

Reason (R): Since the third law is true at every instant, the total impulse on the first object is equal and opposite to that on the second.

- a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false. d) A is false but R is true.

14. **Assertion:** Change in internal energy is zero if the temperature is constant, irrespective of the process being cyclic or non-cyclic. [1]

Reason: For all process change in internal energy $\Delta U = nC_v\Delta T$ is independent of path.

- a) Assertion and reason both are correct statements and reason is correct explanation for assertion. b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
c) Assertion is correct statement but reason is wrong statement. d) Assertion is wrong statement but reason is correct statement.

15. **Assertion:** Even when orbit of a satellite is elliptical, its plane of rotation passes through the centre of earth. [1]

Reason: According to law of conservation of angular momentum plane of rotation of satellite always remain same.

- a) Assertion and reason both are correct statements and reason is correct explanation for assertion. b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
c) Assertion is correct statement but reason is wrong statement. d) Assertion is wrong statement but reason is correct statement.

16. **Assertion (A):** If $\vec{P} \cdot \vec{Q} = |\vec{P} \times \vec{Q}|$, then angle between \vec{P} and \vec{Q} is $\frac{\pi}{2}$. [1]

Reason (R): If angle between \vec{P} and \vec{Q} is $\frac{\pi}{2}$, then dot product is zero.

- a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false. d) A is false but R is true.

Section B

17. A brass rod 1 metre long is firmly clamped in the middle and one end is stroked by a resined cloth. What is the pitch of the note you will hear? Young's modulus for brass = 10^{12} dyn cm^{-2} and density = 9 g cm^{-3} . [2]
18. Find the value of 60 J per min on a system that has 100 g, 100 cm and 1 min as the base units. [2]
19. Calculate the dimensions of force and impulse taking velocity, density and frequency as basic quantities. [2]
20. A uniform rope of length L, resting on a frictionless horizontal surface is pulled at one end by a force F. What is the tension in the rope at a distance l from the end where the force is applied? [2]
21. A man can jump 1.5 m high on earth. Calculate the height he may be able to jump on a planet whose density is one fourth that of the earth and Whose radius is one-third of the earth. [2]

OR

In an imaginary planetary system, the central star has the same mass as our sun, but is brighter so that only a planet twice the distance between the earth and the sun can support life. Assuming biological evolution (including aging

process etc.) on that planet similar to ours, what would be the average life span of a **human** on that planet in terms of its natural year? The average life span of a human on the earth may be taken to be 70 years.

Section C

22. A U-tube is made up of capillaries of bore 1 mm and 2 mm respectively. The tube is held vertically and partially filled with a liquid of surface tension 49 dyne cm^{-1} and zero contact angle. Calculate the density of the liquid, if the difference in the levels of the meniscus is 1.25 cm. Take $g = 980 \text{ cms}^{-2}$. [3]
23. A thermometer has the wrong calibration. It reads the melting point of ice -10°C . It reads 60°C in place of 50° . Calculate the temperature of the boiling point of water on this scale. [3]
24. In a car race, car A takes time t less than car B and passes the finishing point with a velocity v more than the velocity with which car B passes the point. Assuming that the cars start from rest and travel with constant accelerations a_1 and a_2 , show that $v = t\sqrt{a_1 a_2}$. [3]
25. A truck starts from rest and accelerates uniformly at 2.0 ms^{-2} . At $t = 10 \text{ s}$, a stone is dropped by a person standing on the top of the truck (6 m high from the ground). What are the
i. velocity, and
ii. acceleration of the stone at $t = 11\text{s}$? (Neglect air resistance.) [3]
26. A sample of gas ($\gamma = 1.5$) is compressed adiabatically from a volume of 1600 cm^3 to 400 cm^3 . If the initial pressure is 150 kPa, what is the final pressure, and how much work is done on the gas in the process? [3]
27. State three basic laws of motion. Show that the first law of motion gives the definition of force and the second law of motion gives the measure of force. [3]
28. Two soap bubbles have radii in the ratio 2 : 3. Compare the excess of pressure inside these bubbles. Also compare the works done in blowing these bubbles. [3]

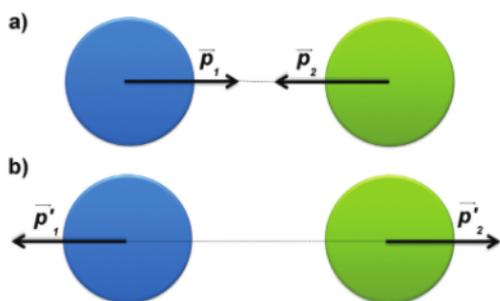
OR

Glycerine flows steadily through a horizontal tube of length 1.5 m and radius 1.0 cm. If the amount of glycerine collected per second at one end is $4.0 \times 10^{-3} \text{ kgs}^{-1}$, what is the pressure difference between the two ends of the tube? (Density of glycerine = $1.3 \times 10^3 \text{ kgm}^{-3}$ and viscosity of glycerine = 0.83 Pa s). [You may also like to check if the assumption of laminar flow in the tube is correct].

Section D

29. **Read the text carefully and answer the questions:** [4]

The kinetic energy of an object is the energy associated with the object which is under motion. It is defined as "the energy required by a body to accelerate from rest to stated velocity." It is a vector quantity and the momentum of an object is the virtue of its mass. It is defined as the product of mass and velocity. It is a vector quantity. The relation between them is given by $E = \frac{P^2}{2m}$. In case of the elastic collision both of these quantities remain constant.



