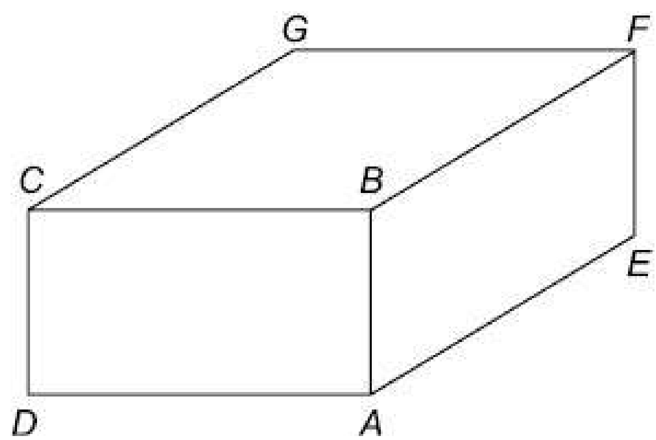
2. ABCDEFGH is a cube. If the length of the diagonals *DF*, *AG* & *CE* are equal to the sides of a triangle, then the circumradius of that triangle would be



(a) Equal to the side of the cube

(b) $\sqrt{3}$ times the side of the cube

(c) $1/\sqrt{3}$ times the side of the cube

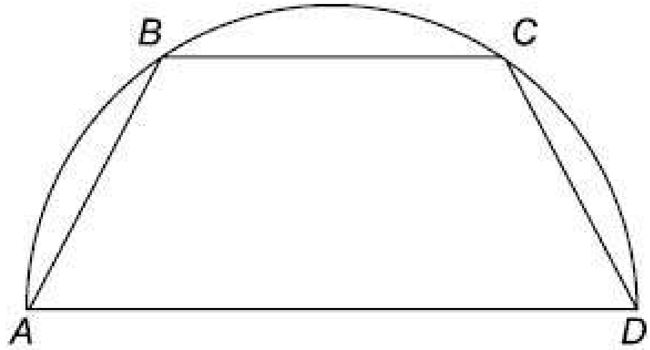
(d) Indeterminate

Solution: If we assume the side of the cube to be a the triangle will be an equilateral triangle with side $a\sqrt{3}$. (we get this using Pythagoras theorem). Also, we know that the circumradius of an equilateral triangle is $1/\sqrt{3}$ times the side of the triangle.

Hence, in this case the circumradius would be a—equal to the side of the cube.

(Again the only formula used in this question would be the Pythagoras theorem.)

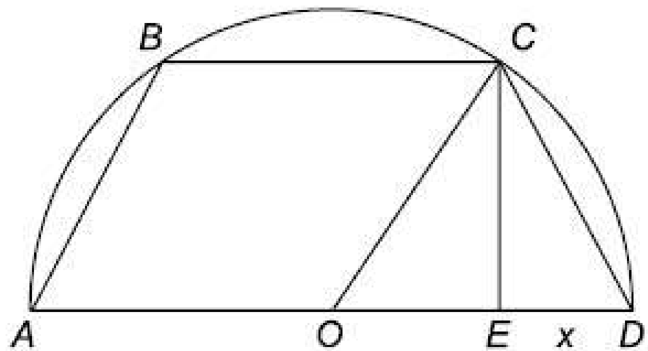
3. On a semicircle (diameter *AD*), chord *BC* is parallel to the diameter *AD*. Also, *AB* = *CD* = 2, while *AD* = 8, what is the length of *BC* ?



(a) 7.5 (b) 7

(c) 7.75 (d) None of these

Solution: Think only of length measuring formulae (Pythagoras theorem is obvious in this case).



If we can find the value of x, we will get the answer for BC as $AD - 2x$. Hence, we need to focus our energies in finding the value of x.

The construction above gives us two right angled triangles (OEC and DEC).

In D OCE, $OC = 4$ (radius) and $OE = (4 - x)$, Then: $(CE)^2 = 8x - x^2$. (Using Pythagoras Theorem)

Then in triangle CED:

$(8x - x^2) + x^2 = 2^2$

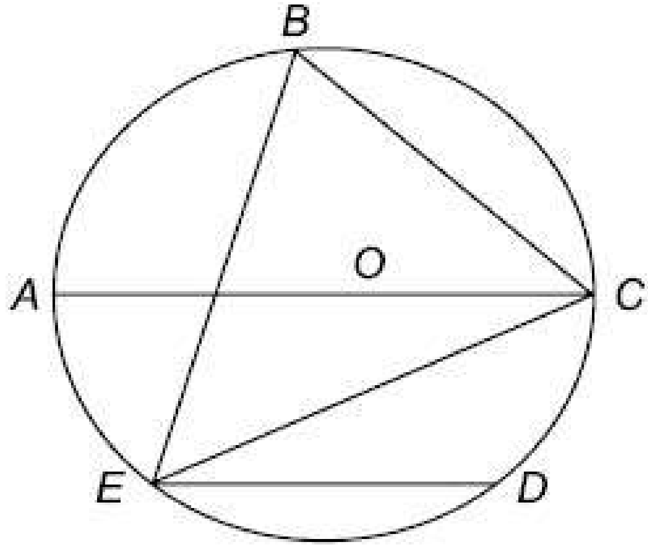
Hence, $x = 0.5$

Thus, $BC = 8 - 2 \times 0.5 = 7$

4. In the given circle, AC is the diameter of the circle. ED is parallel to AC. –CBE = 65°, find –DEC.

(a) 35° (b) 55°

(c) 45° (d) 25°



Solution: Obviously this question has to be solved using only angle measuring tools. Further from the figure, it is obvious that we have to use angle measurement tools related to arcs of circles.

Reacting to the 65° information in the question above, you will get –*EOC* = 130° (Since, the angle at the centre of the circle is twice the angle at any point of the circle).

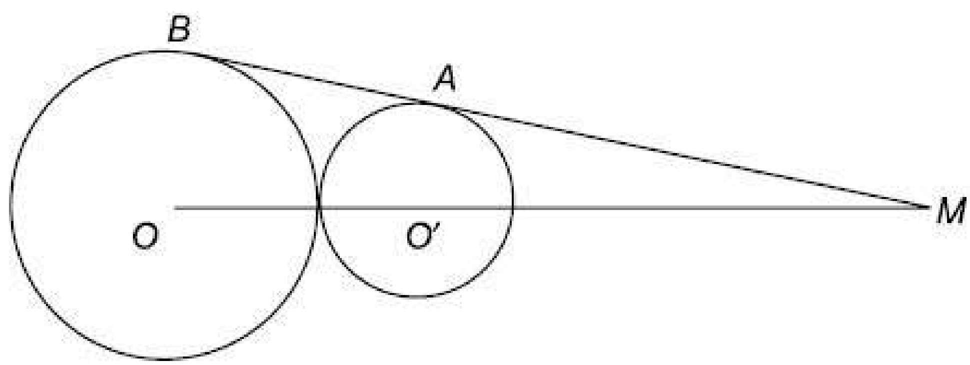
Hence, –*AOE* = 180 – 130 = 50°

This, will be the same as –*COD* since the minor arc *AE* = minor arc *CD*.

Also, –*DEC* = ½ × –*COD*

Hence, –*DEC* = 25°

Directions for Questions 5 to 7: In the figure below, *X* and *Y* are circles with centres *O* and *O*¢ respectively. *MAB* is a common tangent. The radii of *X* and *Y* are in the ratio 4 : 3 and *OM* = 28 cm.



5. What is the ratio of the length of *OO*¢ to that of *O*¢*M* ?

(a) 1 : 4 (b) 1 : 3

(c) 3 : 8 (d) 3 : 4

6. What is the radius of circle *X*?

(a) 2 cm (b) 3 cm

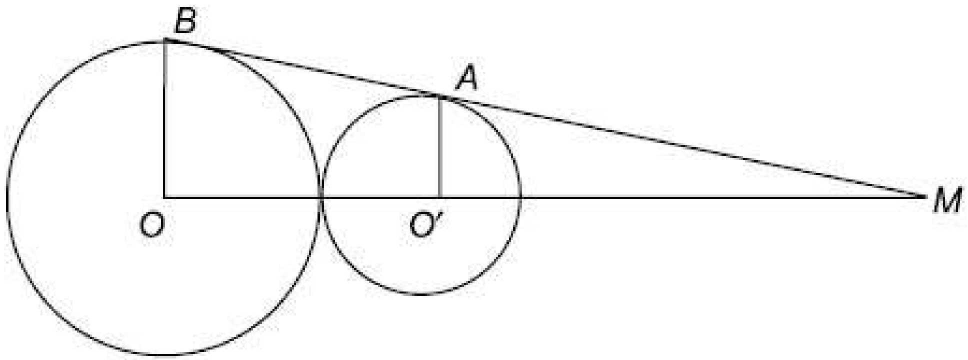
(c) 4 cm (d) 5 cm

7. The length of *AM* is

(a) $8\sqrt{3}$ cm (b) $10\sqrt{3}$ cm

(c) $12\sqrt{3}$ cm (d) $14\sqrt{3}$ cm

Solution: Construct *OB* and *O¢ A* as shown below.



In this construction it is evident that the two right angled triangles formed are similar to each other. i.e. D*OBM* is similar to D*O¢AM*.

Hence, *OM* : *O¢M* = 4 : 3 (since *OB* : *O¢A* = 4 : 3)

Also, *OM* = 28 cm, \ *O¢ M* = 21 cm. Æ *OO¢* = 7 cm. Hence, the radius of circle *X* is 4 cm (Answer to Q. 6).

5. Also: *OO'*= 7 and *O'M* = 21. Hence, required ratio = 1 : 3

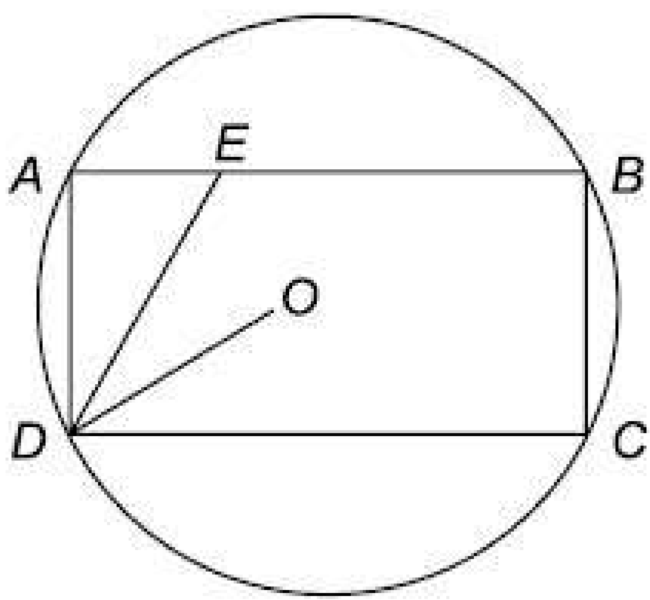
7. *AM* can be found easily using Pythagoras theorem.

$AM^2 = 21^2 - 3^2 = 432$

\ $AM = \sqrt{432} = 12\sqrt{3}$.

(Note: Only similarity of triangles and Pythagoras theorem was used here.)

8. In the figure, *ABCD* is a rectangle inscribed inside a circle with center *O*. Side *AB* > Side *BC*. The ratio of the area of the circle to the area of the rectangle is *p* : ÷3. Also, –*ODC* = –*ADE*. Find the ratio *AE* : *AD*.

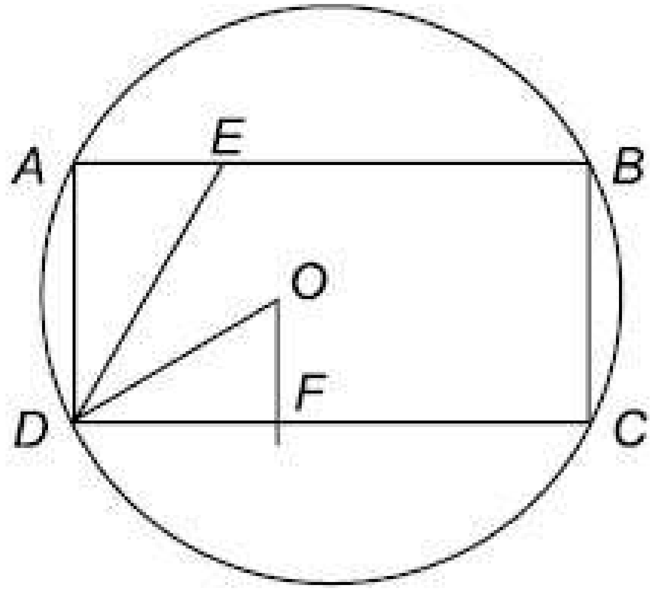


(a) $1 : \sqrt{3}$ (b) $1 : \sqrt{2}$

(c) $2\sqrt{3} : 1$ (d) 1 : 2

Solution: In my experience, questions involving ratios of length typically involves the use of similar triangles. This question is no different.

