3.4 ELECTRIC CURRENT

3.147 The convection current is

$$I = \frac{dq}{dt} \tag{1}$$

here, $dq = \lambda\, dx$, where λ is the linear charge density.

But, from the Gauss' theorem, electric field at the surface of the cylinder,

$$E = \frac{\lambda}{2\pi\varepsilon_0 a}$$

Hence, substituting the value of λ and subsequently of dq in Eqs. (1), we get

$$I = \frac{2E\pi\varepsilon_0 a dx}{dt}$$

$$= 2\pi\varepsilon_0 E a v, \text{ as } \frac{dx}{dt} = v$$

3.148 Since $d << r$, the capacitance of the given capacitor can be calculated using the formula for a parallel plate capacitor. Therefore if the water (permittivity ε) is introduced up to a height x and the capacitor is of length l, we have,

$$C = \frac{\varepsilon\varepsilon_0 \cdot 2\pi r x}{d} + \frac{\varepsilon_0 (l-x) 2\pi r}{d} = \frac{\varepsilon_0 2\pi r}{d}(\varepsilon x + l - x)$$

Hence charge on the plate at that instant, $q = CV$

Again we know that the electric current intensity,

$$I = \frac{dq}{dt} = \frac{d(CV)}{dt}$$

$$= \frac{V\varepsilon_0 2\pi r}{d}\frac{d(\varepsilon x + l - x)}{dt} = \frac{V 2\pi r\varepsilon_0}{d}(\varepsilon - 1)\frac{dx}{dt}$$

But, $$\frac{dx}{dt} = v,$$

So, $$I = \frac{2\pi r\varepsilon_0(\varepsilon - 1)V}{d} v = 0{\cdot}11\ \mu\text{A}$$

3.149 We have, $R_t = R_0(1 + \alpha t)$,(1)

where R_t and R_0 are resistances at $t°$ C and $0°$ C respectively and α is the mean temperature coefficient of resistance.

So, $$R_1 = R_0(1 + \alpha_1 t) \text{ and } R_2 = \eta R_0(1 + \alpha_2 t)$$

(a) In case of series combination, $R = \Sigma R_i$

so $$R = R_1 + R_2 = R_0[(1+\eta) + (\alpha_1 + \eta\alpha_2)t] \tag{1}$$

$$= R_0(1+\eta)\left[1 + 2\frac{\alpha_1 + \eta\alpha_2}{1+\eta}t\right] \tag{2}$$

Comparing Eqs. (1) and (2), we conclude that temperature co-efficient of resistance of the circuit,

$$\alpha = \frac{\alpha_1 + \eta\,\alpha_2}{1+\eta}$$

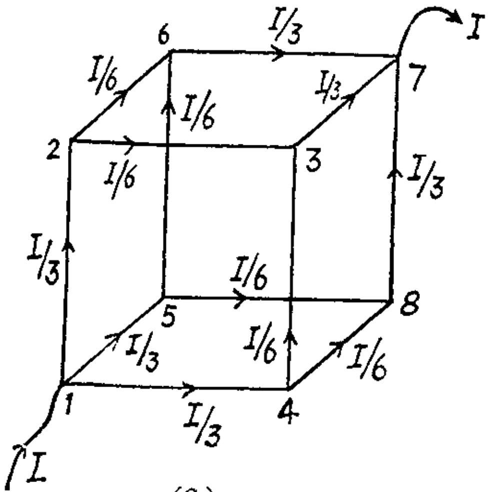
(b) In parallel combination

$$R = \frac{R_0\,(1+\alpha_1 t)\,R_0\,\eta\,(1+\alpha_2 t)}{R_0\,(1+\alpha_1 t) + \eta\,R_0\,(1+\alpha_2 t)} = R'\,(1+\alpha' t),\ \text{where}\ R' = \frac{\eta R_0}{1+\eta}$$

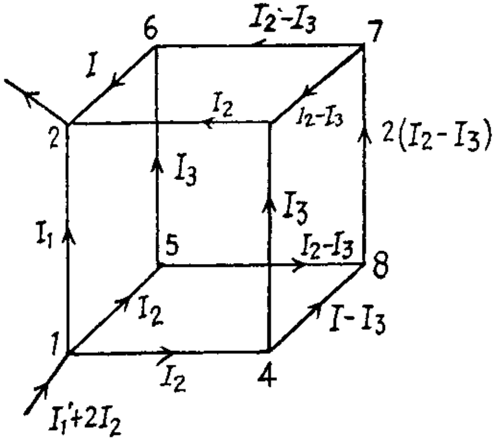
Now, neglecting the terms, proportional to the product of temperature coefficients, as being very small, we get,

$$\alpha' \approx \frac{\eta\,\alpha_1 + \alpha_2}{1+\eta}$$

3.150 (a) The currents are as shown. From Ohm's law applied between 1 and 7 via 1487 (say)



(a)



(b)

$$IR_{eq} = \frac{I}{3}R + \frac{I}{6}R + \frac{I}{3}R = \frac{5}{6}\,RI$$

Thus, $$R_{eq} = \frac{5R}{6}$$

(b) Between 1 and 2 from the loop 14321,

$I_1 R = 2I_2 R + I_3 R$ or, $I_1 = I_3 + 2I_2$

From the loop 48734,

$$(I_2 - I_3)\,R + 2\,(I_2 - I_3)\,R + (I_2 - I_3)\,R = I_3\,R.$$

or, $$4\,(I_2 - I_3) = I_3 \quad \text{or} \quad I_3 = \frac{4}{5}I_2$$

so $$I_1 = \frac{14}{5}I_2$$

Then, $(I_1 + 2I_2)\,R_{eq} = \dfrac{24}{5}I_2\,R_{eq} = I_1 R = \dfrac{14}{5}I_2\,R$

or $$R_{eq} = \frac{7}{12} R$$

(c) Between 1 and 3

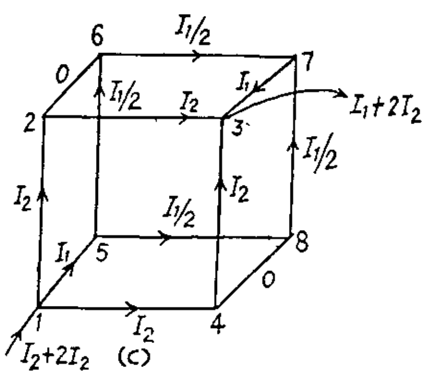
From the loop 15621

$$I_2 R = I_1 R + \frac{I_1}{2} R \quad \text{or,} \quad I_2 = 3\,\frac{I_1}{2}$$

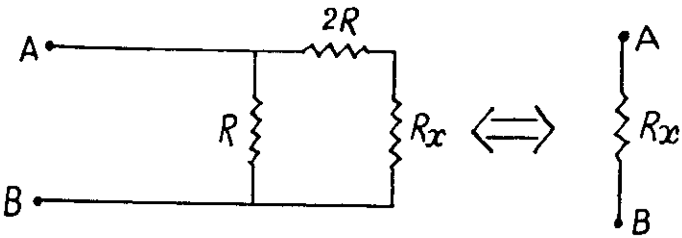
Then, $(I_1 + 2I_2)\, R_{eq} = 4\, I_1\, R_{eq}$

$$= I_2 R + I_2 R = 3 I_1 R$$

Hence, $$R_{eq} = \frac{3}{4} R\ .$$



3.151 Total resistance of the circuit will be independent of the number of cells,



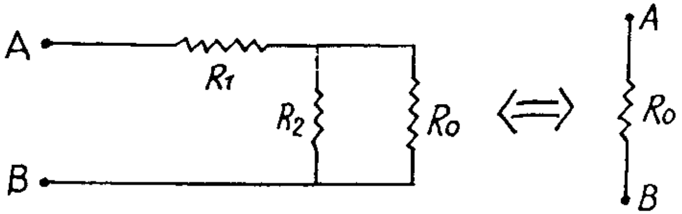
if $$R_x = \frac{(R_x + 2R)\,R}{R_x + 2R + R}$$

or, $$R_x^2 + 2R\,R_x - 2\,R^2 = 0$$

On solving and rejecting the negative root of the quadratic equation, we have,

$$R_x = R\,(\sqrt{3} - 1)$$

3.152 Let R_0 be the resistance of the network,



then, $R_0 = \dfrac{R_0\,R_2}{R_0 + R_2}$ or $R_0^2 - R_0\,R_1 - R_1\,R_2 = 0$

On solving we get,

$$R_0 = \frac{R_1}{2}\left(1 + \sqrt{1 + 4\frac{R_2}{R_1}}\right) = 6\,\Omega$$

3.153 Suppose that the voltage V is applied between the points A and B then

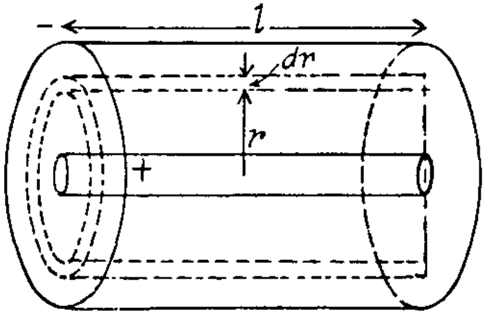
$$V = IR = I_0 R_0,$$

where R is resistance of whole the grid, I, the current through the grid and I_0, the current through the segment AB. Now from symmetry, $I/4$ is the part of the current, flowing through all the four wire segments, meeting at the point A and similarly the amount of current flowing through the wires, meeting at B is also $I/4$. Thus a current $I/2$ flows through the conductor AB, i.e.

