

SQUARE ROOT & CUBE ROOTS

IMPORTANT FACTS AND FORMULAE :

- ◆ **Square Root :** If $x^2 = y$, We say that the square root of y is x and we write, $\sqrt{y} = x$. Thus, $\sqrt{4} = 2$, $\sqrt{9} = 3$, $\sqrt{196} = 14$.
- ◆ **Cube Root :** The cube root of a given number x is the number whose cube is x . We denote the cube root of x by $\sqrt[3]{x}$.
Thus, $\sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2} = 2$, $\sqrt[3]{7 \times 7 \times 7} = 7$ etc.

Note :

1. $\sqrt{xy} = \sqrt{x} \times \sqrt{y}$
2. $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}} = \frac{\sqrt{x}}{\sqrt{y}} \times \frac{\sqrt{y}}{\sqrt{y}} = \frac{\sqrt{xy}}{\sqrt{y}}$.

◆ EXAMPLES ◆

Ex.1 Evaluate $\sqrt{6084}$ by factorization method,

Sol. **Method :** Express the given number as the product of prime factors. Now, take the product of these prime factors choosing one out of every pair of the same primes. This product gives the square root of the given number.

Thus, resolving 6084 into prime factors, we get:

2	6084
2	3042
3	1521
3	507
13	169
	13

$$\therefore 6084 = 2^2 \times 3^2 \times 13^2$$

$$\therefore \sqrt{6084} = (2 \times 3 \times 13) = 78.$$

Ex.2 Find the square root of 1471369.

Sol. **Explanation :** In the given number, mark off the digits in pairs starting from the unit's digit. Each pair and the remaining one digit is called a period.

Now, $1^2 = 1$. One subtracting, we get 0 as remainder. Now, bring down the next period i.e., 47.

Now, trial divisor is $1 \times 2 = 2$ and trial dividend is 47.

So, we take 22 as divisor and put 2 as quotient.

The remainder is 3.

Next, we bring down the next period which is 13.

Now, trial divisor is $12 \times 2 = 24$ and trial dividend is 313. So, we take 241 as dividend and 1 as quotient.

The remainder is 72.

Bring down the next period i.e., 69.

Now, the trial divisor is $121 \times 2 = 242$ and the trial dividend is 7269. So, we take 3 as quotient and 2423 as divisor. The remainder is then zero.

Hence, $\sqrt{1471369} = 1213$.

1	1	471369	(1213)
	1		
22		47	
		44	
241		313	
		241	
2423		7269	
		7269	
			x

Ex.3 Evaluate : $\sqrt{248 + \sqrt{51 + \sqrt{169}}}$.

Sol. Given expression

$$= \sqrt{248 + \sqrt{51+13}} = \sqrt{248 + \sqrt{64}} = \sqrt{248+8} = \sqrt{256} = 16.$$

Ex.4 If $a*b*c = \frac{\sqrt{(a+2)(b+3)}}{c+1}$, then find the value of $6*15*3$.

$$\begin{aligned}\text{Sol. } 6*15*3 &= \frac{\sqrt{(6+2)(15+3)}}{3+1} = \frac{\sqrt{8*18}}{4} = \frac{\sqrt{144}}{4} \\ &= \frac{12}{4} = 3.\end{aligned}$$

Ex.5 Find the value of $\sqrt{1\frac{9}{16}}$.

$$\text{Sol. } \sqrt{1\frac{9}{16}} = \sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4} = 1\frac{1}{4}.$$

Ex.6 What is the square root of 0.0009?

$$\text{Sol. } \sqrt{0.0009} = \sqrt{\frac{9}{10000}} = \frac{\sqrt{9}}{\sqrt{10000}} = \frac{3}{100} = 0.03$$

Ex.7 Evaluate $\sqrt{175.2976}$.

Sol. **Method :** We make even number of decimal places by affixing a zero, If necessary. Now we mark off periods and extract the square root as shown.

1	1	75.2976	(13.24)
	1		
23		75	
		69	
262		629	
		524	
2644		10576	
		10576	
			x

$$\therefore \sqrt{175.2976} = 13.24.$$

Ex.8 What will come in place of question mark in each of the following questions?

$$(i) \frac{\sqrt{32.4}}{?} = 2 \quad (ii) \sqrt{86.49} + \sqrt{5 + (?)^2} = 12.3$$

Sol. (i) Let $\sqrt{\frac{32.4}{x}} = 2$. Then, $\frac{32.4}{x} = 4 \Leftrightarrow 4x = 32.4 \Leftrightarrow x = 8.1$

$$\text{(ii) Let } \sqrt{86.49} + \sqrt{5+x^2} = 12.3. \\ \text{Then, } 9.3 + \sqrt{5+x^2} = 12.3 \Leftrightarrow \sqrt{5+x^2} = 12.3 - 9.3 = 3$$

$$\Leftrightarrow 5+x^2 = 9 \Leftrightarrow x^2 = 9 - 5 = 4 \Leftrightarrow x = \sqrt{4} = 2.$$

Ex.9 Find the value of $\sqrt{\frac{0.289}{0.00121}}$.

$$\text{Sol. } \sqrt{\frac{0.289}{0.00121}} = \sqrt{\frac{0.28900}{0.00121}} = \sqrt{\frac{28900}{121}} = \sqrt{\frac{170}{11}}$$

Ex.10 If $\sqrt{1+\frac{x}{144}} = \frac{13}{12}$, then find the value of x.

$$\text{Sol. } \sqrt{1+\frac{x}{144}} = \frac{13}{12} \Leftrightarrow \left(1+\frac{x}{144}\right) = \left(\frac{13}{12}\right)^2 = \frac{169}{144} - 1 \\ \Leftrightarrow \frac{x}{144} = \frac{25}{144} \Leftrightarrow x = 25.$$

Ex.11 Find the value of $\sqrt{3}$ upto three places of decimal.

Sol.

$$\begin{array}{r} 1 | 3.000000 (1.732) \\ | \\ 27 | 200 \\ | \\ 189 \\ \hline 343 | 1100 \\ | \\ 1029 \\ \hline 3462 | 7100 \\ | \\ 6924 \\ \hline \end{array}$$

$\therefore \sqrt{3} = 1.732.$

Ex.12 If $\sqrt{3} = 1.732$, find the value of $\sqrt{192} - \frac{1}{2}\sqrt{48} - \sqrt{75}$ correct to 3 places of decimal.

$$\text{Sol. } \sqrt{192} - \frac{1}{2}\sqrt{48} - \sqrt{75} \\ = \sqrt{64 \times 3} - \frac{1}{2}\sqrt{16 \times 3} - \sqrt{25 \times 3} \\ = 8\sqrt{3} - \frac{1}{2} \times 4\sqrt{3} - 5\sqrt{3} \\ = 3\sqrt{3} - 2\sqrt{3} = \sqrt{3} = 1.732$$

Ex.13 Evaluate : $\sqrt{\frac{9.5 \times 0.0085 \times 18.9}{0.0017 \times 1.9 \times 0.021}}$.

Sol. Given exp. = $\sqrt{\frac{9.5 \times 0.0085 \times 18.900}{0.0017 \times 1.9 \times 0.021}}$

Now, since the sum of decimal places in the numerator and denominator and under the radial sign is the same, we remove the decimal.

$$\therefore \text{Given exp.} = \sqrt{\frac{95 \times 85 \times 18900}{17 \times 19 \times 21}} = \sqrt{5 \times 5 \times 900}$$

$$= 5 \times 30 = 150.$$

Ex.14 Simplify :

$$\sqrt{[(12.1)^2 - (8.1)^2] \div [(0.25)^2 + (0.25)(19.95)]}$$

Sol. Given exp.

$$= \frac{\sqrt{(12.1+8.1)(12.1-8.1)}}{(0.25)(0.25+19.95)} = \sqrt{\frac{20.2 \times 4}{0.25 \times 20.2}}$$

$$= \sqrt{\frac{4}{0.25}} = \sqrt{\frac{400}{25}} = \sqrt{16} = 4.$$

Ex.15 If $x = 1 + \sqrt{2}$ and $y = 1 - \sqrt{2}$, find the value of (x^2+y^2) .

Sol. $x^2 + y^2 = (1 + \sqrt{2})^2 = 2(1)^2 + (\sqrt{2})^2 = 2 \times 3 = 6.$

Ex.16 Evaluate $\sqrt{0.9}$ upto three place of decimal.

Sol.

$$\begin{array}{r} 9 | 0.\overline{9}\overline{0}\overline{0}\overline{0} (.948 \\ 81 \\ \hline 184 \\ 184 \\ \hline 900 \\ 736 \\ \hline 16400 \\ 15104 \\ \hline \end{array}$$

$$\therefore \sqrt{0.9} = 0.948$$

Ex.17 If $\sqrt{15} = 3.88$, find the value of $\sqrt{\frac{5}{3}}$.

