

Class X Session 2024-25
Subject - Mathematics (Standard)
Sample Question Paper - 2

Time Allowed: 3 hours

Maximum Marks: 80

General Instructions:

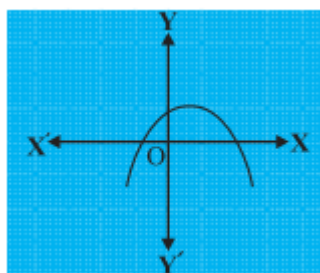
1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

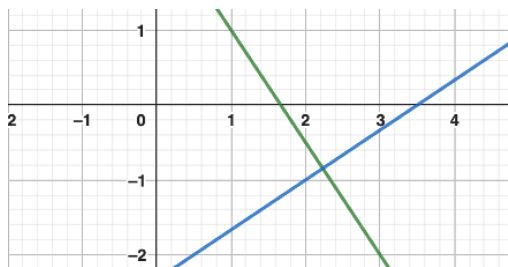
1. If two positive integers m and n can be expressed as $m = x^2y^5$ and $n = x^3y^2$, where x and y are prime numbers, then $\text{HCF}(m, n) =$ [1]

- | | |
|-------------|-------------|
| a) x^2y^2 | b) x^2y^3 |
| c) x^3y^2 | d) x^3y^3 |

2. Find the number of zeroes of $p(x)$ in the graph given below. [1]



- | | |
|------|------|
| a) 3 | b) 0 |
| c) 2 | d) 1 |
3. The pair of linear equations $3x + 2y = 5$ and $2x - 3y = 7$ are [1]



- a) consistent
b) dependent
c) inconsistent
d) independent

4. Which of the following equations has two distinct real roots? [1]

- a) $x^2 + x - 5 = 0$
b) $5x^2 - 3x + 1 = 0$
c) $4x^2 - 3x + 1 = 0$
d) $x^2 + x + 5 = 0$

5. If the second term of an AP is 13 and its fifth term is 25, then its 7th term is [1]

- a) 37
b) 33
c) 38
d) 30

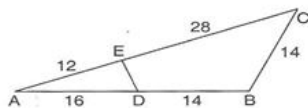
6. A line segment is of length 10 units. If the coordinates of its one end are (2, -3) and the abscissa of the other end is 10, then its ordinate is [1]

- a) -3, 9
b) 9, -6
c) 9, 6
d) 3, -9

7. The vertices of a $\triangle ABC$ are A(2, 1), B(6, -2), C(8, 9). If AD is angle bisector of $\angle BAC$, where D meets on BC, then coordinates of D are _____. [1]

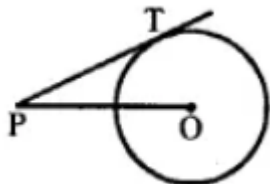
- a) (5, 2)
b) $\left(\frac{14}{3}, \frac{7}{3}\right)$
c) (4, 3)
d) $\left(\frac{20}{3}, \frac{5}{3}\right)$

8. In the given figure if $\triangle AED \sim \triangle ABC$, then DE is equal to [1]



- a) 6.5 cm
b) 5.6 cm
c) 5.5 cm
d) 7.5 cm

9. In the given figure, point P is 26 cm away from the centre O of a circle and the length PT of the tangent drawn from P to the circle is 24 cm. Then, the radius of the circle is [1]



- a) 13 cm
b) 10 cm
c) 15 cm
d) 12 cm

10. The length of the tangent drawn from a point P, whose distance from the centre of a circle is 25 cm, and the radius of the circle is 7 cm, is: [1]

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Let the positive numbers a, b, c be in A.P., then $\frac{1}{bc}, \frac{1}{ac}, \frac{1}{ab}$ are also in A.P. [1]

Reason (R): If each term of an A.P. is divided by a, b, c , then the resulting sequence is also in A.P.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

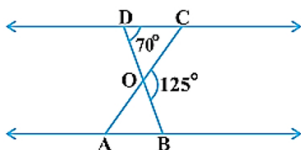
c) A is true but R is false.

d) A is false but R is true.

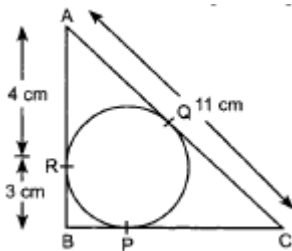
Section B

21. The difference of the square of two numbers is 45. The square of the smaller number is 4 times the larger number. Find the number. [2]

22. In Figure, $\triangle ODC \sim \triangle OBA$, $\angle BOC = 125^\circ$ and $\angle CDO = 70^\circ$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$. [2]



23. In figure, $\triangle ABC$ is circumscribing a circle. Find the length of BC. [2]



24. If $\sin X + \sin^2 X = 1$, prove that $\cos^2 X + \cos^4 X = 1$. [2]

OR

Verify that if $\tan^2 \theta + \sin \theta = \cos^2 \theta$ is an identity or not.

25. Four cows are tethered at the four corners of a square field of side 50 m such that each can graze the maximum unshared area. What area will be left ungrazed? [Take $\pi = 3.14$.] [2]

OR

A sector of a circle of radius 4 cm contains an angle of 30° . Find the area of the sector.

