

## 14. Some Special Integrals

### Exercise 14A

#### 1. Question

Evaluate:

$$\int \frac{dx}{(1-9x)^2}$$

#### Answer

To find:  $\int \frac{dx}{(1-9x)^2}$

Formula Used:  $\int x^n = \frac{x^{n+1}}{n+1} + C$

Let  $y = (1 - 9x) \dots (1)$

Differentiating with respect to  $x$ ,

$$\frac{dy}{dx} = -9$$

i.e.,  $dy = -9 dx$

Substituting in the equation to evaluate,

$$\Rightarrow \int \frac{-9}{y^2}$$

$$\Rightarrow \frac{-1}{9} \int \frac{dy}{y^2}$$

$$\Rightarrow \frac{-1}{9} \times \int y^{-2} dy$$

$$\Rightarrow \frac{-1}{9} \times \frac{y^{-2+1}}{-2+1} + C$$

Simplifying and substituting the value of  $y$  from (1),

$$\Rightarrow \frac{-1}{9} \times \frac{-1}{(1-9x)} + C$$

$$\Rightarrow \frac{1}{9(1-9x)} + C$$

Therefore,

$$\int \frac{dx}{(1-9x)^2} = \frac{1}{9(1-9x)} + C$$

#### 2. Question

Evaluate:

$$\int \frac{dx}{(25-4x^2)}$$

#### Answer

To find:  $\int \frac{dx}{(25-4x^2)}$

Formula Used:  $\frac{dx}{(a^2-x^2)} = \frac{1}{2a} \times \log \left| \frac{a+x}{a-x} \right| + C$

Given equation =  $\int \frac{dx}{4(\frac{25}{4}-x^2)}$

$$\Rightarrow \frac{1}{4} \int \frac{dx}{\left(\left(\frac{5}{2}\right)^2 - x^2\right)} \dots (1)$$

Here  $a = \frac{5}{2}$

Therefore, (1) becomes

$$\Rightarrow \frac{1}{4} \times \frac{1}{5} \times \log \left| \frac{\frac{5}{2} + x}{\frac{5}{2} - x} \right| + C$$

$$\Rightarrow \frac{1}{20} \times \log \left| \frac{5 + 2x}{5 - 2x} \right| + C$$

Therefore,

$$\int \frac{dx}{(25-4x^2)} = \frac{1}{20} \times \log \left| \frac{5 + 2x}{5 - 2x} \right| + C$$

### 3. Question

Evaluate:

$$\int \frac{dx}{(x^2 + 16)}$$

### Answer

To find:  $\int \frac{dx}{(x^2+16)}$

Formula Used:  $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{dx}{4^2 + x^2}$$

Here  $a = 4$

$$\Rightarrow \frac{1}{4} \times \tan^{-1} \left( \frac{x}{4} \right) + C$$

Therefore,

$$\int \frac{dx}{(x^2 + 16)} = \frac{1}{4} \times \tan^{-1} \left( \frac{x}{4} \right) + C$$

### 4. Question

Evaluate:

$$\int \frac{dx}{(4 + 9x^2)}$$

**Answer**

To find:  $\int \frac{dx}{(4+9x^2)}$

Formula Used:  $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$

Rewriting the given equation,

$$\Rightarrow \frac{1}{9} \int \frac{dx}{\left(\frac{4}{9}\right) + x^2}$$

$$\Rightarrow \frac{1}{9} \int \frac{dx}{\left(\frac{2}{3}\right)^2 + x^2}$$

Here  $a = \frac{2}{3}$

$$\Rightarrow \frac{1}{9} \times \frac{3}{2} \times \tan^{-1}\left(\frac{3x}{2}\right) + C$$

$$\Rightarrow \frac{1}{6} \times \tan^{-1}\left(\frac{3x}{2}\right) + C$$

Therefore,

$$\int \frac{dx}{(4+9x^2)} = \frac{1}{6} \times \tan^{-1}\left(\frac{3x}{2}\right) + C$$

**5. Question**

Evaluate:

$$\int \frac{dx}{(50+2x^2)}$$

**Answer**

To find:  $\int \frac{dx}{(50+2x^2)}$

Formula Used:  $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$

Rewriting the given equation,

$$\Rightarrow \frac{1}{2} \int \frac{dx}{25 + x^2}$$

$$\Rightarrow \frac{1}{2} \int \frac{dx}{5^2 + x^2}$$

Here  $a = 5$

$$\Rightarrow \frac{1}{10} \times \tan^{-1}\left(\frac{x}{5}\right) + C$$

Therefore,

$$\int \frac{dx}{(x^2 + 16)} = \frac{1}{10} \times \tan^{-1}\left(\frac{x}{5}\right) + C$$

**6. Question**

Evaluate:

$$\int \frac{dx}{(16x^2 - 25)}$$

**Answer**

To find:  $\int \frac{dx}{(16x^2 - 25)}$

Formula Used:  $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$

Rewriting the given equation,

$$\Rightarrow \frac{1}{16} \int \frac{dx}{x^2 - \left(\frac{25}{16}\right)}$$

$$\Rightarrow \frac{1}{16} \int \frac{dx}{x^2 - \left(\frac{5}{4}\right)^2}$$

Here  $a = \frac{5}{4}$

$$\Rightarrow \frac{1}{16} \times \frac{2}{5} \times \ln \left| \frac{x - \frac{5}{4}}{x + \frac{5}{4}} \right| + C$$

$$\Rightarrow \frac{1}{40} \times \ln \left| \frac{4x - 5}{4x + 5} \right| + C$$

Therefore,

$$\int \frac{dx}{(16x^2 - 25)} = \frac{1}{40} \times \log \left| \frac{4x - 5}{4x + 5} \right| + C$$

## 7. Question

Evaluate:

$$\int \frac{(x^2 - 1)}{(x^2 + 4)} dx$$

**Answer**

To find:  $\int \frac{(x^2 - 1)}{(x^2 + 4)} dx$

Formula Used:  $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + C$

Given equation can be rewritten as the following:

$$\Rightarrow \int \frac{(x^2 + 4 - 5)}{(x^2 + 4)} dx$$

$$\Rightarrow \int \frac{(x^2 + 4)}{(x^2 + 4)} dx - \int \frac{5}{(x^2 + 4)} dx$$

$$\Rightarrow \int dx - 5 \int \frac{1}{(x^2 + 2^2)} dx$$

Here  $a = 2$ ,

$$\Rightarrow x - \frac{5}{2} \tan^{-1} \frac{x}{2} + C$$

Therefore,

$$\int \frac{(x^2 - 1)}{(x^2 + 4)} dx = x - \frac{5}{2} \tan^{-1} \frac{x}{2} + C$$

## 8. Question

Evaluate:

$$\int \frac{x^2}{(9 + 4x^2)} dx$$

## Answer

To find:  $\int \frac{x^2}{(9 + 4x^2)} dx$

Formula Used:  $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + C$

Given equation can be rewritten as the following:

$$\Rightarrow \frac{1}{4} \int \frac{x^2}{(x^2 + \frac{9}{4})} dx$$

$$\Rightarrow \frac{1}{4} \int \frac{x^2 + \frac{9}{4} - \frac{9}{4}}{(x^2 + \frac{9}{4})} dx$$

$$\Rightarrow \frac{1}{4} \int dx - \frac{9}{16} \int \frac{1}{(x^2 + (\frac{3}{2})^2)} dx$$

Here  $a = \frac{3}{2}$ ,

$$\Rightarrow \frac{x}{4} - \left( \frac{9}{16} \times \frac{2}{3} \tan^{-1} \frac{2x}{3} \right) + C$$

$$\Rightarrow \frac{x}{4} - \frac{3}{8} \tan^{-1} \left( \frac{2x}{3} \right) + C$$

Therefore,

$$\int \frac{x^2}{(9 + 4x^2)} dx = \frac{x}{4} - \frac{3}{8} \tan^{-1} \left( \frac{2x}{3} \right) + C$$

## 9. Question

Evaluate:

$$\int \frac{e^x}{(e^{2x} + 1)} dx$$

## Answer

To find:  $\int \frac{e^x}{(e^{2x} + 1)} dx$

Formula Used:  $\int \frac{dx}{1 + x^2} = \tan^{-1} x$

Let  $y = e^x \dots (1)$

Differentiating both sides, we get

$$dy = e^x dx$$

Substituting in given equation,

$$\Rightarrow \int \frac{dy}{y^2 + 1}$$

$$\Rightarrow \tan^{-1} y$$

From (1),

$$\Rightarrow \tan^{-1}(e^x)$$

Therefore,

$$\int \frac{e^x}{(e^{2x} + 1)} dx = \tan^{-1}(e^x) + C$$

### 10. Question

Evaluate:

$$\int \frac{\sin x}{(1 + \cos^2 x)} dx$$

### Answer

To find:  $\int \frac{\sin x}{(1 + \cos^2 x)} dx$

Formula Used:  $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$

Let  $y = \cos x \dots (1)$

Differentiating both sides, we get

$$dy = -\sin x dx$$

Substituting in given equation,

$$\Rightarrow \int \frac{-dy}{1 + y^2}$$

$$\Rightarrow -\tan^{-1} y$$

From (1),

$$\Rightarrow -\tan^{-1}(\cos x)$$

Therefore,

$$\int \frac{\sin x}{(1 + \cos^2 x)} dx = -\tan^{-1}(\cos x) + C$$

### 11. Question

Evaluate:

$$\int \frac{\cos x}{(1 + \sin^2 x)} dx$$

### Answer

To find:  $\int \frac{\cos x}{(1+\sin^2 x)} dx$

Formula Used:  $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$

Let  $y = \sin x \dots (1)$

Differentiating both sides, we get

$$dy = \cos x \, dx$$

Substituting in given equation,

$$\Rightarrow \int \frac{dy}{1+y^2}$$

$$\Rightarrow \tan^{-1} y$$

From (1),

$$\Rightarrow \tan^{-1}(\sin x)$$

Therefore,

$$\int \frac{\cos x}{(1+\sin^2 x)} dx = \tan^{-1}(\sin x) + C$$

## 12. Question

Evaluate:

$$\int \frac{3x^5}{(1+x^{12})} dx$$

## Answer

To find:  $\int \frac{3x^5}{(1+x^{12})} dx$

Formula Used:  $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$

Let  $y = x^6 \dots (1)$

Differentiating both sides, we get

$$dy = 6x^5 \, dx$$

Substituting in given equation,

$$\Rightarrow \int \frac{\frac{1}{2} dy}{1+y^2}$$

$$\Rightarrow \frac{1}{2} \tan^{-1} y + C$$

From (1),

$$\Rightarrow \frac{1}{2} \tan^{-1}(x^6) + C$$

Therefore,

$$\int \frac{3x^5}{(1+x^{12})} dx = \frac{1}{2} \tan^{-1}(x^6) + C$$

## 13. Question

Evaluate:

$$\int \frac{2x^3}{(4+x^8)} dx$$

**Answer**

To find:  $\int \frac{2x^3}{(4+x^8)} dx$

Formula Used:  $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$

Let  $y = x^4 \dots (1)$

Differentiating both sides, we get

$$dy = 4x^3 dx$$

Substituting in given equation,

$$\Rightarrow \int \frac{\frac{1}{2} dy}{4+y^2}$$

$$\Rightarrow \frac{1}{2} \int \frac{1}{2^2+y^2} dy$$

$$\Rightarrow \frac{1}{4} \tan^{-1}\left(\frac{y}{2}\right) + C$$

From (1),

$$\Rightarrow \frac{1}{4} \tan^{-1}\left(\frac{x^4}{2}\right) + C$$

Therefore,

$$\int \frac{2x^3}{(4+x^8)} dx = \frac{1}{4} \tan^{-1}\left(\frac{x^4}{2}\right) + C$$

#### 14. Question

Evaluate:

$$\int \frac{dx}{(e^x + e^{-x})}$$

**Answer**

To find:  $\int \frac{dx}{(e^x + e^{-x})}$

Formula Used:  $\int \frac{dx}{1+x^2} = \tan^{-1} x$

Given equation is:

$$\int \frac{dx}{(e^x + e^{-x})} = \int \frac{e^x dx}{(e^{2x} + 1)} \dots (1)$$

Let  $y = e^x \dots (1)$

Differentiating both sides, we get

$$dy = e^x dx$$



Substituting in (1),

$$\Rightarrow \int \frac{dy}{y^2 + 1}$$

$$\Rightarrow \tan^{-1} y$$

From (1),

$$\Rightarrow \tan^{-1}(e^x)$$

Therefore,

$$\int \frac{dx}{(e^x + e^{-x})} = \tan^{-1}(e^x) + C$$

### 15. Question

Evaluate:

$$\int \frac{x}{(1-x^4)} dx$$

### Answer

$$\text{To find: } \int \frac{x dx}{(1-x^4)}$$

$$\text{Formula Used: } \int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$$

$$\text{Let } y = x^2 \dots (1)$$

Differentiating both sides, we get

$$dy = 2x dx$$

Substituting in given equation,

$$\Rightarrow \int \frac{\frac{1}{2} dy}{1-y^2}$$

Here  $a = 1$ ,

$$\Rightarrow \frac{1}{2} \times \frac{1}{2} \times \log \left| \frac{1+y}{1-y} \right| + C$$

$$\Rightarrow \frac{1}{4} \log \left| \frac{1+y}{1-y} \right| + C$$

From (1),

$$\Rightarrow \frac{1}{4} \log \left| \frac{1+x^2}{1-x^2} \right| + C$$

Therefore,

$$\int \frac{x dx}{(1-x^4)} = \frac{1}{4} \log \left| \frac{1+x^2}{1-x^2} \right| + C$$

### 16. Question

Evaluate:

$$\int \frac{x^2}{(a^6 - x^6)} dx$$

**Answer**

To find:  $\int \frac{x^2 dx}{(a^6 - x^6)}$

Formula Used:  $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$

Let  $y = x^3 \dots (1)$

Differentiating both sides, we get

$$dy = 3x^2 dx$$

Substituting in given equation,

$$\Rightarrow \int \frac{\frac{1}{3} dy}{a^6 - y^2}$$

$$\Rightarrow \frac{1}{3} \int \frac{1}{(a^3)^2 - y^2} dy$$

$$\Rightarrow \frac{1}{3} \times \frac{1}{2a^3} \times \log \left| \frac{a^3 + y}{a^3 - y} \right| + C$$

$$\Rightarrow \frac{1}{6a^3} \log \left| \frac{a^3 + y}{a^3 - y} \right| + C$$

From (1),

$$\Rightarrow \frac{1}{6a^3} \log \left| \frac{a^3 + x^3}{a^3 - x^3} \right| + C$$

Therefore,

$$\int \frac{x^2 dx}{(a^6 - x^6)} = \frac{1}{6a^3} \log \left| \frac{a^3 + x^3}{a^3 - x^3} \right| + C$$

## 17. Question

Evaluate:

$$\int \frac{dx}{(x^2 + 4x + 8)}$$

**Answer**

To find:  $\int \frac{dx}{(x^2 + 4x + 8)}$

Formula Used:  $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{dx}{((x + 2)^2 + 4)}$$

$$\Rightarrow \int \frac{dx}{((x + 2)^2 + 2^2)} \dots (1)$$

Let  $y = x + 2 \dots (2)$

Differentiating both sides,

$$dy = dx$$

Substituting in (1),

$$\Rightarrow \int \frac{dy}{(y^2 + 2^2)}$$

Here  $a = 2$ ,

$$\Rightarrow \frac{1}{2} \tan^{-1} \left( \frac{y}{2} \right) + C$$

From (2),

$$\Rightarrow \frac{1}{2} \tan^{-1} \left( \frac{x+2}{2} \right) + C$$

Therefore,

$$\int \frac{dx}{(x^2 + 4x + 8)} = \frac{1}{2} \tan^{-1} \left( \frac{x+2}{2} \right) + C$$

### 18. Question

Evaluate:

$$\int \frac{dx}{(4x^2 - 4x + 3)}$$

### Answer

To find:  $\int \frac{dx}{(4x^2 - 4x + 3)}$

Formula Used:  $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{dx}{((2x-1)^2 + 2)} \dots (1)$$

Let  $y = 2x - 1 \dots (2)$

Differentiating both sides,

$$dy = 2dx$$

Substituting in (1),

$$\Rightarrow \int \frac{\frac{1}{2} dy}{(y^2 + (\sqrt{2})^2)}$$

Here  $a = \sqrt{2}$ ,

$$\Rightarrow \frac{1}{2} \times \frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{y}{\sqrt{2}} \right) + C$$

