CHAPTER 11

LOGARITHMS

DEFINITION

If $x = a^m$, then $\log_a x = m$, where 'a' and 'x' both are positive real numbers but 'a' not equal to 1 i.e., $a, x > 0$, but $a \neq 1$.

Here log is the short form of logarithm.

$\log_a x$ is read as log of x to the base a.

For example,

(i) Since, $10 = 10^1, 100 = 10^2, 1000 = 10^3$, etc.

Hence, $\log_{10} 10 = 1, \log_{10} 100 = 2, \log_{10} 1000 = 3$, etc.

(ii) Since, $8 = 2^3, 16 = 2^4, 32 = 2^5$, etc.

Hence, $\log_2 8 = 3, \log_2 16 = 4, \log_2 32 = 5$, etc.

(iii) Since, $\frac{1}{8} = (2)^{-3}, \frac{1}{16} = (2)^{-4}$, etc.

Hence, $\log_2 \frac{1}{8} = -3, \log_2 \frac{1}{16} = -4$, etc.

(iv) Since, $0.01 = (10)^{-2}, 0.001 = (10)^{-3}$, etc.

Hence $\log_{10}(0.01) = -2, \log_{10}(0.001) = -3$, etc.

LAWS OF LOGARITHM

(i) $\log_a(m \times n) = \log_a m + \log_a n$

In general,

$\log_a(m \times n \times p \times ....) = \log_a m + \log_a n + \log_a p + ...$

For example:

$\log_2(4 \times 5 \times 6) = \log_2 4 + \log_2 5 + \log_2 6$

Note that

$\log_a m + \log_a n \neq \log_a(m + n)$

(ii) $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$

For example:

$\log_4 \left(\frac{8}{15}\right) = \log_4 8 - \log_4 15$

Note that

$\log_a m - \log_a n \quad \log_a(m - n)$

(iii) $\log_a(m)^n = n \log_a m$

For example:

$\log_3 (5)^4 = 4 \log_3 5$

(iv) $\log_b a = \frac{\log_c a}{\log_c b}$ [Change of base rule]

For example,

$$\log_5 20 = \frac{\log_2 20}{\log_2 5} = \frac{\log_4 20}{\log_4 5} = \frac{\log_7 20}{\log_7 5} = \text{... etc.}$$

(v) $\log_b a = \frac{1}{\log_a b}$

For example,

$$\log_{10} 100 = \frac{1}{\log_{100} 10}$$

(vi) $\log_b a \,.\, \log_c b = \log_c a$ [Chain Rule]

In general,

$\log_b a \,.\, \log_c b \,.\, \log_d c ....\log_n m = \log_n a$

For example,

$\log_{24} 256 \,.\, \log_{10} 24 \,.\, \log_2 10 = \log_2 256$

Remember

- $\log 1 = 0$
- $\log 10 = 1$
- $\log_a a = 1$

Example 1: $\log_a 4 + \log_a 16 + \log_a 64 + \log_a 256 = 10$. **Then** $a = ?$

Solution: The given expression is:

$\log_a (4 \times 16 \times 64 \times 256) = 10 \quad \Rightarrow \log_a (4^1 \times 4^2 \times 4^3 \times 4^4)$

i.e. $\log_a 4^{10} = 10$

Thus, $a = 4$.

Example 2: Find x if $\log x = \log 1.5 + \log 12$

Solution: $\log x = \log 18 \Rightarrow x = 18$

Example 3: Find x, if $\log(2x - 2) - \log(11.66 - x) = 1 + \log 3$

Solution: $\log(2x - 2) - (11.66 - x) = \log 10 + \log 3$

$\Rightarrow \log(2x - 2)/(-11.66 - x) = \log 30$

$\Rightarrow (2x - 2)/(11.66 - x) = 30$

$2x - 2 = 350 - 30x$

Hence, $32x = 352 \Rightarrow x = 11$.

