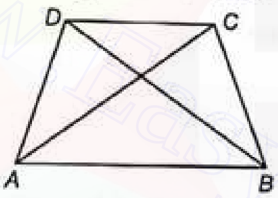
CHAPTER

26

Quadrilateral and Polygon

Quadrilateral

A plane, closed figure bounded by four segments is called quadrilateral. There are different types of quadrilateral so called trapezium, parallelogram, rhombus, rectangle, square.



- Sum of all the angles of a quadrilateral is 360°.
- Here, ABCD is a quadrilateral. $(\angle A, \angle B)$; $(\angle B, \angle C)$; $(\angle C, \angle D)$; $(\angle D, \angle A)$ are four pairs of consecutive angles of quadrilateral *ABCD*.
- *AC* and *BD* are diagonals.
- (AB, BC); (BC, CD); (CD, DA) and (DA, AB) are four pairs of adjacent sides.

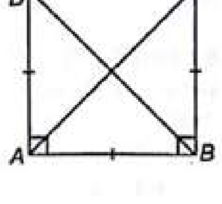
Various Types of Quadrilateral

Parallelogram A quadrilateral in which opposite sides are equal and parallel, then it is a parallelogram, written as ||gm. *e.g.*, square, rectangle, rhombus.

Properties of Parallelogram

(a) Opposite sides are equal.
(b) Opposite angles are equal.
(c) The two diagonals bisect each other.
(d) Diagonal are equal in case of square and rectangle but not in rhombus.
(e) A diagonal of a parallelogram bisects one of the angles of the parallelogram, it also bisects the second angle and then the two diagonals are perpendicular to each other.

Square A parallelogram in which all sides are equal and are parallel. Here, angle between the adjacent sides is 90°.

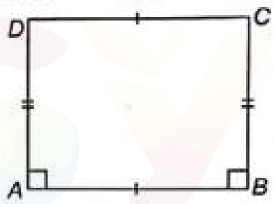


- Diagonals of square are equal.

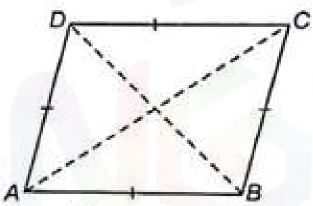
$$AC = BD$$

- Diagonal bisects at right angle.
- Diagonals are perpendicular to each other.

Rectangle The parallelogram in which only opposite side are equal and parallel and angle between adjacent sides is 90°. Diagonals are equal and bisect each other.

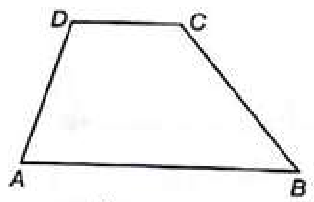


Rhombus A parallelogram in which all sides are equal, is called a rhombus.



- Diagonal bisect each other at 90°.
- Diagonals are not equal.
- Sum of square of sides is equal to sum of the square of diagonal. *i.e.*, $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$.

Trapezium A quadrilateral in which two opposite sides are parallel and other two sides are not parallel. It is called a trapezium.

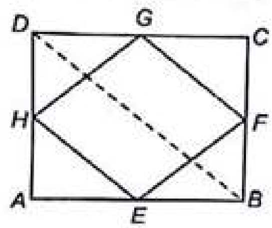


Here, $AB \parallel DC$ and $AD \nparallel BC$

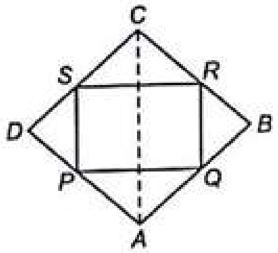
$$AC^2 + BD^2 = BC^2 + AD^2 + 2AB \cdot CD$$

Some Facts about Quadrilaterals

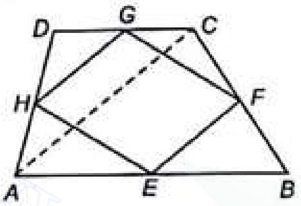
- The quadrilateral formed by joining the mid-points of the consecutive sides of a rectangle is a rhombus. Here, *E, F, G, H* are mid-points of *AB, BC, CD, DA*, respectively, then *EFGH* is a rhombus.



- The quadrilateral formed by joining the mid-point of the consecutive sides of a rhombus is a rectangle. Here, $PQRS$ will be a rectangle.
- The quadrilateral formed by joining the mid-points of the sides of a square, is also a square.



- The figure formed by joining the mid-points of the pairs of consecutive sides of a quadrilateral is a parallelogram.
  Here, $ABCD$ is a quadrilateral while $EFGH$ is a parallelogram.

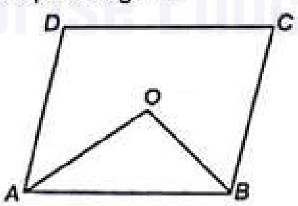


- Two parallelograms on the same base and between same parallel lines have equal areas.
- One parallelogram and one rectangle on the same base and between same parallel lines have equal areas.
- If a rectangle and parallelogram have same dimensions x and y, then area of rectangle > area of parallelogram
- One rectangle/parallelogram and one triangle on the same base and between same parallel lines are related as : Area of rectangle $= 2 \times$ Area of the triangle.
- Two triangles on the same base and between same parallel lines have equal areas.

Examaple 1. In a parallelogram $ABCD$, the bisectors of $\angle A$ and $\angle B$ meet at O. Then, the value of $\angle AOB$ is

(a) 55° (b) 75°
(c) 90° (d) 120°

Sol. (c) As, $ABCD$ is a parallelogram.

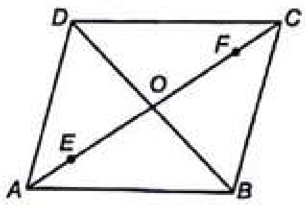


$\therefore \quad \angle A + \angle B = 180°$

$$\frac{1}{2}\angle A + \frac{1}{2}\angle B = 90° \Rightarrow \angle OAB + \angle OBA = 90°$$

$\therefore \quad \angle AOB = 180° - (\angle OAB + \angle OBA) = 180° - 90° = 90°$

Example 2. In the adjoining figure $ABCD$ is a parallelogram and E, F are the centroids of ΔABD and ΔBCD, respectively, then the length of EF is



(a) AE (b) OB (c) $\frac{1}{3}AE$ (d) $\frac{1}{3}FC$

Sol. (a) As E is the centroid of ΔABD and AO is one of its medians.

$\Rightarrow \quad AE : EO = 2 : 1$

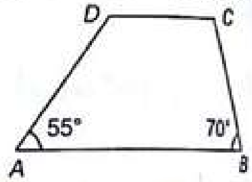
$\therefore \quad EO = \frac{1}{3}OA$

Similarly, $\quad FO = \frac{1}{3}OC$

$\therefore \quad EO + OF = \frac{1}{3}OA + \frac{1}{3}OC = \frac{1}{3}AC = AE$

$\therefore \quad EF = AE$

Example 3. In the adjoining figure, $ABCD$ is a trapezium in which $AB \parallel DC$. If $\angle A = 55°$ and $\angle B = 70°$, then the value of $\angle C$ and $\angle D$ is



(a) 75° and 85°
(b) 90° and 120°
(c) 110° and 125°
(d) 115° and 120°

Sol. (c) As, $AB \parallel CD$

So, $\quad \angle A + \angle D = 180°$

$\Rightarrow \quad \angle D = 180° - 55° = 125°$

Also, $\quad \angle B + \angle C = 180°$

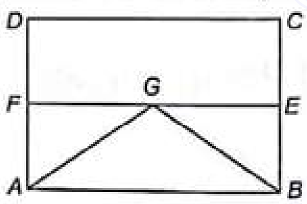
$\Rightarrow \angle C = 180° - \angle B = 180° - 70° = 110°$

So, $\angle C = 110°$ and $\angle D = 125°$

Example 4. If $ABCD$ is a rectangle, E, F are the mid-points of BC and AD, respectively and G is any point on EF, then what will be the area of ΔGAB?

(a) area of rectangle $ABCD$
(b) $\frac{1}{2}$ area of rectangle $ABCD$
(c) $\frac{1}{4}$ area of rectangle $ABCD$
(d) None of the above

Sol. (c) $\because AB \parallel EF \parallel CD$. So, $ABEF$ is a rectangle.



