9 Integration

9.1 INTEGRATION

If $\frac{d}{dx}(F(x)) = f(x)$, then $\int f(x)\, dx = F(x) + C$.

*The function F (x) is called **anti-derivative** or **primitive** or **integral** of the function f (x) and C is called **constant of integration** or **arbitrary constant.***

*The process of finding functions whose derivative is given is called **anti-differentiation** or **integration**.*

9.1.1 Some Elementary Standard Integrals

1. $\int x^n\, dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$

Corollary. $\int dx = x + C$

2. $\int \frac{1}{x}\, dx = \log |x| + C$

3. $\int a^x\, dx = \frac{a^x}{\log a} + C,\ a > 0,\ a \neq 1$

4. $\int e^x\, dx = e^x + C$

5. $\int \sin x\, dx = -\cos x + C$

6. $\int \cos x\, dx = \sin x + C$

7. $\int \sec^2 x\, dx = \tan x + C$

8. $\int \operatorname{cosec}^2 x\, dx = -\cot x + C$

9. $\int \sec x \tan x\, dx = \sec x + C$

10. $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + C.$

(For proofs, see Understanding ISC Mathematics XI)

9.1.2 Four Standard Theorems

Theorem 1. $\frac{d}{dx}\left(\int f(x)\right) dx = f(x)$

Theorem 2. $\int \alpha f(x)\, dx = \alpha \int f(x)\, dx$, *for all* $\alpha \in \mathbf{R}$

Theorem 3. $\int (f_1(x) + f_2(x) - f_3(x) + \ldots)\, dx$

$$= \int f_1(x)\, dx + \int f_2(x)\, dx - \int f_3(x)\, dx + \ldots$$

Theorem 4. $\int f'(g(x))\, g'(x)\, dx = f(g(x)) + C.$

(For proofs, see Understanding ISC Mathematics XI)

ILLUSTRATIVE EXAMPLES

Example 1. *Evaluate the following integrals:*

(i) $\int \left(x - \frac{1}{x}\right)^3 dx$ (ii) $\int \frac{(a^x + b^x)^2}{a^x b^x} dx.$

Solution. (i) $\int \left(x - \frac{1}{x}\right)^3 dx = \int \left(x^3 - \frac{1}{x^3} - 3x.\frac{1}{x}\left(x - \frac{1}{x}\right)\right) dx$

$$= \int \left(x^3 - x^{-3} - 3x + 3.\frac{1}{x}\right)dx$$

$$= \frac{x^4}{4} - \frac{x^{-2}}{-2} - 3.\frac{x^2}{2} + 3 \log |x| + C$$

$$= \frac{x^4}{4} + \frac{1}{2x^2} - \frac{3}{2}x^2 + 3 \log |x| + C.$$

(ii) $\int \frac{(a^x + b^x)^2}{a^x b^x} dx = \int \frac{a^{2x} + b^{2x} + 2a^x b^x}{a^x b^x} dx = \int \left(\left(\frac{a}{b}\right)^x + \left(\frac{b}{a}\right)^x + 2\right) dx$

$$= \frac{\left(\frac{a}{b}\right)^x}{\log \frac{a}{b}} + \frac{\left(\frac{b}{a}\right)^x}{\log \frac{b}{a}} + 2x + C, \; a \neq b.$$

In particular, if $a = b$, then

$$\int \frac{(a^x + b^x)^2}{a^x b^x} dx = \int \frac{(2a^x)^2}{a^x . a^x} dx = \int 4dx = 4x + C.$$

Example 2. *Evaluate the following integrals :*

(i) $\int 9^{\log_3 x} dx$ (ii) $\int e^{3 \log x} (x^{-4}) dx$ (iii) $\int a^{\log x} dx.$

Solution. (i) $\int 9^{\log_3 x} dx = \int (3^2)^{\log_3 x} dx = \int 3^{2\log_3 x} dx$

$$= \int 3^{\log_3 x^2} dx = \int x^2 dx \qquad \left|\because a^{\log_a x} = x\right.$$

$$= \frac{x^3}{3} + C = \frac{1}{3}x^3 + C.$$

(ii) $\int e^{3\log x} (x^{-4}) dx = \int e^{\log x^3} (x^{-4}) dx = \int x^3 . x^{-4} dx \qquad \left|\because e^{\log x} = x\right.$

$$= \int \frac{1}{x} dx = \log x + C.$$

(iii) First we note that $a^{\log x} = x^{\log a}$.