$$I_0 = \frac{I}{2}$$

Hence,
$$R = \frac{R_0}{2}$$

3.154 Let us mentally isolate a thin cylindrical layer of inner and outer radii r and $r + dr$ respectively. As lines of current at all the points of this layer are perpendicular to it, such a layer can be treated as a cylindrical conductor of thickness dr and cross-sectional area $2\pi rl$. So, we have,



$$dR = \rho \frac{dr}{S(r)} = \rho \frac{dr}{2\pi rl}$$

and integrating between the limits, we get,

$$R = \frac{\rho}{2\pi l} \ln \frac{b}{a}$$

3.155 Let us mentally isolate a thin spherical layer of inner and outer radii r and $r + dr$. Lines of current at all the points of the this layer are perpendicular to it and therefore such a layer can be treated as a spherical conductor of thickness dr and cross sectional area $4\pi r^2$. So

$$dR = \rho \frac{dr}{4\pi r^2} \tag{1}$$

And integrating (1) between the limits $[a, b]$, we get,

$$R = \frac{\rho}{4\pi}\left[\frac{1}{a} - \frac{1}{b}\right]$$

Now, for $b \to \infty$, we have

$$R = \frac{\rho}{4\pi a}$$

3.156 In our system, resistance of the medium $R = \frac{\rho}{4\pi}\left[\frac{1}{a} - \frac{1}{b}\right]$,

where ρ is the resistivity of the medium

The current
$$i = \frac{\varphi}{R} = \frac{\varphi}{\frac{\rho}{4\pi}\left[\frac{1}{a} - \frac{1}{b}\right]}$$

Also, $i = \frac{-dq}{dt} = -\frac{d(C\varphi)}{dt} = -C\frac{d\varphi}{dt}$, as capacitance is constant. (2)

So, equating (1) and (2) we get,

$$\frac{\varphi}{\frac{\rho}{4\pi}\left[\frac{1}{a}-\frac{1}{b}\right]} = -C\frac{d\varphi}{dt}$$

or,
$$-\int\frac{d\varphi}{\varphi} = \frac{\Delta t}{\frac{C\rho}{4\pi}\left[\frac{1}{a}-\frac{1}{b}\right]}$$

or,
$$\ln\eta = \frac{\Delta t\,4\pi ab}{C\rho(b-a)}$$

Hence, resistivity of the medium,

$$\rho = \frac{4\pi\,\Delta t\,ab}{C(b-a)\ln\eta}$$

3.157 Let us mentally impart the charge $+q$ and $-q$ to the balls respectively. The electric field strength at the surface of a ball will be determined only by its own charge and the charge can be considered to be uniformly distributed over the surface, because the other ball is at infinite distance. Magnitude of the field strength is given by,

$$E = \frac{q}{4\pi\varepsilon_0 a^2}$$

So, current density $j = \frac{1}{\rho}\frac{q}{4\pi\varepsilon_0 a^2}$ and electric current

$$I = \int\vec{j}\cdot d\vec{S} = jS = \frac{q}{\rho\,4\pi\varepsilon_0 a^2}\cdot 4\pi a^2 = \frac{q}{\rho\varepsilon_0}$$

But, potential difference between the balls,

$$\varphi_+ - \varphi_- = 2\frac{q}{4\pi\varepsilon_0 a}$$

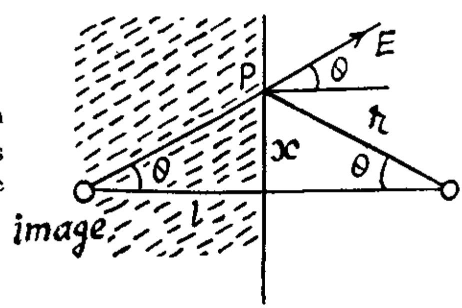
Hence, the sought resistance,

$$R = \frac{\varphi_+ - \varphi_-}{I} = \frac{2q/4\pi\varepsilon_0 a}{q/\rho\varepsilon_0} = \frac{\rho}{2\pi a}$$

3.158 (a) The potential in the unshaded region beyond the conductor as the potential of the given charge and its image and has the form

$$\varphi = A\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

where r_1, r_2 are the distances of the point from the charge and its image. The potential has been taken to be zero on the conducting plane and on the ball



$$\varphi = A\left(\frac{1}{a}-\frac{1}{2l}\right) = V$$

So $A \approx Va$. In this calculation the conditions $a << l$ is used to ignore the variation of φ over the ball.

The electric field at P can be calculated similarly. The charge on the ball is

$$Q = 4\pi\varepsilon_0 Va$$

and
$$E_P = \frac{Va}{r^2}\ 2\cos\theta = \frac{2alV}{r^3}$$

Then $j = \frac{1}{\rho}E = \frac{2alV}{\rho r^3}$ normal to the plane.

(b) The total current flowing into the conducting plane is

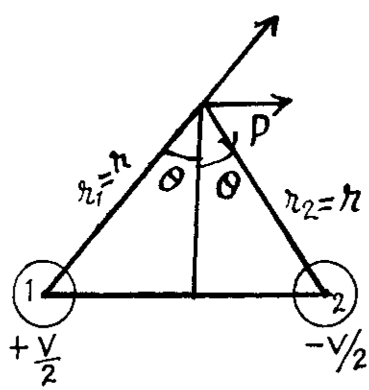
$$I = \int_0^\infty 2\pi x\, dx\, j = \int_0^\infty 2\pi x\, dx \frac{2alV}{\rho\left(\pi^2 + l^2\right)^{3/2}}$$

(On putting $y = x^2 + l^2$)

$$I = \frac{2\pi alV}{\rho}\int_{l^2}^{\infty}\frac{dy}{y^{3/2}} = \frac{4\pi aV}{\rho}$$

Hence
$$R = \frac{V}{I} = \frac{\rho}{2\pi a}$$

3.159 (a) The wires themselves will be assumed to be perfect conductors so the resistance is entirely due to the medium. If the wire is of length L, the resistance R of the medium is $\alpha\frac{1}{L}$ because different sections of the wire are connected in parallel (by the medium) rather than in series. Thus if R_1 is the resistance per unit length of the wire then $R = R_1/L$. Unit of R_1 is ohm-meter.



The potential at a point P is by symmetry and superposition

(for $l >> a$)

$$\varphi = \frac{A}{2}\ln\frac{r_1}{a} - \frac{A}{2}\ln\frac{r_2}{a}$$

$$= \frac{A}{2}\ln\frac{r_1}{r_2}$$

Then $\varphi_1 = \frac{V}{2} = \frac{A}{2}\ln\frac{a}{l}$ (for the potential of 1) or, $$A = -V/\ln\frac{l}{a}$$

and $$\varphi = -\frac{V}{2\ln l/a}\ln r_1/r_2$$

We then calculate the field at a point P which is equidistant from 1 & 2 and at a distance r from both :

Then $$E = \frac{V}{2\ln l/a}\left(\frac{1}{r}\right)\times 2\sin\theta$$

$$= \frac{Vl}{2\ln l/a}\frac{1}{r^2}$$

and $$J = \sigma E = \frac{1}{\rho}\,\frac{V}{2\ln l/a}\,\frac{1}{r^2}$$

(b) Near either wire $$E = \frac{V}{2\ln l/a}\,\frac{1}{a}$$

and $$J = \sigma E = \frac{1}{\rho}\,\frac{V}{2\ln l/a}$$

Then $$I = \frac{V}{R} = L\frac{V}{R_1} = J\,2\pi aL$$

Which gives $$R_1 = \frac{\rho}{\pi}\ln l/a$$

3.160 Let us mentally impart the charges $+q$ and $-q$ to the plates of the capacitor. Then capacitance of the network,

$$C = \frac{q}{\varphi} = \frac{\varepsilon\varepsilon_0\int E_n\,dS}{\varphi} \tag{1}$$

Now, electric current,

$$i = \int \vec{j}\cdot d\vec{S} = \int \sigma E_n\,dS \text{ as } \vec{j}\uparrow\uparrow\vec{E}. \tag{2}$$

Hence, using (1) in (2), we get,

$$i = \frac{C\varphi}{\varepsilon\varepsilon_0}\sigma = \frac{C\varphi}{\rho\,\varepsilon\varepsilon_0} = 1{\cdot}5\,\mu\,\text{A}$$

3.161 Let us mentally impart charges $+q$ and $-q$ to the conductors. As the medium is poorly conducting, the surfaces of the conductors are equipotential and the field configuration is same as in the absence of the medium.

Let us surround, for example, the positively charged conductor, by a closed surface S, just containing the conductor,

then, $$R = \frac{\varphi}{i} = \frac{\varphi}{\int \vec{j}\cdot d\vec{S}} = \frac{\varphi}{\int \sigma E_n\,dS};\ \text{as } \vec{j}\uparrow\uparrow\vec{E}$$

and $$C = \frac{q}{\varphi} = \frac{\varepsilon\varepsilon_0\int E_n\,dS}{\varphi}$$

So, $$RC = \frac{\varepsilon\varepsilon_0}{\sigma} = \rho\,\varepsilon\varepsilon_0$$

3.162 The dielectric ends in a conductor. It is given that on one side (the dielectric side) the electric displacement D is as shown. Within the conductor, at any point A, there can be no normal component of electric field. For if there were such a field, a current will flow towards depositing charge there which in turn will set up countering electric field causing the normal component to vanish. Then by Gauss theorem, we easily derive

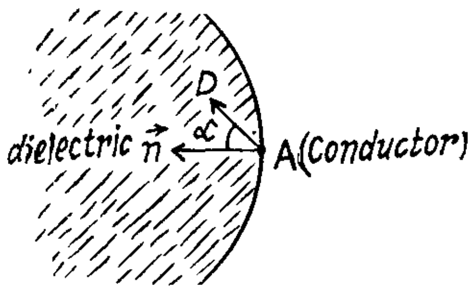
$\sigma = D_n = D \cos \alpha$ where σ is the surface charge density at A.

The tangential component is determined from the circulation theorem

$$\oint \vec{E} \cdot d\vec{r} = 0$$

It must be continuous across the surface of the conductor. Thus, inside the conductor there is a tangential electric field of magnitude,

$$\frac{D \sin \alpha}{\varepsilon_0 \varepsilon} \text{ at } A\ .$$



This implies a current, by Ohm's law, of

$$j = \frac{D \sin \alpha}{\varepsilon_0 \varepsilon\, \rho}$$

3.163 The resistance of a layer of the medium, of thickness dx and at a distance x from the first plate of the capacitor is given by,

$$dR = \frac{1}{\sigma(x)} \frac{dx}{S} \tag{1}$$

Now, since σ varies linearly with the distance from the plate. It may be represented as,

$\sigma = \sigma_1 + \left(\frac{\sigma_2 - \sigma_1}{d}\right) x$, at a distance x from any one of the plate.

