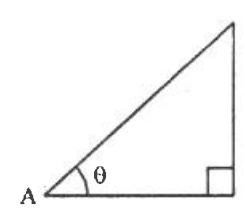
36. त्रिकोणमिती (TRIGONOMETRY)

आवश्यक तथ्य एवं सूत्र

त्रिकोणमितीय अनुपात : माना ΔABC में $\angle B = 90°$ तथा $\angle A = \theta$. तब



(*i*) $\sin\theta = \frac{\text{लम्ब}}{\text{कर्ण}} = \frac{BC}{AC}$. (*ii*) $\cos\theta = \frac{\text{आधार}}{\text{कर्ण}} = \frac{AB}{AC}$.

(*iii*) $\tan\theta = \frac{\text{लम्ब}}{\text{आधार}} = \frac{BC}{AB}$. (*iv*) $\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{AC}{BC}$.

(*v*) $\sec\theta = \frac{1}{\cos\theta} = \frac{AC}{AB}$. (*vi*) $\cot\theta = \frac{1}{\tan\theta} = \frac{AB}{BC}$.

त्रिकोणमितीय समिकायें

(*i*) $\sin^2\theta + \cos^2\theta = 1$ (*ii*) $1 + \tan^2\theta = \sec^2\theta$ (*iii*) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

(*iv*) $\tan\theta = \frac{\sin\theta}{\cos\theta}$ (*v*) $\cot\theta = \frac{\cos\theta}{\sin\theta}$

(*i*) $\sin(90° - \theta) = \cos\theta$ (*ii*) $\cos(90° - \theta) = \sin\theta$ (*iii*) $\tan(90° - \theta) = \cot\theta$

(*iv*) $\operatorname{cosec}(90° - \theta) = \sec\theta$ (*v*) $\sec(90° - \theta) = \operatorname{cosec}\theta$ (*vi*) $\cot(90° - \theta) = \tan\theta$

कुछ विशेष कोणों के त्रिकोणमितीय अनुपात

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\operatorname{cosec}\theta$	$\sec\theta$	$\cot\theta$
0°	0	1	0	परिभाषित नहीं	1	परिभाषित नहीं
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$
90°	1	0	परिभाषित नहीं	1	परिभाषित नहीं	0

योग सूत्र :

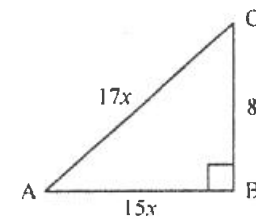
(*i*) $\sin(A + B) = \sin A \cos B + \cos A \sin B$ (*ii*) $\cos(A + B) = \cos A \cos B - \sin A \sin B$

(*iii*) $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$.

साधित उदाहरण

उदाहरण 1. यदि θ न्यूनकोण है तथा $\tan\theta = \frac{8}{15}$ हो, तो $\operatorname{cosec}\theta$ का मान ज्ञात कीजिए.

हल : एक ΔABC लीजिए जिसमें $\angle B = 90°$ तथा $\angle A = \theta°$.



$$\tan\theta = \frac{8}{15} = \frac{8x}{15x} = \frac{\text{लम्ब}}{\text{आधार}} = \frac{BC}{AB}.$$

$$AC^2 = AB^2 + BC^2 = (15x)^2 + (8x)^2 = (225x^2 + 64x^2) = 289x^2$$

$$\Rightarrow AC = \sqrt{289x^2} = 17x$$

$$\Rightarrow \operatorname{cosec}\theta = \frac{AC}{BC} = \frac{17x}{8x} = \frac{17}{8}.$$

उदाहरण 2. $(3\cos^2 30° + \sec^2 30° + 2\cos 0° + 3\sin 90° - \tan^2 60°) = ?$

हल : दिया गया व्यंजक $= 3\times\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{2}{\sqrt{3}}\right)^2 + 2\times 1 + 3\times 1 - (\sqrt{3})^2$

$$= 3\times\frac{3}{4} + \frac{4}{3} + 2 + 3 - 3 = \frac{9}{4} + \frac{4}{3} + 2 = \frac{(27+16+24)}{12} = \frac{67}{12} = 5\frac{7}{12}.$$

उदाहरण 3. निम्नलिखित में से प्रत्येक का मान ज्ञात कीजिए.

$$(i)\ \frac{\sin 15°}{\cos 75°} \qquad (ii)\ \frac{\cos 16°}{\sin 74°} \qquad (iii)\ \frac{\tan 35°}{\cot 55°}$$

हल : (i) $\dfrac{\sin 15°}{\cos 75°} = \dfrac{\sin 15°}{\cos(90°-15°)} = \dfrac{\sin 15°}{\sin 15°} = 1.$

(ii) $\dfrac{\cos 16°}{\sin 74°} = \dfrac{\cos 16°}{\sin(90°-16°)} = \dfrac{\cos 16°}{\cos 16°} = 1.$

(iii) $\dfrac{\tan 35°}{\cot 55°} = \dfrac{\tan 35°}{\cot(90°-35°)} = \dfrac{\tan 35°}{\tan 35°} = 1.$

उदाहरण 4. $\sin(50° + \theta) - \cos(40° - \theta) = ?$

हल : दिया गया व्यंजक

$$= \sin[90° - (40° - \theta)] - \cos(40° - \theta) = \cos(40° - \theta) - \cos(40° - \theta) = 0.$$

