

CBSE

Model Answer Sheet 2014

WITH GRAPH PAPER
केन्द्रीय माध्यमिक शिक्षा बोर्ड, दिल्ली
सीनियर स्कूल सर्टिफिकेट परीक्षा (कक्षा बारहवीं)
परीक्षार्थी प्रवेश-पत्र के अनुसार भरें

विषय Subject : MATHEMATICS

परीक्षा का दिन एवं तिथि
Day & Date of the Examination : THURSDAY, 20/03/2014

उत्तर देने का माध्यम
Medium of answering the paper : ENGLISH

प्रश्न पत्र के ऊपर लिखे कोड को दर्शाए
Write Code No. as written on the
top of Question Paper : 65/2

अतिरिक्त उत्तर-पुस्तिका (ओं) की संख्या
No. of Supplementary answer-book(s) used 1

किसी शारीरिक अक्षमता के प्रभावित हो तो संबंधित वर्ग में ✓ का निशान लगाएं।
If Physically challenged, tick the category

☐ B ☐ D ☐ H ☐ S ☐ C

B = दृष्टिहीन, D = मूक एवं बधिर, H = शारीरिक रूप से विकलांग, S = स्फुरितक, C = डिस्लेक्सिक
B = Blind, D = Deaf & Dumb, H = Physically Handicapped, S = Spastic, C = Dyslexic

क्या लेखन - लिपिक उपलब्ध करवाया गया : हाँ/नहीं
Whether writer provided : Yes/No -

*एक खाने में एक अक्षर लिखें। नाम के प्रत्येक भाग के बीच एक खाना रिक्त छोड़ दें।
यदि परीक्षार्थी का नाम 24 अक्षरों से अधिक है, तो केवल नाम के प्रथम 24 अक्षर ही लिखें।

Each letter be written in one box and one box be left blank between each part of the name. In case Candidate's Name exceed 24 letters, write first 24 letters.

कार्यालय उपयोग के लिए
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केन्द्रीय माध्यमिक शिक्षा बोर्ड, दिल्ली
Central Board of Secondary Education, Delhi

सीनियर स्कूल सर्टिफिकेट परीक्षा (कक्षा बारहवीं)
SENIOR SCHOOL CERTIFICATE EXAMINATION (CLASS XII)



प्रमाणित किया जाता है मैंने/हमने इस उत्तर पुस्तिका का मूल्यांकन प्रश्न पत्र के समुचित सेट के अनुसार और पूर्ण रूप से मूल्यांकन पद्धति के अनुसार किया है।
Certified that I/We have evaluated this answer-book according to the correct set of question paper and strictly as per the marking scheme.

Section - C

24. Let $P(E_1)$ be the probability of a two headed coin to be chosen,
 $P(E_2)$ be the probability of a biased coin showing 75% times head to be chosen
 and $P(E_3)$ be probability of a coin showing 40% tails to be chosen.

Let 'A' be the event of the tossed coin showing heads.

E_1, E_2, E_3 are mutually exclusive and exhaustive events.

By Baye's theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) P\left(\frac{A}{E_1}\right)}{P(E_1) P\left(\frac{A}{E_1}\right) + P(E_2) P\left(\frac{A}{E_2}\right) + P(E_3) P\left(\frac{A}{E_3}\right)} \rightarrow (1)$$

$$P(E_1) P\left(\frac{A}{E_1}\right) + P(E_2) P\left(\frac{A}{E_2}\right) + P(E_3) P\left(\frac{A}{E_3}\right)$$

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

$$P\left(\frac{A}{E_1}\right) = 1$$

[A loaded coin]

$$P\left(\frac{A}{E_2}\right) = \frac{75}{100} = \frac{3}{4}$$

[75% heads]

$$P\left(\frac{A}{E_3}\right) = \frac{60}{100} = \frac{3}{5}$$

[it shows 40% tails
∴ 60% heads]

$$\therefore P\left(\frac{E_1}{A}\right) = \frac{P\left(\frac{A}{E_1}\right)(1)}{\frac{1}{3}(1) + \frac{3}{4}\left(\frac{3}{4}\right) + \frac{1}{5}\left(\frac{3}{5}\right)}$$

[using Baye's
theorem ①]

$$\frac{1}{3}(1) + \frac{3}{4}\left(\frac{3}{4}\right) + \frac{1}{5}\left(\frac{3}{5}\right)$$

$$= \frac{1}{3}$$

$$= \frac{1}{3}$$

$$1 + \frac{3}{4} + \frac{3}{5}$$

$$\frac{20+15+12}{20}$$

$$= \frac{20}{20+27}$$

$$= \frac{20}{47}$$

$$\therefore \text{Probability that it was the two headed coin when the randomly tossed coin showed head} = \frac{30}{47} \left[P\left(\frac{E}{A}\right) \right]$$

$$= \frac{20}{47}$$

25. Let π_1 be $x+y+z=1$

π_2 be $2x+3y+4z=5$

Let the equation of the plane passing through the line of intersection of both planes be π .

$$\pi = \pi_1 + \lambda \pi_2$$

$$\therefore \pi = (x+y+z-1) + \lambda(2x+3y+4z-5) = 0$$

$$\therefore (x+y+z-1) + \lambda(2x+3y+4z-5) = 0$$

$$x(1+2\lambda) + y(1+3\lambda) + z(1+4\lambda) - 1 - 5\lambda = 0$$

$(1+2\lambda)$, $(1+3\lambda)$ and $(1+4\lambda)$ are the direction ratios of the plane π .