Taking logarithm of both sides, we get

$$\log (a^{\log x}) = \log (x^{\log a})$$

$\Rightarrow$ $\log x \log a = \log a \log x$, which is true.

$\therefore$ $\int a^{\log x} dx = \int x^{\log a} dx = \frac{x^{\log a + 1}}{\log a + 1} + C.$

Example 3. *Evaluate the following integrals:*

(i) $\int \tan^2 x \, dx$ (ii) $\int \sqrt{1 - \sin 2x} \, dx.$

Solution. (i) $\int \tan^2 x \, dx = \int (\sec^2 x - 1) dx = \tan x - x + C.$

(*ii*) $\int \sqrt{1-\sin 2x}\, dx = \int \sqrt{\cos^2 x + \sin^2 x - 2\sin x \cos x}\, dx$

$= \int (\cos x - \sin x)\, dx = \sin x - (-\cos x) + C$

$= \sin x + \cos x + C.$

Example 4. *Evaluate the following integrals:*

(*i*) $\int \frac{1}{1+\sec x}\, dx$ (*ii*) $\int \frac{\sin x}{1+\sin x}\, dx.$

Solution. (*i*) $\int \frac{1}{1+\sec x}\, dx = \int \frac{1}{\sec x + 1} \times \frac{\sec x - 1}{\sec x - 1}\, dx = \int \frac{\sec x - 1}{\sec^2 x - 1}\, dx$

$= \int \frac{\sec x - 1}{\tan^2 x}\, dx = \int \left(\frac{\sec x}{\tan^2 x} - \frac{1}{\tan^2 x}\right) dx$

$= \int (\operatorname{cosec} x \cot x - \cot^2 x)\, dx = \int \operatorname{cosec} x \cot x\, dx - \int (\operatorname{cosec}^2 x - 1)\, dx$

$= -\operatorname{cosec} x - (-\cot x - x) + C = -\operatorname{cosec} x + \cot x + x + C.$

(*ii*) $\int \frac{\sin x}{1+\sin x}\, dx = \int \frac{(1+\sin x)-1}{1+\sin x}\, dx = \int 1\, dx - \int \frac{1}{1+\sin x}\, dx$

$= x - \int \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x}\, dx = x - \int \frac{1-\sin x}{\cos^2 x}\, dx$

$= x - \int \left(\frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x}\right) dx = x - \int (\sec^2 x - \sec x \tan x)\, dx$

$= x - (\tan x - \sec x) + C = x - \tan x + \sec x + C.$

Example 5. *Evaluate the following integrals:*

(*i*) $\int \cos^{-1} (\sin x)\, dx$ (*ii*) $\int \tan^{-1} (\sec x + \tan x)\, dx$

(*iii*) $\int \tan^{-1} \left(\sqrt{\frac{1-\sin x}{1+\sin x}}\right) dx.$

Solution. (*i*) $\int \cos^{-1} (\sin x)\, dx = \int \cos^{-1} \left(\cos\left(\frac{\pi}{2} - x\right)\right) dx = \int \left(\frac{\pi}{2} - x\right) dx$

$= \frac{\pi}{2} . x - \frac{x^2}{2} + C.$

(*ii*) $\int \tan^{-1} (\sec x + \tan x)\, dx = \int \tan^{-1} \left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right) dx$

$= \int \tan^{-1} \left(\frac{1+\sin x}{\cos x}\right) dx = \int \tan^{-1} \left(\frac{1-\cos\left(\frac{\pi}{2}+x\right)}{\sin\left(\frac{\pi}{2}+x\right)}\right) dx$