From (2),

$$\Rightarrow \frac{1}{2\sqrt{2}} \tan^{-1} \left( \frac{2x-1}{\sqrt{2}} \right) + C$$

Therefore,

$$\int \frac{dx}{(4x^2 - 4x + 3)} = \frac{1}{2\sqrt{2}} \tan^{-1} \left( \frac{2x-1}{\sqrt{2}} \right) + C$$

### 19. Question

Evaluate:

$$\int \frac{dx}{(2x^2 + x + 3)}$$

### Answer

To find:  $\int \frac{dx}{(2x^2 + x + 3)}$

Formula Used:  $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{dx}{\left( \left( \sqrt{2}x + \frac{1}{2\sqrt{2}} \right)^2 + 3 - \frac{1}{8} \right)}$$

$$\Rightarrow \int \frac{dx}{\left( \left( \sqrt{2}x + \frac{1}{2\sqrt{2}} \right)^2 + \frac{23}{8} \right)} \dots (1)$$

Let  $y = \sqrt{2}x + \frac{1}{2\sqrt{2}} \dots (2)$

Differentiating both sides,

$$dy = \sqrt{2} dx$$

Substituting in (1),

$$\Rightarrow \int \frac{\frac{1}{\sqrt{2}} dy}{\left( y^2 + \left( \frac{\sqrt{23}}{2\sqrt{2}} \right)^2 \right)}$$

Here  $a = \frac{\sqrt{23}}{2\sqrt{2}}$

$$\Rightarrow \frac{1}{\sqrt{2}} \times \frac{2\sqrt{2}}{\sqrt{23}} \tan^{-1} \left( \frac{y \times 2\sqrt{2}}{\sqrt{23}} \right) + C$$

From (2),

$$\Rightarrow \frac{2}{\sqrt{23}} \tan^{-1} \left( \frac{4x+1}{\sqrt{23}} \right) + C$$

Therefore,

$$\int \frac{dx}{(2x^2 + x + 3)} = \frac{2}{\sqrt{23}} \tan^{-1} \left( \frac{4x+1}{\sqrt{23}} \right) + C$$

### 20. Question

Evaluate:

$$\int \frac{dx}{(2x^2 - x - 1)}$$

### Answer

To find:  $\int \frac{dx}{(2x^2-x-1)}$

Formula Used:  $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{dx}{\left( \left( \sqrt{2}x - \frac{1}{2\sqrt{2}} \right)^2 - 1 - \left( \frac{1}{2\sqrt{2}} \right)^2 \right)}$$

$$\Rightarrow \int \frac{dx}{\left( \left( \sqrt{2}x - \frac{1}{2\sqrt{2}} \right)^2 - 1 - \frac{1}{8} \right)}$$

$$\Rightarrow \int \frac{dx}{\left( \left( \sqrt{2}x - \frac{1}{2\sqrt{2}} \right)^2 - \frac{9}{8} \right)}$$

$$\Rightarrow \int \frac{dx}{\left( \left( \sqrt{2}x - \frac{1}{2\sqrt{2}} \right)^2 - \left( \frac{3}{2\sqrt{2}} \right)^2 \right)} \dots (1)$$

$$\text{Let } y = \sqrt{2}x - \frac{1}{2\sqrt{2}} \dots (2)$$

Differentiating both sides,

$$dy = \sqrt{2} dx$$

Substituting in (1),

$$\Rightarrow \int \frac{\frac{1}{\sqrt{2}} dy}{\left( y^2 - \left( \frac{3}{2\sqrt{2}} \right)^2 \right)}$$

$$\text{Here } a = \frac{3}{2\sqrt{2}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{3} \times \log \left| \frac{\frac{3}{2\sqrt{2}} + y}{\frac{3}{2\sqrt{2}} - y} \right| + C$$

$$\Rightarrow \frac{1}{3} \times \log \left| \frac{3 + 2\sqrt{2}y}{3 - 2\sqrt{2}y} \right| + C$$

From (2),

$$\Rightarrow \frac{1}{3} \times \log \left| \frac{3 + 4x - 1}{3 - 4x + 1} \right| + C$$

$$\Rightarrow \frac{1}{3} \log \left| \frac{1 + 2x}{2(1-x)} \right| + C$$

$$\Rightarrow \frac{1}{3} \log \left| \frac{2(x-1)}{2x+1} \right| + C$$

Therefore,

$$\int \frac{dx}{(2x^2-x-1)} = \frac{1}{3} \log \left| \frac{2(x-1)}{2x+1} \right| + C$$

## 21. Question

Evaluate:

$$\int \frac{dx}{(3-2x-x^2)}$$

## Answer

To find:  $\int \frac{dx}{(3-2x-x^2)}$

Formula Used:  $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{-dx}{(x^2+2x-3)}$$

$$\Rightarrow \int \frac{-dx}{(x+1)^2-4}$$

$$\Rightarrow \int \frac{-dx}{(x+1)^2-2^2} \dots (1)$$

Let  $y = x + 1 \dots (2)$

Differentiating both sides wrt  $x$ ,

$$dy = dx$$

Substituting in (1),

$$\Rightarrow \int \frac{-dy}{y^2-2^2}$$

$$\Rightarrow \int \frac{dy}{2^2-y^2}$$

Here  $a = 2$ ,

$$\Rightarrow \frac{1}{4} \log \left| \frac{2+y}{2-y} \right| + C$$

From (2),

$$\Rightarrow \frac{1}{4} \log \left| \frac{x+3}{1-x} \right| + C$$

Therefore,

$$\int \frac{dx}{(3-2x-x^2)} = \frac{1}{4} \log \left| \frac{x+3}{1-x} \right| + C$$

## 22. Question

Evaluate:

$$\int \frac{x}{(x^2+3x+2)} dx$$

## Answer

To find:  $\int \frac{x dx}{(x^2+3x+2)}$

Formula Used:

$$1. \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

$$2. \int \frac{f'(x)}{f(x)} dx = \log |f(x)| + C$$

Using partial fractions,

$$x = A \left( \frac{d}{dx} (x^2 + 3x + 2) \right) + B$$

$$x = A(2x + 3) + B$$

Equating the coefficients of x,

$$1 = 2A$$

$$A = \frac{1}{2}$$

$$\text{Also, } 0 = 3A + B$$

$$B = \frac{-3}{2}$$

Therefore, the given equation becomes,

$$\Rightarrow \int \frac{\frac{1}{2}(2x+3) - \frac{3}{2}}{(x^2 + 3x + 2)} dx$$

$$\Rightarrow \frac{1}{2} \log |x^2 + 3x + 2| - \frac{3}{2} \int \frac{1}{\left( \left( x + \frac{3}{2} \right)^2 + 2 - \left( \frac{3}{2} \right)^2 \right)} dx$$

$$\Rightarrow \frac{1}{2} \log |x^2 + 3x + 2| - \frac{3}{2} \int \frac{1}{\left( \left( x + \frac{3}{2} \right)^2 - \left( \frac{1}{2} \right)^2 \right)} dx$$

$$\Rightarrow \frac{1}{2} \log |x^2 + 3x + 2| - \frac{3}{2} \times \log \left| \frac{x + \frac{3}{2} - \frac{1}{2}}{x + \frac{3}{2} + \frac{1}{2}} \right| + C$$

$$\Rightarrow \frac{1}{2} \log |x^2 + 3x + 2| - \frac{3}{2} \log \left| \frac{x+1}{x+2} \right| + C$$

Therefore,

$$\int \frac{x dx}{(x^2 + 3x + 2)} = \frac{1}{2} \log |x^2 + 3x + 2| - \frac{3}{2} \log \left| \frac{x+1}{x+2} \right| + C$$

### 23. Question

Evaluate:

$$\int \frac{(x-3)}{(x^2 + 2x - 4)} dx$$

### Answer

$$\text{To find: } \int \frac{(x-3) dx}{(x^2 + 2x - 4)}$$

Formula Used:

$$1. \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

$$2. \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + C$$

Using partial fractions,

$$(x - 3) = A \left( \frac{d}{dx}(x^2 + 2x - 4) \right) + B$$

$$x - 3 = A(2x + 2) + B$$

Equating the coefficients of x,

$$1 = 2A$$

$$\Rightarrow A = \frac{1}{2}$$

$$\text{Also, } -3 = 2A + B$$

$$\Rightarrow B = -4$$

Substituting in the given equation,

$$\Rightarrow \int \frac{\frac{1}{2}(2x + 2) - 4}{(x^2 + 2x - 4)} dx$$

$$\Rightarrow \frac{1}{2} \log|x^2 + 2x - 4| - 4 \int \frac{1}{(x + 1)^2 - (\sqrt{5})^2} dx$$

$$\Rightarrow \frac{1}{2} \log|x^2 + 2x - 4| - \left( 4 \times \frac{1}{2\sqrt{5}} \times \log \left| \frac{x + 1 - \sqrt{5}}{x + 1 + \sqrt{5}} \right| \right) + C$$

$$\Rightarrow \frac{1}{2} \log|x^2 + 2x - 4| - \frac{2}{\sqrt{5}} \log \left| \frac{x + 1 - \sqrt{5}}{x + 1 + \sqrt{5}} \right| + C$$

Therefore,

$$\int \frac{(x - 3) dx}{(x^2 + 2x - 4)} = \frac{1}{2} \log|x^2 + 2x - 4| - \frac{2}{\sqrt{5}} \log \left| \frac{x + 1 - \sqrt{5}}{x + 1 + \sqrt{5}} \right| + C$$

## 24. Question

Evaluate:

$$\int \frac{(2x - 3)}{(x^2 + 3x - 18)} dx$$

## Answer

$$\text{To find: } \int \frac{(2x - 3)}{(x^2 + 3x - 18)} dx$$

Formula Used:

$$1. \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

$$2. \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + C$$

Using partial fractions,



$$(2x - 3) = A \left( \frac{d}{dx} (x^2 + 3x - 18) \right) + B$$

$$2x - 3 = A(2x + 3) + B$$

Equating the coefficients of x,

$$2 = 2A$$

$$A = 1$$

$$\text{Also, } -3 = 3A + B$$

$$\Rightarrow B = -6$$

Substituting in the given equation,

$$\Rightarrow \int \frac{(2x + 3) - 6}{(x^2 + 3x - 18)} dx$$

$$\Rightarrow \log|x^2 + 3x - 18| + C_1 - 6 \int \frac{1}{\left(x + \frac{3}{2}\right)^2 - 18 - \left(\frac{3}{2}\right)^2} dx \dots (1)$$

$$\text{Let } I = 6 \int \frac{1}{\left(x + \frac{3}{2}\right)^2 - 18 - \left(\frac{3}{2}\right)^2} dx$$

$$\Rightarrow 6 \int \frac{1}{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2} dx$$

$$\text{Here } a = \frac{9}{2}$$

$$\Rightarrow \frac{6}{9} \times \log \left| \frac{x + \frac{3}{2} - \frac{9}{2}}{x + \frac{3}{2} + \frac{9}{2}} \right| + C_2$$

$$\Rightarrow \frac{2}{3} \times \log \left| \frac{x-3}{x+6} \right| + C_2 \dots (2)$$

Substituting (2) in (1),

$$\Rightarrow \log|x^2 + 3x - 18| - \frac{2}{3} \log \left| \frac{x-3}{x+6} \right| + C$$

Therefore,

$$\int \frac{(2x - 3)}{(x^2 + 3x - 18)} dx = \log|x^2 + 3x - 18| - \frac{2}{3} \log \left| \frac{x-3}{x+6} \right| + C$$

## 25. Question

Evaluate:

$$\int \frac{x^2}{(x^2 + 6x - 3)} dx$$

## Answer

$$\text{To find: } \int \frac{x^2}{(x^2 + 6x - 3)} dx$$

Formula Used:

$$1. \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

$$2. \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + C$$

Given equation can be rewritten as following:

$$\Rightarrow \int \frac{x^2 + (6x - 3) - (6x - 3)}{(x^2 + 6x - 3)} dx$$

$$\Rightarrow \int \frac{(x^2 + 6x - 3) - (6x - 3)}{(x^2 + 6x - 3)} dx$$

$$\Rightarrow x - \int \frac{6x - 3}{x^2 + 6x - 3} dx$$

$$\text{Let } I = \int \frac{6x-3}{x^2+6x-3} dx \dots (2)$$

Using partial fractions,

$$(6x - 3) = A \left( \frac{d}{dx} (x^2 + 6x - 3) \right) + B$$

$$6x - 3 = A (2x + 6) + B$$

Equating the coefficients of x,

$$6 = 2A$$

$$A = 3$$

$$\text{Also, } -3 = 6A + B$$

$$\Rightarrow B = -21$$

Substituting in (1),

$$\Rightarrow \int \frac{3(2x + 6) - 21}{(x^2 + 6x - 3)} dx$$

$$\Rightarrow 3 \times \log|x^2 + 6x - 3| + C_1 - 21 \int \frac{1}{(x + 3)^2 - (\sqrt{12})^2} dx$$

$$\Rightarrow 3 \times \log|x^2 + 6x - 3| + C_1 - 21 \times \frac{1}{2\sqrt{12}} \times \log \left| \frac{x + 3 - \sqrt{12}}{x + 3 + \sqrt{12}} \right| + C_2$$

$$I = 3\log|x^2 + 6x - 3| - \frac{7\sqrt{3}}{4} \times \log \left| \frac{x+3-2\sqrt{3}}{x+3+2\sqrt{3}} \right| + C$$

Therefore,

$$\int \frac{x^2}{(x^2 + 6x - 3)} dx = x - 3\log|x^2 + 6x - 3| + \frac{7\sqrt{3}}{4} \times \log \left| \frac{x + 3 - 2\sqrt{3}}{x + 3 + 2\sqrt{3}} \right| + C$$

## 26. Question

Evaluate:

$$\int \frac{(2x-1)}{(2x^2+2x+1)} dx$$

**Answer**

$$\text{To find: } \int \frac{2x-1}{(2x^2+2x+1)} dx$$

Formula Used:

$$1. \int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$2. \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + C$$

Using partial fractions,

$$(2x - 1) = A \left( \frac{d}{dx} (2x^2 + 2x + 1) \right) + B$$

$$2x - 1 = A (4x + 2) + B$$

Equating the coefficients of x,

$$2 = 4A$$

$$A = \frac{1}{2}$$

$$\text{Also, } -1 = 2A + B$$

$$\Rightarrow B = -2$$

Substituting in the given equation,

$$\Rightarrow \int \frac{\frac{1}{2}(4x + 2) - 2}{(2x^2 + 2x + 1)} dx$$

$$\Rightarrow \frac{1}{2} \log|2x^2 + 2x + 1| - 2 \int \frac{1}{2(x^2 + x + \frac{1}{2})} dx$$

$$\text{Let } I = 2 \int \frac{1}{2(x^2 + x + \frac{1}{2})} dx \dots (1)$$

$$\Rightarrow \int \frac{1}{(x^2 + x + \frac{1}{2})} dx$$

$$\Rightarrow \int \frac{1}{\left( \left( x + \frac{1}{2} \right)^2 + \frac{1}{2} - \left( \frac{1}{2} \right)^2 \right)} dx$$

$$\Rightarrow \int \frac{1}{\left( \left( x + \frac{1}{2} \right)^2 + \frac{1}{2} - \frac{1}{4} \right)} dx$$

$$\Rightarrow \int \frac{1}{\left( \left( x + \frac{1}{2} \right)^2 + \left( \frac{1}{2} \right)^2 \right)} dx$$

$$\text{Here } a = \frac{1}{2}$$

$$\Rightarrow 2 \tan^{-1} \left( \frac{x + \frac{1}{2}}{\frac{1}{2}} \right) + C$$

$$\Rightarrow 2 \tan^{-1}(2x + 1) + C$$

Substituting in (1) and combining with original equation,

$$\Rightarrow \frac{1}{2} \log|2x^2 + 2x + 1| - 2 \tan^{-1}(2x + 1) + C$$

Therefore,

$$\int \frac{2x-1}{(2x^2+2x+1)} dx = \frac{1}{2} \log|2x^2+2x+1| - 2 \tan^{-1}(2x+1) + C$$

## 27. Question

Evaluate:

$$\int \frac{(1-3x)}{(3x^2+4x+2)} dx$$

## Answer

To find:  $\int \frac{1-3x}{(3x^2+4x+2)} dx$

Formula Used:

$$1. \int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$2. \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + C$$