$\therefore$ the plane (π) is $\perp R$ to $[\pi_3 = x - y + z = 0]$
sum of product of direction ratios is zero.

$$\therefore (1+2\lambda)(1) - 1(1+3\lambda) + 1(1+4\lambda) = 0$$

$$\begin{aligned}\therefore 1+2\lambda + 1 - 1 - 3\lambda + 1 + 4\lambda &= 0 \\ 1 &= 3\lambda - 6\lambda \\ &= -3\lambda \\ \lambda &= -1/3\end{aligned}$$

$$\therefore \pi = x(1+2\lambda) + y(1+3\lambda) + z(1+4\lambda) - 1 - 5\lambda = 0$$

$$\Rightarrow x(1 - \frac{2}{3}) + y(1 - \frac{3}{3}) + z(1 - \frac{4}{3}) - 1 + \frac{5}{3} = 0$$

$$x(\frac{1}{3}) + y(0) + z(-\frac{1}{3}) + \frac{2}{3} = 0$$

$$x - z + 2 = 0$$

$\therefore$ The equation of the required plane is $x - z + 2 = 0$

Distance of a point (x_1, y_1, z_1) from a plane $Ax + By + Cz = D$ is given by:

$$d = \frac{|Ax_1 + By_1 + Cz_1 - D|}{\sqrt{A^2 + B^2 + C^2}}$$

$$\therefore \text{Distance from the origin} = \frac{|0 + 0 + 0 + 2|}{\sqrt{1 + 1}}$$

$$= \frac{2}{\sqrt{2}}$$

$$= \sqrt{2} \text{ units}$$

26. The amounts awarded for sincerity, truthfulness and helpfulness are Rs x , Rs y and Rs z respectively.

[awarded by schools A and B]

The equations are :

$$3x + 2y + z = 1600$$

$$4x + y + 3z = 2300$$

$$x + y + z = 900$$

$$\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$$

$A \quad 3 \times 3 \quad X \quad 3 \times 1 \quad B \quad 3 \times 1$

$$\therefore X = A^{-1} B$$

$$\begin{aligned} |A| &= 3(1-3) - 2(4-3) + 1(4-1) \\ &= 3(-2) - 2 + 3 \\ &= -6 - 2 + 3 \\ &= -8 + 3 \\ &= -5 \end{aligned}$$

$$|A| \neq 0 \quad \therefore A^{-1} \text{ exists.}$$

$\therefore$ The equation of the required plane is $x - z + 2 = 0$

Distance of a point (x_1, y_1, z_1) from a plane $Ax + By + Cz = D$ is given by:

$$d = \left| \frac{Ax_1 + By_1 + Cz_1 - D}{\sqrt{A^2 + B^2 + C^2}} \right|$$

$$\begin{aligned} \therefore \text{Distance from the origin} &= \left| \frac{0 + 0 + 0 + 2}{\sqrt{1+1}} \right| \\ &= \frac{2}{\sqrt{2}} \\ &= \sqrt{2} \text{ units} \end{aligned}$$

26. ~~At~~ The amounts awarded for sincerity, truthfulness and helpfulness are Rs x , Rs y and Rs z respectively.
[awarded by schools A and B]

The equations are :

$$3x + 2y + z = 1600$$

$$4x + y + 3z = 2300$$

$$x + y + z = 900$$

$$\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}_{3 \times 3} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3 \times 1} = \begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}_{3 \times 1}$$

$A \qquad \qquad \qquad X \qquad \qquad \qquad B$

$$\therefore X = A^{-1} B$$

$$\begin{aligned} |A| &= 3(1-3) - 2(4-3) + 1(4-1) \\ &= 3(-2) - 2 + 3 \\ &= -6 - 2 + 3 \\ &= -8 + 3 \\ &= -5 \end{aligned}$$

$$|A| \neq 0 \quad \therefore A^{-1} \text{ exists,}$$

$$\therefore X = A^{-1} B$$

$$A^{-1} = \frac{\text{adj}(A)}{|A|}$$

let A_{ij} represent the cofactors of elements in A .

$$A_{11} = -2$$

$$A_{12} = -1$$

$$A_{13} = 3$$

$$A_{21} = -1$$

$$A_{22} = 2$$

$$A_{23} = -1$$

$$A_{31} = 5$$

$$A_{32} = -5$$

$$A_{33} = -5$$

$$\therefore \text{adj}(A) = \begin{matrix} \text{Transpose of co-factor matrix} \\ \text{6x} \end{matrix}$$

$$\therefore \text{adj}(A) = \text{co-factor matrix}^T$$

$$\text{co-factor matrix} = \begin{bmatrix} -2 & -1 & 3 \\ -1 & 2 & -1 \\ 5 & -5 & -5 \end{bmatrix}$$

$$\text{adj}(A) = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\text{adj}(A)}{|A|}$$

[$A \neq 0$; inverse exists]

$$\therefore A^{-1} = \frac{\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}}{-5}$$

$$= \begin{bmatrix} 2/5 & 1/5 & -1 \\ 1/5 & -2/5 & 1 \\ -3/5 & 1/5 & 1 \end{bmatrix}$$

$$\therefore X = A^{-1}B$$

$$= \begin{bmatrix} 2/5 & 1/5 & -1 \\ 1/5 & -2/5 & 1 \\ -3/5 & 1/5 & 1 \end{bmatrix}_{3 \times 3} \begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}_{3 \times 1}$$

$$= \begin{bmatrix} 200 \\ 300 \\ 400 \end{bmatrix}$$

3x1

awards

∴ The amount to be given must
amount to:

RS 200 for sincerity
RS 300 for truthfulness
RS 400 for helpfulness.