Section C

26. A shopkeeper has 120 litres of petrol, 180 litres of diesel and 240 litres of kerosene. He wants to sell oil by filling the three kinds of oils in tins of equal capacity. What should be the greatest capacity of such a tin? [3]

27. Find a quadratic polynomial whose sum and product of the zeroes are $-\frac{21}{8}$ and $\frac{5}{16}$ respectively. Also find the zeroes of the polynomial by factorisation. [3]

28. A fraction is such that if the numerator is multiplied by 3 and the denominator is reduced by 3, we get $\frac{18}{11}$, but if the numerator is increased by 8 and the denominator is doubled, we get $\frac{2}{5}$. Find the fraction. [3]

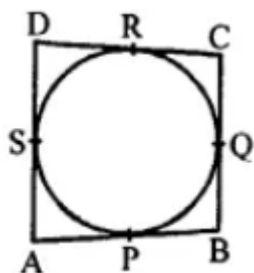
OR

A man has only 20 paise coins and 25 paise coins in his purse. If he has 50 coins in all totalling to ₹ 11.25, how many coins of each kind does he have?

29. Out of the two concentric circles, the radius of the outer circle is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circle. [3]

OR

In the given figure, a quadrilateral ABCD is drawn to circumscribe a circle such that its sides AB, BC, CD and AD touch the circle at P, Q, R and S respectively. If AB = x cm, BC = 7 cm, CR = 3 cm and AS = 5 cm, find x.

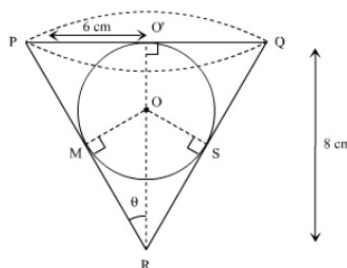


30. Prove that $\sec\theta(1 - \sin\theta)(\sec\theta + \tan\theta) = 1$ [3]
31. Five coins were simultaneously tossed 1000 times and at each toss the number of heads were observed. The number of tosses during which 0,1,2,3,4 and 5 heads were obtained are shown in the table below. Find the mean number of heads per toss. [3]

No. of heads per toss	No. of tosses
0	38
1	144
2	342
3	287
4	164
5	25
Total	1000

Section D

32. A 2-digit number is such that the product of its digits is 24. If 18 is subtracted from the number, the digits interchange their places. Find the number. [5]
- OR
- The product of Tanay's age (in years) five years ago and his age ten years later is 16. Determine Tanay's present age.
33. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio. [5]
34. A conical vessel of radius 6 cm and height 8 cm is completely filled with water. A sphere is lowered into the water and its size is such that when it touches the sides, it is just immersed as shown in Figure. What fraction of water over flows? [5]



OR

A wooden article was made by scooping out a hemisphere from each end of a solid cylinder as shown in the figure. If

the height of the cylinder is 10 cm and its base is of radius 3.5 cm, find the total surface area of the article.



35. Find the mean of the following frequency distribution:

[5]

Class	5 - 15	15 - 25	25 - 35	35 - 45	45 - 55	55 - 65
Frequency	6	11	21	23	14	5

Section E

36. Read the text carefully and answer the questions:

[4]

Akshat's father is planning some construction work in his terrace area. He ordered 360 bricks and instructed the supplier to keep the bricks in such a way that the bottom row has 30 bricks and next is one less than that and so on.



The supplier stacked these 360 bricks in the following manner, 30 bricks in the bottom row, 29 bricks in the next row, 28 bricks in the row next to it, and so on.

- In how many rows, 360 bricks are placed?
- How many bricks are there in the top row?

OR

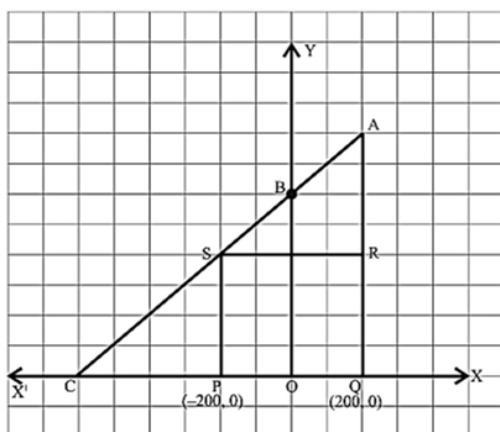
If which row 26 bricks are there?

- How many bricks are there in 10th row?

37. Read the text carefully and answer the questions:

[4]

Jagdish has a field which is in the shape of a right angled triangle AQC. He wants to leave a space in the form of a square PQRS inside the field for growing wheat and the remaining for growing vegetables (as shown in the figure). In the field, there is a pole marked as O.



- Taking O as origin, coordinates of P are $(-200, 0)$ and of Q are $(200, 0)$. PQRS being a square, what are the coordinates of R and S?

- (b) What is the area of square PQRS?

OR

If S divides CA in the ratio $K : 1$, what is the value of K, where point A is (200, 800)?

- (c) What is the length of diagonal PR in square PQRS?

38. **Read the text carefully and answer the questions:**

[4]

Totem poles are made from large trees. These poles are carved with symbols or figures and mostly found in western Canada and northwestern United States.

In the given picture, two such poles of equal heights are standing 28 m apart. From a point somewhere between them in the same line, the angles of elevation of the top of the two poles are 60° and 30° respectively.



- (a) Draw a neat labelled diagram.
(b) Find the height of the poles.

OR

Find the location of the point of observation.

- (c) If the distances of the top of the poles from the point of observation are taken as p and q , then find a relation between p and q .

Solution

Section A

1. (a) x^2y^2

Explanation: $x^2y^5 = y^3(x^2y^2)$

$$x^3y^3 = x(x^2y^2)$$

Therefore HCF (m, n) is x^2y^2

2.

(c) 2

Explanation: The number of zeroes is 2 as the graph intersects the x-axis at two points.

3. (a) consistent

Explanation: Given: $a_1 = 3$, $a_2 = 2$, $b_1 = 2$, $b_2 = -3$, $c_1 = 5$ and $c_2 = 7$

$$\text{Here, } \frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{2}{-3}$$

$$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Therefore, the pair of given linear equations is consistent.

