CLASS - X (CBSE) | MATHEMATICS

SAMPLE PAPER

TIME : 3 HRS. MAX. MARKS : 80

GENERAL INSTRUCTIONS :

- **All questions are compulsory.**
- **The question paper contains two parts A and B.**
- **Both Part-A and Part-B have internal choices.**
- **Part-A consist of two Sections - (I) and (II).**

  Section-(I) has 16 questions of 1 mark each. Internal choices is provided in 5 questions.

  Section-(II) has 4 questions on case study. Each case study has 5 case - based subparts out of which 4 has to be attempted carrying 1 mark for each subpart.
- **Part-B consist of three Sections - (III), (IV) and (V).**

  Section-(III) has 6 questions of 2 marks each. Internal choices is provided in 2 questions.

  Section-(IV) has 7 questions of 3 marks each. Internal choices is provided in 2 questions.

  Section-(V) has 3 questions of 5 marks each. Internal choices is provided in 1 question.

PART-A

SECTION-I

1. Explain why $11 \times 13 \times 15 \times 17 + 17$ is a composite number.

OR

If two positive integers A and B can be expressed as $A = b^2$ and $B = a^3b$, where a, b, are prime numbers, then find LCM (A, B).

2. If α and β are the zeroes of the polynomial $4x^2 + 3x + 7$, then find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.

3. For what value of k, will the system of linear equations $x + 2y = 5$ and $3x + ky - 15 = 0$ has unique solution ?

4. The sum of the digits of a two digit number is 12. The number obtained by interchanging the two digits exceeds the given number by 18. Form a linear equation to represent the condition.

5. Which term of the A.P. 92, 88, 84, 80,.....is 0 ?

OR

Find the next term of the A.P. $\sqrt{27}, \sqrt{48}, \sqrt{75}$, ...

6. Find the values of k for which the following quadratic equation has two equal roots.

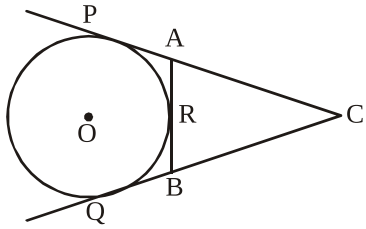
$2x^2 + kx + 3 = 0$

7. Find the roots of the quadratic equation $6x^2 - x - 2 = 0$.

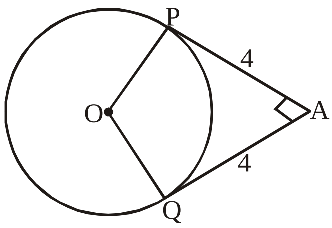
OR

If $r = 3$ is a root of quadratic equation $kr^2 - kr - 3 = 0$, find the value of k.

8. In given figure, CP, CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If CP = 11 cm, BC = 7 cm. Find the length of BR.

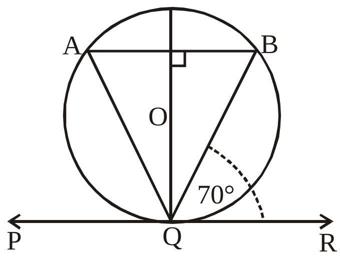


9. In given figure (not to scale), the pair of tangents AP and AQ, drawn from an external point A to a circle with centre O, are perpendicular to each other and length of each tangent is 4 cm, then find the radius of the circle.

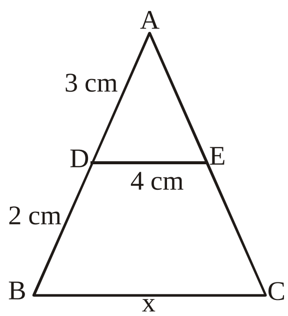


OR

In given figure, PQR is the tangent to a circle at Q whose centre is O. AB is a chord parallel to PR and $\angle BQR = 70°$ then find $\angle OQB$.



10. In the figure given below, if DE $\parallel$ BC, then find x.



11. To divide the line segment AB in the ratio of 2 : 3, first a ray AX is drawn such that angle BAX is an acute angle and then at equal distance, points are marked on the ray AX, then find the minimum number of these points to be marked.

12. If $\cos x = \cos 60° \cos 30° + \sin 60° \sin 30°$, then find the value of x ($0 \le x \le 90°$).

13. If $\cos\theta = \frac{1}{2}$, then find the value of $\cos\theta[\cos\theta - \sec\theta]$.

14. Find the area of circle that can be inscribed in a square of side 6 cm.

15. A solid iron in the form of a cuboid of dimensions 49 cm x 33 cm x 24 cm is melted to form a solid sphere. Find the radius of sphere.