Example 4: Solve for x:

$$\log \frac{75}{35} + 2 \log \frac{7}{5} - \log \frac{105}{x} - \log \frac{13}{25} = 0$$

Solution: $(75/35) \times (49/25) \times (x/105) \times (25/13) = 1$

$\Rightarrow x = 13$

SOME IMPORTANT PROPERTIES

(i) $x = a^m \Rightarrow \log_a x = m$

and $\log_a x = m \Rightarrow x = a^m$

Here equation $x = a^m$ is in exponential form and equation $\log_a x = m$ is in logarithmic form.

(ii) If base of log is not mentioned, then we assume the base as 10.

$\log m = \log_{10} m$

log to the base 10 is called common log.

(iii) Since, $10000 = (100)^2 = (10)^4$

$\therefore \log_{100} 10000 = 2, \log_{10} 10000 = 4$

Thus value of log of a number on different bases is different i.e., value of log of a number depends on its base.

(iv) (a) Since, $a = a^1$, hence $\log_a a = 1$

For example, $\log_5 5 = 1$, $\log_{10} 10 = 1$

Thus log of any number to the same base is always 1.

(b) Since, $1 = a^0$, hence, $\log_a 1 = 0$

For example, $\log_8 1 = 0$

Thus log of 1 to any base always equal to 0.

(v) $a^{(\log_a x)} = x$

For example,

$20^{(\log_{20} 50)} = 50$

(vi) (a) log of zero and negative numbers is not defined.

(b) Base of log is always positive but not equal to 1.

Example 5: Find x, if $0.01^x = 2$

Solution: $x = \log_{0.01} 2 = -\log 2/2.$

Example 6: If log 3 = .4771, find log $(.81)^2$ × log ÷ log 9.