उदाहरण 5. यदि $4\cot\theta = 3$ हो, तो $\left(\dfrac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta}\right) = ?$

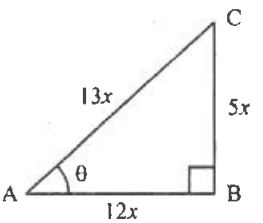
हल : दिया है : $\cot\theta = \frac{3}{4}$.

$$\therefore \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} = \frac{1 - \cot\theta}{1 + \cot\theta}$$ [अंश तथा हर को $\sin\theta$ से भाग देने पर]

$$= \frac{\left(1 - \frac{3}{4}\right)}{\left(1 + \frac{3}{4}\right)} = \frac{(1/4)}{(7/4)} = \frac{1}{7}.$$

उदाहरण 6. यदि $13 \sin\theta = 5$ हो, तो $\dfrac{(5\sin\theta - 2\cos\theta)}{\tan\theta} = ?$

हल : $\sin\theta = \dfrac{5}{13} = \dfrac{5x}{13x} = \dfrac{\text{लम्ब}}{\text{कर्ण}}$.



आधार $= \sqrt{(13x)^2 - (5x)^2} = \sqrt{169x^2 - 25x^2} = \sqrt{144x^2} = 12x.$

$\therefore \cos\theta = \dfrac{12x}{13x} = \dfrac{12}{13}, \tan\theta = \dfrac{5x}{12x} = \dfrac{5}{12}.$

$$\therefore \frac{(5\sin\theta - 2\cos\theta)}{\tan\theta} = \frac{\left(5\times\frac{5}{13} - 2\times\frac{12}{13}\right)}{(5/12)} = \frac{\left(\frac{25}{13} - \frac{24}{13}\right)}{(5/12)} = \left(\frac{1}{13}\times\frac{12}{5}\right) = \frac{12}{65}.$$

प्रश्नमाला 36

निम्नलिखित प्रश्नों में से प्रत्येक में ठीक उत्तर को चिन्हांकित (✓) कीजिए :

1. यदि ΔABC में $\angle B = 90°$ हो, तो निम्न में से सही कथन कौन-सा है ? **(रेलवे, 2006)**

(a) $\dfrac{AB}{AC} = \sin C$ (b) $\dfrac{BC}{AB} = \tan C$ (c) $\dfrac{AC}{AB} = \cos C$ (d) $\dfrac{AB}{BC} = \sec C$

2. यदि $\sin\theta = \dfrac{1}{\sqrt{2}}$ तथा $\cos\theta = \dfrac{1}{\sqrt{2}}$ हो, तो $\cot\theta = ?$ **(रेलवे, 2006)**

(a) 0 (b) 1 (c) $\sqrt{2}$ (d) 2

3. यदि $\sin\theta = \dfrac{\sqrt{5}}{3}$ तथा $\cos\theta = \dfrac{1}{3}$ हो, तो $\tan\theta = ?$ **(रेलवे, 2006)**

(a) $\sqrt{5}$ (b) $\dfrac{\sqrt{5}}{9}$ (c) $\dfrac{3}{\sqrt{5}}$ (d) $\dfrac{\sqrt{5}}{6}$

4. यदि $\tan\theta = \dfrac{3}{4}$ तथा θ एक न्यून कोण हो, तो $\text{cosec}\,\theta = ?$

(a) $\dfrac{5}{4}$ (b) $\dfrac{5}{3}$ (c) $\dfrac{4}{3}$ (d) $\dfrac{4}{5}$

5. यदि $\text{cosec}\,\theta = \sqrt{10}$ हो तथा θ एक न्यून कोण हो, तो $\sec\theta = ?$

(a) $\dfrac{\sqrt{10}}{3}$ (b) $\dfrac{3}{\sqrt{10}}$ (c) $\dfrac{1}{\sqrt{10}}$ (d) $\dfrac{2}{\sqrt{10}}$

6. यदि $\sin\theta = \dfrac{\sqrt{3}}{2}$ हो, तो $(\text{cosec}\,\theta + \cot\theta) = ?$

(a) $2\sqrt{3}$ (b) $(2+\sqrt{3})$ (c) $\sqrt{3}$ (d) $\dfrac{2\sqrt{3}}{3}$

7. यदि $5\tan\theta = 4$ हो, तो $\left(\dfrac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta}\right) = ?$

(a) $\dfrac{2}{5}$ (b) $\dfrac{1}{7}$ (c) $\dfrac{2}{7}$ (d) $\dfrac{5}{7}$

8. यदि $3\cot\theta = 4$ हो, तो $\left(\dfrac{5\sin\theta + 3\cos\theta}{5\sin\theta - 3\cos\theta}\right) = ?$ **(रेलवे, 2002)**

(a) $\dfrac{1}{9}$ (b) $\dfrac{1}{3}$ (c) 3 (d) 9 (e) इनमें से कोई नहीं

9. यदि $\tan\theta = \frac{1}{\sqrt{7}}$ हो, तो $\left(\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}\right) = ?$

(a) $\frac{-3}{4}$ (b) $\frac{-2}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

