CBSE SAMPLE PAPER - 07

Class 11 - Mathematics

Time Allowed: 3 hours Maximum Marks: 80

General Instructions:

- 1. This Question paper contains five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
- 2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
- 3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
- 4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
- 5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
- 6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. The value of $\left(1+\cos\frac{\pi}{9}\right)\left(1+\cos\frac{3\pi}{9}\right)\left(1+\cos\frac{5\pi}{9}\right)\left(1+\cos\frac{7\pi}{9}\right)$ is:

a) $\frac{10}{16}$

b) $\frac{9}{16}$

c) $\frac{5}{16}$

d) $\frac{12}{16}$

2. The age distribution of 400 persons in a colony having median age 32 is given below:

 Age (in years):
 20-25
 25-30
 30-35
 35-40
 40-45
 45-50

 Frequency:
 110
 x
 75
 55
 y
 30

Then, x - y is

a) 20

b) -10

c) 10

d) -20

3. If A and B are two events such that $P(A \cup B) = P(A \cap B)$, then the true relation is:

[1]

[1]

a)
$$P(A) + P(B) = 0$$

b) P(A) + P(B) = P(A) P(B|A)

c) P(A) + P(B) = 2P(A) P(B|A)

d) None of these

4. $\lim_{x \to \frac{\pi}{4}} \frac{\sec^2 x - 2}{\tan x - 1}$ is equal to

[1]

[1]

a) 1

b) 0

c) 2

d) 3

5. A line L passes through the points (1, 1) and (2, 0) and another line M which is perpendicular to L passes through the point (1/2, 0). The area of the triangle formed by these lines with y axis is:

	a) 25/8	b) 25/16	
	c) none of these	d) 25/4	
6.	If a set A has n elements then the total number of sub	osets of A is	[1]
	a) 2n	b) n	
	c) 2 ⁿ	d) n^2	
7.	If z is any complex number such that $ z + 4 \le 3$, the	n the greatest value of $ z + 1 $ is	[1]
	a) 6	b) 3	
	c) 4	d) 5	
8.	Let R be the relation on N defined as by $x + 2y = 8$.	The domain of R is	[1]
	a) {2, 4, 6, 8}	b) {2, 4, 8}	
	c) {1, 2, 3, 4}	d) {2, 4, 6}	
9.	Solve the system of inequalities -2 $\leq 6x$ - $1 \leq 2$		[1]
	a) $-\frac{1}{6} \leq x < \frac{1}{2}$	b) $-\frac{1}{6} < x < \frac{3}{2}$	
	c) none of these	$\mathrm{d}) - \tfrac{1}{7} \leq x > \tfrac{1}{2}$	
10.	cos 75° = ?		[1]
	a) $\frac{(\sqrt{2}-1)}{2\sqrt{2}}$	b) $\frac{(\sqrt{2}+1)}{2\sqrt{2}}$	
	c) $\frac{(\sqrt{3}+1)}{2\sqrt{2}}$	d) $\frac{(\sqrt{3}-1)}{2\sqrt{2}}$	
11.	If ${}^{n}C_{12} = {}^{n}C_{8}$, then n is equal to	oos v ee	[1]
	a) 12	b) 6	
	c) 30	d) 20	
12.	Sum of n terms of the series $\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{3}$	$\overline{32}+$ is	[1]
	a) 2n (n + 1)	b) $\frac{n(n+1)}{\sqrt{2}}$	
	c) 1	d) $\frac{n(n+1)}{2}$	
13.	In Pascal's triangle, each row begins with 1 and ends	in	[1]
	a) -1	b) 0	
	c) 2	d) 1	
14.	If $ x-1 > 5$, then		[1]
	a) $x\in [6,\infty)$	b) $x\in(6,\infty)$	
	c) $x \in (-\infty, -4) \cup ^(6, \infty)$	d) $x\in (-\infty,-4)\cup^(6,\infty]$	
15.	A class has 175 students. The following data shows t	he number of students opting one or more subjects:	[1]
	Mathematics 100, Physics 70, Chemistry 40 Mathematics and Physics 30		
	Mathematics and Chemistry 28		
	Physics and Chemistry 23		

	How many students have opted for Mathematics alone?			
	a) 48	b) 60		
	c) 24	d) 100		
16.	Mark the Correct alternative in the following: If 2 tand	$\alpha = 3 \tan \beta$, then $\tan (\alpha - \beta) =$	[1]	
	a) $\frac{\sin 2\beta}{5-\cos 2\beta}$	b) None of these		
	c) $\frac{\cos 2\beta}{5-\cos 2\beta}$	d) $\frac{\sin 2\beta}{5+\cos 2\beta}$		
17.		= 9 and $ z_2 - 3 - 4i $ = 4. Then, the minimum value of $ z_1 - 3 $	[1]	
	z_2 is			
	a) 1	b) 0		
	c) $\sqrt{2}$	d) 2		
18.	The number of all selections which a student can mak	e for answering one or more questions out of 8 given	[1]	
	questions in a paper, when each question has an altern	ative, is:		
	a) 255	b) 6561		
	c) 6560	d) 256		
19.	Assertion (A): Number of terms in the expansion of	$\left(2x - \frac{4}{x^2}\right)^{10}$ is 11.	[1]	
	Reason (R): Number of terms in the expansion of, $(x + a)^n$ is $n + 1$.			
	a) Both A and R are true and R is the correct explanation of A.	b) Both A and R are true but R is not the correct explanation of A.		
	c) A is true but R is false.	d) A is false but R is true.		
20.	Assertion (A): The relation R on the set A = {1, 2, 3, (2, 3), (3, 1), (3, 2), (3, 3), (3, 4), (4, 3), (4, 4), (5, 5), (6, 4), (6, 5), (7, 1), (1, 1	4, 5, 6} is given by R = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (6, 6)}	[1]	
	Reason (R): The relation R is defined on $A = \{1, 2, 3\}$, 4, 5, 6} as $\{(a, b) : a^2 - b^2 < 9 \}$		
	a) Both A and R are true and R is the correct explanation of A.	b) Both A and R are true but R is not the correct explanation of A.		
	c) A is true but R is false.	d) A is false but R is true.		
	Sec	tion B		
21.	If $f(x) = \log_e(1 - x)$ and $f(x) = [x]$ then find: $(f + g)(x)$		[2]	
22.	Evaluate: $\lim_{x \to 27} \frac{\left(x^{\frac{1}{3}} + 3\right)\left(x^{\frac{1}{3}} - 3\right)}{x - 27}.$		[2]	
23.	Find the equation of the parabola that satisfies the give	en conditions: Vertex (0, 0) Focus (-2, 0)	[2]	
		OR		
24	Find the equations of the hyperbola satisfying the give	en conditions: vertices (0, \pm 5), foci (0, \pm 8).	[2]	
24. 25.	Prove that $A \cap (A \cup B)' = \phi$. Find the equation of a line with slope 2 and the length	of the perpendicular from the origin equal to $\sqrt{5}$	[2] [2]	
		tion C	[-]	
			[3]	

Mathematics, Physics and Chemistry 18

26. If
$$f(x) = \begin{cases} 1+x, & -1 \le x < 0 \\ x^2 - 1, & 0 < x < 2 \\ 2x, & 2 \le x \end{cases}$$

Then, find f(3), f(-2), f(0), $f(\frac{1}{2})$, f(2 - h) and f(-1 + h), where h is very small.