Solution: $2 \log (81/100) \times 2/3 \log (27/10) \div \log 9$

$= 2 [\log 3^4 - \log 100] \times 2/3 [(\log 3^3 - \log 10)] \div 2 \log 3$

$= 2 [\log 3^4 - \log 100] \times 2/3 [(3\log 3 - 1)] \div 2 \log 3$

Substitute $\log 3 = 0.4771 \Rightarrow -0.0552.$

EXERCISE

1. Find the value of $\log_5 10 \times \log_{10} 15 \times \log_{15} 20 \times \log_{20} 25$.
   (a) 5/2 (b) 5
   (c) 2 (d) $\log\left(\frac{5}{2}\right)$
2. If $\log_3 a = 4$, find value of a.
   (a) 27 (b) 3
   (c) 9 (d) 81
3. Find the value of $\log\frac{9}{8} - \log\frac{27}{32} + \log\frac{3}{4}$
   (a) 0 (b) 1
   (c) 3 (d) log (3/4)
4. The value of $\left[\frac{1}{\log_{xy}(xyz)} + \frac{1}{\log_{yz}(xyz)} + \frac{1}{\log_{zx}(xyz)}\right]$ is equal to
   (a) 1 (b) 2
   (c) 3 (d) 4
5. If $\log_2 [\log_3 (\log_2 x)] = 1$, then x is equal to
   (a) 512 (b) 128
   (c) 12 (d) 0
6. Find the value of $\log_{27} \frac{1}{81}$
   (a) – 4/3 (b) – 3
   (c) – 1 (d) – 1/3
7. Find the value of $\frac{8\log_8 8}{2\log_{\sqrt{8}} 8}$
   (a) 1 (b) 2
   (c) 3 (d) 4
8. If $\log_k x \log_5 k = 3$, then find the value of x.
   (a) k^5 (b) $5k^3$
   (c) 243 (d) 125
9. $\log_a\left(\frac{m}{n}\right)$ is equal to
   (a) $\log_a(m-n)$ (b) $\log_a m - \log_a n$
   (c) $\frac{(\log_a m)}{n}$ (d) $\log_a m \div \log_a n$
10. If $\log_5\left[\log_3\left(\log_2 x\right)\right] = 1$ then x is
    (a) 2^{234} (b) 243
    (c) 2^{243} (d) None of these
11. The value of $\left[3\log\left(\frac{81}{80}\right) + 5\log\left(\frac{25}{24}\right) + 7\log\left(\frac{16}{15}\right)\right]$ is
    (a) log 3 (b) log 5
    (c) log 7 (d) log 2
12. If $\log_{10} a + \log_{10} b = c$, then the value of a is
    (a) bc (b) $\frac{c}{b}$
    (c) $\frac{(10)^c}{b}$ (d) $\frac{10b}{c}$
13. If $\log_y x = 8$ and $\log_{10y} 16x = 4$, then find the value of y.
    (a) 1 (b) 2
    (c) 3 (d) 5
14. $\log 0.0867 = ?$
    (a) $\log 8.67 + 2$ (b) $\log 8.67 - 2$
    (c) $\frac{\log 867}{1000}$ (d) $-2\log 8.67$
15. Find x, if $0.01^x = 2$
    (a) log 2/2 (b) 2/log 2
    (c) – 2/log2 (d) – log 2/2
16. If $2^x . 3^{2x} = 100$, then the value of x is ($\log 2 = 0.3010$, $\log 3 = 0.4771$)
    (a) 2.3 (b) 1.59
    (c) 1.8 (d) 1.41
17. $\log_{10} 10 + \log_{10} 10^2 + \ldots\ldots + \log_{10} 10^n$
    (a) $n^2 + 1$ (b) $n^2 - 1$
    (c) $\left(\frac{n^2+n}{3}\right)$ (d) $\frac{n^2+n}{2}$
18. If $\log_{10} a = b$, then find the value of 10^{3b} in terms of a.
    (a) a^3 (b) $3a$
    (a) $a \times 1000$ (d) $a \times 100$
19. If $a = b^x$, $b = c^y$ and $c = a^z$, then the value of xyz is equal to
    (a) –1 (b) 0
    (c) 1 (d) abc
20. If $\log_3 [\log_3 [\log_3 x]] = \log_3 3$, then what is the value of x?
    (a) 3 (b) 27
    (c) 3^9 (d) 3^{27}

21. $\frac{1}{(\log_a bc)+1}+\frac{1}{(\log_b ac)+1}+\frac{1}{(\log_c ab)+1}$ is equal to

(a) 1 (b) 2
(c) 0 (d) abc

22. If a, b, c are three consecutive integers, then $\log(ac+1)$ has the value

(a) $\log b$ (b) $(\log b)^2$
(c) $2\log b$ (d) $\log 2b$

23. The value of $\log_{2\sqrt{3}}(1728)$ is

(a) 3 (b) 5
(c) 6 (d) 9

24. Find the value of $\left(7^3\right)^{-2\log_7 8}$

(a) 8^{-7} (b) 6^{-8}
(c) 8^{-6} (d) None of these

25. If $\log_4 5=a$ and $\log_5 6=b$ then what is the value of $\log_3 2$?

(a) $\frac{1}{2a+1}$ (b) $\frac{1}{2b+1}$
(c) $2ab+1$ (d) $\frac{1}{2ab-1}$

26. If $x=\log_a(bc)$, $y=\log_b(ca)$ and $z=\log_c(ab)$, then which of the following is equal to 1?

(a) $x+y+z$
(b) $(1+x)^{-1}+(1+y)^{-1}+(1+z)^{-1}$
(c) xyz
(d) None of these

27. What is the value of $\log_{32} 27\times\log_{243} 8$?

(a) $\frac{\log 9}{\log 4}$ (b) $\frac{\log 3}{\log 2}$
(c) $\log 27$ (d) None of these

28. $\log a^n/b^n+\log b^n/c^n+\log c^n/a^n$

(a) 1 (b) n
(c) 0 (d) 2

29. Find the value of x and y respectively for

$\log_{10}\left(x^2y^3\right)=7$ and $\log_{10}(x/y)=1$

(a) $x=10, y=100$ (b) $x=100, y=10$
(c) $x=10, y=20$ (d) None of these

30. $\log_2(9-2^x)=10^{\log(3-x)}$, solve for x.

