Class X Session 2023-24
Subject - Mathematics (Basic)
Sample Question Paper - 8

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. Which of the following is an irrational number? **[1]**

   i. $\frac{22}{7}$
   ii. 3.1416
   iii. $3.\overline{1416}$
   iv. 3.141141114...

   a) Option (iv) b) Option (iii)
   c) Option (i) d) Option (ii)

2. The product of two numbers is 1600 and their HCF is 5. The LCM of the numbers is **[1]**

   a) 1600 b) 8000
   c) 1605 d) 320

3. $4x^2 - 20x + 25 = 0$ have **[1]**

   a) Real roots b) No Real roots
   c) Real and Equal roots d) Real and Distinct roots

4. The value of a so that the point (3, a) lies on the line represented by 2x - 3y = 5 is **[1]**

   a) $\frac{1}{3}$ b) – 1
   c) 1 d) $\frac{-1}{3}$

5. $2x^2 + 5\sqrt{3}x + 6 = 0$ have **[1]**

a) Real and equal root

b) Real roots

c) No Real roots

d) Real and Distinct roots

6. If the endpoints of a diameter of a circle are (-4, -3) and (2, 7), then the coordinates of the centre are **[1]**

a) (1, -2)

b) (0, 0)

c) (2, -1)

d) (-1, 2)

7. XY is drawn parallel to the base BC of a $\triangle$ABC cutting AB at X and AC at Y. If AB = 4 BX and YC = 2cm, then AY = **[1]**

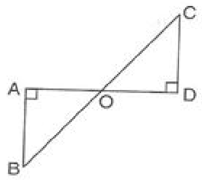
a) 8 cm

b) 4 cm

c) 6 cm

d) 2 cm

8. In the given figure $\Delta ABO \sim \Delta DCO$. If CD = 2cm, AB = 3 cm, OC = 3.2 cm, OD = 2.4 cm, then **[1]**



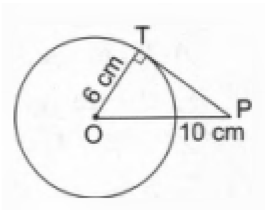
a) OA = 4.3 cm, OB = 3.5 cm.

b) OA = 3 cm, OB = 4 cm.

c) OA = 3.6 cm, OB = 4.8 cm.

d) OA = 3.2 cm, OB = 4.6 cm

9. In the given figure, PT is a tangent to the circle with centre O. If OT = 6 cm and OP = 10 cm, then the length of tangent PT is **[1]**



a) 8 cm

b) 16 cm

c) 10 cm

d) 12 cm

10. If sinα = $\frac{1}{\sqrt{2}}$ and tanβ = 1, then the value of $\cos(\alpha + \beta)$ is **[1]**

a) 3

b) 1

c) 2

d) 0

11. The length of the shadow of a 20 m tall pole on the ground when the sun's elevation is 45° is **[1]**

a) 20 m

b) $20\sqrt{2}$ m

c) 40 m

d) $20\sqrt{3}$ m

12. $\sec^4 A - \sec^2 A$ is equal to **[1]**

a) $\tan^2 A - \tan^4 A$

b) $\tan^4 A - \tan^2 A$

c) $\tan^2 A + \tan^3 A$

d) $\tan^4 A + \tan^2 A$

13. The hour hand of a clock is 6 cm long. The area swept by it between 11.20 am and 11.55 am is **[1]**

a) $10\ cm^2$

b) $11\ cm^2$

c) $5.5\ cm^2$

d) $2.75\ cm^2$

14. If the area of a sector of a circle is $\frac{7}{20}$ of the area of the circle, then the sector angle is equal to **[1]**

a) 110°

b) 100°

c) 130°

d) 126°

15. Three coins are tossed once. The probability of getting two heads and one tale is **[1]**

a) $\frac{3}{8}$

b) $\frac{1}{8}$

c) $\frac{5}{8}$

d) $\frac{1}{4}$

16. If the mode of the data: 64, 60, 48, x, 43, 48, 43, 34 is 43, then x + 3 = **[1]**

a) 45

b) 48

c) 44

d) 46

17. If a sphere is inscribed in a cube, then the ratio of the volume of the cube to the volume of the sphere is **[1]**

a) $6 : \pi$

b) $\pi : 6$

c) $\pi : 4$

d) $4 : \pi$

18. Consider the following distribution : **[1]**

Marks obtained	Number of students
More than or equal to 0	63
More than or equal to 10	58
More than or equal to 20	55
More than or equal to 30	51
More than or equal to 40	48
More than or equal to 50	42

the frequency of the class 30-40 is

a) 4

b) 48

c) 51

d) 3

19. **Assertion (A):** If A(2a, 4a) and B(2a, 6a) are two vertices of an equilateral triangle ABC then the vertex C is given by (2a + $a\sqrt{3}$, 5a). **[1]**

Reason (R): In an equilateral triangle, all the coordinates of three vertices can be rational.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The HCF of two numbers is 5 and their product is 150, then their LCM is 30. **[1]**

Reason (R): For any two positive integers a and b, HCF (a, b) + LCM (a, b) = a $\times$ b.

a) Both A and R are true and R is the correct explanation of A.

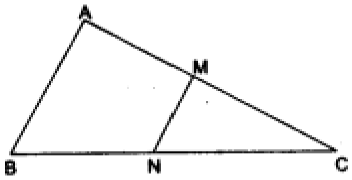
b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

Section B

21. Ten students of class X took part in Mathematics quiz. If the number of girls is 4 more than the number of boys. Represent this situation algebraically and graphically. **[2]**

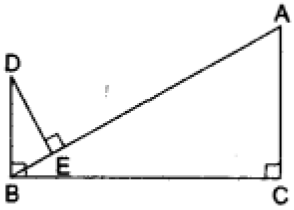
22. In the given figure, $MN||AB$, BC = 7.5 cm, AM = 4 cm and MC = 2 cm. Find the length of BN. **[2]**



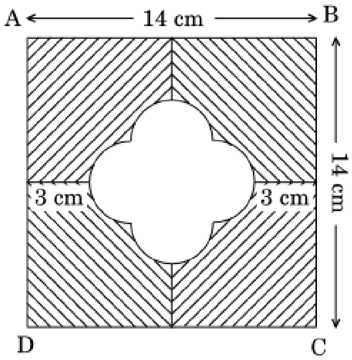
OR

In the given figure, $DB \perp BC$, $DE \perp AB$ and $AC \perp BC$.