27. If the origin is the centroid of the triangle PQR with vertices P(2a, 2, 6), Q(-4, 3b, -10) and R(8, 14, 2c), then find the values of a, b and c. [3]

OR

The coordinates of a point are (3, -2, 5). Write down the coordinates of seven points such that the absolute values of their coordinates are the same as those of the coordinates of the given point.

28. Find
$$(a + b)^4$$
 - $(a - b)^4$. Hence, evaluate $(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4$ [3]

Which of the following is larger? $99^{50} + 100^{50}$ or 101^{50}

29. Evaluate:
$$\sqrt{-7 + 24i}$$
. [3]

OR

Evaluate $\left[rac{1}{1-4i}-rac{2}{1+i}
ight]\left[rac{3-4i}{5+i}
ight]$ to the standard form. Solve the linear inequality: $rac{2x+5}{x-1}>5$

30. Solve the linear inequality:
$$\frac{2x+5}{x-1} > 5$$

31. If
$${}^{22}P_{r+1}$$
: ${}^{20}P_{r+2} = 11:52$. find r. [3]

Section D

32. A bag contains 6 red, 4 white and 8 blue balls. If three balls are drawn at random, find the probability that: [5]

i. one is red and two are white

ii. two are blue and one is red

iii. one is red.

33. Differentiate
$$\frac{\sin x}{x}$$
 from first principle.

[5]

OR

Differentiate $\frac{\cos x}{x}$ from first principle.

34. Prove that:
$$\cos^3 x \sin 3x + \sin^3 x \cos 3x = \frac{3}{4} \sin 4x$$
.

[5]

[5]

OR

Prove that: $\cos 10^{\circ} \cos 30^{\circ} \cos 50^{\circ} \cos 70^{\circ} = \frac{3}{16}$.

35. Calculate the mean deviation about the median for the following data:

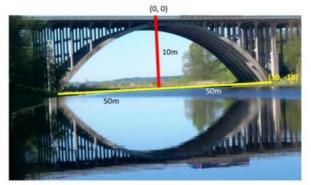
Height (in cm)	95 - 105	105 - 115	115 - 125	125 - 135	135 - 145	145 - 155
Number of boys	9	13	25	30	13	10

Section E

36. Read the text carefully and answer the questions:

[4]

The girder of a railway bridge is a parabola with its vertex at the highest point, 10 m above the ends. Its span is 100 m.



- (i) Find the coordinates of the focus of the parabola.
- (ii) Find the equation of girder of bridge and find the length of latus rectum of girder of bridge.
- (iii) Find the height of the bridge at 20m from the mid-point.

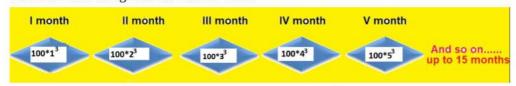
OR

Find the radius of circle with centre at focus of the parabola and passes through the vertex of parabola.

37. Read the text carefully and answer the questions:

[4]

Ratan wants to open an RD for the marriage of his daughter, He visited the branch of SBI at sector 3, Gurgaon. There he made an agreement with the bank.



According to this agreement, he would deposit $\stackrel{?}{\underset{?}{?}}$ 100 \times n³ every month (here n = 1 to 15). Other terms and conditions are as follows:

- I. He has to pay a minimum of six instalments.
- II. If he continues the deposit up to 15 months then the bank will pay 20% extra as a bonus.
- III. If he breaks the deposit after 6 months then the bank will pay 10% extra as a bonus
- IV. If he breaks the deposit after 10 months then the bank will pay 15% extra as a bonus.
- V. No other interest will be paid by the bank.
- (i) How much amount would be accumulated after 15 months?
 - a) ₹ 10,00,000

b) ₹ 11,02,500

c) ₹ 15,00,000

- d) ₹ 14,40,000
- (ii) How much total amount would Ratan get after 15 months?
 - a) ₹ 14,40,000

b) ₹ 13,23,000

c) ₹ 17,28,000

- d) ₹ 15,00,000
- (iii) How much total amount would Ratan get if he breaks the deposit after 10 months?
 - a) ₹ 3,50,000

b) ₹ 3,23,000

c) ₹ 3,47,875

d) ₹ 3,45,875

OR

How much total amount would Ratan get if he breaks the deposit after 6 months?

a) ₹ 60,000

b) ₹ 50,715

c) ₹ 50,000

d) ₹ 65,875

In an University, out of 100 students 15 students offered Mathematics only, 12 students offered Statistics only, 8 students offered only Physics, 40 students offered Physics and Mathematics, 20 students offered Physics and Statistics, 10 students offered Mathematics and Statistics, 65 students offered Physics.



- (i) Find the number of students who offered all the three subjects.
- (ii) Find the number of students who offered mathematics and statistics but not physics.

Solution

CBSE SAMPLE PAPER - 07

Class 11 - Mathematics

Section A

1. **(b)**
$$\frac{9}{16}$$

Explanation: $\frac{9}{16}$

2. **(b)** -10

Explanation: The cumulative frequency distribution is

Age	Number of persons	Cumulative frequency	
20-25	110	110	
25-30	х	110 + x	
30-35	75	185 + x	
35-40	55	240 + x	
40-45	у	240 + x + y	
45-50	30	270 + x + y	
		N = 270 + x + y	

Clearly,
$$270 + x + y = 400$$

$$\Rightarrow$$
 x + y = 130

Since, 32 is the median

: 30-35 is the median class such that

$$l = 30$$
, $h = 5$, $f = 75$, $N = 400$ and $C = 110 + x$

$$\therefore \text{ Median} = 1 + \frac{\frac{N}{2} - C}{f} \times \text{ h}$$

$$\Rightarrow 32 = 30 + \frac{200 - (110 + x)}{75} \times 5$$

$$\Rightarrow 2 = \frac{90 - x}{15} \Rightarrow x = 60$$

$$\therefore x + y = 130 \Rightarrow y = 70$$

$$\therefore x - y = -10$$

3. **(c)**
$$P(A) + P(B) = 2P(A) P(B|A)$$

Explanation:
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cap B) \dots [\because P(A \cap B) = P(A \cup B)]$$

$$\Leftrightarrow 2P(A \cap B) P(A) + P(B)$$

$$\Leftrightarrow 2P(A) \cdot \frac{P(A \cap B)}{P(A)} = P(A) + P(B)$$

$$\Leftrightarrow$$
 2P(A)P(B|A) = P(A) + P(B)

4. **(c)** 2

Explanation: Given,
$$\lim_{x \to \frac{\pi}{4}} \frac{\sec^2 x - 2}{\tan x - 1} = \lim_{x \to \frac{\pi}{4}} \frac{1 + \tan^2 x - 2}{\tan x - 1}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\tan^2 x - 1}{\tan x - 1} = \lim_{x \to \frac{\pi}{4}} \frac{(\tan x + 1)(\tan x - 1)}{(\tan x - 1)}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\tan^2 x - 1}{\tan x - 1} = \lim_{x \to \frac{\pi}{4}} \frac{(\tan x + 1)(\tan x - 1)}{(\tan x - 1)}$$

$$=\lim_{x o rac{\pi}{4}}(an x+1)= an rac{\pi}{4}+1=1+1=2$$

(b) 25/16 5.