Make the following construction:



D*OFD* is similar to D*AED*. Hence, the required ratio *AE* : *AD* = *OF*/*FD*

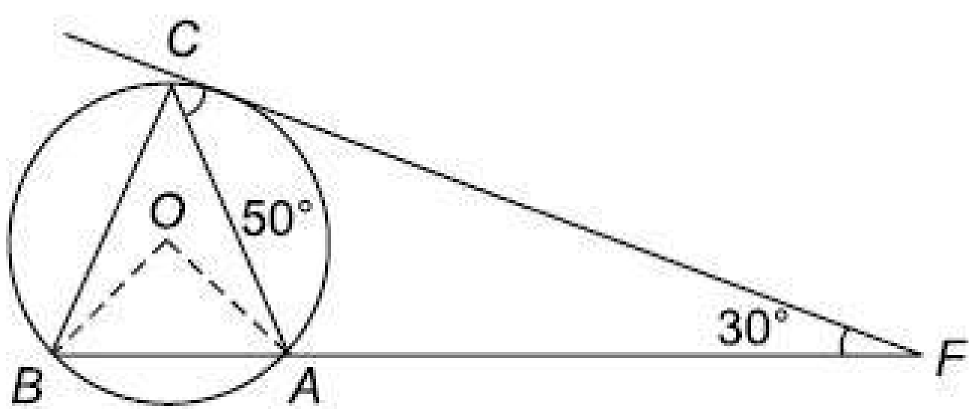
But *OF* = ½ side *BC* while *FD* = ½ side *CD*.

Hence, we need the ratio of the side of the rectangle *BC* : *DC* (This will give the required answer.)

From this point, you can get to the answer through a little bit of unconventional thinking.

The ratio of the area of the circle to that of the rectangle is given as *p* : $\sqrt{3}$. Hence, it is obvious that one of the sides has to have a $\sqrt{3}$ component in it. Hence, options 2 and 4 can be rejected. Also the required ratio has to be less than 1, hence, option (1) is correct.

9. Find –*BOA*.



(a) 100° (b) 150°

(c) 80° (d) Indeterminate

Solution: Obviously this question has to be solved using angle measurement tools.

In order to measure –*BOA*, you could either try to use theorems related to the angle subtended by arcs of a circle or solve using the isosceles D*BOA*.

With this thought in mind start reacting to the information in the question.

– *CAF* = 100°. Hence –*BAC* = 80°

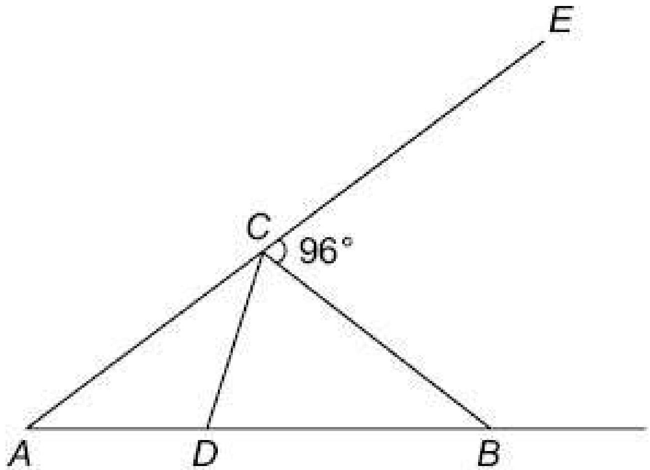
Also, –*OCA* = (90 – *ACF*) = 90 – 50 = 40° = –*OAC* (Since the triangle *OCA* is isosceles)

Hence –*OAB* = 40°

In isosceles D *OAB*, –*OBA* will also be 40°

Hence –*BOA* = 180 – 40 – 40 = 100°

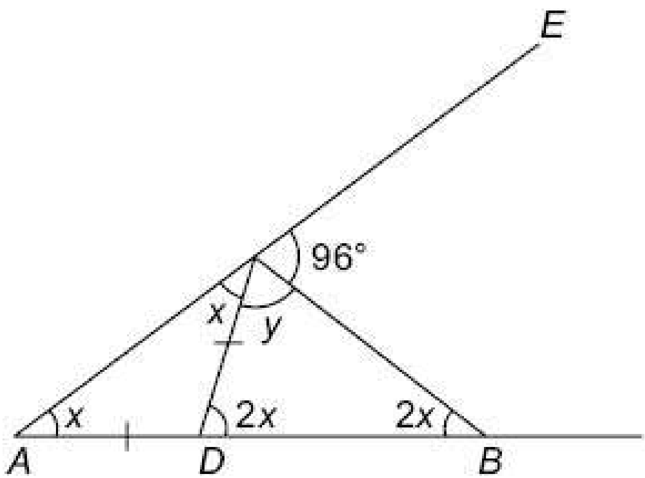
10. In the figure *AD* = *CD* = *BC* and –*BCE* = 96°. How much is –*DBC* ?



(a) 32° (b) 84°

(c) 64° (d) Indeterminate

Solution: Get out your angle measuring formulae and start reacting to the information.



From the figure above, it is clear that

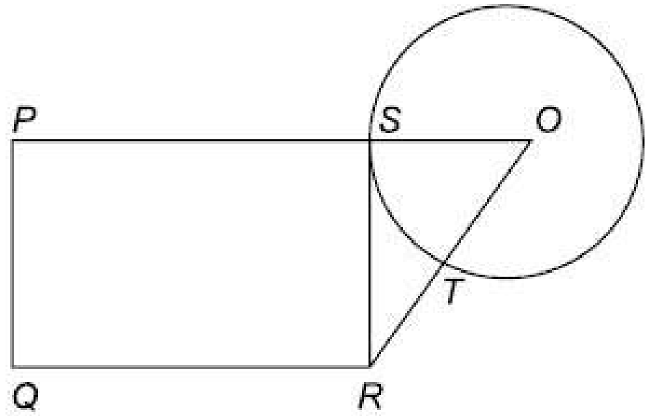
$x + y = 180 - 96 = 84°$

Also $4x + y = 180°$

Solving we get $x = 32°$

Hence, –*DBC* = $2x = 64°$.

11. *PQRS* is a square. *SR* is a tangent (at point *S*) to the circle with centre *O* and *TR* = *OS*. Then the ratio of area of the circle to the area of the square is



(a) $p/3$ (b) 11/7

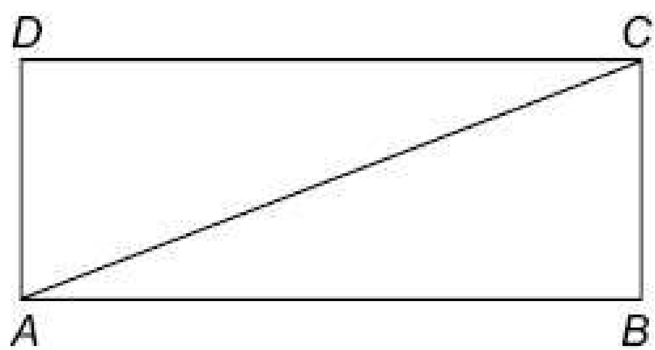
(c) $3/p$ (d) 7/11

Solution: Looking at the options we can easily eliminate option (b) and (d), because in the ratio of the area of the circle to the area of the triangle we cannot eliminate p and hence the answer should contain p.

Further the question is asking for the ratio $\frac{\text{Area of the circle}}{\text{Area of the square}}$ so, p should be in the numerator.

Hence (a).

12. In the adjoining figure, $AC + AB = 5AD$ and $AC - AD = 8$. Then the area of the rectangle $ABCD$ is



(a) 36 (b) 50

(c) 60 (d) Cannot be answered

Solution: Think only of length measuring formulae (Pythagoras Theorem is obvious in this case).

There is no need of forming equations if you have the knowledge of some basic triplets like 3, 4, 5; 5, 12, 13 etc.

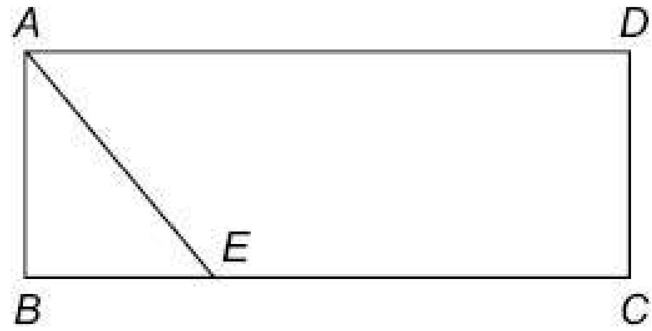
Now looking at the equations given in the question and considering D*CDA* where *AD* is the height and *AC* is the hypotenuse we will easily get,

$AC - AD = 13 - 5 = 8$ and

$AC + AB = 13 + 12 = 25$ i.e. $5AD$

Hence, Area of rectangle is length × breadth i.e. $5 \times 12 = 60$

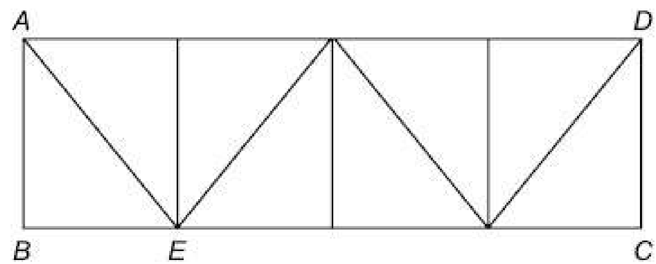
13. In the figure given below. *ABCD* is a rectangle. The area of the isosceles right triangle $ABE = 7$ cm^2, $EC = 3(BE)$. The area of *ABCD* (in cm^2) is



Solution: The key to solve this question is in the visualisation of the construction and the equations.

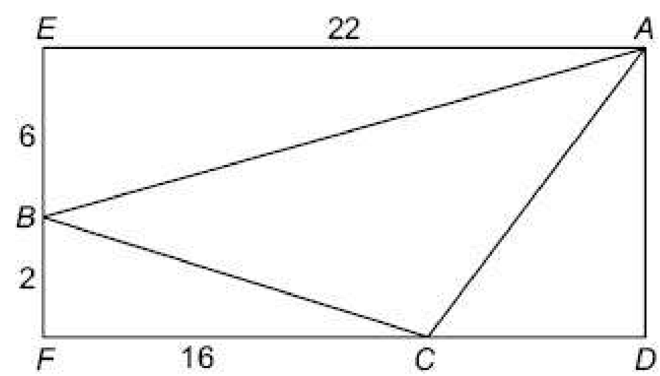
It is given that $EC = 3\ (BE)$ from this we can conclude that the whole side BC can be divided in four equal parts of measurement BE.

Now look at this construction



Each part is of equal area as 7 cm^2. Hence $7 \times 8 = 56$ cm^2

14. In the given figure. $EADF$ is a rectangle and ABC is a triangle whose vertices lie on the sides of $EADF$. $AE = 22$, $BE = 6$ $CF = 16$ and $BF = 2$. Find the length of the line joining the mid–points of the side AB and BC



(a) $4\sqrt{2}$ (b) 5

(c) 3.5 (d) None of these

Solution: Think only of length measuring formulae and triplets.

$EA = 22$ and $FC = 16$, So, $CD = 6$

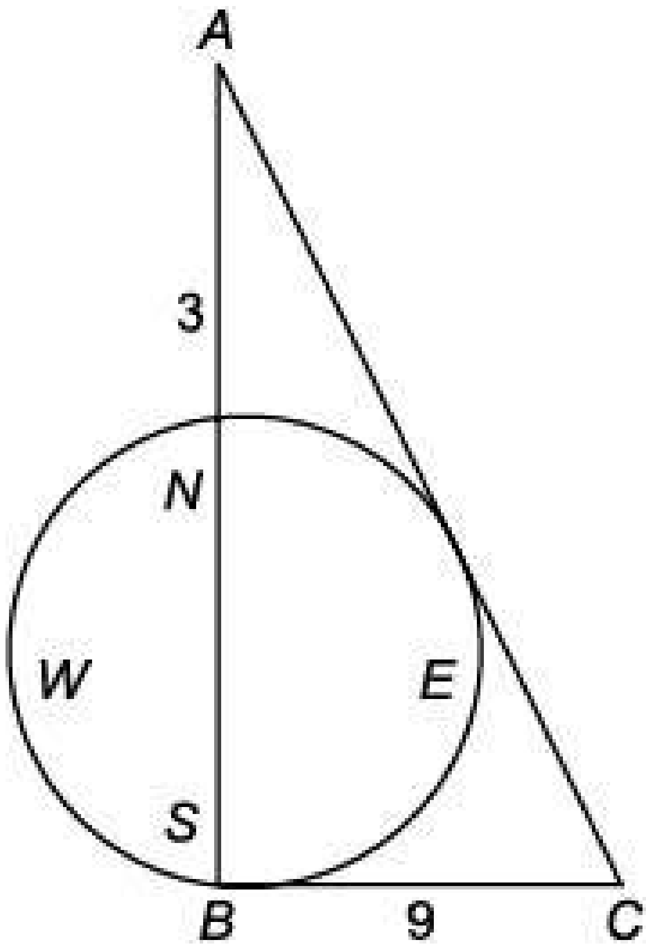
$EF = 8$ So, AD is also 8. Now using the triplet 6, 8, 10 based on basic triplet 3, 4, 5 we will get that $AC = 10$.