(a) 0 (b) 3
(c) Both (a) and (b) (d) 0 and 6

31. What is the value of

$$\left(\log_{\frac{1}{2}} 2\right)\left(\log_{\frac{1}{3}} 3\right)\left(\log_{\frac{1}{4}} 4\right)\ldots\ldots\left(\log_{\frac{1}{1000}} 1000\right)?$$

(a) 1 (b) –1
(c) 1 or –1 (d) 0

32. If $\log_r 6=m$ and $\text{lor}_r 3=n$, then what is $\log_r\left(\frac{r}{2}\right)$ equal to?

(a) $m-n+1$ (b) $m+n-1$
(c) $1-m-n$ (d) $1-m+n$

33. What is the value of

$\left(\frac{1}{3}\log_{10}125-2\log_{10}4+\log_{10}32+\log_{10}1\right)$?

(a) 0 (b) $\frac{1}{5}$
(c) 1 (d) $\frac{2}{5}$

34. What is the value of

$$\frac{1}{2}\log_{10}25-2\log_{10}3+\log_{10}18$$

(a) 2 (b) 3
(c) 1 (d) 0

35. What is the value of $[\log_{10}(5\log_{10}100)]^2$?

(a) 4 (b) 3
(c) 2 (d) 1

36. What is $\log_{10}\left(\frac{3}{2}\right)+\log_{10}\left(\frac{4}{3}\right)+\log_{10}\left(\frac{5}{4}\right)+\ldots\ldots$ upto 8 terms equal to?

(a) 0 (b) 1
(c) $\log_{10}5$ (d) None of these

37. What is the logarithm of 0.0001 with respect to base 10?

(a) 4 (b) 3
(c) –4 (d) –3

38. If $\log_{10} a=p$ and $\log_{10} b=q$, then what is the value of $\log_{10}(a^p p^q)$?

(a) p^2+q^2 (b) p^2-q^2
(c) p^2q^2 (d) $\frac{p^2}{q^2}$

39. What is the value of $\frac{[\log_{13}(10)]}{[\log_{169}(10)]}$? **(CDS)**

(a) $\frac{1}{2}$ (b) 2
(c) 1 (d) $\log_{10}13$

40. The value of $\frac{1}{5}\log_{10}3125-4\log_{10}2+\log_{10}32$ is **(CDS)**

(a) 0 (b) 1
(c) 2 (d) 3

HINTS & SOLUTIONS

1. (c) $\log_5 10 \times \log_{10} 15 \times \log_{15} 20 \times \log_{20} 25.$
$= (\log 10/\log 5) \times (\log 15/\log 10) \times (\log 20/\log 15)$
$\times (\log 25/\log 20)$
$= \log 25/\log 5 = 2\log 5/\log 5 = 2$

2. (d) $\because \log_3 a = 4 \therefore 3^4 = a \Rightarrow a = 81$

3. (a) Given $\log\frac{9}{8} - \log\frac{27}{32} + \log\frac{3}{4} = \log\left(\frac{9}{8} \div \frac{27}{32}\right) + \log\frac{3}{4}$
$= \log\left(\frac{9}{8} \times \frac{32}{27} \times \frac{3}{4}\right) = \log 1 = 0$
$\because \log_a 1 = 0$

4. (b) Given expression
$= \log_{xyz}(xy) + \log_{xyz}(yz) + \log_{xyz}(zx)$
$= \log_{xyz}(xy \times yz \times zx) = \log_{xyz}(xyz)^2$
$= 2\log_{xyz}(xyz) = 2 \times 1 = 2$

5. (a) $\log_2[\log_3(\log_2 x)] = 1 = \log_2 2$
$\Rightarrow \log_3(\log_2 x) = 2$
$\Rightarrow \log_2 x = 3^2 = 9$
$\Rightarrow x = 2^9 = 512$

6. (a) Let $\log_{27}\left(\frac{1}{81}\right) = x$
$\therefore (27)^x = \frac{1}{81}$
$\therefore 3^{3x} = 3^{-4} \Rightarrow 3x = -4 \Rightarrow x = -\frac{4}{3}$