Explanation: The equation of the line joining the two points (x_1, y_1) and (x_2, y_2) is

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

The given points are (1, 1) and (2, 0)

On substituting the values we get

$$\frac{y-1}{0-1} = \frac{x-1}{2-1}$$

On simplifying we get,

$$x + y - 2 = 0$$

The line which is perpendicular to this line is x - y + k = 0

Since it passes through (1/2, 0)

$$(1/2) - 0 = k$$

This implies k = -1/2

Hence the equation of this line is x - y - 1/2 = 0

On solving these twolines we get the point of intersection as (5/4, 3/4)

The point which line x + y - 2 = 0 cuts the Y axis is (0, 2) and the point which the line x - y - 1/2 = 0 cuts the Y axis is (0, -1/2)Hend e the area of the triangle = $[1/2] \times [5/4] \times [5/4] = 25/16$ squnits

Explanation: The total no of subsets = 2^n

Explanation:
$$|z + 1| = |z + 4 - 3| \le |z + 4| + |-3| \dots [\because |z_1 + z_2| \le |z_1| + |z_2|]$$

$$=|z+4|+3\leq 3+3=6\,...[\because |z+4|\leq 3]$$

 \Rightarrow The greatest value of |z + 1| = 6

Explanation: As xRy if x + 2y = 8, therefore, domain of the relation R is given by $x = 8 - 2y \in \mathbb{N}$. When $y = 1, \Rightarrow x = 6$, when y = 2, $\Rightarrow x = 4$, when y = 3, $\Rightarrow x = 2$. Therefore, domain is { 2, 4, 6 }.

9. **(a)**
$$-\frac{1}{6} \le x < \frac{1}{2}$$

Explanation:
$$-2 \le 6x - 1 \le 2$$

$$\Rightarrow$$
 -2 + 1 \leq 6x - 1 + 1 < 2 + 1

$$\Rightarrow$$
 -1 \leq 6x \leq 3

$$\Rightarrow \frac{-1}{6} \le \frac{6x}{6} < \frac{3}{6}$$

$$\Rightarrow \frac{-1}{6} \le \frac{6x}{6} < \frac{3}{6}$$
$$\Rightarrow \frac{-1}{6} \le x < \frac{1}{2}$$

10. **(d)**
$$\frac{(\sqrt{3}-1)}{2\sqrt{2}}$$

Explanation:
$$\cos 75^\circ = \cos(90^\circ - 15^\circ) = \sin 15^\circ = \frac{(\sqrt{3} - 1)}{2\sqrt{2}}$$

Explanation: Here, it is given

$${}^{n}C_{12} = {}^{n}C_{8} [:: {}^{n}C_{r} = {}^{n}C_{n-r}]$$

$${}^{n}C_{12} = {}^{n}C_{n-8}$$

$$\Rightarrow$$
 n = 12 + 8 = 20

12. **(b)**
$$\frac{n(n+1)}{\sqrt{2}}$$

Explanation: Let T_n be the nth term of the given series.

$$T_n = \sqrt{2 \times n^2} = n\sqrt{2}$$

Now, let

S_n be the sum of n terms of the given series.

Thus, we calculate the sum as:

$$S_n = \sqrt{2} \sum_{k=1}^n (k)$$

$$\Rightarrow \mathsf{S}_{\mathsf{n}} = \sqrt{2} \left[\frac{n(n+1)}{2} \right]$$

$$\Rightarrow$$
 S_n = $\frac{n(n+1)}{\sqrt{2}}$

Explanation:

The pascal's triangle is given by

1 2 1

1 3 3 1

14. **(c)**
$$x \in (-\infty, -4) \cup (6, \infty)$$

Explanation:
$$|\mathbf{x} - \mathbf{1}| > 5$$

$$\begin{array}{l} \Rightarrow x-1 < -5 \quad or \quad x-1 > 5 \\ \Rightarrow x-1+1 < -5+1 \quad or \quad x-1+1 > 5+1 \\ \Rightarrow x < -4 \quad or \quad x > 6 \\ \Rightarrow x \in (-\infty, -4) \cup (6, \infty) \end{array} \qquad \begin{array}{l} [\because |x| > a \Leftrightarrow x < -a \quad or \quad x > a] \\ \Rightarrow x < -4 \quad or \quad x > 6 \end{array}$$

(b) 60 15.

Explanation: n(M alone)

=
$$n(M) - n(M \cap C) - n(M \cap P) + n(M \cap P \cap C)$$

= $100 - 28 - 30 + 18 = 60$

16. **(a)**
$$\frac{\sin 2\beta}{5 - \cos 2\beta}$$

Explanation: Given $2 \tan \alpha = 3 \tan \beta$

From here we get, $\tan \alpha = \frac{3}{2} \tan \beta$ ---- (i)

since
$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\left(\frac{3}{2}\tan\beta\right) - \tan\beta}{1 + \left(\frac{3}{2}\tan\beta\right)\tan\beta} \quad \text{[using eq (i)]}$$

$$\tan(\alpha - \beta) = \frac{\left(\frac{3\tan\beta - 2\tan\beta}{2}\right)}{\left(\frac{2+3\tan^2\beta}{2}\right)}$$

$$= \frac{\tan \beta}{2 + 3 \tan^2 \beta} \dots \text{[by using } \tan \theta = \frac{\sin \theta}{\cos \theta} \text{]}$$

$$= \frac{\left(\frac{\sin \beta}{\cos \beta}\right)}{2+3\left(\frac{\sin \beta}{\cos \beta}\right)^2}$$

$$= \frac{\sin \beta \cos \beta}{2 \cos^2 \beta + 3 \sin^2 \beta}$$

$$= \frac{2\cos^2\beta + 3\sin^2\beta}{\sin\beta\cos\beta}$$

$$= \frac{\sin \beta \cos \beta}{2 \cos^2 \beta + 3 (1 - \cos^2 \beta)}$$

$$= \frac{\sin \beta \cos \beta}{2 \cos^2 \beta + 3 - 3 \cos^2 \beta}$$

$$= \frac{2\cos^2\beta + 3 - 3\cos}{\sin\beta\cos\beta}$$

$$= \frac{\sin \beta \cos \beta}{3 - \cos^2 \beta}$$

Multiplying and dividing the equation with 2

$$= \frac{2 \sin \beta \cos \beta}{2(3 - \cos^2 \beta)} \dots [\text{using } \sin 2\theta = 2 \sin \theta \cos \theta]$$
$$= \frac{\sin 2\beta}{6 - 2 \cos^2 \beta}$$

