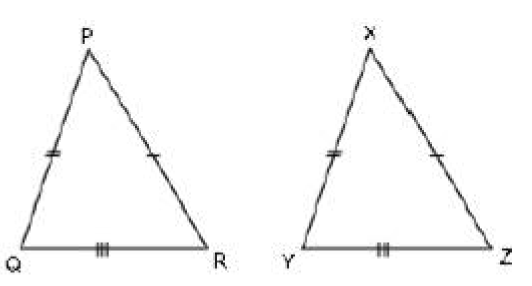
11. Congruency of triangles

Exercise 11.1

1 A. Question

Identify the corresponding sides and corresponding angles in the following congruent triangles:



Answer

Corresponding Sides of the two triangles are

PQ = XY

QR = YZ

RP = ZX

Corresponding Angles of the two triangles are

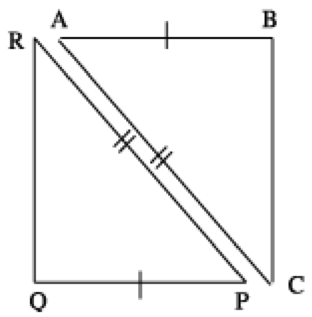
∠PQR = ∠XYZ

∠PRQ = ∠XZY

∠QPR = ∠YXZ

1 B. Question

Identify the corresponding sides and corresponding angles in the following congruent triangles:



Answer

Corresponding Sides of the two triangles are

PQ = AB

QR = BC

RP = CA

Corresponding Angles of the two triangles are

∠PQR = ∠ABC

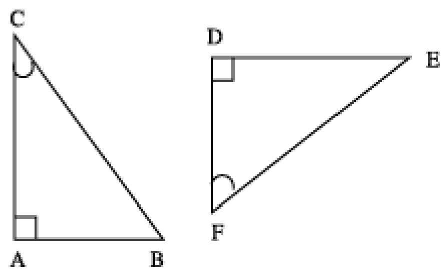
∠PRQ = ∠ACB

∠QPR = ∠CAB

2 A. Question

Pair of congruent triangles and incomplete statements related to them are given below. Observe the figures carefully and fill up the blanks:

In the adjoining figure if ∠C = ∠F, then AB = ______ and BC = ______.



Answer

In the adjoining figure if ∠C = ∠F, then AB = DE and BC = EF.

Since the triangles are congruent the corresponding sides of the triangles are equal

Since ∠C = ∠F

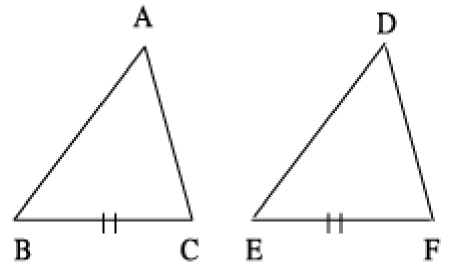
AB = DE

BC = EF

2 B. Question

Pair of congruent triangles and incomplete statements related to them are given below. Observe the figures carefully and fill up the blanks:

In the adjoining figure if BC = EF, then ∠C = ______and ∠A = ______.



Answer

In the adjoining figure if BC = EF, then ∠C = ∠F and ∠A = ∠D.

Since the triangles are congruent the corresponding angles of the triangles are equal

Since BC = EF

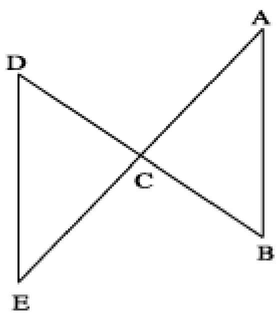
∠C = ∠F

∠A = ∠D

2 C. Question

Pair of congruent triangles and incomplete statements related to them are given below. Observe the figures carefully and fill up the blanks:

In the adjoining figure, if AC = CE and ΔABC ≅ ΔDEC, then ∠D = _______ and ∠A = _______.



Answer

In the adjoining figure, if AC = CE and ΔABC ≅ ΔDEC, then ∠D = ∠B and ∠A = ∠E.

Since the triangles ΔABC and ΔDEC are congruent the corresponding angles of the triangles are equal

Since AC = CE

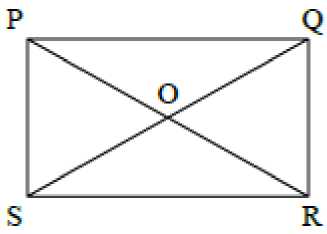
∠D = ∠B

∠A = ∠E

Exercise 11.2

1. Question

In the adjoining figure, PQRS is a rectangle. Identify the congruent triangles formed by the diagonals.



Answer

In Δ POQ and Δ SOR

OP = OR(Diagonals of a rectangle bisect each other)

OQ = OS(Diagonals of a rectangle bisect each other)

PQ = SR(Diagonals are equal in length)

Δ POQ ≅ Δ SOR by S.S.S. axiom of congruency

In Δ POS and Δ QOR

OP = OR(Diagonals of a rectangle bisect each other)

OQ = OS(Diagonals of a rectangle bisect each other)

PS = QR(Diagonals are equal in length)

Δ POS ≅ Δ QOR by S.S.S. axiom of congruency

In Δ PSQ, Δ PQR, Δ QRS and Δ PRS

PS = QR = QR = PS(Opposite sides of rectangle)

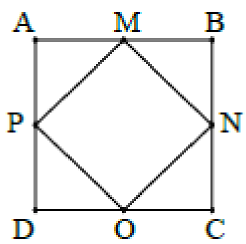
PQ = PQ = SR = SR(Opposite sides of the rectangle)

∠P = ∠Q = ∠R = ∠S (Angles of a rectangle)

Δ PSQ ≅ Δ PQR ≅ Δ QRS ≅ Δ PRS by S.A.S. axiom of congruency

2. Question

In the figure ABCD is a square, M, N, O, and P are the midpoints of sides AB, BC, CD and DA respectively. Identify the congruent triangles.



Answer

In Δ APM, Δ BMN, Δ CNO and Δ DOP

AB = BC = CD = DA (Sides of a square)

Since M, N, O, and P are the midpoints of sides AB, BC, CD and DA

⇒ 2AM = 2BN = 2CO = 2DP

⇒ AM = BN = CO = DP …(i)

⇒ 2AP = 2BM = 2CN = 2DO

⇒ AP = BM = CN = DO …(ii)

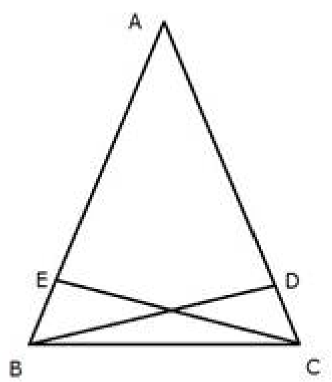
∠PAM = ∠MBN = ∠NCO = ∠ODP(Angles of a square) …(iii)

So Δ APM ≅ Δ BMN ≅ Δ CNO ≅ Δ DOP by S.A.S. axiom of congruency.

3. Question

In a triangle ABC, AB = AC. Points E on AB and D on AC are such that AE = AD. Prove that triangles BCD and CBE are congruent.