Sol. $\sqrt{\frac{5}{3}} = \sqrt{\frac{5 \times 3}{3 \times 3}} = \frac{\sqrt{15}}{3} = \frac{3.88}{3}$

$$= 1.2933 \dots = 1.29\bar{3}.$$

Ex.18 Find the least square number which is exactly divisible by 10, 12, 15 and 18.

Sol. L.C.M. of 10, 12, 15, 18 = 180. Now,

$$180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5.$$

To make it a perfect square, It must be multiplied by 5.

$$\therefore \text{Required number} = (2^2 \times 3^2 \times 5^2) = 900.$$

Ex.19 Find the greatest number of five digits which is a perfect square.

Sol. Greatest number of 5 digits is 99999.

3	99999	(316)
	9	
61	99	
	61	
626	6899	
	3756	
	143	

$$\therefore \text{Required number} = (99999 - 143) = 99856.$$

Ex.20 Find the smallest number that must be added to 1780 to make it a perfect square.

Sol.

4	1780	(42)
	16	
82	180	
	164	
	16	

$$\therefore \text{Required to be added} = (43)^2 - 1780 \\ = 1849 - 1780 = 69.$$

Ex.21 If $\sqrt{2} = 1.4142$, find the value of $\frac{\sqrt{2}}{(2+\sqrt{2})}$.

$$\begin{aligned}\text{Sol. } \frac{\sqrt{2}}{(2+\sqrt{2})} &= \frac{\sqrt{2}}{(2+\sqrt{2})} \times \frac{(2-\sqrt{2})}{(2-\sqrt{2})} = \frac{2\sqrt{2}-2}{(4-2)} \\ &= \frac{2(\sqrt{2}-1)}{2} = (\sqrt{2}-1) = (1.4142-1) = 0.4142.\end{aligned}$$

Ex.22 If $x = \left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \right)$ and $y = \left(\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \right)$, find the value of $(x^2 + y^2)$.

$$\begin{aligned}\text{Sol. } x &= \frac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})} \times \frac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})} = \frac{(\sqrt{5}+\sqrt{3})^2}{(5-3)} \\ &= \frac{5+3+2\sqrt{15}}{2} = 4+\sqrt{15}.\end{aligned}$$

$$\begin{aligned}y &= \frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})} \times \frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})} = \frac{(\sqrt{5}-\sqrt{3})^2}{(5-3)} \\ &= \frac{5+3-2\sqrt{15}}{2} = 4-\sqrt{15}.\end{aligned}$$

$$\begin{aligned}\therefore x^2 + y^2 &= (4+\sqrt{15})^2 + (4-\sqrt{15})^2 \\ &= 2 \times 31 = 62.\end{aligned}$$

Ex.23 Find the cube root of 2744.

Sol. **Method :** Resolve the given number as the product of prime factors and take the product of prime factor. Resolving 2744 as the product of prime factors, we get :

$$\sqrt[3]{2744} = 2^3 \times 7^3.$$

2	2744
2	1327
2	686
7	343
7	49
	7

$$\therefore \sqrt[3]{2744} = 2 \times 7 = 14.$$

Ex.24 By what least number 4320 be multiplied to obtain a number which is a perfect cube ?

Sol. Clearly, $4320 = 2^3 \times 3^3 \times 2^2 \times 5$.

To make it a perfect cube, it must be multiplied by 2×5^2 i.e., 50.

1. $\sqrt{53824} = ?$
 - (A) 202
 - (B) 232
 - (C) 242
 - (D) 332

2. The square root of 64009 is -
 - (A) 253
 - (B) 347
 - (C) 363
 - (D) 803

3. The value of $\sqrt{10 + \sqrt{25 + \sqrt{108 + \sqrt{154 + \sqrt{225}}}}}$ is -
 - (A) 4
 - (B) 6
 - (C) 8
 - (D) 10

4. Evaluate : $\sqrt{41 - \sqrt{21 + \sqrt{19 - \sqrt{9}}}}$ -
 - (A) 3
 - (B) 5
 - (C) 6
 - (D) 6.9

5. $\sqrt{176 + \sqrt{2401}}$ is equal to -
 - (A) 14
 - (B) 15
 - (C) 18
 - (D) 24

6. $\left(\frac{\sqrt{625}}{11} \times \frac{14}{\sqrt{25}} \times \frac{11}{\sqrt{196}} \right)$ is equal to -
 - (A) 5
 - (B) 6
 - (C) 8
 - (D) 11

7. $\left(\sqrt{\frac{225}{729}} - \sqrt{\frac{25}{144}} \right) \div \sqrt{\frac{16}{81}} = ?$
 - (A) $\frac{1}{48}$
 - (B) $\frac{5}{48}$
 - (C) $\frac{5}{16}$
 - (D) None

8. The square root of $(272^2 - 128^2)$ is -
 - (A) 144
 - (B) 200
 - (C) 240
 - (D) 256

9. If $x * y = x = y + \sqrt{xy}$, the value of $6 * 24$ is -
 - (A) 41
 - (B) 42
 - (C) 43
 - (D) 44

10. If $y = 5$, then what is the value of $10y \sqrt{y^3 - y^2}$?
 - (A) $50\sqrt{2}$
 - (B) 100
 - (C) $200\sqrt{5}$
 - (D) 500

11. $\sqrt{110\frac{1}{4}} = ?$
 (A) 10.25 (B) 10.5 (C) 11.15 (D) 19.5
12. $\sqrt{\frac{25}{81} - \frac{1}{9}} = ?$
 (A) $\frac{2}{3}$ (B) $\frac{4}{9}$ (C) $\frac{16}{81}$ (D) $\frac{25}{81}$
13. The digit in the unit's place in the square root of 15876 is -
 (A) 2 (B) 4 (C) 3 (D) None
14. How many two digit-numbers satisfy this property : The last digit (unit's digit) of the square of the two-digit number is 8 ?
 (A) 1 (B) 2 (C) 3 (D) None
15. What is the square root of 0.16 ?
 (A) 0.004 (B) 0.04 (C) 0.4 (D) 4
16. The value of $\sqrt{0.0004761}$ is -
 (A) 0.00021 (B) 0.0021 (C) 0.021 (D) 0.21
17. $\sqrt{0.00004761}$ equals -
 (A) 0.00069 (B) 0.0069 (C) 0.0609 (D) 0.069
18. $1.5^2 \times \sqrt{0.0225} = ?$
 (A) 0.0375 (B) 0.3375 (C) 3.275 (D) 32.75
19. $\sqrt{0.01 + \sqrt{0.0064}} = ?$
 (A) 0.03 (B) 0.3 (C) 0.42 (D) None
20. The value of $\sqrt{0.01} + \sqrt{0.81} + \sqrt{1.21} + \sqrt{0.0009}$ is -
 (A) 2.03 (B) 2.1 (C) 2.11 (D) 2.13
21. $\sqrt{.0025} \times \sqrt{2.25} \times \sqrt{.0001} = ?$
 (A) .000075 (B) .0075 (C) .075 (D) None
22. $\sqrt{1.5625} = ?$
 (A) 1.05 (B) 1.25 (C) 1.45 (D) 1.55
23. If $\sqrt{.00000676} = .0026$, the square root of 67,60,000 is -
 (A) $\frac{1}{26}$ (B) 26 (C) 260 (D) 2600

49. By how much does $\sqrt{12} + \sqrt{18}$ exceed $\sqrt{3} + \sqrt{2}$?

- (A) $\sqrt{2} - 4\sqrt{3}$ (B) $\sqrt{3} + 2\sqrt{2}$
 (C) $2(\sqrt{3} - \sqrt{2})$ (D) $3(\sqrt{3} - \sqrt{2})$

50. $\frac{\sqrt{24} + \sqrt{216}}{\sqrt{96}} = ?$

- (A) $2\sqrt{6}$ (B) 2 (C) $6\sqrt{2}$ (D) $\frac{2}{\sqrt{6}}$

51. The value of $\frac{\sqrt{80} - \sqrt{112}}{\sqrt{45} - \sqrt{63}}$ is -

- (A) $\frac{3}{4}$ (B) $1\frac{1}{3}$ (C) $1\frac{7}{9}$ (D) $1\frac{3}{4}$

52. If $3\sqrt{5} + \sqrt{125} = 17.88$, then what will be the value of $\sqrt{80} + 6\sqrt{5}$?

- (A) 13.41 (B) 20.46 (C) 21.66 (D) 22.36

53. $\sqrt{50} \times \sqrt{98}$ is equal to -

- (A) 63.75 (B) 65.95 (C) 70 (D) 70.25

54. Given $\sqrt{2} = 1.414$. The value of

- $\sqrt{8} + 2\sqrt{32} - 3\sqrt{128} + 4\sqrt{50}$ is -
 (A) 8.246 (B) 8.484 (C) 8.526 (D) 8.876