From Eq. (1)

$$dR = \frac{1}{\sigma_1 + \left(\frac{\sigma_2 - \sigma_1}{d}\right) x} \frac{dx}{S}$$

or,
$$R = \frac{1}{S} \int_0^d \frac{dx}{\sigma_1 + \left(\frac{\sigma_2 - \sigma_1}{d}\right) x} = \frac{d}{S(\sigma_2 - \sigma_1)} \ln \frac{\sigma_2}{\sigma_1}$$

Hence,
$$i = \frac{V}{R} = \frac{S V (\sigma_2 - \sigma_1)}{d \ln \frac{\sigma_2}{\sigma_1}} = 5 \text{ nA}$$

3.164 By charge conservation, current j, leaving the medium (1) must enter the medium (2). Thus

$$j_1 \cos \alpha_1 = j_2 \cos \alpha_2$$

Another relation follows from

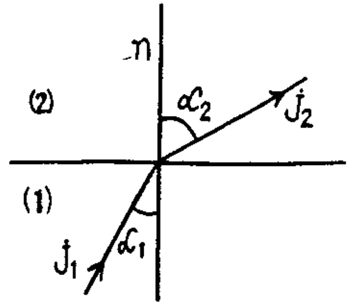
$$E_{1t} = E_{2t}\ ,$$

which is a consequence of $\oint \vec{E} \cdot d\vec{r} = 0$

Thus $\frac{1}{\sigma_1} j_1 \sin\alpha_1 = \frac{1}{\sigma_2} j_2 \ \sin\alpha_2$

or, $$\frac{\tan\alpha_1}{\sigma_1} = \frac{\tan\alpha_2}{\sigma_2}$$

or, $$\frac{\tan\alpha_1}{\tan\alpha_2} = \frac{\sigma_1}{\sigma_2}$$



3.165 The electric field in conductor 1 is

$$E_1 = \frac{\rho_1 I}{\pi R^2}$$

and that in 2 is $E_2 = \frac{\rho_2 I}{\pi R^2}$

Applying Gauss' theorem to a small cylindrical pill-box at the boundary.

$$-\frac{\rho_1 I}{\pi R^2} dS + \frac{\rho_2 I}{\pi R^2} dS = \frac{\sigma\, dS}{\varepsilon_0}$$

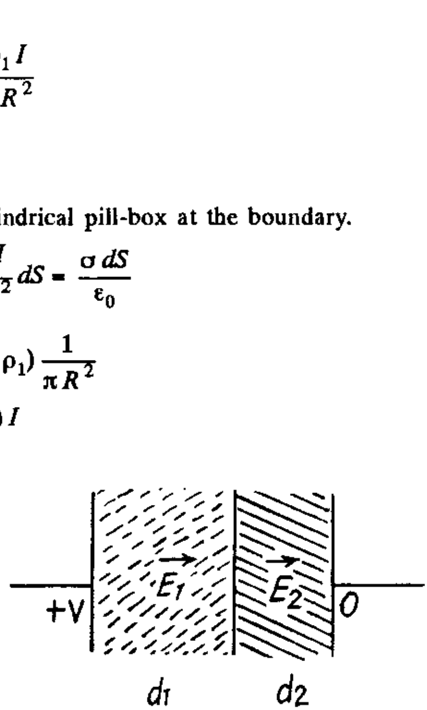
Thus, $$\sigma = \varepsilon_0 \ (\rho_2 - \rho_1) \frac{1}{\pi R^2}$$

and charge at the boundary $= \varepsilon_0 (\rho_2 - \rho_1) I$

3.166 We have, $E_1 d_1 + E_2 d_2 = V$

and by current conservation

$$\frac{1}{\rho_1} E_1 = \frac{1}{\rho_2} E_2$$



Thus, $$E_1 = \frac{\rho_1 V}{\rho_1 d_1 + \rho_2 d_2},$$

$$E_2 = \frac{\rho_2 V}{\rho_1 d_1 + \rho_2 d_2}$$

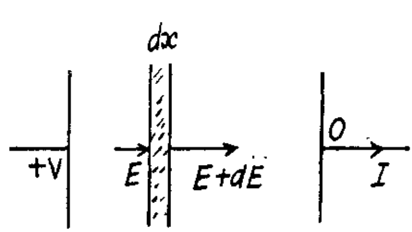
At the boundary between the two dielectrics,

$$\sigma = D_2 - D_1 = \varepsilon_0 \varepsilon_2 E_2 - \varepsilon_0 \varepsilon_1 E_1$$

$$\frac{\varepsilon_0 V}{\rho_1 d_1 + \rho_2 d_2} (\varepsilon_2 \rho_2 - \varepsilon_1 \rho_1)$$

3.167 By current conservation

$$\frac{E(x)}{\rho(x)} = \frac{E(x) + dE(x)}{\rho(x) + d\rho(x)} = \frac{dE(x)}{d\rho(x)}$$



This has the solution,

$$E(x) = C\rho(x) = \frac{I\rho(x)}{A}$$

Hence charge induced in the slice per unit area

$$d\sigma = \varepsilon_0 \frac{I}{A}[\{\varepsilon(x) + d\varepsilon(x)\}\{\rho(x) + d\rho(x)\} - \varepsilon(x)\rho(x)] = \varepsilon_0 \frac{I}{A} d[\varepsilon(x)\rho(x)]$$

Thus, $$dQ = \varepsilon_0 I d[\varepsilon(x)\rho(x)]$$

Hence total charge induced, is by integration,

$$Q = \varepsilon_0 I(\varepsilon_2\rho_2 - \varepsilon_1\rho_1)$$

3.168 As in the previous problem

$$E(x) = C\rho(x) = C(\rho_0 + \rho_1 x)$$

where $$\rho_0 + \rho_1 d = \eta\rho_0 \quad \text{or,} \quad \rho_1 = \frac{(\eta - 1)\rho_0}{d}$$

By integration $$V = \int_0^d C\rho(x)\,dx = C\rho_0 d\left(1 + \frac{\eta - 1}{2}\right) = \frac{1}{2}C\rho_0 d(\eta + 1)$$

Thus $$C = \frac{2V}{\rho_0 d(\eta + 1)}$$

Thus volume density of charge present in the medium

$$= \frac{dQ}{Sdx} = \varepsilon_0\, dE(x)/dx$$

$$= \frac{2\varepsilon_0 V}{\rho_0 d(\eta + 1)} \times \frac{(\eta - 1)\rho_0}{d} = \frac{2\varepsilon_0 V(\eta - 1)}{(\eta + 1)d^2}$$

3.169 (a) Consider a cylinder of unit length and divide it into shells of radius r and thickness dr Different sections are in parallel. For a typical section,

$$d\left(\frac{1}{R_1}\right) = \frac{2\pi r\,dr}{(\alpha/r^2)} = \frac{2\pi r^3\,dr}{\alpha}$$

Integrating, $$\frac{1}{R_1} = \frac{\pi R^4}{2\alpha} = \frac{S^2}{2\pi\alpha}$$

or, $$R_1 = \frac{2\pi\alpha}{S^2}, \quad \text{where } S = \pi R^2$$

(b) Suppose the electric filed inside is $E_z = E_o$ (Z axis is along the axiz of the conductor). This electric field cannot depend on r in steady conditions when other components of E are absent, otherwise one violates the circulation theorem

$$\oint \vec{E}\cdot d\vec{r} = 0$$

The current through a section between radii ($r + dr$, r) is

$$2\pi r dr \frac{1}{\alpha/r^2} E = 2\pi r^3 dr \frac{E}{\alpha}$$

Thus $$I = \int_0^R 2\pi r^3 dr \frac{E}{\alpha} = \frac{\pi R^4 E}{2\alpha}$$

Hence $$E = \frac{2\alpha\pi I}{S^2} \text{ when } S = \pi R^2$$

3.170 The formula is,

$$q = C V_0 (1 - e^{-t/RC})$$

or, $$V = \frac{q}{C} = V_0 (1 - e^{-t/RC}) \quad \text{or,} \quad \frac{V}{V_0} = 1 - e^{-t/RC}$$

or, $$e^{-t/RC} = 1 - \frac{V}{V_0} = \frac{V_0 - V}{V_0}$$

Hence, $t = RC \ln \frac{V_0}{V_0 - V} = RC \ln 10$, if $V = 0{\cdot}9\, V_0$.

Thus $t = 0.6\ \mu S$.

3.171 The charge decays according to the foumula

$$q = q_0 e^{-t/RC}$$

Here, RC = mean life = Half-life/ln 2

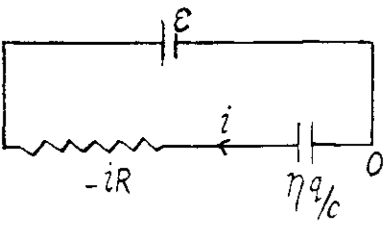
So, half life= $T = RC \ln 2$

But, $$C = \frac{\varepsilon \varepsilon_0 A}{d},\ R = \frac{\rho d}{A}$$

Hence, $$\rho = \frac{T}{\varepsilon \varepsilon_0 \ln 2} = 1.4 \times 10^{13}\ \Omega \cdot \mathrm{m}$$

3.172 Suppose q is the charge at time t. Initially $q = C\xi$, at $t = 0$.

Then at time t,



$$\frac{\eta q}{C} - iR - \xi = 0$$

But, $i = -\frac{dq}{dt}$ (- sign because charge decreases)

So $$\frac{\eta q}{C} + R\frac{dq}{dt} = \xi$$

$$\frac{dq}{dt} + \frac{\eta}{RC}\, q = \frac{\xi}{R}$$

or, $$\frac{d}{dt}\, q\, e^{t\eta/RC} = \frac{\xi}{R} e^{t\eta/RC}$$

or, $$q = \frac{C\xi}{\eta} + A\, e^{-t\eta/RC}$$

$$A = C\xi\left(1 - \frac{1}{\eta}\right), \text{ from } q = C\xi \text{ at } t = 0$$

Hence, $$q = C\xi\left(\frac{1}{\eta} + \left(1 - \frac{1}{\eta}\right) e^{-\eta t/RC}\right)$$

Finally, $$i = -\frac{dq}{dt} = \frac{\xi(\eta - 1)}{R} e^{-t\eta/RC}$$

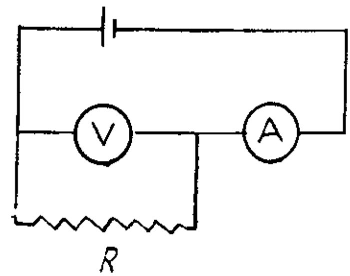
3.173 Let r = internal resistance of the battery. We shall take the resistance of the ammeter to be = 0 and that of voltmeter to be G.