$= \int \tan^{-1} \left(\frac{2\sin^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}\right) dx = \int \tan^{-1} \left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right) dx$

$= \int \left(\frac{\pi}{4}+\frac{x}{2}\right) dx = \frac{\pi}{4} . x + \frac{1}{2} . \frac{x^2}{2} + C = \frac{x}{4} (\pi + x) + C.$

(*iii*) $\int \tan^{-1} \left(\sqrt{\frac{1-\sin x}{1+\sin x}}\right) dx = \int \tan^{-1} \left(\sqrt{\frac{1-\cos\left(\frac{\pi}{2}-x\right)}{1+\cos\left(\frac{\pi}{2}-x\right)}}\right) dx$

$$= \int \tan^{-1}\left(\sqrt{\frac{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}}\right) dx = \int \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right) dx$$

$$= \int \left(\frac{\pi}{4}-\frac{x}{2}\right) dx = \frac{\pi}{4}.x - \frac{1}{2}\cdot\frac{x^2}{2} + C = \frac{\pi}{4}x - \frac{x^2}{4} + C.$$

EXERCISE 9.1

Evaluate the following (1 to 16) integrals:

1. (*i*) $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$ (*ii*) $\int \frac{3+5t-7t^2}{\sqrt{t}}\, dt.$

2. (*i*) $\int (2x-1)(x^2+1)\, dx$ (*ii*) $\int x^2\left(2x-\frac{1}{x}\right)^2 dx.$

3. (*i*) $\int (25)^{\log_5 x}\, dx$ (*ii*) $\int 2^{\log_4 x}\, dx.$

4. (*i*) $\int e^{2\log x}(x^{-3})\, dx$ (*ii*) $\int (e^{x\log a} + e^{a\log x} - e^{a\log a})\, dx.$

5. (*i*) $\int \frac{x^{10}-1}{x-1}\, dx$ (*ii*) $\int \frac{1}{1-\cos 2x}\, dx.$

6. (*i*) $\int \cot^2 x\, dx$ (*ii*) $\int \left(\tan^2 x - 7x^2 + \frac{1}{x}\right) dx.$

7. (*i*) $\int \frac{1}{1+\cos x}\, dx$ (*ii*) $\int \frac{1}{1+\sin x}\, dx.$

8. (*i*) $\int \frac{\sin^2 x}{1+\cos x}\, dx$ (*ii*) $\int \sqrt{1+\sin 2x}\, dx.$

9. (*i*) $\int (2\tan x - 3\cot x)^2\, dx$ (*ii*) $\int \left(x^{3/2} - \cos^2\frac{x}{2}\right) dx.$

10. (*i*) $\int \left(\sqrt{x} - \sin\frac{x}{2}\cos\frac{x}{2} + 5\right) dx$ (*ii*) $\int \frac{\cos 2x}{\sin^2 x\cos^2 x}\, dx.$

11. (*i*) $\int \frac{\sec x}{\sec x + \tan x}\, dx$ (*ii*) $\int \frac{\cos x - \cos 2x}{1-\cos x}\, dx.$

12. (*i*) $\int \frac{\sin x + \cos x}{\sqrt{1+\sin 2x}}\, dx$ (*ii*) $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha}\, dx.$

Hint. (*ii*) $\cos 2x - \cos 2\alpha = (2\cos^2 x - 1) - (2\cos^2\alpha - 1) = 2(\cos^2 x - \cos^2\alpha)$.