55. The approximate value of $\frac{3\sqrt{12}}{2\sqrt{28}} \div \frac{2\sqrt{21}}{\sqrt{98}}$ is -

- (A) 1.0605 (B) 1.0727 (C) 1.6007 (D) 1.6026

56. $\sqrt{\frac{.081 \times .484}{.0064 \times 6.25}}$ is equal to -

- (A) 0.9 (B) 0.99 (C) 9 (D) 99

57. $\sqrt{\frac{0.204 \times 42}{0.07 \times 304}}$ is equal to -

- (A) $\frac{1}{6}$ (B) 0.06 (C) 0.6 (D) 6

58. $\sqrt{\frac{0.081 \times 0.324 \times 4.624}{1.5625 \times 0.0289 \times 72.9 \times 64}}$ is equal to -

- (A) 0.024 (B) 0.24 (C) 2.4 (D) 24

59. $\sqrt{\frac{9.5 \times 0.085}{.0017 \times .19}}$ equals -

- (A) .05 (B) 5 (C) 50 (D) 500

60. The value of $\sqrt{\frac{(0.03)^2 + (0.21)^2 + (0.065)^2}{(0.003)^2 + (0.021)^2 + (0.0064)^2}}$ is -
 (A) 0.1 (B) 10 (C) 10^2 (D) 10^3
61. The square root of $(7+3\sqrt{5})(7-3\sqrt{5})$ is -
 (A) $\sqrt{5}$ (B) 2 (C) 4 (D) $3\sqrt{5}$
62. $\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)^2$ simplifies to -
 (A) $\frac{3}{4}$ (B) $\frac{4}{\sqrt{3}}$
 (C) $\frac{4}{3}$ (D) None of these
63. $\left(\sqrt{2} + \frac{1}{\sqrt{2}}\right)^2$ is equal to -
 (A) $2\frac{1}{2}$ (B) $3\frac{1}{2}$ (C) $4\frac{1}{2}$ (D) $5\frac{1}{2}$
64. If $a = 0.1039$, then the value of $\sqrt{4a^2 - 4a + 1} + 3a$ is -
 (A) 0.1039 (B) 0.2078 (C) 1.1039 (D) 2.1039
65. The square root of $\frac{(0.75)^3}{1-0.75} + [0.75 + (0.75)^2 + 1]$ is -
 (A) 1 (B) 2 (C) 3 (D) 4
66. If $3a = 4b = 6a$ and $a + b + c = 27\sqrt{29}$, then $\sqrt{a^2 + b^2 + c^2}$ is -
 (A) $3\sqrt{29}$ (B) 81
 (C) 87 (D) None of these
67. The square root of $0.\overline{4}$ is -
 (A) $0.\overline{6}$ (B) $0.\overline{7}$ (C) $0.\overline{8}$ (D) $0.\overline{9}$
68. Which one of the following numbers has rational square root ?
 (A) 0.4 (B) 0.09 (C) 0.9 (D) 0.025
69. The value of $\sqrt{0.4}$ is -
 (A) 0.02 (B) 0.2 (C) 0.51 (D) 0.63
70. The value of $\sqrt{0.121}$ is -
 (A) 0.011 (B) 0.11 (C) 0.347 (D) 1.1
71. The value of $\sqrt{0.064}$ is -
 (A) 0.008 (B) 0.08 (C) 0.252 (D) 0.8

72. The value of $\sqrt{\frac{0.16}{0.4}}$ is -
(A) 0.02 (B) 0.2
(C) 0.63 (D) None of these
73. The value of $\frac{1+\sqrt{0.01}}{1-\sqrt{0.1}}$ is close to -
(A) 0.6 (B) 1.1 (C) 1.6 (D) 1.7
74. If $\sqrt{5} = 2.236$, then the value of $\frac{1}{\sqrt{5}}$ is -
(A) .367 (B) .447
(C) .745 (D) None of these
75. If $\sqrt{24} = 4.899$, the value of $\sqrt{\frac{8}{3}}$ is -
(A) 0.544 (B) 1.333 (C) 1.633 (D) 2.666
76. If $\sqrt{6} = 2.449$, then the value of $\frac{3\sqrt{2}}{2\sqrt{3}}$ is -
(A) 0.6122 (B) 0.8163 (C) 1.223 (D) 1.2245
77. If $\sqrt{5} = 2.236$, then the value of $\frac{\sqrt{5}}{2} - \frac{10}{\sqrt{5}} + \sqrt{125}$ is equal to -
(A) 5.59 (B) 7.826 (C) 8.944 (D) 10.062
78. If $2*3 = \sqrt{3}$ and $3*4 = 5$, then the value of $5*12$ is -
(A) $\sqrt{17}$ (B) $\sqrt{29}$ (C) 12 (D) 13
79. The least perfect square number divisible by 3, 4, 5, 6 and 8 is -
(A) 900 (B) 1200 (C) 2500 (D) 3600
80. The least perfect square, which is divisible by each of 21, 36 and 66 is -
(A) 213444 (B) 214344
(C) 214434 (D) 231444
81. The least number by which 294 must be multiplied to make it a perfect square is -
(A) 2 (B) 3 (C) 6 (D) 24
82. Find the smallest number by which 5808 should be multiplied so that the product becomes a perfect square -
(A) 2 (B) 3 (C) 7 (D) 11
83. The least number by which 1470 must be divided to get a number which is a perfect square is -
(A) 5 (B) 6 (C) 15 (D) 30

84. What is the smallest number to be subtracted from 549162 in order to make it a perfect square?
 (A) 28 (B) 36 (C) 62 (D) 81
85. What is the least which should be subtracted from 0.000326 to make it a perfect square ?
 (A) 0.000002 (B) 0.000004
 (C) 0.02 (D) 0.04
86. The smallest number added to 680621 to make the sum a perfect square is -
 (A) 4 (B) 5 (C) 6 (D) 8
87. The greatest four-digit perfect square number, is-
 (A) 9000 (B) 9801 (C) 9900 (D) 9981
88. The least number of digits which is a perfect square is -
 (A) 1000 (B) 1016 (C) 1024 (D) 1036
89. Given $\sqrt{5}=2.2361$, $\sqrt{3}=1.7321$, then $\frac{1}{\sqrt{5}-\sqrt{3}}$ is equal to -
 (A) 1.89 (B) 1.984 (C) 1.9841 (D) 2
90. $\frac{1}{(\sqrt{9}-\sqrt{8})} - \frac{1}{(\sqrt{8}-\sqrt{7})} + \frac{1}{(\sqrt{7}-\sqrt{6})}$
 $- \frac{1}{(\sqrt{6}-\sqrt{5})} + \frac{1}{(\sqrt{5}-\sqrt{4})}$ is equal to -
 (A) 0 (B) $\frac{1}{3}$ (C) 1 (D) 5
91. $\left(2+\sqrt{2}+\frac{1}{2+\sqrt{2}}+\frac{1}{\sqrt{2}+2}\right)$ simplifies to -
 (A) $2-\sqrt{2}$ (B) 2 (C) $2+\sqrt{2}$ (D) $2\sqrt{2}$
92. If $\sqrt{2}=1.4142$, the square root of $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ is nearest to -
 (A) 0.172 (B) 0.414 (C) 0.586 (D) 1.414
93. $\left[\frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}} - \frac{6}{\sqrt{8}-\sqrt{12}} \right] = ?$
 (A) $\sqrt{3}-\sqrt{2}$ (B) $\sqrt{3}+\sqrt{2}$
 (C) $5\sqrt{3}$ (D) 1
94. $\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}$ is equal to -
 (A) 1 (B) 2
 (C) $6-\sqrt{35}$ (D) $6+\sqrt{35}$

95. If $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a+b\sqrt{3}$, then -
 (A) $a = -11, b = -6$ (B) $a = -11, b = 6$
 (C) $a = 11, b = -6$ (D) $a = 6, b = 11$
96. If $\sqrt{2} = 1.414$, the square root of $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ is nearest to -
 (A) 0.172 (B) 0.414
 (C) 0.586 (D) 1.414
97. $\frac{3+\sqrt{6}}{3\sqrt{5}-2\sqrt{12}-\sqrt{32}+\sqrt{50}} = ?$
 (A) 3 (B) $3\sqrt{2}$
 (C) 6 (D) None of these
98. $\left(\frac{2+\sqrt{3}}{2-\sqrt{3}} + \frac{2-\sqrt{3}}{2+\sqrt{3}} + \frac{\sqrt{3}-1}{\sqrt{3}+1} \right)$ simplifies to -
 (A) $16-\sqrt{3}$ (B) $4-\sqrt{3}$
 (C) $2-\sqrt{3}$ (D) $2+\sqrt{3}$
99. If $x = (7 - 4\sqrt{3})$, then the value of $\left(x + \frac{1}{x}\right)$ is -
 (A) $3\sqrt{3}$ (B) $8\sqrt{3}$
 (C) 14 (D) $14 + 8\sqrt{3}$
100. If $x = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ and $y = \frac{\sqrt{3}-1}{\sqrt{3}+1}$, then the value of $(x^2 + y^2)$ is -
 (A) 10 (B) 13 (C) 14 (D) 15
101. If $a = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $b = \frac{\sqrt{5}-1}{\sqrt{5}+1}$, the value of $\left(\frac{a^2+ab+b^2}{a^2-ab+b^2}\right)$ is -
 (A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $\frac{3}{5}$ (D) $\frac{5}{3}$
102. A man plants 15376 apple trees in this garden and arranged them so that there are as many rows as there are apples trees in each row. The number of row is -
 (A) 124 (B) 126 (C) 134 (D) 144
103. A General wishes to draw up his 56581 soldiers in the form of a solid square. After arranging them, he found that some of them are left over. How many are left ?
 (A) 65 (B) 81
 (C) 100 (D) None of these
104. A group of student decided to collect s many paise from each member of the group as is the number of members. If the total collection amounts to Rs. 59.29, The number of members in the group is -
 (A) 54 (B) 67 (C) 77 (D) 87