In the denominator adding and subtracting 1

$$= \frac{\sin 2\beta}{6 - 2\cos^2 \beta + 1 - 1}$$

$$= \frac{\sin 2\beta}{(6 - 1) - (2\cos^2 \beta - 1)} \dots \text{ [using } \cos 2\theta = 2\cos^2 \theta - 1\text{]}$$

$$\tan(\alpha - \beta) = \frac{\sin 2\beta}{5 - \cos 2\beta}$$

17.

Explanation: Clearly $|z_1| = 9$, represents a circle having centre $C_1(0, 0)$ and radius $r_1 = 9$. and $|z_2 - S - 4i| = 4$ represents a circle having centre $C_2(3, 4)$ and radius $r_2 = 4$.

The minimum value of $|z_1-z_2|$ is equals to minimum distance between circles $|z_1|=9$ and $|z_2-3-4i|=4$.

$$\therefore C_1 C_2 = \sqrt{(3-0)^2 + (4-0)^2} = \sqrt{9+16} = \sqrt{25} = 5 \quad \text{and } |\mathbf{r}_1 - \mathbf{r}_2| = |9-4| = 5 \Rightarrow C_1 C_2 = |\mathbf{r}_1 - \mathbf{r}_2|$$

... Circles touches each other internally.

Hence, $|z_1 - z_2|_{min} = 0$

18. **(c)** 6560

Explanation: Since a student can solve every question in three ways- either he can attempt the first alternative or the second alternative or he does not attempt that question

Hence the total ways in which a student can attempt one or more of 8 questions = 3^8

Therefore to find the number of all selections which a student can make for answering one or more questions out of 8 given questions = $3^8 - 1 = 6560$ [we will have to exclude only the case of not answering all the 8 questions]

19. (a) Both A and R are true and R is the correct explanation of A.

Explanation: Assertion:

no. of terms =
$$10 + 1$$

= 11, True

Reason:

no. of term = n + 1, True

A Reason is correct explanation of Assertion.

20. (a) Both A and R are true and R is the correct explanation of A.

Explanation: Both Assertion and reason are true because R defined on the set $A = \{1, 2, 3, 4, 5\}$ by $R = \{(a, b) : |a^2 - b^2| < 9\}$ For example: $|1^2 - 2^2| = 3 < 9$ so $(1, 2) \in R$ and this condition is true for the remaining element of R.

Section B

21. Here we have, $f(x) = log_e(1 - x)$ and f(x) = [x]

Clearly, loge(1 - x) is defined only when 1 - x > 0, i.e., x < 1.

$$\therefore dom(f) = (-\infty, 1).$$

Also,
$$dom(g) = R$$
.

$$\therefore$$
 dom (f) \cap dom (g) = ($-\infty$, 1) \cap R = ($-\infty$, 1).

$$(f + g) : (-\infty, 1) \rightarrow R$$
 is given by

$$(f + g)(x) = f(x) + g(x) = \log_e(1 - x) + [x]$$

22. We have to find
$$\lim_{x\to 27} \frac{\left(x^{\frac{1}{3}}+3\right)\left(x^{\frac{1}{3}}-3\right)}{x-27}$$

We have,

$$\lim_{x \to 27} \frac{\left[x^{\frac{1}{3}} + 3\right] \left[x^{\frac{1}{2}} - 3\right]}{x - 27}$$

$$= \lim_{x \to 27} \left[\frac{\left(x^{\frac{1}{3}} + 3\right) \left(x^{\frac{1}{3}} - 3\right)}{\left(x^{\frac{1}{3}}\right)^{3} - 3^{3}}\right]$$

$$x \rightarrow 27$$

$$\therefore x^{\frac{1}{3}} \rightarrow 3$$

Let
$$y = x^{\frac{1}{3}}$$

$$\lim_{y \to 3} \left[\frac{(y+3)(y-3)}{y^3 - 3^3} \right]$$

$$= \frac{(3+3)}{3 \times 3^{3-1}}$$

$$= \frac{6}{3}$$

$$=\frac{2}{9}$$

$$\Rightarrow$$
 y = 0 \Rightarrow The axis of parabola is along x-axis

So the parabola is of the form $y^2 = 4ax$.

The required equation of parabola is

$$y^2 = 4x - 2x \Rightarrow y^2 = -8x$$
.

OR

Given that vertices of the hyperbola are $(0, \pm 5)$, and the foci are $(0, \pm 8)$.

Thus,
$$a = 5$$
 and $ae = 8$.

Now, using the relation

$$b^2 = a^2 (e^2 - 1)$$
, we get

$$\Rightarrow$$
 b² = 64 - 25

$$\Rightarrow$$
 b² = 39

Therefore, the equation of the hyperbola is $-rac{x^2}{39}+rac{y^2}{25}=1$

24. L.H.S =
$$A \cap (A \cup B)$$

Using De-Morgan's law $(A \cup B)' = (A' \cap B')$

$$\Rightarrow$$
 L.H.S = A \cap (A' \cap B')

$$\Rightarrow$$
 L.H.S = $(A \cap A') \cap (A \cap B')$

We know that $A \cap A' = \phi$

$$\Rightarrow$$
 L.H.S = $\phi \cap (A \cap B')$

We know that intersection of null set with any set is null set only

Suppose (A \cap B') be any set X therefore

$$\Rightarrow$$
 L.H.S = $\phi \cap X$

$$\Rightarrow$$
 L.H.S = ϕ

$$\Rightarrow$$
 L.H.S = R.H.S

Hence proved.

25. Let the y-intercept of the required line be c.

Then, its equation is

$$y = 2x + c$$

$$\Rightarrow$$
 -2x + y = c(i)

Dividing throughout by
$$\sqrt{(\text{ Coefficient of }x)^2+(\text{ Coefficient of }y)^2}$$
, we obtain,
$$-\frac{2}{\sqrt{(-2)^2+1^2}}x+\frac{y}{\sqrt{(-2)^2+1^2}}=\frac{c}{\sqrt{(-2)^2+1^2}}\text{ or, }-\frac{2}{\sqrt{5}}x+\frac{y}{\sqrt{5}}=\frac{c}{\sqrt{5}}$$

This is the normal form of line (i).

Therefore, RHS represents the length of the perpendicular from the origin.

