

# Chapter 1

## Introduction to Control Systems

### CHAPTER HIGHLIGHTS

- Basic Definitions
- Open Loop Control System (OLCS)
- Closed Loop Control System (CLCS)
- Other Classification of Control Systems
- Transfer Function
- Poles and Zeros of Transfer Function
- Sensitivity Analysis
- Effect of Feedback on Disturbance
- Block Diagram
- Signal Flow Graph
- Mason's Gain Formula

### BASIC DEFINITIONS

**System** A set of components or elements connected in a proper sequence to perform a specific task.

**Controller** Controller is an element or subsystem inside or outside the system which regulates the operating condition or response of the system.

**Disturbance** Disturbance is a signal that tends to adversely affect the value of the output of the system. If a disturbance is generated within the system, it is called internal disturbance, while an external disturbance is generated outside the system.

**Control system** A set of components connected in a proper sequence to form a system which provides the desired response.

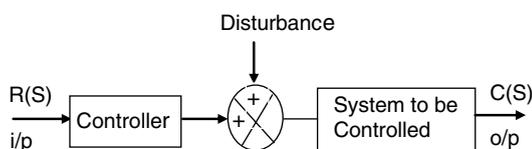
Control system is mainly classified into two types:

- Open-loop control system.
- Closed-loop control system.

### OPEN-LOOP CONTROL SYSTEM (OLCS)

Any physical system in which the output is controlled directly by a controller or actuator without help of feedback is known as open-loop control system. The input of the OLCS is independent of the output.

The output of an open-loop control system is affected not only by input but also by disturbance in it. Open-loop system does not automatically correct the disturbances.



### Advantages

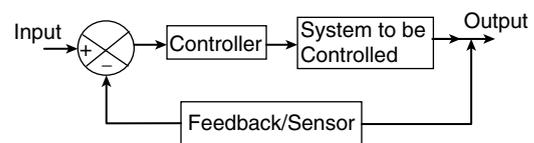
- Simple and easy to design
- Cheap due to less number of components
- Generally, open-loop systems are stable

### Disadvantages

- More sensitive to disturbances and unable to correct the disturbances
- Inaccurate and Unreliable

### CLOSED-LOOP CONTROL SYSTEM (CLCS)

A control system in which input is a function of output in order to maintain the desired value of output is called closed-loop control system.



Since the feedback facilitates the system to automatically correct the system input to meet the desired response, it is also called automatic control system.

### Advantages

- It is accurate and reliable.
- It is less sensitive to disturbances.
- It is accurate even with presence of non-linearities.

## Disadvantages

1. Design is complex and a costly gain of the system.
2. Overall feedback presence is reduced.
3. Feedback may lead to oscillatory response.
4. Improper design of the controller may cause the system to become unstable.

**Table 1.1** Comparison between open-loop and closed-loop system

Open-loop control system	Closed-loop control system
1 Input is independent of the output.	1 Input is dependent on output.
2 Design is easy and cheap.	2 Design is complex and costly.
3 It is inaccurate and unreliable.	3 It is accurate and reliable.
4 It is more sensitive to disturbances.	4 It is less sensitive to disturbances.
5 Feedback does not exist.	5 Feedback exists and reduces the gain of the system. It may also cause oscillations.
6 It is generally stable in operation.	6 Improper design of the controller may cause unstable operation of the system.

## OTHER CLASSIFICATION OF CONTROL SYSTEMS

Depending on the nature of the system, the control system can be classified into the following types.

1. Linear and non-linear control systems
2. Time-variant and time-invariant systems

Depending on the type of signals present at various parts of a feedback control system, it is classified into the following:

1. Continuous and discrete line control systems

### Linear and Non-linear Control Systems

A system that obeys superposition and homogeneity principle is said to be a linear system.

Let  $x_1(t)$  and  $x_2(t)$  be two inputs to a system and  $y_1(t)$  and  $y_2(t)$  be the corresponding outputs. For arbitrary real constants  $k_1$  and  $k_2$ , for input  $k_1x_1(t) + k_2x_2(t)$ , if the output of the system is given by  $k_1y_1(t) + k_2y_2(t)$ , then the system is said to be linear.

Any system that does not obey superposition and homogeneity principle is said to be non-linear.

Physical systems are, in general, non-linear and analysis of such systems is very complicated. Hence, these systems are usually linearized and analysed using linear techniques.

### Timevariant and Time-invariant Control Systems

A system is said to be time variant if its characteristics explicitly depend upon time.

A time invariant system is one whose output does not depend explicitly on time.

If an input signal  $x(t)$  produces an output  $y(t)$ , then any time-shifted input  $x(t + \delta)$ , results in a time-shifted output  $y(t + \delta)$ , then the system is time invariant.

## Continuous Time and Discrete Time Systems

If the signals in all parts of a control system are functions of time, the system is said to be continuous time control system.

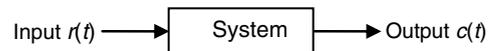
A system in which one or more parts of control systems signals are of the pulse form, it is said to be discrete time control system.

## TRANSFER FUNCTION

Transfer function of a linear time-invariant system is defined as the ratio of the Laplace transform of the output variables to the Laplace transform of the input variables under the assumption that all initial conditions are zero.

(or)

Transfer function is defined as the Laplace transform function of an impulse response of the system when all initial conditions are assumed to be zero.



$$\text{Transfer function} = \frac{L[c(t)]}{L[r(t)]} = \frac{C(s)}{R(s)} \quad \text{with zero initial conditions}$$

Transfer function =  $L[C(t)]$  zero initial conditions and impulse input

### NOTES

1. Transfer function gives mathematical model of all components and individual analysis of various components.
2. Transfer function is independent of input and output of the system.
3. Transfer function is useful in determining the response of the system for any type of input applied.
4. Transfer function is helpful to obtain differential equations related to the system.
5. Transfer function is helpful in stability analysis.

### Limitations of Transfer Function

1. Transfer function is applicable only for linear time-invariant systems.
2. It does not provide any information about the physical structure of the system.
3. Initial condition's effects are completely neglected. Therefore, initial conditions lose their importance.

**Solved Examples**

**Example 1**

The transfer function of the system described by  $5 \frac{d^2 y}{dt^2} + 4 \frac{dy}{dt} = 2 \frac{du}{dt} + 4u$ , with  $u$  as input and  $y$  as output is

- (A)  $\frac{2s+4}{(5s^2+4s)}$
- (B)  $\frac{2s+4}{5s+4}$
- (C)  $\frac{2s+2}{5s^2+4s}$
- (D)  $\frac{2s}{5s^2+4}$

**Solution**

Apply Laplace transform on both sides for the given differential equation with zero initial condition

$$\Rightarrow 5s^2 Y(s) + 4s Y(s) = 2s U(s) + 4 U(s); (5s^2 + 4s) Y(s) = (2s + 4) U(s)$$

$$\frac{Y(s)}{U(s)} = \frac{2s+4}{(5s^2+4s)}$$

**Example 2**

The impulse response of the system is given as  $c(t) = -4e^{-2t} + 6e^{-4t}$ . The step response of the same system for  $t \geq 0$  is equal to

- (A)  $\frac{1}{2}[-1 + e^{-2t}]$
- (B)  $\frac{1}{2}[1 - e^{-2t}]$
- (C)  $\frac{1}{4}[1 - e^{-2t} + 2e^{-4t}]$
- (D)  $\frac{1}{2}[-1 + 4e^{-2t} - 3e^{-4t}]$

**Solution**

Transfer function of the system =  $L \{ \text{Impulse response} \} = \frac{-4}{s+2} + \frac{6}{s+4} = \frac{2s-4}{(s+2)(s+4)}$

Step response of the system  $C(s) = \text{T.F} \times R(s) = \frac{2s-4}{(s+2)(s+4)} \times \frac{1}{s}$

$$c(t) = L^{-1} \left\{ \frac{2s-4}{s(s+2)(s+4)} \right\}$$

$$L^{-1} \left\{ \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4} \right\}$$

$$8A = -4 \Rightarrow A = \frac{-1}{2};$$

$$B = C(s) \times (S+2)$$

at  $S = -2$

$$B = 2 \text{ and}$$

$$C = C(S) \times (S+4)$$

at  $S = -4$

$$C = \frac{-3}{2}$$

$$C(s) = L^{-1} \left\{ \frac{-1}{2s} + \frac{2}{s+2} - \frac{3}{2(S+4)} \right\}$$

$$C(t) = \frac{-1}{2} + 2e^{-2t} - \frac{3}{2}e^{-4t}$$

**Example 3**

A control system is defined by the following differential mathematical relationship

$$9 \frac{d^2 x}{dt^2} + 16 \frac{dx}{dt} + 5x = 12 (1 - e^{-2t})$$

The response of the system as  $t \rightarrow \infty$  is

- (A)  $x = 16$
- (B)  $x = 5$
- (C)  $x = 2.4$
- (D)  $x = -9$

**Solution**

Taking Laplace transform on both sides with zero initial conditions

$$X(s) [9s^2 + 16s + 5] = 12 \left[ \frac{1}{s} - \frac{1}{s+2} \right] = 12 \left[ \frac{2}{s(s+2)} \right]$$

$$\Rightarrow X(s) = \frac{24}{s(s+2)(9s^2+16s+5)}$$

Response of the system  $x(t)$  at  $t \rightarrow \infty = \lim_{t \rightarrow \infty} x(t)$

$$= \lim_{s \rightarrow 0} s X(s) \quad [\because \text{Final value theorem}]$$

$$= \lim_{s \rightarrow 0} s \left[ \frac{24}{s(s+2)(9s^2+16s+5)} \right] = \frac{12}{5} = 2.4$$