4. (a) $x^2 + x - 5 = 0$

Explanation: In equation $x^2 + x - 5 = 0$

$$a = 1, b = 1, c = -5$$

$$\therefore b^2 - 4ac = (1)^2 - 4 \times 1 \times (-5) = 1 + 20 = 21$$

Since $b^2 - 4ac > 0$ therefore, $x^2 + x - 5 = 0$ has two distinct roots.

5.

(b) 33

Explanation: Given: $a_2 = 13$

$$\Rightarrow a + (2 - 1)d = 13$$

$$\Rightarrow a + d = 13 \dots (i)$$

$$\text{And } a_5 = 25$$

$$\Rightarrow a + (5 - 1)d = 25$$

$$\Rightarrow a + 4d = 25 \dots (ii)$$

Solving eq. (i) and (ii),

we get $a = 9$ and $d = 4$

$$\therefore a_7 = a + (7 - 1)d$$

$$= 9 + (7 - 1) \times 4$$

$$= 9 + 6 \times 4$$

$$= 9 + 24 = 33$$

6.

(d) 3, -9

Explanation: Let the ordinate of other end = y

then distance between (2, -3) and (10, y) = 10 units

$$\Rightarrow \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 10$$

$$\Rightarrow \sqrt{(10 - 2)^2 + (y + 3)^2} = 10$$

$$\Rightarrow \sqrt{(8)^2 + (y + 3)^2} = 10$$

Squaring both sides

$$(8)^2 + (y + 3)^2 = (10)^2 \Rightarrow 64 + (y + 3)^2 = 100$$

$$\Rightarrow (y + 3)^2 = 100 - 64 = 36 = (6)^2$$

$$\Rightarrow (y + 3)^2 - (6)^2 = 0 \Rightarrow (y + 3 + 6)(y + 3 - 6)$$

$$= 0 \{ \because a^2 - b^2 = (a + b)(a - b) \}$$

$$\Rightarrow (y + 9)(y - 3) = 0$$

Either $y + 9 = 0$, then $y = -9$

or $y - 3 = 0$, then $y = 3$

$$\therefore y = 3, -9$$

7.

(d) $\left(\frac{20}{3}, \frac{5}{3}\right)$

Explanation: AD is the angle bisector of $\angle BAC$.

So, by the angle bisector theorem in $\triangle ABC$, we have

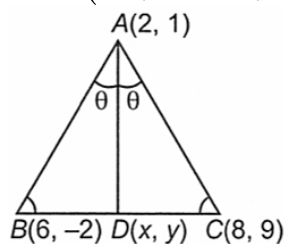
$$\frac{AB}{AC} = \frac{BD}{DC} \dots (i)$$

Now, $AB = 5$ and $AC = 10$

$$\therefore \frac{1}{2} = \frac{BD}{DC} \text{ [using (i)]}$$

Thus, D divides BC in the ratio 1 : 2.

$$\therefore D = \left(\frac{2 \times 6 + 1 \times 8}{2+1}, \frac{-2 \times 2 + 9 \times 1}{2+1} \right) = \left(\frac{20}{3}, \frac{5}{3} \right)$$



8.

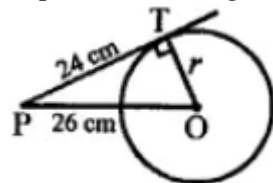
(b) 5.6 cm

Explanation: $\triangle AED \sim \triangle ABC$ (SAS Similarly) $\Rightarrow \frac{12}{30} = \frac{ED}{14} \Rightarrow ED = 5.6 \text{ cm}$

9.

(b) 10 cm

Explanation: In the given figure, point P is 26 cm away from the centre O of the circle.



Length of tangent $PT = 24 \text{ cm}$

Let radius $= r$

In right $\triangle OPT$,

$$OP^2 = PT^2 + OT^2$$

$$\Rightarrow 26^2 = 24^2 + r^2$$

$$\Rightarrow r^2 = 26^2 - 24^2 = 676 - 576 = 100 = (10)^2$$

$$r = 10$$

Radius $= 10 \text{ cm}$

10.

(b) 24 cm

Explanation: 24 cm

11.

(a) 1

Explanation: We have, $(\operatorname{cosec} \theta - \sin \theta)(\sec \theta - \cos \theta)(\tan \theta + \cot \theta)$

$$= \left(\frac{1}{\sin \theta} - \sin \theta \right) \left(\frac{1}{\cos \theta} - \cos \theta \right) \left(\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right)$$

$$= \frac{1 - \sin^2 \theta}{\sin \theta} \times \frac{1 - \cos^2 \theta}{\cos \theta} \times \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}$$

$$= \frac{\cos^2 \theta}{\sin \theta} \times \frac{\sin^2 \theta}{\cos \theta} \times \frac{1}{\sin \theta \cos \theta}$$

$$= \frac{\sin^2 \theta \cos^2 \theta}{\sin^2 \theta \cos^2 \theta} = 1$$

12. (a) $\sqrt{2}$

Explanation: Given: $\sin 45^\circ + \cos 45^\circ$

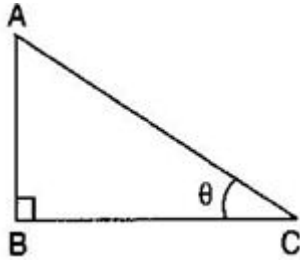
$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$$

$$= \frac{2}{\sqrt{2}} = \sqrt{2}$$

13.