16. The probability of getting a bad egg from a lot of 400 eggs is 0.035. Find the number of bad eggs in the lot.

OR

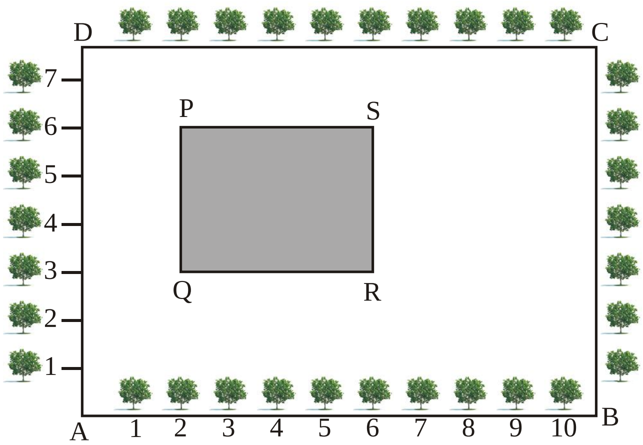
In a throw of two dice, find the probability of getting a sum of 10.

SECTION-II

17. **Case study Based -1**

Tree Plantation to Control Pollution

The class X students of a secondary school in Kota have been alloted a rectangular plot of land for this gardening activity.



Sapling of Rose are planted on the boundary at a distance of 1 m from each other. There is a rectangular gracy lawn in the plot as shown in above figure.

The students sowing seeds of flowering plants on the remaining area of the plot.

(a) Find the coordinates of point Q and S are

(i) (2, 3), (6, 6) (ii) (3, 2), (6, 6) (iii) (2, 3), (5, 5) (iv) None of these

(b) Find the distance between the vertices Q and S.

(i) 3 (ii) 5 (iii) 6 (iv) 7

(c) Find the width of rectangle PQRS.

(i) 3 (ii) 4 (iii) 2 (iv) 5

(d) If the point G divides the line QR in the ratio 1 : 2, then the coordinate of G is

(i) (10, 3) (ii) $\left(\frac{10}{3}, 3\right)$ (iii) $\left(3, \frac{10}{3}\right)$ (iv) None of these

(e) Find the area of the rectangular field PQRS.

(i) 8 sq. units (ii) 10 sq. units (iii) 12 sq. units (iv) 14 sq. units

18. Case study Based -2

Beehive

A beehive is an enclosed cell structure in which some honeybee species of the subgenus Apis live and raise their young. Each cell is in the form of regular hexagonal shape. In a regular hexagon, there are six edges of equal lengths. Taking O as centre and join all the vertices from the centre such that OA = OB = OC = OD = OE = OF.

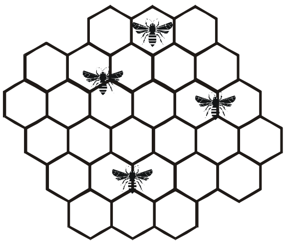


Fig.1

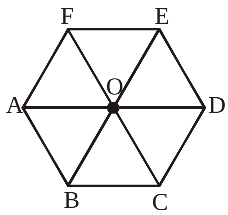


Fig.2

Similarity of triangle

Two triangles are said to be similar, if their all corresponding angles are equal and all corresponding sides are proportional.

(a) Find the number of equilateral triangle in the given figure 2.

(i) 6 (ii) 4 (iii) 3 (iv) 8

(b) If area of two triangles are equal, then it is always

(i) Similar

(ii) Congruent

(iii) Both similar and congruent

(iv) None of the above

(c) How many triangles are similar in the given figure?

(i) 3 (ii) 4 (iii) 6 (iv) 0

(d) Which of the following is not used for proving two triangles similar.

(i) SSS (ii) AA (iii) SAS (iv) SSA

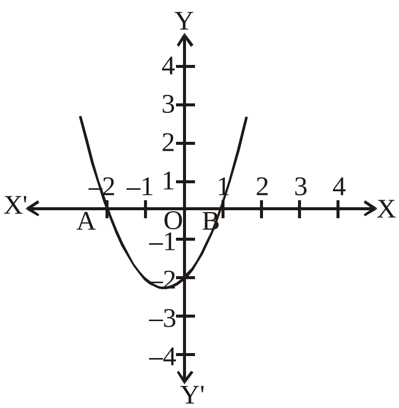
(e) Find the area of the hexagon, if each edge is 'a' unit.

(i) $2\sqrt{3}a^2$ (ii) $3\sqrt{2}a^2$ (iii) $\sqrt{3}a^2$ (iv) $\frac{3\sqrt{3}}{2}a^2$

19. Case study Based -3

Storm in Assam

Few months ago, heavy storm comes out in Assam. Due to this storm thousand of trees breaks and electric pole bent out (or break). Some of the electric poles bent into the shape of parabola which is shown in figure.



(a) Suppose the quadratic polynomial for given curve is $ax^2 + bx + c$, then always a is

(i) > 0 (ii) < 0 (iii) ≥ 0 (iv) ≤ 0

(b) Find the zeroes of the given curve.

(i) 2 and –1 (ii) –2 and 1

(iii) –2 and –1 (iv) None of the above

(c) The polynomial expression of given curve is

(i) $x^2 - x + 2$ (ii) $x^2 + x - 2$ (iii) $x^2 + x - 2$ (iv) None of these

(d) If x = 2, then find the value polynomial.