But, the length of the perpendicular from the origin is given to be $\sqrt{5}$

$$\therefore \left| \frac{c}{\sqrt{5}} \right| = \sqrt{5} \Rightarrow |c| = 5$$

$$\Rightarrow$$
 c = \pm 5

Put, $c = \pm 5$ in (i), we obtain,

 $y = 2x \pm 5$, which is the required equation of line.

Section C

26. We have,
$$f(x) = \left\{ egin{array}{ll} 1+x, & -1 \leq x < 0 \ x^2-1, & 0 < x < 2 \ 2x, & 2 \leq x \end{array}
ight.$$

Now,
$$f(3) = 2 \times 3 = 6$$
 [: $f(x) = 2x$, for $x \ge 2$]

$$f(-2)$$
 = Not defined

$$f(0)$$
 = Not defined

$$f\left(rac{1}{2}
ight) = \left(rac{1}{2}
ight)^2 - 1 = rac{-3}{4} \;\; [\because f(x) = x^2 - 1$$
 , for $0 < x < 2$]

$$f(-1+h) = 1 + (-1+h) = h$$
 [:: $f(x) = 1 + x$ for $-1 \le x < 0$]

$$f(2-h) = (2-h)^2 - 1$$
 [:: $f(x) = x^2 - 1$ for $0 < x < 2$]

$$=4+h^2-4h-1=h^2-4h+3$$

27. Here P(2a, 2, 6), Q(-4, 3b, -10) and R(8, 14, 2c) are vertices of triangle PQR.

$$\therefore$$
 Coordinates of centriod of ΔPQR is $\left(\frac{2a-4+8}{3},\frac{2+3b+14}{3},\frac{6-10+2c}{3}\right)$ = $\left(\frac{2a+4}{3},\frac{3b+16}{3},\frac{2c-4}{3}\right)$

But is it given that coordinates of centroid is (0, 0, 0)

$$\frac{2a+4}{3} = 0 \Rightarrow 2a+4=0$$
 : $a = -2$

OR

Given: Point (3, -2, 5)

To find: the coordinates of 7 more points such that the absolute values of all 8 coordinates are the same

Formula used: Absolute value of any point(x, y, z) is given by,

$$\sqrt{x^2+y^2+z^2}$$

In the formula of absolute value, there is a square of the coordinates.

So when we change the sign of any of the coordinates, it will not affect the absolute value.

Let point A (3, -2, 5)

Remaining 7 points are:

- \Rightarrow Point B (3, 2, 5) (By changing the sign of y coordinate)
- \Rightarrow Point C (-3, -2, 5) (By changing the sign of x coordinate)
- \Rightarrow Point D (3, -2, -5) (By changing the sign of z coordinate)
- \Rightarrow Point E (-3, 2, 5) (By changing the sign of x and y coordinate)
- \Rightarrow Point F (3, 2, -5) (By changing the sign of y and z coordinate)
- \Rightarrow Point G (-3, -2, -5) (By changing the sign of x and z coordinate)
- \Rightarrow Point H (-3, 2, -5) (By changing the sign of x, y and z coordinate)

28.
$$(a+b)^4 = [{}^4C_0a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_3ab^3 + {}^4C_4b^4]$$

and
$$(a-b)^4 = [{}^4C_0a^4 - {}^4C_1a^3b + {}^4C_2a^2b^2 - {}^4C_3ab^3 + {}^4C_4b^4]$$

$$(a+b)^4 - (a-b)^4 = 2[{}^4C_1a^3b + {}^4C_3ab^3]$$

$$=2\left[4a^{3}b\ +4ab^{3}
ight]\ =\ 8ab\left[a^{2}+b^{2}
ight]$$

$$\therefore \left(\sqrt{3} \ + \sqrt{2}\right)^4 \ - \ \left(\sqrt{3} \ - \sqrt{2}\right)^4 \ = \ 8.\sqrt{3}.\sqrt{2}\left[\left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2\right]$$

$$= 8.\sqrt{3}.\sqrt{2}[3+2] = 40.\sqrt{3}.\sqrt{2} = 40\sqrt{6}$$

OR

Let
$$x = 101^{50}$$
 and $y = 100^{50} + 99^{50}$. Then,

$$x-y = 101^{50} - 100^{50} - 99^{50}$$

$$\Rightarrow$$
 x - y = 101⁵⁰ - 99⁵⁰ - 100⁵⁰

$$\Rightarrow$$
 x - y = $(100 + 1)^{50}$ - $(100 - 1)^{50}$ - 100^{50}

$$\Rightarrow$$
 x - y = 2 { 50 C₁ × 100⁴⁹ + 50 C₃ × 100⁴⁷ + ... + 50 C₄₉ × 100} - 100⁵⁰

$$\Rightarrow$$
 x - y = 100^{50} + 2 \times 50 C₃ \times 100^{47} + ... + 2 \times 50 C₄₉ \times 100 - 100^{50}

$$\Rightarrow$$
 x - y = 2 \times ⁵⁰C₃ \times 100⁴⁷ + ... + 2 \times ⁵⁰C₄₉ \times 100

$$\Rightarrow$$
 x - y = a positive integer

$$\Rightarrow$$
 x - y > 0 \Rightarrow x > y \Rightarrow 101⁵⁰ > 100⁵⁰ + 99⁵⁰

29. Let,
$$(a + ib)^2 = -7 + 24i$$

$$\Rightarrow$$
 $a^2 + (bi)^2 + 2abi = -7 + 24i [(a + b)^2 = a^2 + b^2 + 2ab]$

$$\Rightarrow$$
 a² - b² + 2abi = -7 + 24i [i² = -1]

Now, separating real and complex parts, we get

$$\Rightarrow$$
 a² - b² = -7.....eq.1

$$\Rightarrow$$
 2ab = 24

$$\Rightarrow$$
 a = $\frac{12}{h}$eq.2

Now, using the value of a in eq.1, we get

$$\Rightarrow \left(\frac{12}{b}\right)^2 - b^2 = -7$$

$$\Rightarrow 144 - b^4 = -7b^2$$

$$\Rightarrow$$
 b⁴ - 7b² - 144= 0

$$=> (b^2 + 9)(b^2 - 16) = 0$$

$$\Rightarrow$$
 b² = -9 or b² = 16

As b is real no. so, $b^2 = 16$

$$b = 4 \text{ or } b = -4$$

put value of b in equation (2) ==> a = 3 or a = -3

Hence the square root of the complex no. is 3 + 4i and -3 -4i.