The line joining the midpoints of the sides AB and BC will be exactly half the side AC (using similar triangles).

Hence, 5 is the correct answer.

15. A certain city has a circular wall around it and this wall has four gates pointing north, south, east and west. A house stands outside the city, 3 km north of the north gate and it can just be seen from a point nine km east of south gate. What is the diameter of the wall that surrounds the city?

Solution: Make this construction

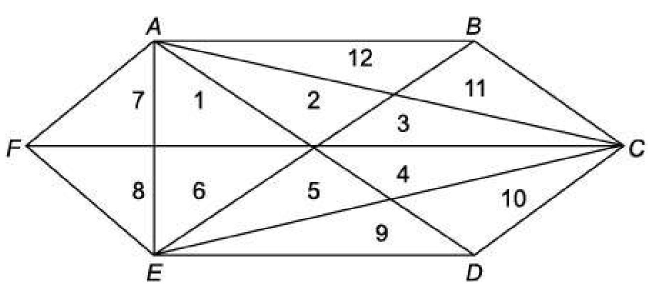


Given $AN = 3$, $BC = 9$ and –B is 90°. Now according to conventional method, we have to use tangent theorem to get to the answer, which will be very long.

Instead if we were to use the Pythagorean triplets again we would easily see the (9, 12, 15) triplet which is based on the basic triplet 3, 4, 5. Here $BC = 9$, Hence, $AC = 15$ and $AB = 12$. Hence, the diameter will be $12 – 3 = 9$ km.

16. Let *ABCDEF* be a regular hexagon: What is the ratio of the area of the triangle *ACE* to that of the hexagon *ABCDEF*?

Solution: Make the following construction:



Now we have to find the ratio $\dfrac{\text{Area of } ACE}{\text{Area of } ABCDEF}$.

In order to do so we use the property of a regular hexagon (that it is a combination of 6 equilateral triangles).

We can easily see that we have divided all 6 equilateral triangles into two equal parts of the same area.

If we number all the equal areas as 1, 2, 3 ... 12 as shown in the above construction we will get the answer as

$$\frac{\text{Sum of area of triangles } 1+2+3+4+5+6}{\text{Sum of area of triangle } 1+2+3\ldots+12}$$

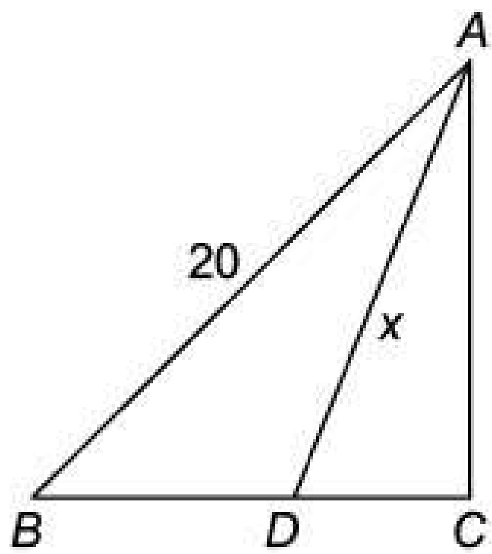
Hence ½.

17. Euclid has a triangle in mind. Its longest side has length 20 and another of its side has length 10. Its area is 80. What is the exact length of its third side?

(a) $\sqrt{260}$ (b) $\sqrt{250}$

(c) $\sqrt{240}$ (d) $\sqrt{270}$

Solution: The solution of the above question is based on the following construction, where *AB* = 20 and *BD* = 10



The question is asking for the exact length *AD*, of triangle *ABD*.

Think only of length measuring formulae (Pythagoras theorem is obvious in this case).

If we extend the side *BD* upto a point *C*, the length *AC* will give the Altitude or height of the D*ABD*. Then we will get:

½ *b* × *h* fi 80 fi ½ × 10 × *h* = 80 fi *h* = 16. i.e. *AC* = 16.

And now as D *ABC* is a right angled triangle, we can easily get the length of *DC* as 2, based on the triplet 12, 16, 20.

Now, if *AC* = 16, *DC* = 2, we can easily get the exact length of *AD* using Pythagoras theorem i.e. *AC* = $\sqrt{2^2+16^2} = \sqrt{260}$

Hence (1).

(*Note:* In this solution we have only used Pythagorean triplet 12, 16, 20 to solve the question. The alternate method for solving this question is through the use of the semi perimeter of the triangle. This will lead to a very cumbersome and long solution to this question. For experimentation purposes you can try this solution for yourself.)

BLOCK REVIEW TESTS

Review Test 1

1. A piece of paper is in the shape of a right angled triangle. This paper is cut along a line that is parallel to the hypotenuse, leaving a smaller triangle. The cut was done in such a manner that there was a 35% reduction in the length of the hypotenuse of the triangle. If the area of the original triangle was 54 square inches before the cut, what is the area (in square inches) of the smaller triangle?

   (a) 26.685 (b) 22.815

   (c) 23.465 (d) 24.365

2. Consider two different cloth-cutting processes. In the first one, *n* square cloth pieces of the same size are cut from a square cloth piece of the size *a*; then a circle of maximum possible area is cut from each of the smaller squares. In the second process, only one circle of maximum possible area is cut from a square of the same size and the process ends there. The cloth pieces remaining after cutting the circles are scrapped in both the processes. The ratio of the total area of .scrap cloth generated in the former to that in the latter is:

   (a) 1 : 1 (b) $\sqrt{2}$: 1

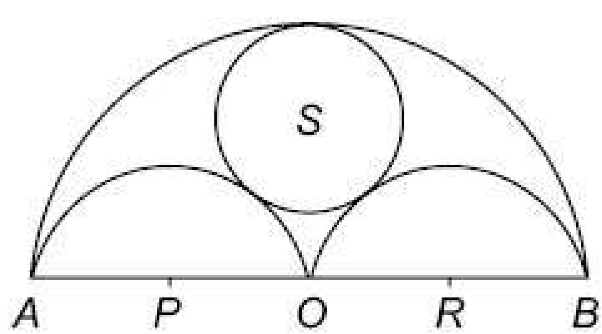
   (c) 2 : 1 (d) $\frac{4n-\pi}{n(4-\pi)}$

3. The length of the circumference of a circle equals the perimeter of a triangle of equal sides. This is also equal to the the perimeter of a square and also the perimeter of a regular pentagon. The areas covered by the circle, triangle, pentagon and square are *c*. *t*, *p* and *s* respectively. Then:

   (a) $s > t > c > p$ (b) $c > p > t > s$

   (c) $c > p > s > t$ (d) $s > p > c > t$

4. Three horses are grazing within a semi-circular field. In the diagram given below, *AB* is the diameter of the semi-circular field with centre at *O*. Horses are tied up at *P*, *R* and *S* such that *PO* and *RO* are the radii of the two semi circles drawn with centers at *P* and *R* respectively. *S* is the centre of the circle touching the two semi-circles with diameters *AO* and *OB*. The horses tied at *P* and *R* can graze within the respective semi-circles and the horse tied at *S* can graze within the circle centred at *S*. The percentage of the area of the semi-circle with diameter *AB* that cannot be grazed by the horses is nearest to:



(a) 16 (b) 24

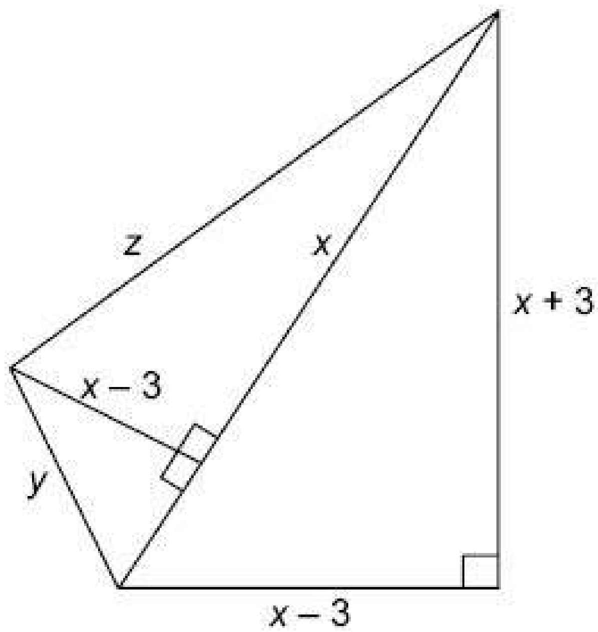
(c) 36 (d) 32

5. A certain city has a circular wall around it with four gates pointing north, south, east and west. A house stands outside the city, three km north of the north gate, and it can just be seen from a point nine km east of the south gate. What is the diameter of the wall that surrounds the city?

(a) 6 km (b) 9 km

(c) 12 km (d) None of these

6. Based on the figure below, what is the value of x, if $y = 10$.



(a) 10 (b) 11

(c) 12 (d) None of these

7. What is the number of distinct triangles with integral valued sides and perimeter as 12?

(a) 6 (b) 5

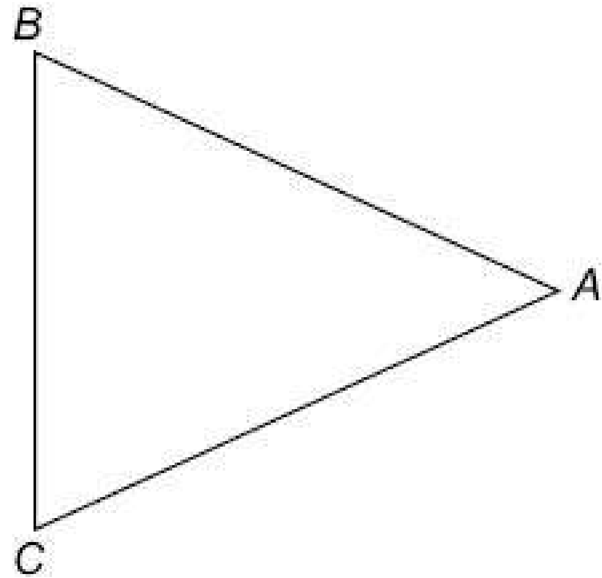
(c) 4 (d) 3

Directions for Questions 8 and 9: Answer the question based on the following information.

A cow is tethered at point *A* by a rope. Neither the rope nor the cow is allowed to enter the triangle *ABC*.

–*BAC* = 30°.

AB = *AC* = 10 m.



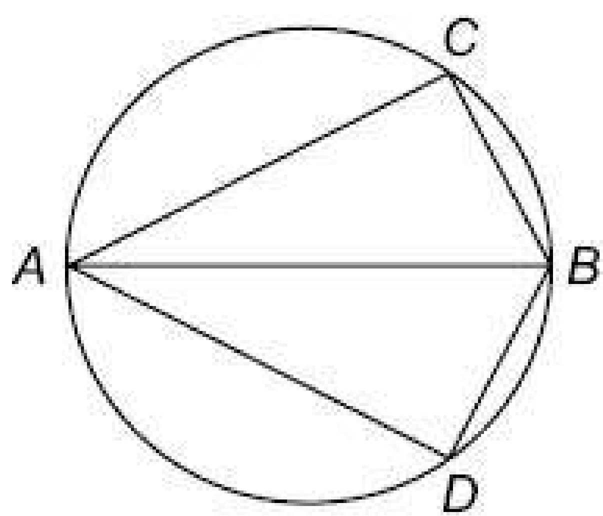
8. What is the area that can be grazed by the cow if the length of the rope is 8 m ?

(a) $134p\frac{1}{3}$ sq. m
(b) $121p$ sq. m
(c) $132p$ sq. m
(d) $\frac{176\pi}{3}$ sq. m

9. What is the area that can be grazed by the cow if the length of the rope is 12 m?

(a) $134p\frac{1}{3}$ sq. m
(b) $121p$ sq. m
(c) $132p$ sq. m
(d) $\frac{176\pi}{3}$ sq. m

10. *AB* is the diameter of the given circle, while points *C* and *D* lie on the circumference as shown. If *AB* is 25 cm, *AC* is 20 cm and *BD* is 7 cm, find the area of the quadrilateral *ACBD*.



(a) 160 sq. cm
(b) 240 sq. cm
(c) 320 sq. cm
(d) None of these

11. The value of each of a set of coins varies as the square of its diameter, if its thickness remains

constant, varies as the thickness, if the diameter remains constant. If the diameter of two coins are in the ratio 3 : 4, what should be the ratio of their thicknesses be if the value of the second is four times that of the first?

(a) 9 : 16
(b) 4 : 9
(c) 16 : 9
(d) 9 : 4

12. If a, b and care the sides of a triangle, and $a^2 + b^2 + c^2 = bc + ca + ab$, then the triangle is:
(a) equilateral
(b) isosceles
(c) right-angled
(d) obtuse-angled

Directions for Questions 13 and 14: Use the following information: *XYZ* is an equilateral triangle in which *Y* is 2 km from *X*. Safdar starts walking from *Y* in a direction parallel to *XZ* and stops when he reaches a point *M* directly east of *Z*. He, then, reverses direction and walks till he reaches a point *N* directly south of *Z*.