(i) 4 (ii) 3 (iii) 2 (iv) –4

(e) If we move the parabola right side of one unit, then find its polynomial expression.

(i) $x^2 - 3x + 2$ (ii) $x^2 + x + 2$ (iii) $x^2 + x - 2$ (iv) $x^2 - x - 2$

20. Case study Based -4

Collection of Weight Data

Now a days, most of the population is suffered from increase of weight. The basic region behind that is due to laziness, eating more junk food and less physical exercise. The central board of education give instruction to all the school that collect the weight data of each student.

During the medical check up of 35 students of a class, their weight were recorded as follows.

Weight (in kg)	Number of students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

(a) Find the median class of the given data.
(i) 44 – 46 (ii) 46 – 48 (iii) 48 – 50 (iv) None of these

(b) Calculate the median weight of the given data.
(i) 46.5 (ii) 47.5 (iii) 46 (iv) 47

(c) Find the mean of the given data
(i) 45 (ii) 45.8 (iii) 46.2 (iv) 46.8

(d) Find the modal class of the given data.
(i) 46 – 48 (ii) 44 – 46 (iii) 48 – 50 (iv) 50 – 52

(e) For the above data, which one is greater among median and mean
(i) Median (ii) Mean (iii) Both are equal (iv) None of these

PART-B

SECTION-III

21. Find the LCM of 72, 80 and 120 using the fundamental theorem of arithmetic.

22. If the point A(4, 3) and B(x, 5) are on the circle with centre O(2, 3). Find the value of x.

OR

Three consecutive vertices of a parallelogram ABCD are A(1, 2), B(1, 0) and C(4, 0). Find the fourth vertex D.

23. Find the zeroes of the polynomial $x^2 + x - 6$. Also verify the relationship between the coefficients and zeroes of the polynomials.

24. Draw a pair of tangents to a circle of radius 3.5 cm which are perpendicular to each other.

25. Prove that : $\dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A} = 2\sec A$

OR

If $\sqrt{3}\ \tan\theta = 3\sin\theta$, find the value of $\sin^2\theta - \cos^2\theta$.

26. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

SECTION-IV

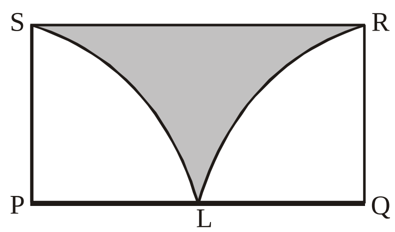
27. Prove that $3-\sqrt{5}$ is an irrational number, given that $\sqrt{5}$ is irrational.

28. Find the value of x : $\dfrac{1}{a+b+x} = \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{x},\ a, b, x \neq 0$

OR

Solve the equation for x : $\dfrac{x+3}{x-2} - \dfrac{1-x}{x} = \dfrac{17}{x}$

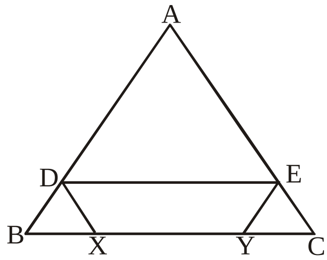
29. RS is a rectangle in which length is two times the breadth and L is mid point of PQ. With P and Q as centres, draw two quadrants as shown in fig. 5. Find the ratio of the area of rectangle PQRS to the area of shaded portion.



30. In an equilateral triangle ABC, D is a point on side BC such that BD = $\frac{1}{3}$BC. Prove that $9AD^2 = 7AB^2$.

OR

In given figure ΔABC, X and Y are two points lying on the side BC such that BX = CY. If DX || AC and YE || AB, then prove that DE || BC.



31. Find the median of the following frequency distribution :

Classes :	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency :	7	8	20	8	7

32. The angle of elevation of the top of a tower at a point on the ground is 45°. After going 40 m towards the foot of the tower, the angle of elevation of the top of tower changes to 60°. Find the height of the tower. (Use $\sqrt{3}$ = 1.73)

33. Calculate the mode of the following data :

Classes	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Frequency	5	10	18	30	20	12	5

SECTION-V

34. An aircraft is flying at a constant height with a speed of 360 km/hour. From a point on the ground, the angle of elevation at an instant was observed to be 45°. After 20 seconds, the angle of elevation was observed to be 30°. Determine the height at which the aircraft is flying, (use $\sqrt{3}$ = 1.732)

OR

A boy 2 m tall is standing at some distance from a 30 m tall building. The angle of elevation from his eyes of the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

35. An iron pillar has some part in the form of a right circular cylinder and the remaining in the form of a right circular cone. The radius of the base of each of the cone and the cylinder is 8 cm. The cylindrical part is 240 cm high and conical part is 36 cm high. Find the weight of the pillar if 1 cu cm of iron weighs 7.5 grams.

