

Factors of Algebraic Expressions

Exercise 76:

Solution 1:

- $7p = 7 \times p$
 \therefore Factors of $7p$ are 7 and p .
- $6m = 6 \times m = 2 \times 3 \times m$
 \therefore Factors of $6m$ are 2, 3, and m .
- $9xy = 9 \times x \times y = 3 \times 3 \times x \times y$
 \therefore Factors of $9xy$ are 3, 3, x , and y .
- $22ab = 22 \times a \times b = 2 \times 11 \times a \times b$
 \therefore Factors of $22ab$ are 2, 11, a , and b .
- $p^2q = p \times p \times q$
 \therefore Factors of p^2q are p , p , and q .
- $10xy^2 = 2 \times 5 \times x \times y \times y$
 \therefore Factors of $10xy^2$ are 2, 5, x , y , and y .
- $5a^2 = 5 \times a \times a$
 \therefore Factors of $5a^2$ are 5, a , and a .
- $15m^2n = 3 \times 5 \times m \times m \times n$
 \therefore Factors of $15m^2n$ are 3, 5, m , m , and n .
- $30a^2b^2 = 2 \times 3 \times 5 \times a \times a \times b \times b$
 \therefore Factors of $30a^2b^2$ are 2, 3, 5, a , a , b , and b .
- $12x^3 = 2 \times 2 \times 3 \times x \times x \times x$
 \therefore Factors of $12x^3$ are 2, 2, 3, x , x , and x .

Exercise 77:

Solution 1:

- $8m, 4m^2n$
 $8m = 2 \times 2 \times 2 \times m$
 $4m^2n = 2 \times 2 \times m \times m \times n$
- $3x^2y, 12xy^2$
 $3x^2y = 3 \times x \times x \times y$
 $12xy^2 = 2 \times 2 \times 3 \times x \times y \times y$
- $15a^2bc, 5ab, 20abc^2$
 $15a^2bc = 3 \times 5 \times a \times a \times b \times c$
 $5ab = 5 \times a \times b$
 $20abc^2 = 2 \times 2 \times 5 \times a \times b \times c \times c$

Solution 2:

- $4p^2q, 16pq^2$
 $4p^2q = 2 \times 2 \times p \times p \times q$
 $16pq^2 = 2 \times 2 \times 2 \times 2 \times p \times q \times q$
 \therefore Common factors $= 2 \times 2 \times p \times q = 4pq$
- $18x^3y^2, 12x^3y$
 $18x^3y^2 = 2 \times 3 \times 3 \times x \times x \times x \times y \times y$
 $12x^3y = 2 \times 2 \times 3 \times x \times x \times x \times y$
 \therefore Common factors $= 2 \times 3 \times x \times x \times x \times y = 6x^3y$
- $7a^3b^2c, 28a^2bc^2$
 $7a^3b^2c = 7 \times a \times a \times a \times b \times b \times c$
 $28a^2bc^2 = 2 \times 2 \times 7 \times a \times a \times b \times c \times c$
 \therefore Common factors $= 7 \times a \times a \times b \times c = 7a^2bc$
- $8x^3y^2, 10x^2y^3, 6x^2y$
 $8x^3y^2 = 2 \times 2 \times 2 \times x \times x \times x \times y \times y$
 $10x^2y^3 = 2 \times 5 \times x \times x \times y \times y \times y$
 $6x^2y = 2 \times 3 \times x \times x \times y$
 \therefore Common factors $= 2 \times x \times x \times y = 2x^2y$
- $24mnp^2, 22m^2p^2, 30m^2n^2p$
 $24mnp^2 = 2 \times 2 \times 2 \times 3 \times m \times n \times p \times p$
 $22m^2p^2 = 2 \times 11 \times m \times m \times p \times p$
 $30m^2n^2p = 2 \times 3 \times 5 \times m \times m \times n \times n \times p$
 \therefore Common factors $= 2 \times m \times p = 2mp$

Solution 3:

- $6m^2n^2, 10m^2n$
Common factors: $2m^2n$
- $38a^3b^2, 57ab^2$
Common factors: $19ab^2$
- $11x^2y^3, xy^2$
Common factors: xy^2
- $35p^2q^2r, 40q^3r^2, 50pq^2r$
Common factors: $5q^2r$
- $15x^3y^3, 39x^2z^2, 48xy^2z^3$
Common factors: $3x$

Exercise 78:

Solution 1:

- $4a + 8b = 4 \times a + 4 \times 2 \times b$
 $= 4(a + 2b)$
- $5m + 15n = 5 \times m + 5 \times 3 \times n$
 $= 5(m + 3n)$
- $abp - abq = ab \times p - ab \times q$
 $= ab(p - q)$
- $x^2 + x^3 = x^2 + x^2 \times x$
 $= x^2(1 + x)$
- $mnx + mny = mn \times x + mn \times y$
 $= mn(x + y)$
- $4x^2y + 3xy^2 = 4 \times x \times x \times y + 3 \times x \times y \times y$
 $= xy(4x + 3y)$
- $15p^2q - 20q = 3 \times 5 \times p \times p \times q - 4 \times 5 \times q$
 $= 5q(3p^2 - 4)$
- $a^2bc + abc^2 = a \times a \times b \times c + a \times b \times c \times c$
 $= abc(a + c)$
- $18m^2n - 27m^3 = 2 \times 9 \times m^2 \times n - 3 \times 9 \times m^2 \times m$
 $= 9m^2(2n - 3m)$
- $24p^3q^2 + 28p^2q^3 = 4 \times 6 \times p^2 \times p \times q^2 + 4 \times 7 \times p^2 \times q^2 \times q$
 $= 4p^2q^2(6p + 7q)$

Exercise 79:

Solution 1:

- $ab + cd + ac + bd$
 $= ab + ac + bd + cd$
 $= a(b + c) + d(b + c)$
 $= (a + d)(b + c)$
- $2x^2 + 4x^3 + 2x + 1$
 $= 4x^3 + 2x^2 + 2x + 1$
 $= 2x^2(2x + 1) + 1(2x + 1)$
 $= (2x + 1)(2x^2 + 1)$
- $ax + bx - ay - by$
 $= ax - ay + bx - by$
 $= a(x - y) + b(x - y)$
 $= (x - y)(a + b)$
- $y - 1 + y^3 - y^2$
 $= y - 1 + y^3 - y^2$

- $$= 1(y - 1) + y^2(y - 1)$$
- $$= (y - 1)(1 + y^2)$$
- $b^2 + bc + ab + ac$
 $= b^2 + bc + ab + ac$
 $= b(b + c) + a(b + c)$
 $= (b + c)(b + a)$
 - $2x^2 + xy - 2xy^2 - y^3$
 $= 2x^2 - 2xy^2 + xy - y^3$
 $= 2x(x - y^2) + y(x - y^2)$
 $= (x - y^2)(2x + y)$
 - $12pm + 18qm + 6pn + 9nq$
 $= 12pm + 6pn + 18qm + 9nq$
 $= 6p(2m + n) + 9q(2m + n)$
 $= (2m + n)(6p + 9q)$
 $= (2m + n)3(2p + 3q)$
 $= 3(2p + 3q)(2m + 3n)$
 - $m^3 + m^2 + m + 1$
 $= m^3 + m^2 + m + 1$
 $= m^2(m + 1) + 1(m + 1)$
 $= (m^2 + 1)(m + 1)$
 - $am + an + al + bm + bl + bn$
 $= am + an + al + bm + bl + bn$
 $= a(m + n + l) + b(m + n + l)$
 $= (a + b)(m + n + l)$
 - $3y^3 - 6y^2 + 4y - 8$
 $= 3y^3 - 6y^2 + 4y - 8$
 $= 3y^2(y - 2) + 4(y - 2)$
 $= (3y^2 + 4)(y - 2)$