13. (*i*) $\int \frac{1}{\sin^2 x\cos^2 x}\, dx$ (*ii*) $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x\cos^2 x}\, dx.$

14. (*i*) $\int \cot^{-1}\left(\frac{\sin 2x}{1-\cos 2x}\right) dx$ (*ii*) $\int \tan^{-1}\left(\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}\right) dx.$

15. (*i*) $\int \sin^{-1}\left(\frac{2\tan x}{1+\tan^2 x}\right) dx$ (*ii*) $\int \cos^{-1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right) dx.$

16. (*i*) $\int \tan^{-1}(\operatorname{cosec} x - \cot x)\, dx$ (*ii*) $\int \tan^{-1}(\operatorname{cosec} x + \cot x)\, dx.$

17. If $f'(x) = a\sin x + b\cos x$, $f'(0) = 4$, $f(0) = 3$ and $f\left(\frac{\pi}{2}\right) = 5$, find $f(x)$.

9.2 INTEGRATION BY SUBSTITUTION

Some functions can be integrated directly by the use of standard integrals while there exist some functions which cannot be integrated directly but can be reduced to the standard integrals by proper substitution *i.e.* by the introduction of new variable. The method of evaluating an integral by reducing it to standard form by a substitution is called **integration by substitution.**

Remark. The method of substitution is essentially the use of theorem 4, namely
$\int f'(g(x))\, g'(x)\, dx = f(g(x)) + C.$

9.2.1 Theorem. ***If*** $\int f(x)\, dx = F(x) + C$, ***then***

$$\int f(ax+b)\, dx = \frac{F(ax+b)}{a} + C,\ a \neq 0.$$

Proof. Put $ax + b = t$, then $a\, dx = dt \Rightarrow dx = \frac{1}{a}dt$,

$$\therefore \quad \int f(ax+b)\, dx = \int f(t) \cdot \frac{1}{a}dt = \frac{1}{a}\int f(t)\, dt$$

$$= \frac{1}{a}F(t) + C = \frac{F(ax+b)}{a} + C,\ a \neq 0.$$

Hence, we have the following important rule:

If the integral of a function of x is known, then if x is multiplied by a constant and to the product is added another constant, the integral is of the same form but it is divided by the coefficient of x.

Remark. Long division

In a rational function, if the degree of the numerator of the integrand is equal to or greater than that of denominator, divide the numerator by the denominator until the degree of the remainder is less than that of the denominator.

In this context, remember that

$$\frac{\text{numerator}}{\text{denominator}} = \text{quotient} + \frac{\text{remainder}}{\text{denominator}}.$$

ILLUSTRATIVE EXAMPLES

Example 1. *Evaluate the following integrals:*

(i) $\int \frac{1}{\sqrt{3x-2}}\, dx$ (ii) $\int 10^{2x+1}\, dx$ (iii) $\int \sqrt{1+\sin x}\, dx.$

Solution. (*i*) $\int \frac{1}{\sqrt{3x-2}}\, dx = \int (3x-2)^{-1/2}\, dx = \frac{(3x-2)^{1/2}}{\frac{1}{2}\cdot 3} + C$

$$= \frac{2}{3}\sqrt{3x-2} + C.$$

(*ii*) $\int 10^{2x+1}\, dx = \frac{10^{2x+1}}{\log 10 \,.\, 2} + C = \frac{10^{2x+1}}{2\log 10} + C.$

(*iii*) $\int \sqrt{1+\sin x}\, dx = \int \sqrt{\cos^2\frac{x}{2} + \sin^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}\, dx$

$$= \int \left(\cos\frac{x}{2} + \sin\frac{x}{2}\right) dx = \frac{\sin\frac{x}{2}}{\frac{1}{2}} + \frac{-\cos\frac{x}{2}}{\frac{1}{2}} + C$$

$$= 2\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right) + C.$$

Example 2. *Find all the primitives of the following functions:*

(i) $5^{-2x}\, e^{-2x}$ (ii) $\frac{1}{\sqrt{2x+3}+\sqrt{2x-3}}$ (iii) $\frac{x^2}{1-x}.$

Solution. (*i*) $\int 5^{-2x}\, e^{-2x}\, dx = \int (5e)^{-2x}\, dx = \frac{(5e)^{-2x}}{\log 5e\,(-2)} + C$

$$= -\frac{(5e)^{-2x}}{2\log 5e} + C.$$

(*ii*) $\int \frac{1}{\sqrt{2x+3}+\sqrt{2x-3}}\,dx = \int \frac{1}{\sqrt{2x+3}+\sqrt{2x-3}} \times \frac{\sqrt{2x+3}-\sqrt{2x-3}}{\sqrt{2x+3}-\sqrt{2x-3}}\,dx$