OR

$$\begin{bmatrix}
\frac{1}{1-4i} - \frac{2}{1+i}
\end{bmatrix} \begin{bmatrix}
\frac{3-4i}{5+i}
\end{bmatrix} = \begin{bmatrix}
\frac{1+i-2+8i}{(1-4i)(1+i)}
\end{bmatrix} \begin{bmatrix}
\frac{3-4i}{5+i}
\end{bmatrix} \\
= \begin{bmatrix}
\frac{-1+9i}{1+i-4i-4i^2}
\end{bmatrix} \begin{bmatrix}
\frac{3-4i}{5+i}
\end{bmatrix} = \begin{bmatrix}
\frac{-1+9i}{5-3i}
\end{bmatrix} \begin{bmatrix}
\frac{3-4i}{5+i}
\end{bmatrix} \\
= \frac{-3+4i+27i-36i^2}{25+5i-15i-3i^2} = \frac{33+31i}{28-10i} \times \frac{28+10i}{28+10i} \\
= \frac{924+330i+868i+310i^2}{(28)^2-(10i)^2} = \frac{614+1198i}{784+100} (::i^2 = -1) \\
= \frac{2(307+599i)}{884} = \frac{307+599i}{442}$$

30. We have,
$$\frac{2x+5}{x-1} > 5 \Rightarrow \frac{2x+5}{x-1} - 5 > 0$$

$$\Rightarrow \frac{2x+5-5x+5}{x-1} > 0 \Rightarrow \frac{-3x+10}{x-1} > 0$$

$$\Rightarrow \frac{-3x+10}{(x-1)} \times (x-1)^2 > 0 \times (x-1)^2$$

$$\Rightarrow (-3x+10)(x-1) > 0$$

 \Rightarrow (3x - 10) (x - 1) < 0 [Multiplying by - 1 on both sides to make coefficient of x positive]

Thus, product of (3x - 10) and (x - 1) will be negative.

Case I: If
$$3x - 10 > 0$$
 and $x - 1 < 0$
 $\Rightarrow 3x > 10$ and $x < 1 \Rightarrow x > \frac{10}{3}$ and $x < 1$

So, this is impossible, since system of inequalities have no common solution.

Thus, there is no solution of given inequality in this case.

Case II: If
$$3x - 10 < 0$$
 and $x - 1 > 0$

$$\Rightarrow 3x < 10 \text{ and } x > 1 \Rightarrow x < \frac{10}{3} \text{ and } x > 1$$

$$\Rightarrow 1 < x < \frac{10}{3} \Rightarrow x \in \left(1, \frac{10}{3}\right)$$

$$-\infty \longleftrightarrow 0$$

$$1$$

$$\frac{10}{3}$$

Hence, the solution of given inequality is, $1 < x < \frac{10}{3}$

$$\begin{array}{l} 31. \, \mathrm{Here}^{\ 22} P_{r+1} \colon \ ^{20} P_{r+2} = 11 \colon 52 \\ \Rightarrow \frac{22!}{(21-r)!} \times \frac{(18-r)!}{20!} = \frac{11}{52} \\ \Rightarrow = \frac{22 \times 21 \times 20!}{(21-r)(20-r)(19-r)(18-r)!} \times \frac{(18-r)!}{20!} = \frac{11}{52} \\ \Rightarrow \frac{22 \times 21}{(21-r)(20-r)(19-r)} = \frac{11}{52} \\ \Rightarrow (21-r)(20-r)(19-r) = 2 \times 21 \times 52 \\ \Rightarrow (21-r)(20-r)(19-r) = 14 \times 13 \times 12 \\ \Rightarrow (21-r)(20-r)(19-r) = (21-7)(20-7)(19-7) \\ \Rightarrow r = 7 \end{array}$$

Section D

32. Bag contains:

- 6 -Red balls
- 4 -White balls
- 8 -Blue balls

Since three ball are drawn,

$$n(S) = {}^{18}C_3$$

i. Let E be the event that one red and two white balls are drawn.

$$\therefore n(E) = {}^{6}C_{1} \times {}^{4}C_{2}$$

$$\therefore P(E) = {}^{6}C_{1} \times {}^{4}C_{2} = {}^{6 \times 4 \times 3} \times {}^{3 \times 2} \times {}^{3 \times 2} \times {}^{18 \times 17 \times 16}$$

$$P(E) = {}^{3}_{68}$$

ii. Let E be the event that two blue balls and one red ball was drawn.

$$\therefore n(E) = {}^{8}C_{2} \times {}^{6}C_{1}$$

$$P(E) = \frac{{}^{8}C_{2} \times {}^{6}C_{1}}{{}^{18}C_{3}} = \frac{8 \times 7}{2} \times 6 \times \frac{3 \times 2 \times 1}{18 \times 17 \times 16} = \frac{7}{34}$$

$$P(E) = \frac{7}{34}$$

iii. Let E be the event that one of the ball must be red.

$$\therefore$$
 E = {(R,W,B) or (R,W,W) or (R,B,B)}

$$\therefore n(E) = {}^{6}C_{1} \times {}^{4}C_{1} \times {}^{8}C_{1} + {}^{6}C_{1} \times {}^{4}C_{2} + {}^{6}C_{1} \times {}^{8}C_{2}$$

$$\therefore P(E) = {}^{6C_{1} \times {}^{4}C_{1} \times {}^{8}C_{1} + {}^{6}C_{1} \times {}^{4}C_{2} + {}^{6}C_{1} \times {}^{8}C_{2}}{{}^{18}C_{3}} = \frac{{}^{6 \times 4 \times 8 +} \frac{6 \times 4 \times 3}{2 \times 1} + \frac{6 \times 8 \times 7}{2 \times 1}}{\frac{18 \times 17 \times 16}{3 \times 2 \times 1}}$$

$$=\frac{396}{816}=\frac{33}{68}$$

33. Let
$$f(x) = \frac{\sin x}{x}$$

By using first principle of derivative,

By using first principle of derivative,
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\therefore f'(x) = \lim_{h \to 0} \frac{\frac{\sin(x+h)}{x+h} - \frac{\sin x}{x}}{h}$$

$$= \lim_{h \to 0} \frac{x \sin(x+h) - (x+h) \sin x}{x(x+h) \times h}$$

$$= \lim_{h \to 0} \frac{x [\sin(x+h) - \sin x] - h \sin x}{h \cdot x(x+h)}$$

$$= \lim_{h \to 0} \frac{x \left[2 \cdot \cos\left(\frac{x+h+x}{2}\right) \cdot \sin\left(\frac{x+h-x}{2}\right) \right] - h \sin x}{h \cdot x(x+h)}$$

$$\left[\because \sin C - \sin D = 2 \cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right) \right]$$

$$= \lim_{h \to 0} \frac{x \left[2 \cdot \sin\frac{h}{2} \cdot \cos\left(x + \frac{h}{2}\right) \right] - h \sin x}{h \cdot x(x+h)}$$

$$= \lim_{h \to 0} \frac{\sin\left(\frac{h}{2}\right)}{h \cdot x(x+h)} \cdot \lim_{h \to 0} \frac{\cos\left(x + \frac{h}{2}\right)}{(x+h)} - \lim_{h \to 0} \frac{\sin x}{x(x+h)}$$