13. Then *M* is
(a) 3 km east and 1 km north of *X*
(b) 3 km east and $\sqrt{3}$ km north of *X*
(c) $\sqrt{3}$ km east and 1 km south of *X*
(d) $\sqrt{3}$ km west and 3 km north of *X*

14. The total distance walked by Safdar is:
(a) 3 km
(b) 4 km
(c) $2\sqrt{3}$ km
(d) 6 km

15. The sides of a triangle are 5, 12 and 13 units. A rectangle is constructed, which has an area twice the area of the triangle, and has a width of 10 units. Then, the perimeter of the rectangle is:
(a) 30 units
(b) 36 units
(c) 32 units
(d) None of these

16. The diameter of a hollow cone is equal to the diameter of a spherical ball. If the ball is placed at the base of the cone, what portion of the ball will be inside the cone?
(a) 50%
(b) less than 50%
(c) more than 50%
(d) 100%

17. The length of a ladder is exactly equal to the height of the wall it is leaning against. If lower end of the ladder is kept on a platform of height 3 feet and the platform is kept 9 feet away from the wall, the upper end of the ladder coincides with the top of the wall. Then, the height of the wall is:
(a) 12 feet
(b) 15 feet
(c) 18 feet
(d) 16 feet

18. A cube of side 12 cm is painted red on all the faces and then cut into smaller cubes, each of side 3 cm. What is the total number of smaller cubes having none of their faces painted?

(a) 16
(b) 8
(c) 12
(d) None of these

19. From a circular sheet of paper with a radius 20 cm, four circles of radius 5 cm each are cut out. What is the ratio of the cut to the uncut portion?

(a) 1 : 3
(b) 4 : 1
(c) 3 : 1
(d) 4 : 3

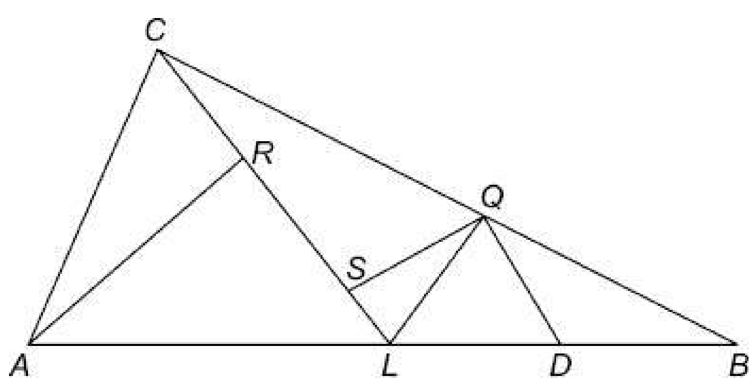
20. Four friends Mani, Sunny, Honey & Funny start from four towns Mindain, Sindain, Hindain and Findain respectively. The four towns are at the four corners of an imaginary rectangle. They meet at a point which falls inside this imaginary rectangle. At that point three of them (Mani, Sunny and Honey had travelled distances of 40 m, 50 m and 60 m respectively). The maximum distance that Funny could have travelled is near about:

(a) 67 m
(b) 53 m
(c) 24 m
(d) Cannot be determined

21. Each side of a given polygon is parallel to either the X or the Y axis. A corner of such a polygon is said to be convex if the internal angle is 90° or concave if the internal angle is 270°. If the number of convex corners in such a polygon is 29, the number of concave corners must be

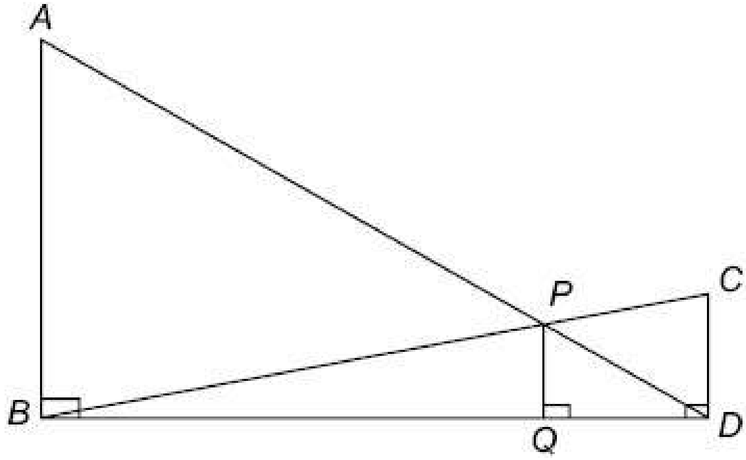
(a) 20
(b) 0
(c) 25
(d) 26

22. In the figure (not drawn to scale) given below, L is a point on AB such that $AL : LB = 4 : 3$. LQ is parallel to AC and QD is parallel to CL. In DARC, –ARC = 90°, and in D LQS, –LSQ = 90°. What is ratio $AL : LD$?



(a) 3 : 7
(b) 4 : 3
(c) 7 : 3
(d) 8 : 3

23. In the diagram given below, –ABC = –CDB = –PQD = 90°. If $AB : CD = 3 : 1$, the ratio of $CD : PQ$ is:



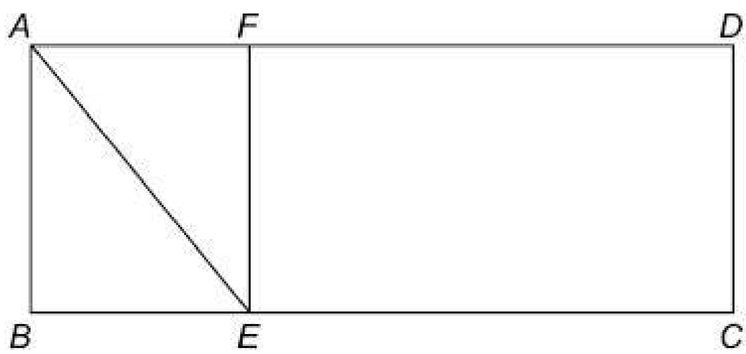
(a) 1 : 0.666

(b) 1 : 0.75

(c) 1 : 0.54

(d) None of the above

24. In the figure given below, *ABCD* is a rectangle. The area of the isosceles right triangle *ABE* = 9 cm^2. *EC* = 3(*BE*). The area of *ABCD* (in cm^2) is:



(a) 27

(b) 36

(c) 54

(d) 72

25. The length of the common chord of two circles of radii 15 cm and 20 cm whose centres are 25 cm apart, is (in cm):

(a) 24

(b) 10

(c) 12

(d) 20

Review Test 2

1. A square tin sheet of side 24 inches is converted into a box with open top in the following steps: The sheet is placed horizontally. Then, equal sized squares, each of side x inches, are cut from the four corners of the sheet. Finally, the four resu1ting sides are bent vertically upwards in the shape of a box. If x is an Integer, then what value of x maximizes the volume of the box?

   (a) 6 (b) 8

   (c) 2 (d) 4

2. Let S_1 be a square of side a. Another square S_2 is formed by joining the mid-points of the sides of S_1. The same process is applied to S_2 to form yet another square S_3, and so on. If $A_1, A_2, A_3, \ldots\ldots$ be the areas and $P_1, P_2, P_3, \ldots\ldots$ be the perimeters of $S_1, S_2, S_3, \ldots\ldots$, respectively, then the ratio $\frac{A_1 + A_2 + A_3 + \ldots}{P_1 + P_2 + P_3 + \ldots}$ equals:

   (a) $1/2(1 + \sqrt{2})$ (b) $a/2(2 - \sqrt{2})$

   (c) $a/2(2 + \sqrt{2})$ (d) $a/2(1 + 2\sqrt{2})$

3. There are two concentric circles such that the area of the outer circle is four times the area of the inner circle. Let A, B and C be three distinct points on the perimeter of the outer circle such that AB and AC are tangents to the inner circle. If the area of the triangle ABC is $\frac{9\sqrt{3}}{\pi}$ then what would be the area of the outer circle?

   (a) 9/4 (b) 9/*p*

   (c)1. (d) None of these

4. Four horses are tethered at four corners of a square plot of side 42 meters so that the adjacent horses can just reach one another. There is a small circular pond of area 20 m^2 at the center. Find the ungrazed area.

   (a) 378 m^2 (b) 388 m^2

   (c) 358 m^2 (d) 368 m^2

5. A square, whose side is 2 meters, has its corners cut away so as to form an octagon with all sides equal. Then length of each side of the octagon, in metres is:

   (a) $\frac{\sqrt{2}}{\sqrt{2}+1}$ (b) $\frac{2}{\sqrt{2}+1}$

   (c) $\frac{2}{\sqrt{2}-1}$ (d) $\frac{\sqrt{2}}{\sqrt{2}-1}$

6. A rectangular pool 30 metres wide and 50 metres long is surrounded by a walkway of uniform width. If the area of the walkway is 516 square meters, how wide, in metres, is the walkway?

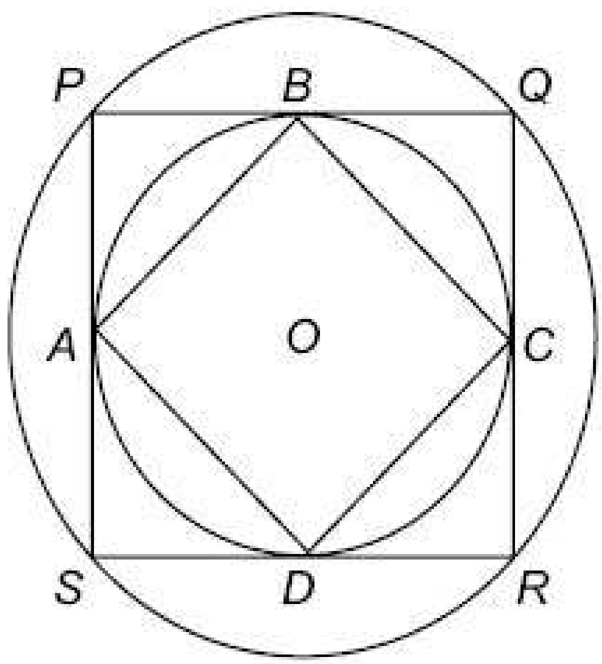
   (a) 1 (b) 2

   (c) 3 (d) None of these

7. A farmer has decided to build a wire fence along one straight side of his property. For this purpose he has planned to place several fence-posts at 12 m intervals, with posts fixed at both ends of the side. After he bought the posts and wire, he found that the number of posts he had bought was 5 less than required. However, he discovered that the number of posts he had bought would be just sufficient if he spaced them 16 m apart. What is the length of the side of his property and how many posts did he buy?

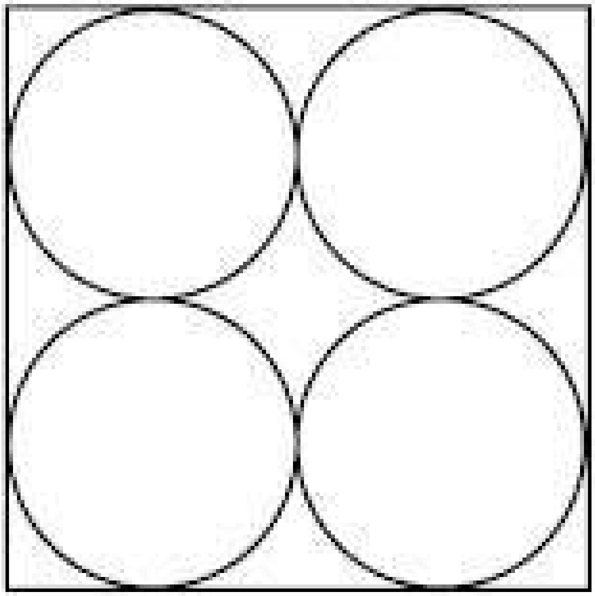
(a) 200 m, 15
(b) 200 m, 16
(c) 240 m, 15
(d) 240 m, 16

8. The figure below shows two concentric circles with centre *O*. *PQRS* is a square inscribed in the outer circle. It also circumscribes the inner circle, touching it at points *B*, *C*, *D* and *A*. What is the ratio of the perimeter of the po1ygon *ABCD* to that of the outer circle?



(a) 4/*p*
(b) 2/3*p*
(c) 2/*p*
(d) 1/*p*

9. Four identical coins are placed in a square. For each coin, the ratio of area to circumference is same as the ratio of circumference to area. Then, find the area of the square that is covered by the coins.



(a) 16*p* (b) 32*p*

(c) 8*p* (d) Insufficient data

10. The sum of the areas of two circles, which touch each other externally, is 153*p*. If the sum of their radii is 15, the ratio of the smaller to the larger radius is:

(a) 1 : 4 (b) 1 : 2

(c) 1 : 3 (d) None of these

11. In D*ABC*, points *P*, *Q* and *R* are the mid-points of sides *AB*, *BC* and *CA* respectively. If area of D*ABC*, is 40 sq units, find the area of D*PQR*.