Prove that $\frac{BE}{DE} = \frac{AC}{BC}$



23. In figure 2, find the area of the shaded region, where ABCD is a square of side 14 cm in which four semi-circles of same radii are drawn as shown **[2]**



24. Prove that: $1 + \frac{\cot^2\theta}{1+cosec\theta} = cosec\theta$ **[2]**

25. A chord 10 cm long is drawn in a circle whose radius is $5\sqrt{2}$ cm. Find the areas of both the segments. [Take π = 3.14.] **[2]**

OR

A sector of 56°, cut out from a circle, contains 17.6 cm^2. Find the radius of the circle.

Section C

26. Prove that $3 + 2\sqrt{5}$ is irrational. **[3]**

27. If α, β are zeroes of the quadratic polynomial $x^2 + 9x + 20$, form a quadratic polynomial whose zeroes are (α + 1) and (β + 1). **[3]**

28. Champa went to a **Sale** to purchase some pants and skirts. When her friends asked her how many of each she had bought, she answered, **The number of skirts is two less than twice the number of pants purchased. Also, the number of skirts is four less than four times the number of pants purchased**. Help her friends to find how many pants and skirts Champa bought. **[3]**

OR

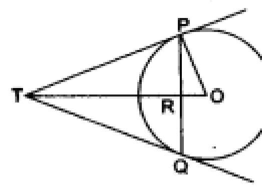
Solve the system of equations graphically:

3x - 4y =7

5x + 2y =3

Shade the region between the lines and the y-axis

29. PQ is a chord of length 4.8 cm of a circle of radius 3 cm. The tangents at P and Q intersect at a point T as shown in the figure. Find the length of TP. **[3]**



30. Prove the following identity : $\frac{1}{\cot^2\theta}+\frac{1}{1+\tan^2\theta}=\frac{1}{1-\sin^2\theta}-\frac{1}{cosec^2\theta}$ **[3]**

OR

Prove that: $\frac{\tan A}{1+\sec A}-\frac{\tan A}{1-\sec A}=2cosecA$

31. The king, queen and jack of club are removed from a deck of 52 cards. Then the cards are well-shuffled. One card is drawn at random from the remaining cards. Find the probability of getting **[3]**

i. a heart
ii. a king
iii. a club
iv. a '10 'of hearts.

Section D

32. Find the value of m for which the quadratic equation $(m+1)y^2-6(m+1)y+3(m+9)=0$, $m\neq-1$ has equal roots. Hence find the roots of the equation. **[5]**

OR

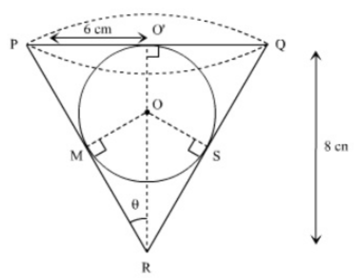
In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/hr and the time of flight was increased by 30 minutes. Find the duration of flight.

33. In a trapezium ABCD, AB | | DC and DC = 2AB. EF | | AB, where E and F lie on BC and AD respectively such that $\frac{BE}{EC}=\frac{4}{3}$. Diagonal DB intersects EF at G. Prove that, 7EF = 11AB. **[5]**

34. A tent is of the shape of a right circular cylinder upto a height of 3 metres and then becomes a right circular cone with a maximum height of 13.5 metres above the ground. Calculate the cost of painting the inner side of the tent at the rate of Rs.2 per square metre, if the radius of the base is 14 metres. **[5]**

OR

A conical vessel of radius 6 cm and height 8 cm is completely filled with water. A sphere is lowered into the water and its size is such that when it touches the sides, it is just immersed as shown in Figure. What fraction of water over flows?



35. The following table shows the marks scored by 140 students in an examination of a certain paper: **[5]**

Marks	0-10	10-20	20-30	30-40	40-50
Number of students	20	24	40	36	20

Calculate the average marks by using all the three methods: direct method, assumed mean deviation and shortcut method.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year.

(i) Find the production during first year.

(ii) Find the production during 8th year.

OR

Find the production during first 3 years.

(iii) In which year, the production is ₹ 29,200.

37. **Read the text carefully and answer the questions:** **[4]**

A satellite image of a colony is shown below. In this view, a particular house is pointed out by a flag, which is situated at the point of intersection of x and y-axes. If we go 2 cm east and 3 cm north from the house, then we reach to a Grocery store. If we go 4 cm west and 6 cm south from the house, then we reach to an Electricians's shop. If we go 6 cm east and 8 cm south from the house, then we reach to a food cart. If we go 6 cm west and 8 cm north from the house, then we reach a bus stand.

Scale:

x-axis : 1 cm = 1 unit

y-axis : 1 cm = 1 unit

(i) What is the distance between the grocery store and food cart?

(ii) What is the distance of the bus stand from the house?

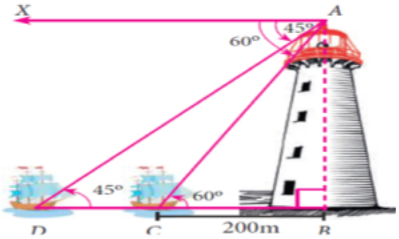
OR

What are the ratio of distances of the house from bus stand to food cart?

(iii) If the grocery store and electricians shop lie on a line, then what will be the ratio of distance of house from grocery store to that from electrician’s shop?

38. **Read the text carefully and answer the questions:** **[4]**

A man is watching a boat speeding away from the top of a tower. The boat makes an angle of depression of 60^o with the man's eye when at a distance of 200 m from the tower. After 10 seconds, the angle of depression becomes 45^o.



(i) What is the approximate speed of the boat (in km/hr), assuming that it is sailing in still water?

(ii) How far is the boat when the angle is 45^o?

OR

What is the height of tower?

(iii) As the boat moves away from the tower, angle of depression will decrease/increase?

Solution

Section A

1. **(a)** Option (iv)

   Explanation: 3.141141114 is an irrational number because it is a non-repeating and non-terminating decimal.

2. **(d)** 320

   Explanation: Let the two numbers be x and y.