- (i) Two masses of 1 gm and 4gm are moving with equal linear momentum. The ratio of their kinetic energy is:

c) kT

d) k/T

(iii) The mean free path is the:

a) length of the container that contains the gas

b) mean of the square of the average distance between two successive collisions

c) the average distance covered by a molecule between two successive collisions

d) none of these

(iv) The law of equipartition of energy is applicable to the system whose constituents are:

a) none of these

b) in random motion

c) in orderly motion

d) in rest

OR

Thermochemical calorie is equal to

a) 41.48 joule

b) 4.148 joule

c) 4148 joule

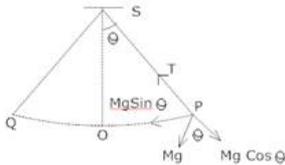
d) 414.8 joule

Section E

31. One end of a V-tube containing mercury is connected to a suction pump and the other end to atmosphere. The two arms of the tube are inclined to horizontal at an angle of 45° each. A small pressure difference is created between two columns when the suction pump is removed. Will the column of mercury in V-tube execute simple harmonic motion? Neglect capillary and viscous forces. Find the time period of oscillation. [5]

OR

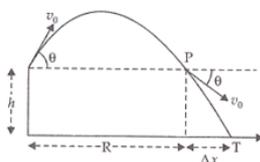
What is Simple pendulum? Find an expression for the time period and frequency of a simple pendulum?



32. i. What is projectile motion? [5]
ii. The maximum range of projectile is $\frac{2}{\sqrt{3}}$ times actual range. What is the angle of projection for the actual range?
iii. Two balls are thrown with the same initial velocity at angles α and $(90^\circ - \alpha)$ with the horizontal. What will be the ratio of the maximum heights attained by them? When will this ratio be equal to 1?

OR

A gun can fire shells with maximum speed v_0 and the maximum horizontal range that can be achieved is $R = \frac{v_0^2}{g}$. If a target farther away by distance Δx (beyond R) has to be hit with the same gun as shown in the figure here, show that it could be achieved by raising the gun to a height at least $h = \Delta x \left[1 + \frac{\Delta x}{R} \right]$



[Hint: This problem can be approached in two different ways:

- i. Refer to the diagram: target T is at the horizontal distance $x = R + \Delta x$ and below the point of projection $y = -h$.
- ii. From point P in the diagram: Projection at speed v_0 at an angle θ below horizontal with height h and horizontal range Δx .]

33. Prove the result that the velocity v of translation of a rolling body (like a ring, disc, cylinder or sphere) at the bottom of an inclined plane of a height h is given by $v^2 = \frac{2gh}{(1+k^2/R^2)}$. [5]

Using dynamical consideration (i.e. by consideration of forces and torques). Note k is the radius of gyration of the body about its symmetry axis, and R is the radius of the body. The body starts from rest at the top of the plane.

OR

Derive an expression for the moment uniform solid sphere about its any diameter, write the expression for its moment of its tangent.

Solution

Section A

1. (a) 4

Explanation: There are three rules on determining how many significant figures are in a number:

- Non-zero digits are always significant.
- Any zeros between two significant digits are significant.
- A final zero or trailing zeros in the decimal portion ONLY are significant.

So keeping these rules in mind, there are 4 significant digits.

2.

(d) 4

Explanation: Beat frequency = $f_2 - f_1 = \frac{\omega_2 - \omega_1}{2\pi}$
 $= \frac{2008\pi - 2000\pi}{2\pi} = 4 \text{ Hz}$

3.

(d) zero

Explanation: As there is no external force and the two bodies move due to mutual force of attraction, so $v_{CM} = 0$.

4.

(c) Poiseuille's equation

Explanation: Poiseuille's formula gives the volume of a liquid flowing out per second through a horizontal capillary tube of length l , radius r , under a pressure difference p applied across its ends.

$$Q = \frac{V}{t} = \frac{\pi p r^4}{8\eta l}$$

5.

(c) $\frac{1}{2} GmM \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$

Explanation: Required kinetic energy

= Final T.E. - Initial T.E.

$$= -\frac{GMm}{2R_2} - \left(-\frac{GMm}{2R_1} \right)$$

$$= \frac{1}{2} GmM \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

6.

(b) ultrasound

Explanation: SONAR emits ultrasound.

7.

(b) motion along z-axis

Explanation: Given, at $t=0$ s, position of an object is $(-1, 0, 3)$ and at $t=5$ s, its coordinate is $(-1, 0, 4)$. So, there is no change in x and y -coordinates, while z -coordinate changes from 3 to 4. So, the object is in motion along z -axis.

8.

(d) 120 cm

Explanation: The fundamental frequency of closed pipe = Second overtone of an open pipe

$$\Rightarrow \frac{v}{4L} = 3 \cdot \frac{v}{2L'}$$

$$\Rightarrow L' = 6L = 6 \times 20 \text{ cm} = 120 \text{ cm}$$

9. (a) all of these

Explanation: all of these

10.