$$\frac{3200 + 2300 - 900}{5}$$

$$= \frac{5500 - 900}{5}$$

$$= \frac{1100 - 900}{5}$$

$$= 200$$

$$\frac{1600 - 4600 + 900}{5}$$

$$= \frac{-3000 + 900}{5}$$

$$= \frac{-600 + 900}{5} = 300$$

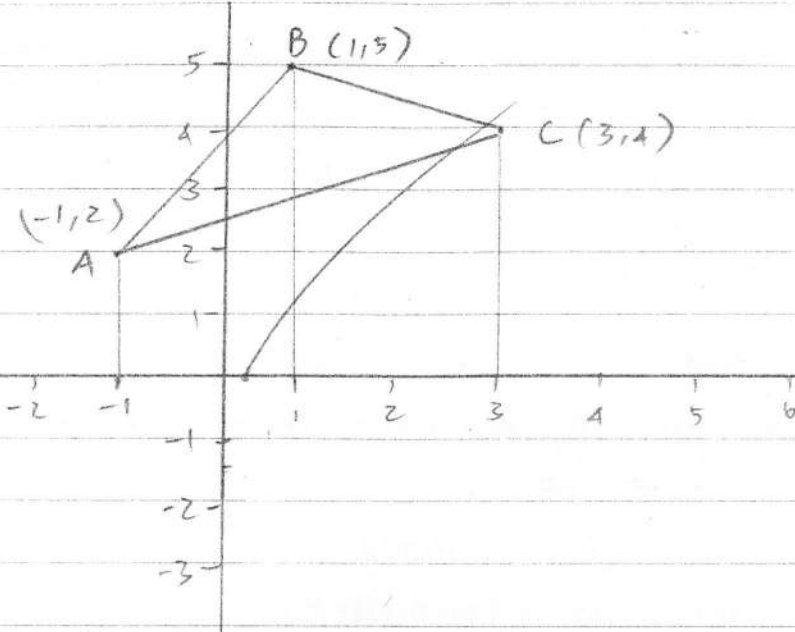
$$= \frac{-4800 + 2300 + 900}{5}$$

$$= \frac{-2500 + 900}{5}$$

* Another value which can be considered for awards is kindness. Kindness is an important quality in students that makes them good human beings. True Friendship can also be rewarded by schools.

Hence, kindness and true friendship are the other two values that can be rewarded.

27.



$$\frac{1}{2} \times 2 \times (5+4)$$

$$\frac{1}{2} \times 2 \times (2+4)$$

$$2(6) = 12$$

Equation of AB: $(y-5) = \frac{3}{2}(x-1)$

$$2y-10 = 3x-3$$

$$3x-2y+7=0$$

$$y = \frac{3x+7}{2}$$

Eq. of BC: $(y-5) = \frac{1}{-2}(x-1)$

$$-2y+10 = x-1$$

$$x+2y-11=0$$

$$y = \frac{11-x}{2}$$

Equation of A is

$$(y-4) = \frac{2}{1} (x-3)$$

$$2y - 8 = x - 3$$

$$x - 2y + 5 = 0$$

$$y = \frac{x+5}{2}$$

(I)

Area under AB is

$$A(I) = \int_{-1}^1 y \, dx$$

$$= \int_{-1}^1 \frac{1}{2} (3x+7) \, dx$$

$$= \frac{1}{2} \left(\frac{3x^2}{2} + 7x \right) \Big|_{-1}^1$$

$$= \frac{1}{2} \left[\frac{3}{2} + 7 - \left(\frac{3}{2} - 7 \right) \right]$$

$$= 7 \text{ sq. units}$$

II

Area under B:

$$A(II) = \int_1^3 y \, dx$$

$$= \int_1^3 \left(\frac{11-x}{2} \right) dx$$

$$= \frac{1}{2} \left[\frac{11x}{1} - \frac{x^2}{2} \right]_1^3$$

$$= \frac{1}{2} \left[33 - \frac{9}{2} - 11 + \frac{1}{2} \right]$$

$$= \frac{1}{2} \left[22 - \frac{8}{2} \right]$$

$$= \frac{1}{2} (22 - 4)$$

$$= 9 \text{ sq. units}$$

III

Area under A:

$$A(III) = \int_{-1}^3 y \, dx$$

$$= \int_{-1}^3 \frac{x+5}{2} dx$$

$$= \frac{1}{2} \int_{-1}^3 (x+5) dx$$

$$= \frac{1}{2} \left(\frac{x^2}{2} + 5x \right) \Big|_{-1}^3$$

$$= \frac{1}{2} \left[\frac{9}{2} + 15 - \frac{1}{2} + 5 \right]$$

$$= \frac{1}{2} [20 + 4] \quad \left(= \frac{24}{2} = 12 \right)$$

$$= 12 \text{ sq. units}$$

$$\therefore \text{Required area of the triangle} = A(I) + A(II) - A(III)$$

$$= (7 + 9 - 12) \text{ sq. units}$$

$$= (16 - 12) \text{ sq. units}$$

$$= 4 \text{ sq. units}$$

$$28. \quad I = \int (\sqrt{\cot x} + \sqrt{\tan x}) \, dx$$

$$I = \int \sqrt{\tan x} (1 + \cot x) \, dx$$

$$\sqrt{\tan x} = t$$

$$\tan x = t^2$$

$$\cot x = \frac{1}{t^2}$$

$$\sec^2 x \, dx = 2t \, dt$$

$$dx = \frac{2t \, dt}{1+t^4}$$

$$\sec^2 x = 1 + \tan^2 x$$

$$= 1 + t^4$$

$$\therefore I = \int t \left(1 + \frac{1}{t^2}\right) \frac{2t \, dt}{1+t^4}$$

$$= \int t \left(\frac{t^2+1}{t^2}\right) \frac{2t \, dt}{1+t^4}$$

$$= \int \frac{2t^2 (t^2+1)}{t^2 (1+t^4)} \, dt$$

$$= 2 \int \frac{(t^2+1)}{(t^4+1)} \, dt$$

($\div$ by $+2$)

$$\therefore I = 2 \int \frac{(1 + 1/t^2) dt}{(t^2 + 1/t^2)}$$

$$= 2 \int \frac{(1 + 1/t^2) dt}{(t - \frac{1}{t})^2 + 2}$$

Putting $t - \frac{1}{t} = u$

$$(1 + \frac{1}{t^2}) dt = du$$

$$\therefore I = 2 \int \frac{du}{u^2 + 2}$$

$$\therefore I = 2 \int \frac{du}{u^2 + (\sqrt{2})^2}$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + C_1$$