   It is given that: $x \times y = 1600$

   HCF = 5

   We know, HCF $\times$ LCM = $x \times y$

   $\Rightarrow 5 \times$ LCM = 1600

   $\therefore$ LCM = $\frac{1600}{5}$ = 320

3. **(c)** Real and Equal roots

   Explanation: D = $b^2 - 4ac$

   D = $(-20)^2 - 4 \times 4 \times 25$

   D = 400 - 400

   D = 0. Hence Real and equal roots.

4. **(a)** $\frac{1}{3}$

   Explanation: $2x - 3y = 5$

   $\Rightarrow 2 \times 3 - 3 \times a = 5$

   $\Rightarrow 6 - 3a = 5$

   $\Rightarrow a = \frac{1}{3}$

5. **(d)** Real and Distinct roots

   Explanation: Comparing the given equation to the below equation

   $ax^2 + bx + c = 0$

   a = 2, b = $5\sqrt{3}$, c = 6

   D = $b^2 - 4ac$

   D = $(5\sqrt{3})^2 - 4 \times 2 \times 6$

   D = 75 - 48

   D = 27

   D > 0.

   If $b^2 - 4ac > 0$, then the equation has real and distinct roots.

6. **(d)** (-1, 2)

   Explanation: Let the coordinates of centre O be (x, y).

   The endpoints of a diameter of the circle are A(- 4, - 3) and B(2, 7).

   Since centre is the midpoint of diameter.

   $\therefore x = \frac{x_1+x_2}{2} = \frac{-4+2}{2} = \frac{-2}{2}$ = -1 and

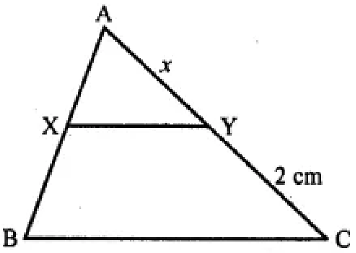
   $y = \frac{y_1+y_2}{2} = \frac{-3+7}{2} = \frac{4}{2}$ = 2

   Therefore, the coordinates of the centre O is (-1, 2)

7. **(c)** 6 cm

   Explanation: In ΔABC, XY || BC

   AB = 4BX, YC = 2 cm



$\therefore AB = 4BX \Rightarrow AX + BX = 4BX$

$\Rightarrow \mathrm{AX} = 4\mathrm{BX} - \mathrm{BX} = 3\mathrm{BX}$

Let AY = x

$\because$ In ΔABC, XY ∥ BC

$\frac{AX}{BX} = \frac{AY}{CY} \Rightarrow \frac{3BX}{BX} = \frac{x}{2}$

$\Rightarrow \frac{3}{1} = \frac{x}{2} \Rightarrow x = 3 \times 2 = 6$

$\therefore \mathrm{AY} = 6\mathrm{cm}$

8.

(c) OA = 3.6 cm, OB = 4.8 cm.

Explanation: $\frac{AB}{DC} = \frac{AO}{DO} = \frac{BO}{CO}$ (cpst)

$\frac{3}{2} = \frac{BO}{3.2} = \frac{AO}{2.4} \Rightarrow$ OA = 3.6cm, OB = 4.8cm

9. **(a)** 8 cm

Explanation: In the given figure, PT is tangent to the circle with centre O and radius

OT = 6 cm OP = 10 cm

OT is the radius and PT is the tangent

OT $\perp$ TP

Now, in right $\triangle$OPT,

$OP^2 = OT^2 + PT^2$ (Pythagoras Theorem)

$\Rightarrow (10)^2 = (6)^2 + PT^2$

$\Rightarrow 100 = 36 + PT^2$

$\Rightarrow PT^2 = 100 - 36 = 64 = (8)^2$.

PT = 8 cm

10.

(d) 0

Explanation: Given: sinα = $\frac{1}{\sqrt{2}}$

$\Rightarrow \sin\alpha = \sin 45^o$

$\Rightarrow \alpha = 45^o$

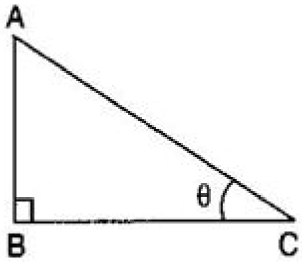
And tanβ = 1

$\Rightarrow \tan\beta = \tan 45^o$

$\Rightarrow \beta = 45^o$

$\therefore \cos(\alpha + \beta) = \cos(45^o + 45^o) = \cos 90^o = 0$

11. **(a)** 20 m

Explanation: 

Given: Height of pole = AB = 20 m

And the angle of elevation $\theta = 45^\circ$

Let length of shadow of pole = BC = x meters

$\therefore \tan 45^{\circ} = \frac{AB}{BC}$
$\Rightarrow 1 = \frac{20}{x}$
$\Rightarrow x = 20$ m
Therefore, the length of the shadow of the pole is 20 m.

12.

(d) $\tan^4 A + \tan^2 A$

Explanation: We have, $\sec^4 A - \sec^2 A = \sec^2 A (\sec^2 A - 1)$

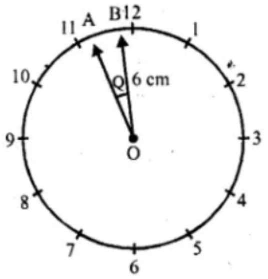
$= (1 + \tan^2 A) \tan^2 A$

$= \tan^2 A + \tan^4 A$

$= \tan^4 A + \tan^2 A$

13.

(c) 5.5 cm^2
Explanation: Length of hour hand of a clock (r) = 6 cm
Time 11.20 am to 11.55 am = 35 minute = $\frac{35}{60}$h



$\therefore$ In 1 hour the hour hand rotates 30°.
Thus, central angle of the sector = $30 \times \frac{35}{60} = 17.5^{\circ}$
$\therefore$ Area of the sector swept by the hour hand = $\frac{17.5}{360} \times \frac{22}{7} \times 6 \times 6$ cm^2
$= \frac{2.5 \times 22}{10} cm^2 = 5.5 cm^2$

14.

(d) 126°
Explanation: We have given that area of the sector is $\frac{7}{20}$ of the area of the circle.
Therefore, area of the sector $= \frac{7}{20} \times$ area of the circle
$\therefore \frac{\theta}{360} \times \pi r^2 = \frac{7}{20} \times \pi r^2$
Now we will simplify the equation as below,
$\frac{\theta}{360} = \frac{7}{20}$
Now we will multiply both sides of the equation by 360,
$\therefore \theta = \frac{7}{20} \times 360$
$\therefore \theta = 126$
Therefore, sector angle is 126°.