Exercise 80:

Solution 1(1):

$$y^2 + 8y + 16$$

$$= (y)^2 + 2 \times 4 \times y + (4)^2$$

$$= (y + 4)^2$$

$$= (y + 4)(y + 4)$$

Solution 1(2):

$$\begin{aligned} & 1 - 8a + 16a^2 \\ &= (1)^2 - 2 \times 4 \times a + (4a)^2 \\ &= (1 - 4a)^2 \\ &= (1 - 4a)(1 - 4a) \end{aligned}$$

Solution 1(3)

$$\begin{aligned} & x^2 - 4x + 4 \\ &= (x)^2 - 2 \times 2 \times x + (2)^2 \\ &= (x - 2)^2 \\ &= (x - 2)(x - 2) \end{aligned}$$

Solution 1(4):

$$\begin{aligned} & n^2 - 2mn + m^2 \\ &= (n)^2 - 2 \times n \times m + (m)^2 \\ &= (n - m)^2 \\ &= (n - m)(n - m) \end{aligned}$$

Solution 1(5):

$$\begin{aligned} & 36 - 12p + p^2 \\ &= (6)^2 - 2 \times 6 \times p + (p)^2 \\ &= (6 - p)^2 \\ &= (6 - p)(6 - p) \end{aligned}$$

Solution 1(6):

$$\begin{aligned} & q^2 + 12q + 36 \\ &= (q)^2 + 2 \times q \times 6 + (6)^2 \\ &= (q + 6)^2 \\ &= (q + 6)(q + 6) \end{aligned}$$

Solution 1(7):

$$\begin{aligned} & 49 + 28y + 4y^2 \\ &= (7)^2 + 2 \times 7 \times 2y + (2y)^2 \\ &= (7 + 2y)^2 \\ &= (7 + 2y)(7 + 2y) \end{aligned}$$

Solution 1(8):

$$\begin{aligned} & 16m^2 - 24mn + 9n^2 \\ &= (4m)^2 - 2 \times 4m \times 3n + (3n)^2 \\ &= (4m - 3n)^2 \\ &= (4m - 3n)(4m - 3n) \end{aligned}$$

Solution 1(9):

$$\begin{aligned} & 4b^2 + 12ab + 9a^2 \\ &= (2b)^2 + 2 \times 2b \times 3a + (3a)^2 \\ &= (2b + 3a)^2 \\ &= (2b + 3a)(2b + 3a) \end{aligned}$$

Solution 1(10):

$$\begin{aligned} & 9x^2 - 24xy + 16y^2 \\ &= (3x)^2 - 2 \times 3x \times 4y + (4y)^2 \\ &= (3x - 4y)^2 \\ &= (3x - 4y)(3x - 4y) \end{aligned}$$

Solution 1(11):

$$\begin{aligned} & a^2 - 10a + 25 \\ &= (a)^2 - 2 \times a \times 5 + (5)^2 \\ &= (a - 5)^2 \\ &= (a - 5)(a - 5) \end{aligned}$$

Solution 1(12):

$$\begin{aligned} & 36 + 36y + 9y^2 \\ &= (6)^2 + 2 \times 6 \times 3y + (3y)^2 \\ &= (6 + 3y)^2 \\ &= (6 + 3y)(6 + 3y) \end{aligned}$$

Solution 1(13):

$$\begin{aligned} & 16m^2 - 40mn + 25n^2 \\ &= (4m)^2 - 2 \times 4m \times 5n + (5n)^2 \\ &= (4m - 5n)^2 \\ &= (4m - 5n)(4m - 5n) \end{aligned}$$

Solution 1(14):

$$\begin{aligned} & 1 + 12q + 36q^2 \\ &= (1)^2 + 2 \times 1 \times 6q + (6q)^2 \\ &= (1 + 6q)^2 \\ &= (1 + 6q)(1 + 6q) \end{aligned}$$