$$= (1) \cdot \frac{\cos x}{x} - \frac{\sin x}{x^2} \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1 \right]$$

$$= \frac{\cos x}{x} - \frac{\sin x}{x^2}$$

OR

We have to find the derivative of $f(x) = \frac{\cos x}{x}$

Derivative of a function f(x) is given by $f'(x) = \lim_{h \to 0} = \frac{f(x+h) - f(x)}{h}$ {where h is a very small positive number}

$$\therefore$$
 Derivative of $f(x) = \frac{\cos x}{x}$ is given as $f'(x) = \lim_{h \to 0} = \frac{f(x+h) - f(x)}{h}$

$$\Rightarrow f(\mathbf{x}) = \lim_{h \to 0} \frac{\frac{\cos(x+h)}{x+h} - \frac{\cos x}{x}}{h}$$

$$\Rightarrow f(\mathbf{x}) = \lim_{h \to 0} \frac{\frac{x\cos(x+h) - (x+h)\cos x}{x}}{h} = \lim_{h \to 0} \frac{x\cos(x+h) - (x+h)\cos x}{h(x)(x+h)}$$

Using the algebra of limits we have:
$$\Rightarrow f(\mathbf{x}) = \lim_{h \to 0} \frac{x \cos(x+h) - (x+h) \cos x}{h} \times \lim_{h \to 0} \frac{1}{x(x+h)}$$

$$\Rightarrow f(\mathbf{x}) = \lim_{h \to 0} \frac{x \cos(x+h) - (x+h) \cos x}{h} \times \frac{1}{x(x+o)}$$

$$\Rightarrow f(\mathbf{x}) = \frac{1}{x^2} \lim_{h \to 0} \frac{x \cos(x+h) - (x+h) \cos x}{h}$$

$$\Rightarrow f(\mathbf{x}) = \frac{1}{x^2} \lim_{h \to 0} \frac{x \cos(x+h) - x \cos x - h \cos x}{h}$$

Using the algebra of limits, we have

$$\Rightarrow f(\mathbf{x}) = \frac{1}{\mathbf{x}^2} \left\{ \lim_{h \to 0} \frac{-h\cos x}{h} + \lim_{h \to 0} \frac{x\cos(x+h) - x\cos x}{h} \right\}$$
$$\Rightarrow f(\mathbf{x}) = \frac{1}{\mathbf{x}^2} \left\{ -\lim_{h \to 0} \cos x + \lim_{h \to 0} \frac{x(\cos(x+h) - \cos x)}{h} \right\}$$

Using the algebra of limits we have

: f'(x) =
$$-\frac{\cos x}{x^2} + \frac{1}{x} \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$$

We can't evaluate the limits at this stage only as on putting value it will take $\frac{0}{0}$ form. So, we need to do little modifications.

Use:
$$\cos A - \cos B = -2 \sin \left(\frac{(A+B)}{2}\right) \sin \left(\frac{(A-B)}{2}\right)$$

$$\therefore f'(x) = -\frac{\cos x}{x^2} + \frac{1}{x} \lim_{h \to 0} \frac{-2 \sin \left(\frac{2x+h}{2}\right) \sin \left(\frac{h}{2}\right)}{h}$$

$$\Rightarrow f'(x) = -\frac{\cos x}{x^2} + \frac{1}{x} \lim_{h \to 0} \frac{\sin \left(x + \frac{h}{2}\right) \sin \left(\frac{h}{2}\right)}{\frac{h}{2}}$$

Using algebra of limits:

$$\Rightarrow f'(x) = -\frac{\cos x}{x^2} + \frac{1}{x} \lim_{h \to 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \times \lim_{h \to 0} \sin\left(x + \frac{h}{2}\right)$$

By using the formula we get : $\lim_{x \to 0} \frac{\sin x}{x} = 1$

$$\Rightarrow f'(x) = -\frac{\cos x}{x^2} + \frac{1}{x} \lim_{h \to 0} \sin\left(x + \frac{h}{2}\right)$$

Put the value of h to evaluate the limit:

:
$$f'(x) = -\frac{\cos x}{x^2} + \frac{1}{x} \times \sin(x+0) = -\frac{\cos x}{x^2} - \frac{\sin x}{x}$$

Derivative of f(x) =
$$(\cos x)/x$$
 is $-\frac{\cos x}{x^2} - \frac{\sin x}{x}$

34. We have to prove that $\cos^3 x \sin 3x + \sin^3 x \cos 3x = \frac{3}{4} \sin 4x$.

We know that,

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$\Rightarrow 4\cos^3\theta = \cos 3\theta + 3\cos \theta$$

$$\Rightarrow \cos^3 \theta = \frac{\cos 3\theta + 3\cos \theta}{4} \dots$$
 (i)

And similarly

$$\Rightarrow \sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

$$\Rightarrow 4 \sin^3 \theta = 3\sin \theta - \sin 3\theta$$

$$\Rightarrow \sin^3 \theta = \frac{3 \sin \theta - \sin 3\theta}{4} \dots (ii)$$

Now,

 $LHS = \cos^3 x \sin 3x + \sin^3 x \cos 3x$

Substituting the values from equation (i) and (ii), we get

$$\Rightarrow \left(\frac{\cos 3x + 3\cos x}{4}\right)\sin 3x + \left(\frac{\cos 3x - 3\cos x}{4}\right)\cos 3x$$
$$= \frac{1}{4}(\sin 3x \cos 3x + 3\sin 3x \cos x + 3\sin x \cos 3x - \sin x)$$

$$= \frac{1}{4} (\sin 3x \cos 3x + 3 \sin 3x \cos x + 3 \sin x \cos 3x - \sin 3x \cos 3x)$$

$$= \frac{1}{4} [3(\sin 3x \cos x + \sin x \cos 3x) + 0]$$

$$=\frac{1}{4}(3\sin(3x+x))$$

$$(as \sin(x+y) = \sin x \cos y + \cos x \sin y)$$

$$\Rightarrow \frac{3}{4} \sin 4x$$

$$LHS = RHS$$

Hence Proved

OR

cos 10° cos 30° cos 50° cos 70° =
$$\frac{3}{16}$$

LHS = cos 10° cos 30° cos 50° cos 70°
= cos 30° cos 10° cos 50° cos 70°
= $\frac{\sqrt{3}}{2}$ (cos 10° cos 50° cos 70°)
= $\frac{\sqrt{3}}{2}$ (cos 10° cos 50°) cos 70°

$$=\frac{\sqrt{3}}{4}$$
 (2 cos 10° cos 50°) cos 70° [Multiplying and dividing by 2]

$$= \frac{\sqrt[4]{3}}{4} \cos 70^{\circ} \left\{ \cos (50^{\circ} + 10^{\circ}) + \cos (10^{\circ} - 50^{\circ}) \right\} \text{ [Using 2 cos A cos B = cos (A + B) + cos (A - B)]}$$