15. **(a)** $\frac{3}{8}$
Explanation: Total outcomes = {HHH, TTT, HHT, HTH, HTT, THH, THT, TTH} = 8
Number of possible outcomes = 3
$\therefore$ Required Probability = $\frac{3}{8}$

16.

(d) 46
Explanation: Mode of 64, 60, 48, x, 43, 48, 43, 34 is 43
$\because$ By definition mode is a number which has maximum frequency which is 43
$\therefore$ x = 43
$\because$ x + 3 = 43 + 3 = 46

17. **(a)** 6 : π
Explanation: Let side of cube be a
Here, side of cube= diameter of sphere

so, radius of sphere = $\frac{a}{2}$

The volume of cube : volume of sphere

$a^3 : \frac{4}{3}\pi r^3$

$a^3 : \frac{4}{3}\pi(\frac{a}{2})^3$

$3 \times 8 \times a^3 : 4\pi a^3$

$6 : \pi$

18.

(d) 3

Explanation:

Marks Obtained	Number of students	f
0-10	(63-58)=5	5
10-20	(58-55)=3	3
20-30	(55-51)=4	4
30-40	(51-48)=3	3
40-50	(48-42)=6	6
50...	42=42	42

Hence, frequency in the class interval 30 - 40 is 3.

19.

(c) A is true but R is false.

Explanation: Let, A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are all rational coordinates,

$$\text{ar}(\Delta ABC) = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$= \frac{\sqrt{3}}{4}\left[(x_1 - x_2)^2 + (y_1 - y_2)^2\right]$

LHS = rational

RHS = irrational

Hence, (x_1, y_1) (x_2, y_2) and (x_3, y_3) cannot be all rational.

20.

(c) A is true but R is false.

Explanation: We have,

LCM (a, b) $\times$ HCF (a, b) = a $\times$ b

LCM $\times$ 5 = 150

LCM = $\frac{150}{5}$ = 30

Section B

21. Formulation: Let the number of girls be x and the number of boys be y.

It is given that total ten students took part in the quiz.

$\therefore$ Number of girls + Number of boys = 10

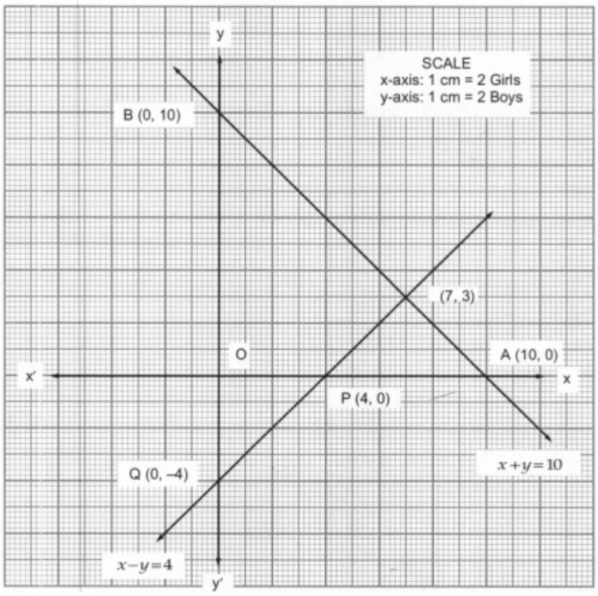
i.e. x + y = 10

It is also given that the number of girls is 4 more than the number of boys.

$\therefore$ Number of girls = Number of boys + 4

i.e. x = y + 4

or, x - y = 4



22. According to quesiton it is given that In $\triangle ABC$,

$MN \| AB$

Therefore by Thale's theroem

$\frac{MC}{AC} = \frac{NC}{BC}$

$\Rightarrow \frac{MC}{AM+MC} = \frac{NC}{BC}$

$\Rightarrow \frac{2}{4+2} = \frac{x}{7.5}$ (when NC = x cm)

$\Rightarrow \quad x = \frac{2\times 7.5}{6}$

$= \frac{15}{6}$ = 2.5

$\Rightarrow$ NC = 2.5 cm

Hence, BN = BC - NC

= (7.5 - 2.5) cm

= 5 cm

OR

In $\triangle BED$ and $\triangle ACB$, we have

$\angle BED = \angle ACB = 90^\circ$

$\because \angle B + \angle C = 180^0$

$\therefore BD \| AC$

$\angle EBD = \angle CAB$ (Alternate angles)

Therefore, by AA similarity theorem, we get

$\triangle$BED $\sim$ $\triangle$ACB

$\Longrightarrow \frac{BE}{AC} = \frac{DE}{BC}$

$\Longrightarrow \frac{BE}{DE} = \frac{AC}{BC}$

23. Given ABCD is a square of side 14 cm each

and four semicircles of same radii

To find : area of shaded region proof

Area of semicircle with radius 2 cm = $\frac{1}{2}\pi r^2$

$= \frac{1}{2} \times 3.14 \times (2)^2$

$= \frac{1}{2} \times 3.14 \times 4$

$= 3.14 \times 2\text{cm}^2$

In figure, there are 4 semi-circles of same radius 2 cm

So, Area of 4 Semi-circle $= 4 \times 3.14 \times 2\text{cm}^2$

$= 8 \times 3.14$

= 25.12 cm ^{2}

Area of smaller square PQRS = (side)^2

$=(4)^2$

=16

Area of shaded region = Area of square ABCD - Area of square PQRS- Area of 4 semi-circles

= 196 - 16- 25.12

= 154.88 cm^2

24. L.H.S. = $1 + \frac{cot^2\theta}{1+cosec\theta}$

$= \frac{(1+cosec\theta+cosec^2\theta-1)}{1+cosec\theta}$

$= \frac{cosec\theta(1+cosec\theta)}{(1+cos\theta)}$

$= cosec\theta$ = R.H.S.

25. 