Answer



In Δ BCD and Δ CBE we have

AB = AC

AE = AD

Subtracting the above two equations we get

AB-AE = AC-AD

⇒ Be = CD

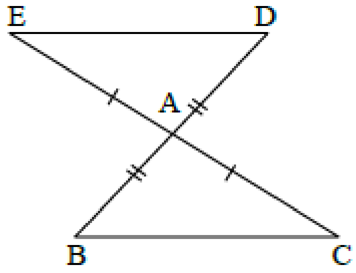
∠EBC = ∠DCB(Base angle of an isosceles triangle)

BC = BC(Common side)

So Δ BCD and Δ CBE are congruent to each other by S.A.S. axiom of congruency.

4. Question

In the adjoining figure, the sides BA and CA have been produced such that BA = AD and CA = AE. Prove that DE || BC. [hint:-use the concept of alternate angles.]



Answer

In Δ AED and Δ ABC we have

BA = AD(Given)

CA = AE(Given)

∠EAD = ∠BAC (Vertically Opposite)

So Δ AED and Δ ABC are congruent by S.A.S. axiom of congruency

So we can say

∠AED = ∠ACB(Corresponding parts of Congruent triangles)

So ∠AED & ∠ACB forms a pair of alternate interior angles

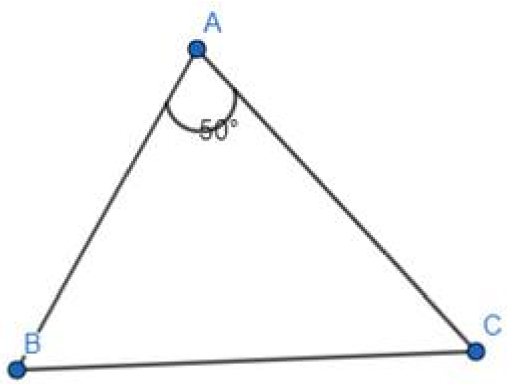
Hence DE || BC

Exercise 11.3

1. Question

In a ΔABC, AB = AC and ∠A = 50°. Find ∠B and ∠C.

Answer



In Δ ABC since AB = AC , so the triangle is isosceles

In an isosceles triangle, the base angles are always equal

So ∠B = ∠C = x(Let's assume)

Since the sum of interior angles of a triangle is 180^0

∠A + ∠B + ∠C = 180^0

⇒ 2x = 180^0-50^0 = 130^0

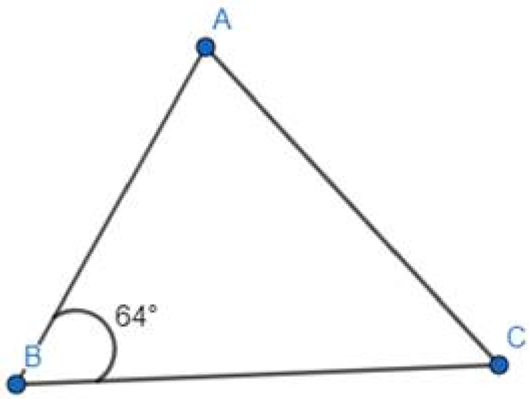
⇒ x = 65^0

∠B = ∠C = 65^0

2. Question

In ΔABC, AB = BC and ∠B = 64°. Find ∠C.

Answer



In Δ ABC since AB = BC ,so the triangle is isosceles

In an isosceles triangle the base angles are always equal

So ∠A = ∠C = x(Let's assume)

Since sum of interior angles of a triangle is 180^0

∠A + ∠B + ∠C = 180^0

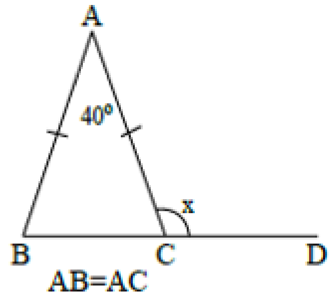
⇒ 2x = 180^0-64^0 = 116^0

⇒ x = 58^0

∠A = ∠C = 58^0

3 A. Question

In each of the following figure, find the value of x:



Answer

Δ ABC is isosceles with AB = AC

$\angle BAC = 40^0$(Given)

In an isosceles triangle since the base angles are equal so

$\angle ABC = \angle ACB = y$ (Let's assume)

$\angle ABC + \angle ACB + \angle BAC = 180^0$(Sum of interior angles of a triangle)

$\Rightarrow 2y = 180^0 - 40^0 = 140^0$

$\Rightarrow y = 70^0$

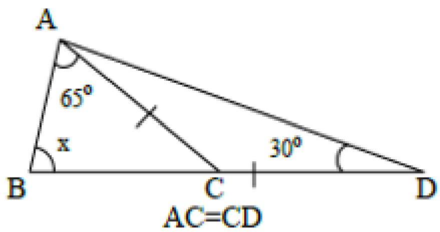
$\Rightarrow \angle ACB = 70^0$

$\angle ACB + x = 180^0$(Angle on a straight line)

$\Rightarrow x = 180^0 - 70^0 = 110^0$

3 B. Question

In each of the following figure, find the value of x:



Answer

Δ ACD is isosceles where AC = CD

$\angle DAC = \angle CDA = 30^0$

$\angle BAD = \angle DAC + \angle CAB$

$\Rightarrow \angle BAD = 65^0 + 30^0 = 95^0$

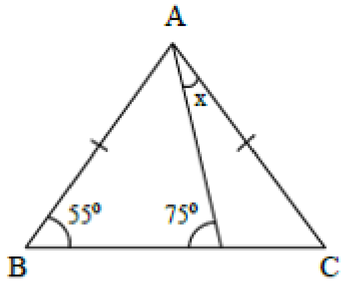
In Δ ABD,

$\angle ABD + \angle BDA + \angle DAB = 180^0$ (Sum of interior angles of a triangle)

$x = 180^0 - (95^0 + 30^0) = 55^0$

3 C. Question

In each of the following figure, find the value of x:



AB=AC

Answer

Δ ABC is isosceles with AB = AC

$\angle ABC = \angle ACB = 55^0$ (Base angles are equal)

$\angle ABC + \angle ACB + \angle BAC = 180^0$(Sum of interior angles of a triangle)

$\Rightarrow \angle BAC = 180^0 - (55^0 + 55^0) = 70^0$

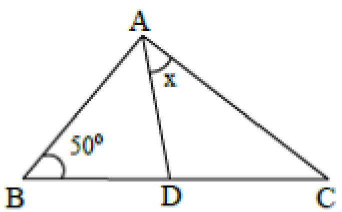
$\angle BAC - x = 180^0 - (55^0 + 75^0)$

$\Rightarrow -x = 180^0 - 200^0 = -20^0$

$\Rightarrow x = 20^0$

3 D. Question

In each of the following figure, find the value of x:



BD=DC=AD

Answer

Δ ABD and Δ ADC is isosceles

$\angle ABD = \angle BAD = 50^0$(Base angle of an isosceles triangle)

$\angle DAC = \angle DCA = x$ (Base angle of an isosceles triangle)

In Δ ABC

∠ABC + ∠ACB + ∠BAC = 180^0(Sum of interior angles of a triangle)

$50^0 + 50^0 + 2x = 180^0$

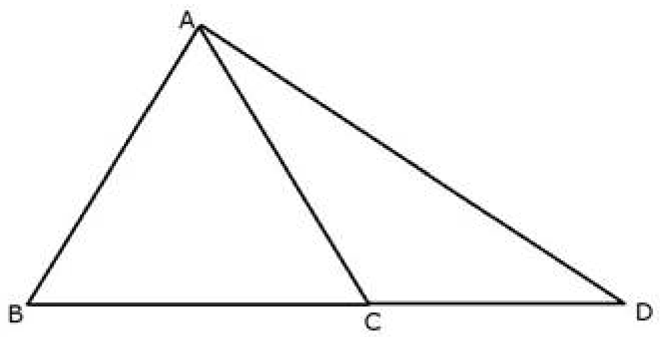
⇒ $2x = 80^0$

⇒ $x = 40^0$

4. Question

Suppose ABC is an equilateral triangle. Its base BC is produced to D such that BC = CD. Calculate (i) ∠ACD and (ii) ∠ADC.