36. A man travels 600 km partly by train and partly by car. It takes 8 hours and 40 minutes if he travels 320 km by train and the rest by car. It would take 30 minutes more if he travels 200 km by train and the rest by car. Find the speed of the train and the car separately.

MATHEMATICS SAMPLE PAPER

ANSWER AND SOLUTIONS

PART-A

SECTION-I

1. $11 \times 13 \times 15 \times 17 + 17$

$= 17[11 \times 13 \times 15 + 1]$

Since, the given number has more than two factors.

1, 17 and number itself. Hence, it is a composite number.

OR

$A = b^2$ and $B = a^3b$

LCM (A, B) $= a^3b^2$

2. Given polynomial $4x^2 + 3x + 7$

$\alpha + \beta = -\frac{3}{4}$ .....(1)

$\alpha\beta = \frac{7}{4}$ ....(2)

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

$= \frac{-\frac{3}{4}}{\frac{7}{4}}$ [From (1) and (2)]

$= \frac{-3}{7}$

3. Given equations $x + 2y = 5$ and $3x + ky = 15$

For unique solution $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\frac{1}{3} \neq \frac{2}{k}$

$k \neq 6$

Thus, for $k \neq 6$, the system of equation will have unique solution.

4. Let one's digit of the number be x and ten's digit by y

According to question

$x + y = 12$

$10x + y - (10y + x) = 18$

$9(x - y) = 18$

$x - y = 2$

Thus, required equation are

$x + y = 12$

$x - y = 2$

5. Let n^{th} term of the A.P. be 0

$a_n = 0$

$a + (n - 1)d = 0$

Here, $a = 92$, $d = 88 - 92 = -4$

$\Rightarrow 92 + (n - 1)(-4) = 0$

$\Rightarrow 92 = (n - 1)4$

$\Rightarrow 23 = n - 1$

$\Rightarrow n = 24$

Thus, 24^{th} term of the A.P. is 0.

OR

Here, $a = \sqrt{27} = 3\sqrt{3}, d = \sqrt{48} - \sqrt{27}$

$= 4\sqrt{3} - 3\sqrt{3} = \sqrt{3}$

$a_4 = a + 3d$

$= 3\sqrt{3} + 3\sqrt{3} = 6\sqrt{3} = \sqrt{108}$

So, next term is $\sqrt{108}$

6. Given, equation $2x^2 + kx + 3 = 0$

Here, $a = 2, b = k, c = 3$

For equal roots

$D = b^2 - 4ac = 0$

$k^2 - 4 \times 2 \times 3 = 0$

$k^2 = 4 \times 2 \times 3$

$k = \pm 2\sqrt{6}$

7. Given equation $6x^2 - x - 2 = 0$

$6x^2 - 4x + 3x - 2 = 0$

$2x(3x - 2) + 1(3x - 2) = 0$

$(3x - 2)(2x + 1) = 0$

$3x - 2 = 0$ or $2x + 1 = 0$

$x = \frac{2}{3}$ or $x = -\frac{1}{2}$

Thus, roots are $\frac{2}{3}$ and $-\frac{1}{2}$

OR

$r = 3$ is root of equation $kr^2 - kr - 3 = 0$

$\Rightarrow k.3^2 - k.3 - 3 = 0$

$\Rightarrow 9k - 3k = 3$

$\Rightarrow 6k = 3$

$\Rightarrow k = \frac{1}{2}$

8. CP = CQ [Length of tangents from an external point to a circle are equal]

CQ = 11 cm [$\because$ CP = 11 cm]

BQ = CQ – CB

= 11 – 7

= 4 cm

BR = BQ [Length of tangents from an external point to a circle are equal]

BR = 4 cm

9. AP = AQ = 4 cm

$\angle PAQ = 90°$ [Given]

Now, $OP \perp AP$, $OQ \perp AQ$

[Line joining the centre to point of contact of tangent is perpendicular to tangent.]

In quadrilateral OPQA

$\angle P + \angle Q + \angle A + \angle O = 360°$

$90° + 90° + 90° + \angle O = 360°$

$\angle O = 90°$

Also, OP = OQ (Radius of same circle)

Thus, OPAQ in a square

$\Rightarrow$ Radius of circle = 4 cm.