Given Radius = r = $5\sqrt{2}$cm

= OA = OB

Length of chord AB = 10 cm

In $\triangle OAB$, OA = OB = $= 5\sqrt{2}$

AB = 10cm

OA^2 + OB^2 = $= (5\sqrt{2})^2 + (5\sqrt{2})^2$

= 50 + 50 = 100 = (AB)^2

Pythagoras theorem is satisfied OAB is right triangle

= angle subtended by chord = $\angle AOB = 90^\circ$

Area of segment (minor) = shaded region

= area of sector - area of $\triangle OAB$

$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times OA \times OB$

$= \frac{90}{360} \times \frac{22}{7}(5\sqrt{2})^2 - \frac{1}{2} \times 5\sqrt{2} \times 5\sqrt{2}$

$= \frac{275}{7} - 25 = \frac{100}{7}\text{cm}^2$

Area of major segment = (area of circle) - (area of minor segment)

$= \pi r^2 - \frac{100}{7}$

$= \frac{22}{7} \times (5\sqrt{2})^2 - \frac{100}{7}$

$= \frac{1100}{7} - \frac{100}{7}$

$= \frac{1000}{7}\text{cm}^2$

OR

$\theta = 56^\circ$ and let r be the radius of the circle

Area of sector $= \frac{\pi r^2\theta}{360^\circ}$

$= 17.6cm^2$

$\Rightarrow \frac{22}{7} \times r^2 \times \frac{56^\circ}{360^\circ}$ = 17.6

$r^2 = \left(\frac{17.6\times360\times7}{22\times56}\right)\text{cm}^2$

$r^2 = 36cm^2$

$\Rightarrow r = \sqrt{36}\text{cm}$

r = 6 cm

Hence radius = 6cm

Section C

26. Let us assume, to the contrary, that is $3 + 2\sqrt{5}$ rational.

That is, we can find coprime integers a and b $(b \neq 0)$ such that

$3 + 2\sqrt{5} = \frac{a}{b}$ Therefore, $\frac{a}{b} - 3 = 2\sqrt{5}$

$\Rightarrow \frac{a-3b}{b} = 2\sqrt{5}$

$\Rightarrow \frac{a-3b}{2b} = \sqrt{5} \Rightarrow \frac{a}{2b} - \frac{3}{2}$

Since a and b are integers,

We get $\frac{a}{2b} - \frac{3}{2}$ is rational, also so $\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational.

This contradiction arose because of our incorrect assumption that $3 + 2\sqrt{5}$ is rational.

So, we conclude that $3 + 2\sqrt{5}$ is irrational.

27. $\because \alpha$ and β are zeroes of given polynomial

So, $x^2 + 9x + 20 = 0$

$x^2 + 4x + 5x + 20 = 0$

x(x + 4) + 5(x + 4) = 0

(x + 5)(x + 4) = 0

x = -5 and x = -4

$\therefore \alpha$ = -5 and β = -4

Now, α + 1 = -4 and β + 1 = -3

So, product of zeroes= (-4) $\times$ (-3) = 12

Sum of zeroes = -7

Now polynomial = x^2 - (sum of zeroes)x + (product of zeroes)

Polynomial = $x^2 + 7x + 12$

28. Let us denote the number of pants by x and the number of skirts by y.

Then the equations formed are:

y = 2x - 2 ………… (i)

y = 4x - 4………..(ii)

From (i)

When x = 2, then y = 2

When x = 1, then y = 0

x	2	1
y	2	0

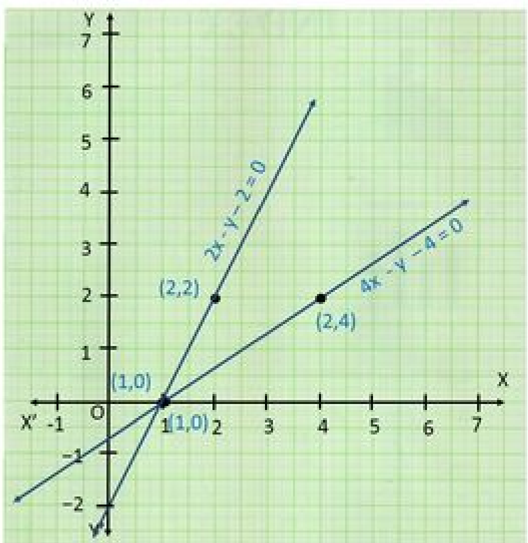
From (ii)

When x = 2, then y = 4

When x = 1, then y = 0

x	2	1
y	4	0

The graphs of two equations of line is shown below.



From the graph, the lines intersect at point (1, 0)

Thus, the value of x = 1 and y = 0

Hence, the number of pants she purchased are 2 and the number of skirts she purchased are 0.

OR

$3x - 4y = 7 \; and \; 5x + 2y = 3$

The given system of linear equation is $3x - 4y = 7 \; and \; 5x + 2y = 3$

Now, $3x - 4y = 7$

$y = \frac{3x-7}{4}$

When x = 1 then, y = -1

When x = -3 then y = -4

x	1	-3
y	-1	-4

Now, 5x + 2y = 3

$y = \frac{3-5x}{2}$

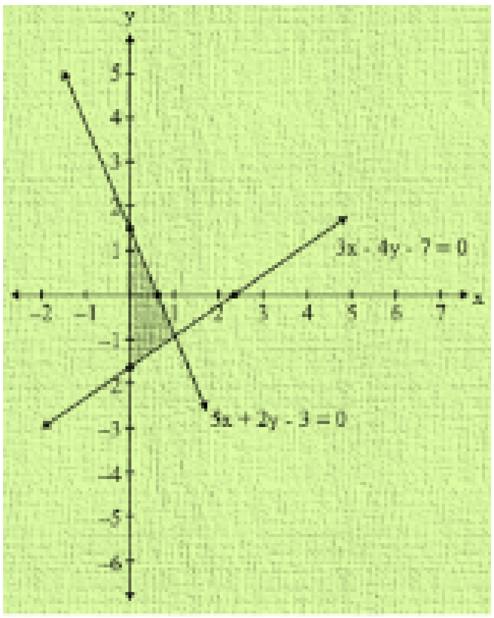
When x = 1 then, y = -1

When x = 3 then y = -6

Thus, we have the following table

x	1	3
y	-1	-6

Graph of the given system of equations are

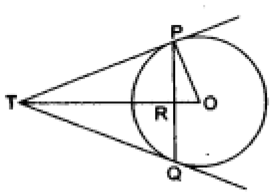


Clearly the two lines intersect at A(1, -1)

Hence, x = 1 and y = -1 is the solution of the given system of equations.

29. Given, Chord PQ = 4.8 cm , radius of circle = 3 cm

Also, The tangents at P and Q intersect at a point T as shown in the figure.We have to find the length of TP.