Initially, $V = \xi - Ir$, $I = \frac{\xi}{r + G}$

So, $$V = \xi \frac{G}{r+G} \quad (1)$$

After the voltmeter is shunted

$$\frac{V}{\eta} = \xi - \frac{\xi r}{r + \frac{RG}{R+G}} \text{ (Voltmeter)} \quad (2)$$



and $$\frac{\xi}{r + \frac{RG}{R+G}} = \eta \frac{\xi}{r+G} \text{ (Ammeter)} \quad (3)$$

From (2) and (3) we have

$$\frac{V}{\eta} = \xi - \frac{\eta r \xi}{r+G} \quad (4)$$

From (1) and (4)

$$\frac{G}{\eta} = r + G - \eta r \text{ or } G = \eta r$$

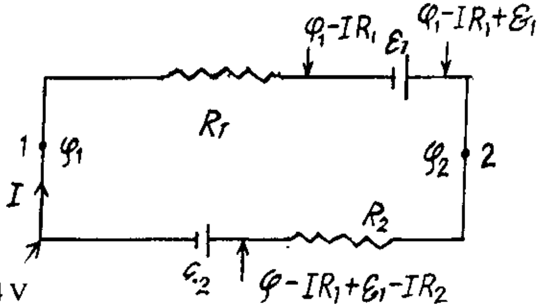
Then (1) gives the required reading

$$\frac{V}{\eta} = \frac{\xi}{\eta+1}$$

3.174 Assume the current flow, as shown. Then potentials are as shown. Thus,

$$\varphi_1 = \varphi_1 - IR_1 + \xi_1 - IR_2 - \xi_2$$

or, $$I = \frac{\xi_1 - \xi_2}{R_1 + R_2}$$



And $$\varphi_2 = \varphi_1 - IR_1 + \xi_1$$

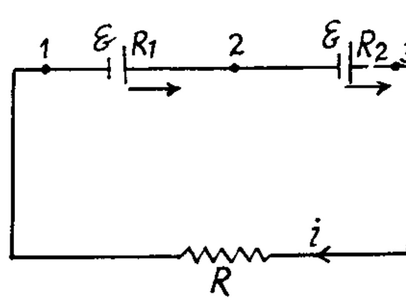
So, $$\varphi_1 - \varphi_2 = -\xi_1 + \frac{\xi_1 - \xi_2}{R_1 + R_2} R_1$$

$$= -(\xi_1 R_2 + \xi_2 R_1)/(R_1 + R_2) = -4 \text{ V}$$

3.175 Let, us consider the current i, flowing through the circuit, as shown in the figure. Applying loop rule for the circuit, $-\Delta\varphi = 0$

$$-2\xi + iR_1 + iR_2 + iR = 0$$

or, $$i(R_1 + R_2 + R) = 2\xi$$



or, $$i = \frac{2\xi}{R + R_1 + R_2}$$

Now, if $$\varphi_1 - \varphi_2 = 0$$

$$-\xi + iR_1 = 0$$

or, $$\frac{2\xi R_1}{R + R_1 + R_2} = \xi \text{ and } 2R_1 = R_2 + R + R_1$$

or, $R = R_1 - R_2$, which is not possible as $R_2 > R_1$

Thus, $$\varphi_2 - \varphi_3 = -\xi + iR_2 = 0$$

or, $$\frac{2\xi R_2}{R + R_1 + R_2} = \xi$$

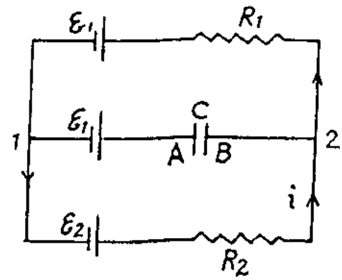
So, $R = R_2 - R_1$, which is the required resistance.

3.176 (a) Current, $i = \dfrac{N\xi}{NR} = \dfrac{N\alpha R}{NR} = \alpha$, as $\xi = \alpha R$ (given)

(b) $\varphi_A - \varphi_B = n\xi - niR = n\alpha R - n\alpha R = 0$

3.177 As the capacitor is fully charged, no current flows through it. So, current

$$i = \frac{\xi_2 - \xi_1}{R_1 + R_2} \text{ (as } \xi_2 > \xi_1\text{)}$$



And hence, $\varphi_A - \varphi_B = \xi_1 - \xi_2 + iR_2$

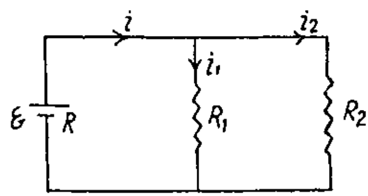
$$= \xi_1 - \xi_2 + \frac{\xi_2 - \xi_1}{R_1 + R_2} R_2$$

$$= \frac{(\xi_1 - \xi_2) R_1}{R_1 + R_2} = -0{\cdot}5\text{ V}$$

3.178 Let us make the current distribution, as shown in the figure.

Current $i = \dfrac{\xi}{R + \dfrac{R_1 R_2}{R_1 + R_2}}$ (using loop rule)

So, current through the resistor R_1,



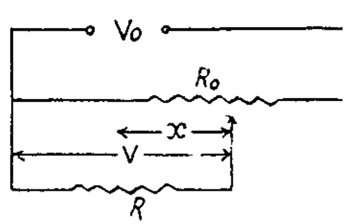
$$i_1 = \frac{\xi}{R + \dfrac{R_1 R_2}{R_1 + R_2}} \frac{R_2}{R_1 + R_2}$$

$$= \frac{\xi R_2}{R R_1 + R R_2 + R_1 R_2} = 1{\cdot}2\text{ A}$$

and similary, current through the resistor R_2,

$$i_2 = \frac{\xi}{R + \dfrac{R_1 R_2}{R_1 + R_2}} \frac{R_1}{R_1 + R_2} = \frac{\xi R_1}{R R_1 + R_1 R_2 + R R_2} = 0{\cdot}8\text{ A}$$

3.179 Total resistance $= \dfrac{l - x}{l} R_0 + \dfrac{R \cdot \dfrac{x R_0}{l}}{R + \dfrac{x R_0}{l}}$



$$= \frac{l - x}{l} R_0 + \frac{x R R_0}{l R + x R_0}$$

$$= R_0 \left[\frac{l - x}{l} + \frac{x R}{l R + x R_0} \right]$$

Then $V = V_0 \frac{xR}{lR + xR_0} \Big/ \left(1 - \frac{x}{l} + \frac{xR}{xR_0 + lR}\right) = V_0 R x \Big/ \left\{lR + R_0 x\left(1 - \frac{x}{l}\right)\right\}$

For $R >> R_0, V = V_0 \frac{x}{l}$

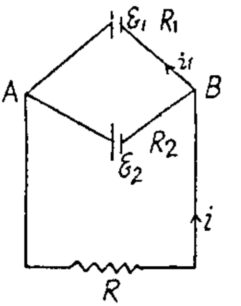
3.180 Let us connect a load of resistance R between the points A and B (Fig.)

From the loop rule, $\Delta \varphi = 0$, we obtain

$$iR = \xi_1 - i_1 R_1 \quad (1)$$

and $$iR = \xi_2 - (i - i_1) R_2$$

or $$i(R + R_2) = \xi + i_1 R_2 \quad (2)$$



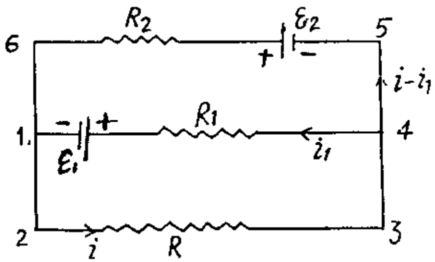
Solving Eqs. (1) and (2), we get

$$i = \frac{\xi_1 R_1 + \xi_2 R_2}{R_1 + R_2} \Big/ R + \frac{R_1 R_2}{R_1 + R_2} = \frac{\xi_0}{R + R_0} \quad (3)$$

where $\xi_0 = \frac{\xi_1 R_1 + \xi_2 R_2}{R_1 + R_2}$ and $R_0 = \frac{R_1 R_2}{R_1 + R_2}$

Thus one can replace the given arrangement of the cells by a single cell having the emf ξ_0 and internal resistance R_0.