(c) $GMK = 4\pi^2$

Explanation: $\frac{mv^2}{r} = \frac{GMm}{r^2}$

$$\Rightarrow v^2 = \frac{GM}{r}$$

$$\Rightarrow \left(\frac{2\pi}{T}\right)^2 = \frac{GM}{r}$$

$$\Rightarrow T^2 = \frac{4\pi^2}{GM} r^3 = Kr^3$$

$$\therefore K = \frac{4\pi^2}{GM}$$

$$= GMK = 4\pi^2$$

11. (d) $\frac{4\pi MR^2}{5T}$
Explanation: $L = I\omega = \frac{2}{5}MR^2 \cdot \frac{2\pi}{T} = \frac{4\pi MR^2}{5T}$
12. (c) 167 kJ/kg
Explanation: Heat given out by freezing liquid
 = Heat used in melting of ice
 $6 \times L = 3 \times 3.34 \times 10^2$
 $L = 1.67 \times 10^2 \text{ kJ/kg} = 167 \text{ kJ/kg}$
13. (a) Both A and R are true and R is the correct explanation of A.
Explanation: Mutual impulsive forces act at the time of the collision. So, it does not depend on the nature of the force. In all collisions, total linear momentum is conserved since the force exerted on the first particle is equal and opposite to the force exerted on the second particle (from Newton's third law of motion). So, assertion and reason both are true and the reason explains the assertion.
14. (c) Assertion is correct statement but reason is wrong statement.
Explanation: Assertion is correct statement but reason is wrong statement.
15. (a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
Explanation: As no torque is acting on the planet its angular momentum must stay constant in magnitude as well as direction. Therefore, plane of rotation must pass through the centre of the earth.
16. (d) A is false but R is true.
Explanation: A is false but R is true.
 If $\vec{P} \cdot \vec{Q} = |\vec{P} \times \vec{Q}|$
 or $PQ \cos \theta = PQ \sin \theta$
 or $\tan \theta = 1$ or $\theta = \frac{\pi}{4}$
 So, $\vec{P} \cdot \vec{Q} = |\vec{P} \times \vec{Q}|$, only when angle between \vec{P} and \vec{Q} is 45° .
 Also, $\vec{P} \cdot \vec{Q} = PQ \cos \frac{\pi}{2} = 0$

Section B

17. Here $Y = 10^{12} \text{ dyn cm}^{-2}$, $\rho = 9 \text{ g cm}^{-3}$
 Speed of sound in the brass rod
 $v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{10^{12}}{9}} = \frac{10^6}{3} \text{ cm s}^{-1}$
 Length of rod, $L = 1 \text{ m} = 200 \text{ cm}$
 Fundamental note, $\nu = \frac{v}{\lambda} = \frac{v}{2L} = \frac{10^6}{3 \times 200} = 1666.67 \text{ Hz}$
18. We are given that, $P = \frac{60 \text{ joule}}{1 \text{ min}} = \frac{60 \text{ joule}}{60 \text{ s}} = 1 \text{ watt}$
 which is the SI unit of power.

Also the dimensional formula of power = $[ML^2T^{-3}]$

SI	New System
$n_1 = 1$	$n_2 = ?$
$M_1 = 1 \text{ kg} = 1000 \text{ g}$	$M_2 = 10 \text{ g}$
$L_1 = 1 \text{ m} = 100 \text{ cm}$	$L_2 = 100 \text{ cm}$

$$T_1 = 1 \text{ s}$$

$$T_2 = 1 \text{ min} = 60 \text{ s}$$

Using the formula $n_2 = n_1 \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c$, we have

$$n_2 = 1 \left[\frac{1000}{100} \right]^{-1} \left[\frac{100}{100} \right]^{-2} \left[\frac{1}{60} \right]^{-3}$$

$$= 2.16 \times 10^6$$

Therefore, $60 \text{ J min}^{-1} = 2.16 \times 10^6$

That is the value of 60 J per minute in new units of power.

19. $v = \text{LT}^{-1}$, $\rho = \text{ML}^{-3}$, $\nu = \text{T}^{-1}$

Solving for M, L and T in terms of v , ρ and ν , we get

$$T = v^{-1}, L = v\nu^{-1} l, M = \rho v^3 \nu^{-3}$$

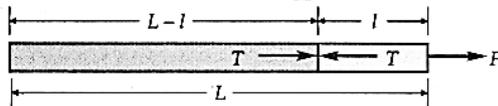
$$[\text{Force}] = \text{MLT}^{-2} = \rho v^3 \nu^{-3} v \nu^{-1} v^2 = \rho v^4 \nu^{-2}$$

$$[\text{Impulse}] = \text{Force} \times \text{time} = \rho v^4 \nu^{-2} v^{-1} = \rho v^4 \nu^{-3}$$

20. Let M be the mass of uniform rope of length L. Then

$$\text{Mass per unit length of rope} = \frac{M}{L}$$

$$\text{Acceleration in the rope} = \frac{F}{M}$$



Let T be the tension in the rope at a distance l from the end where the force F is applied.

Mass of length (L - l) of the rope is

$$M' = \frac{M}{L}(L - l)$$

As tension T is the only force on the length (L - l) of the rope, so

$$T = M' \times \frac{F}{M} = \frac{M}{L}(L - l) \times \frac{F}{M} = \left(1 - \frac{l}{L}\right) F$$

21. Radius of earth = R

density of earth = ρ

$$\text{acceleration due to gravity} = g = \frac{GM}{R^2}$$

$$M = \frac{4}{3} \pi R^3 \rho$$

$$g = \frac{4}{3} \pi G R \rho$$

acceleration due to gravity on the planet

$$g' = \frac{4}{3} \pi G R' \rho'$$

The gain in PE at the highest point will be same in both cases. Hence

$$m g' h' = m g h$$

$$h' = \frac{m g h}{m g'} = \frac{m \times \frac{4}{3} \pi G R \rho h}{m \frac{4}{3} \pi G R' \rho'}$$

$$= \frac{R \rho h}{R' \rho'} = \frac{3R' \times 4\rho' \times 1.5}{R \times \rho'}$$

$$= 18 \text{ m}$$

OR

According to Kepler's law of periods,

$$\left(\frac{T_1}{T_2} \right)^2 = \left(\frac{R_1}{R_2} \right)^3$$

Here,

T_1 = Average life span of a human on the earth = 70 years.

T_2 = Average life span of a human on the planet = ?