Let TR = y and TP= x

We know that the perpendicular drawn from the center to the chord bisects It.Hence, PR = RQ ....(1)

Since, PQ = 4.8

or, PR + RQ = 4.8

or, PR + PR = 4.8 [from (1)]

So, PR = 2.4

Now, in right triangle POR, Using Pythagoras theorem, we have

$PO^2 = OR^2 + PR^2$

$\Rightarrow 3^2 = OR^2 + (2.4)^2$

$\Rightarrow OR^2 = 3.24$

$\Rightarrow OR = 1.8$

Now, in right triangle TPR ,By Using Pythagoras theorem, we have

$TP^2 = TR^2 + PR^2$

$\Rightarrow x^2 = y^2 + (2.4)^2$

$\Rightarrow x^2 = y^2 + 5.76$ ...(2)

Again, In right triangle TPO By Using Pythagoras theorem, we have,

$TO^2 = TP^2 + PO^2$

$\Rightarrow (y + 1.8)^2 = x^2 + 3^2$

$\Rightarrow y^2 + 3.6y + 3.24 = x^2 + 9$

$\Rightarrow y^2 + 3.6y = x^2 + 5.76$ ......(3)

Solving (2) and (3), we get

x = 4cm and y = 3.2cm

$\therefore$ TP = 4cm.

30. LHS

$= \frac{1}{\cot^2\theta} + \frac{1}{1+\tan^2\theta}$

$= \tan^2\theta + \frac{1}{\sec^2\theta}$

$= \tan^2\theta + \cos^2\theta$

$= (\sec^2\theta - 1) + \cos^2\theta$

$= \sec^2\theta - (1 - \cos^2\theta)$

$= \sec^2\theta - \sin^2\theta$

$= \frac{1}{\cos^2\theta} - \sin^2\theta$

$= \frac{1}{1-\sin^2\theta} - \frac{1}{cosec^2\theta}$

= R.H.S

Hence proved.

OR

L.H.S.

$= \frac{\tan A}{1+\sec A} - \frac{\tan A}{1-\sec A}$

$= \tan A\left(\frac{1}{1+\sec A} - \frac{1}{1-\sec A}\right)$

$= \tan A\left(\frac{1-\sec A-1-\sec A}{1-\sec^2 A}\right)$ [Taking L.C.M.]

$= \tan A\left(\frac{-2\sec A}{-\tan^2 A}\right)$ [Since, $\sec^2 A - \tan^2 A = 1$]

$= \frac{2\sec A}{\tan A}$

$= \frac{2\times 1}{\cos A} \times \frac{\cos A}{\sin A}$

$= \frac{2}{\sin A}$

= 2 cosecA

= R.H.S. Proved.

31. According to the question,

Cards removed = king, queen and jack of clubs $= 3$

$\therefore$ Cards left $= 52 - 3 = 49$

$Probability = \frac{favourable\ outcomes}{Total\ outcomes}$

i. Number of hearts $= 13$

   $\therefore$ Probability of drawing a heart = $\frac{13}{49}$

ii. Total number of kings $= 4$

   Number of kings left $= 4 - 1 = 3$

   $\therefore$ Probability of drawing a king = $\frac{3}{49}$

iii. Number of clubs left $= 13 - 3 = 10$

Probability of drawing a club = $\frac{10}{49}$

iv. There is only one '10' of hearts.

$\therefore$ Probability of drawing one '10' of hearts = $\frac{1}{49}$

Section D

32. In equation $(m+1)y^2 - 6(m+1)y + 3(m+9) = 0$

A = m + 1, B = - 6(m + 1), C = 3(m + 9)

For equal roots, $D = B^2 - 4AC = 0$

$36(m+1)^2 - 4(m+1) \times 3(m+9) = 0$

$\Rightarrow 3\left(m^2 + 2m + 1\right) - (m+1)(m+9) = 0$

$\Rightarrow 2m^2 - 4m - 6 = 0$

$\Rightarrow m^2 - 2m - 3 = 0$

$\Rightarrow m^2 - 3m + m - 3 = 0$

$\Rightarrow m(m-3) + 1(m-3) = 0$

$\Rightarrow (m-3)(m+1) = 0$

$\therefore$ m = -1, 3

Neglecting $m \neq -1$

$\therefore m = 3$

$\therefore$ the equation becomes $4y^2 - 24y + 36 = 0$

$\Rightarrow y^2 - 6y + 9 = 0$

$\Rightarrow (y-3)(y-3) = 0$

$\Rightarrow (y-3) = 0 \quad and \quad (y-3) = 0$

$\therefore$ roots are y = 3,3

OR

Let the original speed of the aircraft be x km/hr.

Then, new speed = (x- 200) km/hr.

Duration of flight at original speed = $\frac{600}{x} hrs$

Duration of flight at reduced speed = $\frac{600}{x-200} hrs$

According to the question

$\frac{600}{x-200} - \frac{600}{x} = \frac{1}{2}$

$\implies \frac{600x - 600x + 120000}{x(x-200)} = \frac{1}{2}$

$\implies \frac{120000}{x^2 - 200x} = \frac{1}{2}$

$\implies$ x^2 - 200x -240000 = 0

$\implies$ x^2 - 600 x + 400 x - 240000 = 0

$\implies$ x (x - 600) + 400 (x - 600) = 0

$\implies$ (x- 600) (x + 400) = 0

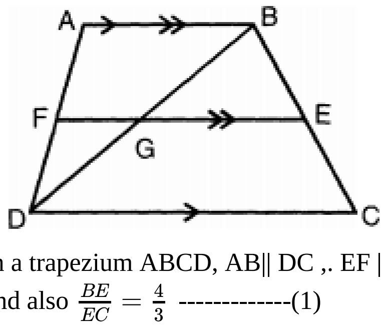
Either x - 600 =0 or x + 400 = 0

$\implies$ x = 600, -400

since Speed cannot be negative. So x = 600

So, original speed of the aircraft was 600 km /hr.

Hence, duration of flight = $\frac{600}{x} hrs = \frac{600}{600} hrs = 1hr$

33. 