(a) 10 sq. units (b) $10\sqrt{3}$ sq. units

(c) 5 sq. units (d) None of these

12. A wooden box (open at the top) of thickness 0.5 cm, length 21 cm, width 11 cm and height 6 cm is painted on the inside. The expenses of painting are ` 140. What is the rate of painting per square centimeter?

(a) ` 1.4 (b) ` 0.5

(c) ` 1 (d) ` 2

13. Which one of the following cannot be the ratio of angles in a right-angled triangle?

(a) 1 : 2 : 3 (b) 1 : 1 : 2

(c) 1 : 3 : 5 (d) None of these

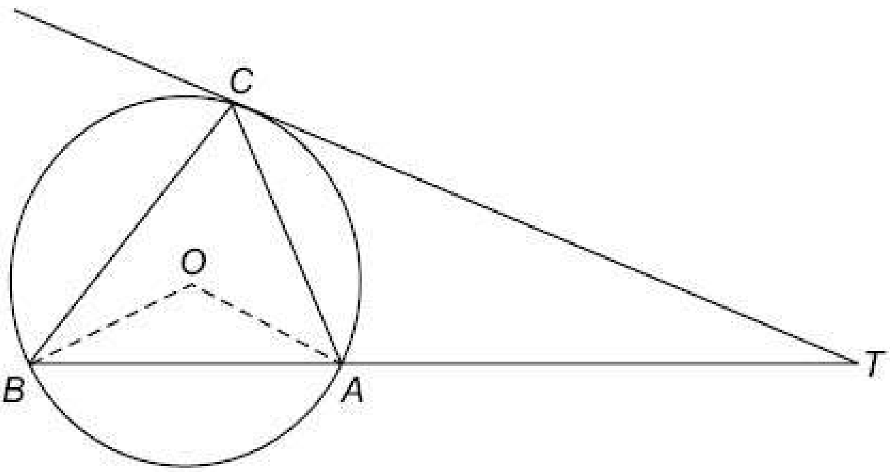
14. A slab of ice 10 inches in length, 15 inches in breadth, and 3 inches thick was melted and re-solidified in the form of a rod of 10 inches diameter. The length of such a rod, in inches, is nearest to:

(a) 5 (b) 5.7

(c) 4.5 (d) 5.5

15. In the figure given below (not drawn to scale), *A*, *B* and *C* are three points on a circle with centre *O*. The chord *BA* is extended to a point *T* such that *CT* becomes a tangent to the circle at point *C*. If

–*A TC* = 20° and –*ACT* = 40°, then the angle –*BOA* is:



(a) 160°

(b) 150°

(c) 140°

(d) Not possible to determine

16. A vertical tower *OP* stands at the center *O* of a square *ABCD*. Let x and y denote the height of the tower *OP* and the side of the square *AB* respectively. Suppose –*APB* = 60° then the relationship between x and y can be expressed as:

(a) $2y^2 = x^2$ (b) $2x^2 = y^2$

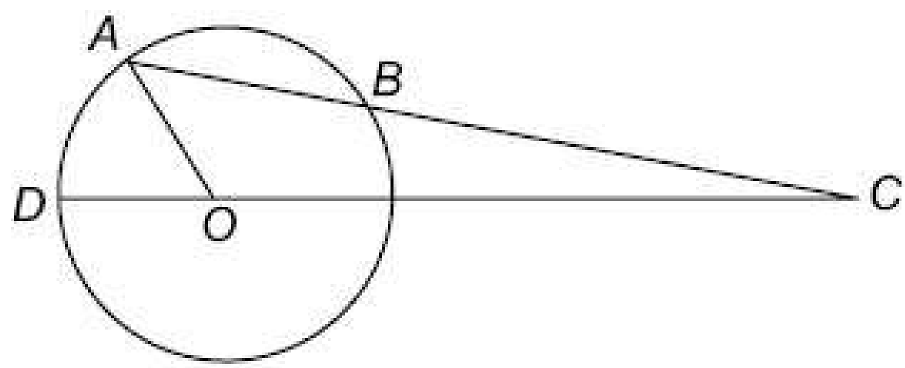
(c) $3y^2 = 2x^2$ (d) $3x^2 = 2y^2$

17. In a triangle *XYZ*, *XY* = 6, *YZ* = 8 and *XZ* = 10. A perpendicular dropped from *Y*, meets the side *XZ* at *M*. A circle of radius *YM* (with center *Y*) is drawn. If the circle cuts *XY* and *YZ* at *P* and *Q* respectively, then *QZ* : *XP* is equal to:

(a) 1 : 1 (b) 2 : 3

(c) 1 : 4 (d) 8 : 3

18. In the figure below, *AB* is the chord of a circle with center *O*. *AB* is extended to *C* such that *BC* = *OB*. The straight line *CO* is produced to meet the circle at *D*. If –*ACD* = 20° and –*AOD* = x degrees then the value of x is:



(a) 20 (b) 40

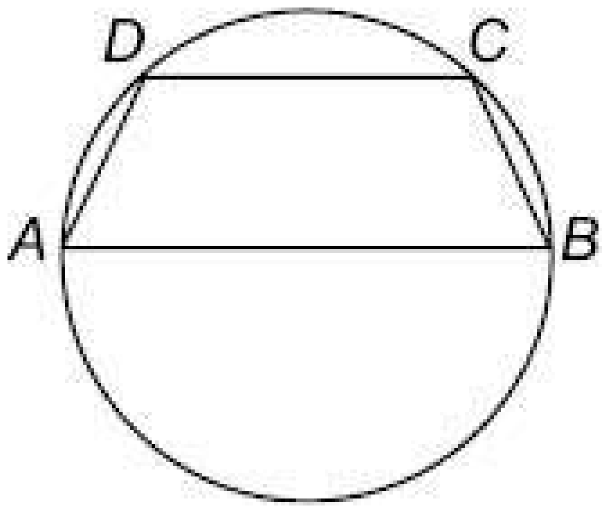
(c) 60 (d) None of these

19. The area of the triangle whose vertices are (a, a), $(a + 1, a + 1)$, $(a + 2, a)$ is:

(a) a^3 (b) 1

(c) $2a$ (d) $2^{1/2}$

20. In the given figure, AB is the diameter of the circle and points C and D are on the circumference such that $–CAD = 40°$ and $–CBA = 60°$. What is the measure of $–ACD$?



(a) 20° (b) 30°

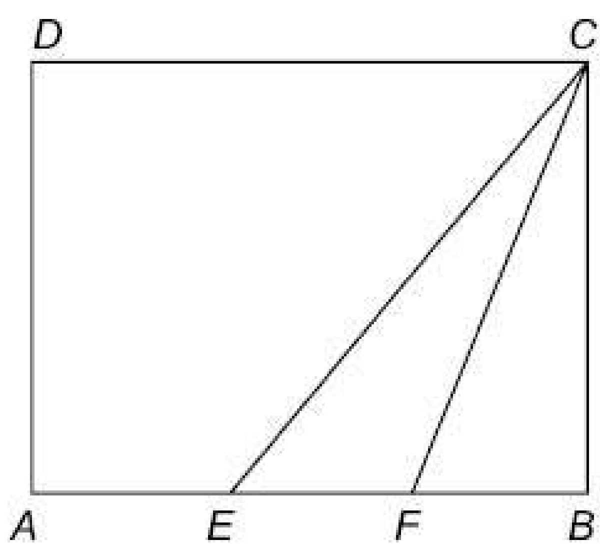
(c) 40° (d) 50°

21. A ladder leans against a vertical wall. The top of the ladder is 12 m above the ground. When the bottom of the ladder is moved 4 m farther away from the wall, the top of the ladder rests against the foot of the wall. What is the length of the ladder?

(a) 16 m (b) 15 m

(c) 20 m (d) 17 m

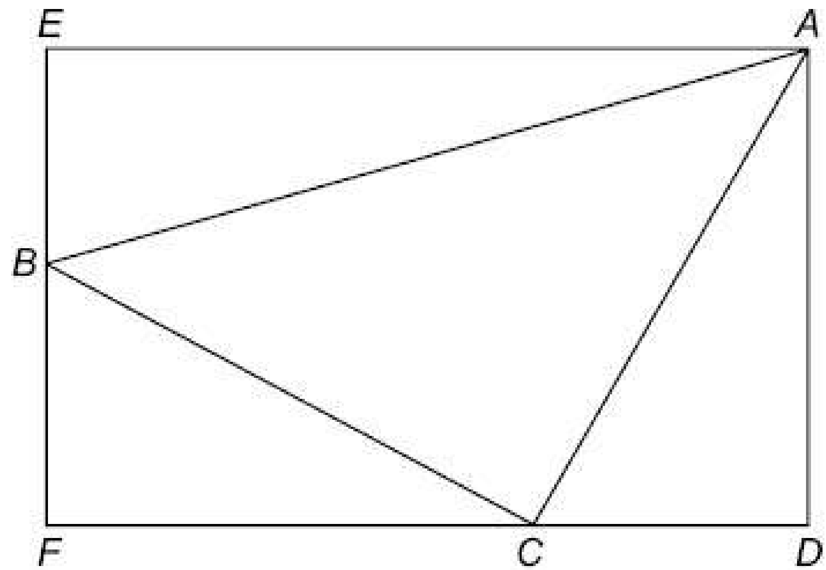
22. In the below diagram, $ABCD$ is a rectangle with $AE = 2EF = 3FB$. What is the ratio of the area of the rectangle to that of the triangle CEF?



(a) 11 : 3 (b) 22 : 3

(c) 11 : 6 (4) None of these

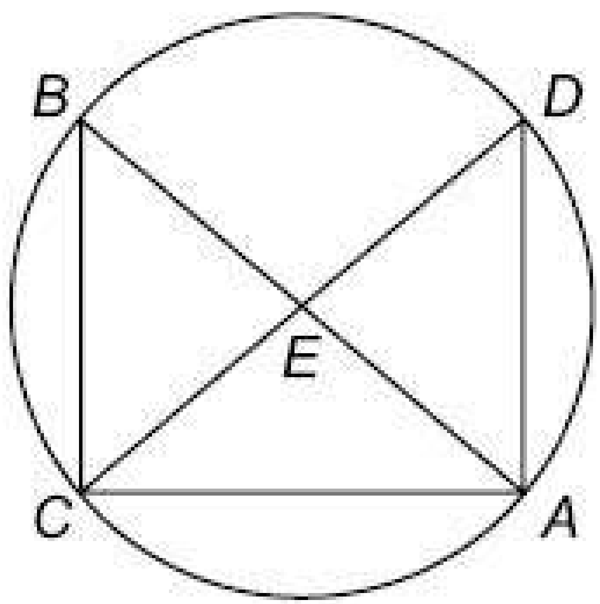
23. In the given figure, *EADF* is a rectangle and *ABC* is a triangle whose vertices lie on the sides of *EADF*. *AE* = 30, *BE* = 12, *CF* = 18 and *BF* = 4. Find the length of the line joining the mid-points of the sides *AB* and *BC*.



(a) 5 (b) 10

(c)15 (d) None of these

24. In the figure, points *A*, *B*, *C* and *D* lie on the circle. *AD* = 44 and *BC* = 11. What is the ratio of the area of D *CBE* to that of the triangle D *ADE*?



(a) 1 : 4 (b) 1 : 16

(c) 1 : 16 (d) Data insufficient

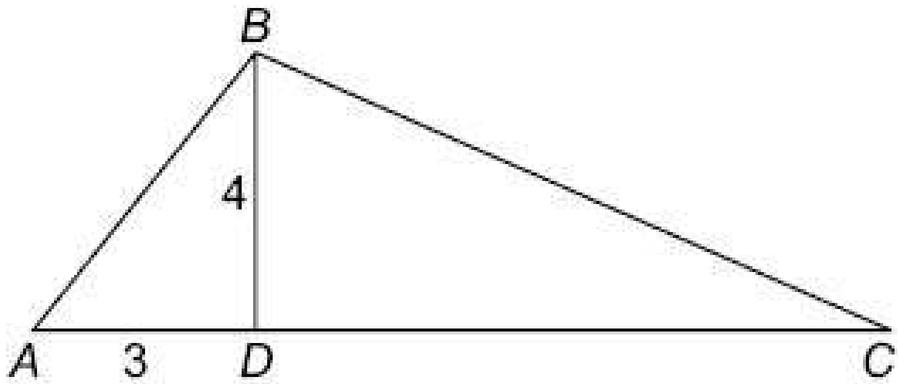
25. In a rectangle, the difference between the sum of the adjacent sides and the diagonal is two-thirds the length of the shorter side. What is the ratio of the shorter to the longer side?