10. यदि $\sin A : \cos A = 4 : 7$ हो, तो $\left(\frac{7\sin A - 3\cos A}{7\sin A + 2\cos A}\right) = ?$

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{3}{14}$ (d) $\frac{3}{2}$

11. यदि $(\tan\theta + \cot\theta) = 5$ हो, तो $(\tan^2\theta + \cot^2\theta) = ?$

(a) 25 (b) 27 (c) 23 (d) 24

12. $\frac{\sin 30^\circ \cos 45^\circ}{\tan 60^\circ} = ?$ **(रेलवे, 2006)**

(a) $\frac{2}{\sqrt{3}}$ (b) $2\sqrt{3}$ (c) $\sqrt{\frac{2}{3}}$ (d) $\frac{\sqrt{6}}{12}$

13. $\frac{(\cos 60^\circ + \sin 60^\circ)}{(\cos 60^\circ - \sin 60^\circ)} = ?$ **(एम०बी०ए० 2002)**

(a) $(\sqrt{3}-2)$ (b) $-(\sqrt{3}+2)$ (c) $(\sqrt{3}+2)$ (d) -1

14. $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ = ?$

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) 1 (d) 2

15. $\frac{\sin 30^\circ - \cos 60^\circ + \tan 45^\circ}{\cos 30^\circ - \tan 45^\circ + \sin 90^\circ} = ?$

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{2\sqrt{3}}{3}$ (c) $\frac{4\sqrt{3}}{3}$ (d) $\sqrt{3}$

16. $\sin^2 60^\circ + \cos^2 30^\circ + \cot^2 45^\circ + \sec^2 60^\circ - \text{cosec}^2 30^\circ = ?$

(a) $\frac{5}{2}$ (b) $\frac{3}{2}$ (c) $\frac{1}{2}$ (d) $\frac{7}{2}$

17. $(2\sin^2 30^\circ - 3\cos^2 45^\circ + \tan^2 60^\circ) = ?$

(a) 1 (b) 2 (c) 4 (d) 5

18. $\left(\sin^2 30^\circ \cos^2 45^\circ + 4\tan^2 30^\circ + \frac{1}{2}\sin^2 90^\circ - 2\cos^2 90^\circ\right) = ?$

(a) $\frac{15}{8}$ (b) $\frac{13}{12}$ (c) $\frac{47}{24}$ (d) $\frac{49}{24}$

19. यदि $x \tan 45^\circ \cos 60^\circ = \sin 60^\circ \cot 60^\circ$ हो, तो $x = ?$

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{3}$ (c) $\frac{1}{2}$ (d) 1

20. यदि $\tan^2 45^\circ - \cos^2 60^\circ = x \sin 45^\circ \cos 45^\circ \tan 60^\circ$ हो, तो $x = ?$ **(रेलवे, 2006)**

(a) $\frac{1}{2}$ (b) $\frac{3}{\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) 2

21. यदि $x \sin^2 60° - \frac{1}{2} \sec 60° \tan^2 30° + \frac{4}{3} \sin^2 45° \tan^2 60° = 0$ हो, तो $x = ?$

(*a*) $-2\frac{2}{9}$ (*b*) $-4\frac{2}{9}$ (*c*) $-8\frac{2}{9}$ (*d*) $2\frac{2}{9}$

22. $\frac{\sin 19°}{\cos 71°} + \frac{\cos 73°}{\sin 17°} = ?$

(*a*) 0 (*b*) $\frac{1}{2}$ (*c*) 1 (*d*) 2

23. $\sin 20° \cos 70° + \cos 20° \sin 70° = ?$

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) इनमें से कोई नहीं

24. $\cos 64° \cos 26° - \sin 64° \sin 26° = ?$

(*a*) 0 (*b*) 1 (*c*) $2 \sin 26°$ (*d*) $2 \cos 64°$

25. $\sin 53° \cos 37° + \cos 53° \sin 37° = ?$

(*a*) 2 (*b*) $\frac{3}{2}$ (*c*) 1 (*d*) 0

26. $\sin^2 25° + \sin^2 65° = ?$

(*a*) 1 (*b*) 0 (*c*) $2 \sin^2 65°$ (*d*) $2 \cos^2 65°$

27. $\cos^2 75° + \cos^2 15° = ?$

(*a*) 0 (*b*) 1 (*c*) $\frac{2}{\sqrt{3}}$ (*d*) $\frac{\sqrt{3}}{2}$

28. $\sin^2 20° + \sin^2 70° - \tan^2 45° = ?$

(*a*) 0 (*b*) 2 (*c*) 1 (*d*) $\frac{1}{2}$

29. $\cos 74° - \sin 16° = ?$

(*a*) 0 (*b*) 1 (*c*) $2 \sin 16°$ (*d*) इनमें से कोई नहीं

30. $\tan 35° \tan 40° \tan 45° \tan 50° \tan 55° = ?$

(*a*) 0 (*b*) $\frac{\sqrt{3}}{2}$ (*c*) 1 (*d*) इनमें से कोई नहीं

31. $\cos 0° = ?$

(*a*) 0 (*b*) 1 (*c*) परिभाषित नहीं (*d*) इनमें से कोई नहीं

32. यदि $\sin 42° = x$ हो, तो $\cos 48° = ?$

(*a*) $-x$ (*b*) $x + 1$ (*c*) x (*d*) $\frac{1}{x}$

33. $\left(\frac{3\pi}{5}\right)$ radians $= ?$ **(एम०बी०ए० 2001)**

(*a*) 54° (*b*) 81° (*c*) 100° (*d*) 108°

34. यदि $(\cos \theta + \sec \theta) = \frac{5}{2}$ हो, तो $(\cos^2 \theta + \sec^2 \theta) = ?$