7. (b) $\frac{8\log_8 8}{2\log_{\sqrt{8}} 8} = \frac{8 \times 1}{2\log_{\sqrt{8}}\left(\sqrt{8}\right)^2} = \frac{8}{4\log_{\sqrt{8}}\sqrt{8}} = \frac{8}{4} = 2$

8. (d) Given, $\log_5 k \log_k x = 3$
$\frac{\log k}{\log 5} \cdot \frac{\log x}{\log k} = 3 \Rightarrow \frac{\log x}{\log 5} = 3$
$\Rightarrow \log x = 3\log 5 \Rightarrow \log x = \log 5^3$
$\Rightarrow x = 5^3 \Rightarrow x = 125$

9. (b) $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

10. (c) $\log_5\left[\log_3\left(\log_2 x\right)\right] = 1 = \log_5 5$
$\Rightarrow \log_3\left(\log_2 x\right) = 5 = \log_3 3^5$
$\Rightarrow \log_2 x = 3^5 = 243$
$\Rightarrow 2^{243} = x$

11. (d) $3\log\frac{81}{80} + 5\log\frac{25}{24} + 7\log\frac{16}{15}$
$= \log\left[\left(\frac{81}{80}\right)^3 \times \left(\frac{25}{24}\right)^5 \times \left(\frac{16}{15}\right)^7\right]$
$= \log\left(\frac{3^{12} \times 5^{10} \times 2^{28}}{2^{12} \times 5^3 \times 2^{15} \times 3^5 \times 3^7 \times 5^7}\right)$
$= \log 2$

12. (c) $\log_{10} a + \log_{10} b = c$
$\Rightarrow \log_{10}(ab) = c$
$\Rightarrow 10^c = ab$
$\Rightarrow a = \frac{(10)^c}{b}$

13. (d) $\log_y x = 8 \Rightarrow y^8 = x$ ...(1)
$\log_{10y} 16x = 4 \Rightarrow 10^4 y^4 = 16x$ ...(2)
Dividing (2) by (1) $10^4 y^{-4} = 16 \Rightarrow y = 5$

14. (b) $\log 0.0867 = \log(8.67/100) = \log 8.67 - \log 100$
$\log 8.67 - 2$

15. (d) $x = \log_{0.01} 2 = -\log 2/2.$

16. (b) $2^x . 3^{2x} = 100$
$\Rightarrow x\log 2 + 2x\log 3 = \log 100$
$\Rightarrow x(0.3010 + 2 \times 0.4771) = 2$
$\Rightarrow x = \frac{1}{1.2552} = 1.59$

17. (d) $\log_{10}10 + \log_{10}10^2 + \ldots\ldots + \log_{10}10^n$
$= 1 + 2 + 3 + \ldots\ldots + n = \frac{n(n+1)}{2}$

18. (a) $\log_{10} a = b \Rightarrow 10^b = a \Rightarrow$ By definition of logs.
Thus $10^{3b} = (10^b)^3 = a^3$.

19. (c) $a = b^x, b = c^y, c = a^z$

$\Rightarrow x = \log_b a, y = \log_c b, z = \log_a c$

$\Rightarrow xyz = (\log_b a) \times (\log_c b) \times (\log_a c)$

$\Rightarrow xyz = \left(\frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log a}\right) = 1.$

20. (d) Consider $\log_3 [\log_3[\log_3 x]] = \log_3 3$

$\Rightarrow \log_3 [\log_3 x] = 3$

$\Rightarrow \log_3 x = 3^3$

$\Rightarrow \log_3 x = 27 \Rightarrow x = 3^{27}$

21. (a) $\frac{1}{(\log_a bc)+1} + \frac{1}{(\log_b ac)+1} + \frac{1}{(\log_c ab)+1}$

$= \frac{1}{\log_a bc + \log_a a} + \frac{1}{\log_b ac + \log_b b} + \frac{1}{\log_c ab + \log_c c}$

$= \frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc}$

$= \log_{abc} a + \log_{abc} b + \log_{abc} c$

$\log_{abc} abc = 1$

22. (c) a, b, c are consecutive integers

$\therefore b = a + 1$ and $c = a + 2$

$\therefore \log(ac + 1) = \log[a(a + 2) + 1]$

$= \log[(b - 1)(b - 1 + 2) + 1]$

$[\because a = b - 1]$

$= \log b^2 = 2 \log b$

23. (c) Let $\log_{2\sqrt{3}}(1728) = x$.