(a) $\sqrt{3}$: 2 (b) 1 : $\sqrt{3}$

(c) 2 : 5 (d) 3 : 4

Review Test 3

1. In the figure given below, if angle $ABC = 90$, and BD is perpendicular to AC, & $BD = 4$ cm and $AD = 3$ cm, what will be the length of BC



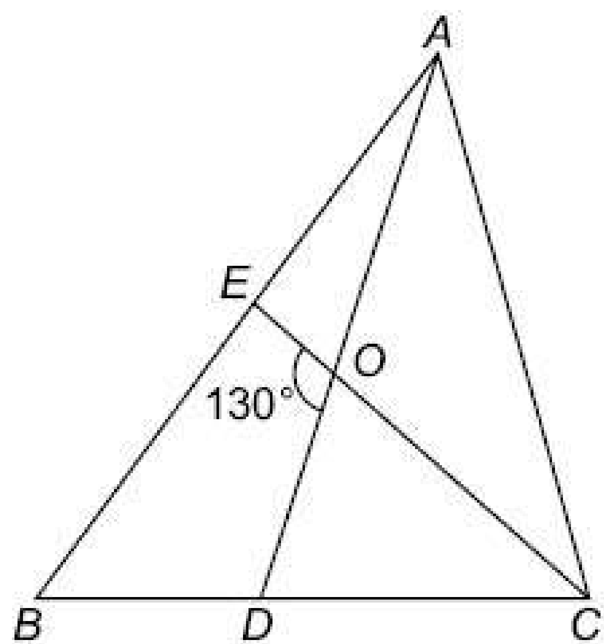
(a) 13 (b) 15

(c) 10 (d) 9

2. In the figure below, the measure of an angle formed by the bisectors of two angles in a triangle ABC is 130 find the measure of an angle B.

(a) 40 (b) 45

(c) 50 (d) 80

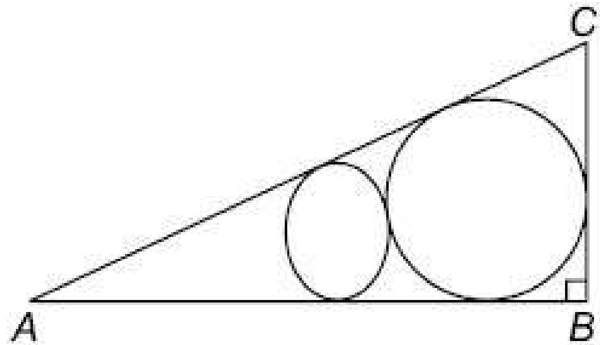


3. The perimeter of right triangle is 36 and the sum of the square of its sides is 450. The area of the right triangle is

(a) 42 (b) 54

(c) 62 (d) 100

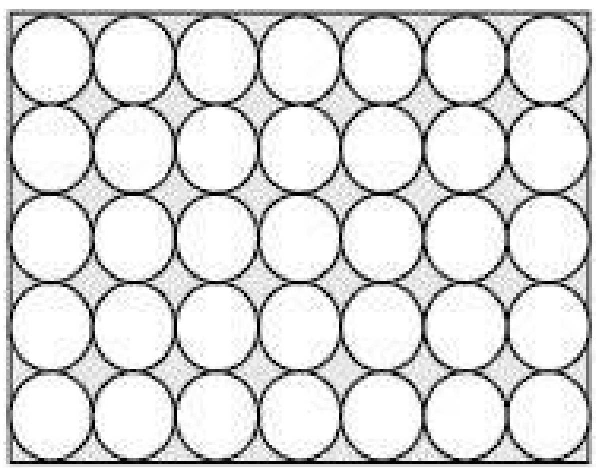
4. The circles are tangent to one another and each circle is tangent to the sides of the right triangle ABC with right angle ABC. If the larger circle has radius 12 and the smaller circle has radius 3, what is the area of the triangle?



(a) 420 (b) 620

(c) 540 (d) 486

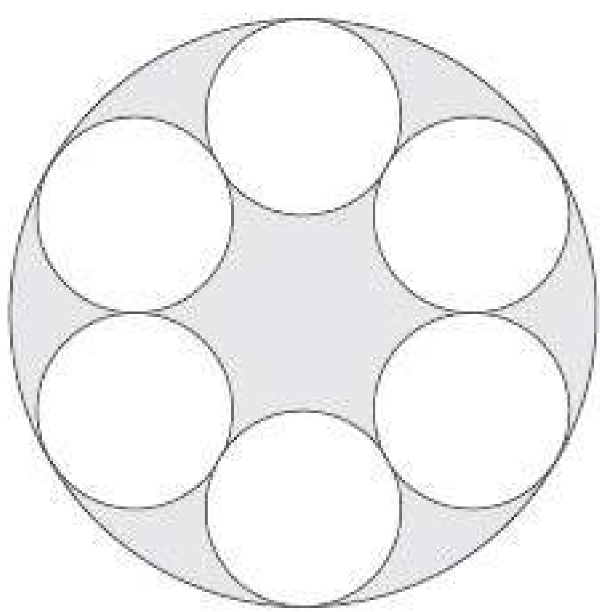
5. All the circles are tangent to one another / or the sides of the rectangle. All circles have radius 1. What is the area of the shaded region to the nearest whole unit, i.e. the region outside all the circles but inside the rectangle?



(a) 27 (b) 28

(c) 29 (d) 30

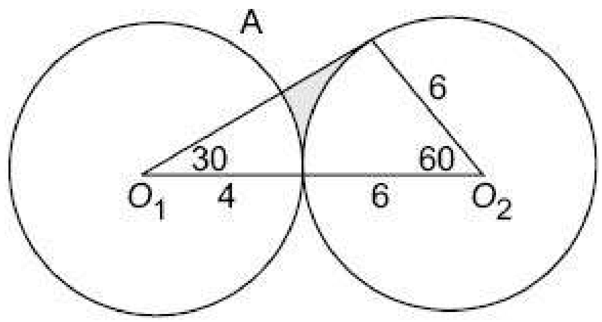
6. Given below are six congruent circles drawn internally tangent to a circle of a radius 21; each smaller circle is also tangent to each of its adjacent circles. Find the shaded area between the circle and the six smaller circles.



(a) 136*p* (b) 196*p*

(c) 180*p* (d) 147*p*

7. In the figure below, O_1 and O_2 are centers of the circles O_1A is the circle centers at O_2.

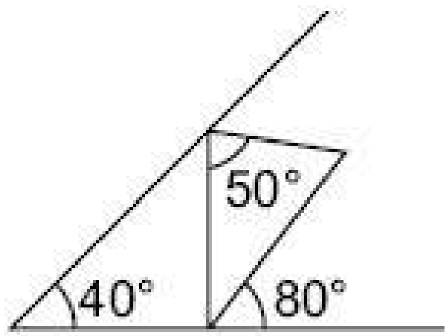


Find the area of the shaded region.

(a) 22 – 7*p* (b) $24\sqrt{2}$ – 7*p*

(c) 18 – 9*p* (d) 24 – *p*22/3

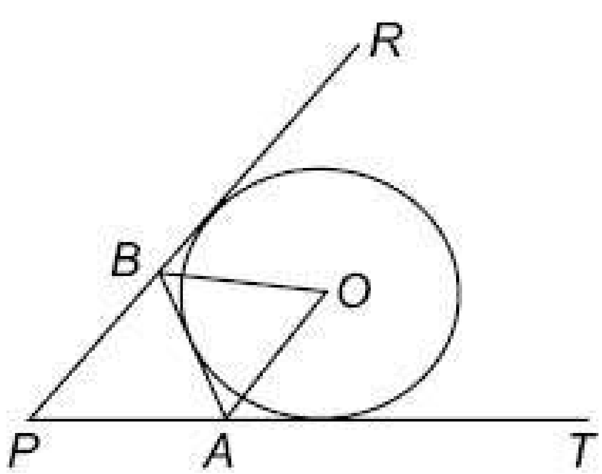
8. In the figure, which of the following is correct?



(a) $x = 60$ (b) $x = 70$

(c) $x = 10$ (d) $x = 90$

9. Triangle *PAB* is formed by three tangents to circle *O* and angle *APB* = 40; then the angle *BOA* equals

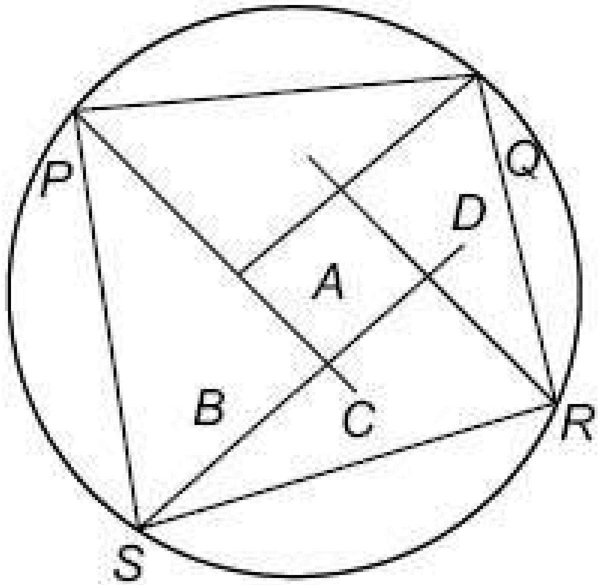


(a) 70 (b) 55

(c) 60 (d) 50

10. *PQRS* is a cyclic quadrilateral. The angle bisector of angle *P*, *Q*, *R* and *S* intersect at *A*, *B*, *C* and

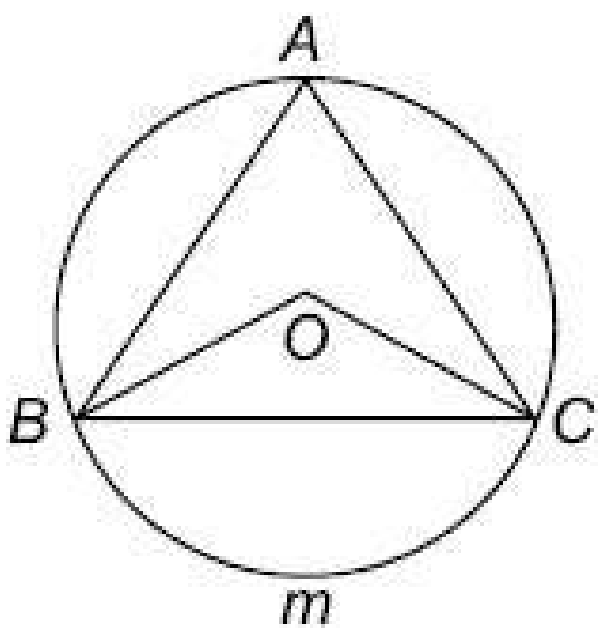
D as shown in the figure below. Then these four points form a quadrilateral *ABCD* is a:



(a) Rectangle (b) square

(c) rhombus (d) cyclic quadrilateral

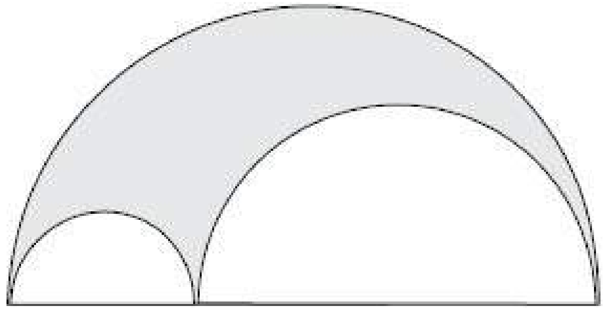
11. In the given figure, *O* is the center of the circle; angle *BOC* = $m°$, angle *BAC* = $n°$.then which of the following is correct?



(a) $m + n = 90$ (b) $m + n = 180$

(c) $2m + n = 180$ (d) $m + 2n = 180$

12. Find the area of shaded portion given that the circles with centers *O* and *O*¢ are 6 cm and 18 cm in diameter respectively and *ACB* is a semi circle.



(a) 54 p cm^2 (b) 27 p cm^2

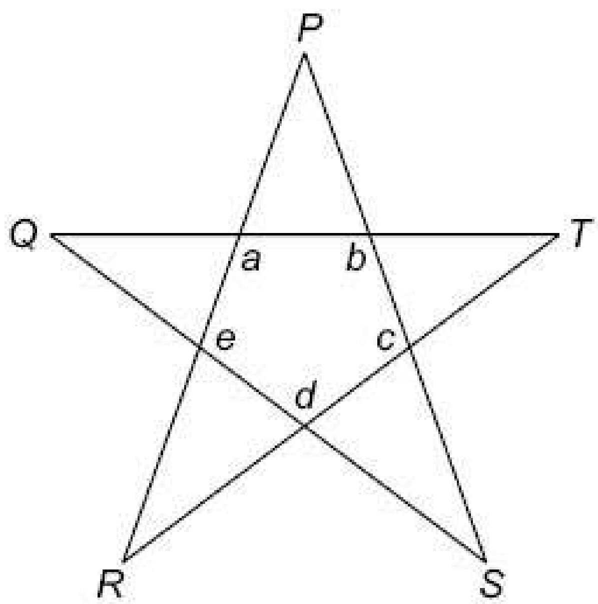
(c) 36 p cm^2 (d) 18 p cm^2

13. There are two spheres and one cube. The cube is inside the bigger sphere and the smaller sphere is inside the cube. Find the ratio of surface areas of the bigger sphere to the smaller sphere?