$\therefore$ Area of $\Delta AGB = \frac{1}{2}$ (area of rectangle $ABEF$)

$$\therefore \quad = \frac{1}{2} \times \left(\frac{1}{2} \times \text{area of rectangle } ABCD\right)$$

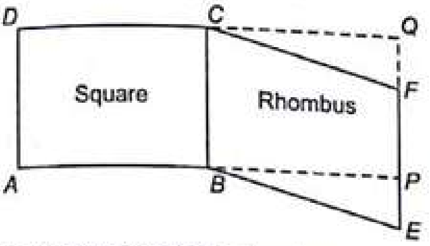
$$= \frac{1}{4} \times (\text{area of rectangle } ABCD)$$

Example 5. If a square and a rhombus stand on the same base, then the ratio of the areas of the square and rhombus is

(a) 1 : 2 (b) 2 : 1
(c) 1 : 3 (d) 1 : 1

Sol. (d) $ABCD$ is a square and $BCFE$ is the rhombus on the same base BC.

Since, ΔBPE and ΔCQF are equal in area.



Therefore, both square and rhombus have equal are.

$\therefore$ Ratio of their areas = 1 : 1

Polygons

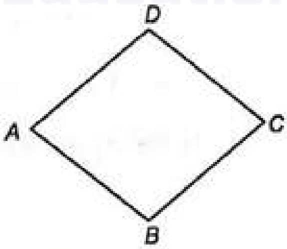
A polygon is a closed, plane figure bounded by 'n' straight lines ($n \geq 3$). Each of the n line segment forming the polygon is called its sides.

A polygon may be a triangle, quadrilateral, pentagon etc. Polygons are classified according to the number of sides as given below

Number of sides	Name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Novagon
10	Decagon

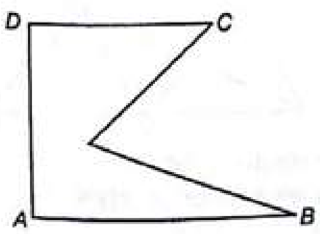
Regular Polygon A polygon is called a regular polygon if all its sides are equal and all angles are equal.

Convex Polygon A polygon is said to be a convex polygon, if each side of the polygon, the line containing that side has all the other vertices on the same side of it.



- None of its interior angle is more than 180°.

Concave Polygon A polygon in which atleast one of the interior angle is more than 180°. Then, the polygon is said to be a concave polygon.



Important Results

- If there is a polygon of n sides ($n \geq 3$), then we can cut it into $(n-2)$ triangles with common vertex. Then, sum of the interior angles of a polygon of n sides is $(2n-4) \times 90°$ or $2(n-2) \times 90°$.
- Each exterior angle of a regular polygon of n sides is $\left(\frac{360}{n}\right)^\circ$.
- Each interior angle of a regular polygon of n sides is $\frac{(n-2) \times 180°}{n}$ or interior angle $= 180° - $ (exterior angle).
- The sum of all the exterior angle formed by producing the sides of a convex polygon in the same order is equal to 360°.
- Number of diagonals of a polygon of n sides is $\frac{n(n-1)}{2} - n$.
- Area of polygon $= \frac{na^2}{4} \cot\left(\frac{180}{n}\right)$ where n = number of sides and a = side length.
- Radius of incircle of a polygon $= \frac{\text{Area of polygon}}{\text{Semi-perimeter of polygon}}$

Example 6. A polygon has 35 diagonals. Then, the number of sides of that polygon is

(a) 7 (b) 10 (c) 11 (d) 12

Sol. (b) Let number of sides be n, then

$$\frac{n(n-1)}{2} - n = 35 \therefore \frac{n^2 - n - 2n}{2} = 35$$

$$\Rightarrow n^2 - 3n - 70 = 0 \Rightarrow n^2 - 10n + 7n - 70 = 0$$

$$\Rightarrow n(n-10) + 7(n-10) = 0 \Rightarrow (n-10)(n+7) = 0$$

$$\Rightarrow n = 10 \text{ and } n \neq -7$$

$\therefore$ The polygon must have 10 sides.

Example 7. The angles of a hexagon are $x°$, $(x-5)°$, $(x-5)°$, $(2x-5)°$, $(2x-5)°$ and $(2x+20)°$. Then, the value of x is

(a) 60° (b) 75° (c) 80° (d) 90°

Sol. (c) Sum of interior angles of a hexagon = 720°

$$\therefore x° + (x-5)° + (x-5)° + (2x-5)° + (2x-5)° + (2x+20)° = 720°$$

$$\therefore 9x = 720° = \frac{720°}{9} = 80°$$

Example 8. The difference between the interior and exterior angles of a regular polygon is 60°. Then, how many sides are there in that polygon?

(a) 5 (b) 6 (c) 7 (d) 8

Sol. (b) Here, (interior angle) − (exterior angle) = 60°

$$\Rightarrow \frac{(n-2) \times 180}{n} - \frac{360}{n} = 60 \Rightarrow \frac{1}{n}[(n-2) \times 180 - 360] = 60$$

$$\Rightarrow \frac{1}{n}[180n - 360 - 360] = 60 \Rightarrow \frac{1}{n}[180n - 720] = 60$$

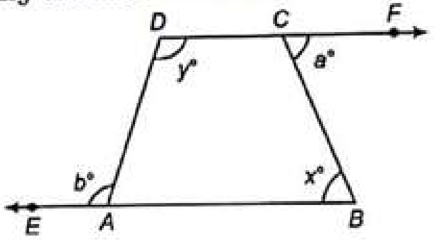
$$\Rightarrow 180n - 720 = 60n \Rightarrow 180n - 60n = 720$$

$$\Rightarrow 120n = 720 \therefore n = 6$$

Therefore, the polygon contains 6 sides.

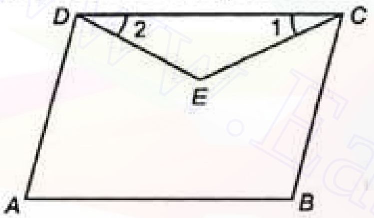
Exercise

1. The sides BA and DC of quadrilateral $ABCD$ are produced as shown in figure. Then, which of the following statement is correct?



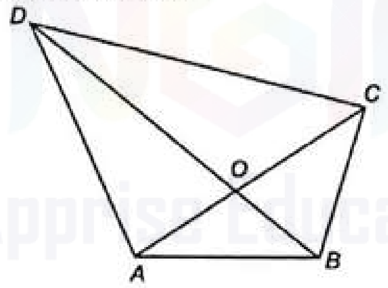
(a) $2x° + y° = a° + b°$ (b) $x° + \frac{1}{2}y° = \frac{a° + b°}{2}$
(c) $x° + y° = a° + b°$ (d) $x° + a° = y° + b°$

2. In the quadrilateral $ABCD$, the line segments bisecting $\angle C$ and $\angle D$ meet at E. Then, the correct statement is



(a) $\angle A + \angle B = \angle CED$ (b) $\angle A + \angle B = 2\angle CED$
(c) $\angle A + \angle B = 3\angle CED$ (d) None of these

3. If $ABCD$ is a quadrilateral whose diagonals AC and BD intersects of O, then



(a) $(AB + BC + CD + DA) < (AC + BD)$
(b) $(AB + BC + CD + DA) > 2(AC + BD)$
(c) $(AB + BC + CD + DA) > (AC + BD)$
(d) $(AB + BC + CD + DA) = 2(AC + BD)$

4. If area of a parallelogram with sides p and q is R and that of a rectangle with sides p and q is S, then
(a) $R > S$ (b) $R < S$
(c) $R = S$ (d) None of these

5. An acute angle made by a side of a parallelogram with other pair of parallel sides is 30°. If the distance between these parallel sides is 10 cm, the other side is
(a) 10 cm (b) $10\sqrt{3}$ cm
(c) 20 cm (d) None of these

6. Two parallelogram stand on equal bases and between the same parallel. The ratio of their areas is
(a) 1:2 (b) 2:1 (c) 1:3 (d) 1:1

7. If $ABCD$ is a rhombus, then
(a) $AC^2 + BD^2 = 4AB^2$ (b) $AC^2 + BD^2 = AB^2$
(c) $AC^2 + BD^2 = 2AB^2$ (d) $2(AC^2 + BD^2) = 3AB^2$