118. If $x = \sqrt[3]{2\frac{93}{125}}$, then value of x is -
 (A) $1\frac{2}{5}$ (B) $2\frac{1}{5}$
 (C) $3\frac{4}{5}$ (D) $4\frac{1}{5}$
119. A number is 64 time of the square of its reciprocal. The number is -
 (A) 10 (B) 4 (C) 2 (D) 16
120. The smallest perfect square number exactly divisible by 4, 5, 6, 15, 18, is -
 (A) 1800 (B) 225 (C) 361 (D) 900
121. A gardner plants 17956 trees in such a way that there are as many rows as there are trees in each row. The number of trees in a row are -
 (A) 136 (B) 164 (C) 134 (D) 166
122. $\sqrt{2\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}}$ is equal to -
 (A) 0 (B) 2 (C) 1 (D) $2^{\frac{31}{32}}$
123. The greatest six digit number which is perfect square is -
 (A) 998004 (B) 998006
 (C) 998049 (D) 998001
124. Divide the number 26244 by the smallest number so that the quotient is a perfect cube, so the smallest number is -
 (A) 4 (B) 6 (C) 36 (D) 16
125. The perfect cube nearest to 2750 is -
 (A) 2749 (B) 2747
 (C) 2744 (D) 2754
126. The least number which must be added to 4931 to make it perfect square is -
 (A) 110 (B) 120
 (C) 130 (D) None
127. The least number which must be added to 4931 to make it perfect square is -
 (A) 20 (B) 30 (C) 40 (D) 50
128. Least perfect square of 6 digits is -
 (A) 998000 (B) 998001
 (C) 998002 (D) None of these
129. A $8 \times 6 \times 4 \text{ cm}^3$ metallic cube is melted. Find the minimum volume of molten metal which should be added to mould it into a cube whose edge is an x, where x is an integer -
 (A) 20 (B) 21 (C) 23 (D) 24
130. Value of $\sqrt[3]{392} \times \sqrt[3]{448}$ is -
 (A) 50 (B) 52 (C) 54 (D) 56

-
131. The smallest number by which 137592 should be multiplied to make it perfect cube -
(A) 1183 (B) 1180
(C) 1181 (D) None of these
132. The volume of two cubes are in the ratio of 343 : 1331, the ratio of their edges is -
(A) 7 : 10 (B) 7 : 11
(C) 7 : 12 (D) None of these

ANSWER KEY

Q.No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	B	A	A	C	B	A	D	C	B	D	B	B	D	D	C	B	B	B	B	D
Q.No	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	D	B	D	B	D	B	D	D	C	B	A	D	C	B	B	A	A	B	C	A
Q.No	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	C	A	D	C	A	A	D	C	B	B	D	C	B	A	B	D	A	C	B	
Q.No	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	B	C	C	C	B	C	A	B	D	C	C	C	C	B	C	D	B	D	D	A
Q.No	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
Ans.	C	B	D	D	A	A	B	C	C	D	B	B	C	D	C	B	D	A	C	C
Q.No	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
Ans.	B	A	C	C	B	B	B	A	D	A	C	C	C	A	A	C	A	B	D	
Q.No	121	122	123	124	125	126	127	128	129	130	131	132								
Ans.	C	D	D	C	C	A	C	B	D	D	A	B								

HINTS & SOLUTION

1.

$$\begin{array}{r} 2 | \overline{53824} \text{ (232)} \\ \underline{4} \\ \overline{43} \quad \overline{138} \\ \underline{43} \quad \underline{129} \\ \overline{462} \quad \overline{924} \\ \underline{462} \quad \underline{924} \\ \times \end{array}$$

$$\therefore \sqrt{53824} = 232.$$

3. Given exp.

$$\begin{aligned} &= \sqrt{10 + \sqrt{25 + \sqrt{108 + \sqrt{154 + 15}}}} \\ &= \sqrt{10 + \sqrt{25 + \sqrt{108 + \sqrt{169}}}} \\ &= \sqrt{10 + \sqrt{25 + \sqrt{108 + 13}}} = \sqrt{10 + \sqrt{25 + \sqrt{121}}} \\ &= \sqrt{10 + \sqrt{25 + \sqrt{11}}} = \sqrt{10 + \sqrt{36}} = \sqrt{10 + 6} \\ &= \sqrt{16} = 4. \end{aligned}$$

4. Given exp.

$$\begin{aligned} &= \sqrt{41 - \sqrt{21 + \sqrt{19 - 3}}} = \sqrt{41 - \sqrt{21 + \sqrt{16}}} \\ &= \sqrt{41 - \sqrt{21 + 4}} = \sqrt{41 - 5} = \sqrt{36} = 6. \end{aligned}$$

5. Given exp.

$$= \sqrt{176 + 49} = \sqrt{225} = 15, \quad \begin{array}{r} 4 | \overline{2401} \text{ (49)} \\ \underline{16} \\ \overline{89} \quad \overline{801} \\ \underline{89} \quad \underline{801} \\ \times \end{array}$$

7. Given exp. = $\left(\frac{\sqrt{225}}{\sqrt{729}} - \frac{\sqrt{25}}{\sqrt{144}} \right) \div \frac{\sqrt{16}}{\sqrt{81}}$

$$= \left(\frac{15}{108} \times \frac{9}{4} \right) \div \frac{5}{16}.$$

9. $6*24 = 6 + 24 + \sqrt{6 \times 24} = 30 + \sqrt{144}$
 $= 30 + 12 = 42.$

10. $10y\sqrt{y^3 - y^2} = 10 \times 5\sqrt{5^3 - 5^2}$
 $= 50 \times \sqrt{125 - 25} = 50 \times \sqrt{100} = 50 \times 10 = 500$

13.

$$\begin{array}{r} 1 | \overline{15876} (126) \\ \quad 1 \\ \hline 22 | \overline{58} \\ \quad 44 \\ \hline 246 | \overline{1476} \\ \quad 1476 \\ \hline \end{array}$$

$$\therefore \sqrt{15876} = 126$$

14. A number ending in 8 can never be a perfect square.

17. $\sqrt{0.00004761} = \sqrt{\frac{4761}{10^8}} = \frac{\sqrt{4761}}{\sqrt{10^8}} = \frac{69}{10^4}$
 $= \frac{69}{10000} = 0.0069$

18. $1.5^2 \times \sqrt{0.0225} = 1.5^2 \times \sqrt{\frac{225}{10000}}$
 $= 2.225 \times \frac{15}{100} = 2.25 \times 0.15 = 0.3375$

19. $\sqrt{0.01 + \sqrt{0.0064}} = 1.5^2 \times \sqrt{\frac{225}{10000}}$
 $= 2\sqrt{0.01 + \sqrt{0.0064}} = \sqrt{0.01 + \sqrt{\frac{64}{10000}}}$
 $= \sqrt{0.01 + \frac{8}{100}} + \sqrt{0.01 + 0.08} = \sqrt{0.09} = 0.3.$

20. Given exp. $= \sqrt{\frac{1}{100}} + \sqrt{\frac{81}{100}} + \sqrt{\frac{121}{100}} + \sqrt{\frac{9}{10000}}$
 $= \frac{1}{10} + \frac{9}{10} + \frac{11}{10} + \frac{3}{100}$
 $= 0.1 + 0.9 + 1.1 + 0.03 = 2.13.$

21. Given Exp. $= \sqrt{\frac{25}{10000}} \times \sqrt{\frac{255}{100}} \times \sqrt{\frac{1}{10000}}$
 $= \frac{5}{100} \times \frac{15}{10} \times \frac{1}{100} = \frac{75}{100000} = 0.00075.$

22.

$$\begin{array}{r} 1 | \overline{1.5625} (1.25) \\ \quad 1 \\ \hline 22 | \overline{56} \\ \quad 44 \\ \hline 245 | \overline{1225} \\ \quad 1225 \\ \hline \end{array}$$

23. $\sqrt{6760000} = \sqrt{0.00000676 \times 10^{12}}$
 $= \sqrt{0.00000676} \times \sqrt{10^{12}} = .0026 \times 10^6 = 2600.$