R_1 = Distance between the earth and the planet = $2 R_2$

R_2 = Distance between the earth and the sun.

$$\therefore \left(\frac{70}{T_2} \right)^2 = \left(\frac{2R_2}{R_2} \right)^3$$

$$\text{or } \frac{70 \times 70}{T_2^2} = 8$$

$$\text{or } T_2^2 = \frac{70 \times 70}{8}$$

$$\therefore T_2 = \frac{70}{\sqrt{8}} = 25 \text{ planet years}$$

Section C

22. Here $h_1 = \frac{2\sigma \cos \theta}{r_1 \rho g}$ and $h_2 = \frac{2\sigma \cos \theta}{r_2 \rho g}$

$$\therefore h_1 - h_2 = \frac{2\sigma \cos \theta}{\rho g} \left[\frac{1}{r_1} - \frac{1}{r_2} \right]$$

$$\rho = \frac{2\sigma \cos \theta}{(h_1 - h_2)g} \left[\frac{1}{r_1} - \frac{1}{r_2} \right]$$

Now $r_1 = \frac{1}{2} \text{ mm} = 0.05 \text{ cm}$,

$r_2 = \frac{2}{2} \text{ mm} = 0.1 \text{ cm}$,

$\sigma = 49 \text{ dyne cm}^{-1}$,

$h_1 - h_2 = 1.125 \text{ cm}$, $\theta = 0^\circ$, $g = 980 \text{ cm s}^{-2}$

$$\therefore \rho = \frac{2 \times 49 \times \cos 0^\circ}{1.125 \times 980} \left[\frac{1}{0.05} - \frac{1}{0.1} \right]$$

$$= \frac{2 \times 49 \times 1}{1.125 \times 980} \times 10 = 0.8 \text{ g cm}^{-3}$$

23. Let

θ_1 = Lower fixed point on faulty thermometer

θ_2 = Reading on faulty thermometer

n = number of divisions between upper and lower fixed points

Now

$$\frac{C}{100} = \frac{\theta_2 - \theta_1}{n} \dots (i)$$

In first case,

$\theta_2 = -10^\circ \text{ C}$, $C = 0^\circ \text{ C}$

$$\therefore 0 = \frac{-10 - \theta_1}{n} \text{ or } \theta_1 = -10^\circ \text{ C}$$

In the second case,

$\theta_2 = 60^\circ \text{ C}$, $C = 50^\circ \text{ C}$

$$\therefore \frac{50}{100} = \frac{60 - \theta_1}{n} \text{ or } \frac{1}{2} = \frac{60 - (-10)}{n} = \frac{70}{n}$$

or $n = 140$

As boiling point of water on Celsius scale is 100° C so putting $C = 100$ in equation (i), we get

$$\frac{100}{100} = \frac{\theta_2 - (-10)}{140} \text{ or } \theta_2 = 130^\circ \text{ C}$$

\therefore The boiling point of water on a faulty thermometer

= 130° C

24. Let s be the distance covered by each car.

Let the times taken by the two cars to complete the journey be t_1 and t_2 , and their velocities at the finishing point be v_1 and v_2 respectively. According to the problem,

$$v_1 - v_2 = v \text{ and } t_2 - t_1 = t$$

When $u = 0$, $s = \frac{0 + v}{2} \times t = \frac{v}{2} \cdot t$

$$\therefore s = \frac{v_1 t_1}{2} = \frac{v_2 t_2}{2}$$

Also, $s = \frac{1}{2} a_1 t_1^2 = \frac{1}{2} a_2 t_2^2$

Hence $\frac{v}{t} = \frac{v_1 - v_2}{t_2 - t_1} = \frac{t_1 - t_2}{t_2 - t_1} = \frac{2s(t_2 - t_1)}{t_1 t_2 (t_2 - t_1)} = \frac{2s}{t_1 t_2}$

$$= \sqrt{\frac{4s^2}{t_1^2 t_2^2}} = \sqrt{\frac{2s}{t_1^2} \cdot \frac{2s}{t_2^2}} = \sqrt{a_1 a_2}$$

or $v = t \sqrt{a_1 a_2}$

25. Given, the truck starts from rest

therefore, the initial velocity of the truck, $u = 0$

Acceleration of the truck, ' a ' = 2 m/s^2

Time, $t = 10 \text{ s}$

Now from the first equation of motion,

the final velocity, ' v ' is

$$v = u + at$$

$$\Rightarrow v = 0 + 2 \text{ m/s}^2 \times 10 \text{ s} = 20 \text{ m/s}$$

at $t = 10\text{s}$

The velocity is 20 m/s

i. At $t = 11\text{ s}$,

The horizontal component of the velocity remains the same, in the absence of air resistance,

Thus, $v_x = 20\text{ m/s}$

According to the first equation of motion, The vertical component of velocity of the stone is given by,

$$v_y = u + a_y \delta t$$

where,

$$\delta t = 11\text{ s} - 10\text{ s} = 1\text{ s} \text{ and}$$

since the direction is vertical the acceleration acting on it is due to the gravity.

$$\text{Thus } a_y = g = 10\text{ m/s}^2$$

$$\Rightarrow v_y = 0 + 10\text{ m/s}^2 \times 1\text{ s} = 10\text{ m/s}$$

The final resultant velocity of the stone is given as,

$$v_{\text{res}} = (v_x^2 + v_y^2)^{1/2}$$

$$\Rightarrow v_{\text{res}} = (20^2 + 10^2)^{1/2} = \sqrt{500}\text{ m/s}$$

$$\Rightarrow v_{\text{res}} = 22.36\text{ m/s}$$

Let us suppose that the angle made by the resultant velocity with the horizontal velocity, v_x is θ ,

Thus,

$$\tan \theta = \left(\frac{v_y}{v_x} \right)$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{10}{20} \right)$$

$$\Rightarrow \theta = \tan^{-1}(0.5) = 26.57^\circ$$

The velocity of the stone at $t = 11\text{ s}$ is 22.36 m/s and is at angle 26.57° with the horizontal.

ii. When the stone is dropped from the truck, the horizontal force provided by the truck acting on the stone becomes zero. The only force and thus, the acceleration, that remains is that in the vertical direction i.e. acceleration due to gravity.