Rewriting the given equation,

$$\Rightarrow - \int \frac{3x-1}{(3x^2+4x+2)} dx$$

Using partial fractions,

$$(3x-1) = A \left( \frac{d}{dx} (3x^2+4x+2) \right) + B$$

$$3x-1 = A(6x+4) + B$$

Equating the coefficients of x,

$$3 = 6A$$

$$A = \frac{1}{2}$$

$$\text{Also, } -1 = 4A + B$$

$$\Rightarrow B = -3$$

Substituting in the original equation,

$$\Rightarrow - \int \frac{\frac{1}{2}(6x+4) - 3}{(3x^2+4x+2)} dx$$

$$\Rightarrow -\frac{1}{2} \log|3x^2+4x+2| + 3 \int \frac{1}{3 \left( x^2 + \frac{4}{3}x + \frac{2}{3} \right)} dx$$

$$\text{Let } I = 3 \int \frac{1}{3 \left( x^2 + \frac{4}{3}x + \frac{2}{3} \right)} dx$$

$$\Rightarrow \int \frac{1}{\left( x^2 + \frac{4}{3}x + \frac{2}{3} \right)} dx$$

$$\Rightarrow \int \frac{1}{\left( \left( x + \frac{2}{3} \right)^2 + \frac{2}{3} - \frac{4}{9} \right)} dx$$

$$\Rightarrow \int \frac{1}{\left( \left( x + \frac{2}{3} \right)^2 + \left( \frac{\sqrt{2}}{3} \right)^2 \right)} dx$$

Here  $a = \frac{\sqrt{2}}{3}$

$$\Rightarrow \frac{3}{\sqrt{2}} \tan^{-1} \left( \frac{x + \frac{2}{3}}{\frac{\sqrt{2}}{3}} \right) + C$$

$$\Rightarrow \frac{3}{\sqrt{2}} \tan^{-1} \left( \frac{3x + 2}{\sqrt{2}} \right) + C$$

Substituting in (1) and combining with original equation,

$$\Rightarrow -\frac{1}{2} \log|3x^2 + 4x + 2| + \frac{3}{\sqrt{2}} \tan^{-1} \left( \frac{3x + 2}{\sqrt{2}} \right) + C$$

Therefore,

$$\int \frac{1 - 3x}{(3x^2 + 4x + 2)} dx = -\frac{1}{2} \log|3x^2 + 4x + 2| + \frac{3}{\sqrt{2}} \tan^{-1} \left( \frac{3x + 2}{\sqrt{2}} \right) + C$$

## 28. Question

Evaluate:

$$\int \frac{2x}{(2 + x - x^2)} dx$$

## Answer

To find:  $\int \frac{2x}{(2+x-x^2)} dx$

Formula Used:

$$1. \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

$$2. \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + C$$

Rewriting the given equation,

$$\Rightarrow -2 \int \frac{x}{(x^2 - x - 2)} dx$$

Using partial fractions,

$$x = A \left( \frac{d}{dx} (x^2 - x - 2) \right) + B$$

$$x = A (2x - 1) + B$$

Equating the coefficients of x,

$$1 = 2A$$

$$A = \frac{1}{2}$$

$$\text{Also, } 0 = -A + B$$

$$B = \frac{1}{2}$$

Substituting in the original equation,

$$\Rightarrow -2 \int \frac{\frac{1}{2}(2x-1) + \frac{1}{2}}{(x^2 - x - 2)} dx$$

$$\Rightarrow -\log|x^2 - x - 2| - \int \frac{1}{(x^2 - x - 2)} dx$$

$$\text{Let } I = \int \frac{1}{(x^2 - x - 2)} dx$$

$$\Rightarrow \int \frac{1}{\left(\left(x - \frac{1}{2}\right)^2 - 2 - \frac{1}{4}\right)} dx$$

$$\Rightarrow \int \frac{1}{\left(\left(x - \frac{1}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right)} dx$$

$$\text{Here } a = \frac{3}{2}$$

$$\Rightarrow \frac{1}{3} \log \left| \frac{x - \frac{1}{2} - \frac{3}{2}}{x - \frac{1}{2} + \frac{3}{2}} \right| + C$$

$$\Rightarrow \frac{1}{3} \log \left| \frac{x-2}{x+1} \right| + C$$

Substituting for I and combining with the original equation,

$$\Rightarrow -\log|x^2 - x - 2| + \frac{1}{3} \log \left| \frac{x-2}{x+1} \right| + C$$

Therefore,

$$\int \frac{2x}{(2+x-x^2)} dx = -\log|x^2 - x - 2| + \frac{1}{3} \log \left| \frac{x-2}{x+1} \right| + C$$

or

$$\int \frac{2x}{(2+x-x^2)} dx = -\log|2+x-x^2| + \frac{1}{3} \log \left| \frac{1+x}{2-x} \right| + C$$

## 29. Question

Evaluate:

$$\int \frac{dx}{(1 + \cos^2 x)}$$

## Answer

$$\text{To find: } \int \frac{1}{(1 + \cos^2 x)} dx$$

Formula Used:

$$1. \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$2. \sec^2 x = 1 + \tan^2 x$$

Dividing the given equation by  $\cos^2 x$  in the numerator and denominator gives us,

$$\Rightarrow \int \frac{\sec^2 x \, dx}{1 + \sec^2 x} \dots (1)$$

Let  $y = \tan x$

$$dy = \sec^2 x \, dx \dots (2)$$

$$\text{Also, } y^2 = \tan^2 x$$

$$\text{i.e., } y^2 = \sec^2 x - 1$$

$$\sec^2 x = y^2 + 1 \dots (3)$$

Substituting (2) and (3) in (1),

$$\Rightarrow \int \frac{dy}{1 + y^2 + 1}$$

$$\Rightarrow \int \frac{dy}{y^2 + (\sqrt{2})^2}$$

$$\Rightarrow \frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{y}{\sqrt{2}} \right) + C$$

Since  $y = \tan x$ ,

$$\Rightarrow \frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{\tan x}{\sqrt{2}} \right) + C$$

Therefore,

$$\int \frac{1}{(1 + \cos^2 x)} dx = \frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{\tan x}{\sqrt{2}} \right) + C$$

### 30. Question

Evaluate:

$$\int \frac{dx}{(2 + \sin^2 x)}$$

### Answer

$$\text{To find: } \int \frac{1}{(2 + \sin^2 x)} dx$$

Formula Used:

$$1. \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$2. \sec^2 x = 1 + \tan^2 x$$

Dividing the given equation by  $\cos^2 x$  in the numerator and denominator gives us,

$$\Rightarrow \int \frac{\sec^2 x \, dx}{2 \sec^2 x + \tan^2 x} \dots (1)$$

Let  $y = \tan x$

$$dy = \sec^2 x \, dx \dots (2)$$

$$\text{Also, } y^2 = \tan^2 x$$

$$\text{i.e., } y^2 = \sec^2 x - 1$$

$$\sec^2 x = y^2 + 1 \dots (3)$$

Substituting (2) and (3) in (1),

$$\Rightarrow \int \frac{dy}{2y^2 + 2 + y^2}$$

$$\Rightarrow \int \frac{dy}{3y^2 + 2}$$

$$\Rightarrow \frac{1}{3} \int \frac{dy}{y^2 + \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2}$$

$$\Rightarrow \frac{1}{3} \times \frac{\sqrt{3}}{\sqrt{2}} \tan^{-1} \left( \frac{y\sqrt{3}}{\sqrt{2}} \right) + C$$

Since  $y = \tan x$ ,

$$\Rightarrow \frac{1}{\sqrt{6}} \tan^{-1} \left( \frac{\sqrt{3} \tan x}{\sqrt{2}} \right) + C$$

Therefore,

$$\int \frac{1}{(2 + \sin^2 x)} dx = \frac{1}{\sqrt{6}} \tan^{-1} \left( \frac{\sqrt{3} \tan x}{\sqrt{2}} \right) + C$$

### 31. Question

Evaluate:

$$\int \frac{dx}{(a^2 \cos^2 x + b^2 \sin^2 x)}$$

### Answer

To find:  $\int \frac{dx}{(a^2 \cos^2 x + b^2 \sin^2 x)}$

Formula Used:

$$1. \sec^2 x = 1 + \tan^2 x$$

$$2. \int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

Dividing by  $\cos^2 x$  in the numerator and denominator,

$$\Rightarrow \int \frac{\sec^2 x \, dx}{a^2 + b^2 \tan^2 x}$$

Let  $y = \tan x$

$$dy = \sec^2 x \, dx$$

Therefore,

$$\Rightarrow \int \frac{dy}{a^2 + b^2 y^2}$$

$$\Rightarrow \frac{1}{b^2} \int \frac{dy}{\left(\frac{a}{b}\right)^2 + y^2}$$



$$\Rightarrow \frac{1}{b^2} \times \frac{b}{a} \tan^{-1} \frac{yb}{a} + C$$

Since  $y = \tan x$ ,

$$\Rightarrow \frac{1}{ab} \tan^{-1} \left( \frac{b \tan x}{a} \right) + C$$

Therefore,

$$\int \frac{dx}{(a^2 \cos^2 x + b^2 \sin^2 x)} = \frac{1}{a^2} \tan^{-1} \left( \frac{b}{a} \tan x \right) + C$$

### 32. Question

Evaluate:

$$\int \frac{dx}{(\cos^2 x - 3 \sin^2 x)}$$

### Answer

To find:  $\int \frac{dx}{(\cos^2 x - 3 \sin^2 x)}$

Formula Used:

$$1. \sec^2 x = 1 + \tan^2 x$$

$$2. \int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$$

Dividing by  $\cos^2 x$  in the numerator and denominator,

$$\Rightarrow \int \frac{\sec^2 x \, dx}{1 - 3 \tan^2 x}$$

Let  $y = \tan x$

$$dy = \sec^2 x \, dx$$

Therefore,

$$\Rightarrow \int \frac{dy}{1 - 3y^2}$$

$$\Rightarrow \frac{1}{3} \int \frac{dy}{\left(\frac{1}{\sqrt{3}}\right)^2 - y^2}$$

$$\Rightarrow \frac{1}{3} \times \frac{\sqrt{3}}{2} \log \left| \frac{\frac{1}{\sqrt{3}} + y}{\frac{1}{\sqrt{3}} - y} \right| + C$$

$$\Rightarrow \frac{1}{2\sqrt{3}} \log \left| \frac{1 + y\sqrt{3}}{1 - y\sqrt{3}} \right| + C$$

Since  $y = \tan x$ ,

$$\Rightarrow \frac{1}{2\sqrt{3}} \log \left| \frac{1 + \sqrt{3} \tan x}{1 - \sqrt{3} \tan x} \right| + C$$

Therefore,

$$\int \frac{dx}{(\cos^2 x - 3 \sin^2 x)} = \frac{1}{2\sqrt{3}} \log \left| \frac{1 + \sqrt{3} \tan x}{1 - \sqrt{3} \tan x} \right| + C$$

### 33. Question

Evaluate:

$$\int \frac{dx}{(\sin^2 x - 4 \cos^2 x)}$$

### Answer

To find:  $\int \frac{dx}{(\sin^2 x - 4 \cos^2 x)}$

Formula Used:

$$1. \sec^2 x = 1 + \tan^2 x$$

$$2. \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

Dividing by  $\cos^2 x$  in the numerator and denominator,

$$\Rightarrow \int \frac{\sec^2 x dx}{\tan^2 x - 4}$$

Let  $y = \tan x$

$$dy = \sec^2 x dx$$

Therefore,

$$\Rightarrow \int \frac{dy}{y^2 - 2^2}$$

$$\Rightarrow \frac{1}{4} \log \left| \frac{y-2}{y+2} \right| + C$$

Since  $y = \tan x$ ,

$$\Rightarrow \frac{1}{4} \log \left| \frac{\tan x - 2}{\tan x + 2} \right| + C$$

Therefore,

$$\int \frac{dx}{(\sin^2 x - 4 \cos^2 x)} = \frac{1}{4} \log \left| \frac{\tan x - 2}{\tan x + 2} \right| + C$$

### 34. Question

Evaluate:

$$\int \frac{dx}{(\sin x \cos x + 2 \cos^2 x)}$$

### Answer

To find:  $\int \frac{dx}{(\sin x \cos x + 2 \cos^2 x)}$

Formula Used:

$$1. \sec^2 x = 1 + \tan^2 x$$

$$2. \int \frac{1}{x} dx = \log x + C$$

Dividing by  $\cos^2 x$  in the numerator and denominator,

$$\Rightarrow \int \frac{\sec^2 x \, dx}{\tan x + 2}$$

Let  $y = \tan x$

$$dy = \sec^2 x \, dx$$

Therefore,

$$\Rightarrow \int \frac{dy}{y + 2}$$

$$\Rightarrow \log |y + 2| + C$$

Since  $y = \tan x$ ,

$$\Rightarrow \log |\tan x + 2| + C$$

Therefore,

$$\int \frac{dx}{(\sin x \cos x + 2 \cos^2 x)} = \log |\tan x + 2| + C$$

### 35. Question

Evaluate:

$$\int \frac{\sin 2x}{(\sin^4 x + \cos^4 x)} dx$$

### Answer

$$\text{To find: } \int \frac{\sin 2x \, dx}{(\sin^4 x + \cos^4 x)}$$

Formula Used:

$$1. \sec^2 x = 1 + \tan^2 x$$

$$2. \int \frac{1}{1+x^2} dx = \tan^{-1} x + C$$

$$3. \sin 2x = 2 \sin x \cos x$$

Rewriting the given equation,

$$\Rightarrow \int \frac{2 \sin x \cos x}{\sin^4 x + \cos^4 x} dx$$

Dividing by  $\cos^4 x$  in the numerator and denominator,

$$\Rightarrow \int \frac{2 \tan x \sec^2 x \, dx}{\tan^4 x + 1}$$

Let  $y = \tan x$

$$dy = \sec^2 x \, dx$$

Therefore,

$$\Rightarrow \int \frac{2y}{y^4 + 1} dy$$

Let  $z = y^2$

$$dz = 2y \, dy$$

$$\Rightarrow \int \frac{dz}{1+z^2}$$

$$\Rightarrow \tan^{-1} z + C$$

$$\text{Since } z = y^2,$$

$$\Rightarrow \tan^{-1}(y^2) + C$$

$$\text{Since } y = \tan x,$$

$$\Rightarrow \tan^{-1}(\tan^2 x) + C$$

Therefore,

$$\int \frac{\sin 2x \, dx}{(\sin^4 x + \cos^4 x)} = \tan^{-1}(\tan^2 x) + C$$

### 36. Question

Evaluate:

$$\int \frac{(2 \sin 2\phi - \cos \phi)}{(6 - \cos^2 \phi - 4 \sin \phi)} d\phi$$

### Answer

$$\text{To find: } \int \frac{(2 \sin 2\phi - \cos \phi)}{(6 - \cos^2 \phi - 4 \sin \phi)} d\phi$$

Formula Used:

$$1. \sec^2 x = 1 + \tan^2 x$$

$$2. \int \frac{1}{1+x^2} dx = \tan^{-1} x + C$$

$$3. \sin 2x = 2 \sin x \cos x$$

Rewriting the given equation,

$$\Rightarrow \int \frac{4 \sin \phi \cos \phi - \cos \phi}{6 - \cos^2 \phi - 4 \sin \phi} d\phi$$

$$\Rightarrow \int \frac{\cos \phi (4 \sin \phi - 1)}{6 - (1 - \sin^2 \phi) - 4 \sin \phi} d\phi$$

$$\Rightarrow \int \frac{\cos \phi (4 \sin \phi - 1)}{5 + \sin^2 \phi - 4 \sin \phi} d\phi$$

$$\text{Let } y = \sin \phi$$

$$dy = \cos \phi \, d\phi$$

Substituting in the original equation,

$$\Rightarrow \int \frac{4y-1}{y^2-4y+5} dy \dots (1)$$

Using partial fraction,

$$4y - 1 = A \left( \frac{d}{dy} (y^2 - 4y + 5) \right) + B$$

$$4y - 1 = A (2y - 4) + B$$

Equating the coefficients of y,

$$4 = 2A$$

$$A = 2$$

$$\text{Also, } -1 = -4A + B$$

$$B = 7$$

Substituting in (1),

$$\Rightarrow \int \frac{2(2y-4)+7}{y^2-4y+5} dy$$

$$\Rightarrow 2 \log|y^2-4y+5| + 7 \int \frac{1}{((y-2)^2+1)} dy$$

$$\Rightarrow 2 \log|y^2-4y+5| + 7 \tan^{-1}(y-2) + C$$

But  $y = \sin \phi$

$$\Rightarrow 2 \log|\sin^2 \phi - 4 \sin \phi + 5| + 7 \tan^{-1}(\sin \phi - 2) + C$$