$= \int \frac{\sqrt{2x+3}-\sqrt{2x-3}}{(2x+3)-(2x-3)}\,dx = \frac{1}{6}\int ((2x+3)^{1/2}-(2x-3)^{1/2})\,dx$

$= \frac{1}{6}\left(\frac{(2x+3)^{3/2}}{\frac{3}{2}.2} - \frac{(2x-3)^{3/2}}{\frac{3}{2}.2}\right) + C = \frac{1}{18}\left((2x+3)^{3/2}-(2x-3)^{3/2}\right) + C.$

(*iii*) $\int \frac{x^2}{1-x}\,dx$

$$\begin{array}{r|l} 1-x & x^2 \quad (-x-1 \\ & x^2 - x \\ & - \quad + \\ \hline & x \\ & x-1 \\ & - \quad + \\ \hline & 1 \end{array}$$

$= \int \left(-x-1+\frac{1}{1-x}\right)dx$

$= -\frac{x^2}{2} - x + \frac{\log|1-x|}{-1} + C$

$= -\left(\frac{x^2}{2}+x+\log|1-x|\right) + C.$

Example 3. *Find all the anti-derivatives of the following functions:*

(*i*) $\frac{2x}{(2x+1)^2}$ (*ii*) $(3x+1)\sqrt{2x-1}$ (*iii*) $\frac{x+2}{\sqrt{3x+1}}$.

Solution. (*i*) $\int \frac{2x}{(2x+1)^2}\,dx = \int \frac{(2x+1)-1}{(2x+1)^2}\,dx = \int \left(\frac{1}{2x+1} - \frac{1}{(2x+1)^2}\right)dx$

$= \int \left(\frac{1}{2x+1} - (2x+1)^{-2}\right)dx = \frac{\log|2x+1|}{2} - \frac{(2x+1)^{-1}}{-1.2} + C$

$= \frac{1}{2}\left(\log|2x+1| + \frac{1}{2x+1}\right) + C.$

(*ii*) $\int (3x+1)\sqrt{2x-1}\,dx = \int \left(\frac{3}{2}(2x-1)+\frac{5}{2}\right)\sqrt{2x-1}\,dx$

$= \frac{3}{2}\int (2x-1)^{3/2}\,dx + \frac{5}{2}\int (2x-1)^{1/2}\,dx$

$= \frac{3}{2}\cdot\frac{(2x-1)^{5/2}}{\frac{5}{2}.2} + \frac{5}{2}\cdot\frac{(2x-1)^{3/2}}{\frac{3}{2}\cdot 2} + C$

$= \frac{3}{10}(2x-1)^{5/2} + \frac{5}{6}(2x-1)^{3/2} + C.$

(*iii*) $\int \frac{x+2}{\sqrt{3x+1}}\,dx = \int \frac{\frac{1}{3}(3x+1)+\frac{5}{3}}{\sqrt{3x+1}}\,dx$

$= \int \left(\frac{1}{3}(3x+1)^{1/2} + \frac{5}{3}(3x+1)^{-1/2}\right)dx$

$= \frac{1}{3}\cdot\frac{(3x+1)^{3/2}}{\frac{3}{2}\cdot 3} + \frac{5}{3}\cdot\frac{(3x+1)^{1/2}}{\frac{1}{2}\cdot 3} + C$

$= \frac{2}{27}(3x+1)^{3/2} + \frac{10}{9}\sqrt{3x+1} + C.$

Example 4. *Evaluate the following integrals :*

$$(i) \int \sin^3 x \, dx \quad (ii) \int \cos^4 x \, dx \quad (iii) \int \sin 4x \cos 7x \, dx.$$