(a) 3 : 1 (b) 2 : 1

(c) 4 : 1 (d) 2 : 1

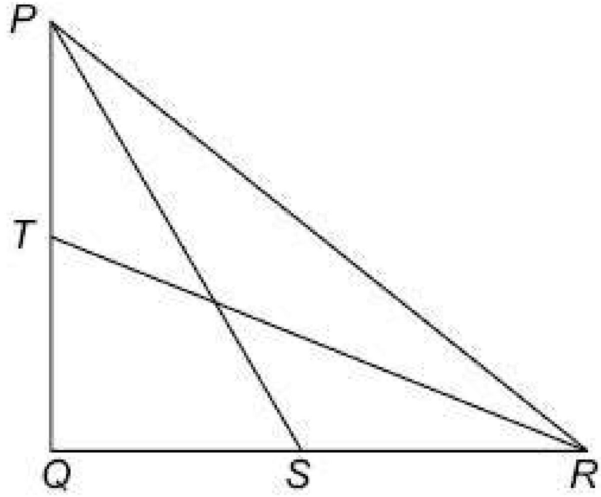
14. In the adjoining figure, a star is shown. What is the sum of the angles P, Q, R, S and T?



(a) 240 (b) 180

(c) 120 (d) Can't be determined

15. In the figure given below PS & RT are the medians each measuring 4 cm. triangle PQR is right angled at Q. what is the area of the triangle PQR?



(a) 5.2 (b) 6.4

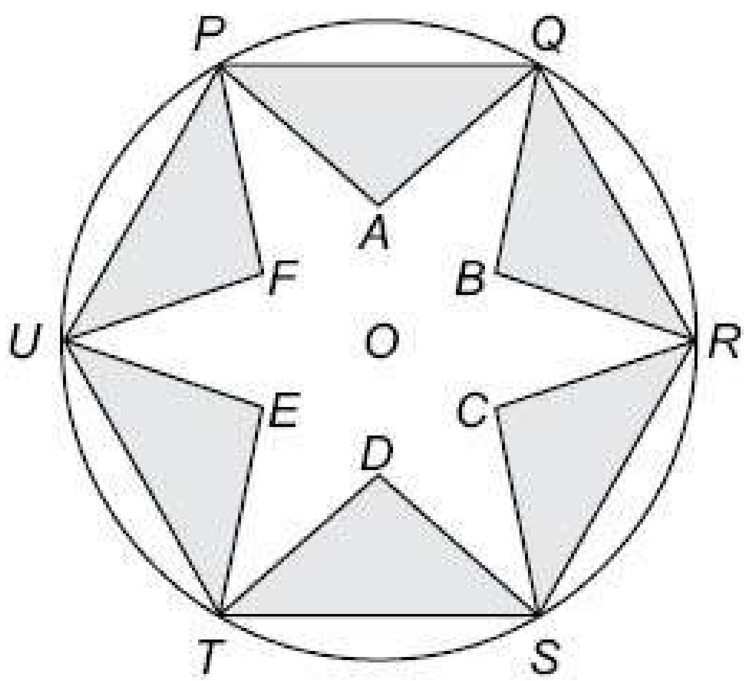
(d) 6.2 (d) 7.2

16. The area of the largest triangle that can be inscribed in a semi circle whose radius is

(a) $2R^2$ (b) $3R^2$

(c) R^2 (d) $3R^2/2$

17. *R* is *O* is the center of the circle having radius (*OP*) = *r*. *PQRSTU* is a regular hexagon and *PAQBRCSDTEUFP* is a regular six pointed star.

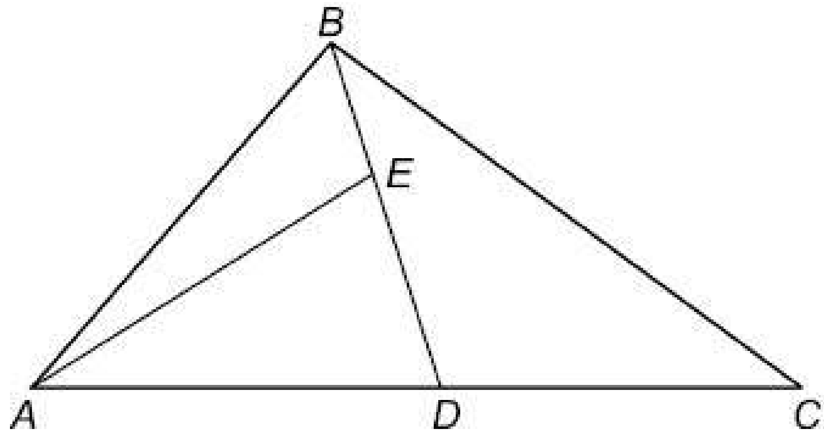


Find the perimeter of hexagon *PQRSTU*.

(a) $12r$ (b) $9r$

(c) $6r$ (d) $8r$

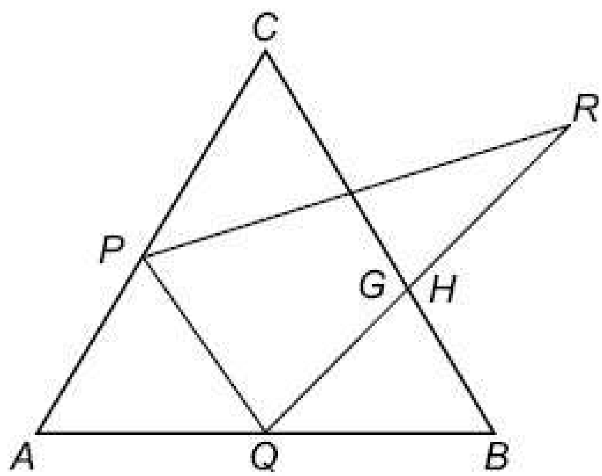
18. In the given figure *ABC* is a triangle in which *AD* = 3*CD* and *E* lies on *BD*, *DE* = 2*BE*. What is the ratio of area of triangle *ABE* and area of triangle *ABC*?



(a) 1/12 (b) 1/8

(c) 1/6 (d) 1/10

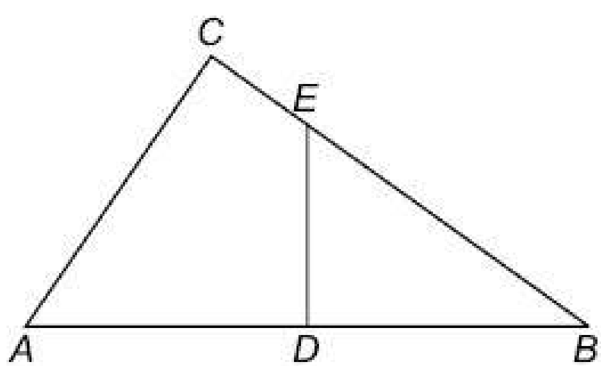
19. In the given figure, P and Q are the mid points of AC and AB. Also, $PG = GR$ and $HQ = HR$. what is the ratio of area of triangle PQR: area of triangle ABC?



(a) 1/2 (b) 2/3

(c) 3/5 (d) 1/3

20. In the given figure, it is given that angle $C = 90$, $AD = DB$, DE is perpendicular to $AB = 20$, and $AC = 12$. The area of quadrilateral $ADEC$ is:



(a) 37(1/2) (b) 75

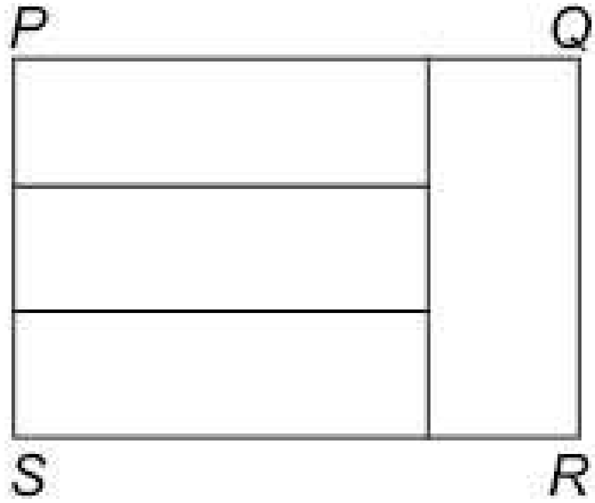
(c) 48 (d) 58(1/2)

Review Test 4

1. Rectangle *PQRS* contains 4 congruent rectangles. If the smaller dimension of one of the small rectangles is 4 units, what is the area of rectangle *PQRS* in square units?

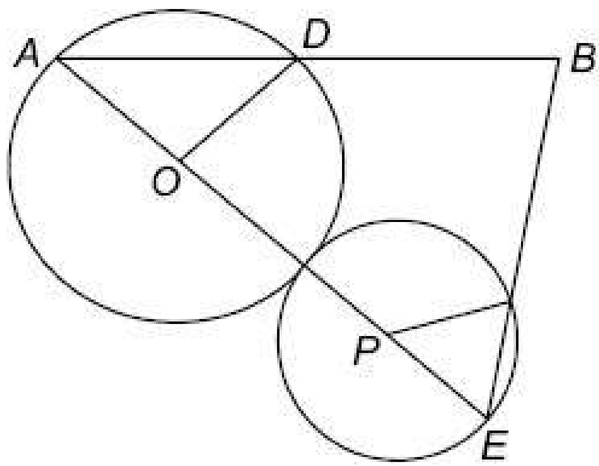
(a) 144 (b) 172

(c) 156 (d) 192



2. If the radii of the circles with centers *O* and *P*, as shown below are 4 and 2 units respectively. Find the area of triangle *ABC*.

Given: angle *DOA* = angle *EPC* = 90



(a) 36 (b) 62

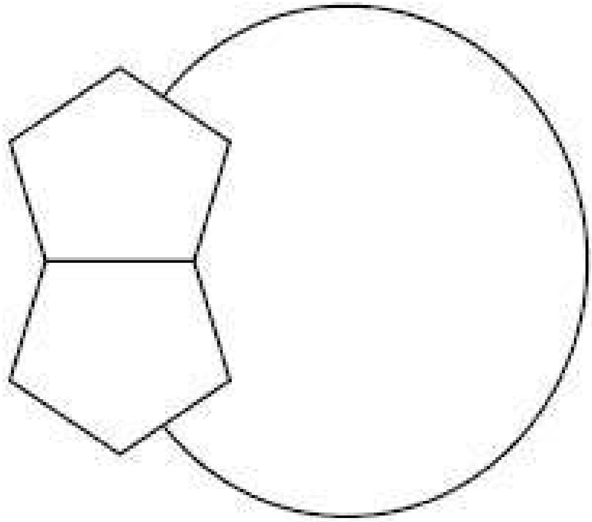
(c) 18 (d) 48

3. A cube of side 16 cm is painted red on all the faces and then cut into smaller cubes, each of side 4 cm. What is the total number of smaller cubes having none of their faces painted?

(a) 16 (b) 8

(c) 12 (d) 24

4. Identical regular pentagons are placed together side by side to form a ring in the manner shown. The diagram shows the first two pentagons. How many are needed to make a full ring?