Therefore,

$$\begin{aligned} \int \frac{(2 \sin 2\phi - \cos \phi)}{(6 - \cos^2 \phi - 4 \sin \phi)} d\phi \\ = 2 \log|\sin^2 \phi - 4 \sin \phi + 5| + 7 \tan^{-1}(\sin \phi - 2) + C \end{aligned}$$

### 37. Question

Evaluate:

$$\int \frac{dx}{(\sin x - 2 \cos x)(2 \sin x + \cos x)}$$

### Answer

$$\text{To find: } \int \frac{dx}{(\sin x - 2 \cos x)(2 \sin x + \cos x)}$$

Formula Used:

$$1. \sec^2 x = 1 + \tan^2 x$$

$$2. \int \frac{1}{x} dx = \log x + C$$

Dividing by  $\cos^2 x$  in the numerator and denominator,

$$\Rightarrow \int \frac{\sec^2 x dx}{(\tan x - 2)(2 \tan x + 1)}$$

Let  $y = \tan x$

$$dy = \sec^2 x dx$$

Therefore,

$$\Rightarrow \int \frac{dy}{(y-2)(2y+1)} \dots (1)$$

Let

$$\frac{1}{(y-2)(2y+1)} = \frac{A}{(y-2)} + \frac{B}{(2y+1)}$$

$$1 = A(2y+1) + B(y-2)$$

When  $y = 0$ ,

$$1 = A - 2B \dots (2)$$

When  $y = 1$ ,

$$1 = 3A - B \Rightarrow 2 = 6A - 2B \dots (3)$$

Solving (2) and (3),

$$1 = 5A$$

$$A = \frac{1}{5}$$

$$\text{So, } B = \frac{-2}{5}$$

(1) becomes,

$$\Rightarrow \int \frac{\frac{1}{5}}{(y-2)} + \frac{\frac{-2}{5}}{(2y+1)}$$

$$\Rightarrow \frac{1}{5} \log|y-2| - \frac{2}{5} \log|2y+1| \times \frac{1}{2} + C$$

Since  $y = \tan x$ ,

$$\Rightarrow \frac{1}{5} \log|\tan x - 2| - \frac{1}{5} \log|2 \tan x + 1| + C$$

$$\Rightarrow \frac{1}{5} \log \left| \frac{\tan x - 2}{2 \tan x + 1} \right| + C$$

Therefore,

$$\int \frac{dx}{(\sin x - 2 \cos x)(2 \sin x + \cos x)} = \frac{1}{5} \log \left| \frac{\tan x - 2}{2 \tan x + 1} \right| + C$$

### 38. Question

Evaluate:

$$\int \frac{(1-x^2)}{(1+x^4)} dx$$

### Answer

$$\text{To find: } \int \frac{(1-x^2)}{(1+x^4)} dx$$

$$\text{Formula used: } \int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

On dividing by  $x^2$  in the numerator and denominator of the given equation,

$$\Rightarrow \int \frac{\frac{1}{x^2} - 1}{\frac{1}{x^2} + x^2} dx$$

$$\Rightarrow \int \frac{\frac{1}{x^2} - 1}{\frac{1}{x^2} + x^2 + 2 - 2} dx$$

$$\Rightarrow \int \frac{-\left(1 - \frac{1}{x^2}\right)}{\left(x + \frac{1}{x}\right)^2 - 2} dx$$

Let  $y = x + \frac{1}{x}$

Differentiating wrt  $x$ ,

$$dy = \left(1 - \frac{1}{x^2}\right) dx$$

Substituting in the original equation,

$$\Rightarrow \int \frac{-dy}{y^2 - (\sqrt{2})^2}$$

$$\Rightarrow \frac{-1}{2\sqrt{2}} \log \left| \frac{y - \sqrt{2}}{y + \sqrt{2}} \right| + C$$

Substituting for  $y = x + \frac{1}{x}$  and taking reciprocal of the value within logarithm, we get

$$\Rightarrow \frac{1}{2\sqrt{2}} \log \left| \frac{x + \frac{1}{x} + \sqrt{2}}{x + \frac{1}{x} - \sqrt{2}} \right| + C$$

$$\Rightarrow \frac{1}{2\sqrt{2}} \log \left| \frac{\sqrt{2}x + x^2 + 1}{\sqrt{2}x - x^2 + 1} \right| + C$$

Therefore,

$$\int \frac{(1 - x^2)}{(1 + x^4)} dx = \frac{1}{2\sqrt{2}} \log \left| \frac{\sqrt{2}x + x^2 + 1}{\sqrt{2}x - x^2 + 1} \right| + C$$

### 39. Question

Evaluate:

$$\int \frac{(x^2 + 1)}{(x^4 + x^2 + 1)} dx$$

### Answer

To find:  $\int \frac{(x^2 + 1)}{(x^4 + x^2 + 1)} dx$

Formula used:  $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$

On dividing by  $x^2$  in the numerator and denominator of the given equation,

$$\Rightarrow \int \frac{1 + \frac{1}{x^2}}{x^2 + 1 + \frac{1}{x^2}} dx$$

$$\Rightarrow \int \frac{\left(1 + \frac{1}{x^2}\right)}{\left(x - \frac{1}{x}\right)^2 + 3} dx$$

Let  $y = x - \frac{1}{x}$

Differentiating wrt x,

$$dy = \left(1 + \frac{1}{x^2}\right) dx$$

Substituting in the original equation,

$$\Rightarrow \int \frac{dy}{y^2 + (\sqrt{3})^2}$$

$$\Rightarrow \frac{1}{\sqrt{3}} \tan^{-1} \frac{y}{\sqrt{3}} + C$$

Substituting for  $y = x - \frac{1}{x}$

$$\Rightarrow \frac{1}{\sqrt{3}} \tan^{-1} \left( \frac{x - \frac{1}{x}}{\sqrt{3}} \right) + C$$

$$\Rightarrow \frac{1}{\sqrt{3}} \tan^{-1} \left( \frac{x^2 - 1}{\sqrt{3}x} \right) + C$$

Therefore,

$$\int \frac{(x^2 + 1)}{(x^4 + x^2 + 1)} dx = \frac{1}{\sqrt{3}} \tan^{-1} \left( \frac{x^2 - 1}{\sqrt{3}x} \right) + C$$

#### 40. Question

Evaluate:

$$\int \frac{dx}{(\sin^4 x + \cos^4 x)}$$

#### Answer

To find:  $\int \frac{dx}{(\sin^4 x + \cos^4 x)}$

Formula used:

$$1. \sec^2 x = 1 + \tan^2 x$$

$$2. \int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

Dividing by  $\cos^4 x$  in the numerator and denominator of the given equation,

$$\Rightarrow \int \frac{\sec^4 x}{(\tan^4 x + 1)} dx$$

$$\Rightarrow \int \frac{\sec^2 x (1 + \tan^2 x)}{(1 + \tan^4 x)} dx$$

Let  $y = \tan x$

$$dy = \sec^2 x dx$$

Substituting in the original equation,

$$\Rightarrow \int \frac{1 + y^2}{1 + y^4} dy$$



Dividing by  $y^2$  in the numerator and denominator,

$$\Rightarrow \int \frac{y^{-2} + 1}{y^{-2} + y^2} dy$$

$$\Rightarrow \int \frac{1 + y^{-2}}{y^2 + y^{-2} - 2 + 2} dy$$

$$\Rightarrow \int \frac{1 + y^{-2}}{\left(y - \frac{1}{y}\right)^2 + 2} dy$$

Let  $z = y - \frac{1}{y}$

$$dz = \left(1 + \frac{1}{y^2}\right) dy$$

Therefore,

$$\Rightarrow \int \frac{dz}{z^2 + (\sqrt{2})^2}$$

$$\Rightarrow \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{z}{\sqrt{2}}\right) + C$$

Substituting for z,

$$\Rightarrow \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{y - \frac{1}{y}}{\sqrt{2}}\right) + C$$

$$\Rightarrow \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{y^2 - 1}{y\sqrt{2}}\right) + C$$

Substituting for  $y = \tan x$ ,

$$\Rightarrow \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{\tan^2 x - 1}{\sqrt{2} \tan x}\right) + C$$

Therefore,

$$\int \frac{dx}{(\sin^4 x + \cos^4 x)} = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{\tan^2 x - 1}{\sqrt{2} \tan x}\right) + C$$

## Exercise 14B

### 1. Question

Evaluate:

$$\int \frac{dx}{\sqrt{16 - x^2}}$$

### Answer

Formula to be used -  $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + c$  where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{16 - x^2}}$$

$$= \int \frac{dx}{\sqrt{4^2 - x^2}}$$

$$= \sin^{-1} \frac{x}{4} + c, c \text{ being the integrating constant}$$

## 2. Question

Evaluate:

$$\int \frac{dx}{\sqrt{1-9x^2}}$$

## Answer

Formula to be used -  $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + c$  where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{1-9x^2}}$$

$$= \int \frac{dx}{\sqrt{9\left\{\left(\frac{1}{9}\right) - x^2\right\}}}$$

$$= \frac{1}{3} \int \frac{dx}{\sqrt{1^2 - \left(\frac{x}{3}\right)^2}}$$

$$= \frac{1}{3} \sin^{-1} \frac{x}{\frac{1}{3}} + c$$

$$= \frac{1}{3} \sin^{-1} 3x + c, c \text{ being the integrating constant}$$

## 3. Question

Evaluate:

$$\int \frac{dx}{\sqrt{15-8x^2}}$$

## Answer

Formula to be used -  $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + c$  where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{15-8x^2}}$$

$$= \int \frac{dx}{\sqrt{15\left\{1 - \left(\frac{\sqrt{8}}{\sqrt{15}}x\right)^2\right\}}}$$

$$= \frac{1}{\sqrt{15}} \int \frac{dx}{\sqrt{1^2 - \left(\frac{\sqrt{8}}{\sqrt{15}}x\right)^2}}$$

$$= \frac{1}{\sqrt{15}} \sin^{-1} \frac{x}{\left(\frac{\sqrt{15}}{\sqrt{8}}\right)} + c$$

$$= \frac{1}{\sqrt{15}} \sin^{-1} \frac{\sqrt{8}}{\sqrt{15}} x + c, c \text{ being the integrating constant}$$

#### 4. Question

Evaluate:

$$\int \frac{dx}{\sqrt{x^2 - 4}}$$

#### Answer

Formula to be used -  $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$  where c is the integrating constant

$$\begin{aligned} \therefore \int \frac{dx}{\sqrt{x^2 - 4}} \\ &= \int \frac{dx}{\sqrt{x^2 - 2^2}} \\ &= \log|x + \sqrt{x^2 - 4}| + c, c \text{ being the integrating constant} \end{aligned}$$

#### 5. Question

Evaluate:

$$\int \frac{dx}{\sqrt{4x^2 - 1}}$$

#### Answer

Formula to be used -  $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$  where c is the integrating constant

$$\begin{aligned} \therefore \int \frac{dx}{\sqrt{4x^2 - 1}} \\ &= \int \frac{dx}{\sqrt{(2x)^2 - 1^2}} \\ &= \frac{1}{2} \log|2x + \sqrt{4x^2 - 1}| + c, c \text{ being the integrating constant} \end{aligned}$$

#### 6. Question

Evaluate:

$$\int \frac{dx}{\sqrt{9x^2 - 7}}$$

#### Answer

Formula to be used -  $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$  where c is the integrating constant

$$\begin{aligned} \therefore \int \frac{dx}{\sqrt{9x^2 - 7}} \\ &= \int \frac{dx}{\sqrt{(3x)^2 - \sqrt{7}^2}} \\ &= \log|3x + \sqrt{9x^2 - 7}| + c, c \text{ being the integrating constant} \end{aligned}$$

## 7. Question

Evaluate:

$$\int \frac{dx}{\sqrt{x^2 - 9}}$$

**Answer**

Formula to be used -  $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$  where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{x^2 - 9}}$$

$$= \int \frac{dx}{\sqrt{x^2 - 3^2}}$$

$$= \log|x + \sqrt{x^2 - 9}| + c, c \text{ being the integrating constant}$$

## 8. Question

Evaluate:

$$\int \frac{dx}{\sqrt{1 + 4x^2}}$$

**Answer**

Formula to be used -  $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$  where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{1 + 4x^2}}$$

$$= \int \frac{dx}{\sqrt{(2x)^2 + 1^2}}$$

$$= \frac{1}{2} \log|2x + \sqrt{4x^2 + 1}| + c, c \text{ being the integrating constant}$$

## 9. Question

Evaluate:

$$\int \frac{dx}{\sqrt{9 + 4x^2}}$$

**Answer**

Formula to be used -  $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$  where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{9 + 4x^2}}$$

$$= \int \frac{dx}{\sqrt{(2x)^2 + 3^2}}$$

$$= \frac{1}{2} \log|2x + \sqrt{4x^2 + 9}| + c, c \text{ being the integrating constant}$$

## 10. Question

Evaluate:

$$\int \frac{x}{\sqrt{9-x^4}} dx$$

### Answer

Tip -  $d(x^2) = 2x dx$  i.e.  $x dx = (1/2) \times d(x^2)$

Formula to be used -  $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + c$  where  $c$  is the integrating constant

$$\begin{aligned} & \therefore \int \frac{x dx}{\sqrt{9-x^4}} \\ &= \frac{1}{2} \int \frac{d(x^2)}{\sqrt{3^2 - (x^2)^2}} \\ &= \frac{1}{2} \sin^{-1} \frac{x^2}{3} + c, \text{ } c \text{ being the integrating constant} \end{aligned}$$

### 11. Question

Evaluate:

$$\int \frac{3x^2}{\sqrt{9-16x^6}} dx$$

### Answer

Tip -  $d(x^3) = 3x^2 dx$  so,  $d(4x^3) = 4 \times 3x^2 dx$  i.e.  $3x^2 dx = (1/4) d(4x^3)$

Formula to be used -  $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + c$  where  $c$  is the integrating constant

$$\begin{aligned} & \therefore \int \frac{3x^2 dx}{\sqrt{9-16x^6}} \\ &= \frac{1}{4} \int \frac{d(4x^3)}{\sqrt{3^2 - (4x^3)^2}} \\ &= \frac{1}{4} \sin^{-1} \frac{4x^3}{3} + c, \text{ } c \text{ being the integrating constant} \end{aligned}$$

### 12. Question

Evaluate:

$$\int \frac{\sec^2 x}{\sqrt{16 + \tan^2 x}} dx$$

### Answer

Tip -  $d(\tan x) = \sec^2 x dx$

Formula to be used -  $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$  where  $c$  is the integrating constant

$$\begin{aligned} & \therefore \int \frac{\sec^2 x dx}{\sqrt{16 + \tan^2 x}} \\ &= \int \frac{d(\tan x)}{\sqrt{4^2 + (\tan x)^2}} \end{aligned}$$

$$= \log|\tan x + \sqrt{16 + \tan^2 x}| + c, c \text{ being the integrating constant}$$

### 13. Question

Evaluate:

$$\int \frac{\sin x}{\sqrt{4 + \cos^2 x}} dx$$

### Answer

Tip -  $d(\cos x) = -\sin x dx$  i.e.  $\sin x dx = -d(\cos x)$

Formula to be used -  $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$  where  $c$  is the integrating constant

$$\therefore \int \frac{\sin x dx}{\sqrt{4 + \cos^2 x}}$$

$$= \int \frac{-d(\cos x)}{\sqrt{(\cos x)^2 + 2^2}}$$

$$= -\log|\cos x + \sqrt{4 + \cos^2 x}| + c, c \text{ being the integrating constant}$$

### 14. Question

Evaluate:

$$\int \frac{\cos x}{\sqrt{9\sin^2 x - 1}} dx$$

### Answer

Tip -  $d(\sin x) = \cos x dx$  so,  $d(3\sin x) = 3\cos x dx$  i.e.  $\cos x dx = (1/3)d(3\sin x)$

Formula to be used -  $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$  where  $c$  is the integrating constant

$$\therefore \int \frac{\cos x dx}{\sqrt{9\sin^2 x - 1}}$$

$$= \frac{1}{3} \int \frac{d(3\sin x)}{\sqrt{(3\sin x)^2 - 1^2}}$$

$$= \frac{1}{3} \log|\cos x + \sqrt{4 + \cos^2 x}| + c, c \text{ being the integrating constant}$$