(c) 60°

Explanation:



Given: Height of the pole = $AB = h$ meters And the length of the shadow of the pole = $BC = \frac{h}{\sqrt{3}}$ meters $\therefore \tan \theta = \frac{h}{\frac{h}{\sqrt{3}}}$

$$\Rightarrow \tan \theta = \sqrt{3} \Rightarrow \tan \theta = \tan 60^\circ \Rightarrow \theta = 60^\circ$$

14. (a) 231 cm^2

Explanation: The angle subtended by the arc = 60°

So, area of the sector = $\left(\frac{60^\circ}{360^\circ}\right) \times \pi r^2 \text{ cm}^2$

$$= \left(\frac{441}{6}\right) \times \left(\frac{22}{7}\right) \text{ cm}^2$$

$$= 231 \text{ cm}^2$$

15.

(d) $\frac{p}{720} \times 2\pi R^2$

Explanation: Area of the sector of angle p of a circle with radius R

$$= \frac{\theta}{360} \times \pi r^2 = \frac{p}{360} \times \pi R^2$$

$$= \frac{p}{2(360)} \times 2\pi R^2 = \frac{p}{720} \times 2\pi R^2$$

16. (a) $\frac{1}{2}$

Explanation: $\frac{1}{2}$

17.

(d) $\frac{1}{12}$

Explanation: Total number of outcomes = 36

Favorable outcomes for sum greater than 10 are $\{(5,6), (6,5), (6,6)\}$

Number of favorable outcomes = 3

$$P = \frac{3}{36} = \frac{1}{12}$$

18.

(d) 46

Explanation: Mode of 64, 60, 48, x , 43, 48, 43, 34 is 43

\therefore By definition mode is a number which has maximum frequency which is 43

$$\therefore x = 43$$

$$\therefore x + 3 = 43 + 3 = 46$$

19.

(c) A is true but R is false.

Explanation: A is true but R is false.

20. (a) Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

Section B

21. Let the larger number be x and the smaller number be y . Then,

$$x = x^2 - y^2 = 45 \dots\dots(i)$$

$$\text{and } y^2 = 4x \dots (ii)$$

substituting $y^2 = 5x$ in (i), we have

$$x^2 - 4x = 45$$

$$\Rightarrow x^2 - 4x - 45 = 0$$

$$\Rightarrow x^2 - 9x + 5x - 45 = 0$$

$$\Rightarrow x(x - 9) + 5(x - 9) = 0$$

$$\Rightarrow (x - 0)(x + 5) = 0$$

$$\Rightarrow \text{either } x - 9 = 0 \text{ or } x + 5 = 0$$

$$\Rightarrow x = 9 \text{ or } x = -5$$

\therefore smaller number is $\sqrt{36}$ or $\sqrt{-20}$ [From (ii)]

\Rightarrow smaller number is $= \pm 6$ [$\because \sqrt{-20}$ is not real]

\therefore Larger number = 9 and smaller number = ± 6

22. From the given figure,

$$\angle DOC + 125^\circ = 180^\circ \text{ [linear pair]}$$

$$\angle DOC = 55^\circ$$

Now, in $\triangle DOC$,

$$\angle DCO + \angle ODC + \angle DOC = 180^\circ \text{ [angle sum property of a triangle]}$$

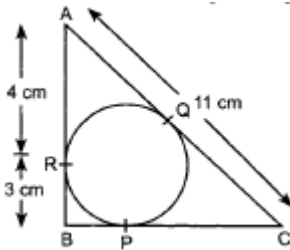
$$\angle DCO + 70^\circ + 55^\circ = 180^\circ$$

$$\angle DCO = 55^\circ$$

Now, $\triangle ODC \cong \triangle OBA$ [given]

$$\therefore \angle OAB = \angle OCD = 55^\circ$$

23. Given,



$$AR = 4 \text{ cm.}$$

$$\text{Also, } AR = AQ \Rightarrow AQ = 4 \text{ cm}$$

$$\text{Now, } QC = AC - AQ$$

$$= 11 \text{ cm} - 4 \text{ cm} = 7 \text{ cm} \dots (i)$$

$$\text{Also, } BP = BR$$

$$\therefore BP = 3 \text{ cm and } PC = QC$$

$$\therefore PC = 7 \text{ cm [From (i)]}$$

$$BC = BP + PC$$

$$= 3 \text{ cm} + 7 \text{ cm}$$

$$= 10 \text{ cm}$$

24. Given $\sin X + \sin^2 X = 1 \dots (i)$

$$\Rightarrow \sin X = 1 - \sin^2 X = \cos^2 X \dots (ii)$$

Now we show that $\cos^2 X + \cos^4 X = 1$

$$\text{L.H.S} = \cos^2 X + \cos^4 X$$

$$= 1 - \sin^2 X + (1 - \sin^2 X)^2 \text{ [Using (ii)]}$$

$$= \sin X + \sin^2 X \text{ [Using (ii)]}$$

$$= 1 \text{ [Using (i)]}$$

$$= \text{R.H.S}$$

OR

$$\tan^2 \theta + \sin \theta = \cos^2 \theta$$

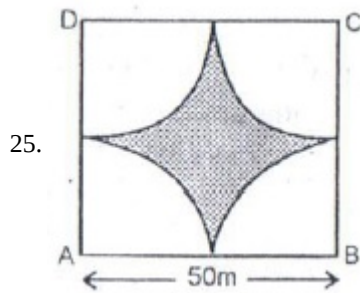
Taking $\theta = 45^\circ$, we have

$$\text{L.H.S.} = \tan^2 45^\circ + \sin 45^\circ = (1)^2 + \frac{1}{\sqrt{2}} = 1 + \frac{1}{\sqrt{2}} = \frac{\sqrt{2}+1}{\sqrt{2}}$$

$$\text{R.H.S.} = \cos^2 45 = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$$

$$\Rightarrow \text{L.H.S.} \neq \text{R.H.S.}$$

Hence given expression is not an identity.