Solution. (*i*) $\int \sin^3 x \, dx = \int \frac{3 \sin x - \sin 3x}{4} dx$ $\quad | \sin 3x = 3 \sin x - 4 \sin^3 x$

$$= \frac{3}{4} (-\cos x) - \frac{1}{4}\left(-\frac{\cos 3x}{3}\right) + C$$

$$= \frac{1}{12} (\cos 3x - 9 \cos x) + C.$$

(*ii*) $\int \cos^4 x \, dx = \int (\cos^2 x)^2 \, dx = \int \left(\frac{1 + \cos 2x}{2}\right)^2 dx$

$$= \frac{1}{4} \int (1 + 2 \cos 2x + \cos^2 2x) \, dx = \frac{1}{4} \int \left(1 + 2\cos 2x + \frac{1 + \cos 4x}{2}\right) dx$$

$$= \frac{1}{8} \int (3 + 4 \cos 2x + \cos 4x) \, dx = \frac{1}{8}\left[3x + 4 . \frac{\sin 2x}{2} + \frac{\sin 4x}{4}\right] + C$$

$$= \frac{1}{32} (12x + 8 \sin 2x + \sin 4x) + C.$$

(*iii*) $\int \sin 4x \cos 7x \, dx = \frac{1}{2} \int 2 \sin 4x \cos 7x \, dx$

$$= \frac{1}{2} \int (\sin (4x + 7x) + \sin (4x - 7x)) \, dx$$

$$= \frac{1}{2} \int (\sin 11x - \sin 3x) \, dx = \frac{1}{2} . \left(-\frac{\cos 11x}{11} - \left(\frac{\cos 3x}{3}\right)\right) + C$$

$$= \frac{1}{66} (-3 \cos 11x + 11 \cos 3x) + C.$$

Example 5. *Evaluate the following integers :*

$$(i) \int (\sin^6 x + \cos^6 x) \, dx \quad (ii) \int \frac{1 + \cos 4x}{\cot x - \tan x} dx.$$

Solution. (*i*) $\int (\sin^6 x + \cos^6 x) \, dx = \int ((\sin^2 x)^3 + (\cos^2 x)^3) \, dx$

$$= \int ((\sin^2 x + \cos^2 x)^3 - 3 \sin^2 x \cos^2 x (\sin^2 x + \cos^2 x)) \, dx$$

$[\because a^3 + b^3 = (a + b)^3 - 3ab (a + b)]$

$$= \int (1^3 - 3 \sin^2 x \cos^2 x.1) \, dx = \int \left(1 - \frac{3}{4} (2 \sin x \cos x)^2\right) dx$$

$$= \int \left(1 - \frac{3}{4} \sin^2 2x\right) dx = \int \left(1 - \frac{3}{4} . \frac{1 - \cos 4x}{2}\right) dx$$

$$= \frac{1}{8} \int (5 + 3 \cos 4x) \, dx = \frac{1}{8} \left(5x + 3 \frac{\sin 4x}{4}\right) + C$$

$$= \frac{1}{32} [20x + 3 \sin 4x] + C.$$

(*ii*) $\int \frac{1 + \cos 4x}{\cot x - \tan x} dx = \int \frac{2\cos^2 2x}{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}} dx = \int \frac{2\cos^2 2x . \sin x \cos x}{\cos^2 x - \sin^2 x} dx = \int \frac{\cos^2 2x . \sin 2x}{\cos 2x} dx$

$$= \int \cos 2x \sin 2x \, dx = \frac{1}{2} \int 2 \sin 2x \cos 2x \, dx$$

$$= \frac{1}{2} \int \sin 4x \, dx = \frac{1}{2} . \left(-\frac{\cos 4x}{4}\right) + C = -\frac{1}{8} \cos 4x + C.$$