3.181 Make the current distribution, as shown in the diagram.



Now, in the loop 12341, applying $-\Delta \varphi = 0$

$$iR + i_1 R_1 + \xi_1 = 0 \quad (1)$$

and in the loop 23562,

$$iR - \xi_2 + (i - i_1) R_2 = 0 \quad (2)$$

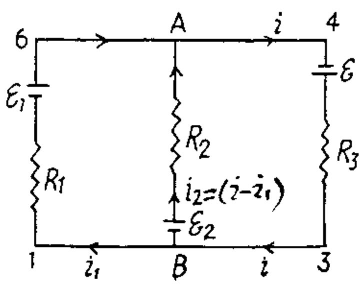
Solving (1) and (2), we obtain current through the resistance R,

$$i = \frac{(\xi_2 R_1 - \xi_1 R_2)}{R R_1 + R R_2 + R_1 R_2} = 0{\cdot}02 \text{ A}$$

and it is directed from left to the right

3.182 At first indicate the currents in the branches using charge conservation (which also includes the point rule).

In the loops 1 BA 61 and B34AB from the loop rule, $-\Delta \varphi = 0$, we get, respectively



$$-\xi_2 + (i - i_1)R_2 + \xi_1 - i_1 R_1 = 0 \quad (1)$$

$$iR_3 + \xi_3 - (i - i_1) R_2 + \xi_2 = 0 \quad (2)$$

On solving Eqs (1) and (2), we obtain

$$i_1 = \frac{(\xi_1 - \xi_2) R_3 + R_2 (\xi_1 + \xi_3)}{R_1 R_2 + R_2 R_3 + R_3 R_1} \approx 0{\cdot}06 \text{ A}$$

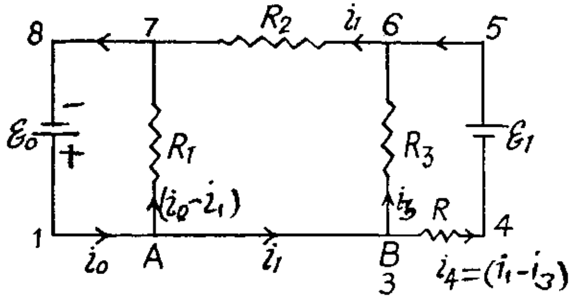
Thus $\varphi_A - \varphi_B = \xi_2 - i_2 R_2 \approx 0{\cdot}9$ V

3.183 Indicate the currents in all the branches using charge conservation as shown in the figure. Applying loop rule, $-\Delta\varphi = 0$ in the loops 1A781, 1B681 and B456B, respectively, we get

$\xi_0 = (i_0 - i_1)\, R_1$ (1)

$i_3 R_3 + i_1 R_2 - \xi_0 = 0$ (2) and

$(i_1 - i_3) R - \xi - i_3 R_3 = 0$ (3)



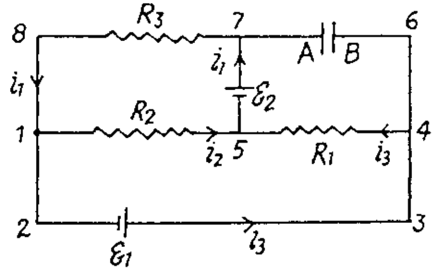
Solving Eqs. (1), (2) and (3), we get the sought current

$$(i_1 - i_3) = \frac{\xi\,(R_2 + R_3) + \xi_0 R_3}{R\,(R_2 + R_3) + R_2 R_3}$$

3.184 Indicate the currents in all the branches using charge conservation as shown in the figure. Applying the loop rule ($-\Delta\varphi = 0$) in the loops 12341 and 15781, we get

$$-\xi_1 + i_3 R_1 - (i_1 - i_3) R_2 = 0 \quad (1)$$

$$\text{and} \quad (i_1 - i_3) R_2 - \xi_2 + i_1 R_3 = 0 \quad (2)$$



Solving Eqs. (1) and (2), we get

$$i_3 = \frac{\xi_1\ (R_2 + R_3) + \xi_2\ R_2}{R_1 R_2 + R_2 R_3 + R_3 R_1}$$

Hence, the sought p.d.

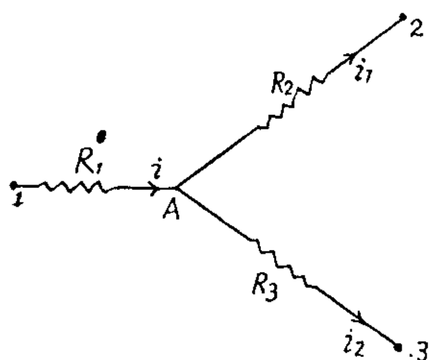
$$\varphi_A - \varphi_B = \xi_2 - i_3 R_1$$

$$= \frac{\xi_2 R_3 (R_1 + R_2) - \xi_1\ R_1\ (R_2 + R_3)}{R_1 R_2 + R_2 R_3 + R_3 R_1} = -1\ \text{V}$$

3.185 Let us distribute the currents in the paths as shown in the figure.

Now, $\varphi_1 - \varphi_2 = i R_1 + i_1 R_2$ (1)

and $\varphi_1 - \varphi_3 = i R_1 + (i - i_1) R_3$ (2)



Simplifying Eqs. (1) and (2) we get

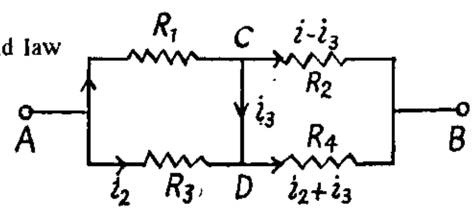
$$i = \frac{R_3\ (\varphi_1 - \varphi_2) + R_2\ (\varphi_1 - \varphi_3)}{R_1 R_2 + R_2 R_3 + R_3 R_1} = 0{\cdot}2\ \text{A}$$

3.186 Current is as shown. From Kirchhoff's Second law

$$i_1 R_1 = i_2 R_3\ ,$$

$$i_1 R_1 + (\, i_1 - i_3\,) R_2 = V\ ,$$

$$i_2 R_3 + (\, i_3 + i_2\,) R_4 = V$$



Eliminating i_2

$$i_1(R_1+R_2)-i_3R_2=V$$

$$i_1\frac{R_1}{R_3}(R_3+R_4)+i_3R_4=V$$

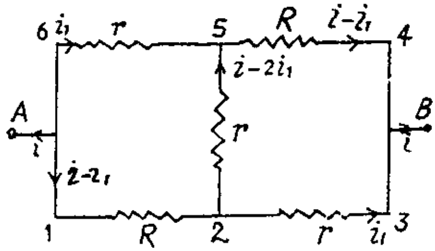
Hence
$$i_3\left[R_4(R_1+R_2)+\frac{R_1R_2}{R_3}(R_3+R_4)\right]$$

$$=V\left[(R_1+R_2)-\frac{R_1}{R_3}(R_3+R_4)\right]$$

or,
$$i_3=\frac{R_3(R_1+R_2)-R_1(R_3+R_4)}{R_3R_4(R_1+R_2)+R_1R_2(R_3+R_4)}$$

On substitution we get $i_3=1{\cdot}0$ A from C to D

3.187 From the symmetry of the problem, current flow is indicated, as shown in the figure.



Now, $\varphi_A-\varphi_B=i_1r+(i-i_1)R$ (1)

In the loop 12561, from $-\Delta\varphi=0$

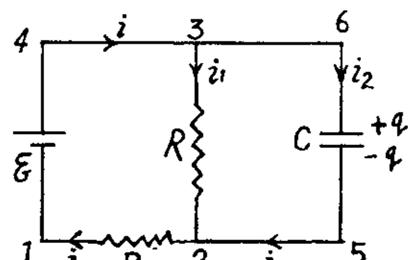
$$(i-i_1)R+(i-2i_1)r-i_1r=0$$

or,
$$i_1=\frac{(R+r)}{3r+R}i \quad (2)$$

Equivalent resistance between the terminals A and B using (1) and (2),

$$R_0=\frac{\varphi_A-\varphi_B}{i}=\frac{\left[\frac{R+r}{3r+R}(r-R)+R\right]i}{i}=\frac{r(3R+r)}{3r+R}$$

3.188 Let, at any moment of time, charge on the plates be $+q$ and $-q$ respectively, then voltage across the capacitor, $\varphi=q/C$ (1)



Now, from charge conservation,

$$i=i_1+i_2,\text{ where } i_2=\frac{dq}{dt} \quad (2)$$

In the loop 65146, using $-\Delta\varphi=0$.

$$\frac{q}{C}+\left(i_1+\frac{dq}{dt}\right)R-\xi=0 \quad (3)$$

[using (1) and (2)]

In the loop 25632, using $-\Delta\varphi=0$

$$-\frac{q}{C}+i_1R=0 \quad \text{or,} \quad i_1R=\frac{q}{C} \quad (4)$$

From (1) and (2),

$$\frac{dq}{dt}R = \xi_1 - \frac{2q}{C} \quad \text{or,} \quad \frac{dq}{\xi - \frac{2q}{C}} = \frac{dt}{R} \tag{5}$$

On integrating the expression (5) between suitable limits,

$$\int_0^q \frac{dq}{\xi - \frac{2q}{C}} = \frac{1}{R}\int_0^t dt \quad \text{or,} \quad -\frac{C}{2}\ln\frac{\xi - \frac{2q}{C}}{\xi} = \frac{t}{R}$$

Thus $$\frac{q}{C} = V = \frac{1}{2}\xi\left(1 - e^{-2t/RC}\right)$$

3.189 (a) As current i is linear function of time, and at $t = 0$ and Δt, it equals i_0 and zero respectively, it may be represented as,

$$i = i_0\left(1 - \frac{t}{\Delta t}\right)$$

Thus $$q = \int_0^{\Delta t} i\,dt = \int_0^{\Delta t} i_0\left(1 - \frac{t}{\Delta t}\right)dt = \frac{i_0\,\Delta t}{2}$$

So, $$i_0 = \frac{2q}{\Delta t}$$

Hence, $$i = \frac{2q}{\Delta t}\left(1 - \frac{t}{\Delta t}\right)$$

The heat generated.

$$H = \int_0^{\Delta t} i^2 R\,dt = \int_0^{\Delta t}\left[\frac{2q}{\Delta t}\left(1 - \frac{t}{\Delta t}\right)\right]^2 R\,dt = \frac{4q^2 R}{3\,\Delta t}$$

(b) Obviously the current through the coil is given by

$$i = i_0\left(\frac{1}{2}\right)^{t/\Delta t}$$

Then charge $$q = \int_0^{\infty} i dt = \int_0^{\infty} i_0\,2^{-t/\Delta t}\,dt = \frac{i_0\,\Delta t}{\ln 2}$$