Response of the system at  $t \rightarrow \infty = 2.4$

**Example 4**

The impulse response of an initially relaxed linear system is  $e^{-3t}u(t)$ . To produce a response of  $te^{-3t}u(t)$ , the input must be equal to

- (A)  $3e^{-t}u(t)$
- (B)  $\frac{1}{3}e^{-3t}u(t)$
- (C)  $e^{-3t}u(t)$
- (D)  $te^{-3t}u(t)$

**Solution**

Impulse response =  $e^{-3t}u(t)$

Transfer function =  $L [\text{Impulse response}]$

$$= L [e^{-3t}u(t)] = \frac{1}{s+3}$$

Response of the system

$$[C(s)] = L [te^{-3t}u(t)] = \frac{1}{(s+3)^2}$$

$$\text{Input } R(s) = \frac{C(s)}{\text{Transfer Function}}$$

$$= \frac{1/(s+3)^2}{1/(s+3)} = \frac{1}{s+3}$$

$$\text{Input } r(t) = L^{-1}[R(s)] = e^{-3t}u(t)$$

**Example 5**

A linear time-invariant system has an impulse response  $e^{2t}$ ,  $t > 0$ . If the initial conditions are zero and input is  $e^{8t}$ , then output for  $t > 0$  is

- (A)  $\frac{1}{6}[e^{8t} - e^{2t}]$       (B)  $e^{5t}$   
 (C)  $e^{8t} + e^{2t}$       (D) None of these

**Solution**

Transfer function =  $L$  [Impulse response]

$$= L [e^{2t}] = \frac{1}{s-2}$$

Input applied  $R(s) = L[e^{8t}] = \frac{1}{s-8}$

Response of the system = T.F  $\times$  R(s)

$$= \frac{1}{s-2} \times \frac{1}{s-8}$$

Output of the system

$$C(t) = L^{-1} \left[ \frac{1}{(s-2)(s-8)} \right] = L^{-1} \left[ \frac{\frac{1}{6}}{s-8} - \frac{\frac{1}{6}}{s-2} \right]$$

$$C(t) = \left( \frac{e^{8t} - e^{2t}}{6} \right)$$

**Example 6**

Let  $x(t)$  be the input to a linear, time-invariant system. The required output is  $8x(t - 4)$ . The transfer function of the system should be

- (A)  $8e^{j8\pi f}$       (B)  $4e^{-j8\pi f}$   
 (C)  $8e^{-j8\pi f}$       (D)  $4e^{j8\pi f}$

**Solution**

Required output of the system

$$y(t) = 8x(t - 4)$$

$$\Rightarrow Y(s) = 8e^{-4s} X(s)$$

Transfer function

$$\frac{Y(s)}{X(s)} = 8e^{-4s} = 8e^{-4\omega j} [\because s = j\omega]$$

Transfer function =  $8e^{-j8\pi f}$  [ $\because \omega = 2\pi f$ ]

**Poles and Zeros of Transfer Function**

**Transfer Function**

It is defined as the ratio of Laplace transform of the response to the Laplace transform of the excitation or input, with all initial conditions as zero.



$$G(s) = \frac{C(s)}{R(s)}$$

The impulse response of the system

$$H(s) = C(s) = G(s)$$

The transfer function of a linear control system can be expressed as

$$G(s) = \frac{K(s+b_0)(s+b_1)(s+b_2)\dots}{(s+a_0)(s+a_1)(s+a_2)\dots}$$

Characteristic equation  $1 + G(s) H(s) = 0$

$$(s+a_0)(s+a_1)(s+a_2)\dots + k(s+b_0)(s+b_1)(s+b_2)\dots = 0$$

If  $K = 0$

Poles of the system is  $S = -a_0, -a_1, -a_2 \dots$  etc.

If  $K = \infty$

Zeros of the system is

$$s = -b_0, -b_1, -b_2 \dots$$

Where  $k$  is gain factor of the transfer function.

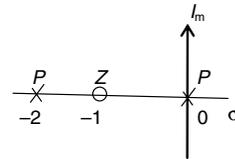
**Example 7**

Consider the unity feedback open-loop system transfer function  $G(s) = \frac{s+1}{s(s+2)}$ . Draw the p-z location and explain about the stability.

**Solution**

Poles are at  $S = 0, -2$

Zero at  $S = -1$



Impulse response of the system  $H(s) = \frac{s+1}{s(s+2)}$

$$h(t) = L^{-1} \left\{ \frac{A}{s} + \frac{B}{s+2} \right\}$$

$$\therefore h(t) = \{A + B.e^{-2t}\}u(t)$$

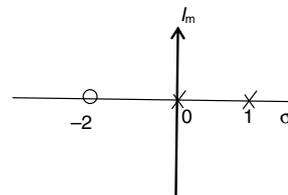
it is a stable system

(exponentially decaying function)

**Example 8**

Consider the impulse response of the system is  $H(s) = \frac{(s+2)}{s(s-1)}$ . Explain about the stability of system.

**Solution**



$$h(t) = \{A + Be^t\}u(t)$$

If  $t \rightarrow \infty$

$h(t) = \infty$  {unbounded for bounded input}

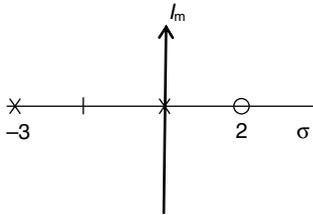
so the system is unstable. [exponentially increasing function]

**Example 9**

If  $H(s) = \frac{(s-2)}{s(s+3)}$ , then the system stability is \_\_\_\_\_

**Solution**

**P-Z location:**



$$h(t) = (A + B.e^{-3t}) u(t)$$

It is an exponentially decaying function  
 If  $t \rightarrow \infty; h(t) = A$   
 For bounded input bounded output  
 Therefore, the system is stable.

**NOTE**

System stability depends on the poles location but not zeros.

If system poles located only in left-hand side of S-plane (non-repeated at origin and Imaginary axis), then the system is called stable system, otherwise, it is unstable.

**SENSITIVITY ANALYSIS**

**Sensitivity**

Sensitivity is the ratio of the percentage change in the function to the percentage change in the parameter.

$$\text{Sensitivity} = \frac{\text{Percentage Change in } F(s)}{\text{Percentage Change in } P(s)}$$

where  $F(s)$  is the function and  $P(s)$  is the parameter.

**Transfer Function Sensitivity with Respect to Parameter Variation**

Transfer function =  $T(s)$ .

Forward path gain =  $G(s)$

Transfer function sensitivity with respect to parameter variation =  $S_G^T$

$$S_G^T = \frac{\frac{\partial T}{T} \times 100}{\frac{\partial G}{G} \times 100} = \frac{\partial T}{\partial G} \times \frac{G}{T}$$

For open-loop system  $S_G^T = 1$

For closed-loop system  $S_G^T = \frac{1}{1+GH}$  (Negative feedback)

**NOTE**

1. The sensitivity of closed-loop system with respect to variation in  $G$  is reduced by a factor  $(1 + GH)$  as compared to that of an open loop system.

**Sensitivity of  $T$  with Feedback Parameters**

Feedback gain =  $H$

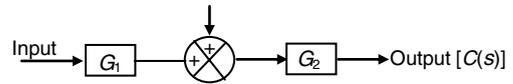
Sensitivity of  $T$  with feedback parameters =  $S_H^T$

$$S_H^T = \frac{\partial T}{\partial H} \times \frac{H}{T} = \frac{-GH}{1+GH}$$

**NOTE**

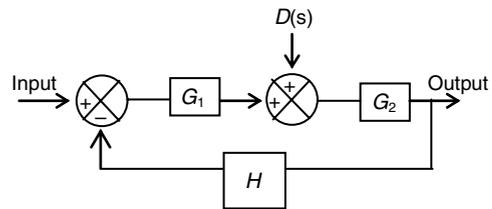
2. Closed-loop system is more sensitive to the feedback parameters variation than that of the forward path parameters variation.