24. Given exp.
 $= \sqrt{\frac{18225}{10^2}} + \sqrt{\frac{18225}{10^4}} + \sqrt{\frac{18225}{10^6}} + \sqrt{\frac{18225}{10^8}}$
 $= \frac{\sqrt{18225}}{10} + \frac{\sqrt{18225}}{10^2} + \frac{\sqrt{18225}}{10^3} + \frac{\sqrt{18225}}{10^4}$
 $= \frac{135}{10} + \frac{135}{100} + \frac{135}{1000} + \frac{135}{10000}$
 $= 13.5 + 1.35 + 0.135 + 0.0135 = 14.9985$

25. Given exp. $= \sqrt{1.30} + \sqrt{1300} + \sqrt{0.0130}$
 $= \sqrt{\frac{130}{100}} + \sqrt{13 \times 100} + \sqrt{\frac{130}{10000}}$
 $= \frac{\sqrt{130}}{10} + \sqrt{13} \times 10 + \frac{\sqrt{130}}{100}$
 $= \frac{11.40}{10} + 3.605 \times 10 + \frac{11.40}{100}$
 $= 1.14 + 36.05 + 0.114 = 37.304.$

26. $\frac{52}{x} = \sqrt{\frac{169}{289}} \Leftrightarrow \frac{52}{x} = \frac{13}{17} \Leftrightarrow x = \left(\frac{52 \times 17}{13} \right) = 38$

27. Let the missing number be x
Then, $x^2 = 15 \times 136 \Leftrightarrow x = \sqrt{15 \times 135}$
 $= \sqrt{15^2 \times 3^2} = 15 \times 3 = 45.$

28. Let $\frac{4\frac{1}{2}}{x} = \frac{x}{32}$.
Then, $x^2 = 32 \times \frac{9}{2} = 144 \Leftrightarrow x = \sqrt{144} = 12$

29. Let $\frac{x}{\sqrt{128}} = \frac{\sqrt{162}}{x}$.
Then, $x^2 = \sqrt{128 \times 162} = \sqrt{64 \times 2 \times 18 \times 9}$
 $= \sqrt{8^2 \times 6^2 \times 3^2} = 8 \times 6 \times 3 = 144$

30. $\frac{0.13}{p^2} = 13 \Leftrightarrow p^2 \cdot \frac{0.13}{13} = \frac{1}{100} \Leftrightarrow p$
 $= \sqrt{\frac{1}{100}} = \frac{1}{10} = 0.1.$

31. Let the required number be x . Then,

$$\frac{x}{\sqrt{0.25}} = 25 \Leftrightarrow \frac{x}{0.5} = 25 \Leftrightarrow x = 25 \times 0.5 = 12.5$$

32. $\sqrt{3^n} = 729 = 3^6 \Leftrightarrow (\sqrt{3^n})^2 = (3^6)^2 \Leftrightarrow 3^n = 13^{12} \Leftrightarrow n = 12$

33. $\sqrt{18 \times 14 \times x} = 84 \Leftrightarrow 18 \times 14 \times x = 84 \times 84 \Leftrightarrow x = \frac{84 \times 84}{18 \times 14} = 28$

34. Let $28\sqrt{x} + 1426 = 3 \times 718$

35. Let $\sqrt{\frac{x}{169}} = \frac{54}{39}$. Then, $\frac{\sqrt{x}}{13} = \frac{54}{39} \Leftrightarrow \sqrt{x} = \left(\frac{54}{39} \times 13\right) = 18 \Leftrightarrow x = (18)^2 = 324$

36. $\frac{\sqrt{x}}{\sqrt{441}} = 0.02 \Leftrightarrow \sqrt{x} = \left(\frac{54}{39} \times 13\right) = 18 \Leftrightarrow x = (0.42)^2 = 0.1764.$

37. Let $\sqrt{\frac{.0169}{x}} = 0.2$. Then, $\frac{.0196}{x} = 0.04 \Leftrightarrow x = \frac{.0196}{.04} = \frac{1.96}{4} = .49.$

38. Let $\sqrt{0.0169 \times x} = 1.3$. Then, $0.0169x = (1.3)^2 = 1.69 \Leftrightarrow x = \frac{1.69}{0.0169} = 100.$

39. $37 + \sqrt{.0615 + x} = 37.25 \Leftrightarrow \sqrt{.0615 + x} = 0.25 \Leftrightarrow .0615 + x = (0.25)^2 = 0.0625 \Leftrightarrow x = 0.001 = \frac{1}{10^3} = 10^{-3}.$

40. $\sqrt{(x-1)(y+2)} = 7 \Rightarrow (x-1)(y+2) = (7)^2 \Rightarrow (x-1) = 7 \text{ and } (y+2) = 7$

41. $\frac{\sqrt{a}}{\sqrt{b}} = \frac{.004 \times .4}{\sqrt{.04 \times .4}} \Rightarrow \frac{a}{b} = \frac{.004 \times .4 \times .004 \times .4}{.04 \times .4} = \frac{.0000064}{.04} \Rightarrow \frac{a}{b} = \frac{.00064}{4} = .00016 = \frac{16}{10^5} = 16 \times 10^{-5}$

42. Let the number be x . Then,

$$\frac{3}{5}x^2 = 126.15 \Leftrightarrow x^2 = \left(126.15 \times \frac{5}{3}\right) = 210.25 \Leftrightarrow x = \sqrt{210.25} = 14.5$$

$$43. \quad \sqrt{\frac{0.361}{0.00169}} = \sqrt{\frac{0.36100}{0.00169}} = \sqrt{\frac{36100}{169}} = \frac{190}{13}.$$

$$44. \quad \sqrt{\frac{48.4}{0.289}} = \sqrt{\frac{48.400}{0.289}} = \sqrt{\frac{48400}{289}} = \frac{220}{17} = 12\frac{16}{17}$$

$$45. \quad \sqrt{1 + \frac{x}{169}} = \frac{x}{27} \Rightarrow \sqrt{\frac{169+x}{169}} = \frac{27+x}{27} \Rightarrow \frac{28+x}{27} = \frac{27+x}{27} \Rightarrow 28+x = 27+x \Rightarrow x = 1.$$

$$46. \quad \sqrt{1 + \frac{55}{729}} = 1 + \frac{x}{27} \Rightarrow \sqrt{\frac{784}{729}} = \frac{27+x}{27} \Rightarrow \frac{28}{27} = \frac{27+x}{27} \Rightarrow 27+x = 28 \Rightarrow x = 1$$

47.	$\begin{array}{r l} & 2.\overline{000000} (1.414 \\ \hline & 1 \\ 24 & \overline{100} \\ & 96 \\ \hline 281 & \overline{400} \\ & 281 \\ \hline 2824 & \overline{11900} \\ & 11296 \\ \hline & \end{array}$
	$\therefore \sqrt{2} = 1.414.$

$$48. \quad 2\sqrt{2} - \sqrt{75} + \sqrt{12} = 2\sqrt{9 \times 3} - \sqrt{25 \times 3} + \sqrt{4 \times 3} \\ = 6\sqrt{3} - 5\sqrt{3} + 2\sqrt{3} = 3\sqrt{3}.$$

$$\begin{aligned}
 49. \quad & (\sqrt{12} + \sqrt{18}) - (\sqrt{3} + \sqrt{2}) \\
 &= (\sqrt{4 \times 3} + \sqrt{9 \times 2}) - (\sqrt{3} + \sqrt{2}) \\
 &= (2\sqrt{3} + 3\sqrt{2}) - (\sqrt{3} + \sqrt{2}) \\
 &= (2\sqrt{3} - \sqrt{3}) + (3\sqrt{2} - \sqrt{2}) = \sqrt{3} + 2\sqrt{2}.
 \end{aligned}$$

$$50. \quad \frac{\sqrt{24} + \sqrt{216}}{\sqrt{96}} = \frac{\sqrt{4 \times 6} + \sqrt{36 \times 6}}{\sqrt{16 \times 6}} \\ = \frac{2\sqrt{6} + 6\sqrt{6}}{4\sqrt{6}} = \frac{8\sqrt{6}}{4\sqrt{6}} = 2.$$

$$51. \frac{\sqrt{80} - \sqrt{112}}{\sqrt{45} - \sqrt{163}} = \frac{\sqrt{16 \times 5} - \sqrt{16 \times 7}}{\sqrt{9 \times 5} - \sqrt{9 \times 7}} \\ = \frac{4\sqrt{5} - 4\sqrt{7}}{3\sqrt{5} - 3\sqrt{7}} = \frac{4(\sqrt{5} - \sqrt{7})}{3(\sqrt{5} - \sqrt{7})} = \frac{4}{3} = 1\frac{1}{3}$$

52. $3\sqrt{5} + \sqrt{125} = 17.88 \Rightarrow 3\sqrt{5} + \sqrt{25 \times 5} = 17.88$
 $\Rightarrow 3\sqrt{5} + 5\sqrt{5} = 17.88 \Rightarrow 8\sqrt{5}$
 $= 17.88 \Rightarrow \sqrt{5} = 2.235$
 $\therefore \sqrt{80} + 6\sqrt{5} = \sqrt{16 \times 5} + 6\sqrt{5}$
 $= 4\sqrt{5} + 6\sqrt{5} = 10\sqrt{5} = (10 \times 2.235) = 22.35$