(*a*) $\frac{33}{4}$ (*b*) $\frac{21}{4}$ (*c*) $\frac{29}{4}$ (*d*) $\frac{17}{4}$

35. यदि $\cos\theta = \frac{5}{13}$ हो तथा θ एक न्यून कोण हो, तो $\frac{\cos\theta + 5\cot\theta}{\text{cosec}\,\theta - \cos\theta} = ?$

(*a*) $\frac{155}{109}$ (*b*) $\frac{169}{109}$ (*c*) $\frac{395}{109}$ (*d*) $\frac{385}{109}$

36. $(\cos\theta - \sin\theta)^2 + (\cos\theta + \sin\theta)^2 = ?$

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) $4\sin\theta\cos\theta$

37. $\left\{\frac{\sin\theta}{(1+\cos\theta)} + \frac{(1+\cos\theta)}{\sin\theta}\right\} = ?$

(*a*) 2 (*b*) $2\,\text{cosec}\,\theta$ (*c*) $4\sec\theta\,\text{cosec}\,\theta$ (*d*) इनमें से कोई नहीं

38. $\frac{1}{(1+\tan^2\theta)} + \frac{1}{(1+\cot^2\theta)} = ?$

(*a*) $\frac{1}{4}$ (*b*) $\frac{1}{2}$ (*c*) 1 (*d*) 2 (*e*) इनमें से कोई नहीं

39. $\sqrt{\frac{1-\cos A}{1+\cos A}} = ?$

(*a*) $(\text{cosec}\,A - \cot A)$ (*b*) 0

(*c*) $(\sec A - \cot A)$ (*d*) 1

40. $\sqrt{\frac{1-\sin A}{1+\sin A}} = ?$

(*a*) $(\cos A - \tan A)$ (*b*) $(\cos A - \cot A)$ (*c*) $(\sec A - \cot A)$ (*d*) $(\sec A - \tan A)$

41. $\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} = ?$

(*a*) $\sec\theta$ (*b*) $2\sec\theta$ (*c*) $\frac{1}{2}\cos\theta$ (*d*) $2\cos\theta$ (*e*) इनमें से कोई नहीं

42. $\frac{\sin A + \sin B}{\cos A - \cos B} + \frac{\cos A + \cos B}{\sin A - \sin B} = ?$ **(रेलवे, 2003)**

(*a*) 0 (*b*) $\sin A\cos B$ (*c*) $2\cos A\cos B$ (*d*) $\tan A\tan B$

43. यदि θ न्यून कोण हो, तो $(\sin\theta + \cos\theta)$ का अधिकतम मान क्या है ?

(*a*) 1 (*b*) $\sqrt{2}$ (*c*) 2 (*d*) $2\sqrt{2}$

44. $\tan 1° \tan 2° \tan 3° \ldots\ldots \tan 45° \tan 46° \ldots\ldots \tan 87° \tan 88° \tan 89° = ?$

(*a*) 0 (*b*) 1 (*c*) $\sqrt{3}$ (*d*) $\frac{1}{\sqrt{3}}$

45. $\frac{\tan 50° + \sec 50°}{\cot 40° + \text{cosec}\,40°} = ?$

(*a*) 1 (*b*) 0 (*c*) $2\sin 50°$ (*d*) $2\,\text{cosec}\,40°$

46. $\frac{\sin\theta}{(1-\cos\theta)} = ?$

(*a*) $(\sec\theta + \tan\theta)$ (*b*) $(\text{cosec}\,\theta + \cot\theta)$ (*c*) $(\cos\theta + \sin\theta)$ (*d*) इनमें से कोई नहीं

47. $\sin^2 60° + \cos^2 30° + \cot^2 45° + \sec^2 60° - \text{cosec}^2 30° = ?$

(*a*) 2 (*b*) $\frac{3}{2}$ (*c*) 3 (*d*) $\frac{5}{2}$

48. $\tan 90° = ?$

(*a*) 1 (*b*) 0 (*c*) परिभाषित नहीं है (*d*) इनमें से कोई नहीं

49. $\cot 90° = ?$

(*a*) 1 (*b*) 0 (*c*) परिभाषित नहीं है (*d*) इनमें से कोई नहीं

50. $\frac{\sin\theta}{\cos(90°-\theta)} + \frac{\cos\theta}{\sin(90°-\theta)} = ?$