**Effect of Feedback on Disturbance Open-loop System**



Output due to disturbance =  $G_2 D(s)$ .

**Closed-loop System**



Output due to disturbance =  $\frac{G_2}{1 + G_1 G_2 H} \cdot D(s)$ .

**NOTES**

1. Negative feedback reduces the effect of noise on output by a factor of  $1 + G_1 G_2 H$  as compared to that of open loop systems.
2. For positive feedback

$$\text{Output due to disturbance} = \frac{G_2}{1 - G_1 G_2 H} \cdot D(s)$$

Effect of disturbance on output of the system increases as compared to open-loop and negative feedback closed-loop control systems.

**Example 10**

A negative feedback system has an amplifier of gain 10 with  $\pm 1\%$  tolerance in the forward path, and an alternator of a value  $\frac{9}{10}$  in the feedback path. The overall system gain is approximately.

- (A)  $10 \pm 1\%$
- (B)  $9 \pm 1\%$
- (C)  $1 \pm 0.1\%$
- (D)  $9 \pm 0.1\%$

**Solution**

$$\text{Overall gain of the system without tolerance} = \frac{10}{1 + \frac{90}{10}} = \frac{1}{10}$$

$$\text{Sensitivity of gain with tolerance } (S_G^T) = \frac{1}{1 + GH} \text{ (For negative feedback)}$$

$$= \frac{1}{1 + \frac{90}{10}} = \frac{1}{10}$$

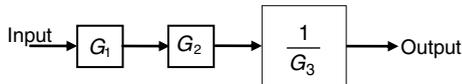
$$(S_G^T) = \frac{\% \text{ Change in gain}}{\% \text{ Change in } G}$$

$$\begin{aligned} \% \text{ Change in gain} &= \frac{1}{10} \times \text{Change in } G \\ &= \frac{1}{10} \times 1\% = 0.1\% \end{aligned}$$

Overall gain with tolerance =  $1 \pm 0.1\%$

**Example 11**

Errors associated with each respective subsystems  $G_1$ ,  $G_2$  and  $G_3$  are 4, 5, and 8. The error associated with the output is



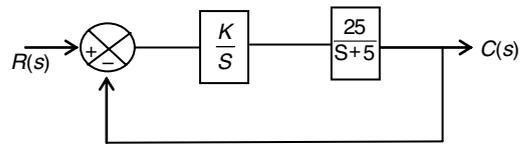
- (A) 160      (B) 17      (C) 1      (D)  $\frac{5}{2}$

**Solution**

$$\begin{aligned} \text{Total error} &= 4 \times S_{G_1}^T + 5 \times S_{G_2}^T + 8 \times S_{G_3}^T \\ &= 4 + 5 - 8 \quad \left[ \because S_{G_1}^T = 1, S_{G_2}^T = 1, S_{G_3}^T = -1 \right] \\ &= 1 \end{aligned}$$

**Example 12**

The sensitivity of transfer function  $\frac{C(s)}{R(s)}$  to variation in parameter  $K$  if system operating frequency  $\omega = 5$  rad/sec and  $K = 1$  is



- (A) 1.41      (B) 2.82      (C) 0      (D)  $\frac{1}{5}$

**Solution**

$$\text{Transfer Function T.F} = \frac{C(s)}{R(s)} = \frac{25K}{s^2 + 5s + 25K}$$

$$\begin{aligned} \text{Sensitivity of T.F with } K (S_K^T) &= \frac{\partial T}{\partial K} \times \frac{K}{T} \\ &= \frac{s(s+5)}{s^2 + 5s + 25K} \end{aligned}$$

If  $K = 1$  and  $\omega = 5$  rad/sec

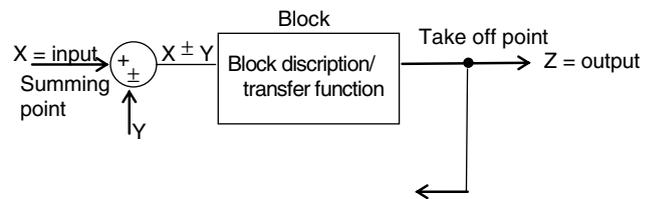
$$|S_K^T| = 1.41$$

**BLOCK DIAGRAM**

Block diagram is a pictorial representation of system between input and output. Block diagram, along with transfer function, is used to describe the cause and effect relationship throughout the system.

The different elements in a block diagram are as follows:

1. Block Physical description or transfer function of a subsystem
2. Summing point Addition or subtraction of all incoming signals
3. Take-off point Measurement or sensing a signal
4. Line with arrow Represents unidirectional signal flow and connectivity between subsystems

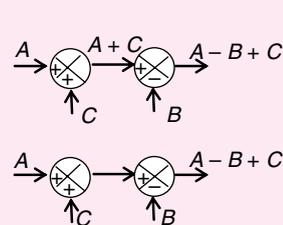
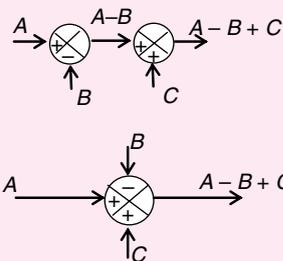


**Block Diagram Reduction Techniques**

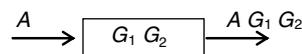
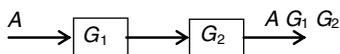
**Original diagram**

**Equivalent diagram**

1. Associative law



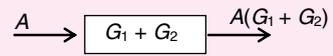
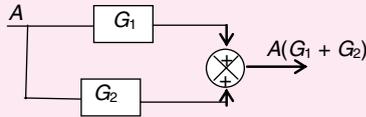
2. Blocks in series or combining blocks in cascade.



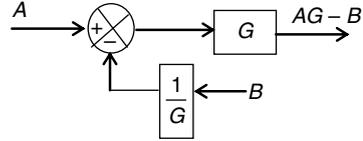
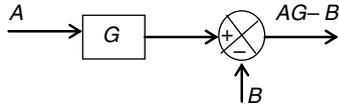
**Original diagram**

**Equivalent diagram**

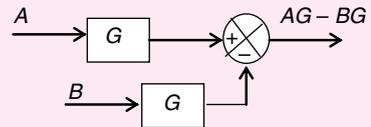
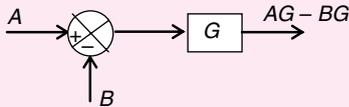
**3. Blocks in parallel**



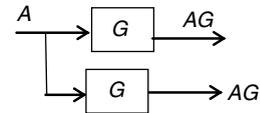
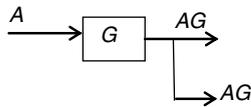
**4. Shifting summing point behind the block**



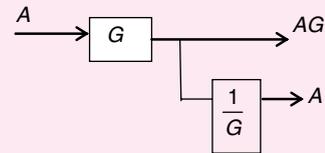
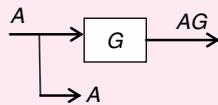
**5. Shifting summing point beyond the block**



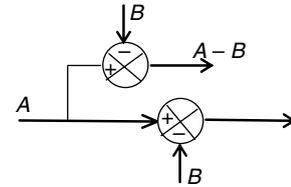
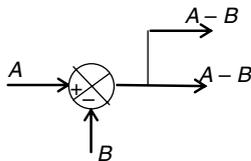
**6. Shifting a take-off point behind the block**



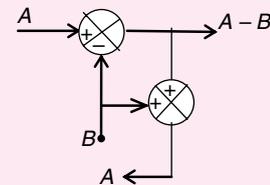
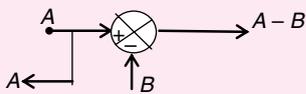
**7. Shifting a take-off point beyond the block**



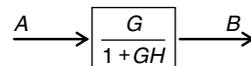
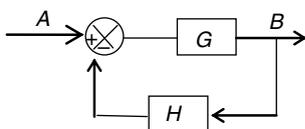
**8. Shifting take-off point after summing point**



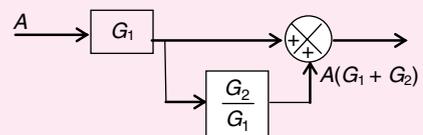
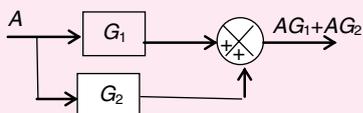
**9. Shifting take-off point before summing point**



**10. Removing minor feedback loop**



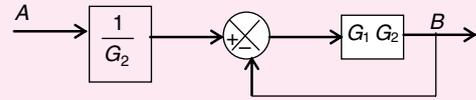
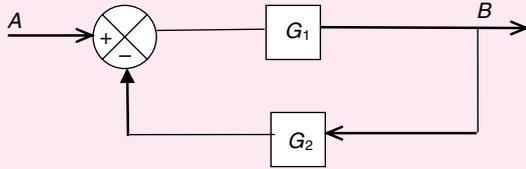
**11. Removing block-in forward path**