53. $\sqrt{50} \times \sqrt{98} = \sqrt{50 \times 98} = \sqrt{4900} = 70.$

54. Given exp.
 $= \sqrt{4 \times 2} + 2\sqrt{16 \times 2} - 3\sqrt{64 \times 2} + 4\sqrt{25 \times 2}$
 $= 2\sqrt{2} + 8\sqrt{2} - 24\sqrt{2} + 20\sqrt{2}$
 $= 6\sqrt{2} = 6 \times 1.414 = 8.484.$

55. Given exp.
 $= \frac{3\sqrt{2}}{2\sqrt{28}} \times \frac{\sqrt{98}}{2\sqrt{21}} = \frac{2\sqrt{4 \times 3}}{3\sqrt{4 \times 7}} \times \frac{\sqrt{49 \times 2}}{2\sqrt{21}}$
 $= \frac{6\sqrt{3}}{4\sqrt{7}} \times \frac{7\sqrt{3}}{4\sqrt{7}} \times \frac{7\sqrt{2}}{2\sqrt{21}}$
 $= \frac{21\sqrt{6}}{4\sqrt{7 \times 21}} = \frac{21\sqrt{6}}{28\sqrt{3}} = \frac{3}{4}\sqrt{2}$
 $= \frac{3}{4} \times 1.414 = 3 \times 0.3535 = 1.0605$

56. Sum of decimal places in the numerator and denominator under the radical sign being the same, we remove the decimal.

\therefore Given exp. $= \sqrt{\frac{81 \times 484}{64 \times 625}} = \frac{9 \times 22}{8 \times 25} = 0.99.$

57. Given exp. $= \sqrt{\frac{204 \times 42}{7 \times 34}} = \sqrt{36} = 6.$

58. Given exp.
 $= \sqrt{\frac{81 \times 324 \times 4624}{15625 \times 289 \times 729 \times 64}} = \frac{9 \times 18 \times 68}{125 \times 17 \times 27 \times 8}$
 $= \frac{3}{125} = 0.024.$

59. Given exp. $= \sqrt{\frac{9.5 \times 08500}{.19 \times .0017}} = \sqrt{\frac{95 \times 82500}{19 \times 17}}$
 $= \sqrt{5 \times 500} = \sqrt{2500} = 50$

60. Given exp. $= \sqrt{\frac{(0.03)^2 + (0.21)^2 + (0.065)^2}{\left(\frac{0.03}{10}\right)^2 + \left(\frac{0.21}{10}\right)^2 + \left(\frac{0.065}{10}\right)^2}}$
 $= \sqrt{\frac{100[(0.03)^2 + (0.21)^2 + (0.065)^2]}{(0.03)^2 + (0.21)^2 + (0.065)^2}} = \sqrt{100} = 10.$

61. $\sqrt{(7+3\sqrt{5})(7-3\sqrt{5})} = \sqrt{(7)^2 - (3\sqrt{5})^2}$
 $= \sqrt{49-45} = \sqrt{4} = 2.$

62. $\left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)^2 = (\sqrt{3})^2 + \left(\frac{1}{\sqrt{3}}\right)^2 - 2 \times \sqrt{3} \times \frac{1}{\sqrt{3}}$
 $= 3 + \frac{1}{3} - 2 = 1 + \frac{1}{3} = \frac{4}{3}.$

63. $\left(\sqrt{2}+\frac{1}{\sqrt{2}}\right)^2 = (\sqrt{2})^2 + \left(\frac{1}{\sqrt{2}}\right)^2 - 2 \times \sqrt{2} \times \frac{1}{\sqrt{2}}$
 $= 2 + \frac{1}{2} + 2 = 4 + \frac{1}{2} = 4\frac{1}{2}.$

64. $\sqrt{4a^2 - 4a + 1 + 3a}$
 $= \sqrt{(1)^2 + (2a)^2 - 2 \times 1 \times 2a + 3a}$
 $= \sqrt{(1-2a)^2} + 3a = (1-a) + 3a$
 $= (1+a) = (1+0.1039) = 1.1039.$

65. $\sqrt{\frac{(0.75)^3}{(1-0.75)} + [0.75 + (0.75)^2 + 1]}$
 $= \sqrt{\frac{(0.75)^3 + (1-0.75)[(1)^2 + (0.75)^2 + 1 \times 0.75]}{1-0.75}}$
 $= \sqrt{\frac{(0.75)^3 + [(1)^3 - (0.75)^3]}{1-0.75}} = \sqrt{\frac{1}{0.25}} =$
 $= \sqrt{\frac{100}{25}} = \sqrt{4} = 2.$

66. $4a = 6c \Rightarrow b = \frac{2}{3}c \text{ and } 3a = 4b \Rightarrow a = \frac{4}{3}b$
 $= \frac{4}{3}\left(\frac{3}{2}c\right) = 2c.$

$$\begin{aligned} a + b + c &= 27\sqrt{29} \Rightarrow 2c + \frac{3}{2}c + c \\ &= 27\sqrt{29} \Rightarrow \frac{9}{2}c = 27\sqrt{29} \Rightarrow c = 6\sqrt{29}. \end{aligned}$$

$$\begin{aligned} \therefore \sqrt{a^2 + b^2 + c^2} &= \sqrt{(a+b+c)^2 - 2(ab+bc+ca)} \\ &= \sqrt{(27\sqrt{29})^2 - 2\left(2c \times \frac{3}{2}c + \frac{3}{2}c \times c + c \times 2c\right)} \\ &= \sqrt{(729 \times 29) - 2\left(3c^2 + \frac{3}{2}c^2 + 2c^2\right)} \\ &= \sqrt{(729 \times 29) - 2 \times \frac{13}{2}c^2} \\ &= \sqrt{(729 \times 29) - 13 \times (6\sqrt{29})^2} \end{aligned}$$

$$\begin{aligned}
 &= \sqrt{(729 \times 29) - 13 \times (6\sqrt{29})^2} \\
 &= \sqrt{29(729 - 468)} \\
 &= \sqrt{29 \times 261} = \sqrt{29 \times 29 \times 9} = 29 \times 3 = 87.
 \end{aligned}$$

67. $\sqrt{0.4} = \sqrt{\frac{4}{9}} = \frac{2}{3} = 0.666\dots = 0.\overline{6}$.

68. $\sqrt{0.09} = \sqrt{\frac{9}{100}} = \frac{3}{10} = 0.3$, Which is rational.
 $\therefore 0.09$ has rational square root.

69.

$$\begin{array}{r}
 6 | 0.4\overline{00000} (.63 \\
 \quad 36 \\
 \hline
 123 | 400 \\
 \quad 369 \\
 \hline
 \end{array}$$

70.

$$\begin{array}{r}
 3 | 0.1\overline{21000} (.347 \\
 \quad 9 \\
 \hline
 64 | 310 \\
 \quad 256 \\
 \hline
 687 | 5400 \\
 \quad 4809 \\
 \hline
 \end{array}$$

71.

$$\begin{array}{r}
 2 | 0.0\overline{64000} (.252 \\
 \quad 4 \\
 \hline
 45 | 240 \\
 \quad 225 \\
 \hline
 502 | 1500 \\
 \quad 1006 \\
 \hline
 \end{array}$$

72. $\sqrt{\frac{0.16}{0.4}} = \sqrt{\frac{0.16}{0.4}} = \sqrt{\frac{16}{40}} = \sqrt{\frac{4}{10}} = \sqrt{0.4} = 0.63$.

73. $\frac{1 + \sqrt{0.01}}{1 - \sqrt{0.1}} = \frac{1 + 0.1}{1 - 0.316} = \frac{1.1}{0.684}$

$$\begin{array}{r}
 3 | 0.1\overline{00000} (.316 \\
 \quad 9 \\
 \hline
 61 | 100 \\
 \quad 60 \\
 \hline
 62 | 3900 \\
 \quad 3750 \\
 \hline
 \end{array}
 = \frac{1100}{684} = 1.6.$$

74. $\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5} = \frac{2.236}{5} = 0.447$.

75. $\sqrt{\frac{8}{3}} = \sqrt{\frac{8 \times 3}{3 \times 3}} = \frac{\sqrt{24}}{3} = \frac{4.899}{3} = 1.633$.