Shaded area = area of square $- 4$ (area of sector)

$$\begin{aligned} &= \left[(50 \times 50) - \frac{4 \times \pi \times (25)^2 \times 90}{360} \right] m^2 \\ &= [2500 - 3.14 \times 25 \times 25] m^2 \\ &= [2500 - 1962.5] m^2 \\ &= 537.5 m^2 \end{aligned}$$

OR

Radius of circle = 25m

$$\theta = 90^\circ$$

$$\begin{aligned} \therefore \text{Area of sector} &= \frac{\theta}{360^\circ} \times \pi r^2 \\ &= \frac{90^\circ}{360^\circ} \times \pi \times 25 \times 25 \\ &= \frac{3\pi}{4} cm^2 \end{aligned}$$

Section C

26. The required greatest capacity is the HCF of 120, 180 and 240.

$$240 = 180 \times 1 + 60$$

$$180 = 60 \times 3 + 0$$

HCF is 60.

Now HCF of 60, 120

$$120 = 60 \times 2 + 0$$

\therefore HCF of 120, 180 and 240 is 60.

\therefore The required capacity is 60 litres.

27. We know, quadratic polynomial = $x^2 - (\text{Sum of zeroes})x + \text{Product of zeroes}$

$$\text{Given, Sum of zeroes} = -\frac{21}{8} \text{ and Product of zeroes} = \frac{5}{16}$$

$$\therefore \text{Quadratic Polynomial} = x^2 + \frac{21}{8}x + \frac{5}{16}$$

$$= \frac{1}{16}(16x^2 + 42x + 5)$$

$$\Rightarrow \text{Quadratic polynomial is } 16x^2 + 42x + 5$$

Now, we rewrite the polynomial as $16x^2 + 2x + 40x + 5$

$$= 2x \cdot (8x + 1) + 5 \cdot (8x + 1)$$

$$= (2x + 5) \cdot (8x + 1)$$

$$\text{Now, for Zeros, } (8x + 1) \cdot (2x + 5) = 0$$

$$\Rightarrow x = -\frac{1}{8}, -\frac{5}{2}$$

28. Let us suppose that the numerator be x and denominator be y

Therefore, the fraction is $\frac{x}{y}$.

Then, according to the given conditions, we have

$$\frac{3x}{y-3} = \frac{18}{11} \text{ and } \frac{x+8}{2y} = \frac{2}{5}$$

$$\Leftrightarrow 11x = 6y - 18 \text{ and } 5x + 40 = 4y$$

$$\Leftrightarrow 11x - 6y + 18 = 0 \text{ and } 5x - 4y + 40 = 0$$

By cross multiplication, we have

$$\frac{x}{(-6) \times 40 - (-4) \times 18} = \frac{-y}{11 \times 40 - 5 \times 18} = \frac{1}{11 \times (-4) - 5 \times (-6)}$$

$$\Rightarrow \frac{x}{-240+72} = \frac{-y}{440-90} = \frac{1}{-44+30}$$

$$\Rightarrow \frac{x}{-168} = \frac{y}{-350} = \frac{1}{-14}$$

$$\Rightarrow x = \frac{-168}{-14} \text{ and } y = \frac{-350}{-14}$$

$$\Rightarrow x = 12 \text{ and } y = 25$$

Therefore, the fraction is $\frac{12}{25}$.

OR

Let the number of 20 paisa coins be x and that of 25 paisa coins be y . Then,

$$x + y = 50 \dots (i)$$

Total value of 20 paisa coins = $20x$ paisa

Total value of 25 paisa coins = $25y$ paisa

$$\therefore 20x + 25y = 1125 \dots (\text{because Rs}11.25 = 1125 \text{ paisa})$$

$$\Rightarrow 4x + 5y = 225 \dots (ii)$$

Thus, we get the following system of linear equations

$$x + y - 50 = 0$$

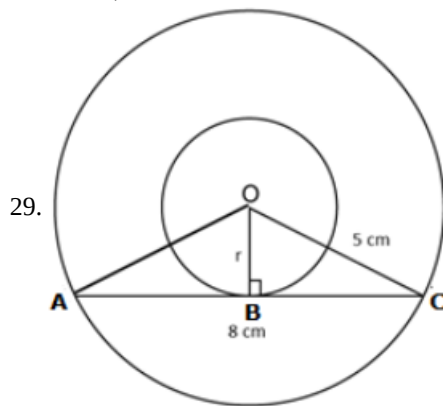
$$4x + 5y - 225 = 0$$

By using cross-multiplication, we have

$$\frac{x}{-225+250} = \frac{-y}{-225+200} = \frac{1}{5-4}$$

$$\Rightarrow \frac{x}{25} = \frac{y}{25} = \frac{1}{1} \Rightarrow x = 25 \text{ and } y = 25$$

Hence, there are 25 coins of each kind.



Since AC is a tangent to the inner circle.

$$\angle OBC = 90^\circ$$

AC is a chord of the outer circle.

We know that, the perpendicular drawn from the centre to a chord of a circle, bisects the chord.

$$AC = 2BC$$

$$8 = 2BC$$

$$\Rightarrow BC = 4 \text{ cm}$$

In $\triangle OBC$,

By Pythagoras theorem,

$$OC^2 = OB^2 + BC^2$$

$$\Rightarrow 5^2 = r^2 + 4^2$$

$$\Rightarrow r^2 = 5^2 - 4^2$$

$$\Rightarrow r^2 = 25 - 16$$

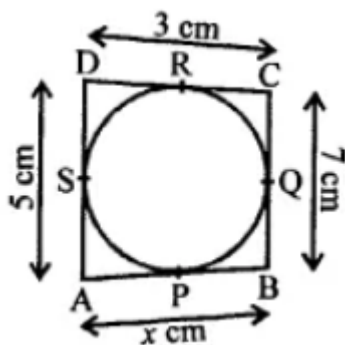
$$\Rightarrow r^2 = 9 \text{ cm}$$

$$\Rightarrow r = 3 \text{ cm}$$

OR

In the given figure, a quadrilateral ABCD is circumscribed a circle touching its sides at P, Q, R and S respectively.

