Magnetostatic Field

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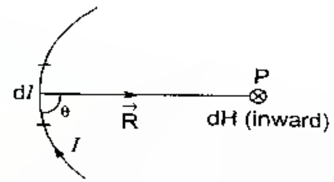
Biot-Savart's Law

The Biot-Savart's law states that the differential magnetic field intensity dH produced at a point P, by the differential current element Idl is proportional to the product Idl and the sine of the angle θ between the element and the line joining P to the element and is inversely proportional to the square of the distance R between P and the element.



$$dH \propto \frac{Idl\sin\alpha}{R^2} \text{ A/m}$$

$$dH = \frac{Idl \times a_r}{4\pi R^2} = \frac{Idl \times \vec{R}}{4\pi|\vec{R}|^3}$$

$$H = \oint \frac{Idl \times a_r}{4\pi R^2} \text{ A/m}$$

where, a_r = unit vector pointing from the different element of current to the point of interest.

Magnetic Field Intensity for Distributed Current Source

(i) Line current

$$\vec{H} = \int_L \frac{Idl \times a_r}{4\pi R^2}$$

(ii) Surface current

$$\vec{H} = \int_S \frac{K \cdot dS \times a_r}{4\pi R^2}$$

where, K = surface current density

(ii) Volume current

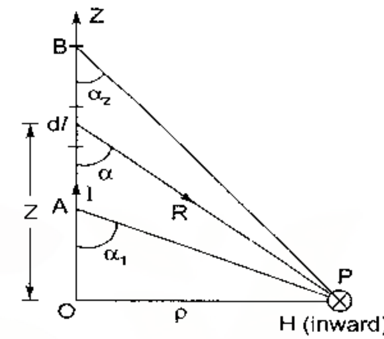
$$\vec{H} = \int_V \frac{J \cdot dV \times a_r}{4\pi R^2}$$

where, J = volume current density

Magnetic field intensity due to straight current carrying filamentary conductor

(i) Finite length AB

$$\vec{H} = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)a_\phi$$



(ii) Semi-infinite length

$$\vec{H} = \frac{I}{4\pi\rho}a_\phi$$

(iii) Infinite length

$$\vec{H} = \frac{I}{2\pi\rho}a_\phi$$

$$a_\phi = a_l \times a_\rho$$

where, a_l is unit vector along the line current and a_ρ is unit vector along the perpendicular line from the line current to the field point.

Ampere's Circuit Law

The closed line integral of static magnetic field intensity $\vec{H}$, integrated over any closed curve 'C' is always equal to total current enclosed within the closed curve 'C'.

$$\oint \vec{H} \cdot \vec{dl} = I_{enc} \text{ A}$$

$$\oint \vec{H} \cdot \vec{dl} = \iint \vec{J} \cdot \vec{dS}$$ .... Ampere's circuit law in the integral form

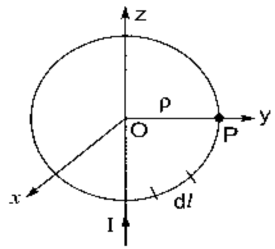
- The Curl of static magnetic field intensity $\vec{H}$ at any point in the electromagnetic region is equal to volume current density $\vec{J}$ present at that point.

$$\nabla \times \vec{H} = J$$ .... Ampere's circuit law in point form of differential form

Note:

- $\nabla \times \vec{H} = J$ is the also known as Maxwell's third equation.
- Ampere's circuit law is applicable irrespective of shape of the closed curve 'C'.
- The magnetostatic field is not conservative as $\nabla \times H = J \neq 0$.

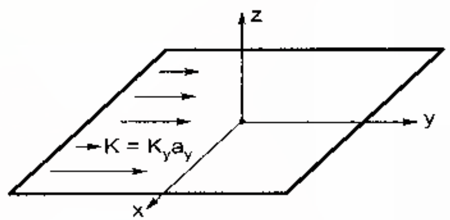
Magnetic Field Intensity Due to Infinite Line Current



$$\vec{H} = \frac{1}{2\pi\rho} a_\phi$$

Magnetic Field Intensity Due to Infinite Sheet of Current

$$\vec{H} = \frac{1}{2}\vec{K} \times a_n$$



where, a_n = Unit normal

K = uniform current density

$$H = \begin{cases} \frac{1}{2} K_y a_x; & z > 0 \\ -\frac{1}{2} K_y a_x; & z < 0 \end{cases}$$

Magnetic Flux Density

$$\vec{B} = \mu_0 \vec{H} \quad \text{Wb/m}^2$$

where, μ_0 = permeability of free space.