76. $\frac{3\sqrt{2}}{2\sqrt{3}} = \frac{3\sqrt{2}}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{2} = \frac{2.449}{2} = 1.2245.$

77.
$$\begin{aligned}\frac{\sqrt{5}}{2} - \frac{10}{\sqrt{5}} + \sqrt{125} &= \frac{(\sqrt{5})^2 - 20 + 2\sqrt{5} \times 5\sqrt{5}}{2\sqrt{5}} \\&= \frac{2 - 20 + 50}{2\sqrt{5}} = \frac{35}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\&= \frac{35\sqrt{5}}{10} = \frac{7}{2} \times 2.236 = 7 \times 1.118 = 7.826.\end{aligned}$$

78. Clearly, $a + b = \sqrt{a^2 + b^2}$
 $\therefore 5*12 = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13.$

79. L.C.M. of 3, 4, 5, 6, 8 is 120. Now,
 $120 = 2 \times 2 \times 2 \times 3 \times 5.$
To make it a perfect square, it must be multiplied by $2 \times 3 \times 5.$
So, required number
 $= 2^2 \times 2^2 \times 3^2 \times 5^2 = 3600.$

80. L.C.M. of 21, 36, 66 = 2772. Now,
 $2772 = 2 \times 2 \times 3 \times 3 \times 7 \times 11.$
To make it a perfect square, It must be multiplied by $7 \times 11.$
So required number $= 2^2 \times 3^2 \times 7^2 \times 11^2$
 $= 213444.$

81. $294 = 7 \times 7 \times 2 \times 3$ i.e., 6.
 \therefore Required number is = 6.

82. $5808 = 2 \times 2 \times 2 \times 2 \times 3 \times 11 \times 11$
 $= 2^2 \times 2^2 \times 3 \times 11^2 = 213444.$
To make it a perfect square, It must be multiplied by 3.

83. $1470 = 7 \times 7 \times 5 \times 6.$ To make it a perfect square, it must be divided by 5×6 , i.e., 30.

84.
$$\begin{array}{r} 7 \mid \overline{549162} (\ 741 \\ \hline 49 \\ \hline 144 \quad 591 \\ \hline 1481 \quad 576 \\ \hline 1562 \\ \hline 1481 \\ \hline 81 \end{array}$$

\therefore Required number to be subtracted = 81.

85. $0.000326 = \frac{326}{10^6}$

$$\begin{array}{r} 1 \mid \overline{326} (\ 18 \\ \hline 1 \\ \hline 28 \quad 226 \\ \hline 224 \\ \hline 2 \end{array}$$

$$\therefore \text{Required number to be subtracted} = \frac{2}{10^6}$$

$$= 0.000002.$$

86.

$$\begin{array}{r} 8 | \overline{680621} \text{ (824)} \\ \underline{64} \\ 162 \quad 406 \\ \underline{324} \\ 1644 \quad 8221 \\ \underline{6576} \\ 1645 \end{array}$$

$$\therefore \text{Number to be added} = (825)^2 - 680621$$

$$= 680625 - 680621 = 4.$$

87. Greatest number of four digits is 9999.

$$\begin{array}{r} 9 | \overline{9999} \text{ (99)} \\ \underline{81} \\ 189 \quad 1899 \\ \underline{1701} \\ 198 \end{array}$$

$$\therefore \text{Required number} = (9999 - 198) = 9801.$$

88. Least number of four digits is 1000.

$$\begin{array}{r} 3 | \overline{1000} \text{ (31)} \\ \underline{9} \\ 61 \quad 100 \\ \underline{61} \\ 39 \end{array}$$

$$\therefore (31)^2 < 1000 < (32)^2. \text{ Hence, required number} = (32)^2 = 1024.$$

$$89. \quad \frac{1}{(\sqrt{5}-\sqrt{3})} = \frac{1}{(\sqrt{5}-\sqrt{3})} \times \frac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})}$$

$$= \frac{(2.2361+1.7321)}{2} = \frac{3.9682}{2} = 1.9841$$

90. Given exp.

$$\begin{aligned} &= \frac{1}{(\sqrt{9}-\sqrt{8})} \times \frac{(\sqrt{9}+\sqrt{8})}{(\sqrt{9}+\sqrt{8})} - \frac{1}{(\sqrt{8}-\sqrt{7})} \\ &\quad \times \frac{(\sqrt{8}+\sqrt{7})}{(\sqrt{8}+\sqrt{7})} + \frac{1}{(\sqrt{7}-\sqrt{6})} \times \frac{(\sqrt{7}+\sqrt{6})}{(\sqrt{7}+\sqrt{6})} \\ &\quad - \frac{1}{(\sqrt{6}-\sqrt{5})} \times \frac{(\sqrt{6}+\sqrt{5})}{(\sqrt{6}+\sqrt{5})} \\ &\quad + \frac{1}{(\sqrt{5}-\sqrt{4})} \times \frac{(\sqrt{5}+\sqrt{4})}{(\sqrt{5}+\sqrt{4})} \end{aligned}$$

$$\begin{aligned}
 &= \frac{(\sqrt{9} + \sqrt{8})}{(9-8)} - \frac{(\sqrt{8} + \sqrt{7})}{(8-7)} + \frac{(\sqrt{7} + \sqrt{6})}{(7-6)} \\
 &\quad - \frac{(\sqrt{6} + \sqrt{5})}{(6-5)} - \frac{(\sqrt{5} + \sqrt{4})}{(5-4)} \\
 &= (\sqrt{9} + \sqrt{8}) - (\sqrt{8} + \sqrt{7}) + (\sqrt{7} + \sqrt{6}) \\
 &\quad - (\sqrt{6} + \sqrt{5}) + (\sqrt{5} + \sqrt{4}) = (\sqrt{9} + \sqrt{4}) \\
 &= 3 + 2 = 5.
 \end{aligned}$$

91. Given exp. = $(2 + \sqrt{2}) + \frac{1}{(2+\sqrt{2})} \times \frac{(2-\sqrt{2})}{(2-\sqrt{2})}$

$$\begin{aligned}
 &\quad - \frac{1}{(2-\sqrt{2})} \times \frac{(2+\sqrt{2})}{(2+\sqrt{2})} \\
 &= (2+\sqrt{2}) + \frac{(2-\sqrt{2})}{(4-2)} - \frac{(2+\sqrt{2})}{(4-2)} \\
 &= (2+\sqrt{2}) + \frac{1}{2}(2-\sqrt{2}) - \frac{1}{2}(2+\sqrt{2}) = 2.
 \end{aligned}$$

92. $\frac{7}{(3+\sqrt{2})} = \frac{7}{(3+\sqrt{2})} \times \frac{(3-\sqrt{2})}{(3-\sqrt{2})}$

$$\begin{aligned}
 &= \frac{7(3-\sqrt{2})}{(9-2)} = (3-\sqrt{2}) \\
 &= (3 - 1.4142) = 1.5858.
 \end{aligned}$$

93. Given exp.

$$\begin{aligned}
 &= \frac{3\sqrt{2}}{(\sqrt{6}-\sqrt{3})} \times \frac{(\sqrt{6}+\sqrt{3})}{(\sqrt{6}+\sqrt{3})} - \frac{4\sqrt{3}}{(\sqrt{6}-\sqrt{2})} \\
 &\quad \times \frac{(\sqrt{6}+\sqrt{2})}{(\sqrt{6}+\sqrt{2})} - \frac{6}{2(\sqrt{2}-\sqrt{3})} \\
 &= \frac{3\sqrt{2}(\sqrt{6}+\sqrt{3})}{(6-3)} - \frac{4\sqrt{3}(\sqrt{6}+\sqrt{2})}{(6-2)} \\
 &\quad + \frac{3}{(\sqrt{3}-\sqrt{2})} \times \frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})} \\
 &= \sqrt{2} + (\sqrt{6} + \sqrt{3}) - \sqrt{3}(\sqrt{6} + \sqrt{2}) + 3(\sqrt{3} + \sqrt{2}) \\
 &= \sqrt{12} + \sqrt{6} - \sqrt{18} - \sqrt{6} + 3\sqrt{3} + 3\sqrt{2} \\
 &= 2\sqrt{3} - 3\sqrt{2} + 3\sqrt{3} + 3\sqrt{2} = 5\sqrt{3}.
 \end{aligned}$$

94. $\frac{\sqrt{7} + \sqrt{5}}{\sqrt{7} - \sqrt{5}} = \frac{\sqrt{7} + \sqrt{5}}{\sqrt{7} - \sqrt{5}} \times \frac{(\sqrt{7} + \sqrt{5})}{(\sqrt{7} + \sqrt{5})}$

$$\begin{aligned}
 &= \frac{(\sqrt{7} + \sqrt{5})^2}{(7-5)} = \frac{7+5+2\sqrt{35}}{2} \\
 &= \frac{12+2\sqrt{35}}{2} = 6 + \sqrt{35}.
 \end{aligned}$$

95. $a + b\sqrt{3} = \frac{(5+2\sqrt{3})}{(7+4\sqrt{3})} \times \frac{(7-4\sqrt{3})}{(7-4\sqrt{3})}$

$$= \frac{35 - 20\sqrt{3} + 14\sqrt{3} - 24}{(7)^2 - (4\sqrt{3})^2} = \frac{11 - 6\sqrt{3}}{49 - 48} = 11 - 6\sqrt{3}$$

$$\therefore a = 11, b = -6.$$

96. $\frac{\sqrt{2}-1}{\sqrt{2}+1} = \frac{(\sqrt{2}-1)}{(\sqrt{2}+1)} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)} = (\sqrt{2}-1)^2.$

$$\therefore \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}} = (\sqrt{2}-1) = (1.414-1) = 0.414.$$