(*a*) 2 (*b*) 0 (*c*) $\frac{1}{4}$ (*d*) 1

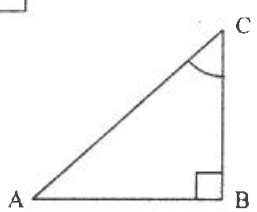
उत्तरमाला

1. (*a*)	**2.** (*b*)	**3.** (*a*)	**4.** (*b*)	**5.** (*a*)	**6.** (*c*)	**7.** (*b*)	**8.** (*d*)	**9.** (*d*)	**10.** (*a*)
11. (*c*)	**12.** (*d*)	**13.** (*b*)	**14.** (*c*)	**15.** (*b*)	**16.** (*a*)	**17.** (*b*)	**18.** (*c*)	**19.** (*d*)	**20.** (*c*)
21. (*a*)	**22.** (*d*)	**23.** (*b*)	**24.** (*a*)	**25.** (*c*)	**26.** (*a*)	**27.** (*b*)	**28.** (*a*)	**29.** (*a*)	**30.** (*c*)
31. (*b*)	**32.** (*c*)	**33.** (*d*)	**34.** (*d*)	**35.** (*d*)	**36.** (*c*)	**37.** (*b*)	**38.** (*c*)	**39.** (*a*)	**40.** (*d*)
41. (*b*)	**42.** (*a*)	**43.** (*b*)	**44.** (*b*)	**45.** (*a*)	**46.** (*b*)	**47.** (*d*)	**48.** (*c*)	**49.** (*b*)	**50.** (*b*)

दिये गये प्रश्नों के हल

1. $\angle C$ के लिए, आधार $= BC$, लम्ब $= AB$ तथा कर्ण $= AC$.

$\therefore \sin C = \frac{\text{लम्ब}}{\text{कर्ण}} = \frac{AB}{AC}$.



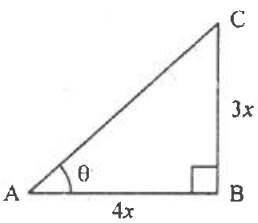
2. $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{(1/\sqrt{2})}{(1/\sqrt{2})} = 1$.

3. $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{(\sqrt{5}/3)}{(1/3)} = \sqrt{5}$.

4. $\tan\theta = \frac{3x}{4x} = \frac{\text{लम्ब}}{\text{आधार}} = \frac{BC}{AB}$.

$\therefore AC^2 = (AB^2 + BC^2) = (4x)^2 + (3x)^2 = 25x^2$

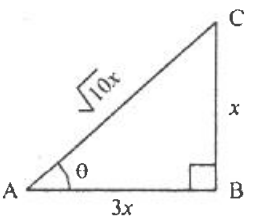
$\Rightarrow AC = \sqrt{25x^2} = 5x \Rightarrow \text{cosec}\,\theta = \frac{AC}{BC} = \frac{5x}{3x} = \frac{5}{3}$.



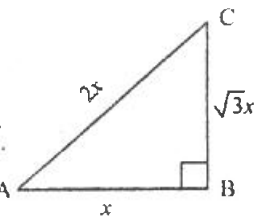
5. $\text{cosec}\,\theta = \frac{\sqrt{10}x}{x} = \frac{\text{कर्ण}}{\text{लम्ब}} = \frac{AC}{BC}$

$AB^2 = (AC^2 - BC^2) = (10x^2 - x^2) = 9x^2 \Rightarrow AB = 3x$.

$\therefore \sec\theta = \frac{AC}{AB} = \frac{\sqrt{10}x}{3x} = \frac{\sqrt{10}}{3}$.



6. $\sin\theta = \frac{\sqrt{3}}{2} = \frac{\sqrt{3}x}{2x} = \frac{\text{लम्ब}}{\text{कर्ण}} = \frac{BC}{AC}$



$AB^2 = (AC^2 - BC^2) = (4x^2 - 3x^2) = x^2 \Rightarrow AB = x.$

$\therefore (\text{cosec}\,\theta + \cot\theta) = \left(\frac{AC}{BC} + \frac{AB}{BC}\right) = \left(\frac{2x}{\sqrt{3}x} + \frac{x}{\sqrt{3}x}\right) = \left(\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right) = \frac{3}{\sqrt{3}} = \sqrt{3}.$

7. दिया है : $\tan\theta = \frac{4}{5}.$

अब, $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta} = \frac{5\tan\theta - 3}{5\tan\theta + 3}$ [अंश तथा हर को $\cos\theta$ से भाग देने पर]

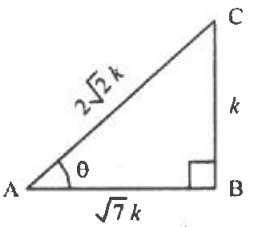
$$= \frac{\left(5\times\frac{4}{5} - 3\right)}{\left(5\times\frac{4}{5} + 3\right)} = \frac{(4-3)}{(4+3)} = \frac{1}{7}.$$

8. दिया है : $\cot\theta = \frac{4}{3}.$

$\therefore \frac{5\sin\theta + 3\cos\theta}{5\sin\theta - 3\cos\theta} = \frac{5 + 3\cot\theta}{5 - 3\cot\theta}$ [अंश तथा हर को $\sin\theta$ से भाग देने पर]

$$= \frac{5 + 3\times\frac{4}{3}}{5 - 3\times\frac{4}{3}} = \frac{5+4}{5-4} = \frac{9}{1} = 9.$$