### 15. Question

Evaluate:

$$\int \frac{e^x}{\sqrt{4 + e^{2x}}} dx$$

### Answer

Tip -  $d(e^x) = e^x dx$

Formula to be used -  $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$  where  $c$  is the integrating constant

$$\therefore \int \frac{e^x dx}{\sqrt{4 + e^{2x}}}$$

$$= \int \frac{d(e^x)}{\sqrt{2^2 + (e^x)^2}}$$

$$= \log|e^x + \sqrt{4 + e^{2x}}| + c, c \text{ being the integrating constant}$$

### 16. Question

Evaluate:

$$\int \frac{2e^x}{\sqrt{4 - e^{2x}}} dx$$

### Answer

Tip -  $d(e^x) = e^x dx$

Formula to be used -  $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + c$  where  $c$  is the integrating constant

$$\therefore \int \frac{2e^x dx}{\sqrt{4 - e^{2x}}}$$

$$= 2 \int \frac{d(e^x)}{\sqrt{2^2 - (e^x)^2}}$$

$$= 2 \sin^{-1} \left( \frac{e^x}{2} \right) + c, c \text{ being the integrating constant}$$

### 17. Question

Evaluate:

$$\int \frac{dx}{\sqrt{1 - e^x}}$$

### Answer

Formula to be used -  $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$  where  $c$  is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{1 - e^x}}$$

$$= \int \frac{dx}{\sqrt{e^x(e^{-x} - 1)}}$$

$$= \int \frac{e^{-\frac{x}{2}} dx}{\sqrt{e^{-x} - 1}}$$

$$= \int \frac{e^{-\frac{x}{2}} dx}{\sqrt{(e^{-\frac{x}{2}})^2 - 1^2}}$$

Tip - Assuming  $e^{-\frac{x}{2}} = a$ ,  $-\frac{1}{2} e^{-\frac{x}{2}} dx = da$  i.e.  $e^{-\frac{x}{2}} dx = -2da$

$$\therefore \int \frac{e^{-\frac{x}{2}} dx}{\sqrt{(e^{-\frac{x}{2}})^2 - 1^2}}$$

$$= \int \frac{-2da}{\sqrt{a^2 - 1^2}}$$

$$= -2\log|a + \sqrt{a^2 - 1}| + c$$

$$= -2\log|e^{\frac{-x}{2}} + \sqrt{e^{-x} - 1}| + c, c \text{ being the integrating constant}$$

### 18. Question

Evaluate:

$$\int \sqrt{\frac{a-x}{a+x}} dx$$

### Answer

Tip - Taking  $x = a\cos 2\theta$ ,

$$dx = -2a \sin 2\theta d\theta \text{ and } \theta = \frac{1}{2} \cos^{-1} \frac{x}{a}$$

$$x = a\cos 2\theta \text{ i.e } \cos 2\theta = \frac{x}{a}$$

$$\therefore \sin 2\theta = \sqrt{1 - \frac{x^2}{a^2}}$$

$$\therefore \int \sqrt{\frac{a-x}{a+x}} dx$$

$$= \int \sqrt{\frac{a - a\cos 2\theta}{a + a\cos 2\theta}} \times (-2a \sin 2\theta d\theta)$$

$$= \int \sqrt{\frac{a(1 - \cos 2\theta)}{a(1 + \cos 2\theta)}} \times (-2a \sin 2\theta d\theta)$$

Formula to be used -  $\cos 2\theta = 1 - 2\sin^2 \theta = 2\cos^2 \theta - 1$

$$\sin 2\theta = 2\sin \theta \cos \theta$$

$$\therefore \int \sqrt{\frac{1 - \cos 2\theta}{1 + \cos 2\theta}} \times (-2a \sin 2\theta d\theta)$$

$$= \int \sqrt{\frac{2\sin^2 \theta}{2\cos^2 \theta}} \times (-2a \sin 2\theta d\theta)$$

$$= \int \frac{\sin \theta}{\cos \theta} \times (-2a \times 2\sin \theta \cos \theta d\theta)$$

$$= -2a \int 2\sin^2 \theta d\theta$$

$$= -2a \int 1 - \cos 2\theta d\theta$$

$$= -2a \left[ \theta - \frac{\sin 2\theta}{2} \right]$$

$$= -2a \left[ \theta - \frac{\sin 2\theta}{2} \right] + c$$



$$= -2a \left[ \frac{1}{2} \cos^{-1} \frac{x}{a} - \frac{\sqrt{1 - \frac{x^2}{a^2}}}{2} \right] + c$$

$$= -a \cos^{-1} \frac{x}{a} + a \sqrt{1 - \frac{x^2}{a^2}} + c$$

$$= a \sin^{-1} \frac{x}{a} + \sqrt{a^2 - x^2} + c, c \text{ being the integrating constant}$$

### 19. Question

Evaluate:

$$\int \frac{dx}{\sqrt{x^2 + 6x + 5}}$$

### Answer

Formula to be used -  $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$  where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{x^2 + 6x + 5}}$$

$$= \int \frac{dx}{\sqrt{(x^2 + 2 \times x \times 3 + 3^2) + 5 - 3^2}}$$

$$= \int \frac{dx}{\sqrt{(x + 3)^2 - 2^2}}$$

$$= \log|(x + 3) + \sqrt{x^2 + 6x + 5}| + c, c \text{ being the integrating constant}$$

### 20. Question

Evaluate:

$$\int \frac{dx}{\sqrt{(2-x)^2 + 1}}$$

### Answer

Tip -  $d(2 - x) = -dx$  i.e.  $dx = -d(2 - x)$

Formula to be used -  $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$  where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{(2-x)^2 + 1}}$$

$$= \int \frac{-d(2-x)}{\sqrt{(2-x)^2 + 1}}$$

$$= -\log|(2-x) + \sqrt{(2-x)^2 + 1}| + c$$

$$= -\log|(2-x) + \sqrt{x^2 - 4x + 5}| + c, c \text{ being the integrating constant}$$

### 21. Question

Evaluate:

$$\int \frac{dx}{\sqrt{(x-3)^2 + 1}}$$

**Answer**

Formula to be used -  $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$  where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{(x-3)^2 + 1}}$$

$$= \log|(x-3) + \sqrt{(x-3)^2 + 1}| + c$$

$$= \log|(x-3) + \sqrt{x^2 - 6x + 10}| + c, c \text{ being the integrating constant}$$

## 22. Question

Evaluate:

$$\int \frac{dx}{\sqrt{x^2 - 6x + 10}}$$

**Answer**

Formula to be used -  $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$  where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{x^2 - 6x + 10}}$$

$$= \int \frac{dx}{\sqrt{(x-3)^2 + 1}}$$

$$= \log|(x-3) + \sqrt{(x-3)^2 + 1}| + c$$

$$= \log|(x-3) + \sqrt{x^2 - 6x + 10}| + c, c \text{ being the integrating constant}$$

## 23. Question

Evaluate:

$$\int \frac{dx}{\sqrt{2 + 2x - x^2}}$$

**Answer**

Formula to be used -  $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + c$  where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{2 + 2x - x^2}}$$

$$= \int \frac{dx}{\sqrt{3 - (x^2 - 2x + 1)}}$$

$$= \int \frac{dx}{\sqrt{(\sqrt{3})^2 - (x-1)^2}}$$

$$= \sin^{-1} \left( \frac{x-1}{\sqrt{3}} \right) + c, c \text{ being the integrating constant}$$

#### 24. Question

Evaluate:

$$\int \frac{dx}{\sqrt{8-4x-2x^2}}$$

#### Answer

Formula to be used -  $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + c$  where c is the integrating constant

$$\begin{aligned} &\therefore \int \frac{dx}{\sqrt{8-4x-2x^2}} \\ &= \int \frac{dx}{\sqrt{10-2(x^2+2x+1)}} \\ &= \int \frac{dx}{\sqrt{(\sqrt{10})^2-2(x+1)^2}} \\ &= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{(\sqrt{5})^2-(x+1)^2}} \\ &= \frac{1}{\sqrt{2}} \sin^{-1} \left( \frac{x+1}{\sqrt{5}} \right) + c, c \text{ being the integrating constant} \end{aligned}$$

#### 25. Question

Evaluate:

$$\int \frac{dx}{\sqrt{16-6x-x^2}}$$

#### Answer

Formula to be used -  $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + c$  where c is the integrating constant

$$\begin{aligned} &\therefore \int \frac{dx}{\sqrt{16-6x-x^2}} \\ &= \int \frac{dx}{\sqrt{25-(x^2+6x+9)}} \\ &= \int \frac{dx}{\sqrt{(5)^2-(x+3)^2}} \\ &= \sin^{-1} \left( \frac{x+3}{5} \right) + c, c \text{ being the integrating constant} \end{aligned}$$

#### 26. Question

Evaluate:

$$\int \frac{dx}{\sqrt{7-6x-x^2}}$$

#### Answer

Formula to be used -  $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + c$  where c is the integrating constant

$$\begin{aligned} & \therefore \int \frac{dx}{\sqrt{7-6x-x^2}} \\ &= \int \frac{dx}{\sqrt{16-(x^2+6x+9)}} \\ &= \int \frac{dx}{\sqrt{(4)^2-(x+3)^2}} \\ &= \sin^{-1} \left( \frac{x+3}{4} \right) + c, c \text{ being the integrating constant} \end{aligned}$$

## 27. Question

Evaluate:

$$\int \frac{dx}{\sqrt{x-x^2}}$$

## Answer

Formula to be used -  $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + c$  where c is the integrating constant

$$\begin{aligned} & \therefore \int \frac{dx}{\sqrt{x-x^2}} \\ &= \int \frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2 - (x^2 - 2 \times x \times \frac{1}{2} + \left(\frac{1}{2}\right)^2)}} \\ &= \int \frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2 - (x - \frac{1}{2})^2}} \\ &= \sin^{-1} \left( \frac{x - \frac{1}{2}}{\frac{1}{2}} \right) + c \\ &= \sin^{-1}(2x - 1) + c, c \text{ being the integrating constant} \end{aligned}$$

## 28. Question

Evaluate:

$$\int \frac{dx}{\sqrt{8+2x-x^2}}$$

## Answer

Formula to be used -  $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + c$  where c is the integrating constant

$$\begin{aligned} & \therefore \int \frac{dx}{\sqrt{8+2x-x^2}} \\ &= \int \frac{dx}{\sqrt{9-(x^2-2x+1)}} \end{aligned}$$

$$= \int \frac{dx}{\sqrt{(3)^2 - (x-1)^2}}$$

$$= \sin^{-1}\left(\frac{x-1}{3}\right) + c, c \text{ being the integrating constant}$$

### 29. Question

Evaluate:

$$\int \frac{dx}{\sqrt{x^2 - 3x + 2}}$$

### Answer

Formula to be used -  $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$  where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{x^2 - 3x + 2}}$$

$$= \int \frac{dx}{\sqrt{x^2 - 2 \times x \times \frac{3}{2} + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 2}}$$

$$= \int \frac{dx}{\sqrt{\left(x - \frac{3}{2}\right)^2 - \frac{1}{4}}}$$

$$= \log\left|\left(x - \frac{3}{2}\right) + \sqrt{x^2 - 3x + 2}\right| + c, c \text{ being the integrating constant}$$

### 30. Question

Evaluate:

$$\int \frac{dx}{\sqrt{2x^2 + 3x - 2}}$$

### Answer

Formula to be used -  $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$  where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{2x^2 + 3x - 2}}$$

$$= \int \frac{dx}{\sqrt{2\left(x^2 + 2 \times x \times \frac{3}{4} + \left(\frac{3}{4}\right)^2\right) - \frac{7}{8}}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\left(x + \frac{3}{4}\right)^2 - \left(\frac{\sqrt{7}}{4}\right)^2}}$$

$$= \frac{1}{\sqrt{2}} \log\left|\left(x + \frac{3}{4}\right) + \sqrt{2x^2 + 3x - 2}\right| + c, c \text{ being the integrating constant}$$

### 31. Question

Evaluate:

$$\int \frac{dx}{\sqrt{2x^2 + 4x + 6}}$$

### Answer

Formula to be used -  $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$  where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{2x^2 + 4x + 6}}$$

$$= \int \frac{dx}{\sqrt{2(x^2 + 2x + 1) + 4}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{(x+1)^2 + (\sqrt{2})^2}}$$

$$= \frac{1}{\sqrt{2}} \log|(x+1) + \sqrt{2x^2 + 4x + 6}| + c, c \text{ being the integrating constant}$$

### 32. Question

Evaluate:

$$\int \frac{dx}{\sqrt{1 + 2x - 3x^2}}$$

### Answer

Formula to be used -  $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + c$  where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{1 + 2x - 3x^2}}$$

$$= \int \frac{dx}{\sqrt{\left(1 - \frac{1}{3}\right) - 3\left(x^2 - 2 \times x \times \frac{1}{3} + \left(\frac{1}{3}\right)^2\right)}}$$

$$= \int \frac{dx}{\sqrt{\left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2 - 3\left(x - \frac{1}{3}\right)^2}}$$

$$= \frac{1}{\sqrt{3}} \int \frac{dx}{\sqrt{\left(\frac{\sqrt{2}}{3}\right)^2 - \left(x - \frac{1}{3}\right)^2}}$$

$$= \frac{1}{\sqrt{3}} \sin^{-1} \left( \frac{x - \frac{1}{3}}{\frac{\sqrt{2}}{3}} \right) + c$$

$$= \frac{1}{\sqrt{3}} \sin^{-1} \left( \frac{3x-1}{\sqrt{2}} \right) + c, c \text{ being the integrating constant}$$

### 33. Question

Evaluate:

$$\int \frac{dx}{\sqrt{x}\sqrt{5-x}}$$

**Answer**

Formula to be used -  $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + c$  where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{5x-x^2}}$$

$$= \int \frac{dx}{\sqrt{\left(\frac{5}{2}\right)^2 - \left(x^2 - 2 \times x \times \frac{5}{2} + \left(\frac{5}{2}\right)^2\right)}}$$

$$= \int \frac{dx}{\sqrt{\left(\frac{5}{2}\right)^2 - \left(x - \frac{5}{2}\right)^2}}$$

$$= \sin^{-1} \left( \frac{x - \frac{5}{2}}{\frac{5}{2}} \right) + c$$

$$= \sin^{-1} \left( \frac{2x-5}{5} \right) + c, c \text{ being the integrating constant}$$

### 34. Question

Evaluate:

$$\int \frac{dx}{\sqrt{3+4x-2x^2}}$$

**Answer**

Formula to be used -  $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + c$  where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{3+4x-2x^2}}$$

$$= \int \frac{dx}{\sqrt{5-2(x^2-2x+1)}}$$

$$= \int \frac{dx}{\sqrt{(\sqrt{5})^2 - 2(x-1)^2}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\left(\frac{\sqrt{5}}{2}\right)^2 - (x-1)^2}}$$

$$= \frac{1}{\sqrt{2}} \sin^{-1} \left( \frac{x-1}{\frac{\sqrt{5}}{2}} \right) + c$$

$$= \frac{1}{\sqrt{2}} \sin^{-1} \left( \frac{\sqrt{2}(x-1)}{\sqrt{5}} \right) + c, c \text{ being the integrating constant}$$

### 35. Question

Evaluate:

$$\int \frac{x^2}{\sqrt{x^6 + 2x^3 + 3}} dx$$

### Answer

Tip -  $d(x^3) = 3x^2 dx$  i.e.  $x^2 dx = (1/3)d(x^3)$

Formula to be used -  $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$  where  $c$  is the integrating constant

$$\therefore \int \frac{x^2 dx}{\sqrt{x^6 + 2x^3 + 3}}$$

$$= \int \frac{\frac{1}{3} d(x^3)}{\sqrt{(x^3)^2 + 2x^3 + 3}}$$

$$= \frac{1}{3} \int \frac{d(x^3)}{\sqrt{(x^3 + 1)^2 + (\sqrt{2})^2}}$$

$$= \frac{1}{3} \log|(x^3 + 1) + \sqrt{x^6 + 2x^3 + 3}| + c, c \text{ being the integrating constant}$$