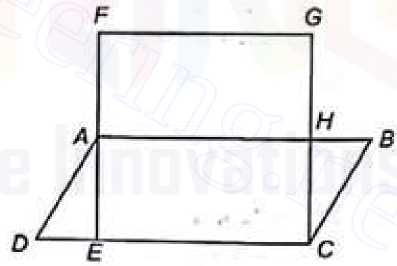
8. A point O in the interior of a rectangle $ABCD$ is joined with each of the vertices A, B, C and D. Then,
(a) $OB + OD = OC + OA$ (b) $OB^2 + OA^2 = OC^2 + OD^2$
(c) $OB \cdot OD = OC \cdot OA$ (d) $OB^2 + OD^2 = OC^2 + OA^2$

9. In a trapezium $ABCD$, if $AB \| CD$, then $AC^2 + BD^2$ is equal to
(a) $BC^2 + AD^2 + 2AB \cdot CD$ (b) $AB^2 + CD^2 + 2AD \cdot BC$
(c) $AB^2 + CD^2 + 2AB \cdot CD$ (d) $BC^2 + AD^2 + 2BC \cdot AD$

10. The parallel sides of a trapezium are p and q respectively. The line joining the mid-points of its non-parallel sides will be
(a) $\sqrt{pq}$ (b) $\frac{2pq}{p+q}$ (c) $\frac{(p+q)}{2}$ (d) $\frac{1}{2}(p-q)$

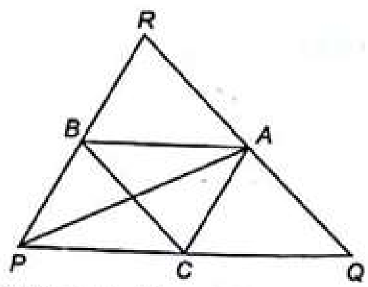
11. $ABCD$ is a trapezium in which $AB \| CD$ and $AB = 2CD$. Its diagonals intersects each other at O, then the ratio of the areas of the ΔAOB and ΔCOD is
(a) 1:2 (b) 2:1
(c) 4:1 (d) 1:4

12. In the figure, $ABCD$ is a parallelogram with $AD = a$ units, $DC = 2a$ units and $DE : EC = 1:2$. $CEFG$ is a rectangle with $FE = 3\,AE$. What is the ratio of the area of a parallelogram and the rectangle?



(a) 1:1 (b) 1:2 (c) $\frac{a^2}{5}:1$ (d) 2:1

13. In the figure, $AB \| PQ$, $AC \| PR$ and $BC \| QR$, then AP is



(a) perpendicular to QR
(b) the angle bisector of $\angle QPR$
(c) a median of $\angle PQR$
(d) None of the above

14. How many points "P" in the plane of a rhombus $ABCD$ are such that "P" is equidistant from the sides of $ABCD$?
(a) 4 (b) 2 (c) 1 (d) 0

15. Let $LMNP$ be a parallelogram and NR be perpendicular to LP. If the area of the parallelogram is six times the area of ΔRNP and $RP = 6$ cm what is LR equal to? **(CDS 2011 I)**
(a) 15 cm (b) 12 cm (c) 9 cm (d) 8 cm

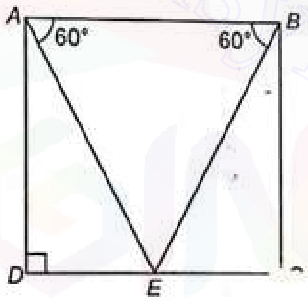
Directions (Q.Nos. 16-18) *Let ABCD be a quadrilateral. Let the diagonals AC and BD meet at O. Let the perpendicular drawn from A to CD, meet CD at E. Further, AO : OC = BO : OD, AB = 30 cm, CD = 40 cm and the area of the quadrilateral ABCD is 1050 sq cm.* **(CDS 2010 II)**

16. What is the value of BE?
(a) 30 cm (b) $30\sqrt{2}$ cm
(c) $30\sqrt{3}$ cm (d) None of these

17. What is the area of the ΔADC?
(a) 300 cm^2 (b) 450 cm^2
(c) 600 cm^2 (d) None of these

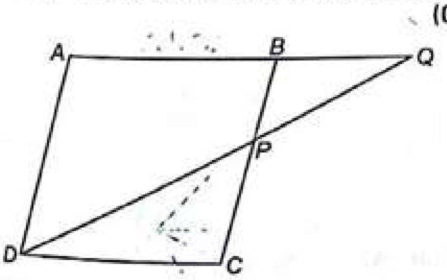
18. What is the value of $\angle AEB$?
(a) 30° (b) 45°
(c) 60° (d) None of these

19. In the given figure $ABCD$ is a quadrilateral with AB parallel to DC and AD parallel to BC, ADC is a right angle. If the perimeter of the ΔABE is 6 units, what is the area of the quadrilateral? **(CDS 2010 II)**



(a) $2\sqrt{3}$ sq units
(b) 4 sq units
(c) 3 sq units
(d) $4\sqrt{3}$ sq units

20. In the figure given below, $ABCD$ is a parallelogram. P is a point in BC such that $PB : PC = 1 : 2$. DP produced meets AB produced at Q. If the area of the ΔBPQ is 20 sq units, what is the area of the ΔDCP? **(CDS 2010 II)**



(a) 20 sq units (b) 30 sq units
(c) 40 sq units (d) None of these

21. $ABCD$ is a square, P, Q, R and S are points on the sides AB, BC, CD and DA, respectively such that $AP = BQ = CR = DS$. What is the value of $\angle SPQ$? **(CDS 2010 I)**
(a) 30° (b) 45° (c) 60° (d) 90°

22. The middle points of the parallel sides AB and CD of a parallelogam $ABCD$ are P and Q, respectively. If AQ and CP divide the diagonal BD into three parts BX, XY and YD, then which one of the following is correct? **(CDS 2010 I)**
(a) $BX \neq XY \neq YD$ (b) $BX = YD \neq XY$
(c) $BX = XY = YD$ (d) $XY = 2BX$

23. A parallelogram and a rectangle stand on the same base and on the same side of the base with the same height. If l_1, l_2 be the perimeters of the parallelogram and the rectangle respectively, then which one of the following is correct? **(CDS 2010 I)**
(a) $l_1 < l_2$ (b) $l_1 = l_2$
(c) $l_1 > l_2$ but $l_1 \neq 2l_2$ (d) $l_1 = 2l_2$

24. Two similar parallelograms have corresponding sides in the ratio $1 : k$. What is the ratio of their areas? **(CDS 2010 I)**
(a) $1 : 3k^2$ (b) $1 : 4k^2$ (c) $1 : k^2$ (d) $1 : 2k^2$

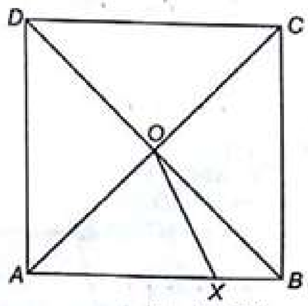
25. Consider the following statements in respect of a quadrilateral
I. The line segments joining the mid-points of the two pairs of opposite sides bisect each other at the point of intersection.
II. The area of the quadrilateral formed by joining the mid-points of the four adjacent sides is half of the total area of the quadrilateral. **(CDS 2010 I)**

Which of the statements given above is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

26. Let $WXYZ$ be a square. Let P, Q and R be the mid-points of WX, XY and ZW, respectively and K, L be the mid-points of PQ and PR, respectively. What is the value of $\dfrac{\text{area of } \Delta PKL}{\text{area of square } WXYZ}$? **(CDS 2010 I)**
(a) $\frac{1}{32}$ (b) $\frac{1}{16}$ (c) $\frac{1}{8}$ (d) $\frac{1}{64}$

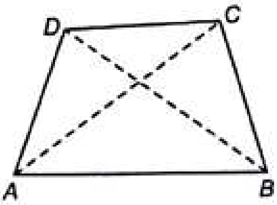
27. ABC is a triangle in which $AB = AC$. Let BC be produced to D. From a point E on the line AC let EF be a straight line such that EF is parallel to AB. Consider the quadrilateral $ECDF$ thus formed. If $\angle ABC = 65°$ and $\angle EFD = 80°$, then what is the value of $\angle FDC$? **(CDS 2009 II)**
(a) 43° (b) 41° (c) 37° (d) 35°

28. In the given figure, $ABCD$ is a square in which $AO = AX$. What is $\angle XOB$? **(CDS 2009 II)**



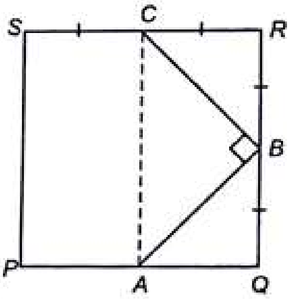
(a) 22.5° (b) 25°
(c) 30° (d) 45°

29. In the adjoining figure, $ABCD$ is a quadrilateral in which AB is the longest side and CD is the shortest side, then



(a) $\angle C > \angle A$ and $\angle D > \angle B$
(b) $\angle C > \angle A$ and $\angle B > \angle D$
(c) $\angle C < \angle A$ and $\angle D < \angle B$
(d) $\angle C < \angle A$ and $\angle D = \angle B$

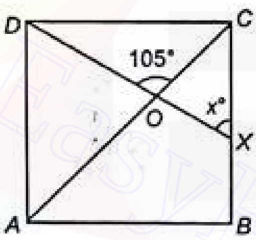
30. In the given figure, $PQRS$ is a square and $\angle ABC = 90°$. If $AQ = BR = SC$, then consider the following statements.



