II PUC Mock Paper II JAN 2020
Subject: II PUC Mathematics (35)

Duration: 3 hours 15 minutes **Max. Marks: 100**

PART-A

I. Answer all the TEN questions: **10X1=10**

1. An operator * on z^+ is defined as a*b= $|a-b| \forall a, b \in z^+ is *$ is binary operation $z^{+?}$
2. Find the value of $\sin^{-1}$ (Sin $^{2\pi}/_3$).
3. Define skew symmetric matrix.
4. Find x, for which $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$
5. If y = sin (x^2 + 5). Find dy/dx.
6. Evaluate $\int e^x \left(\frac{x-1}{x^2} \right) dx.$
7. Find unit vector in the direction of vector $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}.$
8. Write the direction cosines of $\hat{i} + \hat{j} + \hat{k}.$
9. Define feasible region of a L.P.P.
10. If P(A) = $\frac{7}{13}, P(B) = \frac{9}{13} \& P(A \cap B) = \frac{4}{13}$, *find* P(A/ B).

PART-B

II. Answer any TEN questions: **10X2=20**

11. Find fog, if f:R→R & g: R→R are given by f(x) = cos x and g(x) = $3x^2$.
12. Prove that $3\sin^{-1}x = \text{Sin}^{-1}(3x - 4x^3)$ xε[-1/2, ½].
13. [x] is greater integer function f(x) = $|x| \& g(x) = [x]$, f:R→R & g : R → R find fog(-1/2) & g o f (-1/2)
14. Find the equation of line joining (3 1) & (9, 3) using determined.
15. Find the slope of the tangent to the curve $y = \frac{x-1}{x-2}.\ x \neq 2\ at\ x = 10$
16. Find $\frac{dy}{dx}$ *if* $ax + by^2 = \cos$ y .
17. Find the approximate change in the volume of a cube of side x meters caused by increasing the side by 3 %.
18. Integrate $\frac{\cot^6 \sqrt{x} \operatorname{cosec}^2 \sqrt{x}}{\sqrt{x}}$ with respect to x.
19. Evaluate $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx.$
20. Find the order and degree, of the differential equation $\frac{d^4y}{dx^4} + Sin\left(\frac{d^3y}{dx^3} \right) = 0.$
21. If two vectors $\vec{a} \& \vec{b}$ such that $|\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a}.\vec{b} = 4,$ *find* $|\vec{a} - \vec{b}|$.
22. Find the area of the parallelogram whose adjacent sides are determined by the vectors. $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}. \& \vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}.$

23. Show that the lines $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$ & $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ are perpendicular to each other.
24. Find the probability distribution of number of heads in two tosses of a coin.

PART-C

III. Answer any TEN questions: 10X3=30

25. Let Z be the set of all integers & R is the relation on z defined as R = {(a,b): a, bεz & a-b is divisible by 5} Prove that R is an equivalence relation.
26. Prove that $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$.
27. Using elementary transformations, find the inverse of the matrix A = $\begin{bmatrix}1 & 3\\ 2 & 7\end{bmatrix}$.
28. Differentiate $\sin^2 x$ with respect to $e^{\cos x}$.
29. Find 2 positive numbers. x and y such that x+y = 60 and xy^3 is maximum.
30. Find all the points of local maxima and minima of the function $f(x) = 2x^3 - 6x^2 + 6x + 5$.
31. Evaluate $\int \sin 3x \cos 4x\, dx$.
32. Evaluate $\int e^x \sin x \, dx$.
33. Determine the area of the region bounded by $y^2 = x$ and the lines x=1, x = 4 and x-axis.
34. Find the equation of the curve passing through the point (1,1), given that the slope of the tangent to the curve at any point is $\frac{x}{y}$.
35. Find the equation of plane that contains the point (1,-1,2) & is perpendicular to each of the planes 2x + 3y – 2z = 5 & x +2y -3z = 8.
36. Prove that $[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}] = 2\,[\vec{a}\ \vec{b}\ \vec{c}]$.
37. Find the equation of the plane passing through the intersection of the planes 3x – y + 2 z – 4 = 0 x + y + z -2 = 0 and the point (2, 2,1).
38. A bag contains 4 red and 4 block balls another bag contains 2 red and 6 block balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

PART-D

IV. Answer any SIX of the following: 6X5=30

39. Let A = R- {3} & B= R- {1}. consider the function f : A→B defined by f(x) $=\frac{x-2}{x-3}$ show that f is invertible & write the inverse of f.
40. If A = $\begin{bmatrix}1 & +1 & -1\\ 2 & 0 & 3\\ 3 & -1 & 2\end{bmatrix}$ $B=\begin{bmatrix}1 & 3\\ 0 & 2\\ -1 & 4\end{bmatrix}$ & $C=\begin{bmatrix}1 & 2 & 3 & -4\\ 2 & 0 & -2 & 1\end{bmatrix}$ find A(BC) & (AB) C & show that A(BC) = (AB)C
41. Solve by matrix method.
    x + y + z = 6
    y+3z=11
    x-2y+z=0
42. If $y = Ae^{mx} + Be^{nx}$. Prove that $\frac{d^2y}{dx^2}-(m+n)\frac{dy}{dx}+mny=0$.

43. The length x of a rectangle is decreasing at the rate of 3cm/min and the width y is increasing at the rate of 2cm/min. When x = 10 cm and y = 6 cm find the rate of change of permeter and the area of rectangle.

44. Find the integral of $\frac{1}{\sqrt{a^2+x^2}}$ w.r.to x and hence evaluate $\frac{3x^2}{\sqrt{x^6+1}}dx$.

45. Using imtergation find the area of the region bounced by the trinangle whose vertices are (-1, 0) (1,3) and (3,2).

46. Solove the differential equations x log x $\frac{dy}{dx}+y=\frac{2}{3}\log x$.

47. Derive the equation of a line in space passing through two given points both in vector and cartesian sorm**.**

48. If a fair coin is tossed 10 times. Find the probabality of
i) exactly six heads &
ii) atleast six heads.

PART-E

V. Answer any one of the following: 1X10=10

49. a) minimize & maximize Z = 3x + 9y. subject to the constraints. x+ 3y ≤ 60, x + y ≥ 10, x ≤ y & x ≥ 0, y ≥ 0, by graphical method.

b) Find the value of k of f(x) = $\frac{1-\cos 2x}{1+\cos 2x}$ When x ≠0 and f (x) = k when x = 0 is a continuous function.

50. a) Prove that $\int_0^a f(x)dx = \int_0^a f(\text{a}-\text{x})dx$ then evaluate $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$.

b) Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$.

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