So, $$i_0 = \frac{q\ln 2}{\Delta t}$$

And hence, heat generated in the circuit in the time interval $t\,[0, \infty]$,

$$H = \int_0^{\infty} i^2 R\,dt = \int_0^{\infty}\left[\frac{q\ln 2}{\Delta t}2^{-t/\Delta t}\right]^2 R\,dt = -\frac{q^2\ln 2}{2\,\Delta t}R$$

3.190 The equivalent circuit may be drawn as in the figure.

Resistance of the network = $R_0 + (R/3)$

Let, us assume that e.m.f. of the cell is ξ, then current

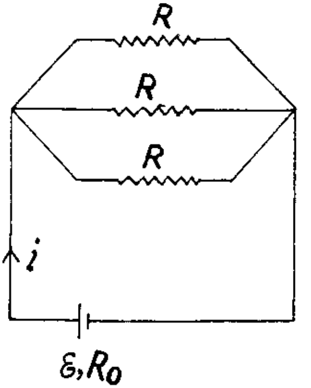
$$i = \frac{\xi}{R_0 + (R/3)}$$

Now, thermal power, generated in the circuit

$$P = i^2 R/3 = \frac{\xi^2}{(R_0 + (R/3))^2}(R/3)$$

For P to be maximum, $\frac{dP}{dR} = 0$, which yields

$$R = 3R_0$$

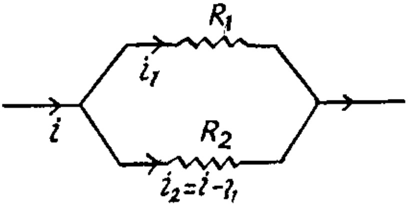


3.191 We assume current conservation but not Kirchhoff's second law. Then thermal power dissipated is

$$P(i_1) = i_1^2 R_1 + (i - i_1)^2 R_2$$

$$= i_1^2 (R_1 + R_2) - 2ii_1R_2 + i^2R_2$$

$$= [R_1 + R_2]\left[i_1 - \frac{R_2}{R_1 + R_2} i\right]^2 + i^2 \frac{R_1R_2}{R_1 + R_2}$$



The resistances being positive we see that the power dissipated is minimum when

$$i_1 = i\frac{R_2}{R_1 + R_2}$$

This corresponds to usual distribution of currents over resistance joined is parallel.

3.192 Let, internal resistance of the cell be r, then

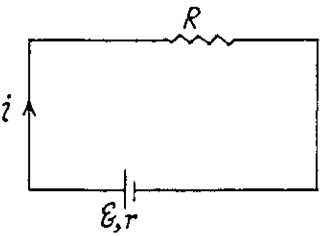
$$V = \xi - ir \qquad (1)$$

where i is the current in the circuit. We know that thermal power generated in the battery.

$$Q = i^2 r \qquad (2)$$

Putting r from (1) in (2), we obtain,

$$Q = (\xi - V)\, i = 0{\cdot}6\text{ W}$$

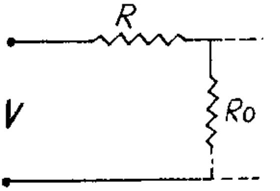


In a battery work is done by electric forces (whose origin lies in the chemical processes going on inside the cell). The work so done is stored and used in the electric circuit outside. Its magnitude just equals the power used in the electric circuit. We can say that net power developed by the electric forces is

$$P = -IV = -2\cdot 0\text{W}$$

Minus sign means that this is generated not consumed.

3.193 As far as motor is concerned the power delivered is dissipated and can be represented by a load, R_0. Thus



$$I = \frac{V}{R + R_0}$$

and $$P = I^2 R_0 = \frac{V^2 R_0}{(R_0 + R)^2}$$

This is maximum when $R_0 = R$ and the current I is then

$$I = \frac{V}{2R}$$

The maximum power delivered is

$$\frac{V^2}{4R} = P_{max}$$

The power input is $\frac{V^2}{R + R_0}$ and its value when P is maximum is $\frac{V^2}{2R}$

The efficiency then is $\frac{1}{2} = 50\%$

3.194 If the wire diameter decreases by δ then by the information given

$P =$ Power input $= \frac{V^2}{R} =$ heat lost through the surface, H.

Now, $H \propto (1 - \delta)$ like the surface area and

$$R \propto (1 - \delta)^{-2}$$

So, $$\frac{V^2}{R_0}(1 - \delta)^2 = A\,(1 - \delta) \quad \text{or,} \quad V^2 (1 - \delta) = \text{constant}$$

But $\quad V \propto 1 + \eta \quad$ so $\quad (1 + \eta)^2 (1 - \delta) = \text{Const} = 1$

Thus $$\delta = 2\eta = 2\%$$

3.195 The equation of heat balance is

$$\frac{V^2}{R} - k\,(T - T_0) = C\,\frac{dT}{dt}$$

Put $$T - T_0 = \xi$$

So, $$C\,\dot{\xi} + k\,\xi = \frac{V^2}{R} \quad \text{or,} \quad \dot{\xi} + \frac{k}{C}\,\xi = \frac{V^2}{C\,R}$$

or, $$\frac{d}{dt}\left(\xi e^{kt/c}\right) = \frac{V^2}{C\,R}\,e^{kt/c}$$

or, $$\xi\, e^{kt/c} = \frac{V^2}{kR}\,e^{kt/c} + A$$

where A is a constant. Clearly

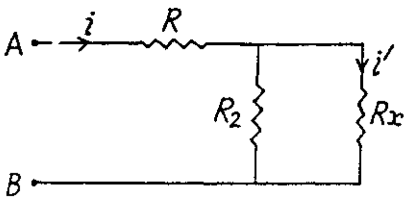
$$\xi = 0 \text{ at } t = 0, \text{ so } A = -\frac{V^2}{kR} \text{ and hence,}$$

$$T = T_0 + \frac{V^2}{kR}\left(1 - e^{-kt/C}\right)$$

3.196 Let, $\varphi_A - \varphi_B = \varphi$

Now, thermal power generated in the resistance R_x,

$$P = i'^2 R_x = \left[\frac{\varphi}{R_1 + \frac{R_2 R_x}{R_2 + R_x}} \frac{R_2}{R_2 + R_x} \right]^2 R_x,$$



For P to be independent of R_x,

$$\frac{dP}{dR_x} = 0, \text{ which yeilds}$$

$$R_x = \frac{R_1 R_2}{R_1 + R_2} = 12\ \Omega$$

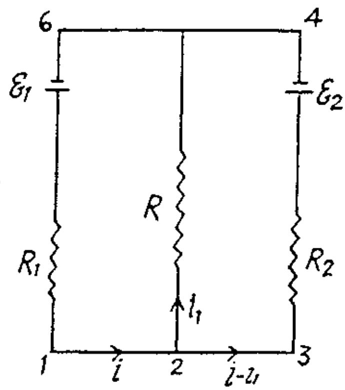
3.197 Indicate the currents in the circuit as shown in the figure.

Appying loop rule in the closed loop 12561, $-\Delta\varphi = 0$ we get

$$i_1 R - \xi_1 + i R_1 = 0 \qquad (1)$$

and in the loop 23452,

$$(i - i_1) R_2 + \xi_2 - i_1 R = 0 \qquad (2)$$



Solving (1) and (2), we get,

$$i_1 = \frac{\xi_1 R_2 + \xi_2 R_1}{R R_1 + R_1 R_2 + R R_2}$$

So, thermal power, generated in the resistance R,

$$P = i_1^2 R = \left[\frac{\xi_1 R_2 + \xi_2 R_1}{R R_1 + R_1 R_2 + R R_2} \right]^2 R$$

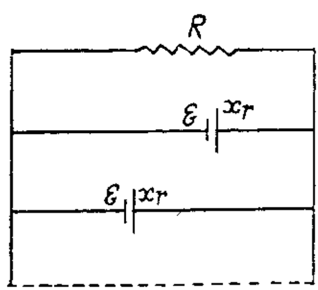
For P to be maximum, $\frac{dP}{dR} = 0$, which fields

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

Hence,
$$P_{max} = \frac{(\xi_1 R_2 + \xi_2 R_1)^2}{4 R_1 R_2 (R_1 + R_2)}$$

3.198 Let, there are x number of cells, connected in series in each of the n parallel groups

then, $n x = N$ or, $x = \frac{N}{n}$ (1)



Now, for any one of the loop, consisting of x cells and the resistor R, from loop rule

$$i R + \frac{i}{n} x r - x \xi = 0$$

So, $i = \frac{x \xi}{R + \frac{xr}{n}} = \frac{\frac{N}{n}\xi}{R + \frac{N r}{n^2}}$, using (1)

Heat generated in the resistor R,

$$Q = i^2 R = \left(\frac{N n \xi}{n^2 R + N R}\right)^2 R \qquad (2)$$

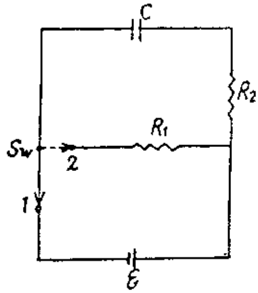
and for Q to be maximum, $\frac{dQ}{dn} = 0$, which yields

$$n = \sqrt{\frac{Nr}{R}} = 3$$

3.199 When switch 1 is closed, maximum charge accumulated on the capacitor,

$$q_{max} = C\xi, \qquad (1)$$

and when switch 2 is closed, at any arbitrary instant of time,



$$(R_1 + R_2)\left(-\frac{dq}{dt}\right) = q/C,$$

because capacitor is discharging.

$$\text{or,} \quad \int_{q_{max}}^{q} \frac{1}{q}\, dq = -\frac{1}{(R_1 + R_2)\, C} \int_0^t dt$$

Integrating, we get

$$\ln \frac{q}{q_{max}} = \frac{-t}{(R_1 + R_2)\, C} \quad \text{or,} \quad q = q_{max}\, e^{\frac{-t}{(R_1 + R_2)\, C}} \qquad (2)$$

Differentiating with respect to time,

$$i(t) = \frac{dq}{dt} = q_{max}\, e^{\frac{-t}{(R_1 + R_2) C}} \left(-\frac{1}{(R_1 + R_2)\, C}\right)$$

$$\text{or,} \qquad i(t) = \frac{C\xi}{(R_1 + R_2)\, C}\, e^{\frac{-t}{(R_1 + R_2)\, C}}$$