### 36. Question

Evaluate:

$$\int \frac{(2x + 3)}{\sqrt{x^2 + x + 1}} dx$$

### Answer

Formula to be used -  $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$  where  $c$  is the integrating constant

$$\therefore \int \frac{(2x + 3)}{\sqrt{x^2 + x + 1}} dx$$

$$= \int \frac{(2x + 1) + 2}{\sqrt{x^2 + x + 1}} dx$$

$$= \int \frac{(2x + 1)}{\sqrt{x^2 + x + 1}} dx + \int \frac{2}{\sqrt{x^2 + x + 1}} dx$$

Tip - Assuming  $x^2 + x + 1 = a^2$ ,  $(2x + 1)dx = 2ada$

$$\therefore \int \frac{(2x + 1)}{\sqrt{x^2 + x + 1}} dx$$

$$= \int \frac{2ada}{a}$$

$$= \int 2da$$

$$= 2a + c_1$$

$$= 2\sqrt{x^2 + x + 1} + c_1$$



$$\begin{aligned}
& \therefore \int \frac{2}{\sqrt{x^2 + x + 1}} dx \\
&= 2 \int \frac{dx}{\sqrt{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}} \\
&= 2 \log \left| \left(x + \frac{1}{2}\right) + \sqrt{x^2 + x + 1} \right| + c_2 \\
&\therefore \int \frac{(2x + 1)}{\sqrt{x^2 + x + 1}} dx + \int \frac{2}{\sqrt{x^2 + x + 1}} dx \\
&= 2\sqrt{x^2 + x + 1} + 2 \log \left| \left(x + \frac{1}{2}\right) + \sqrt{x^2 + x + 1} \right| + c, c \text{ is the integrating constant}
\end{aligned}$$

### 37. Question

Evaluate:

$$\int \frac{(5x + 3)}{\sqrt{x^2 + 4x + 10}} dx$$

### Answer

Formula to be used -  $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$  where  $c$  is the integrating constant

$$\begin{aligned}
& \therefore \int \frac{(5x + 3)}{\sqrt{x^2 + 4x + 10}} dx \\
&= \int \frac{\frac{5}{2} \times (2x + 4) - 7}{\sqrt{x^2 + 4x + 10}} dx \\
&= \frac{5}{2} \int \frac{(2x + 4)}{\sqrt{x^2 + 4x + 10}} dx - \int \frac{7}{\sqrt{x^2 + 4x + 10}} dx
\end{aligned}$$

Tip - Assuming  $x^2 + 4x + 10 = a^2$ ,  $(2x + 4)dx = 2ada$

$$\begin{aligned}
& \therefore \frac{5}{2} \int \frac{(2x + 4)}{\sqrt{x^2 + 4x + 10}} dx \\
&= \frac{5}{2} \int \frac{2ada}{a} \\
&= \frac{5}{2} \int 2da \\
&= 5a + c_1 \\
&= 5\sqrt{x^2 + 4x + 10} + c_1 \\
&\therefore \int \frac{7}{\sqrt{x^2 + 4x + 10}} dx \\
&= 7 \int \frac{dx}{\sqrt{(x + 2)^2 + (\sqrt{6})^2}} \\
&= 7 \log \left| (x + 2) + \sqrt{x^2 + 4x + 10} \right| + c_2
\end{aligned}$$

$$\therefore \frac{5}{2} \int \frac{(2x + 4)}{\sqrt{x^2 + 4x + 10}} dx - \int \frac{7}{\sqrt{x^2 + 4x + 10}} dx$$

$$= 5\sqrt{x^2 + 4x + 10} - 7 \log|(x + 2) + \sqrt{x^2 + 4x + 10}| + c, c \text{ is the integrating constant}$$

### 38. Question

Evaluate:

$$\int \frac{(4x + 3)}{\sqrt{2x^2 + 2x - 3}} dx$$

### Answer

Formula to be used -  $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$  where c is the integrating constant

$$\therefore \int \frac{(4x + 3)}{\sqrt{2x^2 + 2x - 3}} dx$$

$$= \int \frac{(4x + 2) + 1}{\sqrt{2x^2 + 2x - 3}} dx$$

$$= \int \frac{(4x + 2)}{\sqrt{2x^2 + 2x - 3}} dx + \int \frac{1}{\sqrt{2x^2 + 2x - 3}} dx$$

Tip - Assuming  $2x^2 + 2x - 3 = a^2$ ,  $(4x + 2)dx = 2ada$

$$\therefore \int \frac{(4x + 2)}{\sqrt{2x^2 + 2x - 3}} dx$$

$$= \int \frac{2ada}{a}$$

$$= \int 2da$$

$$= 2a + c_1$$

$$= 2\sqrt{2x^2 + 2x - 3} + c_1$$

$$\therefore \int \frac{1}{\sqrt{2x^2 + 2x - 3}} dx$$

$$= \int \frac{dx}{\sqrt{2\left(x + \frac{1}{2}\right)^2 - \left(\sqrt{\frac{7}{2}}\right)^2}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\left(x + \frac{1}{2}\right)^2 - \left(\frac{\sqrt{7}}{2}\right)^2}}$$

$$= \frac{1}{\sqrt{2}} \log \left| \left(x + \frac{1}{2}\right) + \sqrt{x^2 + x - \frac{3}{2}} \right| + c_2$$

$$\therefore \int \frac{(4x + 2)}{\sqrt{2x^2 + 2x - 3}} dx + \int \frac{1}{\sqrt{2x^2 + 2x - 3}} dx$$

$$= 2\sqrt{2x^2 + 2x - 3} + \frac{1}{\sqrt{2}} \log \left| \left( x + \frac{1}{2} \right) + \sqrt{x^2 + x - \frac{3}{2}} \right| + c, \text{ c is the integrating constant}$$

### 39. Question

Evaluate:

$$\int \frac{(3-2x)}{\sqrt{2+x-x^2}} dx$$

### Answer

Formula to be used -  $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + c$  where c is the integrating constant

$$\begin{aligned} & \therefore \int \frac{(3-2x)}{\sqrt{2+x-x^2}} dx \\ &= \int \frac{(1-2x) + 2}{\sqrt{2+x-x^2}} dx \\ &= \int \frac{(1-2x)}{\sqrt{2+x-x^2}} dx + \int \frac{2}{\sqrt{2+x-x^2}} dx \end{aligned}$$

Tip - Assuming  $2+x-x^2 = a^2$ ,  $(1-2x)dx = 2ada$

$$\therefore \int \frac{(1-2x)}{\sqrt{2+x-x^2}} dx$$

$$= \int \frac{2ada}{a}$$

$$= 2a + c_1$$

$$= 2\sqrt{2+x-x^2} + c_1$$

$$\therefore \int \frac{2}{\sqrt{2+x-x^2}} dx$$

$$= 2 \int \frac{dx}{\sqrt{\left(\frac{3}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2}}$$

$$= 2 \sin^{-1} \frac{\left(x - \frac{1}{2}\right)}{\left(\frac{3}{2}\right)} + c_2$$

$$= 2 \sin^{-1} \left( \frac{2x-1}{3} \right) + c_2$$

$$\therefore \int \frac{(1-2x)}{\sqrt{2+x-x^2}} dx + \int \frac{2}{\sqrt{2+x-x^2}} dx$$

$$= 2\sqrt{2+x-x^2} + 2 \sin^{-1} \left( \frac{2x-1}{3} \right) + c, \text{ c is the integrating constant}$$

### 40. Question

Evaluate:

$$\int \frac{(x+2)}{\sqrt{2x^2+2x-3}} dx$$

## Answer

Formula to be used -  $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$  where  $c$  is the integrating constant

$$\begin{aligned} &\therefore \int \frac{(x+2)}{\sqrt{2x^2+2x-3}} dx \\ &= \int \frac{\frac{1}{4} \times (4x+2) + \frac{3}{2}}{\sqrt{2x^2+2x-3}} dx \\ &= \frac{1}{4} \int \frac{(4x+2)}{\sqrt{2x^2+2x-3}} dx + \frac{3}{2} \int \frac{1}{\sqrt{2x^2+2x-3}} dx \end{aligned}$$

Tip - Assuming  $2x^2 + 2x - 3 = a^2$ ,  $(4x+2)dx = 2ada$

$$\begin{aligned} &\therefore \frac{1}{4} \int \frac{(4x+2)}{\sqrt{2x^2+2x-3}} dx \\ &= \frac{1}{4} \int \frac{2ada}{a} \\ &= \frac{1}{2} \int da \\ &= \frac{a}{2} + c_1 \\ &= \frac{\sqrt{2x^2+2x-3}}{2} + c_1 \\ &\therefore \frac{3}{2} \int \frac{1}{\sqrt{2x^2+2x-3}} dx \\ &= \frac{3}{2} \int \frac{dx}{\sqrt{2\left(x+\frac{1}{2}\right)^2 - \left(\sqrt{\frac{7}{2}}\right)^2}} \\ &= \frac{3}{2\sqrt{2}} \int \frac{dx}{\sqrt{\left(x+\frac{1}{2}\right)^2 - \left(\frac{\sqrt{7}}{2}\right)^2}} \\ &= \frac{3}{2\sqrt{2}} \log \left| \left(x+\frac{1}{2}\right) + \sqrt{x^2+x-\frac{3}{2}} \right| + c_2 \\ &\therefore \frac{1}{4} \int \frac{(4x+2)}{\sqrt{2x^2+2x-3}} dx + \frac{3}{2} \int \frac{1}{\sqrt{2x^2+2x-3}} dx \\ &= \frac{\sqrt{2x^2+2x-3}}{2} + \frac{3}{2\sqrt{2}} \log \left| \left(x+\frac{1}{2}\right) + \sqrt{x^2+x-\frac{3}{2}} \right| + c, c \text{ is the integrating constant} \end{aligned}$$

## 41. Question

Evaluate:

$$\int \frac{(3x+1)}{\sqrt{5-2x-x^2}} dx$$

### Answer

Formula to be used -  $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + c$  where c is the integrating constant

$$\begin{aligned} &\therefore \int \frac{(3x + 1)}{\sqrt{5 - 2x - x^2}} dx \\ &= \int \frac{3(x + 1) - 2}{\sqrt{5 - 2x - x^2}} dx \\ &= \int \frac{3(x + 1)}{\sqrt{5 - 2x - x^2}} dx - \int \frac{2}{\sqrt{5 - 2x - x^2}} dx \end{aligned}$$

Tip - Assuming  $5 - 2x - x^2 = a^2$ ,  $(-2 - 2x)dx = 2ada$  i.e.  $(x + 1)dx = -ada$

$$\begin{aligned} &\therefore \int \frac{3(x + 1)}{\sqrt{5 - 2x - x^2}} dx \\ &= -3 \int \frac{ada}{a} \\ &= -3a + c_1 \\ &= -3\sqrt{5 - 2x - x^2} + c_1 \\ &\therefore \int \frac{2}{\sqrt{5 - 2x - x^2}} dx \\ &= 2 \int \frac{dx}{\sqrt{(\sqrt{6})^2 - (x + 1)^2}} \\ &= 2 \sin^{-1} \frac{(x + 1)}{\sqrt{6}} + c_2 \\ &\therefore \int \frac{3(x + 1)}{\sqrt{5 - 2x - x^2}} dx - \int \frac{2}{\sqrt{5 - 2x - x^2}} dx \\ &= -3\sqrt{5 - 2x - x^2} - 2 \sin^{-1} \left( \frac{x+1}{\sqrt{6}} \right) + c, c \text{ is the integrating constant} \end{aligned}$$

### 42. Question

Evaluate:

$$\int \frac{(6x + 5)}{\sqrt{6 + x - 2x^2}} dx$$

### Answer

Formula to be used -  $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + c$  where c is the integrating constant

$$\begin{aligned} &\therefore \int \frac{(6x + 5)}{\sqrt{6 + x - 2x^2}} dx \\ &= \int \frac{\frac{6}{4}(4x - 1) + \frac{13}{2}}{\sqrt{6 + x - 2x^2}} dx \\ &= \frac{3}{2} \int \frac{(4x - 1)}{\sqrt{6 + x - 2x^2}} dx + \frac{13}{2} \int \frac{1}{\sqrt{6 + x - 2x^2}} dx \end{aligned}$$

Tip - Assuming  $6 + x - 2x^2 = a^2$ ,  $(1 - 4x)dx = 2ada$  i.e.  $(4x - 1)dx = -2ada$

$$\therefore \frac{3}{2} \int \frac{(4x-1)}{\sqrt{6+x-2x^2}} dx$$

$$= -\frac{3}{2} \int \frac{2ada}{a}$$

$$= -3a + c_1$$

$$= -3\sqrt{6+x-2x^2} + c_1$$

$$\therefore \frac{13}{2} \int \frac{1}{\sqrt{6+x-2x^2}} dx$$

$$= \frac{13}{2} \int \frac{dx}{\sqrt{\left(\frac{7}{2\sqrt{2}}\right)^2 - 2\left(x - \frac{1}{4}\right)^2}}$$

$$= \frac{13}{2\sqrt{2}} \int \frac{dx}{\sqrt{\left(\frac{7}{4}\right)^2 - \left(x - \frac{1}{4}\right)^2}}$$

$$= \frac{13}{2\sqrt{2}} \sin^{-1} \frac{\left(x - \frac{1}{4}\right)}{\left(\frac{7}{4}\right)} + c_2$$

$$= \frac{13}{2\sqrt{2}} \sin^{-1} \left( \frac{4x-1}{7} \right) + c_2$$

$$\therefore \frac{3}{2} \int \frac{(4x-1)}{\sqrt{6+x-2x^2}} dx + \frac{13}{2} \int \frac{1}{\sqrt{6+x-2x^2}} dx$$

$$= -3\sqrt{6+x-2x^2} + \frac{13}{2\sqrt{2}} \sin^{-1} \left( \frac{4x-1}{7} \right) + c, \text{ c is the integrating constant}$$

### 43. Question

Evaluate:

$$\int \sqrt{\frac{1+x}{x}} dx$$

### Answer

Formula to be used -  $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$  where c is the integrating constant

$$\int \sqrt{\frac{1+x}{x}} dx$$

$$= \int \sqrt{\frac{(1+x)^2}{x(1+x)}} dx$$

$$= \int \frac{1+x}{\sqrt{x^2+x}} dx$$

$$= \int \frac{\frac{1}{2}(2x+1) + \frac{1}{2}}{\sqrt{x^2+x}} dx$$

$$= \frac{1}{2} \int \frac{2x + 1}{\sqrt{x^2 + x}} dx + \frac{1}{2} \int \frac{dx}{\sqrt{x^2 + x}}$$

Tip - Taking  $x^2 + x = a^2$ ,  $(2x + 1)dx = 2ada$

$$\therefore \frac{1}{2} \int \frac{2x + 1}{\sqrt{x^2 + x}} dx$$

$$= \frac{1}{2} \int \frac{2ada}{a}$$

$$= a + c_1$$

$$= \sqrt{x^2 + x} + c_1$$

$$\therefore \frac{1}{2} \int \frac{1}{\sqrt{x^2 + x}} dx$$

$$= \frac{1}{2} \int \frac{dx}{\sqrt{\left(x + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$$

$$= \frac{1}{2} \log \left| \left(x + \frac{1}{2}\right) + \sqrt{x^2 + x} \right| + c_2$$

$$\therefore \frac{1}{2} \int \frac{2x + 1}{\sqrt{x^2 + x}} dx + \frac{1}{2} \int \frac{dx}{\sqrt{x^2 + x}}$$

$$= \sqrt{x^2 + x} + \frac{1}{2} \log \left| \left(x + \frac{1}{2}\right) + \sqrt{x^2 + x} \right| + c, c \text{ is the integrating constant}$$

#### 44. Question

Evaluate:

$$\int \frac{(x + 2)}{\sqrt{x^2 + 5x + 6}} dx$$

#### Answer

Formula to be used -  $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$  where  $c$  is the integrating constant

$$\int \frac{(x + 2)}{\sqrt{x^2 + 5x + 6}} dx$$

$$= \int \frac{\frac{1}{2}(2x + 5) - \frac{1}{2}}{\sqrt{x^2 + 5x + 6}} dx$$

$$= \frac{1}{2} \int \frac{2x + 5}{\sqrt{x^2 + 5x + 6}} dx - \frac{1}{2} \int \frac{dx}{\sqrt{x^2 + 5x + 6}}$$