$$\therefore I = 2 \int \frac{du}{u^2 + (\sqrt{2})^2}$$

$$= 2 \times \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{u}{\sqrt{2}} \right) + C$$

$$= \sqrt{2} \tan^{-1} \left(\frac{u}{\sqrt{2}} \right) + C$$

$$= \sqrt{2} \tan^{-1} \left[\frac{t - \frac{1}{t}}{\sqrt{2}} \right] + C$$

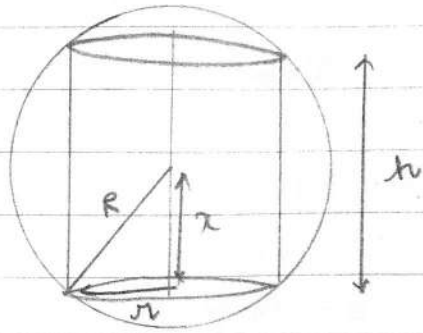
$$= \sqrt{2} \tan^{-1} \left[\frac{\sqrt{\tan x} - \frac{1}{\sqrt{\tan x}}}{\sqrt{2}} \right] + C$$

$$= \sqrt{2} \tan^{-1} \left[\frac{\tan x - 1}{\sqrt{2 \tan x}} \right] + C$$

$$\therefore I = \int (\sqrt{1+t} + \sqrt{\tan x}) dx$$

$$= \sqrt{2} \tan^{-1} \left[\frac{\tan x - 1}{\sqrt{2 \tan x}} \right] + C$$

29.



$$h = 2x$$

The radius of the sphere is R .

Let radius of cylinder be r , and its height be h .

$$\text{Let } x \text{ be the } x = \frac{h}{2} \rightarrow (1)$$

$$R^2 = x^2 + r^2 \quad r^2 = R^2 - x^2 \rightarrow (2)$$

$$\begin{aligned} \text{Volume} &= \pi r^2 h \\ &= \pi (R^2 - x^2) 2x \quad [\text{from (1) and (2)}] \end{aligned}$$

$$\therefore V(x) = \pi (R^2 - x^2) 2x$$

$$V'(x) = 0 \quad (\because \text{Maximum Volume})$$

$$V'(x) = \pi(R^2 - x^2) \cdot 2 + \pi(2x) [-2x] = 0$$

$$\therefore \pi(R^2 - x^2) \cdot 2 + \pi(2x)(-2x) = 0$$

$$R^2 - x^2 - 2x^2 = 0$$

$$R^2 = 3x^2$$

$$R = \sqrt{3} x$$

$$x = \frac{R}{\sqrt{3}}$$

→ (3)

$$h^2 = R^2 - x^2$$

$$= R^2 - \frac{R^2}{3}$$

$$= \frac{2R^2}{3}$$

∴ volume :

$$\therefore h = 2x$$

(from (1))

$$\therefore h = \frac{2R}{\sqrt{3}}$$

(from (3))

Hence ; proved. // (that height is $\frac{2R}{\sqrt{3}}$)

Second derivative test:

$$V''(x) = 2\pi(-2x) + \pi(-8x)$$

$$= -4\pi x - 8\pi x$$

$$= -12\pi x$$

$$< 0$$

$$\therefore V''(x) < 0$$

$\therefore P$ is a point of maxima.

$$\therefore \text{Maximum Volume} = \pi r^2 h$$

$$= \pi \frac{2R^2}{3} \times \frac{2R}{\sqrt{3}}$$

$$= \frac{4\pi R^3}{3\sqrt{3}} \text{ cubic units.}$$

Section - B

11. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\frac{2x}{a^2} - \frac{2y}{b^2} \frac{dy}{dx} = 0$$

$$\frac{x}{a^2} = \frac{y}{b^2} \left(\frac{dy}{dx} \right)$$

$$\therefore \frac{dy}{dx} = \frac{b^2}{a^2} \frac{x}{y}$$

At any point (x_0, y_0) on the curve,

$$\therefore \frac{dy}{dx} = \frac{b^2}{a^2} \frac{x_0}{y_0}$$

$$\therefore \text{At } (\sqrt{2}a, b) \quad \frac{dy}{dx} = \frac{b^2}{a^2} \frac{\sqrt{2}a}{b}$$
$$= \frac{\sqrt{2}b}{a}$$

23.

90 RS

Scale =

X axis $\rightarrow 1 \text{ cm} \rightarrow 5 \text{ units}$
 Y axis $\rightarrow 1 \text{ cm} \rightarrow 5 \text{ units}$
 $x:10$; $y:10$ units

Let x be the no. of
 pieces of A and y be
 the no. of pieces of B to
 be manufactured