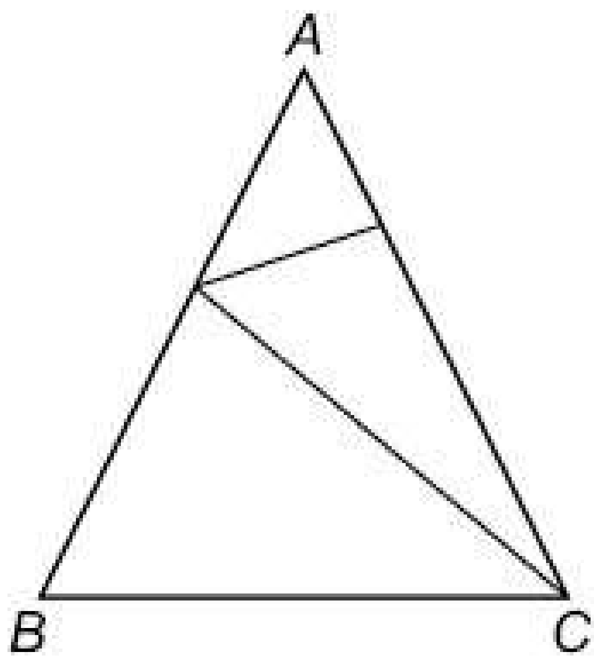
(a) 9 (b) 10

(c) 11 (d) 12

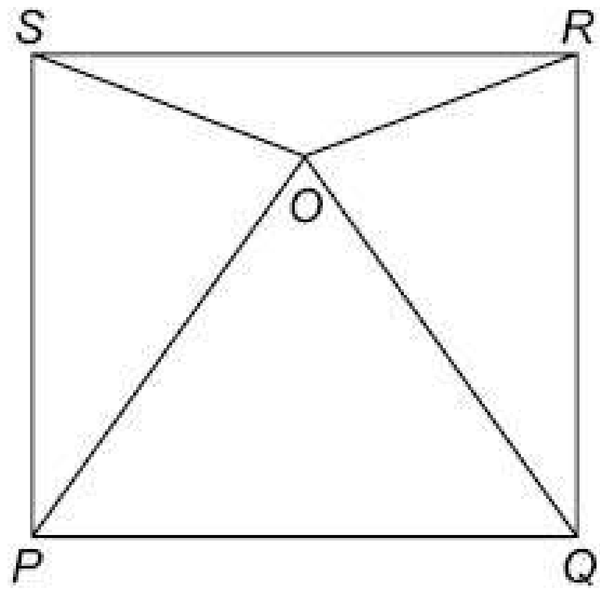
5. Find angle *EBC* + angle *ECB* from the given figure, given *ADE* is an equilateral triangle and angle *DCE* = 20°

(a) 160 (b) 140

(c) 100 (d) 120



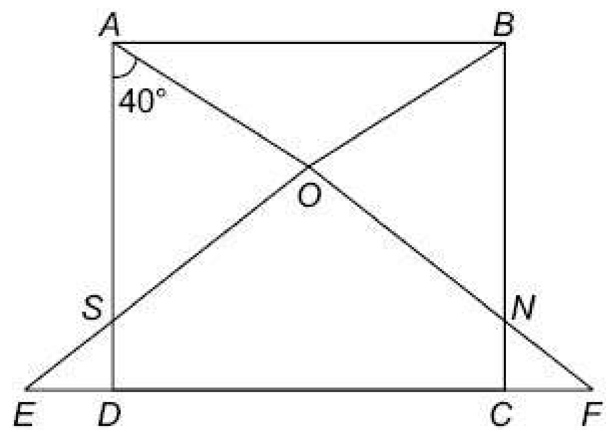
6. *PQRS* is a square and *POQ* is an equilateral triangle. What is the value of angle *SOR*?



(a) 150 (b) 120

(c) 125 (d) 100

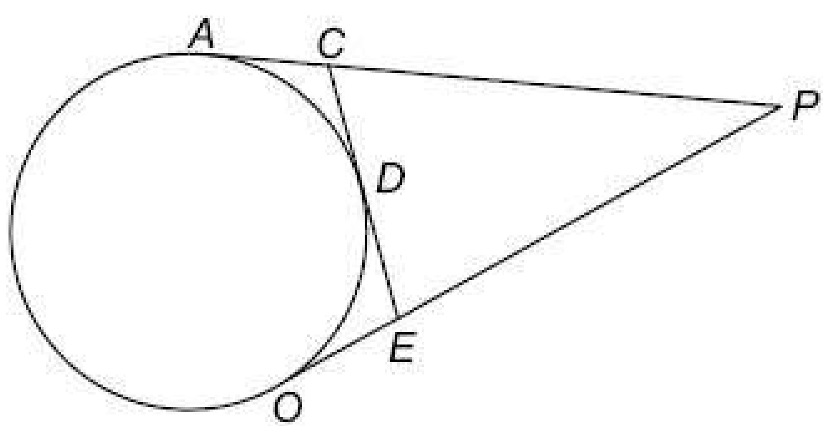
7. In the following figure *ABCD* is a square, angle *DAO* = 40 then find angle *BNO*.



(a) 50 (b) 60

(c) 30 (d) 40

8. From an external point *P*, tangents *PA* and *PB* are drawn to a circle with center *O*. If *CO* is the tangent to the circle at a point *E* and *PA* = 14 cm, find the perimeter of D*CPD*.



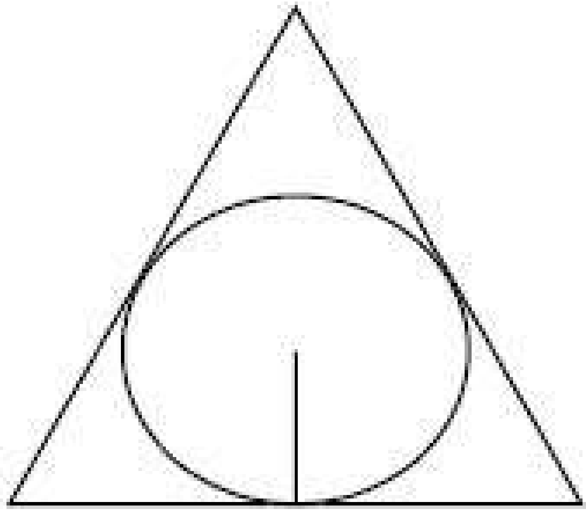
(a) 21 (b) 28

(c) 24 (d) 25

9. A circle is inscribed in an equilateral triangle; the radius of the circle is 2 cm. Find the area of triangle.

(a) $12\sqrt{3}$ (b) $15\sqrt{3}$

(c) $12\sqrt{2}$ (d) $18\sqrt{3}$

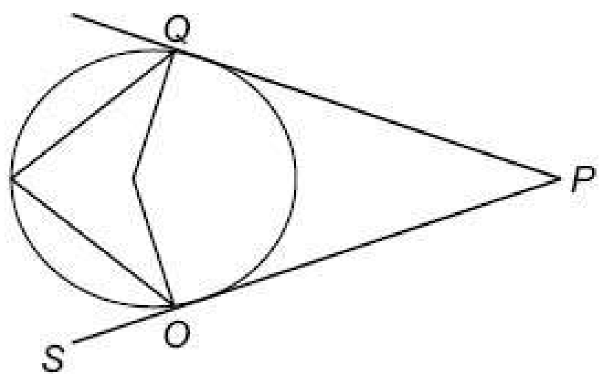


10. If interior angle of a regular polygon is 168∞, then find no. of sides in that polygon.

(a) 10 (b) 20

(c) 30 (d) 25

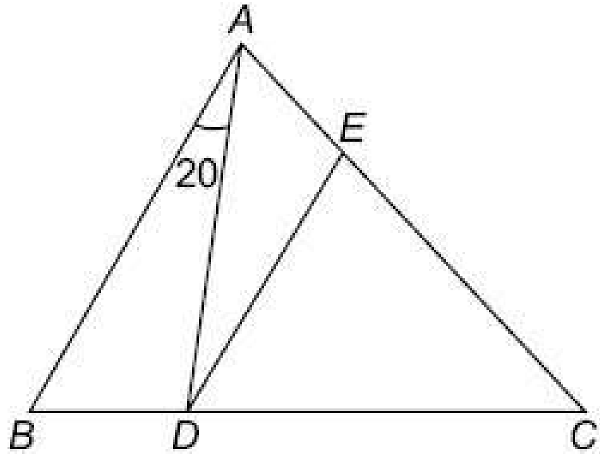
11. In the given figure, *PQ* and *PR* are two tangents to the circle, whose center is *O*. If angle *QPR* = 40∞, find angle *QSR*.



(a) 60 (b) 70

(c) 80 (d) 50

12. In the figure *AB*\\ *DE*, angle *BAD* = 20∞ and angle *DAE* = 30∞, and *DE* = *EC*. Then *x* equals:

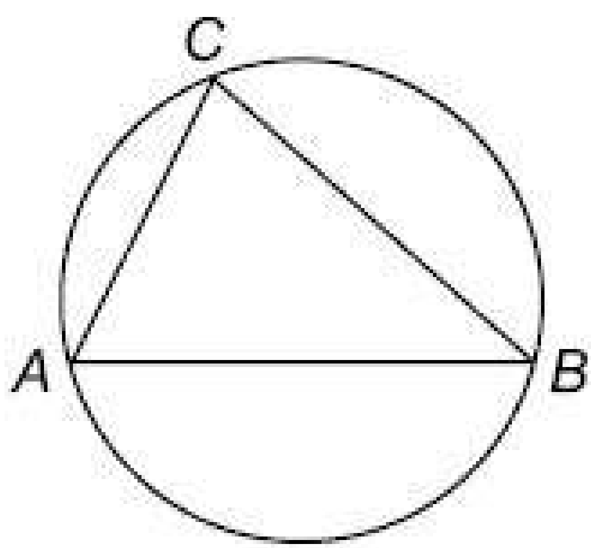


(a) 60
(b) 65
(c) 75
(d) 70

13. There is an equilateral triangle of side 32 cm. The mid-points of the sides are joined to form another triangle, whose mid-points are again joined to form still another triangle. This process is continued for 'n' number of times. The sum of the perimeters of all the triangles is 180 cm. Find the value of n.

(a) 4
(b) 5
(c) 8
(d) 3

14. There is a circle of diameter AB and radius 26 cm. If chord CA is 10 cm long, find the ratio of area of triangle ABC to the remaining area of circle.



(a) 0.60
(b) 0.30
(c) 0.29
(d) 0.52

15. Ram Singh has a rectangular plot of land of dimensions 30 m * 40 m. He wants to construct a unique swimming pool which is in the shape of an equilateral triangle. Find the area of the largest swimming pool which he can have?

(a) $300\sqrt{3}$ sq cm
(b) $225\sqrt{3}$ sq cm
(c) 300 sq cm
(d) 225÷ sq cm

16. The perimeter of a triangle is 105 cm. The ratio of its altitudes is 3 : 5 : 6. Find the sides of the triangle.

(a) 72, 46, 36
(b) 62, 28, 41
(c) 30, 60, 25
(d) 50, 30, 25

17. Rizwan gave his younger sister a rectangular sheet of paper. He halved it by folding it at the mid point of its longer side. The piece of paper again became a rectangle whose longer and shorter sides were in the same proportion as the longer and shorter sides of the original rectangle. If the shorter side of the original rectangle was 4 cm, find the diagonal of the smaller rectangle?

(a) $3(3)^{1/2}$
(b) $5(3)^{1/2}$
(c) $4(3)^{1/2}$
(d) $2(3)^{1/2}$

18. A rectangular hall, 50 m in length and 75 m in width has to be paved with square tiles of equal size. What is the minimum number of tiles required?

(a) 4
(b) 5
(c) 6
(d) 8

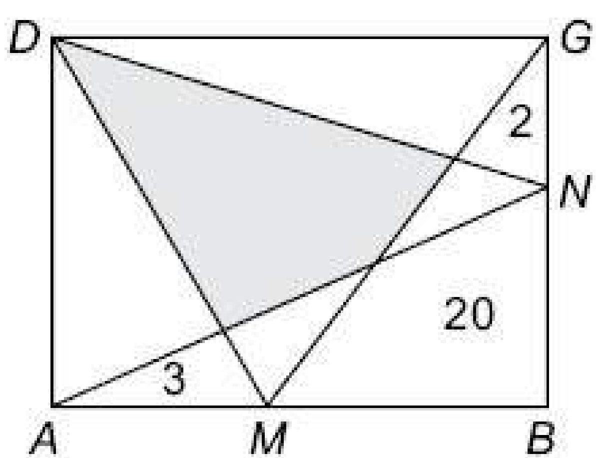
19. In triangle *ABC* we have angle *A* = 100 degree and *B* = *C* = 40. The side *AB* is produced to a point *D* so that *B* lies between *A* and *D* and *AD* = *BC*. Then angle *BCD* =?

(a) 20°
(b) 10°
(c) 30°
(4) 40°

20. The sides of a cyclic quadrilateral are 9,10,12 and 16. If one of its diagonals is 14, then find the other diagonal?

(a) 16
(b) 17
(c) 18
(d) 19

21. Segments starting with points *M* and *N* and ending with vertices of the rectangle *ABCD* divide the given figure into eight parts (see the figure). The areas of three parts of the rectangle are indicated in the picture. What is the area of the shaded region?



(a) 25
(b) 40

(c) 29 (d) 20

ANSWER KEY

Review Test 1

1. (b)	2. (a)	3. (c)	4. (c)
5. (b)	6. (b)	7. (d)	8. (d)
9. (c)	10. (d)	11. (b)	12. (a)
13. (b)	14. (d)	15. (c)	16. (b)
17. (b)	18. (b)	19. (a)	20. (a)
21. (c)	22. (c)	23. (b)	24. (d)
25. (a)			

Review Test 2

1. (d)	2. (c)	3. (c)	4. (c)
5. (a)	6. (c)	7. (d)	8. (c)
9. (a)	10. (a)	11. (a)	12. (c)
13. (c)	14. (b)	15. (a)	16. (b)
17. (d)	18. (c)	19. (b)	20. (a)
21. (c)	22. (b)	23. (b)	24. (b)
25. (b)			

Review Test 3

1. (b)	2. (d)	3. (b)	4. (d)
5. (d)	6. (d)	7. (d)	8. (d)
9. (a)	10. (d)	11. (a)	12. (b)
13. (a)	14. (b)	15. (b)	16. (c)
17. (c)	18. (a)	19. (a)	20. (d)

Review Test 4

1. (d)	2. (a)	3. (b)	4. (b)
5. (c)	6. (a)	7. (d)	8. (b)
9. (a)	10. (c)	11. (b)	12. (b)
13. (a)	14. (c)	15. (a)	16. (d)
17. (c)	18. (c)	19. (b)	20. (c)
21. (a)			