I. $QB = RC$
II. $AB = BC$
III. $\angle BAC = 45°$
IV. $AC = PS$

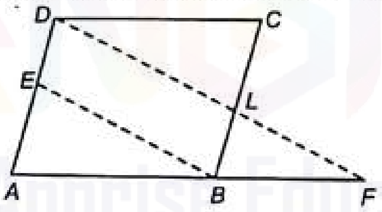
(a) Only I and IV are correct
(b) I, II and III are correct
(c) II, III and IV are correct
(d) All are correct

31. In the given figure, $ABCD$ is a square. A line segment DX cuts the side BC at X and the diagonal AC at O such that $\angle COD = 105°$ and $\angle OXC = x$. The value of x is



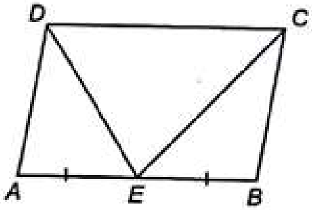
(a) 40° (b) 60°
(c) 80° (d) 85°

32. In the given figure, $ABCD$ is a parallelogram and E is the mid-point of AD. A line through D, drawn parallel to EB, meets AB produced at F and BC at L. Then,



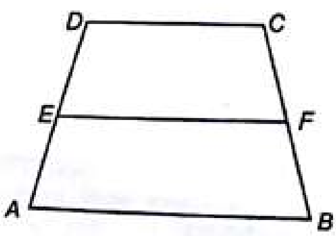
(a) $AF = 2DC$ (b) $2AF = 3DC$
(c) $2DF = 3DL$ (d) $AF + DF = 3DC + DL$

33. In the given figure, $ABCD$ is a parallelogram, E is the mid-point of AB and CE bisects $\angle BCD$. Then, which of the following is correct statement?



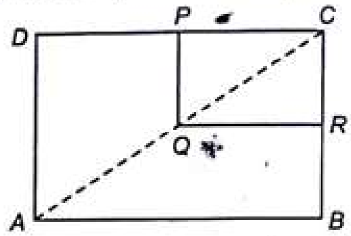
(a) $AE = AD$
(b) $DE = EC$
(c) $AD = EC$
(d) $DE + EC = DC$

34. Let $ABCD$ be a trapezium in which $AB \parallel DC$ and let E be the mid-point of AD. Let F be a point on BC such that $EF \parallel AB$. Then, consider the following statements.



(a) F is the mid-point of BC (b) $EF = \frac{1}{2}(AB + DC)$
(c) Both (a) and (b) (d) None of these

35. In the figure given below, $ABCD$ and $PQRC$ are rectangles, where Q is the mid-point of AC, then

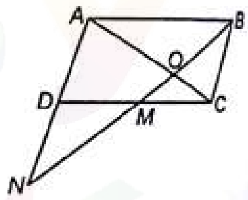


(a) $DP = PC$ (b) $\frac{1}{3}PR = \frac{3}{2}DB$
(c) $DP = PQ$ (d) $DP = \frac{1}{2}AC$

36. Two sides of a parallelogram are 10 cm and 15 cm. If the altitude corresponding to the side of length 15 cm is 5 cm, then what is the altitude to the side of length 10 cm? **(CDS 2009 I)**

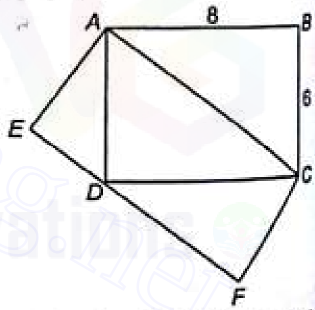
(a) 5 cm (b) 7.5 cm
(c) 10 cm (d) 15 cm

37. In the given figure, M is the mid-point of the side CD of the parallelogram $ABCD$. What is $ON : OB$? **(CDS 2009 I)**



(a) 3 : 2 (b) 2 : 1
(c) 3 : 1 (d) 5 : 2

38. $ABCD$ is a rectangle of dimensions 8 units and 6 units. $AEFC$ is a rectangle drawn in such a way that diagonal AC of the first rectangle is one side and side opposite to it is touching the first rectangle at D as shown in the given figure. What is the ratio of the area of rectangle $ABCD$ to that of $AEFC$? **(CDS 2008 II)**



(a) 2 (b) 3/2 (c) 1 (d) 8/9

39. $ABCD$ is a square. The diagonals AC and BD meet at O. Let K, L be the points on AB such that $AO = AK$, $BO = BL$. If $\theta = \angle LOK$, then what is the value of $\tan\theta$? **(CDS 2008 II)**

(a) $1/\sqrt{3}$ (b) $\sqrt{3}$
(c) 1 (d) 1/2

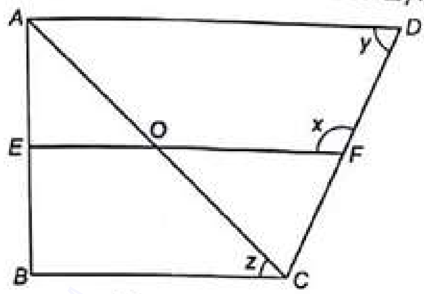
40. **Assertion (A)** If the side of a rhombus is 10 cm. Its diagonals should have values 16 cm and 12 cm.
Reason (R) The diagonals of a rhombus cut at right angles. **(CDS 2007 II)**

(a) A and R are correct and R is correct explanation of A
(b) A and R are correct but R is not correct explanation of A
(c) A is correct but R is wrong.
(d) A is wrong but R is correct.

41. An equilateral triangle and a regular hexagon are inscribed in a given circle. If a and b are the lengths of their sides respectively, then which one of the following is correct? **(CDS 2007 II)**

(a) $a^2 = 2b^2$ (b) $b^2 = 3a^2$ (c) $b^2 = 2a^2$ (d) $a^2 = 3b^2$

42. $ABCD$ is a trapezium in which EF is parallel to BC. $\angle x = 120°$ and $\angle z = 50°$, then what is $\angle y$? **(CDS 2007 II)**



(a) 50° (b) 60° (c) 70° (d) 80°

43. An obtuse angle made by a side of a parallelogram $PQRS$ with other pair of parallel sides is 150°. If the perpendicular distance between these parallel sides (PQ and SR) is 20 cm, what is the length of the side RQ? **(CDS 2007 I)**

(a) 40 cm (b) 50 cm (c) 60 cm (d) 70 cm

44. The measure of each angle of a regular hexagon is

(a) 90° (b) 120° (c) 105° (d) 135°

45. Each interior angle of a regular polygon is 150°. The number of sides of the polygon is

(a) 4 (b) 8 (c) 12 (d) 16

46. How many diagonals are there in a octagon?

(a) 8 (b) 16 (c) 18 (d) 20

47. If one of the interior angles of a regular polygon is found to be $\frac{9}{8}$ times of one of the interior angles of a regular hexagon, then the number of sides of the polygon is

(a) 8 (b) 14 (c) 12 (d) 10

48. Four angles of a 8-sided figure are each 154°. If the remaining four angles are equal, then the measure of each angle.