Then, $\left(2\sqrt{3}\right)^x = 1728 = (12)^3$

\ $= \left[\left(2\sqrt{3}\right)^x\right]^3 = \left(2\sqrt{3}\right)^6$.

$\therefore x = 6$, *i.e.*, $\log_{2\sqrt{3}}(1728) = 6$.

24. (c) $\left(7^3\right)^{-2\log_7 8} = 7^{-6\log_7 8} = 7^{\left(\log_7 8^{-6}\right)}$

$= 8^{-6} = \frac{1}{8^6}$

25. (d) $\log_4 5 = a$ and $\log_5 6 = b$

$\Rightarrow \log_4 5 \times \log_5 6 = ab$

$\Rightarrow \log_4 6 = ab \Rightarrow \frac{1}{2}\log_2 6 = ab$

$\Rightarrow (1 + \log_2 3) = 2ab$

$\Rightarrow \log_2 3 = 2ab - 1$

$\Rightarrow \log_3 2 = \frac{1}{2ab - 1}$

26. (b) $x = \log_a bc \therefore 1 + x = \log_a abc$

$y = \log_b ca \therefore 1 + y = \log_b abc$

$z = \log_c bc \therefore 1 + z = \log_c abc$

$\therefore$

$\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z} = \log_{abc} a + \log_{abc} b + \log_{abc} c$

$= \log_{abc} abc = 1$

27. (d) $\log_{32} 27 \times \log_{243} 8 = \log_{2^5} 3^3 \times \log_{3^5} 2^3$

$= \frac{3}{5}\log_2 3 \times \frac{3}{5}\log_3 2$

$= \left(\frac{3}{5}\right)^2 \log_2 3 \times \log_3 2 = \frac{9}{25}$

28. (c) $\log\left(a^n b^n c^n / a^n b^n c^n\right) = \log 1 = 0$

29. (b) Best way is to go through options

Alternatively: $\log_{10} x^2 y^3 = 7$

$\Rightarrow x^2 y^3 = 10^7$ ...(1)

and $\log_{10}\left(\frac{x}{y}\right) = 1$

$\Rightarrow \frac{x}{y} = 10$ ...(2)

$\therefore \frac{x^2 y^3}{(x/y)^2} = \frac{10^7}{(10)^2} \Rightarrow y^5 = 10^5$

$\Rightarrow y = 10 \therefore x = 100$

30. (a) For $x = 0$, we have LHS

$\log_2 8 = 3$.

RHS: $10^{\log 3} = 3$

We do not get LHS = RHS for either $x = 3$ or $x = 6$.

Thus, option (a) is correct.

31. (b) $\left(\log_{\frac{1}{2}} 2\right)\left(\log_{\frac{1}{3}} 3\right)\left(\log_{\frac{1}{4}} 4\right).....\left(\log_{\frac{1}{1000}} 1000\right)$

$$= \left(\frac{\log 2}{\log \frac{1}{2}}\right)\left(\frac{\log 3}{\log \frac{1}{3}}\right)\left(\frac{\log 4}{\log \frac{1}{4}}\right).....\left(\frac{\log 1000}{\log\left(\frac{1}{1000}\right)}\right)$$

$\left(\because \log_b a = \frac{\log a}{\log b}\right)$

$$= \left(\frac{\log 2}{-\log 2}\right)\left(\frac{\log 3}{-\log 3}\right)\left(\frac{\log 4}{-\log 4}\right).....\left(\frac{\log 1000}{-\log 1000}\right)$$