AB = x cm, BC = 7 cm, CR = 3 cm and AS = 5 cm



A circle touches the sides of a quadrilateral ABCD.

$$AB + CD = BC + AD \dots(i)$$

Now, AP and AS are tangents to the circle

$$AP = AS = 5 \text{ cm} \dots(ii)$$

$$\text{Similarly, } CQ = CR = 3 \text{ cm}$$

$$BP = BQ = x - 5 = 4$$

$$BQ = BC - CQ = 7 - 3 = 4 \text{ cm}$$

$$x - 5 = 4$$

$$\Rightarrow x = 4 + 5 = 9 \text{ cm}$$

$$30. \text{ LHS} = \sec\theta(1 - \sin\theta)(\sec\theta + \tan\theta)$$

$$= \left[\sec\theta - \frac{\sin\theta}{\cos\theta} \right] \times (\sec\theta + \tan\theta)$$

$$= (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)$$

$$= \sec^2\theta - \tan^2\theta = 1 = \text{RHS}$$

31.	No of heads per toss	No of tosses	$f_i x_i$
	0	38	0
	1	144	144
	2	342	684
	3	287	861
	4	164	656
	5	25	125
		$\sum f_i = 1000$	$\sum f_i x_i = 2470$

$$\text{Mean number of heads per toss} = \frac{\sum f_i x_i}{\sum f_i} = \frac{2470}{1000} = 2.47$$

Therefore, Mean = 2.47

Section D

32. Let the ten's digit be x and the one's digit be y.

The number will be $10x + y$

Given, a product of digits is 24

$$\therefore xy = 24$$

$$\text{or, } y = \frac{24}{x} \dots(i)$$

Given that when 18 is subtracted from the number, the digits interchange their places.

$$\therefore 10x + y - 18 = 10y + x$$

$$\text{or, } 9x - 9y = 18$$

Substituting y from equation (i) in equation (ii), we get

$$9x - 9\left(\frac{24}{x}\right) = 18$$

$$\text{or, } x - \frac{24}{x} = 2$$

$$\text{or, } x^2 - 24 - 2x = 0$$

$$\text{or, } x^2 - 2x - 24 = 0$$

$$\text{or, } x^2 - 6x + 4x - 24 = 0$$

$$\text{or, } x(x - 6) + 4(x - 6) = 0$$

$$\text{or, } (x - 6)(x + 4) = 0$$

$$\text{or, } x - 6 = 0 \text{ and } x + 4 = 0$$

$$\text{or, } x = 6 \text{ and } x = -4$$

Since, the digit cannot be negative, so, $x = 6$

Substituting $x = 6$ in equation (i), we get

$$y = \frac{24}{6} = 4$$

$$\therefore \text{The number} = 10(6) + 4 = 60 + 4 = 64$$

OR

Let the present age of Tanay be x years

By the question,

$$(x - 5)(x + 10) = 16$$

$$\text{or, } x^2 + 5x - 50 = 16$$

$$\text{or, } x^2 + 5x - 66 = 0$$

$$\text{or, } x^2 + 11x - 6x - 66 = -66$$

$$x(x + 1) - 6(x - 11) = 0$$

$$(x + 11)(x - 6) = 0$$

$$= -11, 6$$

Rejecting $x = -11$, as age cannot be negative.

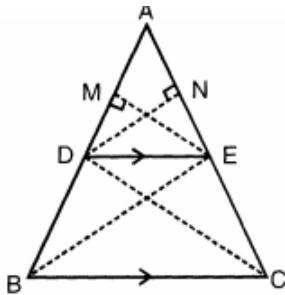
\therefore Present age of Tanay is 6 years.

33. Given: ABC is a triangle in which $DE \parallel BC$.

To prove: $\frac{AD}{BD} = \frac{AE}{CE}$

Construction: Draw $DN \perp AE$ and $EM \perp AD$., Join BE and CD.

Proof :



In $\triangle ADE$,

$$\text{Area of } \triangle ADE = \frac{1}{2} \times AE \times DN \dots(i)$$

In $\triangle DEC$,

$$\text{Area of } \triangle DCE = \frac{1}{2} \times CE \times DN \dots(ii)$$

Dividing equation (i) by equation (ii),

$$\Rightarrow \frac{\text{area}(\triangle ADE)}{\text{area}(\triangle DEC)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times CE \times DN}$$

$$\Rightarrow \frac{\text{area}(\triangle ADE)}{\text{area}(\triangle DEC)} = \frac{AE}{CE} \dots(iii)$$

Similarly, In $\triangle ADE$,

$$\text{Area of } \triangle ADE = \frac{1}{2} \times AD \times EM \dots(iv)$$

In $\triangle DEB$,

$$\text{Area of } \triangle DEB = \frac{1}{2} \times EM \times BD \dots(v)$$

Dividing equation (iv) by equation (v),

$$\Rightarrow \frac{\text{area}(\triangle ADE)}{\text{area}(\triangle DEB)} = \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times BD \times EM}$$

$$\Rightarrow \frac{\text{area}(\triangle ADE)}{\text{area}(\triangle DEB)} = \frac{AD}{BD} \dots(vi)$$

$\triangle DEB$ and $\triangle DEC$ lie on the same base DE and between two parallel lines DE and BC.