Original diagram

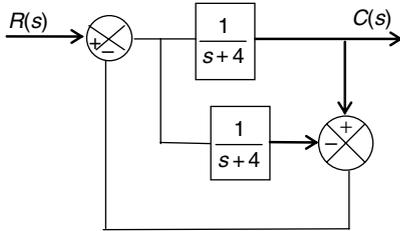
Equivalent diagram

12. Removing block in feedback path



Example 13

The transfer function  $\frac{C(s)}{R(s)}$  of the system shown is



- (A)  $\frac{1}{s+4}$     (B)  $\frac{1}{s+8}$     (C)  $\frac{1}{s+3}$     (D)  $\frac{2}{s+8}$

Solution

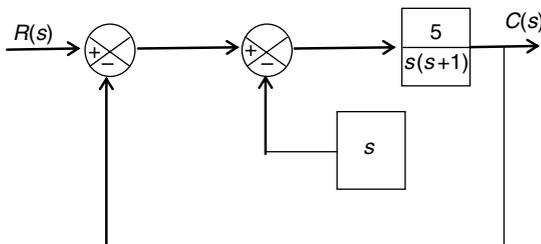
Forward path  $\rightarrow \frac{1}{s+4}$

Loops  $\rightarrow \frac{1}{s+4}, -\frac{1}{s+4}$

$$\begin{aligned} \text{Transfer function} &= \frac{P_1}{1-L_1-L_2} \\ &= \frac{1/s+4}{1-\frac{1}{s+4}+\frac{1}{s+4}} = \frac{1}{s+4} \end{aligned}$$

Example 14

For the system shown in figure the transfer function  $\frac{C(s)}{R(s)}$  is equal to



- (A)  $\frac{5}{s^2+4s+5}$     (B)  $\frac{5}{s^2-4s+5}$   
 (C)  $\frac{s^2}{s^2+6s+5}$     (D)  $\frac{5}{s^2+6s+5}$

Solution

Forward path  $\rightarrow \frac{5}{s(s+1)}$

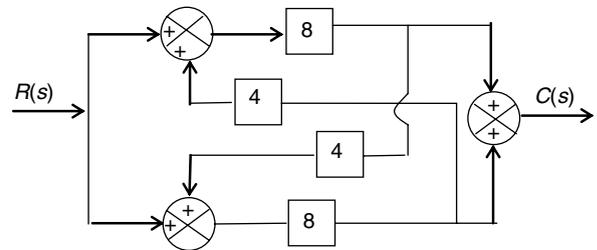
Loops  $\rightarrow \frac{-5s}{s(s+1)}, \frac{-5}{s(s+1)}$

$$\begin{aligned} \text{Transfer function} &= \frac{5/s(s+1)}{1+\frac{5s}{s(s+1)}+\frac{5}{s(s+1)}} \\ &= \frac{5}{s^2+s+5s+5} \end{aligned}$$

Transfer function  $= \frac{5}{s^2+6s+5}$

Example 15

The overall transfer function of the system in figure is



- (A)  $\frac{31}{16}$     (B)  $\frac{16}{31}$     (C)  $\frac{-16}{31}$     (D)  $-\frac{32}{15}$

Solution

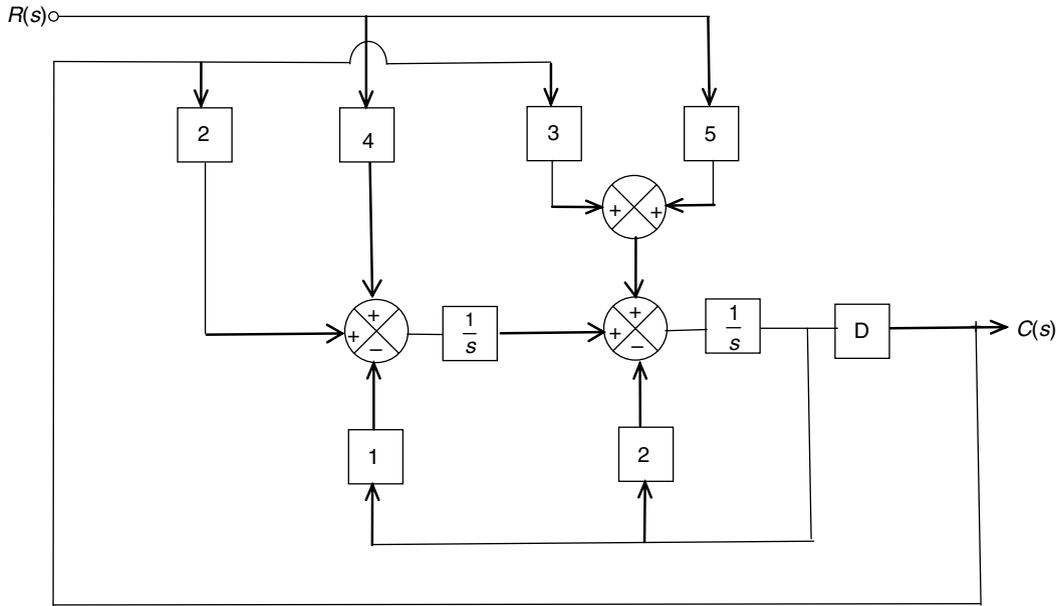
Forward paths  $\rightarrow 8, 8-4-8, 8-4-8, 8$

$$\begin{aligned} \text{Transfer function} &= \frac{8+(8 \times 4 \times 8)+8+(8 \times 4 \times 8)}{1-(8 \times 4 \times 8 \times 4)} = \\ &= \frac{16(1+32)}{1-32 \times 32} \end{aligned}$$

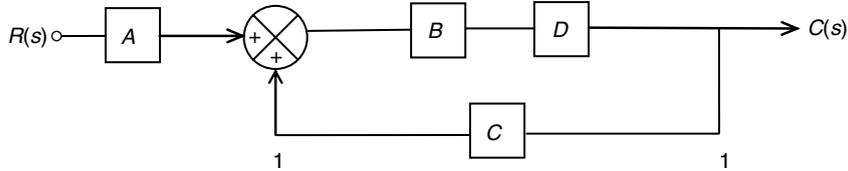
Transfer function  $= \frac{16}{1-32} = \frac{-16}{31}$

**Example 16**

The system shown in the figure below:



can be reduced to the form



(A)  $A = 2 + 3s, B = \frac{1}{s^2 + 2s + 1}, C = 4s + 5$

(B)  $A = 4s + 5, B = \frac{1}{s^2 + 2s + 1}, C = 2 + 3s$

(C)  $A = 2 + 3s, B = \frac{4s + 5}{s^2 + 2s + 5}, C = 4s + 5$

(D)  $A = 4s + 5, B = 2 + 3s, C = 1/s^2 + 2s + 5$

**Solution**

Forward paths  $\rightarrow 4 - \frac{1}{s} - \frac{1}{s} - D \left[ \frac{4D}{s^2} \right] \rightarrow 5 - \frac{1}{s} - D \left[ \frac{5D}{s} \right]$

Loops  $\rightarrow \frac{1}{s} - 2 \left[ -2/s \right]$

$\frac{1}{s} - 1 - \frac{1}{s} \left[ -1/s^2 \right]$

$\frac{1}{s} - D - 2 - \frac{1}{s} \left[ 2D/s^2 \right]$

$\frac{1}{s} - D - 3 \left[ 3D/s \right]$

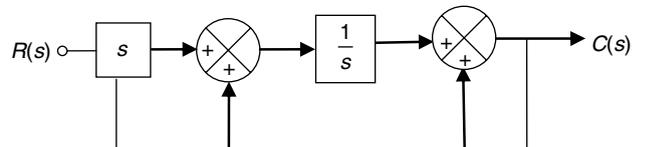
Transfer function =  $\frac{\frac{4D}{s^2} + \frac{5D}{s}}{1 + \frac{2}{s} + \frac{1}{s^2} - \frac{2D}{s^2} - \frac{3D}{s}} = \frac{(4s + 5)D}{(s^2 + 2s - 1) - (2D + 3Ds)}$

$B \cdot \frac{AD}{1 - BCD} = \frac{1}{s^2 + 2s + 1} \cdot \frac{(4s + 5)D}{1 - \frac{(2 + 3s)}{s^2 + 2s + 1} \cdot D}$

$\therefore A = (4s + 5), B = \frac{1}{s^2 + 2s + 1}, C = (2 + 3s)$

**Example 17**

For the block diagram shown in figure, the transfer function is equal to



- (A)  $\frac{s}{2s + 1}$  (B)  $s + \frac{1}{s}$  (C)  $\frac{s}{s^2 + 1}$  (D)  $\frac{2s + 1}{s}$

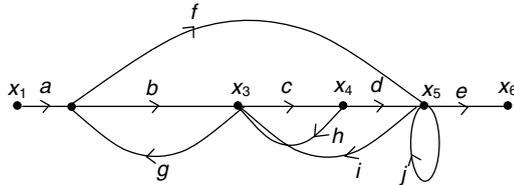
**Solution**

Forward paths  $\rightarrow 1, \frac{1}{s}, 1$

Transfer function =  $1 + \frac{1}{s} + 1 = \frac{2s + 1}{s}$

**SIGNAL FLOW GRAPH**

A signal flow graph is a graphical representation of simultaneous algebraic equations.