OR

AB || PR

$\angle ABQ = \angle BQR$ (Alternate $\angle$s)

$\angle ABQ = 70°$

$\angle OQB = 180° - 90° - \angle ABQ = 20°$

10. In Δs ADE and ABC

$\angle ADE = \angle ABC$ [Corresponding angles]

$\angle AED = \angle ACB$ [Corresponding angles]

$\Delta ADE \sim \Delta ABC$ [by AA]

$\frac{AD}{AB} = \frac{DE}{BC}$

$\frac{3}{5} = \frac{4}{x}$

$x = \frac{20}{3}$ cm

11. Minimum number of points of be marked = 5

12. $\cos x = \cos 60° \cos 30° + \sin 60° \sin 30°$

$\cos x = \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}$

$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

$= \cos 30°$

$x = 30°$

13. $\cos\theta = \frac{1}{2}$

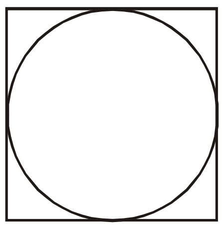
$\Rightarrow \sec\theta = 2$

Now, $\cos\theta(\cos\theta - \sec\theta) = \frac{1}{2}\left(\frac{1}{2} - 2\right)$

$= \frac{1}{2}\left(\frac{1-4}{2}\right) = \frac{-3}{4}$

14. Diameter of circle = side of square = 6 cm

Radius of circle = 3 cm



Area of circle = $\frac{22}{7} r^2 = \frac{22}{7} \times 3 \times 3 = \frac{198}{7}$

$= 28.28 \text{ cm}^2$

15. Volume of sphere = Volume of cuboid

$$\frac{4}{3}\pi r^3 = 49 \times 33 \times 24$$

r = 21 cm

16. Number of bed eggs = 0.035 × 400 = 14

OR

Number of favourable outcomes = 3

Total outcomes = 36

$$\text{Probability} = \frac{3}{36} = \frac{1}{12}$$

SECTION-II

17. (a) (i) The coordinates of points Q and S are (2, 3) and (6, 6)

(b) (ii) Distance between the vertices Q and S is

$$8 = \sqrt{(6-2)^2+(6-3)^2} = \sqrt{4^2+3^2}$$

$$= \sqrt{16+9} = \sqrt{25} = 5\,\text{units}$$

(c) (i) As the coordinates of points Q and P are (2, 3) and (2, 6).

∴ The width of the rectangle = 6 – 3 = 3

(d) (ii) By using internal division formula.

Coordinate of

$$G = \left(\frac{1\times 6+2\times 2}{1+2}, \frac{1\times 3+2\times 3}{1+2}\right)$$

$$= \left(\frac{6+4}{3}, \frac{3+6}{3}\right)$$

$$= \left(\frac{10}{3}, \frac{9}{3}\right) = \left(\frac{10}{3}, 3\right)$$

(e) (iii) Area of rectangle PQRS

= Length × Breadth

= 4 × 3 = 12 sq. units

18. (a) (i) Total number of equilateral triangle in the given figure is 6.

(b) (iv) If area of two triangle are equal, then it is not necessary to be similar or congruent.

(c) (iii) As we know that there are six equilateral triangle all have equal side.

Hence, we get six similar triangles.

(d) (iv) 'SSA' property is not used for making triangles similar.

(e) (iv) Area of hexagon = 6 × Area of one equilateral triangle having side a.

$$= 6 \times \frac{\sqrt{3}}{4} \times (a)^2$$

$$= \frac{3\sqrt{3}}{2}(a)^2$$

19. (a) (i) Here, we see that shape of the parabola is upward, So, in quadratic polynomial $ax^2 + bx + c$, a is always > 0.

(b) (ii) Given curve intersect the x-axis at two point i.e. –2 and 1.

Hence, zeroes of given curve are –2 and 1.

(c) (ii) Since, zeroes of given polynomial are –2 and 1.

∴ Polynomial expression,

$p(x) = x^2$ – (sum of zeroes)x + product of zeroes

$= x^2 - (-2 + 1)x + (-2)(1)$

$= x^2 + x - 2$

(d) (i) We have, $p(x) = x^2 + x - 2$

When x = 2, then $p(2) = 2^2 + 2 - 2 = 4$

(e) (iv) If we move that parabola right side of one unit, then zeroes polynomial becomes –1 and 2.

Now, polynomial expression

$= x^2$ – (sum of zeroes)x + product of zeroes

$= x^2 - (-1 + 2)x + (-1) \times 2 = x^2 - x - 2$

20.

Weight (in kg)	Number of students	cf	x_i	$f_i x_i$
38 – 40	3	3	39	117
40 – 42	2	5	41	82
42 – 44	4	9	43	172
44 – 46	5	14	45	225
46 – 48	14	28	47	658
48 – 50	4	32	49	196
50 – 52	3	35	51	153
	$\Sigma f = 35$			$\Sigma f_i x_i = 1603$

(a) (ii) N = 35, $\frac{N}{2} = \frac{35}{2}$ = 17.5 median class 46 – 48

(b) (i) Here ℓ = 46, $\frac{N}{2}$ = 17.5, cf = 14, f = 14

Median = $46 + \frac{17.5 - 14}{14} \times 2$

$= 46 + \frac{3.5}{14} \times 2$

= 46.5

(c) (ii) Mean = $\frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{1603}{35}$ = 45.8