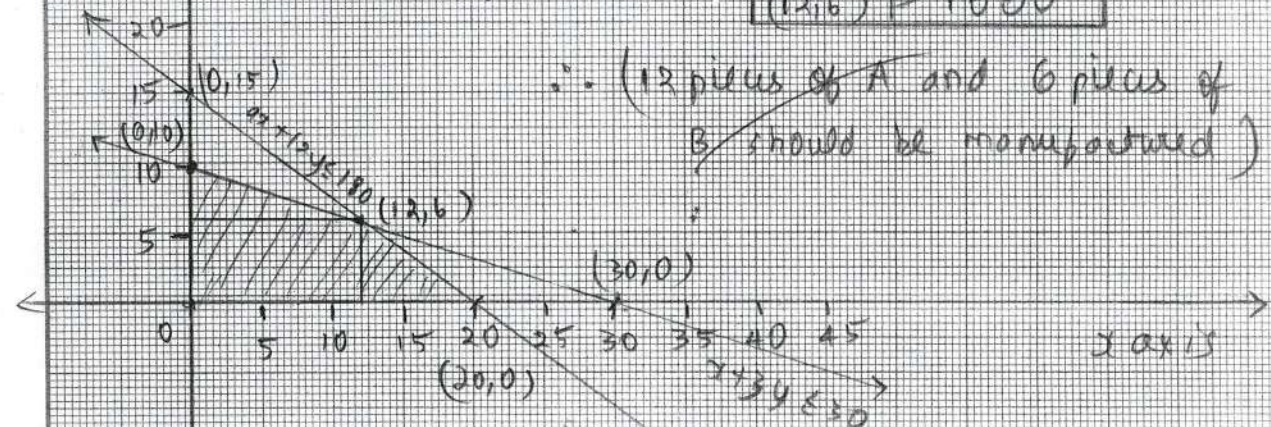
$$4x + 12y \leq 180$$

$$x + 3y \leq 30$$

$$Z = 80x + 120y$$

	x	y
Profit	RS 80	RS 120
fabricating hrs	4 hrs	12 hrs
finishing hrs	1 hr	3 hrs

(unit, profit)	Z
(0, 0)	RS 0
(0, 10)	RS 1200
(12, 6)	RS 1680
(30, 0)	RS 2400



$\therefore$ (12 pieces of A and 6 pieces of B should be manufactured)

Maximum profit
 $= 12(80) + 6(120)$
 $= \underline{\underline{RS 1680 \text{ per week}}}$

A $\rightarrow$ 12 pieces (per week)
 B $\rightarrow$ 6 pieces (per week)
 Max. profit = RS 1680 per week

Equation of tangent at $(\sqrt{2}a, b)$:

$$(y - b) = \frac{\sqrt{2}b}{a} (x - \sqrt{2}a)$$

$$(y - b)a = \sqrt{2}bx - 2ab$$

$$ay - by$$

$$ay - ab = \sqrt{2}bx - 2ab$$

$$\sqrt{2}bx - ay = ab$$

$$\therefore (\sqrt{2}a, b) \text{ lies on } \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{2a^2}{a^2} - \frac{b^2}{b^2} = 1$$

($\div$ by ab)

$$\frac{\sqrt{2}bx}{ab} - \frac{ay}{ab} = 1$$

$$\therefore \frac{\sqrt{2}x}{a} - \frac{y}{b} = 1 \quad (\text{or}) \quad \sqrt{2}xb - ay = ab$$

(Eq. of tangent at $(\sqrt{2}a, b)$)

Equation of normal at $(\sqrt{2}a, b)$:

$$(y-b) = -\frac{dx}{dy} (x-\sqrt{2}a)$$

$$\frac{dy}{dx} = \frac{\sqrt{2}b}{a}$$

$$\therefore -\frac{dx}{dy} = -\frac{a}{\sqrt{2}b}$$

$$\therefore (y-b) = \frac{-a}{\sqrt{2}b} (x-\sqrt{2}a)$$

$$\sqrt{2}by - \sqrt{2}b^2 = -ax + \sqrt{2}a^2$$

$$ax + \sqrt{2}by = \sqrt{2}(a^2 + b^2)$$

$$\frac{ax}{\sqrt{2}} + by = (a^2 + b^2) \quad \text{[Equation of normal at } (\sqrt{2}a, b)\text{]}$$

$$12. \quad I = \int_0^{\pi} \frac{4x \sin x \, dx}{1 + \cos^2 x}$$

$$I = 4 \int_0^{\pi} \frac{x \sin x \, dx}{1 + \cos^2 x} \rightarrow \textcircled{1}$$

$$\int_0^a f(x) \, dx = \int_0^a f(a-x) \, dx$$

$$\therefore I = 4 \int_0^{\pi} \frac{(\pi-x) \sin(\pi-x) \, dx}{1 + \cos^2(\pi-x)}$$

$$= 4 \int_0^{\pi} \frac{(\pi-x) \sin x \, dx}{1 + \cos^2 x} \rightarrow \textcircled{2}$$

Adding $\textcircled{1}$ and $\textcircled{2}$,

$$2I = 4 \int_0^{\pi} \frac{\pi \sin x \, dx}{1 + \cos^2 x}$$

$$\frac{2I}{2 \times \pi} = \int_0^{\pi} \frac{\sin x \, dx}{1 + \cos^2 x}$$

$$\frac{I}{2\pi} = \int_0^{\pi} \frac{\sin x \, dx}{1 + \cos^2 x}$$

$$\int_0^{2a} f(x) \, dx = 2 \int_0^a f(x) \, dx \quad [\text{if } f(2a-x) = f(x)]$$

$$\therefore \frac{I}{2\pi} = 2 \int_0^{\pi/2} \frac{\sin x \, dx}{1 + \cos^2 x}$$

$$\frac{I}{4\pi} = \int_0^{\pi/2} \frac{\sin x \, dx}{1 + \cos^2 x}$$

$$\cos x = t$$

$$\text{if } x=0 \quad t=1 \\ x=\pi/2 \quad t=0$$

$$-\sin x \, dx = dt$$

$$\therefore \frac{I}{4\pi} = \int_1^0 \frac{-dt}{1+t^2}$$

$$= \int_0^1 \frac{dt}{1+t^2} \\ = \left[\tan^{-1} t \right]_0^1$$

$$\left[\because \int_a^b f(x) \, dx = - \int_b^a f(x) \, dx \right]$$

$$\frac{I}{4\pi} = \frac{\pi}{4}$$

$$\therefore I = \pi^2$$

$$\therefore \int_0^{\pi} \frac{4x \sin x}{1 + \cos^2 x} dx = \pi^2$$

$$13. \quad y = p e^{ax} + q e^{bx} \rightarrow (1)$$

$$\frac{dy}{dx} = a p e^{ax} + b q e^{bx} \rightarrow (2)$$

From (1),

$$p e^{ax} = y - q e^{bx}$$

Substituting in (2),

$$\frac{dy}{dx} = a(y - q e^{bx}) + b q e^{bx}$$

$$\frac{d^2 y}{dx^2} = a^2 p e^{ax} + b^2 q e^{bx} \rightarrow (3)$$

Required to prove:

$$\frac{d^2y}{dx^2} - (a+b) \frac{dy}{dx} + aby = 0$$

Evaluating LHS,

$$\text{LHS} = \frac{d^2y}{dx^2} - (a+b) \frac{dy}{dx} + aby$$

$$= a^2 p e^{ax} + b^2 q e^{bx} - (a+b) [a p e^{ax} + b q e^{bx}]$$

[From (1), (2),
and (3)]

$$- ab (p e^{ax} + q e^{bx})$$

$$= a^2 p e^{ax} + b^2 q e^{bx} - a^2 p e^{ax} - b^2 q e^{bx}$$

$$= ab q e^{bx} - ab p e^{ax} - ab p e^{ax} - ab q e^{bx}$$

$$= 0 \quad (\text{cancelling all terms})$$

$$\text{RHS} = 0$$

$$\text{Hence } \text{LHS} = \text{RHS}$$

$$\therefore \frac{d^2y}{dx^2} - (a+b) \frac{dy}{dx} + aby = 0 \quad \text{Hence, proved,}$$

$$14. \quad \tan^{-1} \left[\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right] = \frac{\pi}{4} - \frac{\cos^{-1}x}{2}$$

$$\text{LHS} = \tan^{-1} \left[\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right]$$

$$x = \cos 2\theta \rightarrow (1)$$

$$\text{LHS} = \tan^{-1} \left[\frac{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}} \right]$$

$$= \tan^{-1} \left[\frac{\sqrt{2} \cos \theta - \sqrt{2} \sin \theta}{\sqrt{2} \cos \theta + \sqrt{2} \sin \theta} \right]$$

$$\therefore 1 + \cos 2\theta = 2 \cos^2 \theta$$

$$1 - \cos 2\theta = 2 \sin^2 \theta$$

$$\text{LHS} = \tan^{-1} \left[\frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} \right]$$

($\div$ numerator and denominator by $\cos \theta$)

$$\therefore \text{LHS} = \tan^{-1} \left[\frac{1 - \tan \theta}{1 + \tan \theta} \right]$$

$$\tan \left(\frac{\pi}{4} - \theta \right) = \frac{1 - \tan \theta}{1 + \tan \theta}$$

$$\therefore \text{LHS} = \tan^{-1} \left[\tan \left(\frac{\pi}{4} - \theta \right) \right]$$

$$= \frac{\pi}{4} - \theta$$

$$x = \cos 2\theta$$

$$\therefore 2\theta = \cos^{-1} x$$

$$\theta = (\cos^{-1} x) \times \frac{1}{2}$$

$$= \frac{\pi}{4} - \frac{1}{2} \cos^{-1} x$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

(hence; proved)

$$15. \quad (1+x^2) \frac{dy}{dx} + y = e^{\tan^{-1}x}$$

$$\frac{dy}{dx} + \frac{y}{(1+x^2)} = \frac{e^{\tan^{-1}x}}{1+x^2}$$

$$\frac{dy}{dx} + P y = Q$$

(where P, Q are functions of x)

$$\Rightarrow y e^{\int P dx} = \int Q e^{\int P dx} \cdot dx + C_1$$

$$\text{integrating factor} = e^{\int P dx} = e^{\int \frac{1}{1+x^2} dx} = e^{\tan^{-1}x}$$

$$\therefore y e^{\tan^{-1}x} = \int \frac{e^{\tan^{-1}x}}{1+x^2} \cdot e^{\tan^{-1}x} \cdot dx + C$$

→ ①

$$\text{let } I \text{ be } \int \frac{e^{\tan^{-1}x} \cdot e^{\tan^{-1}x}}{1+x^2} dx$$

$$\therefore I = \int \frac{e^{\tan^{-1}x} \cdot e^{\tan^{-1}x}}{1+x^2} dx$$

$$\text{Putting } \tan^{-1}x = t$$

$$\therefore x = \tan t \Rightarrow 1+x^2 = \sec^2 t$$

$$\therefore dx = \sec^2 t dt$$

$$\therefore I = \int \frac{e^t \cdot e^t \sec^2 t}{\sec^2 t} dt$$

$$= \int e^{2t} dt$$

$$= \frac{1}{2} e^{2t} + C_1$$

$$= \frac{1}{2} e^{2 \tan^{-1}x} + C_2$$

substituting in ①,

$$y e^{\tan^{-1} x} = \frac{1}{2} e^{2 \tan^{-1} x} + C_2 + C_1$$

$$= \frac{1}{2} e^{2 \tan^{-1} x} + C \quad (C = C_2 + C_1)$$



$$\therefore y = \frac{1}{2} \frac{e^{2 \tan^{-1} x}}{e^{\tan^{-1} x}} + \frac{C}{e^{\tan^{-1} x}}$$

$$y = \frac{1}{2} e^{\tan^{-1} x} + \frac{C}{e^{\tan^{-1} x}}$$

16. $\vec{AB} = \cancel{4i^{\wedge}} - y^{\wedge} - k^{\wedge} - 4i^{\wedge} - 5y^{\wedge} - k^{\wedge}$

$$= -4i^{\wedge} - 6j^{\wedge} - 2k^{\wedge}$$

$$\vec{AC} = -i^{\wedge} + 4y^{\wedge} + 3k^{\wedge}$$

$$\vec{AD} = -8i^{\wedge} - y^{\wedge} + 3k^{\wedge}$$

$\vec{AB}$, $\vec{AC}$ and $\vec{AD}$ are initial vectors.

For $\vec{AB}$, $\vec{AC}$ and $\vec{AD}$ to be coplanar,

$$\vec{AB} \cdot [\vec{AC} \times \vec{AD}] = 0$$

$$\therefore \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = \vec{AB} \cdot [\vec{AC} \times \vec{AD}]$$

$$= -4(12 + 3) + 6(-3 + 24) - 2(-3 + 24)$$

$$= -4(15) + 6(21) - 2(21)$$

$$= -60 + 126 - 42$$

$$= 126 - 126$$

$$= 0$$

$$\therefore \vec{AB} \cdot [\vec{AC} \times \vec{AD}] = 0$$

$\therefore \vec{AB}, \vec{AC}, \vec{AD}$, A, B, C and D are coplanar points.