**Node** A node is a point representing a variable or signal.  
**Transmittance** This is a real gain or complex gain between two nodes.

**Branch** A branch is a directed line segment joining two nodes.

**Input node or source** A source is a node that has only outgoing variables. (node  $x_1$ )

**Output node or sink** A sink is a node that has only incoming branches. (node  $x_6$ )

**Mixed node** A mixed node has both incoming and outgoing branches. (nodes  $x_2, x_3, x_4, x_5$ )

**Forward path** A forward path is a path from an input node (source) to an output node (sink) that does not cross any nodes or branch more than once. ( $x_1 - x_2 - x_3 - x_4 - x_5 - x_6$  and  $x_1 - x_2 - x_5 - x_6$ )

**Loop** A loop is a closed path with no node or branch repeated more than once. ( $x_2 - x_3 - x_2, -x_3 - x_4 - x_3, x_3 - x_4 - x_5 - x_3, x_5 - x_5$ ) and  $x_2 - x_5 - x_2$ ).

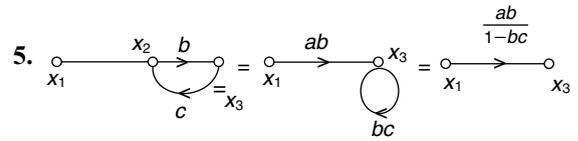
**Loop gain** The loop gain is the product of the branch transmittances of a loop. ( $bg, ch, cd, i, f$ , and  $fg$ )

**Non-touching loops** Loops are non-touching if they do not possess any common nodes.

**Signal flow graph algebra**

Signal flow graph for a system can be reduced to obtain the transfer function of the system using the following rules.

1.  $\Rightarrow x_2 = ax_1$
2.  $\Rightarrow x_3 = abx_1$
3.  $x_2 = (a + b) x_1$
4.  $x_4 = \dots$



**Mason's gain formula**

The transfer function of a signal flow graph can be found from Mason's gain formula as follows:

$$M = \frac{1}{\Delta} \sum_k P_k \Delta_k$$

$M \rightarrow$  Overall gain of the system

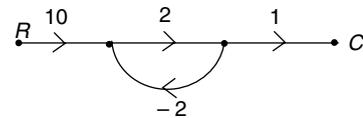
$P_k \rightarrow$  Gain of the  $k^{\text{th}}$  forward path

$\Delta = 1 - (\text{sum of all individual loop gains}) + (\text{sum of gain products of all possible combinations of two non-touching loops}) - (\text{sum of gain products of all possible combinations of three non touching loops}) + \dots$

$\Delta_k \rightarrow$  same as  $\Delta$  but formed by loops not touching the  $k^{\text{th}}$  forward path

**Example 18**

In the signal flow graph shown in figure,  $\frac{C}{R}$  is equals



- (A)  $-\frac{20}{3}$       (B) 4      (C) 2      (D) 18

**Solution**

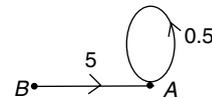
Forward paths  $\rightarrow 10 - 2 - 1$  (20)

Loops  $\rightarrow 2 - -2$  (-4)

Transfer function =  $\frac{20}{1 + 4} = 4$

**Example 19**

In the signal flow graph shown in figure,  $A = TB$ .  $T$  is equal to



- (A) 2.5      (B) 5.5      (C) 5      (D) 10

**Solution**

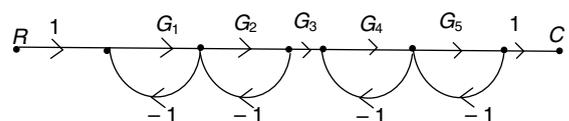
Forward path  $\rightarrow 5$ ,

Loop  $\rightarrow 0.5$

Transfer function =  $\frac{A}{B} = \frac{5}{1 - 0.5} = 10 \Rightarrow A = 10 B$

**Example 20**

The  $\frac{C}{R}$  for the signal flow graph in figure is



- (A)  $\frac{G_1 G_2 G_3 G_4 G_5}{1 + G_1 + G_2 + G_3 + G_4}$
- (B)  $\frac{G_1 G_2 G_3 G_4 G_5}{(1 + G_1 + G_2 + G_3 + G_4 + G_1 G_2 G_4 G_5)}$
- (C)  $\frac{G_1 G_2 G_3 G_4 G_5}{(1 + G_1 + G_2)(1 + G_4 + G_5)}$
- (D)  $\frac{G_1 G_2 G_4 G_5}{(1 + G_1)(1 + G_2)(1 + G_4)(1 + G_5)}$

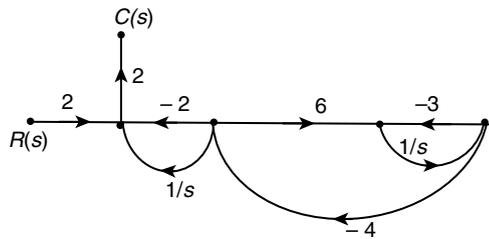
**Solution**

Given signal flow graph can be considered as two series signal flow graphs

$$\text{Transfer function} = \frac{G_1 G_2 G_3 G_4 G_5}{(1 + G_1 + G_2)(1 + G_4 + G_5)}$$

**Example 21**

The signal flow graph of a system is shown in figure. The transfer function  $\frac{C(s)}{R(s)}$  of the system is



- (A)  $\frac{s + 27}{s^2 + 29s + 6}$
- (B)  $\frac{6s}{s^2 + 29s + 6}$
- (C)  $\frac{27s}{s^2 + 6s + 6}$
- (D)  $\frac{4(s + 27)}{s^2 + 29s + 6}$

**Solution**

Forward path  $\rightarrow 2 - 2(4)$

$$\text{Loops} \rightarrow -2 - \frac{1}{s} \left( -\frac{2}{s} \right)$$

$$6 - \frac{1}{s} - 4 \left( \frac{-24}{s} \right) = \frac{1}{s} - (-3) \left( \frac{-3}{s} \right)$$

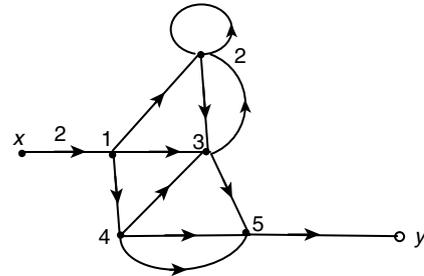
Now touching loops pair  $\rightarrow \left( \frac{-2}{s}, \frac{-3}{s} \right)$

Non-touching loops to forward path  $\rightarrow \frac{-24}{s}, \frac{-3}{s}$

$$\begin{aligned} \text{Transfer function} &= \frac{4 \left( 1 + \frac{24}{s} + 3/s \right)}{1 + \frac{2}{s} + \frac{24}{s} + \frac{3}{s} + \frac{6}{s^2}} \\ &= 4(s + 27)/(s^2 + 29s + 6) \end{aligned}$$

**Example 22**

The signal flow graph shown in the figure has – forward paths and self loops



- (A) 4, 4
- (B) 4, 1
- (C) 3, 3
- (D) 3, 1

**Solution**

Forward path  $\rightarrow x - 1 - 2 - 3 - 5 - y$

$$X - 1 - 3 - 5 - y$$

$$X - 1 - 4 - 3 - 5 - y$$

$$X - 1 - 4 - 5 - y$$

Total  $\rightarrow 4$

Self-loop  $\rightarrow 2 - 2$

Total  $\rightarrow 1$

**EXERCISES**

**Practice Problems I**

**Direction for questions 1 to 15:** Select the correct alternative from the given choices.

1. Which of the following statements are true?

- (i) In a closed-loop system, the effect of non-linearities is reduced.
- (ii) Feedback in closed loop may lead to oscillatory response.
- (iii) Feedback cannot control dynamics of the system.
- (iv) Open-loop systems are stable.