Therefore, the acceleration of the stone is 10 m/s^2 and it is in the downward direction.

26. Here $\gamma = 1.5$, $V_1 = 1600\text{ cm}^3$, $V_2 = 400\text{ cm}^3$, $P_1 = 150\text{ kPa}$, $P_2 = ?$

For an adiabatic process,

$$P_2 V_2^\gamma = P_1 V_1^\gamma$$

$$\therefore P_2 = P_1 \left(\frac{V_1}{V_2} \right)^\gamma = 150 \left(\frac{1600}{400} \right)^{1.5} = 1200\text{ kPa}$$

Work done in the adiabatic compression,

$$\begin{aligned} W_{\text{adi}} &= \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} \\ &= \frac{150 \times 10^3 \times 1600 \times 10^{-6} - 1200 \times 10^3 \times 400 \times 10^{-6}}{1.5 - 1} \\ &= \frac{240 - 480}{0.5} = -480\text{ J} \end{aligned}$$

27. **Newton's First Law of Motion** also known as **Law of Inertia** states that every object persists to stay in uniform motion in a straight line or in the state of rest unless an external force acts upon it. In a simpler form, the first law of motion may also be stated as "If the net external force on a body is zero, its acceleration is zero. Acceleration can be non-zero only if there is a net external force on the body".

Newton's Second Law of Motion states that force is equal to the change in momentum per change in time. For a constant mass, force equals mass times acceleration, i.e.

$$F = ma$$

Thus, $\vec{F} \propto \frac{\Delta \vec{p}}{\Delta t}$ or $\vec{F} = k \frac{\Delta \vec{p}}{\Delta t}$, where k is a constant of proportionality, Δp is the change in momentum and $p = mv$.

Newton's Third Law of Motion: It states that "For every action, there is an equal and opposite reaction".

According to the first law of motion, in the absence of an external force, a body will maintain its position of rest or state of uniform motion along a straight line. Thus, to change the position of rest or uniform motion of a body, we shall have to apply an external force. If the external force is large enough, it may change the state of rest or of uniform motion. However, if the magnitude of the force is small then it may not be able to change that state. Hence, "force is that external cause (push or pull) which changes or tries to change the state of rest or of uniform motion along a straight line of a given body".

Also, we know that,

$$F = ma$$

where F is the vector sum of all forces acting on the body, m is the mass of body and equation can be regarded as a statement of Newton's 2nd law of motion.

This relation can be used to have the measure of a force.

28. If R_1 and R_2 are the radii of the two bubbles, then $\frac{R_1}{R_2} = \frac{2}{3}$

Let σ be the surface tension of the soap solution.

Excess pressure inside the bubble of radius R_1 ,

$$p_1 = \frac{4\sigma}{R_1}$$

Excess pressure inside the bubble of radius R_2 ,

$$p_2 = \frac{4\sigma}{R_2}$$

$$\therefore \frac{p_1}{p_2} = \frac{4\sigma}{R_1} \times \frac{R_2}{4\sigma} = \frac{R_2}{R_1} = \frac{3}{2} = 3 : 2$$

Work done in blowing up the two soap bubbles is

$$W_1 = 2 \times 4\pi R_1^2 \times \sigma$$

$$\text{and } W_2 = 2 \times 4\pi R_2^2 \times \sigma$$

$$\therefore \frac{W_1}{W_2} = \frac{R_1^2}{R_2^2} = \left(\frac{2}{3}\right)^2 = 4 : 9$$

OR

Length of the horizontal tube is given by, $l = 1.5$ m

Radius of the tube is, $r = 1$ cm = 0.01 m

Diameter of the tube is given by, $d = 2r = 0.02$ m

Glycerine is flowing at a rate of $4.0 \times 10^{-3} \text{ kgs}^{-1}$.

$$M = 4.0 \times 10^{-3} \text{ kgs}^{-1}$$

Density of Glycerine is given by, $\rho = 1.3 \times 10^3 \text{ kgm}^{-3}$

Viscosity of Glycerine is given by, $\eta = 0.83$ Pa s

Volume of Glycerine flowing per sec is given by :

$$\begin{aligned} V &= \frac{M}{\rho} \\ &= \frac{4.0 \times 10^{-3}}{1.3 \times 10^3} \\ &= 3.08 \times 10^{-6} \text{ m}^3 \text{ s}^{-1} \end{aligned}$$

According to Poiseville's formula, we have the relation for the rate of flow:

$$V = \frac{\pi p r^4}{8\eta l}$$

Where, p is the pressure difference between the two ends of the tube

$$\begin{aligned} \therefore p &= \frac{V 8\eta l}{\pi r^4} \\ &= \frac{3.08 \times 10^{-6} \times 8 \times 0.83 \times 1.5}{\pi \times (0.01)^4} \\ &= 9.8 \times 10^2 \text{ Pa} \end{aligned}$$

Reynolds' number is given by the relation:

$$\begin{aligned} R &= \frac{4\rho V}{\pi d \eta} \\ &= \frac{4 \times 1.3 \times 10^3 \times 3.08 \times 10^{-6}}{\pi \times (0.02) \times 0.83} = 0.3 \end{aligned}$$

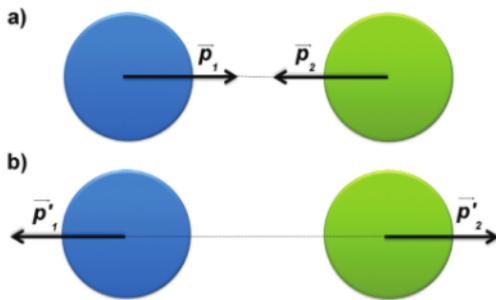
Reynolds' number is about 0.3. Hence, the flow is laminar.