Solution 1(15):

$$\begin{aligned} & 4m^2 + 36m + 81 \\ &= (2m)^2 + 2 \times 2m \times 9 + (9)^2 \\ &= (2m + 9)^2 \\ &= (2m + 9)(2m + 9) \end{aligned}$$

Solution 1(16):

$$\begin{aligned} & 64 - 48n + 9n^2 \\ &= (8)^2 - 2 \times 8 \times 3n + (3n)^2 \\ &= (8 - 3n)^2 \\ &= (8 - 3n)(8 - 3n) \end{aligned}$$

Solution 1(17):

$$\begin{aligned} & 81a^2 - 72ab + 16b^2 \\ &= (9a)^2 - 2 \times 9a \times 4b + (4b)^2 \\ &= (9a - 4b)^2 \\ &= (9a - 4b)(9a - 4b) \end{aligned}$$

Solution 1(18):

$$\begin{aligned} & 4p^2 + 24pq + 36q^2 \\ &= (2p)^2 + 2 \times 2p \times 6q + (6q)^2 \\ &= (2p + 6q)^2 \\ &= (2p + 6q)(2p + 6q) \end{aligned}$$

Exercise 81:**Solution 1(1):**

$$\begin{aligned} & x^2 - y^2 \\ &= (x)^2 - (y)^2 \\ &= (x + y)(x - y) \end{aligned}$$

Solution 1(2):

$$\begin{aligned} & m^2 - 25^2 \\ &= (m)^2 - (5)^2 \\ &= (m + 5)(m - 5) \end{aligned}$$

Solution 1(3):

$$\begin{aligned} & 1 - p^2 \\ &= (1)^2 - (p)^2 \\ &= (1 + p)(1 - p) \end{aligned}$$

Solution 1(4):

$$\begin{aligned}16 - a^2 \\&= (4)^2 - (a)^2 \\&= (4 + a)(4 - a)\end{aligned}$$

Solution 1(5):

$$\begin{aligned}36r^2 - 1 \\&= (6r)^2 - (1)^2 \\&= (6r + 1)(6r - 1)\end{aligned}$$

Solution 1(6):

$$\begin{aligned}121 - 49n^2 \\&= (11)^2 - (7n)^2 \\&= (11 + 7n)(11 - 7n)\end{aligned}$$

Solution 1(7):

$$\begin{aligned}4x^2 - 81y^2 \\&= (2x)^2 - (9y)^2 \\&= (2x + 9y)(2x - 9y)\end{aligned}$$

Solution 1(8):

$$\begin{aligned}169p^2 - 81r^2 \\&= (13p)^2 - (9r)^2 \\&= (13p + 9r)(13p - 9r)\end{aligned}$$

Solution 1(9):

$$\begin{aligned} & m^2 - \frac{49}{64} \\ &= (m)^2 - \left(\frac{7}{8}\right)^2 \\ &= \left(m + \frac{7}{8}\right)\left(m - \frac{7}{8}\right) \end{aligned}$$

Solution 1(10):

$$\begin{aligned} & \frac{r^2}{s^2} - \frac{81}{100} \\ &= \left(\frac{r}{s}\right)^2 - \left(\frac{9}{10}\right)^2 \\ &= \left(\frac{r}{s} + \frac{9}{10}\right)\left(\frac{r}{s} - \frac{9}{10}\right) \end{aligned}$$

Solution 1(11):

$$\begin{aligned} & \frac{16}{25} - \frac{y^2}{x^2} \\ &= \left(\frac{4}{5}\right)^2 - \left(\frac{y}{x}\right)^2 \\ &= \left(\frac{4}{5} + \frac{y}{x}\right)\left(\frac{4}{5} - \frac{y}{x}\right) \end{aligned}$$

Solution 1(12):

$$\begin{aligned} & 1 - \frac{36m^2}{49n^2} \\ &= (1)^2 - \left(\frac{6m}{7n}\right)^2 \\ &= \left(1 + \frac{6m}{7n}\right)\left(1 - \frac{6m}{7n}\right) \end{aligned}$$