Answer



Δ ABC is an equilateral triangle

CD is drawn such that CD = BC

∠ACB = 60^0(Interior angle of an equilateral triangle)

∠ACD = 180^0-60^0 = 120^0(Angle on a straight line)

In an equilateral triangle since all sides are equal so

AC = CD

Hence Δ ACD is isosceles

Since base angles of an isosceles triangle is equal so

∠CAD = ∠CDA = x(Let us assume)

∠ACD + ∠CAD + ∠CDA = 180^0(Sum of interior angles of a triangle)

⇒ $2x = 180^0 - 120^0 = 60^0$

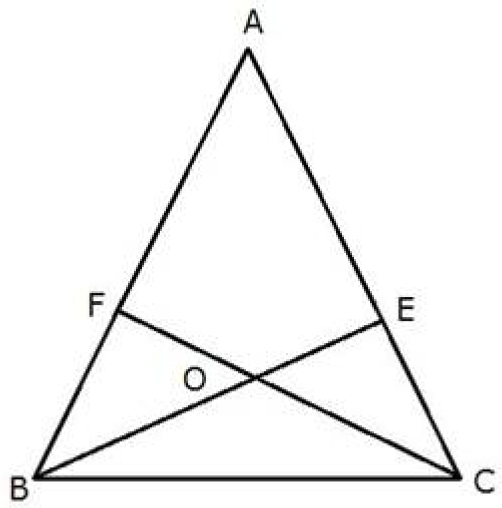
⇒ $x = 30^0$

∠ADC = 30^0

5. Question

Show that the perpendiculars drawn from the vertices of the base of an isosceles triangle to the opposite sides are equal.

Answer



Let Δ ABC be the isosceles triangle

BE and CF are the two perpendiculars drawn which cuts at O

In Δ BFC and Δ BEC

∠FBC = ∠ECB (Base angles of isosceles triangle)

∠BFC = ∠BEC = 90^0 (Perpendiculars)

BC = BC(Common side)

So Δ BFC and Δ BEC are congruent to each other by R.H.S. axiom of congruency

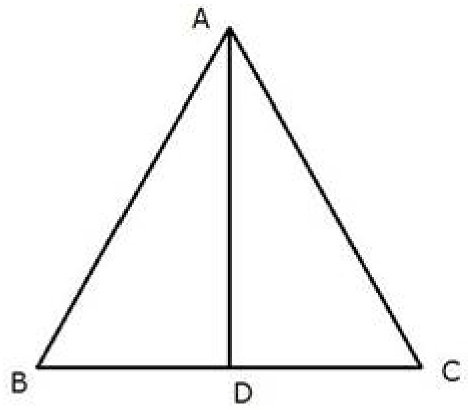
BE = FC (Corresponding Part of Congruent Triangle)

Hence its proved

6. Question

Prove that a ΔABC is an isosceles triangle if the altitude AD from A on BC bisects BC.

Answer



Δ ABC is isosceles with AB = AC

AD is the altitude on BC

In Δ ABD and Δ ACD

∠ABD = ∠ACD (Base angles of an isosceles triangle)

∠ADB = ∠ADC (AD is a perpendicular)

AD = AD (Common side)

So Δ ABD and Δ ACD are congruent to each other by A.A.S. axiom of congruency

BD = DC(Corresponding Parts of Congruent Triangles)

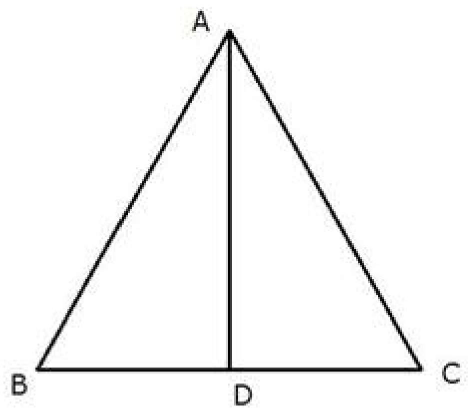
Since BD = DC so D is the midpoint of BC

So altitude AD bisects BC

7. Question

Suppose a triangle is equilateral. Prove that it is equiangular.

Answer



Δ ABC is an equilateral triangle

Let AD be the perpendicular from A on BC

In Δ ABD and Δ ACD

AB = AC (ΔABC is equilateral)

∠ADB = ∠ADC (AD is a perpendicular)

AD = AD (Common side)

So Δ ABD and Δ ACD are congruent to each other by S.A.S. axiom of congruency

∠ABD = ∠ACD (Corresponding Parts of Congruent Triangles)

∠BAD = ∠CAD (Corresponding Parts of Congruent Triangles)

Since the triangle is equilateral and ∠ABD = ∠ACD, so

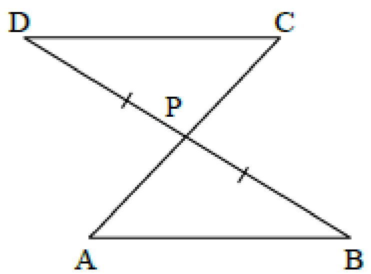
∠ABD = ∠ACD = ∠BAC

Hence the triangle is equiangular

Exercise 11.4

1. Question

In the given figure, If AB || DC and P is the midpoint of BD, prove that P is also the midpoint of AC.



Answer

In Δ DPC and Δ APB we have

DP = PB(P is the midpoint of BD)

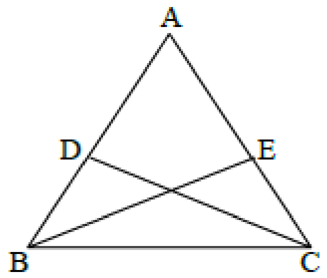
∠DPC = ∠APB(Vertically Opposite)

∠DCP = ∠PAB (Alternate interior angle)

So Δ DPC and Δ APB are congruent to each other by A.A,S. axiom of congruency

2. Question

In the adjacent figure, CD and BE are altitudes of an isosceles triangle ABC with AC = AB. Prove that AE = AD.



Answer

In Δ ABC BE and CD are the perpendiculars drawn on sides AC and AB respectively

In Δ BDC and Δ BEC

∠BDC = ∠BEC = 90^0 (Perpendiculars)

∠DBC = ∠ECB(Base angle of the isosceles triangle)

BC = BC(Common side)

So Δ BDC and Δ BEC are congruent to each other by A.A.S. axiom of congruency

DB = EC (Corresponding Part of Congruent Triangle) ...(i)

AB = AC (Given) ...(ii)

Subtracting (i) from (ii) we get

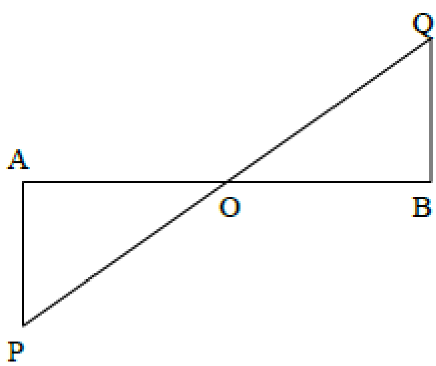
AB-DB = AC-EC

⇒ AD = AE

Hence Proved

3. Question

In the figure, AP and BQ are perpendiculars to the line segment AB and AP = BQ. Prove that O is the midpoint of line segment AB as well as PQ.