Section D

29. Read the text carefully and answer the questions:

The kinetic energy of an object is the energy associated with the object which is under motion. It is defined as "the energy required by a body to accelerate from rest to stated velocity." It is a vector quantity and the momentum of an object is the virtue of its mass. It is defined as the product of mass and velocity. It is a vector quantity. The relation between them is given by $E = \frac{P^2}{2m}$. In

case of the elastic collision both of these quantities remain constant.



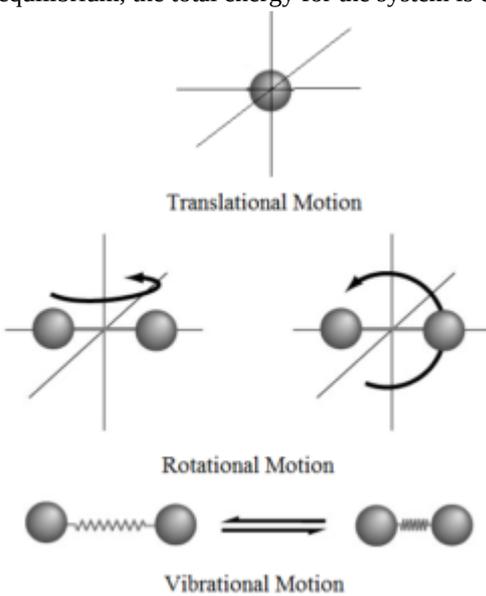
- (i) **(b)** 4:1
Explanation: 4:1
- (ii) **(c)** 125%
Explanation: 125%
- (iii) **(d)** light object
Explanation: light object

OR

- (b)** no work is done on it
Explanation: no work is done on it
- (iv) **(d)** positive
Explanation: positive

30. Read the text carefully and answer the questions:

The number of independent ways by which a dynamic system can move, without violating any constraint imposed on it, is called the number of **degrees of freedom**. According to the law of equipartition of energy, for any dynamic system in thermal equilibrium, the total energy for the system is equally divided among the degree of freedom.



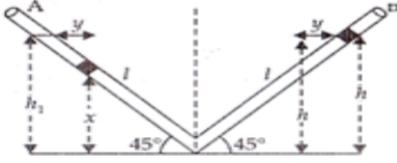
- (i) **(c)** $1 + 2/n$
Explanation: $1 + 2/n$
- (ii) **(c)** kT
Explanation: kT
- (iii) **(c)** the average distance covered by a molecule between two successive collisions
Explanation: the average distance covered by a molecule between two successive collisions
- (iv) **(b)** in random motion
Explanation: in random motion

OR

- (b)** 4.148 joule
Explanation: 4.148 joule

Section E

31. Let the liquid column in both arms of the V-tube were at h_0 heights initially. Now due to pressure difference the liquid columns in A arm is pressed by x and in arm B is lifted by x (so difference in vertical height between two levels = $2x$) Consider an element of liquid of height dx inside the tube.



Then its $dm = \text{volume} \times \text{density} = A \cdot dx \rho$ (where, A = area of cross-section of tube, ρ = density of the liquid inside the tube)

Potential energy of the right arm with dm elementary mass column = $(dm) gh$

Potential energy of dm elementary mass in left arm column = $A\rho g x dx$ (putting the value of $dm = A \cdot dx \cdot \rho$ and $h = x$)

$$\therefore \text{Total potential energy in left column} = \int_0^{h_1} A\rho g x dx = A\rho g \left[\frac{x^2}{2} \right]_0^{h_1}$$

$$= A\rho g \frac{h_1^2}{2}$$

$$\text{From above given figure } \sin 45^\circ = \frac{h_1}{l} \therefore h_1 = h_2 = l \sin 45^\circ = \frac{l}{\sqrt{2}}$$

$$\therefore h_1^2 = h_2^2 = \frac{l^2}{2}$$

$$\therefore \text{Potential energy in the left column} = A\rho g \frac{l^2}{4}$$

$$\text{Similarly potential energy in right column} = A\rho g \frac{l^2}{4}$$

$$\therefore \text{Total potential energy} = A\rho g \frac{l^2}{4} + A\rho g \frac{l^2}{4} = \frac{A\rho g l^2}{2}$$

Due to pressure difference, left element moves towards right side by ' y ' units and the same element rises in the right arm by ' y ' units.