- (A) i, ii, and iv
- (B) i, ii, and iii
- (C) ii, iii, and iv
- (D) i and iii

- 2. The transfer function of a linear system is the
  - (A) ratio of two quantities which have the same units.
  - (B) ratio of the output to the input.
  - (C) ratio of the Laplace transform of the output to that of the input.
  - (D) ratio of the Laplace transform of the output to that of the input with all initial conditions zeros.
- 3. Which of the following is not a characteristic of negative feedback system?
  - (A) Rejection of disturbance signal
  - (B) High sensitivity to parameter variations

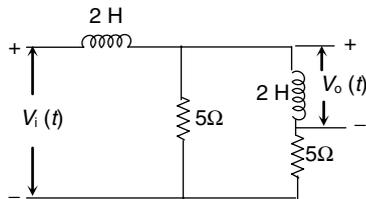
- (C) Reduction in gain  
 (D) Accuracy in tracking steady state value
4. Which one of the following is not a closed loop system?  
 (A) Respiratory system of an animal  
 (B) Execution of a program by a computer  
 (C) Air conditioning system  
 (D) Driving a car
5. If  $\frac{dy}{dx} = x$  represents the equation of an integrator, then which of the following is true?  
 (A) The system is stable.  
 (B) The system is unstable.  
 (C) The system is marginally stable.  
 (D) It cannot be determined.
6. The speed of response of the given three systems will be in the order

$$\text{Given } G_1(s) = \frac{5}{0.5s+1}$$

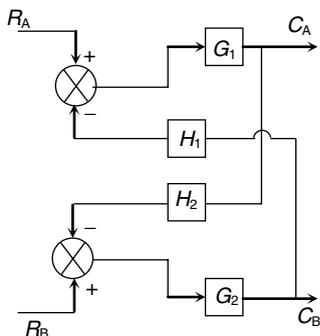
$$G_2(s) = \frac{5}{2s+1}$$

$$G_3(s) = \frac{5}{5s+1}$$

- (A)  $G_1(s) > G_2(s) > G_3(s)$   
 (B)  $G_1(s) = G_2(s) = G_3(s)$   
 (C)  $G_1(s) < G_2(s) < G_3(s)$   
 (D) Cannot be determined
7. Derive the transfer function of the network shown

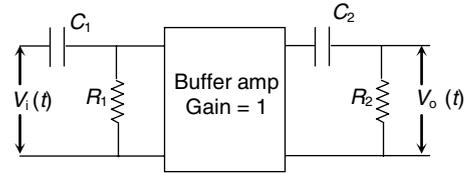


- (A)  $\frac{s}{4s^2 + 30s + 25}$       (B)  $\frac{10s}{4s^2 + 30s + 25}$   
 (C)  $\frac{1}{s(s^2 + 6s + 5)}$       (D)  $\frac{10}{s^2 + 3s + 5}$
8. Find  $\frac{C_A}{R_B}$  of the given system, if  $G_1 = \frac{20}{S}$ ,  $G_2 = \frac{S}{S+1}$ ,  $H_1 = 50s + 1$  and  $H_2 = 0.5s + 1$ .



- (A)  $\frac{20s+1}{25s^2 + 100s + 2}$       (B)  $\frac{5s+1}{25s^2 + 40s + 2}$   
 (C)  $\frac{20}{25s^2 + 100s + 10}$       (D)  $\frac{(5s+1)}{s^2 + 4s + 25}$

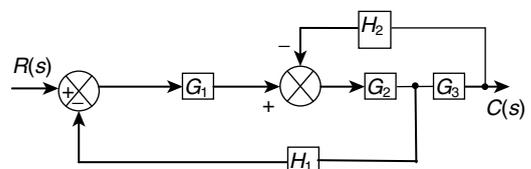
9. Find the transfer function



Given  $R_1 = R_2 = 10 \Omega$

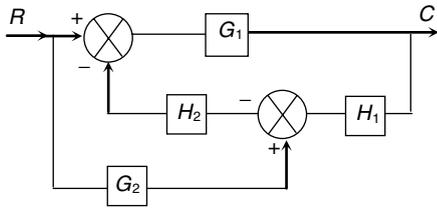
$C_1 = C_2 = 0.1 \mu\text{F}$

- (A)  $\frac{s}{s^2 + 2s + 1}$       (B) = 1  
 (C)  $\frac{s^2}{(s+1)^2}$       (D)  $\frac{(.1s)^2}{(s+10)^2}$
10. The Laplace transform of a function is given by  $\frac{s}{(s+1)^2}$ . Find its impulse response.  
 (A)  $t^t e \cos t$       (B)  $\frac{te^{-t}}{1-t}$   
 (C)  $t e^{-t}$       (D)  $e^{-t} (1-t)$
11. A system is described by  $3 \frac{dc(t)}{dt} + c(t) = r(t-3)$  where  $r(t)$  and  $c(t)$  are the input (Ramp) and output, respectively. The transfer function of the system is  
 (A)  $\frac{1+3s}{s^2(1+6s)}$       (B)  $\frac{1-3s}{(1+3s)}$   
 (C)  $\frac{1-3s}{s^2(1-6s)}$       (D)  $\frac{1-3s}{s(1+3s)}$
12. The transfer function of the system whose block diagram as shown in the figure is given by



- (A)  $\frac{1 + G_1 G_2 G_3}{1 + G_1 G_2 H_1 + G_2 G_3 H_2}$   
 (B)  $\frac{G_1 G_2 G_3}{1 + G_1 G_2 H_1 + G_2 G_3 H_2}$   
 (C)  $\frac{G_1 G_2 G_3}{1 + G_1 G_2 G_3 H_1 H_2}$   
 (D)  $\frac{G_1 G_2 G_3}{1 + G_1 G_3 H_1 + G_1 G_2 H_2}$

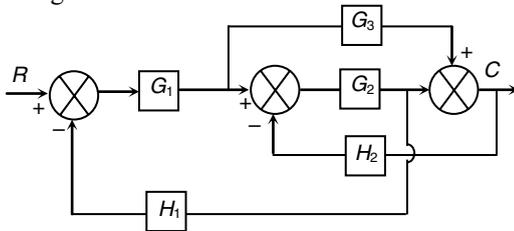
13. Using the block diagram given below, find the transfer function



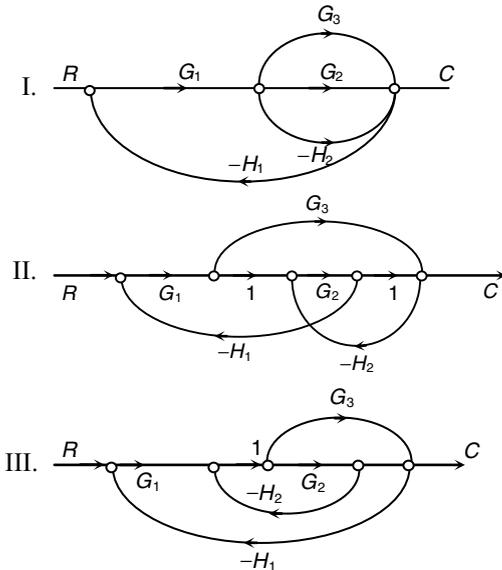
- (A)  $\frac{G_2}{H_2} \left[ \frac{G_1}{1+G_1H_1} \right]$       (B)  $\frac{G_1[1+G_2]}{1+G_1H_1H_2}$   
 (C)  $\frac{[1+G_2H_2]G_1}{1+G_1H_1H_2}$       (D)  $\frac{G_1G_2}{H_2+G_1H_1}$

**Direction for questions 14 and 15:**

The block diagram representation of a closed loop control system is given here.



14. Draw the signal flow graph of the given system.



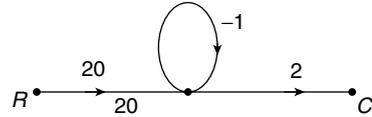
- (A) I and II are true      (B) All are true  
 (C) Only II is true      (D) Only I is true

15. Obtain the transfer function representation of the above block diagram.

- (A)  $\frac{G_1G_2 + G_1G_3}{1 + G_1G_2H_2H_3 + G_2H_2}$   
 (B)  $\frac{G_1G_2 + G_1G_3}{1 + G_2H_2 + G_1G_2H_1 + G_3H_1}$

- (C)  $\frac{G_1G_2 + G_1G_3}{1 + G_2H_2 + G_1G_2H_1}$   
 (D)  $\frac{G_1G_2 + G_1G_3}{G_2H_2 + G_1G_2H_1 + G_2G_3H_2}$

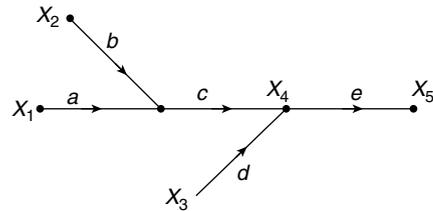
16. Transfer function  $\frac{C}{R}$  of the given signal flow graph is



- (A) 40      (B) 20      (C) 10      (D) 2  
 17. The dynamics equation related to a system is given by  $\frac{d^2c(t)}{dt^2} + 5\frac{dc(t)}{dt} + 8c(t) = 7r(t)$  Where  $r(t)$  is input and  $c(t)$  is output. Transfer function of the system is given by