Answer

In Δ AOP and Δ BOQ

AP = BQ (Given)

∠AOP = ∠BOQ (Vertically Opposite)

∠PAO = ∠OBO (Perpendiculars)

So AOP and Δ BOQ are congruent to each other by A.A.S. axiom of congruency

Hence we can say

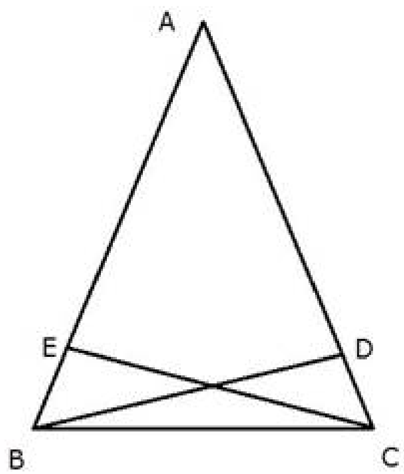
AO = OB(Corresponding parts of Congruent triangles)

PO = OQ(Corresponding parts of Congruent triangles)

4. Question

Suppose ABC is an isosceles triangle with AB = AC; BD and CE are bisectors of ∠B and ∠C. Prove that BD = CE.

Answer



BD and CE are bisectors of ∠B and ∠C

∠ABD = ∠DBC

∠ACE = ∠BCE

Since Δ ABC is isosceles so

∠ABC = ∠ACB …(i)

Since BD and CE are bisectors so

2∠DBC = 2∠ECB

⇒ ∠DBC = ∠ECB …(ii)

BC = BC (Common) …(iii)

From (i) , (ii) and (iii) we can say

Δ BCE and Δ BCD are congruent to each other by A.A.S. axiom of congruency

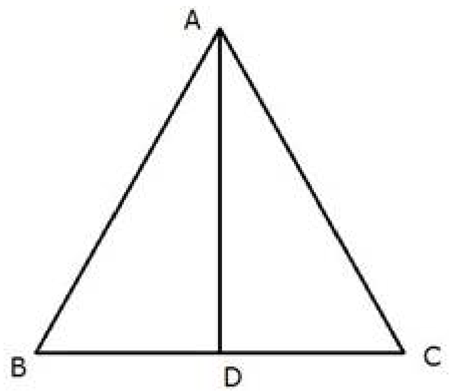
So we can say

BD = EC (Corresponding parts of Congruent triangles)

5. Question

Suppose ABC is an equiangular triangle. Prove that it is equilateral.(You have seen earlier that an equilateral triangle is equiangular. Thus for triangles equiangularity is equivalent to equilaterality.)

Answer



Δ ABC is an equiangular triangle

Let AD be the perpendicular from A on BC

In Δ ABD and Δ ACD

∠ABD = ∠ACD (ΔABC is equiangular)

∠ADB = ∠ADC (AD is a perpendicular)

AD = AD (Common side)

So Δ ABD and Δ ACD are congruent to each other by A.A.S. axiom of congruency

AB = AC (Corresponding Parts of Congruent Triangles)

BD = DC(Corresponding Parts of Congruent Triangles)

Since the triangle is equiangular and AB = AC , so

AB = AC = BC

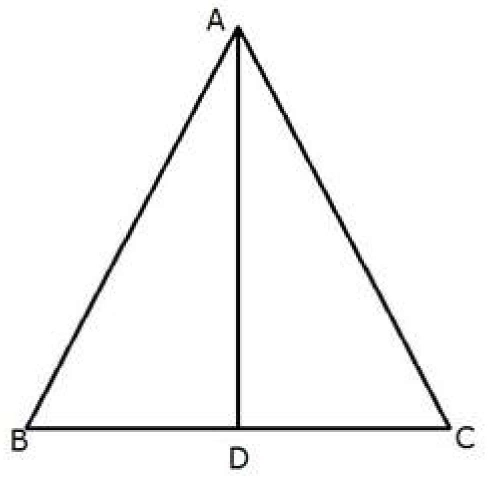
Hence the triangle is equilateral

Exercise 11.5

1. Question

In a triangle, ABC, AC = AB, and the altitude AD bisect BC. Prove that ΔADC ≅ ΔADB.

Answer



In ΔADC and ΔADB

AC = AB(Given)

BD = DC(AD bisects BC)

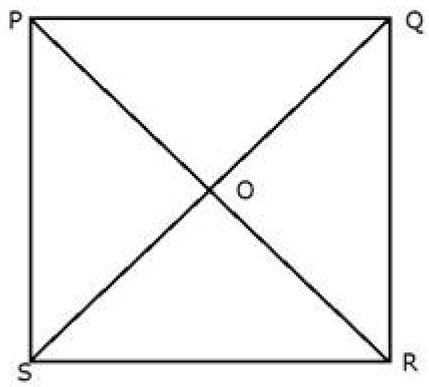
AD = AD(Common)

So ΔADC ≅ ΔADB by S.S.S. axiom of congruency

2. Question

In a square PQRS, diagonals bisect each other at O. Prove that ΔPOQ ≅ ΔQOR ≅ ΔROS ≅ ΔSOP.

Answer



In ΔPOQ, ΔQOR , ΔROS, and ΔSOP

PQ = QR = RS = SP (All sides of a square are equal)

∠POQ = ∠QOR = ∠ROS = ∠SOP = 90^0 (Diagonals of a square bisect at right angle)

PO = QO = RO = SO (Diagonals bisect each other)

So ΔPOQ ≅ ΔQOR ≅ ΔROS ≅ ΔSOP by S.A.S. axiom of congruency

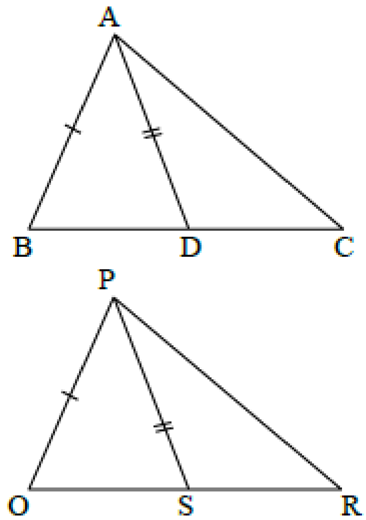
3. Question

In the figure, two sides AB, BC and the median AD of ΔABC are respectively equal to two sides PQ, QR and median PS of ΔPQR. Prove that

(i) ΔADB ≅ ΔPSQ;

(ii) ΔADC ≅ ΔPSR.

Does it follow that triangles ABC and PQR are congruent?