97. Given exp.

$$\begin{aligned} &= \frac{3+\sqrt{6}}{5\sqrt{3}-4\sqrt{3}-4\sqrt{2}+4\sqrt{2}} = \frac{(3+\sqrt{6})}{(\sqrt{3}+\sqrt{2})} \\ &= \frac{(3+\sqrt{6})}{(\sqrt{3}+\sqrt{2})} \times \frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})} \\ &= \frac{3\sqrt{3}-3\sqrt{2}+3\sqrt{2}-2\sqrt{3}}{(3-2)} = \sqrt{3}. \end{aligned}$$

98. Given exp.

$$\begin{aligned} &= \frac{(2+\sqrt{3})}{(2-\sqrt{3})} \times \frac{(2+\sqrt{3})}{(2+\sqrt{3})} + \frac{(2-\sqrt{3})}{(2+\sqrt{3})} \times \frac{(2-\sqrt{3})}{(2-\sqrt{3})} \\ &\quad + \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)} \\ &= \frac{(2+\sqrt{3})^2}{(4-3)} + \frac{(2-\sqrt{3})^2}{(4-3)} + \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)} \\ &= \frac{(2+\sqrt{3})^2}{(4-3)} + \frac{(2-\sqrt{3})^2}{(4-3)} + \frac{(\sqrt{3}-1)^2}{(3-1)} \\ &= [(2+\sqrt{3})^2 + (2-\sqrt{3})^2] + \frac{4-2\sqrt{3}}{2} \\ &= 2(4+3) + 2 - \sqrt{3} = 16 - \sqrt{3}. \end{aligned}$$

99. $x + \frac{1}{x} = (7-4\sqrt{3}) + \frac{1}{(7-4\sqrt{3})} \times \frac{(7+4\sqrt{3})}{(7+4\sqrt{3})}$

$$= (7+4\sqrt{3}) + \frac{(7+4\sqrt{3})}{(49-48)}$$

$$= (7-4\sqrt{3}) + (7+4\sqrt{3}) = 14.$$

100. $x = \frac{(\sqrt{3}+1)}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}-1)} = \frac{(\sqrt{3}+1)^2}{(3-1)}$

$$= \frac{3+1+2\sqrt{3}}{2} = 2 + \sqrt{3}.$$

$$\begin{aligned} y &= \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)} = \frac{(\sqrt{3}-1)^2}{(3+1)} \\ &= \frac{3+1+2\sqrt{3}}{2} = 2 - \sqrt{3}. \end{aligned}$$

$$\therefore x^2 + y^2 = (2 + \sqrt{3})^2 + (2 - \sqrt{3})^2 \\ = 2(4 + 3) = 2 \times 7 = 14.$$

101. $a = \frac{(\sqrt{5}+1)}{(\sqrt{5}-1)} \times \frac{(\sqrt{5}+1)}{(\sqrt{5}-1)} = \frac{(\sqrt{5}+1)^2}{(5-1)}$
 $= \frac{5+2+1\sqrt{5}}{4} = \left(\frac{3+\sqrt{5}}{2} \right).$

$b = \frac{(\sqrt{5}-1)}{(\sqrt{5}+1)} \times \frac{(\sqrt{5}-1)}{(\sqrt{5}-1)} = \frac{(\sqrt{5}-1)^2}{(5-1)}$
 $= \frac{5+1+2\sqrt{5}}{4} = \left(\frac{3+\sqrt{5}}{2} \right).$

$\therefore a^2 + b^2 = \frac{(3+\sqrt{5})^2}{4} + \frac{(3-\sqrt{5})^2}{4}$
 $= \frac{(3+\sqrt{5})^2 + (3-\sqrt{5})^2}{4} = \frac{2(9+5)}{4} = 7.$

102.

$$\begin{array}{r} 1 | 15376 \text{ (} 124 \\ 1 | \\ 22 | 53 \\ 44 | \\ 244 | 976 \\ \hline & 976 \\ \times & \end{array}$$

\therefore Number of row = 124.

103.

$$\begin{array}{r} 1 | 36581 \text{ (} 191 \\ 1 | \\ 29 | 265 \\ 261 | \\ 381 | 481 \\ \hline & 381 \\ \times & 100 \end{array}$$

\therefore Number of men left = 100.

104. Money collected = (59.29×100) paise
 $= 5929$ paise
 \therefore Number of members = $\sqrt{5929} = 77$.

105. $(.000216)^{1/3} = \left(\frac{216}{10^6} \right)^{1/3} = \left(\frac{6 \times 6 \times 6}{10^2 \times 10^2 \times 10^2} \right)$
 $= \frac{6}{10^2} = \frac{6}{100} = .06$.

106. $\sqrt[3]{4 \frac{12}{125}} = \sqrt[3]{4 \frac{512}{125}} = \left(\frac{8 \times 8 \times 8}{5 \times 5 \times 5} \right)^{1/3} = \frac{8}{5} = 1 \frac{3}{5}.$

107. $\sqrt{.000064} = \sqrt{\frac{64}{10^6}} = \frac{8}{10^3} = \frac{8}{1000} = .008.$
 $\therefore \sqrt[3]{\sqrt{.000064}} = \sqrt[3]{.008} = \sqrt[3]{\frac{8}{1000}} = \frac{2}{10} = 0.2$

108. Clearly, 9261 is a perfect cube satisfying the given property.

109. $675 = 5 \times 5 \times 3 \times 3 \times 3.$
To make it a perfect cube, it must be multiplied by 5.

110. $3600 = 2^3 \times 5^2 \times 3^2 \times 2.$
To make it a perfect cube, it must be divided by $5^2 \times 3^2 \times 2$ i.e., 450.

111. (A) Do your self.

112. (C) $\sqrt{\frac{x}{49}} = \frac{4}{7}$ or $\frac{\sqrt{x}}{7} = \frac{4}{7} \Rightarrow \sqrt{x} = 4 \Rightarrow x = 16$

113. (C) Do your self.

114. (C) $\frac{\sqrt{7} + \sqrt{5}}{\sqrt{7} - \sqrt{5}} \times \frac{\sqrt{7} + \sqrt{5}}{\sqrt{7} + \sqrt{5}} = \frac{(\sqrt{7} + \sqrt{5})^2}{7 - 5}$
 $= \frac{7 + 5 + 2\sqrt{35}}{2} = \frac{12 + 2\sqrt{35}}{2} = 6 + \sqrt{35}$
 $= 6 + 5.9160 = 11.9160$

115. (A) $\sqrt{318 + \sqrt{36 + \sqrt{169}}} = \sqrt{3018 + \sqrt{36 + \sqrt{13}}}$
 $= \sqrt{3018 + \sqrt{49}}$
 $= \sqrt{3018 + 7} = \sqrt{3025} = 55$

116. (A) $\sqrt{\frac{x}{5}} = \frac{0.9 \times 0.09}{\sqrt{0.9 \times 0.09}} = 0.9 \times 0.09 = 0.081$

117. (C) $\sqrt{7 + 2\sqrt{10}} = \sqrt{5 + 2 + 2\sqrt{5} + \sqrt{2}}$
 $= \sqrt{(\sqrt{5})^2 + (\sqrt{2})^2 + 2\sqrt{5} \times \sqrt{2}} = \sqrt{(\sqrt{5 + \sqrt{2}})^2}$
 $= \sqrt{5 + \sqrt{2}}$

119. (B) Let number be x so $x = 64 \times \frac{1}{x^2} \Rightarrow x^3 = 64 \Rightarrow x = 4$

120. (D) LCM of 4, 5, 6, 15, 18 = 180 required perfect square = $180 \times 5 = 900$

121. (C)

122. (D) $\sqrt{2\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}} = 2\frac{31}{32}$

123. (D) Greatest number of size digits = 999999 number nearest to it is 998001 which is perfect square.

124. (C) $26244 = 2 \times 2 \times 3 \times 3 \times 3^2 \times 3^2$. Hence required no. to divide is $2 \times 2 \times 3 \times 3 = 36$.

125. (C) $14^3 = 2744$ & $15^3 = 3375$

126. (A)

$$\begin{array}{r} 71 \\ \hline 74931 \\ -49 \\ \hline 14131 \\ -141 \\ \hline 110 \end{array}$$

127. (C)

128. (B)

129. (D) Volume = $8 \times 6 \times 4 = 192$
 Cube of 5 = $5^3 = 125$
 Cube of 6 = $6^3 = 216$
 So $216 - 192 = 24$

130. (D) $\sqrt[3]{392 \times 448}$ do prime factorization & find.

131. (A)

132. (B) $\frac{x^3}{y^3} = \frac{343}{1331} \Rightarrow \frac{x}{y} = \frac{7}{11}$.