(d) (i) Modal class is 46 – 48

(e) (i) Median > Mean

PART-B

SECTION-III

21. 72 = 2 × 2 × 2 × 3 × 3

80 = 2 × 2 × 2 × 2 × 5

120 = 2 × 2 × 2 × 3 × 5

LCM of 72, 80, 120 = 2 × 2 × 2 × 2 × 3 × 3 × 5

= 720

22. Since A and B both are on the same circle with centre O(2, 3))

$\Rightarrow$ OA = OB (Radius of circle)

$\Rightarrow \sqrt{(2-4)^2 + (3-3)^2} = \sqrt{(2-x)^2 + (3-5)^2}$

On squaring

4 = (2 – x) + 4

2 – x = 0

x = 2

OR

Let fourth vertex D be (x, y)

Since diagonals of parallelogram bisect each other

$\Rightarrow$ 1 + 4 = 1 + x

x = 4

2 + 0 = 0 + y

$\Rightarrow$ y = 2

Thus, D(4, 2)

23. $x^2 + x - 6$

$= x^2 + 3x - 2x - 6$

$= x(x + 3) - 2(x + 3)$

$= (x + 3)(x - 2)$

For finding zeros

$(x + 3)(x - 2) = 0$

$\Rightarrow x = -3$, or $x = 2$

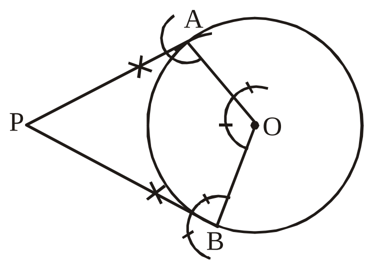
Thus, zeros are –3 and 2

Sum of zeros = –3 + 2 = –1 = $\frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$

Product of zeros = –3 × 2 = –6 = $\frac{\text{constant}}{\text{coefficient of } x^2}$

Hence verified.

24. **Steps of construction**



1. A circle with centre O and radius 3.5 cm is drawn.
2. A radius OB is drawn.
3. OA $\perp$ OB is drawn.
4. AP $\perp$ OA and BP $\perp$ OB is drawn intersecting each other at P.
5. PA and PB are required tangents.

25. LHS

$$\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}$$

$$= \frac{\cos^2 A+(1+\sin A)^2}{\cos A(1+\sin A)}$$

$$= \frac{\cos^2 A+1+\sin^2 A+2\sin A}{\cos A(1+\sin A)}$$

$$= \frac{1+1+2\sin A}{\cos A(1+\sin A)} \quad [\because \sin^2 A + \cos^2 A = 1]$$

$$= \frac{2(1+\sin A)}{\cos A(1+\sin A)}$$

$$= \frac{2}{\cos A} = 2\sec A$$

= RHS

Hence proved

OR

$$\sqrt{3}\tan\theta = 3\sin\theta$$

$$\sqrt{3}\frac{\sin\theta}{\cos\theta} = 3\sin\theta$$

$$\cos\theta = \frac{1}{\sqrt{3}}$$

Now, $\sin^2\theta - \cos^2\theta = 1 - \cos^2\theta - \cos^2\theta$

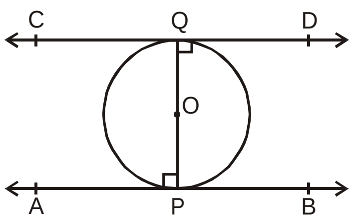
$= 1 - 2\cos^2\theta$

$$= 1-\frac{2}{3}=\frac{1}{3}$$

26. In the figure, PQ is diameter of the given circle and O is its centre.

Let tangents AB and CD be drawn at the end points of the diameter PQ.

Since, the tangents at a point to a circle is perpendicular to the radius through the point.



$\therefore$ PQ $\perp$ AB

$\Rightarrow$ $\angle$APQ = 90° and PQ $\perp$ CD

$\Rightarrow$ $\angle$PQD = 90°

$\Rightarrow$ $\angle$APQ = $\angle$PQD

But they form a pair of alternate angles.

$\therefore$ AB || CD.

Hence, the two tangents are parallel.

SECTION-IV

27. Let us assume, to the contrary, that $3 - \sqrt{5}$ is rational. That is, we can find coprime integers a and b ($b \neq 0$) such that $3 - \sqrt{5} = \frac{a}{b}$, $b \neq 0$,

a, b $\in$ I

Therefore, $\frac{a}{b} - 3 = -\sqrt{5}$

$$\Rightarrow \frac{a-3b}{b} = -\sqrt{5}$$

$$\Rightarrow \frac{a-3b}{2b} = -\sqrt{5} \Rightarrow \frac{a}{2b}-\frac{3}{2} = -\sqrt{5}$$

Since a and b are integers, we get $\frac{a}{2b}-\frac{3}{2}$ is rational, and so $\frac{a-3b}{2b} = -\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $3 - \sqrt{5}$ is rational.

So, we conclude that $3 - \sqrt{5}$ is irrational.