(a) 100° (b) 110° (c) 116° (d) 120°

49. The angles of a pentagon are in the ratio $1:2:3:5:9$, the largest angle is

(a) 81° (b) 135° (c) 243° (d) 249°

50. The exterior angle of a regular polygon is one-third of its interior angle. The number of sides of polygon is

(a) 2 (b) 4 (c) 6 (d) 8

51. The ratio between the number of sides of two regular polygon is 1 : 2 and the ratio between their interior angles 2 : 3. The number of sides of these polygons are respectively

(a) 4, 8 (b) 5, 9 (c) 4, 10 (d) 6, 8

52. The ratio between the number of sides of two polygons is $2:1$ and the ratio between their interior angle is $4:3$. The number of sides of these polygons are, respectively

(a) 8, 4 (b) 10, 5 (c) 12, 6 (d) 14, 7

53. The ratio of an interior angle to the exterior angle of a regular polygon is 5 : 1. The number of sides of polygon is

(a) 10 (b) 11
(c) 12 (d) 14

Answers

1. (c)	**2.** (b)	**3.** (c)	**4.** (b)	**5.** (c)	**6.** (d)	**7.** (a)	**8.** (d)	**9.** (a)	**10.** (c)
11. (c)	**12.** (b)	**13.** (b)	**14.** (c)	**15.** (b)	**16.** (b)	**17.** (c)	**18.** (b)	**19.** (a)	**20.** (d)
21. (d)	**22.** (c)	**23.** (c)	**24.** (c)	**25.** (c)	**26.** (b)	**27.** (d)	**28.** (a)	**29.** (a)	**30.** (d)
31. (b)	**32.** (a)	**33.** (a)	**34.** (c)	**35.** (a)	**36.** (b)	**37.** (b)	**38.** (c)	**39.** (c)	**40.** (a)
41. (d)	**42.** (b)	**43.** (a)	**44.** (b)	**45.** (c)	**46.** (d)	**47.** (a)	**48.** (c)	**49.** (c)	**50.** (d)
51. (a)	**52.** (b)	**53.** (c)							

Hints and Solutions

1. As $\angle A + b° = 180° \Rightarrow \angle A = 180° - b$

Also, $\angle C + a° = 180°$ (linear pair)

$\Rightarrow \angle C = 180° - a°$

But $\angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow (180° - b°) + x° + (180° - a°) + y° = 360°$

$\Rightarrow x° + y° = a° + b°$

2. $\angle 1 = \frac{1}{2}\angle C, \angle 2 = \frac{1}{2}\angle D$

$\angle 1 + \angle 2 + \angle CED = 180°$

$\therefore \angle CED = 180° - (\angle 1 + \angle 2)$

Also, $\angle A + \angle B + \angle C + \angle D = 360°$

$\angle A + \angle B + 2(\angle 1 + \angle 2) = 360°$

$\angle A + \angle B = 360° - 2(\angle 1 + \angle 2)$

$\angle A + \angle B = 2\angle CED$

3. In $\Delta ABC, \Delta ACD, \Delta BCD$ and ΔABD

$AB + BC > AC$

$$CD + DA > AC$$
$$BC + CD > BD$$
$$DA + AB > BD$$

Adding above inequalities

$$2(AB + BC + CD + DA) > 2(AC + BD)$$
$$(AB + BC + CD + DA) > (AC + BD)$$

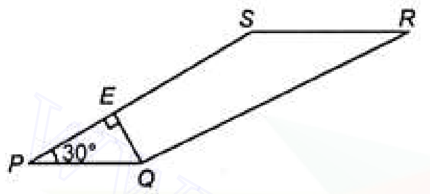
4. Let h be the height of the parallelogram. Then, clearly $h < q$

So, $R = p \times h < p \times q = S$

So, $R < S$

5. Here, $\angle EPQ = 30°$ and also $QE = 10$ cm

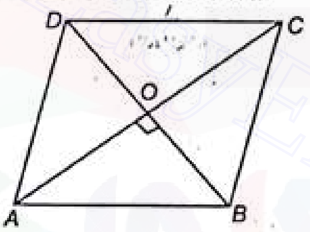
Now, $\text{cosec } 30° = \dfrac{PQ}{QE} = 2$ $\quad (\because \text{coses } 30° = 21)$



$\Rightarrow PQ = 2 \times QE = 2 \times 10 = 20$ cm

6. Two parallelograms on the same base and between the same parallels are equal in area. So, the ratio of their areas is 1:1.

7. As $ABCD$ is a rhombus.



So, $AO = OC = \dfrac{1}{2} AC$

$BO = OD = \dfrac{1}{2} BD$

$\angle AOB = 90°$

$\therefore AB^2 = OA^2 + OB^2$

$AB^2 = \dfrac{AC^2}{4} + \dfrac{BD^2}{4}$

$\Rightarrow 4AB^2 = AC^2 + BD^2$

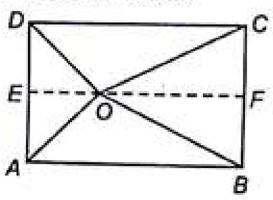
8. Draw $EF \| AB$.

In right angled ΔEOA and ΔOCF,

$OA^2 = OE^2 + AE^2$ and $OC^2 = OF^2 + CF^2$

$\therefore OA^2 + OC^2 = OE^2 + AE^2 + OF^2 + CF^2$ ...(i)

In right angled ΔDEO and ΔOBF,



$OD^2 = OE^2 + DE^2, OB^2 = OF^2 + BF^2$

$\Rightarrow OD^2 + OB^2 = OE^2 + OF^2 + DE^2 + BF^2$ ...(ii)

As $FB = EA$

and $DE = CF$

Here, from Eqs. (i) and (ii), we get

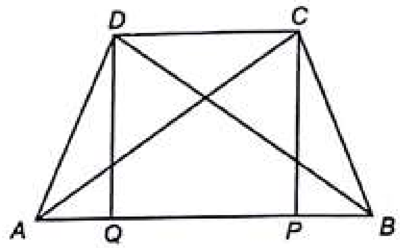
$$OA^2 + OC^2 = OD^2 + OB^2$$

9. In ΔABD, $\angle A$ is acute.

So, $BD^2 = AD^2 + AB^2 - 2AB \cdot AQ$ ...(i)

In ΔABC, $\angle B$ is acute.

So, $AC^2 = BC^2 + AB^2 - 2AB \cdot AD$ ...(ii)



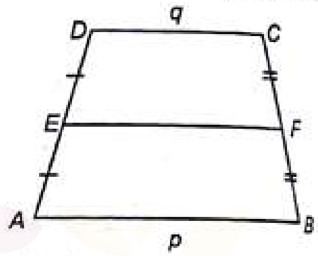
Adding Eqs. (i) and (ii),

$\therefore AC^2 + BD^2 = (BC^2 + AD^2) + 2AB\,(AB - BP - AQ)$

$= (BC^2 + AD^2) + 2AB \cdot PQ$

$= BC^2 + AD^2 + 2AB \cdot CD$ $\quad (\because PQ = DC)$

10. Here, as $ABCD$ is trapezium.

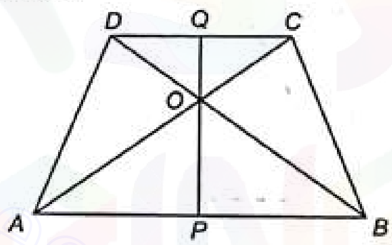


So, $EF = \dfrac{1}{2}(AB + DC)$

$EF = \dfrac{1}{2}(p + q)$

$(\because AB = p, DC = q)$

11. Let PQ be the perpendicular distance between the two parallel sides AB and CD. $\quad (\because PO = 2OQ)$



So, $\dfrac{\text{Area of } \Delta AOB}{\text{Area of } \Delta COD} = \dfrac{\frac{1}{2} AB \times OP}{\frac{1}{2} CD \times OQ}$

$= \dfrac{2CD \times 2OQ}{CD \times OQ} = \dfrac{4}{1}$

12. Area of parallelogram $ABCD$

$= 2$ (area ΔADE) + (area of rectangle $AECH$)

$= 2 \times \dfrac{1}{2} DE \times AE + AE \times EC$

$= DE \times AE + AE \times EC$

$= AE \times [DC]$

$= \dfrac{FE}{3} \times \dfrac{3EC}{2}$

$= \dfrac{FE \times EC}{2}$

$= \dfrac{\text{Area of rectangle } EFGC}{2}$

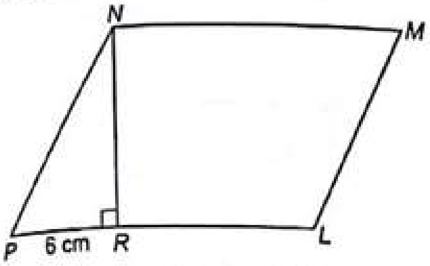
$\left[\because \dfrac{DE}{EC} = \dfrac{1}{2},\ \dfrac{DE + EC}{EC} = \dfrac{3}{2},\ \dfrac{DC}{EC} = \dfrac{3}{2}\right]$