Answer

(i) In Δ ADB and Δ PQS

AB = PQ

AD = PS

BC = QR

Since D and S are midpoints of BC and QR

⇒ 2DC = 2SR

⇒ DC = SR

So Δ ADB and Δ PQS are congruent to each other by S.S.S. axiom of congruency

(ii) In Δ ADC and Δ PSR

AD = PS

BC = QR

Since D and S are midpoints of BC and QR

⇒ 2BD = 2QS

⇒ BD = QS

∠ADB = ∠PSO(Corresponding parts of Congruent triangles)

⇒ 180^0-∠ADB = 180^0-∠PSO

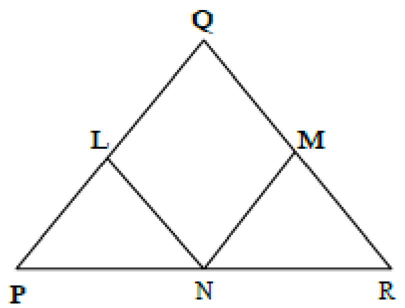
∠ADC = ∠PSR

So Δ ADC and Δ PSR are congruent to each other by S.A.S. axiom of congruency

Yes it follows Δ ABC and Δ PQR are congruent because Δ ABC is the sum of Δ ADB and Δ ADC and Δ PQR is the sum of Δ PQS and Δ PSR

4. Question

In ΔPQR, PQ = QR; L, M, and N are the midpoints of the sides of PQ, QR and RP respectively. Prove that LN = MN.



Answer

In Δ LNP and Δ MNR

LP = MR(Since PQ = QR)

PN = NR(N is a midpoint)

∠LPN = ∠MRN(Base angles of an isosceles triangle)

So Δ LNP and Δ MNR to each other by S.A.S. axiom of congruency

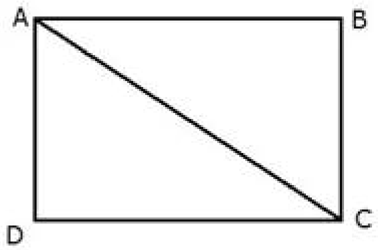
Hence LN = MN (Corresponding parts of Congruent Triangles)

Exercise 11.6

1. Question

Suppose ABCD is a rectangle. Using the RHS theorem, prove that triangles ABC and ADC are congruent.

Answer



ABCD is a rectangle

AC is a diagonal

In Δ ABC and Δ ADC

AD = BC (Opposite sides of a rectangle)

∠ADC = ∠ABC = 90^0(Angle of a rectangle)

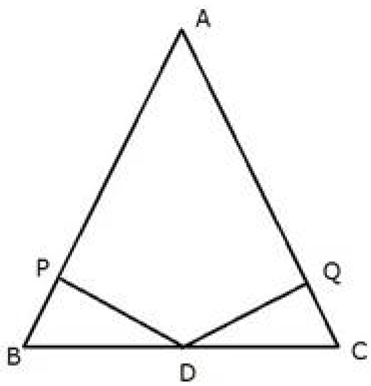
AC = AC (Common side)

So Δ ABC and Δ ADC is congruent by R.H.S. axiom of congruency.

2. Question

Suppose ABC is a triangle and D is the midpoint of BC. Assume that the perpendiculars from D to AB and AC are of equal length. Prove that ABC is isosceles.

Answer



In Δ ABC, D is the midpoint on BC

Let PD and QD be the two perpendiculars drawn from D on AB and AC respectively

In Δ BPD and Δ CQD we have

PD = QD(Given)

∠DPB = ∠DQC = 90^0(Perpendiculars)

BD = DC(D is a midpoint)

So Δ BPD and Δ CQD are congruent by R.H.S. axiom of congruency

So according to Corresponding Parts of Congruent triangles we get

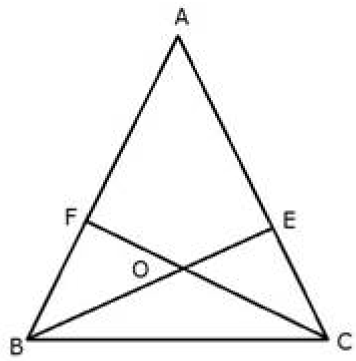
∠PBD = ∠QCD

Since the two angles of ABC are equal so Δ ABC isosceles.

3. Question

Suppose ABC is a triangle in which BE and CF are respectively the perpendiculars to the sides AC and AB. If BE = CF, prove that triangle ABC is isosceles.

Answer



In Δ ABC BE and CF are the perpendiculars drawn on sides AC and AB respectively

Let O be the intersection of the two perpendiculars

In Δ BFC and Δ BEC

BE = FC(Given)

∠BFC = ∠BEC = 90^0 (Perpendiculars)

BC = BC(Common side)

So Δ BFC and Δ BEC are congruent to each other by R.H.S. axiom of congruency

∠FBC = ∠ECB (Corresponding Part of Congruent Triangle)

Since two angles are the same so Δ ABC is isosceles.

Exercise 11.7

1. Question

In a triangle ABC, ∠B = 28° and ∠C = 56°. Find the largest and the smallest sides.

Answer

The side opposite to the smallest angle is the smallest side and the angle opposite to the largest angle is the largest side

Since the sum of interior angles of a triangle is 180^0

$\angle A + \angle B + \angle C = 180^0$

$\Rightarrow \angle A = 180^0 - (28° + 56°)$

$\Rightarrow \angle A = 96^0$

Since $\angle A$ is the largest angle so BC is the largest side

Since $\angle B$ is the smallest angle so AC is the smallest side

2. Question

In a triangle ABC, we have AB = 4cm, BC = 5.6cm and CA = 7.6cm. Write the angles of the triangle in ascending order of measures.

Answer

The side opposite to the smallest angle is the smallest side and the angle opposite to the largest angle is the largest side

The largest side among the three sides of Δ ABC is CA = 7.6 cm

So the largest angle of Δ ABC is $\angle B$

The smallest side among the three sides of Δ ABC is AB = 4 cm

So the smallest angle of Δ ABC is $\angle C$

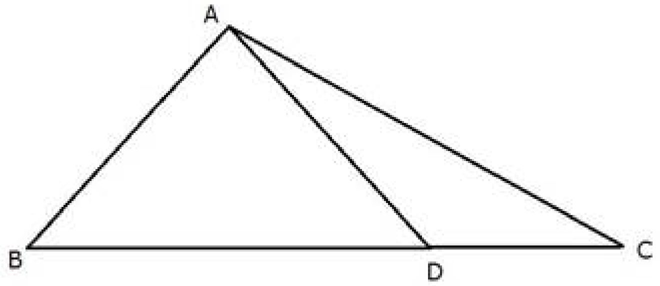
So in ascending order, the angles will be

$\angle C < \angle A < \angle B$

3. Question

Let ABC be a triangle such that $\angle B = 70°$ and $\angle C = 40°$. Suppose D is a point on BC such that AB = AD. Prove that AB > CD.