In a trapezium ABCD, AB|| DC ,. EF || AB and CD=2AB

and also $\frac{BE}{EC} = \frac{4}{3}$ -------------(1)

AB || CD and AB || EF

$\therefore \frac{AF}{FD} = \frac{BE}{EC} = \frac{4}{3}$

In ΔBGE and ΔBDC

$\angle BEG = \angle BCD$ (∵ corresponding angles)

$\angle GBE = \angle DBC$ (Common)

$\therefore \Delta BGE \sim \Delta BDC$ [By AA similarity]

$\Rightarrow \frac{EG}{CD} = \frac{BE}{BC}$ ....(2)

Now, from (1) $\frac{BE}{EC} = \frac{4}{3}$

$\Rightarrow \frac{EC}{BE} = \frac{3}{4}$

$\Rightarrow \frac{EC}{BE} + 1 = \frac{3}{4} + 1$

$\Rightarrow \frac{EC+BE}{BE} = \frac{7}{4}$

$\Rightarrow \frac{BC}{BE} = \frac{7}{4}$ or $\frac{BE}{BC} = \frac{4}{7}$

from equation (2), $\frac{EG}{CD} = \frac{4}{7}$

So $EG = \frac{4}{7}CD$ ......(3)

Similarly, $\Delta DGF \sim \Delta DBA$ (by AA similarity)

$\Rightarrow \frac{DF}{DA} = \frac{FG}{AB}$

$\Rightarrow \frac{FG}{AB} = \frac{3}{7}$

$\Rightarrow FG = \frac{3}{7}AB$ ...(4)

$$\begin{bmatrix} \because \frac{AF}{AD} = \frac{4}{7} = \frac{BE}{BC} \\ \Rightarrow \frac{EC}{BC} = \frac{3}{7} = \frac{DE}{DA} \end{bmatrix}$$

Adding equations (3) and (4),we get,

$EG + FG = \frac{4}{7}CD + \frac{3}{7}AB$

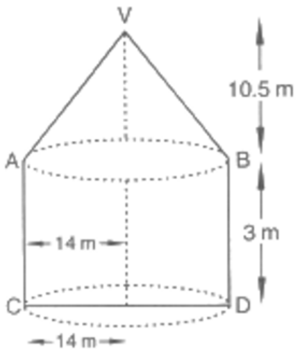
$\Rightarrow EF = \frac{4}{7} \times (2AB) + \frac{3}{7}AB$

$= \frac{8}{7}AB + \frac{3}{7}AB = \frac{11}{7}AB$

$\therefore \quad 7EF = 11AB$

34. Let r metres be the radius of the base of the cylinder and h metres be its heigh$\Rightarrow l_1 = \sqrt{306.25}\text{m} = 17.5\text{m}$

Then, r = 14m and h = 3 m



Now we have Curved surface area of the cylinder ..$= 2\pi rh\text{m}^2 = \left(2 \times \frac{22}{7} \times 14 \times 3\right)\text{m}^2$ = 264 m^2

Let r_1 m be the radius of the base, h_1 m be the height and Z m be the slant height of the cone. Then, r_1 = 14m, h_1 = (13.5 - 3)m = 10.5m

$\therefore I_1 = \sqrt{r_1^2 + h_1^2}$

$\Rightarrow l_1 = \sqrt{14^2 + (10.5)^2}\text{m} = \sqrt{196 + 110.25}\text{m}$

Therefore Curved surface area of the cone $= \pi r_1 l_1$

$= \frac{22}{7} \times 14 \times 17.5\text{m}^2$ = 770m^2

So, Total area which is to be painted = Curved surface area of the cylinder + Curved surface area of the cone....

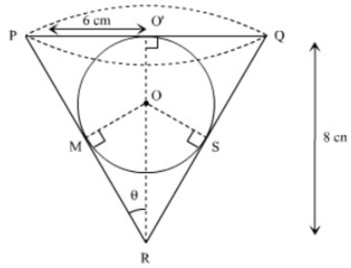
= (264 + 770) m2 = 1034 m^2

Now for Flence, total cost of painting = Rs.(1034 $\times$ 2) = Rs.2068 .

OR

Radius (R) of conical vessel = 6 cm

Height (H) of conical vessel = 8 cm



Volume of conical vessel (V_c) = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3} \times \pi \times 6 \times 6 \times 8$

= $96\pi \text{cm}^3$

Let the radius of the sphere be r cm

In right Δ PO'R by pythagoras theorem We have

$l^2 = 6^2 + 8^2$

l = $\sqrt{36+64}$ = 10 cm

In right triangle MRO

$\sin\theta = \frac{OM}{OR}$

$\Rightarrow \quad \frac{3}{5} = \frac{r}{8-r}$

$\Rightarrow$ 24 - 3r = 5r

$\Rightarrow$ 8r = 24

$\Rightarrow$ r = 3 cm

$\therefore$ V_1 = Volume of the sphere ,= $\frac{4}{3}\pi \times 3^3 \text{cm}^3 = 36\pi \text{cm}^3$

V_2 = Volume of the water = Volume of the cone = $\frac{1}{3}\pi \times 6^2 \times 8\text{cm}^3 = 96\pi \text{cm}^3$

Clearly, volume of the water that flows out of the cone is same as the volume of the sphere i.e., V_1.

$\therefore$ Fraction of the water that flows out $= V_1 : V_2 = 36\pi : 96\pi = 3 : 8$

35. i. Direct method:

Class interval	**Mid value x_i**	**Frequency(f_i)**	**$f_i x_i$**
0 – 10	5	20	100
10 – 20	15	24	360
20 – 30	25	40	1000
30 – 40	35	36	1260
40 – 50	45	20	900
		N = 140	$\sum f_i u_i$ **= 3620**

Mean = $\frac{\sum f_i u_i}{N}$

= $\frac{3620}{140}$

= 25.857

ii. Assumed mean method:

Class interval	**Mid value x_i**	**$u_i = (x_i – A)$**	**Frequency f_i**	**$f_i u_i$**
0 – 10	5	-20	20	-400
10 – 20	15	-10	24	-240
20 – 30	25	0	40	0
30 – 40	35	10	36	360
40 – 50	45	20	20	400
			N = 140	$\sum f_i u_i$ **= 120**

Let the assumed mean is 25

Mean = A + $\left(\frac{\sum f_i u_i}{N}\right)$

= 25 + $\left(\frac{120}{140}\right)$

= 25 + 0.857

= 25.857

iii. Step deviation method:

Class interval	**Mid value x_i**	$d_i = x_i - 25$	$u_i = \frac{(x_i - 25)}{10}$	**Frequency f_i**	$f_i u_i$
0 – 10	5	-20	-2	20	-40
10 – 20	15	-10	-1	24	-24
20 – 30	25	0	0	40	0
30 – 40	35	10	1	36	36
40 – 50	45	20	2	20	40
				N = 140	$\sum f_i u_i$ **= 12**

Let the assumed mean (A) = 25

h = 10

Mean = A + h$\left(\frac{\sum f_i u_i}{N}\right)$

= 25 + 10$\left(\frac{12}{140}\right)$

= 25 + 0.857

= 25.857

Section E

36. **Read the text carefully and answer the questions:**

India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year.