9. $\tan\theta = \frac{BC}{AB} = \frac{1}{\sqrt{7}} = \frac{k}{\sqrt{7}k}.$



$AC^2 = (AB^2 + BC^2) = (7k^2 + k^2) = 8k^2 \Rightarrow AC = 2\sqrt{2}k.$

$\sec\theta = \frac{AC}{AB} = \frac{2\sqrt{2}k}{\sqrt{7}k} = \frac{2\sqrt{2}}{\sqrt{7}}$, $\text{cosec}\,\theta = \frac{AC}{BC} = \frac{2\sqrt{2}k}{k} = 2\sqrt{2}.$

$$\therefore \frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \frac{(2\sqrt{2})^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{(2\sqrt{2})^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2} = \frac{\left(8 - \frac{8}{7}\right)}{\left(8 + \frac{8}{7}\right)} = \frac{48}{64} = \frac{3}{4}.$$

10. माना $\sin A = 4K$. तब $\cos A = 7K$.

$$\therefore \frac{7\sin A - 3\cos A}{7\sin A + 2\cos A} = \frac{7\times 4k - 3\times 7k}{7\times 4k + 2\times 7k} = \frac{(28k - 21k)}{(28k + 14k)} = \frac{7k}{42k} = \frac{1}{6}.$$

11. $(\tan\theta + \cot\theta)^2 = 5^2 \Rightarrow \tan^2\theta + \cot^2\theta + 2\tan\theta.\cot\theta = 25$

$$\Rightarrow \tan^2\theta + \cot^2\theta + 2 = 25 \Rightarrow \tan^2\theta + \cot^2\theta = 23.$$

12. $\frac{\sin 30^\circ \cos 45^\circ}{\tan 60^\circ} = \frac{\frac{1}{2}\times\frac{1}{\sqrt{2}}}{\sqrt{3}} = \frac{1}{(2\sqrt{2})\times\sqrt{3}} = \frac{1}{2\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{6}}{12}.$

13. $\dfrac{(\cos 60^\circ + \sin 60^\circ)}{(\cos 60^\circ - \sin 60^\circ)} = \dfrac{\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)} = \dfrac{(1+\sqrt{3})}{(1-\sqrt{3})} \times \dfrac{(1+\sqrt{3})}{(1+\sqrt{3})} = \dfrac{(1+\sqrt{3})^2}{(1-3)}$

$= \dfrac{(1+3+2\sqrt{3})}{-2} = \dfrac{4+2\sqrt{3}}{-2} = -(2+\sqrt{3}) = -(\sqrt{3}+2).$

14. दिया गया व्यंजक $= \left(\dfrac{\sqrt{3}}{2} \times \dfrac{\sqrt{3}}{2} + \dfrac{1}{2} \times \dfrac{1}{2}\right) = \left(\dfrac{3}{4} + \dfrac{1}{4}\right) = \dfrac{4}{4} = 1.$

15. दिया गया व्यंजक $= \dfrac{\left(\frac{1}{2} - \frac{1}{2} + 1\right)}{\left(\frac{\sqrt{3}}{2} - 1 + 1\right)} = \dfrac{2}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}.$

16. दिया गया व्यंजक $= \left\{\left(\dfrac{\sqrt{3}}{2}\right)^2 + \left(\dfrac{\sqrt{3}}{2}\right)^2 + 1^2 + 2^2 - 2^2\right\} = \left(\dfrac{3}{4} + \dfrac{3}{4} + 1\right) = \dfrac{10}{4} = \dfrac{5}{2}.$

17. दिया गया व्यंजक $= 2 \times \left(\dfrac{1}{2}\right)^2 - 3 \times \left(\dfrac{1}{\sqrt{2}}\right)^2 + (\sqrt{3})^2 = \left(2 \times \dfrac{1}{4}\right) - \left(3 \times \dfrac{1}{2}\right) + 3 = \left(\dfrac{1}{2} - \dfrac{3}{2} + 3\right) = 2.$

18. दिया गया व्यंजक $= \left(\dfrac{1}{2}\right)^2 \times \left(\dfrac{1}{\sqrt{2}}\right)^2 + 4 \times \left(\dfrac{1}{\sqrt{3}}\right)^2 + \dfrac{1}{2} \times (1)^2 - 2 \times 0^2$

$= \left(\dfrac{1}{4} \times \dfrac{1}{2} + 4 \times \dfrac{1}{3} + \dfrac{1}{2} - 0\right) = \left(\dfrac{1}{8} + \dfrac{4}{3} + \dfrac{1}{2}\right) = \dfrac{47}{24}.$

19. $x \times 1 \times \dfrac{1}{2} = \dfrac{\sqrt{3}}{2} \times \dfrac{1}{\sqrt{3}} \Rightarrow x = 1.$

20. $(1)^2 - \left(\dfrac{1}{2}\right)^2 = x \times \dfrac{1}{\sqrt{2}} \times \dfrac{1}{\sqrt{2}} \times \sqrt{3} \Rightarrow x \times \dfrac{\sqrt{3}}{2} = \dfrac{3}{4} \Rightarrow x = \dfrac{\sqrt{3}}{2}.$