$$= \frac{\sqrt{3}}{4} \cos 70^{\circ} \left\{ \cos 60^{\circ} + \cos (-40^{\circ}) \right\}$$

$$= \frac{\sqrt{3}}{4} \cos 70^{\circ} \left[\frac{1}{2} + \cos 40^{\circ} \right] \left[\because \cos 60^{\circ} = \frac{1}{2} \text{ and } \cos (-x) = \cos x \right]$$
$$= \frac{\sqrt{3}}{8} \cos 70^{\circ} + \frac{\sqrt{3}}{4} \cos 70^{\circ} \cos 40^{\circ}$$

$$= \frac{\sqrt[4]{3}}{8} \cos 70^{\circ} + \frac{\sqrt[4]{3}}{4} \cos 70^{\circ} \cos 40^{\circ}$$

$$= \frac{\sqrt{3}}{8} \cos 70^\circ + \frac{\sqrt{3}}{8} (2 \cos 70^\circ \cos 40^\circ)$$

$$= \frac{\sqrt{3}}{8} \left[\cos 70^{\circ} + \cos (70^{\circ} + 40^{\circ}) + \cos (70^{\circ} - 40^{\circ}) \right]$$

$$= \frac{\sqrt{3}}{8} [\cos 70^{\circ} + \cos 110^{\circ} + \cos 30^{\circ}]$$

$$= \frac{\sqrt{3}}{8} [\cos 70^{\circ} + \cos (180^{\circ} - 70^{\circ}) + \frac{\sqrt{3}}{2}] [\because \cos 30^{\circ} = \frac{\sqrt{3}}{2}]$$

$$= \frac{\sqrt{3}}{8} [\cos 70^{\circ} - \cos 70^{\circ} + \frac{\sqrt{3}}{2}] [\because \cos (180^{\circ} - x) = -\cos x]$$

$$= \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2} = \frac{3}{16}$$

$$= RHS$$

Hence proved.

35. First we find the median from the below table.

Class	Frequency f _i	Cumulative frequency (cf)	Midpoint x _i
95 -105	9	9	100
105 - 115	13	22	110
115 - 125	25	47	120
125 - 135	30	77	130
135 - 145	13	90	140
145 - 155	10	100	150
	$N=\Sigma f_i=100$		

Thus N =100 and therefore, $\frac{N}{2}$ = 50

 \Rightarrow median class is 125 - 135

$$\Rightarrow$$
 L = 125, f = 30, h = 10 and c = 47.

Therefore, median = L +
$$\frac{\left(\frac{N}{2}-c\right)}{f} \times h$$

= $\left\{125 + \frac{(50-47)}{30} \times 10\right\}$ = $(125 + 1)$ = 126.

$$M = 126$$
.

Now, we prepare the table given below

f _i	x _i	$ x_i-M $	$f_i \times x_i - M $
9	100	26	234
13	110	16	208
25	120	6	150
30	130	4	120
13	140	14	182
10	150	24	240
N =100			1134

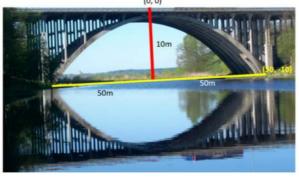
$$\therefore MD(M) = \frac{\sum f_i \times |x_i - M|}{N} = \frac{1134}{100} = 11.34.$$

Hence, the mean deviation about the median is 11.34.

Section E

36. Read the text carefully and answer the questions:

The girder of a railway bridge is a parabola with its vertex at the highest point, 10 m above the ends. Its span is 100 m.



(i) From the diagram equation of parabola is $x^2 = -4ay$

Vertex is 10m hight and spam is 100m

parabola passes through (50, -10)

Hence,
$$50^2 = -4a(-10)$$

$$\Rightarrow$$
 a = $\frac{2500}{40}$ = 62.5

Hence coordinates of focus = (-a, 0) = (-62.5, 0)

(ii) Equation of parabola is x^2 = -4ay and $a=\frac{2500}{40}=62.5$

Equation is
$$x^2 = -4\left(\frac{2500}{40}\right)y$$

$$\Rightarrow$$
 $x^2 = -250v$

Length of latus rectum is $4a = 4 \times 62.5 = 250m$

(iii)Equation parabola $x^2 = -250y$

Coordinates of the point at 20 m from mid point = (20, y)

Substituting in the equation of parabola

$$\Rightarrow$$
 400 = -250v

$$\Rightarrow y = \frac{-400}{250} = -1.6$$

height of the bridge = 10 - 1.6 = 8.4m

OR

vertex of parabola is (0, 0) and focus is (0, -62.5)

$$\Rightarrow$$
 (0, -62.5) is center and (0, 0) is on the circle

$$\Rightarrow$$
 r = 0 - (-62.5) = 62.5 m

37. Read the text carefully and answer the questions:

Ratan wants to open an RD for the marriage of his daughter, He visited the branch of SBI at sector 3, Gurgaon. There he made an agreement with the bank.



According to this agreement, he would deposit \P 100 \times n³ every month (here n = 1 to 15). Other terms and conditions are as follows:

- I. He has to pay a minimum of six instalments.
- II. If he continues the deposit up to 15 months then the bank will pay 20% extra as a bonus.
- III. If he breaks the deposit after 6 months then the bank will pay 10% extra as a bonus
- IV. If he breaks the deposit after 10 months then the bank will pay 15% extra as a bonus.
- V. No other interest will be paid by the bank.
 - (i) **(d)** ₹ 14,40,000

Explanation: ₹ 14,40,000

(ii) (c) ₹ 17,28,000

Explanation: ₹ 17,28,000

Explanation: ₹ 3,47,875

OR

(b) ₹ 50,715

Explanation: ₹ 50,715

38. Read the text carefully and answer the questions:

In an University, out of 100 students 15 students offered Mathematics only, 12 students offered Statistics only, 8 students offered only Physics, 40 students offered Physics and Mathematics, 20 students offered Physics and Statistics, 10 students offered Mathematics and Statistics, 65 students offered Physics.



(i) Let 'x' be number of students opted for all three subjects.

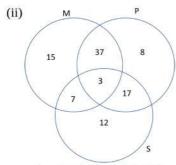
only
$$n(P) = n(P) - n(P \cap M) - n(P \cap S) + n(P \cap M \cap S)$$

$$8 = 65 - 40 - 20 + x$$

$$\Rightarrow$$
 x = 8 - 5 = 3

$$\Rightarrow$$
 n(P \cap M \cap S) = 3

Hence number of students offered for all three subjects = 3



only $n(M \cap S) = n(M \cap S) - n(M \cap S \cap P)$

$$\Rightarrow$$
 only n(M \cap S) = 10 - 3 = 7

Hence the number of students who offered mathematics and statistics but not physics = 7