(i) Let 1st year production of TV = x

Production in 6th year = 16000

t_6 = 16000

t_9 = 22,600

t_6 = a + 5d

t_9 = a + 8d

16000 = x + 5d ...(i)

22600 = x + 8d ...(ii)

\-   -   -

-6600 = - 3d

d = 2200

Putting d = 2200 in equation ...(i)

16000 = x + 5 $\times$ (2200)

16000 = x + 11000

x = 16000 - 11000

x = 5000

$\therefore$ Production during 1st year = 5000

(ii) Production during 8th year is (a + 7d) = 5000 + 7(2200) = 20400

OR

Production during first 3 year = Production in (1^{st} + 2^{nd} + 3^{rd}) year

Production in 1^{st} year = 5000

Production in 2^{nd} year = 5000 + 2200

= 7200

Production in 3^{rd} year = 7200 + 2200

= 9400

$\therefore$ Production in first 3 year = 5000 + 7200 + 9400

= 21,600

(iii) Let in n^{th} year production was = 29,200

$t_n = a + (n - 1)d$

29,200 = 5000 + (n - 1) 2200

29,200 = 5000 + 2200n - 2200

29200 - 2800 = 2200n

26,400 = 2200n

$\therefore n = \frac{26400}{2200}$

n = 12

i.e., in 12^{th} year, the production is 29,200

37. **Read the text carefully and answer the questions:**

A satellite image of a colony is shown below. In this view, a particular house is pointed out by a flag, which is situated at the point of intersection of x and y-axes. If we go 2 cm east and 3 cm north from the house, then we reach to a Grocery store. If we go 4 cm west and 6 cm south from the house, then we reach to an Electricians's shop. If we go 6 cm east and 8 cm south from the house, then we reach to a food cart. If we go 6 cm west and 8 cm north from the house, then we reach a bus stand.

Scale:

x-axis : 1 cm = 1 unit

y-axis : 1 cm = 1 unit

(i) Consider the house is at origin (0, 0), then coordinates of grocery store, electrician's shop, food cart and bus stand are respectively (2, 3), (-4, -6), (6, - 8) and (-6, 8).

Since, grocery store is at (2, 3) and food cart is at (6, -8)

$\therefore$ Required distance = $\sqrt{(6-2)^2 + (-8-3)^2}$

= $\sqrt{4^2 + 11^2} = \sqrt{16 + 121}$ = $\sqrt{137}$ cm

(ii) Consider the house is at origin (0, 0), then coordinates of the grocery store, electrician's shop, food cart and bus stand are respectively (2, 3), (-4, -6), (6, - 8) and (-6, 8).

Required distance

= $\sqrt{(-6)^2 + 8^2} = \sqrt{36 + 64}$ = $\sqrt{100}$ = 10 cm

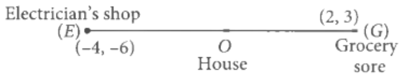
OR

Consider the house is at origin (0, 0), then coordinates of grocery store, electrician's shop, food cart and bus stand are respectively (2, 3), (-4, -6), (6, - 8) and (-6, 8).

Since, (0, 0) is the mid-point of (-6, 8) and (6, -8), therefore both bus stand and food cart are at equal distances from the house. Hence, required ratio is 1 : 1.

(iii) Consider the house is at origin (0, 0), then coordinates of grocery store, electrician's shop, food cart and bus stand are respectively (2, 3), (-4, -6), (6, - 8) and (-6, 8).

Let O divides EG in the ratio k: 1, then



$0 = \frac{2k-4}{k+1}$

$\Rightarrow$ 2k = 4

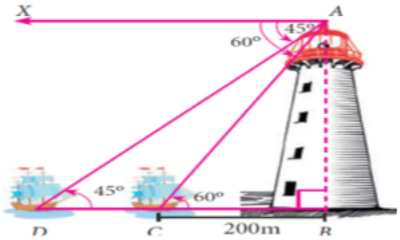
$\Rightarrow$ k = 2

Thus, O divides EG in the ratio 2 : 1

Hence, required ratio = OG : OE i.e., 1 : 2.

38. **Read the text carefully and answer the questions:**

A man is watching a boat speeding away from the top of a tower. The boat makes an angle of depression of 60^o with the man's eye when at a distance of 200 m from the tower. After 10 seconds, the angle of depression becomes 45^o.



(i) In $\triangle$ ABC

$\tan 60^o = \frac{AB}{BC}$

$\sqrt{3} = \frac{AB}{200}$

AB = $200\sqrt{3}$

Now, In $\triangle$ ABD

$\tan 45^o = \frac{AB}{BD}$

$1 = \frac{200\sqrt{3}}{BD}$

BD = $200\sqrt{3}$

$\therefore$ CD = BD - BC

= $200\sqrt{3}$ - 200

= 200 ($\sqrt{3}$ - 1)

= 200 $\times$ (1.732 - 1)

= 200 $\times$ 0.732

= 146.4 m

speed = $\frac{distance}{time}$

= $\frac{146.4}{10}$

= 14.64 m/s

Now,

speed = 14.64 $\times$ $\frac{18}{5}$ km/hr

= 52.7

$\approx$ 53 km/hr

(ii) In $\triangle$ ABD

$\tan 45^o = \frac{AB}{BD}$

$1 = \frac{200\sqrt{3}}{BD}$

BD = $200\sqrt{3}$ m

$\therefore$ CD = $200\sqrt{3}$ - 200

= 200 ($\sqrt{3}$ - 1)

= 200 (1.732 - 1)

= 200 $\times$ 0.732

= 146.4

$\approx$ 147 m

$\therefore$ boat is at a distance of 147 m from its actual position.

OR

In $\triangle$ ABC

tan 60^o = $\frac{AB}{BC}$

$\sqrt{3}$ = $\frac{AB}{200}$

AB = $200\sqrt{3}$ m

Hence, height of tower = $200\sqrt{3}$ m

(iii)As boat moves away from the tower angle of depression decreases.