Tip - Taking  $x^2 + 5x + 6 = a^2$ ,  $(2x + 5)dx = 2ada$

$$\therefore \frac{1}{2} \int \frac{2x + 5}{\sqrt{x^2 + 5x + 6}} dx$$

$$= \frac{1}{2} \int \frac{2ada}{a}$$

$$= a + c_1$$

$$= \sqrt{x^2 + 5x + 6} + c_1$$

$$\therefore -\frac{1}{2} \int \frac{1}{\sqrt{x^2 + 5x + 6}} dx$$

$$= -\frac{1}{2} \int \frac{dx}{\sqrt{\left(x + \frac{5}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$$

$$= -\frac{1}{2} \log \left| \left(x + \frac{5}{2}\right) + \sqrt{x^2 + 5x + 6} \right| + c_2$$

$$\therefore \frac{1}{2} \int \frac{2x + 5}{\sqrt{x^2 + 5x + 6}} dx - \frac{1}{2} \int \frac{dx}{\sqrt{x^2 + 5x + 6}}$$

$$= \sqrt{x^2 + 5x + 6} - \frac{1}{2} \log \left| \left(x + \frac{5}{2}\right) + \sqrt{x^2 + 5x + 6} \right| + c, \text{ c is the integrating constant}$$

## Exercise 14C

### 1. Question

Evaluate the following integrals:

$$\int \sqrt{4 - x^2} dx$$

**Answer**

To Find :  $\int \sqrt{4 - x^2} dx$

Now,  $\int \sqrt{4 - x^2} dx$  can be written as  $\int \sqrt{2^2 - x^2} dx$

Formula Used:  $\int \sqrt{a^2 - x^2} dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$

Since  $\int \sqrt{2^2 - x^2} dx$  is of the form  $\int \sqrt{a^2 - x^2} dx$ ,

$$\text{Hence, } \int \sqrt{2^2 - x^2} dx = \frac{1}{2} x \sqrt{2^2 - x^2} + \frac{2^2}{2} \sin^{-1} \frac{x}{2} + C$$

$$= \frac{1}{2} x \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} + C$$

$$= \frac{1}{2} x \sqrt{4 - x^2} + 2 \sin^{-1} \frac{x}{2} + C$$

$$\text{Therefore, } \int \sqrt{4 - x^2} dx = \frac{1}{2} x \sqrt{4 - x^2} + 2 \sin^{-1} \frac{x}{2} + C$$

### 2. Question

Evaluate the following integrals:

$$\int \sqrt{4 - 9x^2} dx$$

**Answer**

To Find :  $\int \sqrt{4 - 9x^2} dx$

Now,  $\int \sqrt{4 - 9x^2} dx$  can be written as  $\int \sqrt{2^2 - (3x)^2} dx$

Formula Used:  $\int \sqrt{a^2 - x^2} dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$

Since  $\int \sqrt{2^2 - (3x)^2} dx$  is of the form  $\int \sqrt{a^2 - x^2} dx$ ,



$$\text{Hence, } \int \sqrt{2^2 - (3x)^2} dx = \frac{1}{2}(3x)\sqrt{2^2 - (3x)^2} + \frac{2^2}{2} \sin^{-1} \frac{3x}{2} + C$$

$$= \frac{x}{2} \sqrt{4 - 9x^2} + \frac{4}{6} \sin^{-1} \frac{3x}{2} + C$$

$$= \frac{x}{2} \sqrt{4 - 9x^2} + \frac{2}{3} \sin^{-1} \frac{3x}{2} + C$$

$$\text{Therefore, } \int \sqrt{4 - 9x^2} dx = \frac{x}{2} \sqrt{4 - 9x^2} + \frac{2}{3} \sin^{-1} \frac{3x}{2} + C$$

### 3. Question

Evaluate the following integrals:

$$\int \sqrt{x^2 - 2} dx$$

#### Answer

$$\text{To Find : } \int \sqrt{x^2 - 2} dx$$

$$\text{Now, } \int \sqrt{x^2 - 2} dx \text{ can be written as } \int \sqrt{x^2 - (\sqrt{2})^2} dx$$

$$\text{Formula Used: } \int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + C$$

$$\text{Since } \int \sqrt{x^2 - (\sqrt{2})^2} dx \text{ is of the form } \int \sqrt{x^2 - a^2} dx ,$$

$$\text{Hence, } \int \sqrt{x^2 - (\sqrt{2})^2} dx = \frac{x}{2} \sqrt{x^2 - (\sqrt{2})^2} - \frac{(\sqrt{2})^2}{2} \log |x + \sqrt{x^2 - (\sqrt{2})^2}| + C$$

$$= \frac{x}{2} \sqrt{x^2 - 2} - \frac{2}{2} \log |x + \sqrt{x^2 - 2}| + C$$

$$= \frac{x}{2} \sqrt{x^2 - 2} - \log |x + \sqrt{x^2 - 2}| + C$$

$$\text{Therefore, } \int \sqrt{x^2 - 2} dx = \frac{x}{2} \sqrt{x^2 - 2} - \log |x + \sqrt{x^2 - 2}| + C$$

### 4. Question

Evaluate the following integrals:

$$\int \sqrt{2x^2 - 3} dx$$

#### Answer

$$\text{To Find : } \int \sqrt{2x^2 - 3} dx$$

$$\text{Now, } \int \sqrt{2x^2 - 3} dx \text{ can be written as } \int \sqrt{(\sqrt{2}x)^2 - (\sqrt{3})^2} dx$$

$$\text{Formula Used: } \int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + C$$

$$\text{Since } \int \sqrt{(\sqrt{2}x)^2 - (\sqrt{3})^2} dx \text{ is of the form } \int \sqrt{x^2 - a^2} dx ,$$

$$\text{Hence, } \int \sqrt{(\sqrt{2}x)^2 - (\sqrt{3})^2} dx = \frac{\sqrt{2}x}{2} \sqrt{(\sqrt{2}x)^2 - (\sqrt{3})^2} - \frac{(\sqrt{3})^2}{2} \log |\sqrt{2}x + \sqrt{(\sqrt{2}x)^2 - (\sqrt{3})^2}| + C$$

$$= \frac{\sqrt{2}x}{2} \sqrt{2x^2 - 3} - \frac{3}{2} \log |\sqrt{2}x + \sqrt{2x^2 - 3}| + C$$

$$= \frac{x}{2} \sqrt{2x^2 - 3} - \frac{3}{2\sqrt{2}} \log |\sqrt{2x} + \sqrt{2x^2 - 3}| + C$$

$$\text{Therefore, } \int \sqrt{2x^2 - 3} \, dx = \frac{x}{2} \sqrt{2x^2 - 3} - \frac{3}{2\sqrt{2}} \log |\sqrt{2x} + \sqrt{2x^2 - 3}| + C$$

## 5. Question

Evaluate the following integrals:

$$\int \sqrt{x^2 + 5} \, dx$$

### Answer

$$\text{To Find : } \int \sqrt{x^2 + 5} \, dx$$

$$\text{Now, } \int \sqrt{x^2 + 5} \, dx \text{ can be written as } \int \sqrt{x^2 + (\sqrt{5})^2} \, dx$$

$$\text{Formula Used: } \int \sqrt{x^2 + a^2} \, dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log |x + \sqrt{x^2 + a^2}| + C$$

$$\text{Since } \int \sqrt{x^2 + (\sqrt{5})^2} \, dx \text{ is of the form } \int \sqrt{x^2 + a^2} \, dx ,$$

$$\text{Hence, } \int \sqrt{x^2 + (\sqrt{5})^2} \, dx = \frac{x}{2} \sqrt{x^2 + (\sqrt{5})^2} + \frac{(\sqrt{5})^2}{2} \log |x + \sqrt{x^2 + (\sqrt{5})^2}| + C$$

$$= \frac{x}{2} \sqrt{x^2 + 5} + \frac{5}{2} \log |x + \sqrt{x^2 + 5}| + C$$

$$\text{Therefore, } \int \sqrt{x^2 + 5} \, dx = \frac{x}{2} \sqrt{x^2 + 5} + \frac{5}{2} \log |x + \sqrt{x^2 + 5}| + C$$

## 6. Question

Evaluate the following integrals:

$$\int \sqrt{4x^2 + 9} \, dx$$

### Answer

$$\text{To Find : } \int \sqrt{4x^2 + 9} \, dx$$

$$\text{Now, } \int \sqrt{4x^2 + 9} \, dx \text{ can be written as } \int \sqrt{(2x)^2 + 3^2} \, dx$$

$$\text{Formula Used: } \int \sqrt{x^2 + a^2} \, dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log |x + \sqrt{x^2 + a^2}| + C$$

$$\text{Since } \int \sqrt{(2x)^2 + 3^2} \, dx \text{ is of the form } \int \sqrt{x^2 + a^2} \, dx ,$$

$$\text{Hence, } \int \sqrt{(2x)^2 + 3^2} \, dx = \frac{2x}{2} \sqrt{(2x)^2 + 3^2} + \frac{3^2}{2} \log |2x + \sqrt{(2x)^2 + 3^2}| + C$$

$$= \frac{2x}{2} \sqrt{4x^2 + 9} + \frac{9}{2} \log |2x + \sqrt{4x^2 + 9}| + C$$

$$= \frac{x}{2} \sqrt{4x^2 + 9} + \frac{9}{4} \log |2x + \sqrt{4x^2 + 9}| + C$$

$$\text{Therefore, } \int \sqrt{4x^2 + 9} \, dx = \frac{x}{2} \sqrt{4x^2 + 9} + \frac{9}{4} \log |2x + \sqrt{4x^2 + 9}| + C$$

## 7. Question

Evaluate the following integrals:

$$\int \sqrt{3x^2 + 4} dx$$

**Answer**

To Find :  $\int \sqrt{3x^2 + 4} dx$

Now,  $\int \sqrt{3x^2 + 4} dx$  can be written as  $\int \sqrt{(\sqrt{3}x)^2 + 2^2} dx$

Formula Used:  $\int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log |x + \sqrt{x^2 + a^2}| + C$

Since  $\int \sqrt{(\sqrt{3}x)^2 + 2^2} dx$  is of the form  $\int \sqrt{x^2 + a^2} dx$ ,

Hence,  $\int \sqrt{(\sqrt{3}x)^2 + 2^2} dx = \frac{\sqrt{3}x}{2} \sqrt{(\sqrt{3}x)^2 + 2^2} + \frac{2^2}{2} \log |\sqrt{3}x + \sqrt{(\sqrt{3}x)^2 + 2^2}| + C$

$$= \frac{\sqrt{3}x}{2} \sqrt{3x^2 + 4} + \frac{4}{2} \log |\sqrt{3}x + \sqrt{3x^2 + 4}| + C$$

$$= \frac{x}{2} \sqrt{3x^2 + 4} + \frac{2}{\sqrt{3}} \log |\sqrt{3}x + \sqrt{3x^2 + 4}| + C$$

Therefore,  $\int \sqrt{3x^2 + 4} dx = \frac{x}{2} \sqrt{3x^2 + 4} + \frac{2}{\sqrt{3}} \log |\sqrt{3}x + \sqrt{3x^2 + 4}| + C$

## 8. Question

Evaluate the following integrals:

$$\int \cos x \sqrt{9 - \sin^2 x} dx$$

**Answer**

To Find :  $\int \cos x \sqrt{9 - \sin^2 x} dx$

Now, let  $\sin x = t$

$$\Rightarrow \cos x dx = dt$$

Therefore,  $\int \cos x \sqrt{9 - \sin^2 x} dx$  can be written as  $\int \sqrt{3^2 - t^2} dt$

Formula Used:  $\int \sqrt{a^2 - x^2} dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$

Since,  $\int \sqrt{3^2 - t^2} dt$  is in the form of  $\int \sqrt{a^2 - x^2} dx$  with  $t$  as a variable instead of  $x$ .

$$\Rightarrow \int \sqrt{3^2 - t^2} dt = \frac{1}{2} t \sqrt{3^2 - t^2} + \frac{3^2}{2} \sin^{-1} \frac{t}{3} + C$$

$$= \frac{t}{2} \sqrt{9 - t^2} + \frac{9}{2} \sin^{-1} \frac{t}{3} + C$$

Now since  $\sin x = t$  and  $\cos x dx = dt$

$$\Rightarrow \int \cos x \sqrt{9 - \sin^2 x} dx = \frac{\sin x}{2} \sqrt{9 - \sin^2 x} + \frac{9}{2} \sin^{-1} \left( \frac{\sin x}{3} \right) + C$$

## 9. Question

Evaluate the following integrals:

$$\int \sqrt{x^2 - 4x + 2} dx$$

**Answer**

To Find :  $\int \sqrt{x^2 - 4x + 2} dx$

Now,  $\int \sqrt{x^2 - 4x + 2} dx$  can be written as  $\int \sqrt{x^2 - 4x + 2^2 - 2^2 + 2} dx$

i.e.,  $\int \sqrt{(x-2)^2 - 2} dx$

Here , let  $x - 2 = y \Rightarrow dx = dy$

Therefore,  $\int \sqrt{(x-2)^2 - 2} dx$  can be written as  $\int \sqrt{y^2 - (\sqrt{2})^2} dy$

Formula Used:  $\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + C$

Since  $\int \sqrt{y^2 - (\sqrt{2})^2} dy$  is of the form  $\int \sqrt{x^2 - a^2} dx$  with change in variable.

$$\Rightarrow \int \sqrt{y^2 - (\sqrt{2})^2} dy = \frac{y}{2} \sqrt{y^2 - (\sqrt{2})^2} - \frac{(\sqrt{2})^2}{2} \log |y + \sqrt{y^2 - (\sqrt{2})^2}| + C$$

$$= \frac{y}{2} \sqrt{y^2 - 2} - \frac{4}{2} \log |y + \sqrt{y^2 - 2}| + C$$

$$= \frac{y}{2} \sqrt{y^2 - 2} - 2 \log |y + \sqrt{y^2 - 2}| + C$$

Since ,  $x - 2 = y$  and  $dx = dy$

$$\Rightarrow \int \sqrt{(x-2)^2 - 2} dx = \frac{(x-2)}{2} \sqrt{(x-2)^2 - 2} - 2 \log |(x-2) + \sqrt{(x-2)^2 - 2}| + C \text{ Therefore,}$$

$$\int \sqrt{x^2 - 4x + 2} dx = \frac{(x-2)}{2} \sqrt{x^2 - 4x + 2} - 2 \log |(x-2) + \sqrt{x^2 - 4x + 2}| + C$$

## 10. Question

Evaluate the following integrals:

$$\int \sqrt{x^2 + 6x - 4} dx$$

### Answer

To Find :  $\int \sqrt{x^2 + 6x - 4} dx$

Now,  $\int \sqrt{x^2 + 6x - 4} dx$  can be written as  $\int \sqrt{x^2 + 6x + 3^2 - 3^2 - 4} dx$

i.e.,  $\int \sqrt{(x+3)^2 - 13} dx$

Here , let  $x + 3 = y \Rightarrow dx = dy$

Therefore,  $\int \sqrt{(x+3)^2 - 13} dx$  can be written as  $\int \sqrt{y^2 - (\sqrt{13})^2} dy$

Formula Used:  $\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + C$

Since  $\int \sqrt{y^2 - (\sqrt{13})^2} dy$  is of the form  $\int \sqrt{x^2 - a^2} dx$  with change in variable.

$$\Rightarrow \int \sqrt{y^2 - (\sqrt{13})^2} dy = \frac{y}{2} \sqrt{y^2 - (\sqrt{13})^2} - \frac{(\sqrt{13})^2}{2} \log |y + \sqrt{y^2 - (\sqrt{13})^2}| + C$$

$$= \frac{y}{2} \sqrt{y^2 - 13} - \frac{13}{2} \log |y + \sqrt{y^2 - 13}| + C$$

Since ,  $x + 3 = y$  and  $dx = dy$

$$\Rightarrow \int \sqrt{(x+3)^2 - 13} dx = \frac{(x+3)}{2} \sqrt{(x+3)^2 - 13} - \frac{13}{2} \log |(x+3) + \sqrt{(x+3)^2 - 13}| + C$$

Therefore,

$$\int \sqrt{x^2 + 6x - 4} dx = \frac{(x+3)}{2} \sqrt{x^2 + 6x - 4} - \frac{13}{2} \log |(x+3) + \sqrt{x^2 + 6x - 4}| + C$$