Answer



In Δ ABC

AB = AD

$\angle B = 70^0$

$\angle ADB = 70^0$(Base angles of an isosceles triangle)

$\angle ADC = 180^0 - 70^0 = 110^0$(Angles on a straight line)

$\angle C = 40^0$

$\angle DAC = 180^0 - (110^0 + 40^0) = 30^0$

Comparing ∠ACD and ∠DAC we find

∠ACD>∠DAC

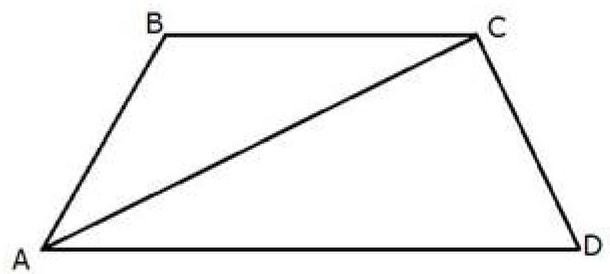
Since the side opposite to the largest angle is the largest so

AB > CD

4. Question

Let ABCD be a quadrilateral in which AD is the largest side and BC is the smallest side. Prove that ∠A <∠C. (Hint: Join AC)

Answer



ABCD is a quadrilateral with AD being the largest and BC being the smallest side

AC is a diagonal of the quadrilateral

Applying the law of Inequality of triangle in Δ BAC

∠BAC < ∠BCA (BC is the shortest side) ...(i)

Applying the law of Inequality of triangle in Δ ACD

∠CAD < ∠DCA (AD is the largest side) ...(ii)

Adding (i) and (ii) we get

∠BAC + ∠CAD < ∠BCA + ∠DCA

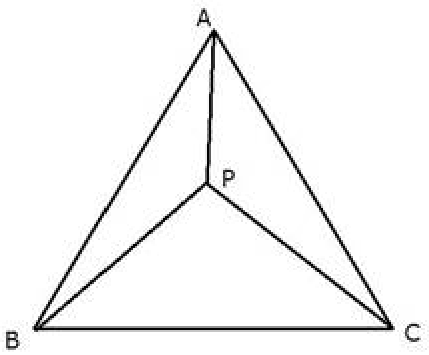
⇒ ∠A < ∠C

Hence Proved

5. Question

Let ABC be a triangle and P be an interior point. Prove that AB + BC + CA < 2 (PA + PB + PC).

Answer



According to triangle inequality the sum of smaller two sides is always greater than the largest side

In ΔAPB

AP + PB>AB …(i)

In ΔAPC

AP + PC>AC …(ii)

In ΔCPB

CP + PB>CB …(iii)

Adding (i),(ii) and (iii) we get

2 (PA + PB + PC)> AB + BC + CA

or changing the sign of inequality we can say

AB + BC + CA < 2 (PA + PB + PC)

Additional Problems 11

1. Question

Fill in the blanks to make the statements true.

(a) In right triangle the hypotenuse is the ______ side.

(b) The sum of three altitudes of a triangle is ______ than its perimeter.

(c) The sum of any two sides of a triangle is ______ than the third side.

(d) If two angles of a triangle are unequal, then the smaller angle has the ______ side opposite to it.

(e) Difference of any two sides of a triangle is _____ than the third side.

(f) If two sides of a triangle are unequal, then the larger side has _____ angle opposite to it.

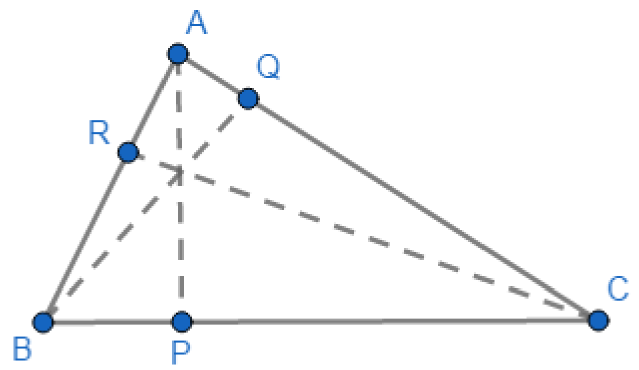
Answer

(a) In the right triangle, the hypotenuse is the **longest** side.

Explanation: The longest side is the side opposite to the largest angle and in a right-angled triangle the largest angle is 90° and the side opposite to it is the hypotenuse.

(b) The sum of three altitudes of a triangle is **less** than its perimeter

Explanation: Consider ΔABC with altitudes as AP, BQ and CR on segments BC, AB and AC respectively



Consider ΔAPC

∠APC = 90° … AP is altitude

∠ACP is some acute angle less than 90°

Hence AC > AP … (i)

Similarly we can prove

BC > CR … (ii)

AB > BQ … (iii)

Adding (i), (ii) and (iii) we get

⇒ AC + BC + AB > AP + CR + BQ

⇒ sum of sides > sum of altitudes

(c) The sum of any two sides of a triangle is **larger** than the third side.

Explanation: if the sum of two sides is equal to the third then the points will be collinear and these points won't form a triangle

(d) If two angles of a triangle are unequal, then the smaller angle has the **smaller** side opposite to it.

Explanation: The side opposite to larger angle is larger. And the side opposite to shorter angle is shorter. So, the smaller angle has smaller side opposite to it.

(e) The difference of any two sides of a triangle is **less** than the third side.

Explanation: if a, b, c are sides of a triangle we know that the sum of two sides is greater than the third

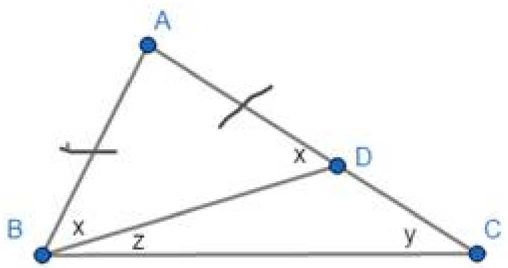
i.e. $a + b > c$

rearranging we get $a > c - b$

(f) If two sides of a triangle are unequal, then the larger side has **larger** angle opposite to it.

Explanation: consider ΔABC with AC the longer side than AB

Construct a line BD to AC from point B such that AB = AD



The angles are as marked

We have to prove that $(x + z) > y$

From figure

$\Rightarrow x + \angle BDC = 180°$ … linear pair of angles ∠ADB and ∠BDC

$\Rightarrow \angle BDC = 180° - x$

$\Rightarrow z + y + \angle BDC = 180°$ … angles of ΔBDC

$\Rightarrow z + y + 180° - x = 180°$

$\Rightarrow z + y = x$

$\Rightarrow x > y$

$\Rightarrow \angle ABD > \angle BDC$

If x is greater than y then x plus something will obviously be greater than y

$\Rightarrow x + z > y$

$\Rightarrow \angle ABC > \angle ACB$

Hence proved

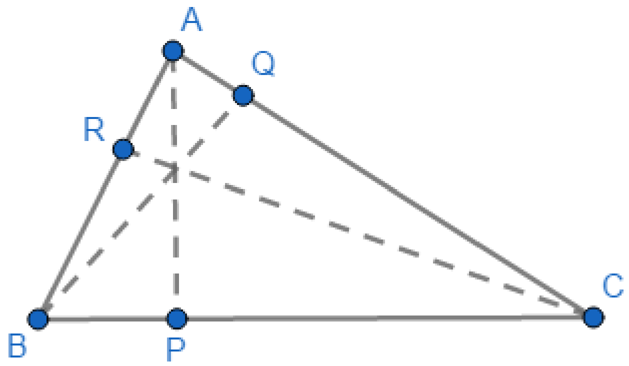
2 A. Question

Justify the following statements with reasons:

The sum of three sides of a triangle is more than the sum of its altitudes.