28. $$\frac{1}{a+b+x}-\frac{1}{x}=\frac{1}{a}+\frac{1}{b}$$

$$\frac{x-(a+b+x)}{x(a+b+x)}=\frac{a+b}{ab}$$

$-ab = x(a + b + x)$

$x^2 + ax + bx + ab = 0$

$x(x + a) + b(x + a) = 0$

$(x + a)(x + b) = 0$

$x = -a$ or $-b$

OR

$$\frac{x+3}{x-2}-\frac{1-x}{x}=\frac{17}{x}$$

$$\frac{x+3}{x-2}=\frac{17}{x}+\frac{1-x}{x}$$

$$\frac{x+3}{x-2}=\frac{17+1-x}{x}$$

$x(x + 3) = (18 - x)(x - 2)$

$x^2 + 3x = 18x - x^2 - 36 + 2x$

$2x^2 - 17x + 36 = 0$

$2x^2 - 9x - 8x + 36 = 0$

$x(2x - 9) - 4(2x - 9) = 0$

$(2x - 9)(x - 4) = 0$

$x = \frac{9}{2}, 4$

29. Let breadth of rectangle be x

Length of rectangle be 2x

Area of rectangle = $x \times 2x = 2x^2$

Area of shaded region = $2x^2 - \frac{1}{2}\pi x^2$

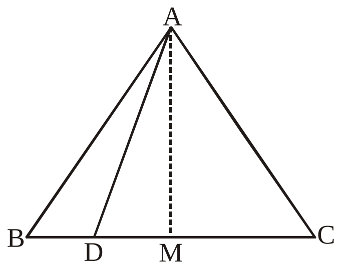
Ratio = $\frac{2x^2}{2x^2 - \frac{1}{2}\pi x^2} = \frac{2\times 2}{4-\pi} = \frac{4}{4-\pi} = \frac{4}{4-\frac{22}{7}}$

$\frac{28}{6} = \frac{14}{3}$

Ratio = 14 : 3

30. AM ⊥ BC is drawn

In ΔADM, by pythagores theroem



$AD^2 = AM^2 + DM^2$

$AD^2 = AB^2 - BM^2 + DM^2$

$AD^2 = AB^2 - (BM^2 - DM^2)$

$= AB^2 - (BM + DM)(BM - DM)$

$= AB^2 - (CM + DM)(BD)$ [$\because$ BM = CM]

$= AB^2 - CD.BD$

$= AB^2 - \frac{2}{3}BC.\frac{1}{3}BC$

$[BD = \frac{1}{3}BC \Rightarrow CD = \frac{2}{3}BC]$

$= AB^2 - \frac{2}{9}BC^2$

$AD^2 = \frac{9AB^2 - 2AB^2}{9}$ [$\because$ AB = BC]

$9AD^2 = 7AB^2$ Hence proved.

OR

In ΔABC, DX || AC

$\frac{BX}{BC} = \frac{BD}{AB}$ ... (1) [By corollary of thales theroem]

In ΔABC, YE || AB

$\frac{CE}{AC} = \frac{CY}{CB}$ ... (2) [By corollary of thales theroem]

From (1) and (2)

$\frac{BD}{AB} = \frac{CE}{AC}$ [$\because$ BX = CY]

$\frac{AB}{BD} = \frac{AC}{CE}$

Subracting 1 from both sides

$\frac{AB - BD}{BD} = \frac{AC - CE}{CE}$

$\frac{AD}{BD} = \frac{AE}{CE}$

By inverse of BPT

DE || BC.

31.

Class	Frequency	cf
0–10	7	7
10–20	8	15
20–30	20	35
30–40	8	43
40–50	7	50

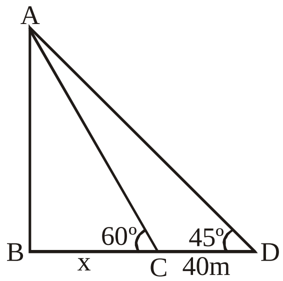
$\frac{N}{2} = \frac{50}{2} = 25$

Median class 20 – 30, cf = 15, f = 20, b = 10

Median = $20 + \frac{25 - 15}{20} \times 10$

= 20 + 5 = 25

32. Let AB be the tower of height h. D and B are points at a distance of 40m from each other and makes angle of elevation 45º and 60º. Let BC = x



In ΔABC

$$\tan 60^\circ = \frac{h}{x}$$

$$\sqrt{3} = \frac{h}{x} \qquad [\because \tan 60^\circ = \sqrt{3}]$$

$$x = \frac{h}{\sqrt{3}} \quad \ldots (1)$$

In ΔABD

$$\tan 45^\circ = \frac{h}{x+40}$$

$$1 = \frac{h}{x+40}$$

$$h = x + 40$$

$$h = \frac{h}{\sqrt{3}} + 40 \qquad \text{(from 1)}$$

$$h - \frac{h}{\sqrt{3}} = 40$$

$$h\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) = 40$$

$$h = \frac{40\sqrt{3}\left(\sqrt{3}+1\right)}{2}$$

$$= 20\sqrt{3}(\sqrt{3} + 1)$$

$$= 20 \times 1.73 \times 2.73 = 94.46 \text{ m}$$

33.