Hence, proved.

$$17. \quad f(x) = x^2 + 2$$

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

$$g(x) = \frac{x}{x-1} \quad (x \neq 1)$$

$$g: \mathbb{R} \rightarrow \mathbb{R}$$

$$f \circ g(x) = f[g(x)] \quad f \circ g \text{ exists.}$$

$$= f\left(\frac{x}{x-1}\right)$$

$\downarrow$
 $(\because \text{Range of } g(x) \text{ and domain of } f(x) \text{ coincide})$

$$= \left(\frac{x}{x-1}\right)^2 + 2$$

$\therefore f \circ g \text{ exists.}$

$$= \frac{x^2}{(x-1)^2} + 2$$

$$= \frac{x^2 + 2(x-1)^2}{(x-1)^2}$$

$$= \frac{x^2 + 2x^2 + 2 - 4x}{(x-1)^2} = \frac{3x^2 - 4x + 2}{(x-1)^2} \quad (x \neq 1)$$

$$\therefore b \circ g(2) = \left(\frac{2}{2-1}\right)^2 + 2$$

$$= 2^2 + 2$$

$$= 4 + 2$$

$$= 6$$

$$g \circ b(x) = g(b(x))$$

$$= g(x^2 + 2)$$

$$= \frac{x^2 + 2}{x^2 + 2 - 1}$$

$$= \frac{x^2 + 2}{x^2 + 1}$$

$g \circ b(x)$ exists

($\because$ range of $b(x)$ and
domain of $g(x)$
coincide)

$\therefore g \circ b(x)$ exists

$$\therefore g \circ b(-3) = \frac{9+2}{9+1}$$

$$= \frac{11}{10}$$

18.

$$p = 3q$$

 $p \rightarrow$ success probability

 $q \rightarrow$ failure probability

$$p + q = 1$$

$$\therefore 3q + q = 1$$

$$q = 1/4$$

$$p = 3/4$$

$$n = 5$$

 Probability of atleast 3 successes = $[P(3) + P(4) + P(5)]$
 $P(3) \rightarrow$ Prob. of 3 succ.

 $P(4) \rightarrow$ Prob. of 4 succ.

 $P(5) \rightarrow$ Prob. of 5 succ.

~~$$P(x) = {}^5C_x \left(\frac{3}{4}\right)^x \left(\frac{1}{4}\right)^{5-x}$$~~

~~$$P(x) = {}^nC_x p^x q^{n-x}$$~~

$$\therefore P(3) = {}^5C_3 \left(\frac{3}{4}\right)^3 \left(\frac{1}{4}\right)^2$$

$$P(5) = {}^5C_5 \left(\frac{3}{4}\right)^5 \left(\frac{1}{4}\right)^0$$

$$P(4) = {}^5C_4 \left(\frac{3}{4}\right)^4 \left(\frac{1}{4}\right)^1$$

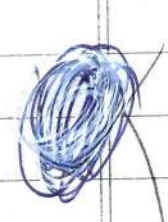
$$\therefore P(3) + P(4) + P(5) = 5 \cancel{3} \left(\frac{3}{4}\right)^{\cancel{3}} \left(\frac{1}{4}\right)^2 + 5 \cancel{4} \left(\frac{3}{4}\right)^{\cancel{4}} \left(\frac{1}{4}\right)^1 + 5 \cancel{5} \left(\frac{3}{4}\right)^{\cancel{5}} \left(\frac{1}{4}\right)^0$$

$$\begin{array}{r} 2 \\ 5 \times 3 \times 3 \\ 1 \times 2 \times 3 \end{array}$$

$$= 10 \left(\frac{3^3}{4^5}\right) + 5 \frac{3^4}{4^5} + \left(\frac{3^5}{4^5}\right)$$

$$\begin{array}{r} 81 \\ \times 5 \\ \hline 405 \end{array}$$

$$= \frac{270 + 405 + 243}{4^5}$$



$$= \frac{918}{4^5} = \frac{409}{2 \times 4^4} = \frac{409}{512} //$$

$$= \frac{459}{512} //$$

$$\begin{array}{r} 81 \\ \times 3 \\ \hline 243 \end{array}$$

$$\begin{array}{r} 270 \\ 405 \\ 243 \\ \hline 918 \end{array}$$

$$\begin{array}{r} 64 \\ \times 4 \\ \hline 256 \end{array}$$

$$19. \quad LHS = \begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix}$$

$$C_1 \rightarrow C_1 + C_2 + C_3$$

$$\Rightarrow LHS = \begin{vmatrix} 2a+2b+2c & c+a & a+b \\ 2p+2q+2r & r+p & p+q \\ 2x+2y+2z & z+x & x+y \end{vmatrix}$$

$$= 2 \begin{vmatrix} a+b+c & c+a & a+b \\ p+q+r & r+p & p+q \\ x+y+z & z+x & x+y \end{vmatrix}$$

$$C_2 \rightarrow C_2 - C_1 \quad C_3 \rightarrow C_3 - C_1$$

$$\therefore \text{LHS} = 2 \begin{vmatrix} a+b+c & -b & -c \\ p+q+r & -q & -r \\ x+y+z & -y & -z \end{vmatrix}$$

$$C_1 \rightarrow C_1 + C_2 + C_3$$

$$\therefore \text{LHS} = 2 \begin{vmatrix} a+b+c-b-c & -b & -c \\ p+q+r-q-r & -q & -r \\ x+y+z-y-z & -y & -z \end{vmatrix}$$

$$= 2(-1)(-1) \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$

(Taking (-1) common from C_2 and C_3)

$$= 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$

hence ; LHS = RHS

hence, proved,

20. $x = a \sin 2t (1 + \cos 2t)$

$$\frac{dx}{dt} = a \left[\sin 2t (-2 \sin 2t) + (1 + \cos 2t) 2 \cos 2t \right]$$

$$= a \left[-2 \sin^2 2t + 2 \cos 2t + 2 \cos^2 2t \right]$$

$$= 2a \left[\cos^2 2t - \sin^2 2t + \cos 2t \right] = 2a \left[\cos 4t + \cos 2t \right]$$