Negative sign is ignored, as we are not interested in the direction of the current.

$$\text{thus,} \qquad i(t) = \frac{\xi}{(R_1 + R_2)}\, e^{\frac{-1}{(R_1 + R_2)\, C}} \qquad (3)$$

When the switch (Sw) is at the position 1, the charge (maximum) accumalated on the capacitor is,

$$q = C\xi$$

When the Sw is thrown to position 2, the capacitor starts discharging and as a result the electric energy stored in the capacitor totally turns into heat energy tho' the resistors R_1 and R_2 (during a very long interval of time). Thus from the energy conservation, the total heat liberated tho' the resistors.

$$H = U_i = \frac{q^2}{2C} = \frac{1}{2} C\xi^2$$

During the process of discharging of the capacitor, the current tho' the resistors R_1 and R_2 is the same at all the moments of time, thus

$$H_1 \propto R_1 \text{ and } H_2 \propto R_2$$

So, $$H_1 = \frac{H\,R_1}{(R_1 + R_2)} \quad (\text{as } H = H_1 + H_2)$$

Hence $$H_1 = \frac{1}{2}\,\frac{CR_1}{R_1 + R_2}\,\xi^2$$

3.200 When the plate is absent the capacity of the condenser is

$$C = \frac{\varepsilon_0 S}{d}$$

When it is present, the capacity is

$$C' = \frac{\varepsilon_0 S}{d\,(1-\eta)} = \frac{C}{1-\eta}$$

(a) The energy increment is clearly.

$$\Delta U = \frac{1}{2}CV^2 - \frac{1}{2}C'V^2 = \frac{C\,\eta}{2\,(1-\eta)}\,V^2$$

(b) The charge on the plate is

$$q_i = \frac{CV}{1-\eta} \text{ initially and } q_f = CV \text{ finally.}$$

A charge $\frac{CV\eta}{1-\eta}$ has flown through the battery charging it and withdrawing $\frac{CV^2\eta}{1-\eta}$ units of energy from the system into the battery. The energy of the capacitor has decreased by just half of this. The remaining half i.e. $\frac{1}{2}\frac{CV^2\eta}{1-\eta}$ must be the work done by the external agent in withdrawing the plate. This ensures conservation of energy.

3.201 Initially, capacitance of the system $= C\,\varepsilon$.

So, initial energy of the system : $U_i = \frac{1}{2}(C\,\varepsilon)\,V^2$

and finally, energy of the capacitor : $U_f = \frac{1}{2}C\,V^2$

Hence capacitance energy increment,

$$\Delta U = \frac{1}{2}C\,V^2 - \frac{1}{2}(C\,\varepsilon)\,V^2 = -\frac{1}{2}\,CV^2\,(\varepsilon - 1) = -0{\cdot}5 \text{ mJ}$$

From energy conservation

$$\Delta U = A_{cell} + A_{agent}$$

(as there is no heat liberation)

But $A_{cell} = (C_f - C_i)\,V^2 = (C - C\varepsilon)V^2$

Hence $A_{\text{agent}} = \Delta U - A_{\text{cell}}$

$$= \frac{1}{2}C(1-\varepsilon)V^2 = 0{\cdot}5 \text{ m J}$$

3.202 If C_0 is the initial capacitance of the condenser before water rises in it then

$$U_i = \frac{1}{2}C_0 V^2, \quad \text{where} \quad C_0 = \frac{\varepsilon_0 2l\pi R}{d}$$

(R is the mean radius and l is the length of the capacitor plates.)

Suppose the liquid rises to a height h in it. Then the capacitance of the condenser is

$$C = \frac{\varepsilon\varepsilon_0 h 2\pi R}{d} + \frac{\varepsilon(l-h)2\pi R}{d} = \frac{\varepsilon_0 2\pi R}{d}(l + (\varepsilon - 1)h)$$

and energy of the capacitor and the liquid (including both gravitational and electrosatic contributions) is

$$\frac{1}{2}\frac{\varepsilon_0 2\pi R}{d}(l + (\varepsilon-1)h)V^2 + \rho g(2\pi R\, hd)\frac{h}{2}$$

If the capacitor were not connected to a battery this energy would have to be minimized. But the capacitor is connected to the battery and, in effect, the potential energy of the whole system has to be minimized. Suppose we increase h by δh. Then the energy of the capacitor and the liquid increases by

$$\delta h\left(\frac{\varepsilon_0 2\pi R}{2d}(\varepsilon-1)V^2 + \rho g(2\pi R d)h\right)$$

and that of the cell diminishes by the quantity A_{cell} which is the product of charge flown and V

$$\delta h \frac{\varepsilon_0(2\pi R)}{d}(\varepsilon - 1)V^2$$

In equilibrium, the two must balance; so

$$\rho g d h = \frac{\varepsilon_0(\varepsilon-1)V^2}{2d}$$

Hence
$$h = \frac{\varepsilon_0(\varepsilon-1)V^2}{2\rho g d^2}$$

3.203 (a) Let us mentally islolate a thin spherical layer with inner and outer radii r and $r + dr$ respectively. Lines of current at all the points of this layer are perpendicular to it and therefore such a layer can be treated as a spherical conductor of thickness dr and cross sectional area $4\pi r^2$. Now, we know that resistance,

$$dR = \rho\frac{dr}{S(r)} = \rho\frac{dr}{4\pi r^2} \tag{1}$$

Integrating expression (1) between the limits,

$$\int_0^R dR = \int_a^b \rho \frac{dr}{4\pi r^2} \quad \text{or,} \quad R = \frac{\rho}{4\pi}\left[\frac{1}{a} - \frac{1}{b}\right] \tag{2}$$

Capacitance of the network, $C = \dfrac{4\pi \varepsilon_0 \varepsilon}{\left[\frac{1}{a} - \frac{1}{b}\right]}$ (3)

and $q = C\varphi \left[\begin{array}{l}\text{where } q \text{ is the charge}\\ \text{at any arbitrary moment}\end{array}\right]$ (4)

also, $\varphi = \left(\dfrac{-dq}{dt}\right)R$, as capacitor is discharging. (5)

From Eqs. (2), (3), (4) and (5) we get,

$$q = \frac{4\pi\varepsilon_0\varepsilon}{\left[\frac{1}{a} - \frac{1}{b}\right]} \frac{\left[-\frac{dq}{dt}\right]\rho\left[\frac{1}{a} - \frac{1}{b}\right]}{4\pi} \quad \text{or,} \quad \frac{dq}{q} = \frac{dt}{\rho\,\varepsilon\,\varepsilon_0}$$

Integrating $\displaystyle\int_{q_0}^{q} -\frac{dq}{q} = \frac{1}{\rho\varepsilon_0\varepsilon}\int_0^t dt = \frac{dt}{\rho\,\varepsilon\varepsilon_0}$

Hence $q = q_0\, e^{\frac{-t}{\rho\,\varepsilon_0\varepsilon}}$

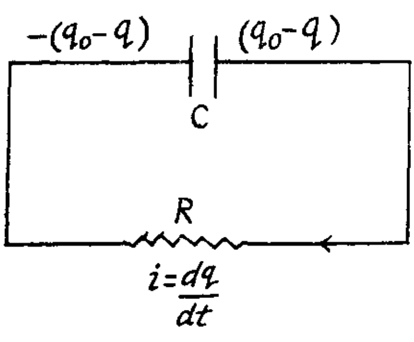
(b) From energy conservation heat generated, during the spreading of the charge,

$$H = U_i - U_f \text{ [because } A_{cell} = 0]$$

$$= \frac{1}{2}\frac{q_0^2}{4\pi\varepsilon_0\varepsilon}\left[\frac{1}{a} - \frac{1}{b}\right] - 0 = \frac{q_0^2}{8\pi\varepsilon_0\varepsilon}\frac{b-a}{ab}$$

3.204 (a) Let, at any moment of time, charge on the plates be $(q_0 - q)$ then current through the resistor, $i = -\dfrac{d(q_0 - q)}{dt}$, because the capacitor is discharging.

or, $i = \dfrac{dq}{dt}$



Now, applying loop rule in the circuit,

$$iR - \frac{q_0 - q}{C} = 0$$

or, $\dfrac{dq}{dt}R = \dfrac{q_0 - q}{C}$

or, $\dfrac{dq}{q_0 - q} = \dfrac{1}{RC}dt$

At $t = 0$, $q = 0$ and at $t = \tau$, $q = q$

So, $$\ln \frac{q_0 - q}{q_0} = \frac{-\tau}{RC}$$

Thus $$q = q_0\left(1 - e^{-\tau/RC}\right) = 0{\cdot}18\ \text{mC}$$

(b) Amount of heat generated = decrement in capacitance energy

$$= \frac{1}{2}\frac{q_0^2}{C} - \frac{1}{2}\frac{\left[q_0 - q_0\left(1 - e^{-\tau/RC}\right)\right]^2}{C}$$

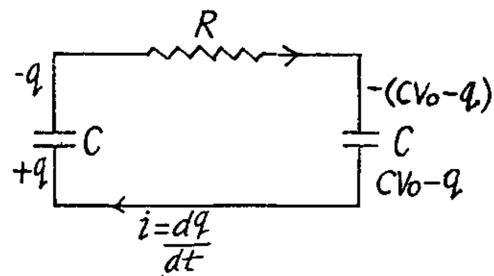
$$= \frac{1}{2}\frac{q_0^2}{C}\left[1 - e^{-\frac{2\tau}{RC}}\right] = 82\ \text{mJ}$$

3.205 Let, at any moment of time, charge flown be q then current $i = \frac{dq}{dt}$

Applying loop rule in the circuit, $-\Delta\varphi = 0$, we get :

$$\frac{dq}{dt}R - \frac{(CV_0 - q)}{C} + \frac{q}{C} = 0$$

or, $$\frac{dq}{CV_0 - 2q} = \frac{1}{RC}\,dt$$



So, $\ln \frac{CV_0 - 2q}{CV_0} = -2\frac{t}{RC}$ for $0 \le t \le t$

or, $$q = \frac{CV_0}{2}\left(1 - e^{\frac{-2t}{RC}}\right)$$

Hence, $$i = \frac{dq}{dt} = \frac{CV_0}{2}\frac{2}{RC}\,e^{-2t/RC} = \frac{V_0}{R}e^{-2t/RC}$$

Now, heat liberaled,

$$Q = \int_0^\infty i^2 R\,dt = \frac{V_0^2}{R^2}R\int_0^\infty e^{\frac{-4t}{RC}}\,dt = \frac{1}{4}CV_0^2$$

3.206 In a rotating frame, to first order in ω, the main effect is a coriolis force $2m\vec{v}' \times \vec{\omega}$.

This unbalanced force will cause electrons to react by setting up a magnetic field $\vec{B}$ so that the magnetic force $e\vec{v}\times\vec{B}$ balances the coriolis force.