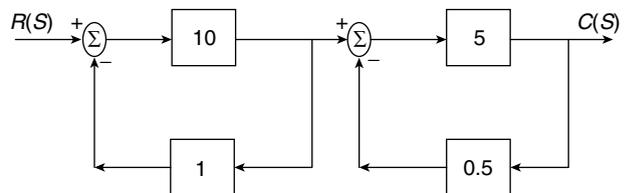
- (A)  $\frac{8}{7s^2 + 5s + 1}$       (B)  $\frac{7}{s^2 + 5s + 8}$   
 (C)  $\frac{5}{s^2 + 7s + 8}$       (D)  $\frac{7}{7s^2 + 5s + 8}$

18. The expression for the following signal flow graph is



- (A)  $[(acX_1 + bcX_2) + dX_3]e$   
 (B)  $[(aX_1 + bX_2)cdX_3]e$   
 (C)  $[(aX_1 + bX_2) + dX_3]e$   
 (D)  $[acX_1 + bcX_2 + dX_3]e$

19. The transfer function  $\frac{C(s)}{R(s)}$  of the block diagram given below is



- (A) 5      (B) 10      (C) 15      (D) 50

**Practice Problems 2**

**Direction for questions 1 to 15:** Select the correct alternative from the given choices.

- In a time-variant system,
  - the system parameters are independent of time.
  - the system parameters are functions of time.
  - the input and output are functions of time.
  - the system parameters depend on time-varying input and output.
- The transfer function of a tachometer has
  - A pole at origin and a zero anywhere in the real axis
  - A zero at origin and pole anywhere in the real axis
  - Only a zero at origin
  - Only a pole at origin
- Match the following

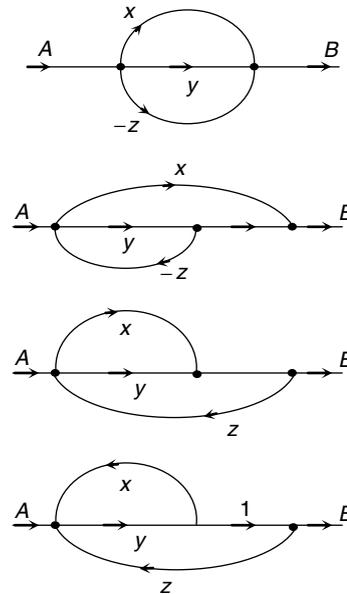
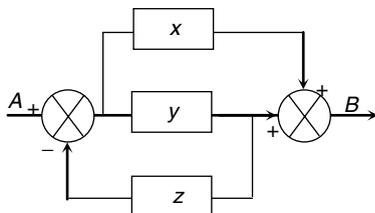
Type of roots	Nature of response term
1. Single root at $s = \sigma$	I $A \sin(\omega t + \beta)$
2. Roots of multiplicity $K$ at the origin	II $(A_1 + a_2 t + \dots) e^{\sigma t}$
3. Roots of multiplicity $K$ , at $s = \sigma$	III $A e^{\sigma t}$
4. Single complex Conjugate root pair on the $j\omega$ axis	IV $A + A_2 t + \dots + A_K t^{K-1}$

- 1 – III, 2 – II, 3 – IV, 4 – I
  - 1 – III, 2 – IV, 3 – II, 4 – I
  - 1 – III, 2 – I, 3 – IV, 4 – II
  - 1 – I, 2 – III, 3 – II, 4 – IV
- A system  $Y = f(x)$  is said to be linear if it satisfies the following properties:
    - $f(x + y) = f(x) + f(y)$
    - $f(\alpha x) = \alpha f(x)$
    - $f(ax_1 + bx_2) = af(x_1) + bf(x_2)$
    - None of these

5. The error transfer function of system is given by

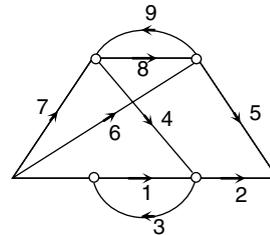
- $E(s) = \frac{G(s)}{1 + G(s)H(s)} - R(s)$
- $E(s) = \frac{R(s)}{1 + G(s)H(s)}$
- $E(s) = \frac{1}{1 + G(s)H(s)}$
- $E(s) = R(s) - G(s)H(s)$

6. The signal flow graph for the given system is

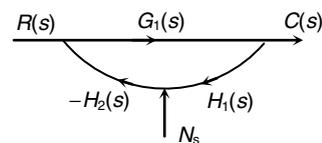


**Direction for questions 7 and 8:**

The SFD of a system is shown below. Study the system to give answers to the following questions.

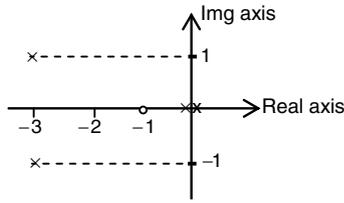


- Find the number of forward paths in the given SFD.
  - 5
  - 4
  - 6
  - 7
- Find the number of independent loops in the system.
  - 4
  - 3
  - 2
  - 5
- The impulse response of a system is given by  $g(t) = e^{-2t}(1 - \cos 2t)$ . Find the transfer function of the system.
  - $\frac{1}{s+2}$
  - $\frac{4}{(s+2)(s^2+4s+8)}$
  - $\frac{8}{(s+2)(s^2+4s+8)}$
  - $\frac{1}{(s^2+4s+8)}$
- For the system shown if the gain is very high, the transfer function  $\frac{C(s)}{R(s)}$  will be



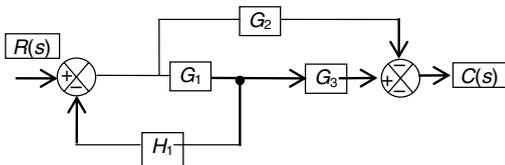
- $\frac{G_1(s)}{1 + G_1(s)H_1(s)H_2(s)}$
- $\frac{G_1(s)}{1 - H_1(s)H_2(s)N(s)}$
- $\frac{1}{H_1(s)H_2(s)}$
- $\frac{1}{1 - H_1(s)H_2(s)}$

11. A transfer function  $G(s)$  has the pole zero plot as shown in the figure. The steady state gain is given as 10. Find the transfer function.



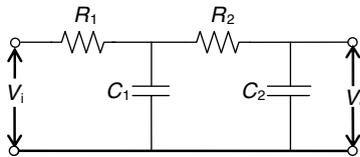
- (A)  $\frac{10(s+1)}{s^2+6s+10}$       (B)  $\frac{10(s-1)}{(s^2+6s+10)s}$   
 (C)  $\frac{10(s+1)}{s^2(s^2+6s+10)}$       (D)  $\frac{10(s+1)s}{s^2+6s+10}$

12.



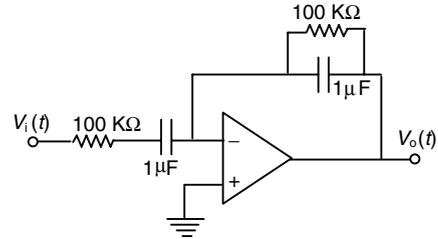
The transfer function of the block diagram shown in the above figure is

- (A)  $\frac{G_1G_2 - G_3}{1 + G_1H}$       (B)  $\frac{G_1G_3 + G_2}{1 + G_1H}$   
 (C)  $\frac{G_1G_2}{1 + G_1H} - G_3$       (D)  $\frac{G_1G_3 - G_2}{1 + G_1H}$
13. The transfer function of the electrical network shown in the figure is



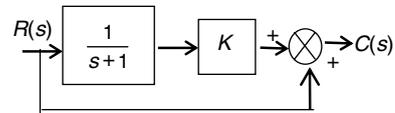
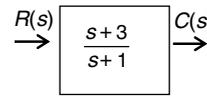
- (A)  $\frac{1}{1 + s[R_1C_1 + R_2C_2 + R_1C_2] + s^2R_1R_2C_1C_2}$   
 (B)  $\frac{1}{1 + s^2[R_1C_1 + R_2C_2 + R_2C_1] + s^4R_1R_2C_1C_2}$   
 (C)  $\frac{1}{1 + s^2[R_1C_1 + R_2C_2 + R_1C_2] + s^4R_1R_2C_1C_2}$   
 (D)  $\frac{1}{1 + s[R_1C_1 + R_2C_2 + R_1C_2] + s^3R_1R_2C_1C_2}$

14. For the system given, find the transfer function.



- (A)  $\frac{(0.1s + 2)s}{(0.1s + 1)^2}$       (B)  $\frac{0.1s}{(0.1s + 2)}$   
 (C)  $\frac{-0.1s}{(0.1s + 1)^2}$       (D) None of these