Magnetic flux through a surface S

$$\psi = \int_S \vec{B} . d\vec{S} \quad \text{Wb}$$

Remember:

- An isolated magnetic charge does not exist. Thus the total flux through a closed surface in a magnetic field must be zero.

$$\oint \vec{B} \cdot d\vec{S} = 0$$

Maxwell's fourth equation: $\nabla \cdot \vec{B} = 0$

Magnetic Energy Density (W_m)

The magnetic energy density represents magnetic energy stored at a point in the electro magnetic region and gives total magnetic energy per unit volume of the given configuration.

$$W_m = \frac{1}{2}\mu \vec{H}^2 = \frac{1}{2}\vec{B}\vec{H}$$

Magnetic Energy

$$W_m = \iiint_V W_m dV = \frac{1}{2} L I^2 \quad \text{Joule}$$

Remember:

The magnetic energy density depends upon

(i) Magnetic field due to given current distribution in the configuration.

(ii) Permeability of the magnetic medium.

Magnetic Scalar and Vector Potentials

The magnetic potential could be scalar V_m or vector $\vec{A}$

Magnetic scalar potential (V_m)

The magnetic scalar potential is only defined in a region where J = 0.

$$\mathbf{H} = -\nabla \mathbf{V_m}$$

Vector Magnetic Potential $\vec{A}$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

For line current

$$A = \int_L \frac{\mu_0 I dl}{4\pi R} \quad \text{Wb/m}$$

For surface current

$$A = \int_S \frac{\mu_0 K dS}{4\pi R}$$

For volume current

$$A = \int_V \frac{\mu_0 J dV}{4\pi R}$$

Remember:

Important identities which must always hold for any scalar field V and vector field $\vec{A}$.

$$\boxed{\nabla \times (\nabla \mathbf{V}) = 0} \text{ and } \boxed{\nabla \cdot (\nabla \times \vec{\mathbf{A}}) = 0}$$

Forces Due to Magnetic Field

Force on a Charged Particle

(i) Magnetic force

$$\boxed{\vec{F}_m = Q(\vec{u} \times \vec{B})} \text{ Newton}$$

where, u = Velocity of moving charge Q
B = Magnetic field

(ii) Electric force

$$\boxed{\vec{F}_e = Q\vec{E}}$$

where, Q = electric charge ; E = electric field intensity

Lorentz force equation

For a moving charge Q in the presence of both electric and magnetic fields

$$\boxed{\vec{F} = \vec{F}_e + \vec{F}_m = Q(\vec{E} + \vec{u} \times \vec{B})}$$

Force on a Current Element

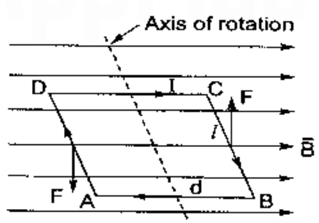
Force on a current carrying conductor

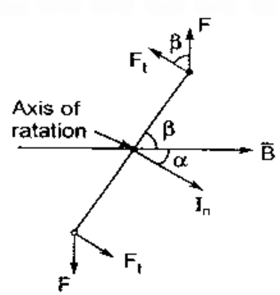
The magnetic field is defined as the force per unit current element.

$$\boxed{\vec{F} = \oint_L I d\vec{l} \times \vec{B}}$$

where $I d\vec{l}$ = Current element of current carrying conductor

Magnetic Torque and Magnetic Moment





Magnetic Torque

$$\boxed{T = BIS \sin\alpha}$$

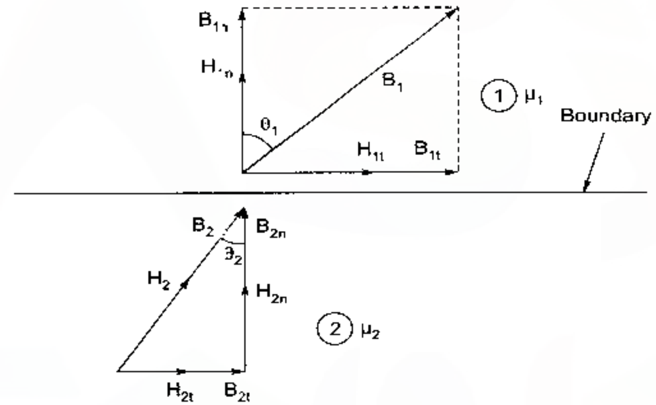
where, S = area of the loop

$$\boxed{\vec{T} = \vec{m} \times \vec{B}}$$

Magnetic Dipole Moment

$$\boxed{\vec{m} = IS i_n}$$

Magnetic Boundary Conditions



Tangential component relation

$$\boxed{\vec{H}_{1t} = \vec{H}_{2t}} \text{ or } \boxed{\frac{\vec{B}_{1t}}{\mu_1} = \frac{\vec{B}_{2t}}{\mu_2}}$$

Normal component relation

$$\boxed{\vec{B}_{1n} = \vec{B}_{2n}} \text{ or } \boxed{\mu_1 \vec{H}_{1n} = \mu_2 \vec{H}_{2n}}$$

Remember:

- The tangential component of H is continuous while that of B is discontinuous at boundary.
- The normal component of B is continuous while that of H is discontinuous at boundary.

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}}$$

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