Then the liquid column length in the left arm becomes by decreasing = $(l - y)$

And the liquid column length in the right arm becomes by increasing = $(l + y)$

Now decreased potential energy of liquid column in the left arm = $A\rho g (l - y)^2 \sin^2 45^\circ$

Similarly increased potential energy of liquid column in the right arm = $A\rho g (l + y)^2 \sin^2 45^\circ$

$$\therefore \text{Total potential energy due to two liquid columns in the left and right arm respectively} = A\rho g \left(\frac{1}{\sqrt{2}} \right)^2 [(l - y)^2 + (l + y)^2]$$

Final potential energy due to difference in liquid columns in the two arms,

$$= \frac{A\rho g}{2} [l^2 + y^2 - 2ly + l^2 + y^2 + 2ly]$$

$$\therefore \text{Final potential energy} = \frac{A\rho g}{2} (2l^2 + 2y^2)$$

Now change in potential energy = Final potential energy due to liquid columns in the two arms - Initial potential energy due to liquid columns in the two arms

$$= \frac{A\rho g}{2} (2l^2 + 2y^2) - \frac{A\rho g l^2}{2}$$

$$= \frac{A\rho g}{2} [2l^2 + 2y^2 - l^2]$$

$$\therefore \text{Change in potential energy} = \frac{A\rho g}{2} (l^2 + 2y^2)$$

If change in velocity (v) of total liquid column be v then change in kinetic energy,

$$\Delta KE = \frac{1}{2} m v^2$$

Again $m = \text{volume} \times \text{density} = (A \cdot 2l) \rho$

$$\therefore \Delta KE = \frac{1}{2} (A \cdot 2l \rho) v^2 = A\rho l v^2$$

$$\therefore \text{Change in Total energy} = \text{change in potential energy} + \text{change in kinetic energy} = \frac{A\rho g}{2} (l^2 + 2y^2) + A\rho l v^2$$

Again, from the law of conservation of energy, total change in energy $\Delta PE + \Delta KE = 0$

$$\therefore \frac{A\rho g}{2} [l^2 + 2y^2] + A\rho l v^2 = 0$$

$$\therefore \frac{A\rho}{2} [g(l^2 + 2y^2) + 2lv^2] = 0$$

$$\therefore \frac{A\rho}{2} \neq 0$$

$$\therefore g(l^2 + 2y^2) + 2lv^2 = 0$$

Differentiating on both sides of the above equation with respect to time, t we get $g \left[0 + 2 \times 2y \frac{dy}{dt} \right] + 2l \cdot 2v \cdot \frac{dv}{dt} = 0$

$$\therefore 4gy \frac{dy}{dt} + 4vl \frac{d^2 y}{dt^2} = 0 \left[\because a = \frac{dv}{dt} = \frac{d^2 y}{dt^2} \right]$$

$$\Rightarrow 4gy \cdot v + 4vl \frac{d^2 y}{dt^2} = 0 \Rightarrow 4v \left[gy + l \cdot \frac{d^2 y}{dt^2} \right] = 0$$

$$\Rightarrow \frac{d^2y}{dt^2} + \frac{g}{l}y = 0 \quad \because 4v \neq 0 \dots(i)$$

It is the equation of a simple harmonic motion and can be compared with the standard equation of a simple harmonic motion i.e.

$$\frac{d^2y}{dt^2} + \omega^2y = 0 \dots(ii) \quad [\omega \text{ is the angular acceleration or angular frequency of the particle executing simple harmonic motion}]$$

Comparing the above two equations (i) and (ii) we get, $\therefore \omega^2 = \frac{g}{l}$

$$\therefore \frac{2\pi}{T} = \sqrt{\frac{g}{l}} \Rightarrow T = 2\pi\sqrt{\frac{l}{g}} \quad [\because \omega = \frac{2\pi}{T}, T \text{ being time period of the simple harmonic motion}]$$

OR

A simple pendulum is the most common example of the body executing S.H.M, it consists of heavy point mass body suspended by a weightless inextensible and perfectly flexible string from rigid support, which is free to oscillate. When a pendulum is displaced sideways from its resting, equilibrium position, it is subject to a restoring force due to gravity that will accelerate it back toward the equilibrium position. When released, the restoring force acting on the pendulum's mass causes it to oscillate about the equilibrium position, swinging back and forth. The time for one complete cycle, a left swing and a right swing, is called the period.

Let m = mass of bob

l = length of a pendulum

Let O is the equilibrium position, $OP = X$

Let θ = small angle through which the bob is displaced.

The forces acting on the bob are:-

i. The weight = Mg acting vertically downwards.

ii. The tension = T in string acting along Ps .

Resolving Mg into 2 components as $Mg \cos \theta$ and $Mg \sin \theta$,

Now, $T = Mg \cos \theta$

Restoring force $F = - Mg \sin \theta$

-ve sign shows force is directed towards mean position.

Let θ = Small, so $\sin \theta \approx \theta = \frac{\text{Arc(op)}}{1} = \frac{x}{1}$

Hence $F = - mg \theta$

$$\Rightarrow F = - mg \frac{x}{l} \rightarrow 3)$$

Now, In S.H.M, $F = kx \rightarrow 4)$

where, k = Spring constant

Equating equation 3) & 4) for F

$$\Rightarrow -kx = -mg \frac{x}{l}$$

$$\Rightarrow \text{Spring factor} = k = \frac{mg}{l}$$

Inertia factor = Mass of bob = m

Now, Time period = T

$$= 2\pi \sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}}$$

$$\Rightarrow T = 2\pi \sqrt{\frac{l}{g}}$$

32. i. When a particle is thrown obliquely near the earth's surface, it moves along a curved path under constant acceleration that is directed towards the centre of the earth (we assume that the particle remains close to the surface of the earth). The path of such a particle is called a projectile and the motion is called projectile motion.

$$\text{ii. } R_{\max} = \frac{2}{\sqrt{3}}R \text{ or } \frac{u^2}{g} = \frac{2}{\sqrt{3}} \times \frac{u^2 \sin 2\theta}{g}$$

$$\text{or } \sin 2\theta = \frac{\sqrt{3}}{2} = \sin 60^\circ \therefore \theta = 30^\circ$$

$$\text{iii. } h_1 = \frac{v^2 \sin^2 \alpha}{2g}$$

$$h_2 = \frac{y^2 \sin 2(90^\circ - \alpha)}{2g} = \frac{y^2 \cos^2 \alpha}{2g}$$

$$\text{Therefore, } \frac{h_1}{h_2} = \frac{v^2 \sin^2 \alpha}{2g} \times \frac{2g}{v^2 \cos^2 \alpha}$$

$$= \frac{\tan^2 \alpha}{1}$$

$$\text{Ratio: } h_1:h_2 = \tan^2 \alpha:1$$

OR

Maximum range of projectile, $R = \frac{V_0^2}{g}$ (i)

We know, maximum range is achieved at $\theta = 45^\circ$

Let the gun is raised to a height h from the horizontal level of target T, so that the projectile can hit the target T.