$\therefore$ Required ratio $= 1 : 2$

13. Here, $ABPC$ is a parallelogram, so AP is the angle bisector of $\angle QPR$.

14. Only one point *i.e.*, the point of intersection of diagonals of the rhombus.

15. By given condition,



Area of parallelogram $= 6 \times$ area of ΔNPR

$\therefore \quad NR \times PL = 6 \times \frac{1}{2} \times NR \times PR$

$\Rightarrow \quad PL = 3PR$

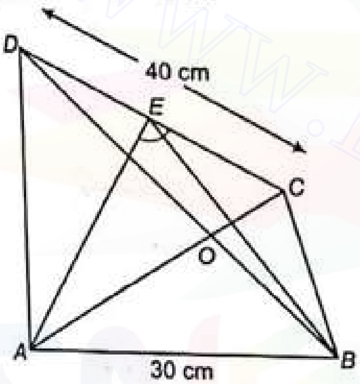
$\Rightarrow \quad PR + RL = 3PR$

$\Rightarrow \quad RL = 2PR = 2 \times 6 = 12$ cm

Solutions (Q.Nos. 16-18)

Given, $\quad AO : OC = BO : OD$

and $AB = 30$ cm and $CD = 40$ cm



$\therefore \quad \Delta AOB \sim \Delta COD$

$\Rightarrow \quad \frac{OC}{OA} = \frac{AB}{CD} = \frac{3}{4}$

$\therefore \quad \angle OAB = \angle OCD$ and $\angle OBA = \angle ODC$

It means $DC \parallel AB$.

So, It is a trapezium.

Area of quadrilateral $ABCD = \frac{1}{2}(AB + CD) \times AE$

$\Rightarrow \quad 1050 = \frac{1}{2}(30 + 40) \times AE$

$\Rightarrow \quad AE = 30$ cm

Also, $\quad \angle BAE = 90°$

16. In right ΔEAB,

$$EB = \sqrt{AE^2 + AB^2} = \sqrt{30^2 + 30^2} = 30\sqrt{2}\text{cm}$$

17. Area of $\Delta ADC = \frac{1}{2} \times CD \times AE = \frac{1}{2} \times 40 \times 30 = 600 \text{ cm}^2$

18. Also, $\quad \angle BAE = 90°, AE = AB = 30$ cm

$\therefore \quad \angle AEB = \angle ABE = 45°$

19. $AB \parallel DC$ and $AD \parallel BC$

In ΔABE,

$\Rightarrow \quad \angle EAB = \angle ABE = 60°$

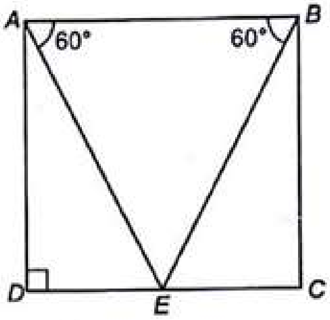
$\angle AEB = 60°$

$\Rightarrow \Delta ABE$ is an equilateral triangle.

Now, perimeter of $\Delta ABE = 6$

$\Rightarrow \quad AB + BE + EA = 6$

$\Rightarrow \quad AB = 2$ units



and in ΔADE,

$AE^2 = AD^2 + ED^2$

$\Rightarrow \quad 4 = AD^2 + 1$

($\because E$ is mid-point of CD)

$\Rightarrow \quad AD = \sqrt{3}$ units

Hence, area of quadrilateral

$ABCD = AB \times AD$

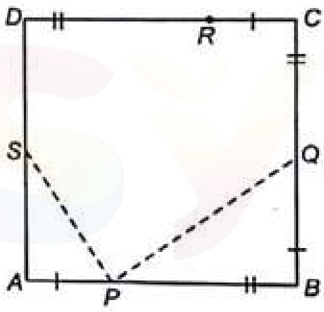
$= 2 \times \sqrt{3} = 2\sqrt{3}$ sq units

20. We know that ratio of the areas of two similar triangles is equal to the ratios of squares of their corresponding sides.

$\therefore \quad \frac{\text{Area}(\Delta BPQ)}{\text{Area}(\Delta DPC)} = \frac{PB^2}{PC^2} \Rightarrow \frac{20}{\text{Area}(\Delta DPC)} = \frac{1}{4}$

$\Rightarrow \quad$ Area $(\Delta DPC) = 80$ sq units

21. In ΔAPS and ΔPBQ, (given)



$PB = AS$

$AP = BQ$

and $\quad \angle A = \angle B = 90°$

$\therefore \quad \Delta APS$ and ΔPBQ are congruent.

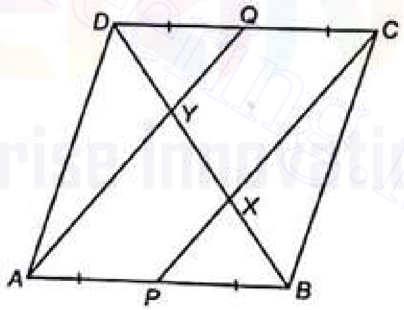
$\therefore \quad SP = PQ$

$\angle SPA = \angle BQP$

and $\quad \angle ASP = \angle BPQ$

$\therefore \quad \angle SPQ = 90°$

22. By properties of parallelogram

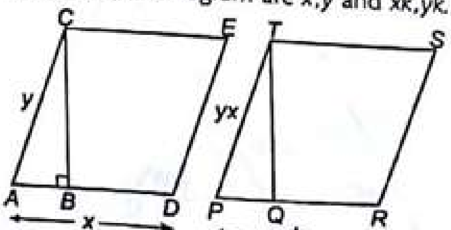


$BX = XY = YD$

23. If a parallelogram and a rectangle stand on the same base and on the same side of the base with the same height, then perimeter of parallelogram is greater than perimeter of rectangle

$\therefore \quad l_1 > l_2$

24. Let the sides of a parallelogram are x, y and xk, yk.



Since, sides of two parallelogram are in $1 : k$.

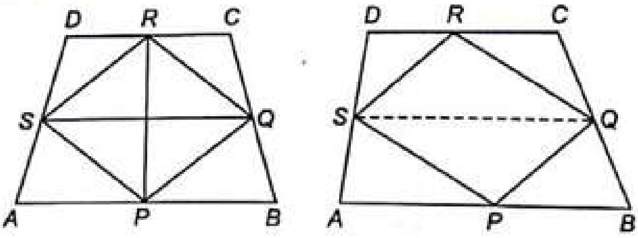
$\therefore \quad \Delta ABC \sim \Delta PQT$

$\therefore \quad \dfrac{AC}{PT} = \dfrac{BC}{QT} \Rightarrow \dfrac{BC}{QT} = \dfrac{y}{yk} = \dfrac{1}{k}$

Let $\quad BC = z$ and $QT = zk$

$\therefore$ Ratio of areas of two similar parallelograms $= \dfrac{x \times z}{xk \times zk} = \dfrac{1}{k^2}$

25. I. PQRS can be shown parallelogram, so the diagonal PR and SQ bisect each other.



II. Area $(RSQ) = \dfrac{1}{2}$ Area $(SQCD)$ ...(i)

and Area $(PSQ) = \dfrac{1}{2}$ Area $(ABQS)$ ...(ii)

On adding Eqs. (i) and (ii), we get

Area (RSQ) + Area $(PSQ) = \dfrac{1}{2}$ [Area $(SQCD)$ + Area $(ABQS)$]

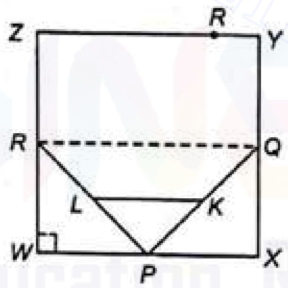
$\Rightarrow$ Area $(PQRS) = \dfrac{1}{2}$ area $(ABCD)$

Hence, both statements are true.