$= (-1) \times (-1) \times (-1) \times ..... \times (-1)$

($\because$ number of factors is odd)

$= -1$

32. (d) Given, $\log_r 6 = m$ and $\log_r 3 = n$

$\because$ $\log_r 6 = \log_r (2 \times 3)$

$= \log_r 2 + \log_r 3$

$\therefore$ $\log_r 3 + \log_r 2 = m$

$\Rightarrow$ $n + \log_r 2 = m$

$\Rightarrow$ $\log_r 2 = m - n$

$\therefore$ $\log_r \left(\frac{r}{2}\right) = \log_r r - \log_r 2$

$= 1 - m + n$

33. (c) $\frac{1}{3}\log_{10} 125 - 2\log_{10} 4 + \log_{10} 32 + \log_{10} 1$

$= \frac{1}{3}\log_{10} (5)^3 - 2\log_{10} (2)^2 + \log_{10} (2)^5 + 0$

$= \log_{10} 5 - 4\log_{10} 2 + 5\log_{10} 2$

$= \log_{10} 5 + \log_{10} 2 = \log_{10} 5 \times 2 = \log_{10} 10 = 1$

34. (c) $\frac{1}{2}\log_{10} 25 - 2\log_{10} 3 + \log_{10} 18$

$= \log_{10} 25^{1/2} - \log_{10} 3^2 + \log_{10} 18$

$= \log_{10} 5 - \log_{10} 9 + \log_{10} 18$

$= \log_{10} \frac{5 \times 18}{9} = \log_{10} \frac{90}{9} = \log_{10} 10 = 1$

35. (d) $[\log_{10} (5 \log_{10} 100)]^2 = [\log_{10} (5 \log_{10} 10^2)]^2$

$= [\log_{10} (10 \log_{10} 10)]^2$

$= [\log_{10} 10]^2$ $(\because \log_{10} 10 = 1)$

$= 1^2 = 1$

36. (c) $\log_{10}\left(\frac{3}{2}\right) + \log_{10}\left(\frac{4}{3}\right) + \log_{10}\left(\frac{5}{4}\right) + ... +$ 8th term

$= \log_{10}\left(\frac{3}{2}\right) + \log_{10}\left(\frac{4}{3}\right) + \log_{10}\left(\frac{5}{4}\right) + ... + \log_{10}\left(\frac{10}{9}\right)$

$= \log_{10}\left(\frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times ..... \times \frac{10}{9}\right)$

$= \log_{10}\left(\frac{10}{2}\right) = \log_{10} 5.$

37. (c) Let $\log_{10} 0.0001 = a$

$a = \log_{10} \frac{1}{(10)^4}$

$= \log_{10} 1 - \log_{10} (10)^4 = 0 - 4 = -4$

38. (a) Given that,

$\log_{10} a = p$ and $\log_{10} b = q$

$\log_{10} (a^p b^q) = \log_{10} a^p + \log_{10} b^q$

$= p \log_{10} a + q \log_{10} b$

$= p \times p + q \times q = p^2 + q^2$

39. (b) $\frac{\log_{13} (10)}{\log_{169} (10)} = \frac{\log_{13} (10)}{\log_{13^2} (10)}$ $\left(\because \log_{a^b} c = \frac{1}{b}\log_a c\right)$

$= \frac{\log_{13} 10}{\frac{1}{2}\log_{13} 10} = \frac{1}{\frac{1}{2}} = 2$

40. (b) $\frac{1}{5}\log_{10} 3125 - 4\log_{10} 2 + \log_{10} 32$

$= \frac{1}{5}\log_{10} (5)^5 - 4\log_{10} 2 + \log_{10} (2)^5$

$= \frac{5}{5}\log_{10} 5 - 4\log_{10} 2 + 5\log_{10} 2$

$= \log_{10} 5 + \log_{10} 2$

$= \log_{10} (5 \times 2)$

$[\log m + \log n = \log mn]$

$= \log_{10} 10 = 1$