$y = b \cos 2t (1 - \cos 2t)$

$$\frac{dy}{dt} = b \left[\cos 2t (-2 \sin 2t) + (1 - \cos 2t) (2 \sin 2t) \right]$$

$$= b \left[-2 \cos 2t \sin 2t + 2 \sin 2t - 2 \sin 2t \cos 2t \right]$$

$$= b \left[2 \sin 2t - 2 \sin 2t \right]$$

$$= 2b (\sin 4t - \sin 2t)$$

$$\therefore \frac{dy}{dx} = \frac{2b}{2a} \frac{(\sin 4t - \sin 2t)}{(\cos 4t + \cos 2t)}$$

$$= \frac{b}{a} \frac{2(\sin t) \cos(3t)}{2(\cos 3t) \cos t}$$

$$= \frac{b}{a} \tan t$$

$$\therefore \left. \frac{dy}{dx} \right|_{t=\pi/4} = \frac{b}{a} \tan \pi/4$$

$$= b/a //$$

Hence; proved //

21.

$$x(1+y^2) dx = y(1+x^2) dy$$

$$\frac{dy}{dx} = \frac{x(1+y^2)}{y(1+x^2)}$$

$$\therefore \int \frac{dy}{1+y^2} = \int \frac{x dx}{1+x^2}$$

$$\text{or } 1+y^2 = t$$

$$2y dy = dt$$

$$y dy = \frac{dt}{2}$$

$$1+x^2 = u$$

$$2x dx = du$$

$$x dx = \frac{du}{2}$$

$$\therefore \int \frac{dt}{t} = \int \frac{du}{u}$$

$$\therefore \log(t) = \log(u) + C$$

$$\therefore \log(1+y^2) = \log(1+x^2) + C$$

$$\therefore \frac{1+y^2}{1+x^2}$$

$$\therefore \log \left(\frac{1+y^2}{1+x^2} \right) = C$$

$$\text{when } y=1, x=0 \Rightarrow \log 2 = C$$

$$\therefore \log \left(\frac{1+y^2}{1+x^2} \right) = \log 2$$

$$\therefore 1+y^2 = 2(1+x^2) //$$

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$$22. L_1: \frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$$

$$L_2: \frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$$

$$(\because \text{eq. of line} = \frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c})$$

Let the eq. of the line be $\frac{x-2}{a} = \frac{y-1}{b} = \frac{z-3}{c}$
(req.)

$$a+2b+3c=0$$

$$-3a+2b+5c=0$$

$$4a-2c=0$$

$$4a=2c$$

$$2a=c \rightarrow (1)$$

$$\therefore a+2b+6a=0$$

$$2b=-7a$$

$$\therefore b = \frac{-7a}{2}$$

$$\rightarrow (2)$$

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$$\therefore \frac{x-2}{a} = \frac{y-1}{b} = \frac{z-3}{c}$$

$$\Rightarrow \frac{x-2}{2} = \frac{y-1}{-7/2} = \frac{z-3}{2} \quad (\text{from (1) and (2)})$$

$$\Rightarrow \frac{x-2}{1} = \frac{y-1}{-7/2} = \frac{z-3}{2}$$

$$\Rightarrow \frac{x-2}{2} = \frac{y-1}{-7} = \frac{z-3}{4} \quad (\text{Cartesian equation})$$

$$\vec{r} = (2\hat{i} + \hat{j} + 3\hat{k}) + \lambda (2\hat{i} - 7\hat{j} + 4\hat{k})$$

(Vector equation)

Section - A

1. $x - y = -1$

$2x - y = 0$

$y = 2x$

$x - 2x = -1$

$x = 1$

$\therefore y = 2$

$\therefore x + y = 3$

2. $12x + 14 = 3x - 4x$

$12x + 14 = -10$

$12x = -24$

$x = -2$

3. $b(x) = \int_0^x t \sin t \, dt$

$$b'(x) = x \sin x$$

$$4. \quad x + 2y = 8$$

$$\text{Range} = \{1, 3, 3\}$$

$$5. \quad \tan^{-1} \left(\frac{x+y}{1-xy} \right) = \pi/4$$

$$x+y = 1-xy$$

$$x+y+xy = 1$$

$$6. \quad 7A - I^3 - A^3 - 3A^2I - 3AI^2$$

$$= 7A - I - A - 3A - 3A$$

$$[\because A^2 = A]$$

$$A^3 = A$$

$$= -I$$

$$\therefore 7A - (I+A)^3 = -I$$

$$7. \quad \frac{x}{3} = -\frac{p}{3}$$

$$\therefore x = -p \quad \therefore p = -\frac{1}{3}$$

$$8. \quad \frac{x-3}{-5} = \frac{y+4}{7} = \frac{z-3}{2}$$

$$\therefore \vec{r} = (+3\hat{i} - 4\hat{j} + 3\hat{k}) + \lambda(-5\hat{i} + 7\hat{j} + 2\hat{k})$$

$$9. \quad \log x = t \quad \frac{dx}{x} = dt$$

$$\therefore \int_1^2 \frac{1}{t} dt$$

$$= \log t \Big|_1^2$$

$$= \log 2$$

$$10. \quad l^2 + m^2 + n^2 = 1$$

$$\frac{1}{2} + 0 + n^2 = 1$$

$$\therefore \vec{a} = \sqrt{2}$$

$$n^2 = \frac{1}{2}$$

$$\therefore \vec{a} = 5\sqrt{2} \frac{1}{\sqrt{2}} \hat{i} + 0 + 5\sqrt{2} \frac{1}{\sqrt{2}} \hat{k}$$

$$= 5\hat{i} + 5\hat{k}$$