ANSWERS

EXERCISE 9.1

1. (*i*) $\frac{x^2}{2} + \log|x| - 2x + C$ (*ii*) $6\sqrt{t} + \frac{10}{3}t^{1/3} - \frac{14}{5}t^{5/2} + C.$
2. (*i*) $\frac{x^4}{2} - \frac{x^3}{3} + x^2 - x + C$ (*ii*) $\frac{4}{5}x^5 - \frac{4}{3}x^3 + x + C.$
3. (*i*) $\frac{1}{3}x^3 + C$ (*ii*) $\frac{2}{3}x^{3/2} + C.$
4. (*i*) $\log x + C$ (*ii*) $\frac{a^x}{\log a} + \frac{x^{a+1}}{a+1} - a^a . x + C.$
5. (*i*) $\frac{x^{10}}{10} + \frac{x^9}{9} + \frac{x^8}{8} + \ldots + x + C$ (*ii*) $-\frac{1}{2}\cot x + C.$
6. (*i*) $-\cot x - x + C$ (*ii*) $\tan x - x - \frac{7}{3}x^3 + \log|x| + C.$
7. (*i*) $-\cot x + \operatorname{cosec} x + C$ (*ii*) $\tan x - \sec x + C.$
8. (*i*) $x - \sin x + C$ (*ii*) $\sin x - \cos x + C.$
9. (*i*) $4\tan x - 9\cot x - 25x + C$ (*ii*) $\frac{2}{5}x^{5/2} - \frac{1}{2}x - \frac{1}{2}\sin x + C.$
10. (*i*) $\frac{2}{3}x^{2/3} + \frac{1}{2}\cos x + 5x + C$ (*ii*) $-\cot x - \tan x + C.$
11. (*i*) $\tan x - \sec x + C$ (*ii*) $2\sin x + x + C.$
12. (*i*) $x + C$ (*ii*) $2(\sin x + x\cos\alpha) + C.$
13. (*i*) $\tan x - \cot x + C$ (*ii*) $\tan x - \cot x - 3x + C.$
14. (*i*) $\frac{1}{2}x^2 + C$ (*ii*) $\frac{1}{2}x^2 + C.$
15. (*i*) $x^2 + C$ (*ii*) $x^2 + C.$
16. (*i*) $\frac{1}{4}x^2 + C$ (*ii*) $\frac{\pi}{2}x - \frac{1}{4}x^2 + C.$
17. $2\cos x + 4\sin x + 1.$

EXERCISE 9.2

1. (*i*) $\frac{2}{3a}(ax+b)^{3/2} + C$ (*ii*) $\frac{(ax+b)^{n+1}}{a(n+1)} + C,\ n \neq -1.$
2. (*i*) $-\frac{1}{3}\log|5-3x| + C$ (*ii*) $2\sqrt{e^x} + C.$
3. (*i*) $-\frac{1}{3}\tan(7-3x) + C$ (*ii*) $\frac{1}{2}(e^{2x} - e^{-2x} + 4x) + C.$
4. (*i*) $\frac{x^3}{6} - \frac{x^2}{8} + \frac{x}{8} - \frac{1}{16}\log|2x+1| + C$ (*ii*) $\frac{x^{3a+1}}{3a+1} + \frac{a^{3x}}{3\log a} + C.$
5. (*i*) $\frac{1}{9}[(2x+3)^{3/2} + (2x)^{3/2}] + C$ (*ii*) $\log|x+1| + \frac{1}{x+1} + C.$
6. (*i*) $\frac{1}{6}(2x-1)^{3/2} + \frac{3}{2}\sqrt{2x-1} + C$ (*ii*) $\frac{2}{45}(3x-2)^{5/2} + \frac{4}{27}(3x-2)^{3/2} + C.$
7. (*i*) $\frac{1}{4}(2x + \sin 2x) + C$ (*ii*) $\frac{1}{32}(12x - 8\sin 2x + \sin 4x) + C.$
8. (*i*) $\frac{1}{8}(4x - \sin(4x+10)) + C$ (*ii*) $\frac{1}{64}(24x + 8\sin 4x + \sin 8x) + C.$
9. (*i*) $2\sqrt{2}\sin\frac{x}{2} + C$ (*ii*) $2\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) + C.$