15. For what value of  $k$ , are the two block diagrams shown below equivalent?

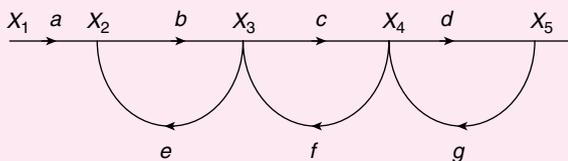


- (A) 2      (B) 1      (C)  $s + 1$       (D)  $s + 2$

PREVIOUS YEARS' QUESTIONS

1. Consider the signal flow graph shown in the figure.

The gain  $\frac{X_5}{X_1}$  is [2004]



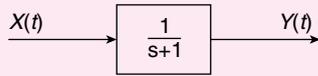
- (A)  $\frac{1 - (be + cf + dg)}{abc}$   
 (B)  $\frac{bedg}{1 - (be + cf + dg)}$

- (C)  $\frac{abcd}{1 - (be + cf + dg) + bedg}$   
 (D)  $\frac{1 - (be + cf + dg) + bedg}{abcd}$

2. Despite the presence of negative feedback, control systems still have problems of instability because the [2005]

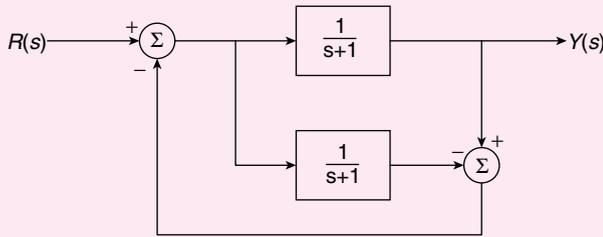
- (A) components used have nonlinearities.  
 (B) dynamic equation of the subsystems are not known exactly.  
 (C) mathematical analysis involves approximations  
 (D) system has a large negative phase angle at high frequencies.

3. In the system shown below,  $x(t) = (\sin t)u(t)$ . In a steady state, the response  $y(t)$  will be **[2006]**



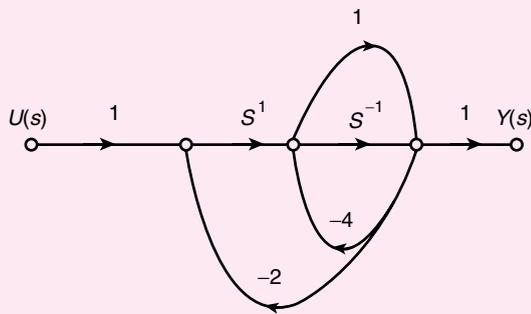
- (A)  $\frac{1}{\sqrt{2}} \sin\left(t - \frac{\pi}{4}\right)$  (B)  $\frac{1}{\sqrt{2}} \sin\left(t + \frac{\pi}{4}\right)$   
 (C)  $\frac{1}{\sqrt{2}} e^{-t} \sin t$  (D)  $\sin t - \cos t$

4. The transfer function  $Y(s)/R(s)$  of the system shown is **[2010]**



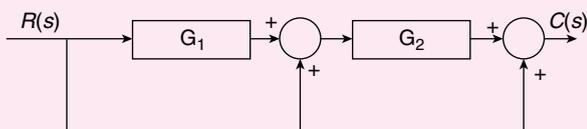
- (A) 0 (B)  $\frac{1}{s+1}$  (C)  $\frac{2}{s+1}$  (D)  $\frac{2}{s+3}$

5. The signal flow graph for a system is given below. The transfer function  $\frac{Y(s)}{U(s)}$  for this system is **[2013]**



- (A)  $\frac{s+1}{5s^2+6s+2}$  (B)  $\frac{s+1}{s^2+6s+2}$   
 (C)  $\frac{s+1}{s^2+4s+2}$  (D)  $\frac{1}{5s^2+6s+2}$

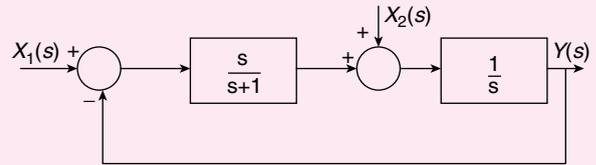
6. Consider the following block diagram in the figure. **[2014]**



The transfer function  $\frac{C(s)}{R(s)}$  is

- (A)  $\frac{G_1 G_2}{1+G_1 G_2}$  (B)  $G_1 G_2 + G_1 + 1$   
 (C)  $G_1 G_2 + G_2 + 1$  (D)  $\frac{G_1}{1+G_1 G_2}$

7. For the following system, **[2014]**



When  $X_1(s) = 0$ , the transfer function  $\frac{Y(s)}{X_2(s)}$  is

- (A)  $\frac{s+1}{s^2}$  (B)  $\frac{1}{s+1}$   
 (C)  $\frac{s+2}{s(s+1)}$  (D)  $\frac{s+1}{s(s+2)}$

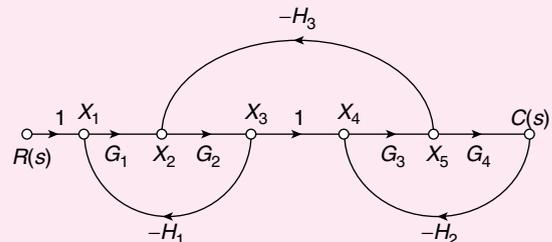
8. Negative feedback in a closed-loop control system DOES NOT **[2015]**

- (A) reduce the overall gain  
 (B) reduce bandwidth  
 (C) improve disturbance rejection  
 (D) reduce sensitivity to parameter variation

9. By performing cascading and/or summing/differencing operations using transfer function blocks  $G_1(s)$  and  $G_2(s)$ , one CANNOT realize a transfer function of the form **[2015]**

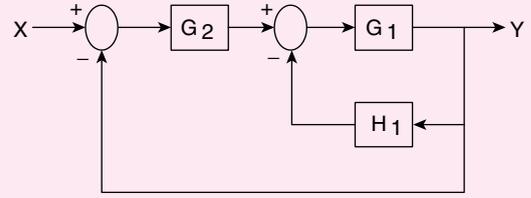
- (A)  $G_1(s) G_2(s)$   
 (B)  $\frac{G_1(s)}{G_2(s)}$   
 (C)  $G_1(s) \left[ \frac{1}{G_1(s)} + G_2(s) \right]$   
 (D)  $G_1(s) \left[ \frac{1}{G_1(s)} - G_2(s) \right]$

10. For the signal flow graph shown in the figure, the value of  $\frac{C(s)}{R(s)}$  is **[2015]**



- (A)  $\frac{G_1 G_2 G_3 G_4}{1 - G_1 G_2 H_1 - G_3 G_4 H_2 - G_2 G_3 H_3 + G_1 G_2 G_3 G_4 H_1 H_2}$
- (B)  $\frac{G_1 G_2 G_3 G_4}{1 + G_1 G_2 H_1 + G_3 G_4 H_2 + G_2 G_3 H_3 + G_1 G_2 G_3 G_4 H_1 H_2}$
- (C)  $\frac{1}{1 + G_1 G_2 H_1 + G_3 G_4 H_2 + G_2 G_3 H_3 + G_1 G_2 G_3 G_4 H_1 H_2}$
- (D)  $\frac{1}{1 - G_1 G_2 H_1 - G_3 G_4 H_2 - G_2 G_3 H_3 + G_1 G_2 G_3 G_4 H_1 H_2}$

11. The response of the system  $G(s) = \frac{s-2}{(s+1)(s+3)}$  to the unit step input  $u(t)$  is  $y(t)$ . The value of  $\frac{dy}{dt}$  at  $t = 0^+$  is \_\_\_\_\_ . [2016]
12. The block diagram of a feedback control system is shown in the figure. The overall closed loop gain  $G$  of the system is [2016]



- (A)  $G = \frac{G_1 G_2}{1 + G_1 H_1}$
- (B)  $G = \frac{G_1 G_2}{1 + G_1 G_2 + G_1 H_1}$
- (C)  $G = \frac{G_1 G_2}{1 + G_1 G_2 H_1}$
- (D)  $G = \frac{G_1 G_2}{1 + G_1 G_2 + G_1 G_2 H_1}$

## ANSWER KEYS

### EXERCISES

#### Practice Problems 1

1. A    2. D    3. B    4. B    5. C    6. A    7. B    8. C    9. C    10. D  
 11. B    12. B    13. C    14. C    15. C    16. B    17. B    18. A    19. D

#### Practice Problems 2

1. B    2. C    3. B    4. C    5. C    6. B    7. A    8. A    9. B    10. C  
 11. C    12. B    13. A    14. C    15. A

#### Previous Years' Questions

1. C    2. A    3. A    4. B    5. A    6. C    7. D    8. B    9. B    10. B  
 11. 1    12. B