21. $x \times \left(\dfrac{\sqrt{3}}{2}\right)^2 - \dfrac{1}{2} \times 2 \times \left(\dfrac{1}{\sqrt{3}}\right)^2 + \dfrac{4}{3} \times \left(\dfrac{1}{\sqrt{2}}\right)^2 \times (\sqrt{3})^2 = 0 \Rightarrow x \times \dfrac{3}{4} - \dfrac{1}{3} + \dfrac{4}{3} \times \dfrac{1}{2} \times 3 = 0$

$\Rightarrow \dfrac{3x}{4} - \dfrac{1}{3} + 2 = 0 \Rightarrow \dfrac{3x}{4} = \left(\dfrac{1}{3} - 2\right) = \dfrac{-5}{3} \Rightarrow x = \dfrac{-5}{3} \times \dfrac{4}{3} = \dfrac{-20}{9} = -2\dfrac{2}{9}.$

22. $\dfrac{\sin 19^\circ}{\cos 71^\circ} + \dfrac{\cos 73^\circ}{\sin 17^\circ} = \dfrac{\sin 19^\circ}{\cos (90^\circ - 19^\circ)} + \dfrac{\cos 73^\circ}{\sin (90^\circ - 73^\circ)} = \left(\dfrac{\sin 19^\circ}{\sin 19^\circ} + \dfrac{\cos 73^\circ}{\cos 73^\circ}\right) = (1+1) = 2.$

23. $\sin 20^\circ \cos 70^\circ + \cos 20^\circ \sin 70^\circ = \sin (20^\circ + 70^\circ) = \sin 90^\circ = 1.$

24. $\cos 64^\circ \cos 26^\circ - \sin 64^\circ \sin 26^\circ = \cos (64^\circ + 26^\circ) = \cos 90^\circ = 0.$

25. $\sin 53^\circ \cos 37^\circ + \cos 53^\circ \sin 37^\circ = \sin (53^\circ + 37^\circ) = \sin 90^\circ = 1.$

26. $\sin^2 25^\circ + \sin^2 65^\circ = \sin^2 25^\circ + \sin^2 (90^\circ - 25^\circ) = \sin^2 25^\circ + \cos^2 25^\circ = 1.$

27. $\cos^2 75^\circ + \cos^2 15^\circ = \cos^2 75^\circ + \cos^2 (90^\circ - 75^\circ) = \cos^2 75^\circ + \sin^2 75^\circ = 1.$

28. $\sin^2 20^\circ + \sin^2 70^\circ - \tan^2 45^\circ = \sin^2 20^\circ + \sin^2 (90^\circ - 20^\circ) - \tan^2 45^\circ$

$= \sin^2 20^\circ + \cos^2 20^\circ - (\tan 45^\circ)^2 = [1 - (1)^2] = 0.$

29. $\cos 74^\circ - \sin 16^\circ = \cos (90^\circ - 16^\circ) - \sin 16^\circ = (\sin 16^\circ - \sin 16^\circ) = 0.$

30. दिया गया व्यंजक $= \tan 35° \tan 55° \tan 40° \tan 50° \tan 45°$

$= \tan 35° \tan (90° - 35°) \tan 40° \tan (90° - 40°) \tan 45°$

$= \tan 35° \cot 35° \tan 40° \cot 40° \tan 45° = (1 \times 1 \times 1) = 1.$

31. हम जानते हैं कि $\cos 0° = 1.$

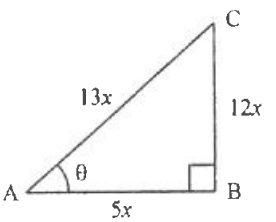
32. $\cos 48° = \cos (90° - 42°) = \sin 42° = x.$

33. $\frac{3\pi}{5}$ radians $= \left(\frac{3}{5} \times 180\right)° = 108°.$

34. $(\cos\theta + \sec\theta)^2 = \left(\frac{5}{2}\right)^2 \Rightarrow \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta = \frac{25}{4}$

$\Rightarrow \cos^2\theta + \sec^2\theta + 2 = \frac{25}{4} \Rightarrow \cos^2\theta + \sec^2\theta = \left(\frac{25}{4} - 2\right) = \frac{17}{4}.$

35. माना $\cos\theta = \frac{5}{13} = \frac{5x}{13x} = \frac{\text{आधार}}{\text{कर्ण}}$



$\therefore (AB = 5x, AC = 13x) \Rightarrow BC^2 = (AC)^2 - (AB)^2 = (13x)^2 - (5x)^2$

$\Rightarrow BC^2 = (169x^2 - 25x^2) = 144x^2 \Rightarrow BC = 12x.$

$$\therefore \frac{\cos\theta + 5\cot\theta}{\operatorname{cosec}\theta - \cos\theta} = \frac{\frac{5x}{13x} + 5 \cdot \frac{5x}{12x}}{\frac{13x}{12x} - \frac{5x}{13x}} = \frac{\frac{5}{13} + \frac{25}{12}}{\frac{13}{12} - \frac{5}{13}} = \frac{(60 + 325)}{(169 - 60)} = \frac{385}{109}.$$

36. दिया गया व्यंजक $= 2(\cos^2\theta + \sin^2\theta) = 2 \times 1 = 2.$

37. दिया गया व्यंजक $= \frac{\sin^2\theta + (1 + \cos\theta)^2}{(1 + \cos\theta)\sin\theta} = \frac{\sin^2\theta + \cos^2\theta + 1 + 2\cos\theta}{(1 + \cos\theta)\sin\theta}$