Total range of projectile must be $R_{\text{total}} = R + \Delta x$

Horizontal component of velocity at A = $v_0 \cos(\theta)$

As A and P are on the same level, the magnitude of velocity will be the same at A and P.

But the direction of velocity will be below horizontal,

So horizontal velocity at P, $v_x = -v_0 \cos(\theta)$

and vertical velocity at P, $v_y = v_0 \sin(\theta)$

Now $h = ut + \frac{1}{2}at^2$

$$h = -v_0 \sin \theta(t) + \frac{1}{2}gt^2 \dots\dots(ii)$$

Consider horizontal motion from A to T, distance $(R + \Delta x) = v_0 \cos \theta \cdot t$

$$t = \frac{R + \Delta x}{v_0 \cos \theta}$$

Substitute t in (ii) we get

$$h = -v_0 \sin \theta \left[\frac{R + \Delta x}{v_0 \cos \theta} \right] + \frac{1}{2}g \frac{(R + \Delta x)^2}{v_0^2 \cos^2 \theta}$$

$$h = -\tan \theta (R + \Delta x) + \frac{1}{2} \left(\frac{g}{v_0^2} \right) \frac{(R + \Delta x)^2}{1/\cos^2 \theta} \quad (\because \theta = 45^\circ \Rightarrow \tan(45) = 1)$$

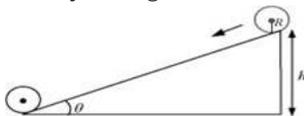
$$h = -(R + \Delta x) + \frac{1}{R} (R^2 + \Delta x^2 + 2R\Delta x) \left[\because \frac{g}{v_0^2} = \frac{1}{R} \right]$$

$$h = -R - \Delta x + R + \frac{\Delta x^2}{R} + 2\Delta x = \Delta x + \frac{\Delta x^2}{R}$$

$$h = \Delta x \left[1 + \frac{\Delta x}{R} \right]$$

Hence Proved.

33. A body rolling on an inclined plane of height h , is shown in the following figure:



m = Mass of the body

R = Radius of the body

K = Radius of gyration of the body

At highest point,

energy of body (E_i) = PE = mgh

At lowest point,

Energy of body (E_f) = linear kinetic energy + rotation kinetic energy

$$= \frac{1}{2} \times mv^2 + \frac{1}{2} \times I\omega^2$$

But $I = mk^2$ and $\omega = \frac{v}{R}$

$$\therefore E_f = \frac{1}{2} (mk^2) \left(\frac{v^2}{R^2} \right) + \frac{1}{2} mv^2$$

$$= \frac{1}{2} mv^2 \frac{k^2}{R^2} + \frac{1}{2} mv^2$$

$$= \frac{1}{2} mv^2 \left(1 + \frac{k^2}{R^2} \right)$$

From the law of conservation of energy, we have:

$$E_i = E_f$$

$$mgh = \frac{1}{2} mv^2 \left(1 + \frac{k^2}{R^2} \right)$$

$$\therefore v = \frac{2gh}{\left(1 + \frac{k^2}{R^2} \right)}$$

Hence, the given result is proved.

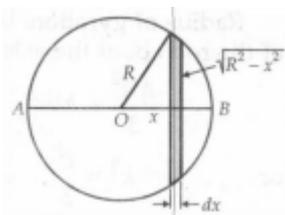
OR

Moment of inertia of a solid sphere about its diameter. Consider a uniform solid sphere of mass M and radius R . We wish to determine its moment of inertia about diameter AB.

$$\text{Volume of the sphere} = \frac{4}{3} \pi R^3$$

$$\text{Mass per unit volume, } \rho = \frac{3M}{4\pi R^3}$$

We can imagine the sphere to be made up of a large number of thin slices placed perpendicular to the diameter AB. Consider one such slice of thickness dx placed at distance x from the centre O.



$$\text{Radius of the elementary slice} = \sqrt{R^2 - x^2}$$

$$\text{Volume of the elementary slice} = \text{Area} \times \text{thickness}$$

$$= \pi \left(\sqrt{R^2 - x^2} \right)^2 \times dx = \pi (R^2 - x^2) dx$$

$$\text{Mass of the elementary slice}$$

$$= \text{Volume} \times \rho = \pi (R^2 - x^2) dx \times \frac{3M}{4\pi R^3}$$

$$= \frac{3M(R^2 - x^2)dx}{4R^3}$$

Moment of inertia of the thin slice about the axis AB passing through its centre and perpendicular to its plane,

$$dI = \frac{1}{2} \text{Mass} \times (\text{radius})^2$$

$$= \frac{1}{2} \cdot \frac{3M(R^2 - x^2)dx}{4R^3} \cdot (R^2 - x^2)$$

$$= \frac{3M(R^2 - x^2)^2 dx}{8R^3}$$