26. Area $(PRQ) = \dfrac{1}{2}$ Area $(WXQR)$

$= \dfrac{1}{2}\left[\dfrac{1}{2}\text{Area}(WXYZ)\right]$

$= \dfrac{1}{4}$ Area$(WXYZ)$ ...(i)



$\dfrac{\text{Area}(PRQ)}{\text{Area}(PLK)} = \dfrac{RP^2}{LP^2}$

(by properties of similar triangle)

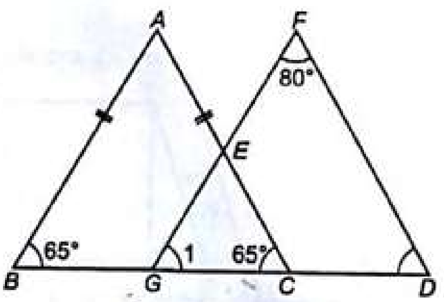
$\Rightarrow \quad \dfrac{\text{Area}(PRQ)}{\text{Area}(PLK)} = \dfrac{(2LP)^2}{LP^2}$

$\Rightarrow \quad$ Area$(PRQ) = 4$ Area (PLK)

$\Rightarrow \quad \dfrac{1}{4}$ Area $(WXYZ) = 4$ Area(PLK) [from Eq. (i)]

$\Rightarrow \quad \dfrac{1}{16}$ Area $(WXYZ)$ = Area $(PLK) \Rightarrow \dfrac{\text{Area}(PLK)}{\text{Area}(WXYZ)} = \dfrac{1}{16}$

27. Here, $\angle B = \angle C = 65°$



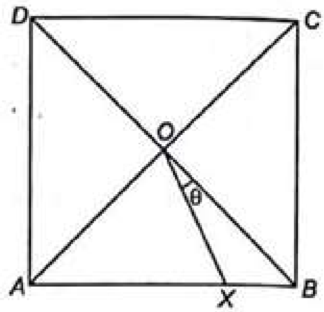
$\angle 1 = \angle B = 65°$ (corresponding angles)

In ΔFGD,

$\angle 1 + \angle F + \angle D = 180°$

$\Rightarrow 65° + 80° + \angle D = 180° \Rightarrow \angle D = 35°$

28. Let $\angle XOB = \theta$



In ΔOXB,

$\angle XOB + \angle OBX + \angle OXB = 180°$

$\Rightarrow \quad \theta + 45° + \angle OXB = 180°$

$\Rightarrow \angle OXB = 180° - 45° - \theta = 135° - \theta$

Here, $\quad \angle OXA + \angle OXB = 180°$

$\Rightarrow \quad \angle OXA + 135° - \theta = 180°$

$\Rightarrow \quad \angle OXA = 45° + \theta$

In ΔOXA,

$AO = OX$

$\therefore \quad \angle OXA = \angle AOX = 45° + \theta$ (given)

Since, $\quad \angle AOX + \angle XOB = 90°$

$\Rightarrow \quad 45° + \theta + \theta = 90°$

$\Rightarrow 2\theta = 45° \Rightarrow \theta = 22.5°$

29. Join AC and BD.

As $\quad AB > AC \Rightarrow \angle ACB > \angle BAC$

Also, $\quad AD > DC$

$\Rightarrow \quad \angle ACD > \angle CAD$

$\therefore \quad \angle ACB + \angle ACD > \angle BAC + \angle CAD$

$\Rightarrow \quad \angle C > \angle A$, similarly $\angle D > \angle B$

30. I. $QR = RS$

$\Rightarrow \quad QR - BR = RS - SC$

$\Rightarrow \quad QB = RC$

II. In ΔRCB and ΔAQB

$\angle R = \angle Q$, $QB = RC$, $AQ = RB$

$\therefore \quad \Delta RCB \cong \Delta AQB$

$\therefore \quad AB = BC$

III. $\because \quad AB = BC$, so $\angle BCA = \angle BAC$

$\angle BCA + \angle BAC = 90°$

$2\angle BAC = 90°$

$\angle BAC = 45°$

IV. $AC \parallel PS$ and $AC = PS$

31. $\angle OCX = 45°$

$\angle COD + \angle COX = 180°$

$\Rightarrow \quad \angle COX = 180° - \angle COD = 180° - 105° = 75°$

In ΔOCX

$\angle OCX + \angle COX + \angle OXC = 180°$

$\Rightarrow \quad 45° + 75° + \angle OXC = 180°$

$\Rightarrow \quad \angle OXC = 180° - 120° = 60° \quad x = 60°$

32. $EB \parallel DL$ and $ED \parallel BL$

$\Rightarrow \quad EBLD$ is a parallelogram.

$\therefore \quad BL = ED = \dfrac{1}{2}AD = \dfrac{1}{2}BC = LC$

Also, in ΔDCL and ΔFBL

$$LC = BL,$$

$\angle DLC = \angle FLB$ and $\angle CDL = \angle BFL$

$\therefore \quad \Delta DCL \cong \Delta FBL$

$\therefore \quad DC = BF$ and $DL = FL$

$\therefore \quad BF = DC = AB$

$\Rightarrow \quad 2AB = 2DC$

$\Rightarrow \quad AF = 2DC \qquad (\because AF = AB)$

33. As $AB \parallel DC$, EC cuts, then

so, $\quad \angle BEC = \angle ECD \qquad (\because \angle ECD = \angle ECB)$

$\Rightarrow \quad EB = BC$

$\Rightarrow \quad AE = AD$

34. Join BD, cutting EF at M.

So, M is mid-point of BD.

$\because E$ is mid-point of AD and $EM \parallel AB$.

$\therefore \quad EM = \frac{1}{2} AB$

Similarly, F is mid-point of BC in ΔBCD.

$\therefore \quad FM = \frac{1}{2} DC$

$\therefore \quad EF = EM + MF = \frac{1}{2}(AB + DC)$

35. $\angle CRQ = \angle CBA = 90°$

$\Rightarrow \quad QR \parallel AB$

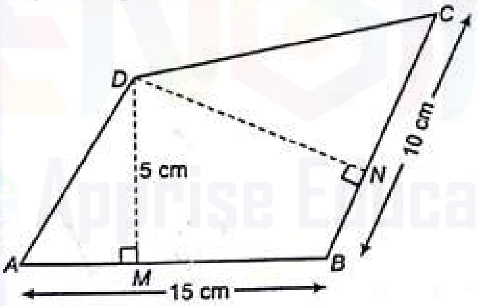
In ΔABC, Q is the mid-point of AC and $QR \parallel AB$.

$\therefore$ R is mid-point of BC.

Similarly, P is the mid-point of DC.

$\therefore \quad DP = PC$

36. Area of parallelogram = base × height $= 15 \times 5 = 75$ sq cm



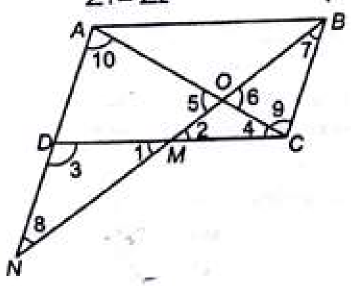
Area of parallelogram = base × height $= 10 \times DN$

$\therefore 10 \times DN = 75 \Rightarrow DN = \frac{75}{10} = 7.5$ cm

37. In ΔDMN and ΔBMC,

$DM = MC \qquad$ (given)

$\angle 1 = \angle 2 \qquad$ (vertically opposite)



$\angle 3 = \angle 4 + \angle 9 \qquad$ (alternate interior angle)

$\Delta DMN = \Delta BMC \qquad$ (as A)

$DN = BC = AD$

So, $\quad AN = 2BC$

$\Rightarrow \quad \frac{AN}{BC} = \frac{2}{1} \qquad$ ...(i)

In ΔOAN and ΔOBC,

$\angle 5 = \angle 6 \qquad$ (vertically opposite)

$\angle 7 = \angle 8 \qquad$ (alternate interior angle)

$\angle 9 = \angle 10 \qquad$ (rest angle)

$\therefore \quad \Delta OAN \sim \Delta OBC$

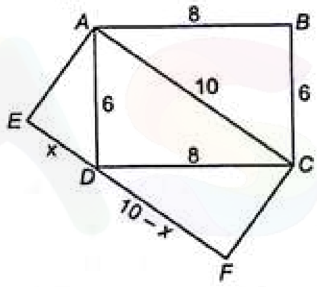
So, the sides will be in same ratio

$$\frac{AN}{BC} = \frac{ON}{OB}$$

$\Rightarrow \quad \frac{2}{1} = \frac{ON}{OB} \qquad$ [from Eq. (i)]

38. Let $ED = x$

Now, $\quad AC = \sqrt{8^2 + 6^2} = 10$



In ΔAED,

$$AE^2 = AD^2 - x^2$$

$$= 36 - x^2 \qquad \text{...(i)}$$

And in ΔCFD,

$$CF^2 = (8)^2 - (10 - x)^2 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$36 - x^2 = 64 - (10 - x)^2 \qquad (\because AE = FC)$