$$\therefore \text{Area}(\triangle DEB) = \text{Area}(\triangle DEC)$$

From equation (iii),

$$\Rightarrow \frac{\text{area}(\triangle ADE)}{\text{area}(\triangle DEB)} = \frac{AE}{CE} \dots(vii)$$

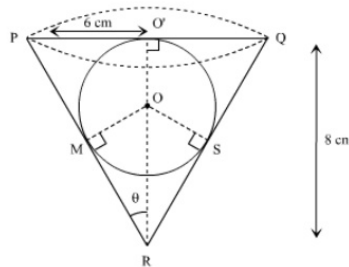
From equation (vi) and equation (vii),

$$\frac{AE}{CE} = \frac{AD}{BD}$$

∴ If a line is drawn parallel to one side of a triangle to intersect the other two sides in two points, then the other two sides are divided in the same ratio.

34. Radius (R) of conical vessel = 6 cm

Height (H) of conical vessel = 8 cm



$$\text{Volume of conical vessel } (V_c) = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \pi \times 6 \times 6 \times 8$$

$$= 96\pi \text{ cm}^3$$

Let the radius of the sphere be r cm

In right ΔPOR by pythagoras theorem We have

$$l^2 = 6^2 + 8^2$$

$$l = \sqrt{36 + 64} = 10 \text{ cm}$$

In right triangle MRO

$$\sin \theta = \frac{OM}{OR}$$

$$\Rightarrow \frac{3}{5} = \frac{r}{8-r}$$

$$\Rightarrow 24 - 3r = 5r$$

$$\Rightarrow 8r = 24$$

$$\Rightarrow r = 3 \text{ cm}$$

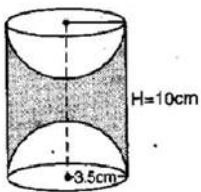
$$\therefore V_1 = \text{Volume of the sphere} = \frac{4}{3}\pi \times 3^3 \text{ cm}^3 = 36\pi \text{ cm}^3$$

$$V_2 = \text{Volume of the water} = \text{Volume of the cone} = \frac{1}{3}\pi \times 6^2 \times 8 \text{ cm}^3 = 96\pi \text{ cm}^3$$

Clearly, volume of the water that flows out of the cone is same as the volume of the sphere i.e., V_1 .

$$\therefore \text{Fraction of the water that flows out} = V_1 : V_2 = 36\pi : 96\pi = 3 : 8$$

OR



$$\text{TSA of the article} = 2\pi rh + 2(2\pi r^2)$$

$$= 2\pi (3.5)(10) + 2[2\pi (3.5)^2]$$

$$= 70\pi + 49\pi$$

$$= 119\pi$$

$$= 119 \times \frac{22}{7}$$

$$= 374 \text{ cm}^2$$

35.	CI	f_i	x_i	d_i	u_i	$f_i u_i$
	5 - 15	6	10	-20	-2	-12
	15 - 25	11	20	-10	-1	-11
	25 - 35	21	30	0	0	0

35 - 45	23	40	10	1	23
45 - 55	14	50	20	2	28
55 - 65	5	60	30	3	15
Total	80				43

$$\text{Mean} = A + \frac{\sum f_i u_i}{\sum f_i} \times h$$

$$= 30 + \frac{43}{80} \times 10$$

$$= 35.375$$

Section E

36. Read the text carefully and answer the questions:

Akshat's father is planning some construction work in his terrace area. He ordered 360 bricks and instructed the supplier to keep the bricks in such a way that the bottom row has 30 bricks and next is one less than that and so on.



The supplier stacked these 360 bricks in the following manner, 30 bricks in the bottom row, 29 bricks in the next row, 28 bricks in the row next to it, and so on.

- (i) Number of bricks in the bottom row = 30. in the next row = 29, and so on.

Therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27, ..., which is an AP with first term, $a = 30$ and common difference, $d = 29 - 30 = -1$

Suppose number of rows is n , then sum of number of bricks in n rows should be 360.

$$\text{i.e. } S_n = 360$$

$$\Rightarrow \frac{n}{2} [2 \times 30 + (n - 1)(-1)] = 360 \quad \{S_n = \frac{n}{2} (2a + (n - 1)d)\}$$

$$\Rightarrow 720 = n(60 - n + 1)$$

$$\Rightarrow 720 = 60n - n^2 + n$$

$$\Rightarrow n^2 - 61n + 720 = 0$$

$$\Rightarrow n^2 - 16n - 45n + 720 = 0 \quad [\text{by factorization}]$$

$$\Rightarrow n(n - 16) - 45(n - 16) = 0$$

$$\Rightarrow (n - 16)(n - 45) = 0$$

$$\Rightarrow (n - 16) = 0 \text{ or } (n - 45) = 0$$

$$\Rightarrow n = 16 \text{ or } n = 45$$

Hence, number of rows is either 45 or 16.

$n = 45$ not possible so $n = 16$

$$a_{45} = 30 + (45 - 1)(-1) \quad \{a_n = a + (n - 1)d\}$$

$$= 30 - 44 = -14 \quad [\because \text{The number of logs cannot be negative}]$$

Hence the number of rows is 16.

- (ii) Number of bricks in the bottom row = 30. in the next row = 29, and so on.

Therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27, ..., which is an AP with first term, $a = 30$ and common difference, $d = 29 - 30 = -1$

Suppose number of rows is n , then sum of number of bricks in n rows should be 360.

Number of bricks on top row are $n = 16$,

$$a_{16} = 30 + (16 - 1)(-1) \quad \{a_n = a + (n - 1)d\}$$

$$= 30 - 15 = 15$$

Hence, and number of bricks in the top row is 15.