Answer

Consider ΔABC with altitudes as AP, BQ and CR on segments BC, AB and AC respectively



Consider ΔAPC

$\angle APC = 90°$ ... AP is altitude

$\angle ACP$ is some acute angle less than $90°$

Hence AC > AP ... (i)

Similarly we can prove

BC > CR ... (ii)

AB > BQ ... (iii)

Adding (i), (ii) and (iii) we get

$\Rightarrow$ AC + BC + AB > AP + CR + BQ

$\Rightarrow$ sum of sides > sum of altitudes

2 B. Question

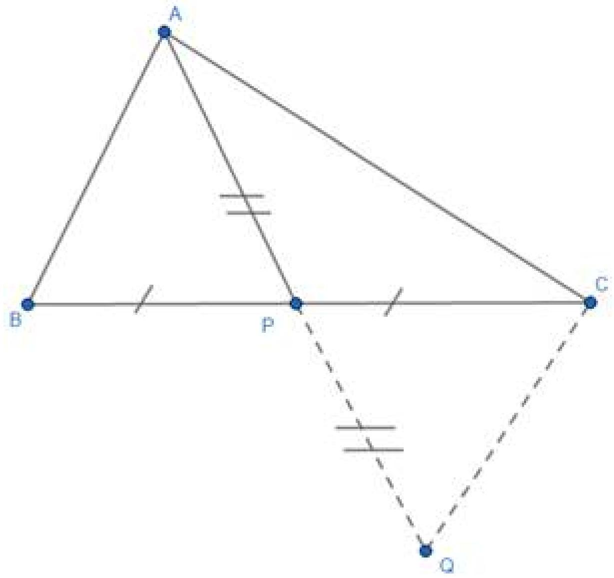
Justify the following statements with reasons:

The sum of any two sides of a triangle is greater than twice the median drawn to the third side.

Answer

Consider ΔABC and AP is the median as shown BP = PC

Construct PQ such that AP = PQ



Consider ΔAPB and ΔQPC

AP = PQ … construction

$\angle APB = \angle QCP$ … vertically opposite angles

BP = PC … AP is median

Thus by SAS test for congruency

$\Delta APB \cong \Delta QPC$

$\Rightarrow$ AB = CQ … corresponding sides of congruent triangles…(i)

Consider ΔAQC

$\Rightarrow AC + CQ > AQ$

Using (i)

$\Rightarrow AC + AB > AQ$

But AQ = AP + PQ and AP = PQ by construction hence AQ = 2AP

$\Rightarrow AC + AB > 2AP$

2 C. Question

Justify the following statements with reasons:

The difference of any two sides of a triangle is less than the third side.

Answer

If a, b, c are sides of a triangle we know that the sum of two sides is greater than the third

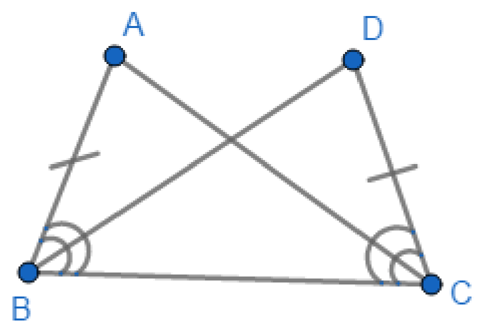
i.e. a + b > c

rearranging we get a > c – b

3. Question

Two triangles ABC and DBC have common base BC. Suppose AB = DC and ∠ABC = ∠BCD. Prove that AC = BD.

Answer



In ΔABC and ΔDBC

BC = BC … common side

∠ABC = ∠DBC … given

AB = CD … given

Therefore, by SAS test for congruency

ΔABC ≅ ΔDCB

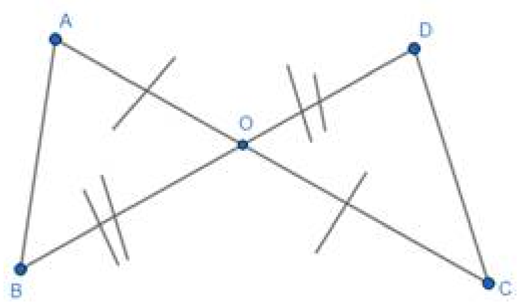
⇒ AC = BD … corresponding sides of congruent triangles

Hence proved AC = BD

4. Question

Let AB and CD be two line segments such that AD and BC intersect at O. Suppose AO = OC and BO = OD. Prove that AB = CD.

Answer



In ΔAOB and ΔCOD

AO = CO … given

∠AOB = ∠COD … vertically opposite angles

BO = OD … given

Therefore, by SAS test for congruency

ΔAOB ≅ ΔCOD

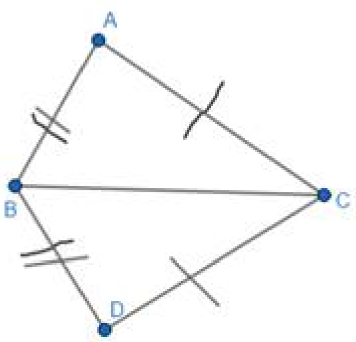
⇒ AB = CD … corresponding sides of congruent triangles

Hence proved AB = CD

5. Question

Let ABC be a triangle. Draw a triangle BDC externally on BC such that AB = BD and AC = CD. Prove that ΔABC ≅ ΔDBC.

Answer



In ΔABC and ΔDCB

AB = BD … given

AC = CB … given

BC = BC … common side

Therefore, by SSS test for congruency

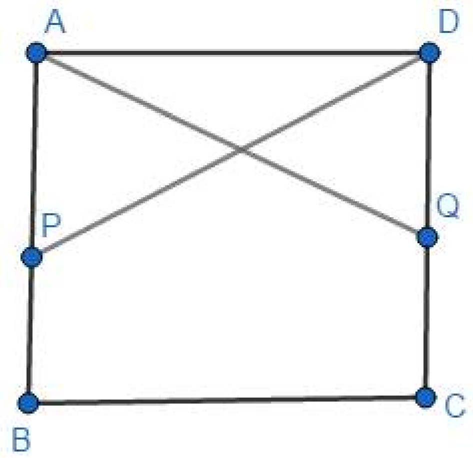
ΔABC ≅ ΔDBC

Hence proved

6. Question

Let ABCD be a square and let points P on AB and Q on DC be such that DP = AQ. Prove that BP = CQ.

Answer



Consider ΔADQ and ΔDAP

DP = AQ … given

∠PAD = ∠QDA … angles of square both 90°

AD = AD … common side

Therefore, by SAS test for congruency

ΔADQ ≅ ΔDAP

⇒ AP = DQ … corresponding sides of congruent triangles (i)

Now AB and CD are sides of square therefore,

AB = CD

⇒ AP + PB = DQ + QC … from figure AB = AP + PB and CD = DQ + QC

But using (i) AP = DQ

⇒ DQ + PB = DQ + QC

⇒ PB = QC

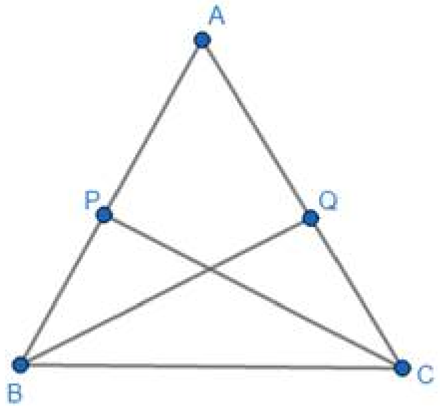
⇒ BP = CQ

Hence proved

7. Question

In a triangle ABC, AB = AC. Suppose P is a point on AB and Q is a point on AC such that AP = AQ. = Prove that ΔAPC ≅ ΔAQB.