$\Rightarrow \quad 36 - x^2 = 64 - (100 + x^2 - 20x)$

$\Rightarrow \quad 20x = 72 \Rightarrow x = \frac{18}{5}$

$\therefore$ From Eq. (i), $AE^2 = 36 - \left(\frac{18}{5}\right)^2$

$$AE^2 = 36 - \frac{324}{25} = \frac{900 - 324}{25}$$

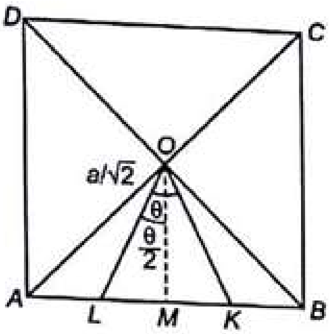
$\Rightarrow \quad AE^2 = \frac{576}{25} \Rightarrow AE = \frac{24}{5}$

$\therefore \quad \frac{\text{Area of rectangle } ABCD}{\text{Area of rectangle } AEFC} = \frac{8 \times 6}{10 \times \frac{24}{5}} = 1$

39. Let sides of a square be a

Then, $\quad AC = a\sqrt{2}$ and $AO = OC = \frac{a}{\sqrt{2}}$

Here, $\quad AM = \frac{a}{2}$



$\therefore \quad LM = \frac{a}{\sqrt{2}} - \frac{a}{2}$ and $OM = \frac{a}{2}$

In ΔOML, $\tan\frac{\theta}{2} = \frac{\frac{a}{\sqrt{2}} - \frac{a}{2}}{\frac{a}{2}} = \frac{\frac{\sqrt{2}-1}{2}}{\frac{1}{2}} = \sqrt{2}-1$

$$\therefore \quad \tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}} = \frac{2(\sqrt{2}-1)}{1-(2+1-2\sqrt{2})}$$

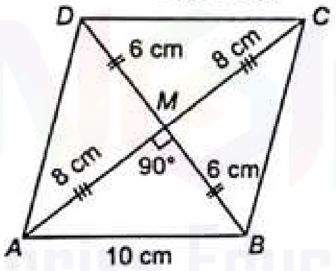
$$= \frac{2(\sqrt{2}-1)}{1-3+2\sqrt{2}} = \frac{2(\sqrt{2}-1)}{2\sqrt{2}-2} \Rightarrow \tan\theta = 1$$

40. $H^2 = P^2 + B^2$

$\therefore \quad AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$

$\Rightarrow \quad (10)^2 + (10)^2 + (10)^2 + (10)^2 = (16)^2 + (12)^2$

$\Rightarrow \quad 400 = 400$

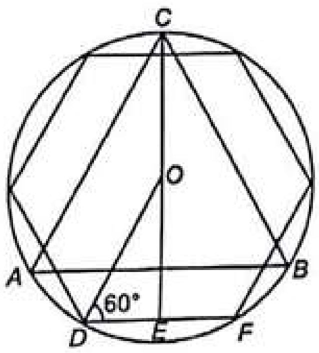


Hence, both (A) and (R) are true and (R) is the correct explanation of (A).

41. We know altitude of equilateral ΔABC is $\frac{\sqrt{3}}{2}a$.

$\therefore$ Length of $OC = \frac{\sqrt{3}}{2}a \times \frac{2}{\sqrt{3}}$

$= \frac{a}{\sqrt{3}} =$ radius



Also, $\quad DF = b$

$\Rightarrow \quad DE = \frac{b}{2}$

In ΔODE, $\quad \cos 60° = \frac{DE}{OD} = \frac{b/2}{a/\sqrt{3}}$

$\Rightarrow \quad \frac{1}{2} = \frac{\sqrt{3}b}{2a}$

$\Rightarrow \quad a = \sqrt{3}b$

$\Rightarrow \quad a^2 = 3b^2$

42. ABCD is a trapezium.

$\therefore \quad AD \| BC, EF \| BC$ (given)

Hence, $\quad EF \| AD$

$\therefore \quad \angle x + \angle y = 180°$ (interior angles)

$\therefore \quad \angle y = 180° - 120° = 60°$

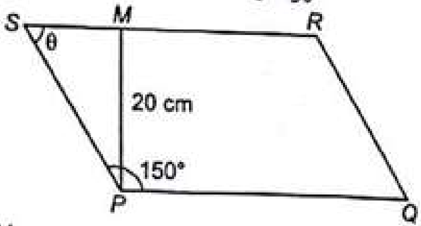
43. Given that, $\angle SPQ = 150°$ and $PM = 20$ cm

In parallelogram $PQRS$,

$\angle RSP + \angle SPQ = 180°$

$\Rightarrow \quad \angle RSP = 180° - 150° = 30°$

$\Rightarrow \quad \theta = 30°$



In ΔPSM,

$\sin\theta = \sin 30° = \frac{PM}{SP} \Rightarrow \frac{1}{2} = \frac{20}{SP} \Rightarrow SP = 40$ cm

$\Rightarrow \quad RQ = SP = 40$ cm

44. Here, $n = 6$

Sum of interior angles of a hexagon

$= (6-2) \times 180° = 4 \times 180° = 720°$

Number of angles = 6

$\therefore \quad$ Each angle $= \frac{720°}{6} = 120°$

45. Each interior angle $= \frac{(n-2)\times 180°}{n}$

$\therefore \quad \frac{(n-2)\times 180°}{n} = 150°$ (given)

$(n-2)180 = n \times 150$

$30n = 360 \Rightarrow n = \frac{360}{30} \Rightarrow n = 12$

46. Number of diagonals of a polygon of n sides

$= \frac{n(n-1)}{2} - n = \frac{8(8-1)}{2} - 8 = 20$

47. Exterior angle of a regular hexagon $= \frac{360°}{6} = 60°$

$\therefore$ Each of interior angles $= 180° - 60° = 120°$

$\therefore$ Each interior angle of the given polygon

$= \frac{9}{8} \times 120° = 135°$

$\therefore$ Each of exterior angle of the given polygon

$= 180° - 135° = 45°$

$\therefore \frac{360°}{n} = 45° \Rightarrow n = \frac{360°}{45°} = 8$

48. Let measure of each angle be x, then

measure of all angles = 1080°

$\therefore \quad 4 \times 154° + 4x = 1080°$

$4x = 1080° - 616° = 464°$

$x = \frac{464°}{4°} = 116°$

49. Sum of all angles of a pentagon $= (n-2) \times 180°$

$= (5-2) \times 180° = 3 \times 180° = 540°$

Let the angle be $x, 2x, 3x, 5x$ and $9x$.

$\therefore \quad x + 2x + 3x + 5x + 9x = 540°$

$20x = 540° \Rightarrow x = 27°$

$\therefore$ The Largest angle $= 9x = 9 \times 27° = 243°$

50. Let number of sides be n

Each exterior angle of regular polygon of n sides $= \frac{360°}{n}$

Each interior angle $= \frac{(n-2)180°}{n}$

So, $\frac{360}{n} = \frac{1}{3}\left[\frac{(n-2) \times 180}{n}\right]$ (by condition)

$\Rightarrow \quad \frac{360}{n} = \frac{(n-2)60}{n}$

$\Rightarrow \quad 360 = (n-2)60 \Rightarrow 6 = n-2 \Rightarrow n = 8$

51. Let the number of sides be n and $2n$

And let their interior angles be $2y°$ and $3y°$.

$\therefore$ Exterior angles are $(180-2y)°$ and $(180-3y)°$

$\therefore \quad \frac{360°}{n} = 180° - 2y \quad \text{...(i)}$

$\frac{360°}{2n} = 180° - 3y \quad \text{...(ii)}$

Solving Eqs. (i) and (ii), we get $n = 4$

$\therefore$ Number of sides of the polygon are 4 and 8.

52. Here, let side be $2n$ and n so equation are

$\frac{360}{2n} + 4y = 180 \quad \text{...(i)}$

$\frac{360}{n} + 3y = 180 \quad \text{...(ii)}$

Solving above equations, we get $n = 5$

So, number of sides are respectively 10 and 5.

53. By condition,

$\frac{\text{Interior angle of a regular polygon}}{\text{Exterior angle a regular polygon}} = \frac{5}{1}$

$\Rightarrow \quad \frac{\frac{(n-2)}{n} \times 180°}{\frac{360°}{n}} = \frac{5}{1}$

$\Rightarrow \quad \frac{(n-2)}{2} = \frac{5}{1}$

$\Rightarrow \quad n - 2 = 10 \Rightarrow n = 12$