OR

Number of bricks in the bottom row = 30. in the next row = 29, and so on.

Therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27, ..., which is an AP with first term, $a = 30$ and common difference, $d = 29 - 30 = -1$.

Suppose number of rows is n , then sum of number of bricks in n rows should be 360.

$$a_n = 26, a = 30, d = -1$$

$$a_n = a + (n - 1)d$$

$$\Rightarrow 26 = 30 + (n - 1) \times -1$$

$$\Rightarrow 26 - 30 = -n + 1$$

$$\Rightarrow n = 5$$

Hence 26 bricks are in 5th row.

(iii) Number of bricks in the bottom row = 30. in the next row = 29, and so on.

therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27, ..., which is an AP with first term, $a = 30$ and common difference, $d = 29 - 30 = -1$.

Suppose number of rows is n , then sum of number of bricks in n rows should be 360

Number of bricks in 10th row $a = 30, d = -1, n = 10$

$$a_n = a + (n - 1)d$$

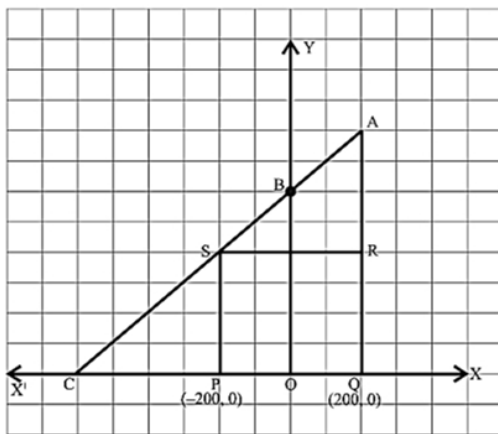
$$\Rightarrow a_{10} = 30 + 9 \times -1$$

$$\Rightarrow a_{10} = 30 - 9 = 21$$

Therefore, number of bricks in 10th row are 21.

37. Read the text carefully and answer the questions:

Jagdish has a field which is in the shape of a right angled triangle AQC. He wants to leave a space in the form of a square PQRS inside the field for growing wheat and the remaining for growing vegetables (as shown in the figure). In the field, there is a pole marked as O.



(i) Since, PQRS is a square

$$\therefore PQ = QR = RS = PS$$

$$\text{Length of PQ} = 200 - (-200) = 400$$

$$\therefore \text{The coordinates of R} = (200, 400)$$

$$\text{and coordinates of S} = (-200, 400)$$

(ii) Area of square PQRS = (side)²

$$= (PQ)^2$$

$$= (400)^2$$

$$= 1,60,000 \text{ sq. units}$$

OR

Since, point S divides CA in the ratio $K : 1$

$$\therefore \left(\frac{Kx_2 + x_1}{K+1}, \frac{Ky_2 + y_1}{K+1} \right) = (-200, 400)$$

$$\Rightarrow \left(\frac{K(200) + (-600)}{K+1}, \frac{K(800) + 0}{K+1} \right) = (-200, 400)$$

$$\Rightarrow \left(\frac{200K - 600}{K+1}, \frac{800K}{K+1} \right) = (-200, 400)$$

$$\therefore \frac{800K}{K+1} = 400$$

$$\Rightarrow 800K = 400K + 400$$

$$\Rightarrow 400K = 400$$

$$\Rightarrow K = 1$$

(iii) By Pythagoras theorem

$$(PR)^2 = (PQ)^2 + (QR)^2$$

$$\begin{aligned}
 &= 1,60,000 + 1,60,000 \\
 &= 3,20,000 \\
 &\Rightarrow PR = \sqrt{3,20,000} \\
 &= 400 \times \sqrt{2} \text{ units}
 \end{aligned}$$

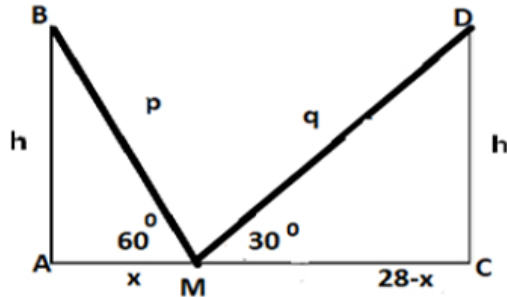
38. Read the text carefully and answer the questions:

Totem poles are made from large trees. These poles are carved with symbols or figures and mostly found in western Canada and northwestern United States.

In the given picture, two such poles of equal heights are standing 28 m apart. From a point somewhere between them in the same line, the angles of elevation of the top of the two poles are 60° and 30° respectively.



- (i) Let AB and CD be the 2 poles and M be a point somewhere between their bases in the same line.



- (ii) $\tan 60^\circ = \frac{h}{x} \Rightarrow h = x\sqrt{3}$
 $\tan 30^\circ = \frac{h}{28-x} \Rightarrow h = \frac{(28-x)}{\sqrt{3}}$
 $\therefore h = 7\sqrt{3} \text{ m}$

OR

$$\begin{aligned}
 \tan 60^\circ &= \frac{7\sqrt{3}}{x} \Rightarrow x = 7\text{m} = \text{AM} \\
 \text{MC} &= 28 - x = 21\text{m}
 \end{aligned}$$

- (iii) BM = p and DM = q

$$\begin{aligned}
 \sin 60^\circ &= \frac{h}{p} \Rightarrow h = \frac{p\sqrt{3}}{2} \\
 \sin 30^\circ &= \frac{h}{q} \Rightarrow h = \frac{q}{2} \\
 \therefore \frac{p\sqrt{3}}{2} &= \frac{q}{2} \Rightarrow q = \sqrt{3}p
 \end{aligned}$$