Answer



In ΔAPC and ΔAQB

AP = AQ … given

∠BAQ = ∠CAP … ∠A common angle

AB = AC … given

Therefore, by SAS test for congruency

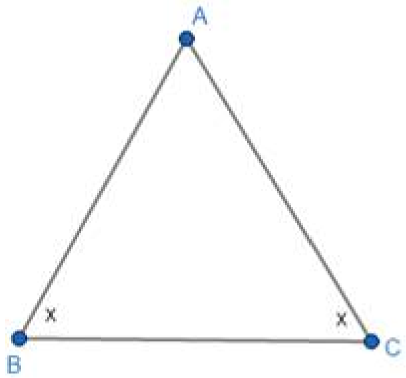
ΔAPC ≅ ΔAQB

Hence proved

8. Question

In an isosceles triangle, if the vertex angle is twice the sum of the base angles, calculate the angles of the triangle.

Answer



Let the isosceles triangle be ΔABC as shown with base as BC and equal sides are AB and AC

In an isosceles triangle the base angles are equal let them be 'x°'

Thus ∠B = x° and ∠C = x°

Given that ∠A = 2 × (∠B + ∠C)

⇒ ∠A = 2 × (x° + x°)

⇒ ∠A = 2 × 2x°

⇒ ∠A = 4x°

Now the sum of all angles of a triangle is 180°

⇒ ∠A + ∠B + ∠C = 180°

⇒ 4x° + x° + x° = 180°

⇒ 6x° = 180°

$$\Rightarrow x^\circ = \frac{180^\circ}{6}$$

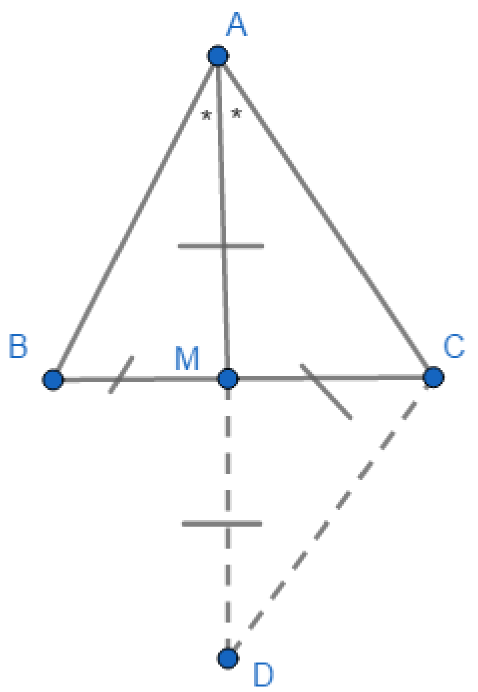
⇒ x = 30°

Therefore, angles of triangle are ∠B = 30°, ∠C = 30° and ∠A = 4x = 4 × 30° = 120°

9. Question

If the bisector of the vertical angle of a triangle bisects the base, show that the triangle is isosceles.

Answer



Consider ΔABC and AM is the bisector of ∠A and AM bisects the base BC so BM = CM

Extend segment AM to D such that AM = MD and join points C and D to from ΔDMC as shown

To prove ΔABC is isosceles we have to prove that AB = AC

Consider ΔAMB and ΔDMC

AM = MD … construction

∠AMB = ∠DMC … vertically opposite angles

BM = MC … AM bisects BC given

Hence by SAS test for congruency

ΔAMB ≅ ΔDMC

⇒ AB = CD … corresponding sides of congruent triangles … (i)

⇒ ∠BAM = ∠CDM … corresponding angles of congruent triangles … (a)

⇒ ∠BAM = ∠MAC … given AM is angle bisector of ∠A … (b)

Thus using (a) and (b) we can conclude that

∠CDM = ∠MAC … (c)

Now consider ΔACD

∠CAD = ∠CDA … from (c)

As the two angles are equal ΔACD is isosceles hence we can say that

⇒ AC = CD … (ii)

Now using (i) and (ii) we can conclude that

⇒ AB = AC

And hence ΔABC is isosceles triangle

10. Question

Suppose ABC is an isosceles triangle with AB = AC. Side BA has produced to D such that BA = AD. Prove that ∠BCD is a right angle.

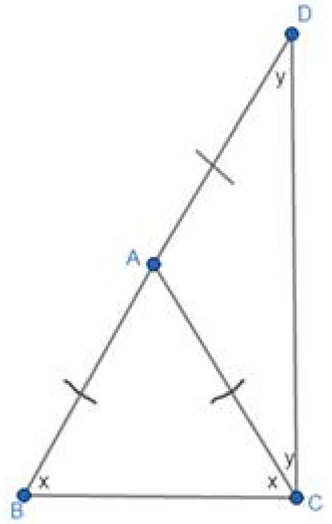
Answer

The figure is as shown

As AB = AC and AB = AD thus AC = AD

Therefore, ΔACD is also isosceles triangle

Let the base angles of ΔABC be x and the base angles of ΔACD be y as shown



In order to prove $\angle BCD$ is a right angle we have to prove that $x + y = 90°$

Consider ΔBCD

$\angle DBC = x$,

$\angle DCB = x + y$ and

$\angle BDC = y$

Sum of angles of a triangle is 180°

$\Rightarrow \angle DBC + \angle DCB + \angle BDC = 180°$

$\Rightarrow x + x + y + y = 180°$

$\Rightarrow 2x + 2y = 180°$

$\Rightarrow 2(x + y) = 180°$

$$\Rightarrow (x + y) = \frac{180°}{2}$$

$\Rightarrow x + y = 90°$

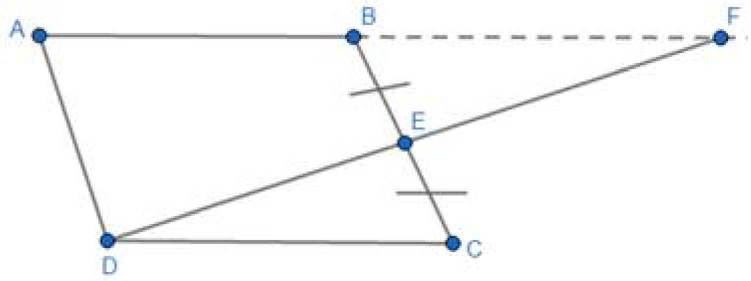
$\Rightarrow \angle DCB = 90°$

Hence proved $\angle BCD$ is a right angle.

11. Question

Let AB, CD be two line segments such that AB || CD and AD ||BC. Let E be the midpoint of BC and let DE extended meet AB in F. Prove that AB = BF.

Answer



Given AB || CD and AD || BC hence ABCD is a parallelogram

⇒ AB = CD … opposite sides of parallelogram (i)

Consider ΔCED and ΔBEF

∠DEC = ∠BEF … vertically opposite angles

BE = EC … given

∠ECD = ∠EBF … alternate angles as AF || CD with transversal BC

Therefore, by ASA test for congruency

ΔCED ≅ ΔBEF

⇒ CD = BF … corresponding sides of congruent triangles

Using (i)

⇒ AB = BF

Hence proved