Thus $$-\frac{e}{2m}\vec{B} = \vec{\omega} \quad \text{or,} \quad \vec{B} = -\frac{2m}{e}\vec{\omega}$$

The flux associated with this is

$$\Phi = N\pi r^2 B = N\pi r^2 \frac{2m}{e}\omega$$

where $N = \frac{l}{2\pi r}$ is the number of turns of the ring. If ω changes (and there is time for the electron to rearrange) then B also changes and so does Φ. An emf will be induced and a current will flow. This is

$$I = N\pi r^2 \frac{2m}{e}\omega/R$$

The total charge flowing through the ballastic galvanometer, as the ring is stopped, is

$$q = N\pi r^2 \Big/ \frac{2m}{e}\omega/R$$

So, $$\frac{e}{m} = \frac{2N\pi r^2\omega}{qR} = \frac{l\,\omega r}{qR}$$

3.207 Let, n_0 be the total number of electorns then, total momentum of electorns,

$$p = n_0 m_e v_d \tag{1}$$

Now, $$I = \rho S_x v_d = \frac{n_0 e}{V} S_x v_d = \frac{ne}{l} v_d \tag{2}$$

Here S_x = Cross sectional area, ρ = electron charge density, V = volume of sample
From (1) and (2)

$$p = \frac{m_e}{e} Il = 0{\cdot}40\ \mu\,\text{Ns}$$

3.208 By definition
$n e v_d = j$ (where v_d is the drift velocity, n is number density of electrons.)

Then $$\tau = \frac{l}{v_d} = \frac{nel}{j}$$

So distance actually travelled

$$S = \langle v \rangle \tau = \frac{nel \langle v \rangle}{j}$$

($\langle v \rangle$ = mean velocity of thermal motion of an electron)

3.209 Let, n be the volume density of electrons, then from $I = \rho S_x v_d$,

$$I = n e S_x \left| \langle \vec{v} \rangle \right| = n e S_x \frac{l}{t}$$

So, $$t = \frac{n e S_x l}{I} = 3\ \mu\text{s}.$$

(b) Sum of electric forces

$$= \left| (nv)\, e \vec{E} \right| = \left| n S l e \rho \vec{j} \right|, \text{ where } \rho \text{ is resistivity of the material.}$$

$$= n S l e \rho \frac{I}{S} = n e l \rho I = 1{\cdot}0\ \mu\text{N}$$

3.210 From Gauss theorem field strength at a surface of a cylindrical shape equals, $\frac{\lambda}{2\pi\varepsilon_0 r}$, where λ is the linear charge density.

Now, $$eV = \frac{1}{2} m_e v^2 \quad \text{or,} \quad v = \sqrt{\frac{2eV}{m_e}} \tag{1}$$

Also, $$dq = \lambda\, dx \quad \text{so,} \quad \frac{dq}{dt} = \lambda \frac{dx}{dt}$$

or, $$I = \lambda v \quad \text{or,} \quad \lambda = \frac{I}{v} = \frac{I}{\sqrt{\frac{2eV}{m_e}}}, \text{ using (1)}$$

Hence $$E = \frac{I}{2\pi\varepsilon_0 r}\sqrt{\frac{m_e}{2eV}} = 32\ \text{V/m}$$

(b) For the point, inside the solid charged cylinder, applying Gauss' theorem,

$$2\pi r h E = \pi r^2 h \frac{q}{\varepsilon_0 \pi R^2 l}$$

or, $$E = \frac{q/l}{2\pi\varepsilon_0 R^2} r = \frac{\lambda r}{2\pi\varepsilon_0 R^2}$$

So,from $$E = -\frac{d\varphi}{dr},$$

$$\int_{\varphi_1}^{\varphi_2} -d\varphi = \int_0^R \frac{\lambda}{2\pi\varepsilon_0 R^2} r\, dr$$

or, $$\varphi_1 - \varphi_2 = \frac{\lambda}{2\pi\varepsilon_0 R^2}\left[\frac{r^2}{2}\right]_0^R = \frac{\lambda}{4\pi\varepsilon_0}$$

Hence, $$\varphi_1 - \varphi_2 = \frac{VI}{4\pi\varepsilon_0}\sqrt{\frac{m_e}{2eV}} = 0{\cdot}80\ \text{V}$$

3.211 Between the plates $\varphi = a x^{4/3}$

or, $$\frac{\partial\varphi}{\partial x} = a \times \frac{4}{3} x^{1/3}$$

$$\frac{d^2\varphi}{\partial x^2} = \frac{4}{9} a x^{-2/3} = -\rho/\varepsilon_0$$

or, $$\rho = -\frac{4\varepsilon_0 a}{9} x^{-2/3}$$

Let the charge on the electron be - e, then $$\frac{1}{2}mv^2 - e\varphi = \text{Const.} = 0,$$

as the electron is initially emitted with neligible energy.

$$v^2 = \frac{2e\varphi}{m}, \quad v = \sqrt{\frac{2e\varphi}{m}}$$

So, $$j = -\rho v = \frac{4\varepsilon_0 a}{9}\sqrt{\frac{2\varphi}{m}}\, x^{-2/3}.$$

(j is measued from the anode to cathode, so the - ve sign.)

3.212 $E = \frac{V}{d}$

So by the definition of the mobility

$$v^+ = u^+_0 \frac{V}{d}, \quad v^- = u^-_0 - \frac{V}{d}$$

and $$j = (n_+ u_0^+ + n_- u_0^-)\frac{eV}{d}$$

(The negative ions move towards the anode and the positive ion towards the cathode and the total current is the sum of the currents due to them.)

On the other hand, in equilibrium $n_+ = n_-$

So, $$n_+ = n_- = \frac{I}{S} \Big/ (u_0^+ + u_0^-)\frac{eV}{d}$$

$$= \frac{I\,d}{e\,V\,S\,(u_0^+ + u_0^-)} = 2{\cdot}3 \times 10^8 \text{ cm}^{-3}$$

3.213 Velocity = mobility × field

or, $v = u\frac{V_0}{l}\sin\omega t$, which is positive for $0 \le \omega t \le \pi$

So, maximum displacement in one direction is

$$x_{max} = \int_0^{\pi} u\frac{V_0}{l}\sin\omega t\, dt = \frac{2uV_0}{l\omega}$$

At $\omega = \omega_0$, $x_{max} = l$, so, $\frac{2uV_0}{l\omega} = l$

Thus $$u = \frac{\omega l^2}{2V_0}$$

3.214 When the current is saturated, all the ions, produced, reach the plate.

Then, $$\dot{n}_i = \frac{I_{sat}}{eV} = 6 \times 10^9 \text{ cm}^{-3}\text{s}^{-1}$$

(Both positive ions and negative ions are counted here)

The equation of balance is, $\frac{dn}{dt} = \dot{n}_i - rn^2$

The first term on the right is the production rate and the second term is the recombination rate which by the usual statistical arguments is proportional to n^2 (= no of positive ions × no. of $-ve$ ion). In equilibrium,

$$\frac{dn}{dt} = 0$$

so, $$n_{eq} = \sqrt{\frac{\dot{n}_i}{r}} = 6 \times 10^7 \text{ cm}^{-3}$$

3.215 Initially $n = n_0 = \sqrt{\dot{n}_i / r}$

Since we can assume that the long exposure to the ionizer has caused equilibrium to be set up. Afer the ionizer is switched off,

$$\frac{dn}{dt} = -rn^2$$

or $$r\,dt = -\frac{dn}{n^2}, \quad \text{or,} \quad rt = \frac{1}{n} + \text{constant}$$

But $$n = n_0 \text{ at } t = 0, \text{ so, } rt = \frac{1}{n} - \frac{1}{n_0}$$

The concentration will decrease by a factor η when

$$r\,t_0 = \frac{1}{n_0/\eta} - \frac{1}{n_0} = \frac{\eta - 1}{n_0}$$

or, $$t_0 = \frac{\eta - 1}{\sqrt{r\,\dot{n}_i}} = 13 \text{ ms}$$

3.216 Ions produced will cause charge to decay. Clearly,

$$\eta\, CV = \text{decrease of charge} = \dot{n}_i\, eA\, dt = \frac{\varepsilon_0 A}{d} V \eta$$

or, $$t = \frac{\varepsilon_0 V \eta}{\dot{n}_i e d^2} = 4{\cdot}6 \textit{ days}$$

Note, that n_i, here, is the number of ion pairs produced.

3.217 If ν = number of electrons moving to the anode at distance x, then

$$\frac{d\nu}{dx} = \alpha \nu \quad \text{or} \quad \nu = \nu_0 e^{\alpha x}$$

Assuming saturation, $I = e\,\nu_0\, e^{\alpha d}$

3.218 Since the electrons are produced uniformly through the volume, the total current attaining saturation is clearly,

$$I = \int_0^d e\,(\dot{n}_i A\, dx)\, e^{\alpha x} = e\,\dot{n}_i A \left(\frac{e^{\alpha d} - 1}{\alpha}\right)$$

Thus, $$j = \frac{I}{A} = e\,\dot{n}_i \left(\frac{e^{\alpha d} - 1}{\alpha}\right)$$