Class	Frequency
0-10	5
10-20	10
20-30	18
30-40	30
40-50	20
50-60	12
60-70	5

Modal class is 30–40

$f_1 = 30$, $f_2 = 20$, $f_0 = 18$, $h = 10$

$$\text{Mode} = \ell + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

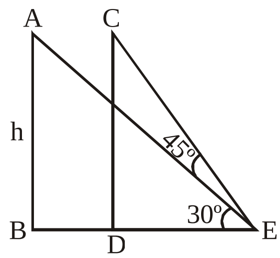
$$= 30 + \frac{30-18}{60-20-18} \times 10$$

$$= 30 + \frac{12}{22} \times 10$$

$$= 30 + 5.46 = 35.46$$

SECTION-V

34. Distance travelled in 20 seconds



$$= 360 \times \frac{2}{3600}$$

$= 2$ km

Let C and A be the positions of areoplanes and BD = 2 km

Let DE = x

In ΔCDE

$$\tan 45^\circ = \frac{CD}{x}$$

$CD = x \qquad [\because \tan 45^\circ = 1]$

In ΔABE

$$\tan 30^\circ = \frac{AB}{BE}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{2+x} \quad [\because AB = CD]$$

$$2 + x = \sqrt{3}x$$

$$2 = (\sqrt{3} - 1)x$$

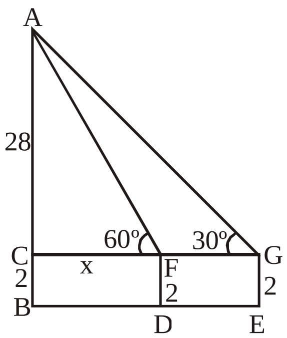
$$x = \frac{2(\sqrt{3}+1)}{2}$$

$$= \sqrt{3} + 1 = 2.732 \text{ km}$$

OR

Let AB be the building & GE and DF be the position of boy. Let CF = x

In ΔACF



$$\frac{28}{x} = \tan 60^\circ$$

$$x = \frac{28}{\sqrt{3}} \quad ...(1) \qquad [\because \tan 60^\circ = \sqrt{3}]$$

In ΔACG

$$\tan 30^\circ = \frac{28}{CG}$$

$$CG = 28\sqrt{3} \qquad [\because \tan 30^\circ = \frac{1}{\sqrt{3}}]$$

$$CF + FG = 28\sqrt{3}$$

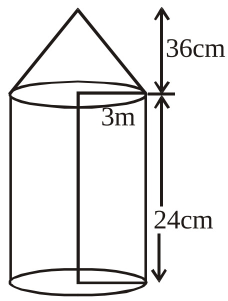
$$FG = 28\sqrt{3} - \frac{28}{\sqrt{3}} \qquad [\text{From (1)}]$$

$$= \frac{28(3-1)}{\sqrt{3}}$$

$$= \frac{28 \times 2\sqrt{3}}{3} = 32.30 \text{ m}$$

35. Volume of pillar

= Volume of cone volume of cylinder



$$= \frac{1}{3}\pi r^2 h + \pi r^2 H$$

$$= \frac{1}{3}\pi\, 8^2 \times 36 + \pi\, 8^2 \times 240$$

$$= \pi\, 8^2\, [12 + 240]$$

$$= \frac{22}{7} \times 64 \times 252$$

$= 22 \times 64 \times 36 = 50688 \text{ cm}^3$

Weight of 1 cu cm of iron = 7.5 g

Weight of pillar = 50688×7.5

= 380160 gm

= 380.16 kg

36. Let speed of train be x km / hr and speed of car be y km / hr

ATQ

$$\frac{320}{x} + \frac{280}{y} = 8 + \frac{40}{60} = \frac{26}{3}$$

$$\frac{200}{x} + \frac{400}{y} = 9 + \frac{10}{60} = \frac{55}{6}$$

Let $\frac{1}{x} = u$, $\frac{1}{y} = v$

$$320u + 280v = \frac{26}{3} \quad ...(1)$$

$$200u + 400v = \frac{55}{6} \quad ...(2)$$

Multiplying equation (1) by 20 & (2) by 32 and subtracting

$$6400\,u + 5600\,v = \frac{520}{3}$$

$$6400u + 12800\,v = \frac{880}{3}$$

$$\underline{-\qquad -\qquad\qquad -}$$

$$7200\,v = 120$$

$$v = \frac{120}{7200} = \frac{1}{60}$$

From (1)

$$320u + 280 \times \frac{1}{60} = \frac{26}{3}$$

$$320u = \frac{26}{3} - \frac{14}{3}$$

$$320u = \frac{12}{3}$$

$$u = \frac{4}{320} = \frac{1}{80}$$

Thus, speed of train = x = $\frac{1}{u}$ = 80 km/hr

Thus, speed of train = y = $\frac{1}{v}$ = 60 km/hr