$= \frac{(2 + 2\cos\theta)}{(1 + \cos\theta)\sin\theta} = \frac{2(1 + \cos\theta)}{(1 + \cos\theta)\sin\theta} = \frac{2}{\sin\theta} = 2\operatorname{cosec}\theta.$

38. दिया गया व्यंजक $= \frac{1}{\sec^2\theta} + \frac{1}{\operatorname{cosec}^2\theta} = (\cos^2\theta + \sin^2\theta) = 1.$

39. दिया गया व्यंजक $= \frac{\sqrt{1 - \cos A}}{\sqrt{1 + \cos A}} \times \frac{\sqrt{1 - \cos A}}{\sqrt{1 - \cos A}} = \frac{(1 - \cos A)}{\sqrt{1 - \cos^2 A}} = \frac{(1 - \cos A)}{\sin A}$

$= \left(\frac{1}{\sin A} - \frac{\cos A}{\sin A}\right) = (\operatorname{cosec} A - \cot A).$

40. दिया गया व्यंजक $= \frac{\sqrt{1 - \sin A}}{\sqrt{1 + \sin A}} \times \frac{\sqrt{1 - \sin A}}{\sqrt{1 - \sin A}} = \frac{(1 - \sin A)}{\sqrt{1 - \sin^2 A}} = \frac{(1 - \sin A)}{\sqrt{\cos^2 A}}$

$= \frac{(1 - \sin A)}{\cos A} = \left(\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right) = (\sec A - \tan A).$

41. $\frac{1 + \sin\theta}{\cos\theta} + \frac{\cos\theta}{1 + \sin\theta} = \frac{(1 + \sin\theta)^2 + \cos^2\theta}{\cos\theta(1 + \sin\theta)} = \frac{1 + \sin^2\theta + 2\sin\theta + \cos^2\theta}{\cos\theta(1 + \sin\theta)}$

$= \frac{2 + 2\sin\theta}{\cos\theta(1 + \sin\theta)} = \frac{2(1 + \sin\theta)}{\cos\theta(1 + \sin\theta)} = \frac{2}{\cos\theta} = 2\sec\theta.$

42. दिया गया व्यंजक $= \frac{(\sin^2 A - \sin^2 B) + (\cos^2 A - \cos^2 B)}{(\cos A - \cos B)(\sin A - \sin B)} = \frac{(\sin^2 A + \cos^2 A) - (\sin^2 B + \cos^2 B)}{(\cos A - \cos B)(\sin A - \sin B)}$

$= \frac{(1 - 1)}{(\cos A - \cos B)(\sin A - \sin B)} = 0.$

43. $(\sin\theta+\cos\theta)$ का मान अधिकतम होगा यदि $\theta = 45°$.

$\therefore$ अभीष्ट मान $= (\sin 45° + \cos 45°) = \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{2}{\sqrt{2}} = \sqrt{2}$.

44. दिया गया व्यंजक $= \tan 1° \tan 89° \tan 2° \tan 88° \ldots\ldots \tan 44° \tan 46° \tan 45°$

$= (\tan 1° \cot 1°)\ (\tan 2° \cot 2°) \ldots\ldots (\tan 44° \cot 44°) \tan 45°$

$= \tan 45° = 1$.

45. $\dfrac{\tan 50° + \sec 50°}{\cot 40° + \operatorname{cosec} 40°} = \dfrac{\tan(90° - 40°) + \sec(90° - 40°)}{\cot 40° + \operatorname{cosec} 40°}$

$= \dfrac{\cot 40° + \operatorname{cosec} 40°}{\cot 40° + \operatorname{cosec} 40°} = 1.$

46. $\dfrac{\sin\theta}{(1-\cos\theta)} = \dfrac{\sin\theta}{(1-\cos\theta)} \times \dfrac{(1+\cos\theta)}{(1+\cos\theta)} = \dfrac{\sin\theta(1+\cos\theta)}{(1-\cos^2\theta)}$

$= \dfrac{\sin\theta(1+\cos\theta)}{\sin^2\theta} = \dfrac{(1+\cos\theta)}{\sin\theta} = \left(\dfrac{1}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta}\right) = (\operatorname{cosec}\theta + \cot\theta).$

47. $\sin^2 60° + \cos^2 30° + \cot^2 45° + \sec^2 60° - \operatorname{cosec}^2 30°$

$= \left\{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + 1^2 + 2^2 - 2^2\right\} = \left(\frac{3}{4} + \frac{3}{4} + 1\right) = \frac{(3+3+4)}{4} = \frac{10}{4} = \frac{5}{2}.$

48. $\tan 90° = \dfrac{\sin 90°}{\cos 90°} = \dfrac{1}{0}$, जो परिभाषित नहीं है.

49. $\cot 90° = \dfrac{\cos 90°}{\sin 90°} = \dfrac{0}{1} = 0.$

50. $\dfrac{\sin\theta}{\cos(90°-\theta)} + \dfrac{\cos\theta}{\sin(90°-\theta)} = \left(\dfrac{\sin\theta}{\sin\theta} + \dfrac{\cos\theta}{\cos\theta}\right) = (1+1) = 2.$