### 11. Question

Evaluate the following integrals:

$$\int \sqrt{2x - x^2} dx$$

### Answer

To Find :  $\int \sqrt{2x - x^2} dx$

Now,  $\int \sqrt{2x - x^2} dx$  can be written as  $\int \sqrt{2x - x^2 - 1^2 + 1^2} dx$

i.e,  $\int \sqrt{1 - (x-1)^2} dx$

Let  $x - 1 = y \Rightarrow dx = dy$

Therefore ,  $\int \sqrt{1 - (x-1)^2} dx$  becomes  $\int \sqrt{1^2 - y^2} dy$

Formula Used:  $\int \sqrt{a^2 - x^2} dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$

Since  $\int \sqrt{1^2 - y^2} dy$  is of the form  $\int \sqrt{a^2 - x^2} dx$  with change in variable,

Hence  $\int \sqrt{1^2 - y^2} dy = \frac{1}{2} y \sqrt{1^2 - y^2} + \frac{1^2}{2} \sin^{-1} \frac{y}{1} + C$

$= \frac{y}{2} \sqrt{1 - y^2} + \frac{1}{2} \sin^{-1} \frac{y}{1} + C$

Here we have  $x - 1 = y$  and  $dx = dy$

$\Rightarrow \int \sqrt{1 - (x-1)^2} dx = \frac{(x-1)}{2} \sqrt{1 - (x-1)^2} + \frac{1}{2} \sin^{-1} \frac{(x-1)}{1} + C$

Therefore ,  $\int \sqrt{2x - x^2} dx = \frac{(x-1)}{2} \sqrt{2x - x^2} + \frac{1}{2} \sin^{-1}(x-1) + C$

### 12. Question

Evaluate the following integrals:

$$\int \sqrt{1 - 4x - x^2} dx$$

### Answer

To Find :  $\int \sqrt{1 - 4x - x^2} dx$

Now,  $\int \sqrt{1 - 4x - x^2} dx$  can be written as  $\int \sqrt{1 - 4x - x^2 - 2^2 + 2^2} dx$

i.e,  $\int \sqrt{5 - (x+2)^2} dx$

Let  $x + 2 = y \Rightarrow dx = dy$

Therefore ,  $\int \sqrt{5 - (x+2)^2} dx$  becomes  $\int \sqrt{(\sqrt{5})^2 - y^2} dy$

Formula Used:  $\int \sqrt{a^2 - x^2} dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$

Since  $\int \sqrt{(\sqrt{5})^2 - y^2} dy$  is of the form  $\int \sqrt{a^2 - x^2} dx$  with change in variable,

$$\begin{aligned}\text{Hence } \int \sqrt{(\sqrt{5})^2 - y^2} dy &= \frac{1}{2}y\sqrt{(\sqrt{5})^2 - y^2} + \frac{(\sqrt{5})^2}{2} \sin^{-1} \frac{y}{\sqrt{5}} + C \\ &= \frac{y}{2}\sqrt{5 - y^2} + \frac{5}{2} \sin^{-1} \frac{y}{\sqrt{5}} + C\end{aligned}$$

Here we have  $x + 2 = y$  and  $dx = dy$

$$\Rightarrow \int \sqrt{5 - (x + 2)^2} dx = \frac{(x+2)}{2} \sqrt{5 - (x + 2)^2} + \frac{5}{2} \sin^{-1} \left( \frac{x+2}{\sqrt{5}} \right) + C$$

$$\text{Therefore, } \int \sqrt{1 - 4x - x^2} dx = \frac{(x+2)}{2} \sqrt{1 - 4x - x^2} + \frac{5}{2} \sin^{-1} \left( \frac{x+2}{\sqrt{5}} \right) + C$$

### 13. Question

Evaluate the following integrals:

$$\int \sqrt{2ax - x^2} dx$$

#### Answer

To Find :  $\int \sqrt{2ax - x^2} dx$

Now,  $\int \sqrt{2ax - x^2} dx$  can be written as  $\int \sqrt{2ax - x^2 - a^2 + a^2} dx$

$$\text{i.e., } \int \sqrt{a^2 - (x - a)^2} dx$$

Let  $x - a = y \Rightarrow dx = dy$

Therefore,  $\int \sqrt{a^2 - (x - a)^2} dx$  becomes  $\int \sqrt{a^2 - y^2} dy$

$$\text{Formula Used: } \int \sqrt{a^2 - x^2} dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

Since  $\int \sqrt{a^2 - y^2} dy$  is of the form  $\int \sqrt{a^2 - x^2} dx$  with change in variable,

$$\begin{aligned}\text{Hence } \int \sqrt{a^2 - y^2} dy &= \frac{1}{2}y\sqrt{a^2 - y^2} + \frac{a^2}{2} \sin^{-1} \frac{y}{a} + C \\ &= \frac{y}{2}\sqrt{a^2 - y^2} + \frac{a^2}{2} \sin^{-1} \frac{y}{a} + C\end{aligned}$$

Here we have  $x - a = y$  and  $dx = dy$

$$\Rightarrow \int \sqrt{a^2 - (x - a)^2} dx = \frac{(x-a)}{2} \sqrt{a^2 - (x - a)^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{x-a}{a} \right) + C$$

$$\text{Therefore, } \int \sqrt{2ax - x^2} dx = \frac{(x-a)}{2} \sqrt{2ax - x^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{x-a}{a} \right) + C$$

### 14. Question

Evaluate the following integrals:

$$\int \sqrt{2x^2 + 3x + 4} dx$$

#### Answer

To Find :  $\int \sqrt{2x^2 + 3x + 4} dx$

$$\text{Now, consider } \int \sqrt{2x^2 + 3x + 4} dx = \int \sqrt{2\left[x^2 + \frac{3}{2}x + 2\right]} dx$$

$$= \sqrt{2} \int \sqrt{x^2 + \frac{3}{2}x + 2} dx$$

$$= \sqrt{2} \int \sqrt{x^2 + \frac{3}{2}x + \left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2 + 2} dx$$

$$= \sqrt{2} \int \sqrt{\left(x + \frac{3}{4}\right)^2 + \frac{23}{16}} dx$$

$$\text{Let } x + \frac{3}{4} = y \Rightarrow dx = dy$$

$$\text{Hence } \sqrt{2} \int \sqrt{\left(x + \frac{3}{4}\right)^2 + \frac{23}{16}} dx \text{ becomes } \sqrt{2} \int \sqrt{y^2 + \left(\frac{\sqrt{23}}{4}\right)^2} dy$$

$$\text{Formula Used: } \int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log |x + \sqrt{x^2 + a^2}| + C$$

Now consider  $\int \sqrt{y^2 + \left(\frac{\sqrt{23}}{4}\right)^2} dy$  which is in the form of  $\int \sqrt{x^2 + a^2} dx$  with change in variable.

$$\Rightarrow \int \sqrt{y^2 + \left(\frac{\sqrt{23}}{4}\right)^2} dy = \frac{y}{2} \sqrt{y^2 + \left(\frac{\sqrt{23}}{4}\right)^2} + \frac{\left(\frac{\sqrt{23}}{4}\right)^2}{2} \log |y + \sqrt{y^2 + \left(\frac{\sqrt{23}}{4}\right)^2}| + C$$

$$= \frac{y}{2} \sqrt{y^2 + \frac{23}{16}} + \frac{23}{32} \log |y + \sqrt{y^2 + \frac{23}{16}}| + C$$

$$\text{Since } x + \frac{3}{4} = y \text{ and } dx = dy$$

$$\Rightarrow \int \sqrt{\left(x + \frac{3}{4}\right)^2 + \frac{23}{16}} dx = \frac{1}{8} (4x + 3) \sqrt{\left(x + \frac{3}{4}\right)^2 + \frac{23}{16}} + \frac{23}{32} \log \left| x + \frac{3}{4} + \sqrt{\left(x + \frac{3}{4}\right)^2 + \frac{23}{16}} \right| + C$$

$$\text{Now, } \sqrt{2} \int \sqrt{\left(x + \frac{3}{4}\right)^2 + \frac{23}{16}} dx = \frac{\sqrt{2}}{8} (4x + 3) \sqrt{\left(x + \frac{3}{4}\right)^2 + \frac{23}{16}} + \frac{23\sqrt{2}}{32} \log \left| x + \frac{3}{4} + \sqrt{\left(x + \frac{3}{4}\right)^2 + \frac{23}{16}} \right| + C$$

Therefore,

$$\int \sqrt{2x^2 + 3x + 4} dx = \frac{1}{8} (4x + 3) \sqrt{2x^2 + 3x + 4} + \frac{23}{32} \log \left| \left(x + \frac{3}{4}\right) + \sqrt{2x^2 + 3x + 4} \right| + C$$

## 15. Question

Evaluate the following integrals:

$$\int \sqrt{x^2 + x} dx$$

### Answer

$$\text{To Find : } \int \sqrt{x^2 + x} dx$$

$$\text{Now, } \int \sqrt{x^2 + x} dx \text{ can be written as } \int \sqrt{x^2 + x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx$$

$$\text{i.e., } \int \sqrt{\left(x + \frac{1}{2}\right)^2 - \frac{1}{4}} dx$$

$$\text{Here, let } x + \frac{1}{2} = y \Rightarrow dx = dy$$

$$\text{Therefore, } \int \sqrt{\left(x + \frac{1}{2}\right)^2 - \frac{1}{4}} dx \text{ can be written as } \int \sqrt{y^2 - \left(\frac{1}{2}\right)^2} dy$$

$$\text{Formula Used: } \int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + C$$

Since  $\int \sqrt{y^2 - (\frac{1}{2})^2} dy$  is of the form  $\int \sqrt{x^2 - a^2} dx$  with change in variable.

$$\Rightarrow \int \sqrt{y^2 - (\frac{1}{2})^2} dy = \frac{y}{2} \sqrt{y^2 - (\frac{1}{2})^2} - \frac{(\frac{1}{2})^2}{2} \log |y + \sqrt{y^2 - (\frac{1}{2})^2}| + C$$

$$= \frac{y}{2} \sqrt{y^2 - \frac{1}{4}} - \frac{1}{8} \log |y + \sqrt{y^2 - \frac{1}{4}}| + C$$

Since ,  $x + \frac{1}{2} = y$  and  $dx = dy$

$$\Rightarrow \int \sqrt{(x + \frac{1}{2})^2 - \frac{1}{4}} dx = \frac{1}{4}(2x + 1) \sqrt{(x + \frac{1}{2})^2 - \frac{1}{4}} - \frac{1}{8} \log |(x + \frac{1}{2}) + \sqrt{(x + \frac{1}{2})^2 - \frac{1}{4}}| + C$$

Therefore,

$$\int \sqrt{x^2 + x} dx = \frac{1}{4}(2x + 1) \sqrt{x^2 + x} - \frac{1}{8} \log |x + \frac{1}{2} + \sqrt{x^2 + x}| + C$$

## 16. Question

Evaluate the following integrals:

$$\int \sqrt{x^2 + x + 1} dx$$

### Answer

To Find :  $\int \sqrt{x^2 + x + 1} dx$

Now,  $\int \sqrt{x^2 + x + 1} dx$  can be written as  $\int \sqrt{x^2 + x + (\frac{1}{2})^2 - (\frac{1}{2})^2 + 1} dx$

$$\text{i.e., } \int \sqrt{(x + \frac{1}{2})^2 + \frac{3}{4}} dx$$

Here , let  $x + \frac{1}{2} = y \Rightarrow dx = dy$

Therefore,  $\int \sqrt{(x + \frac{1}{2})^2 + \frac{3}{4}} dx$  can be written as  $\int \sqrt{y^2 + (\frac{\sqrt{3}}{2})^2} dy$

Formula Used:  $\int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log |x + \sqrt{x^2 + a^2}| + C$

Since  $\int \sqrt{y^2 + (\frac{\sqrt{3}}{2})^2} dy$  is of the form  $\int \sqrt{x^2 + a^2} dx$  with change in variable.

$$\Rightarrow \int \sqrt{y^2 + (\frac{\sqrt{3}}{2})^2} dy = \frac{y}{2} \sqrt{y^2 + (\frac{\sqrt{3}}{2})^2} + \frac{(\frac{\sqrt{3}}{2})^2}{2} \log |y + \sqrt{y^2 + (\frac{\sqrt{3}}{2})^2}| + C$$

$$= \frac{y}{2} \sqrt{y^2 + \frac{3}{4}} + \frac{3}{8} \log |y + \sqrt{y^2 + \frac{3}{4}}| + C$$

Since ,  $x + \frac{1}{2} = y$  and  $dx = dy$

$$\Rightarrow \int \sqrt{(x + \frac{1}{2})^2 + \frac{3}{4}} dx = \frac{1}{4}(2x + 1) \sqrt{(x + \frac{1}{2})^2 + \frac{3}{4}} + \frac{3}{8} \log |(x + \frac{1}{2}) + \sqrt{(x + \frac{1}{2})^2 + \frac{3}{4}}| + C$$

Therefore,

$$\int \sqrt{x^2 + x + 1} dx = \frac{1}{4}(2x + 1) \sqrt{x^2 + x + 1} + \frac{3}{8} \log |x + \frac{1}{2} + \sqrt{x^2 + x + 1}| + C$$

## 17. Question



Evaluate the following integrals:

$$\int (2x - 5)\sqrt{x^2 - 4x + 3} dx$$

**Answer**

To Find :  $\int (2x - 5)\sqrt{x^2 - 4x + 3} dx$

Now, let  $2x - 5$  be written as  $(2x - 4) - 1$  and split

Therefore ,

$$\begin{aligned}\int (2x - 5)\sqrt{x^2 - 4x + 3} dx &= \int \{(2x - 4)\sqrt{x^2 - 4x + 3} - 1\sqrt{x^2 - 4x + 3}\} dx \\ &= \int (2x - 4)\sqrt{x^2 - 4x + 3} dx - \int \sqrt{x^2 - 4x + 3} dx\end{aligned}$$

Now solving,  $\int (2x - 4)\sqrt{x^2 - 4x + 3} dx$

Let  $x^2 - 4x + 3 = u \Rightarrow dx = \frac{du}{(2x-4)}$

Thus,  $\int (2x - 4)\sqrt{x^2 - 4x + 3} dx$  becomes  $\int \sqrt{u} du$

Now ,  $\int \sqrt{u} du = \int u^{\frac{1}{2}} du = \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{2}{3} u^{\frac{3}{2}}$

$$= \frac{2}{3} (x^2 - 4x + 3)^{\frac{3}{2}}$$

Now solving,  $\int \sqrt{x^2 - 4x + 3} dx$

$$\begin{aligned}\int \sqrt{x^2 - 4x + 3} dx &= \int \sqrt{x^2 - 4x + 2^2 - 2^2 + 3} dx \\ &= \int \sqrt{(x - 2)^2 - 1} dx\end{aligned}$$

Let  $x - 2 = y \Rightarrow dx = dy$

Then  $\int \sqrt{(x - 2)^2 - 1} dx$  becomes  $\int \sqrt{y^2 - 1^2} dy$

Formula Used:  $\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + C$

Since  $\int \sqrt{y^2 - 1^2} dy$  is in the form of  $\int \sqrt{x^2 - a^2} dx$  with change in variable.

Hence  $\int \sqrt{y^2 - 1^2} dy = \frac{y}{2} \sqrt{y^2 - 1^2} - \frac{1^2}{2} \log |y + \sqrt{y^2 - 1^2}| + C$

$$= \frac{y}{2} \sqrt{y^2 - 1} - \frac{1}{2} \log |y + \sqrt{y^2 - 1}| + C$$

Now, since  $x - 2 = y$  and  $dx = dy$

$$\int \sqrt{(x - 2)^2 - 1} dx = \frac{(x-2)}{2} \sqrt{(x - 2)^2 - 1} - \frac{1}{2} \log |(x-2) + \sqrt{(x - 2)^2 - 1}| + C$$

Hence  $\int \sqrt{x^2 - 4x + 3} dx = \frac{(x-2)}{2} \sqrt{x^2 - 4x + 3} - \frac{1}{2} \log |(x-2) + \sqrt{x^2 - 4x + 3}| + C$

Therefore ,  $\int (2x - 4)\sqrt{x^2 - 4x + 3} dx - \int \sqrt{x^2 - 4x + 3} dx = \frac{2}{3} (x^2 - 4x + 3)^{\frac{3}{2}}$

$$- \frac{(x-2)}{2} \sqrt{x^2 - 4x + 3} + \frac{1}{2} \log |(x-2) + \sqrt{x^2 - 4x + 3}| + C$$

i. e,  $\int (2x - 5)\sqrt{x^2 - 4x + 3} dx = \frac{2}{3} (x^2 - 4x + 3)^{\frac{3}{2}}$

$$-\frac{(x-2)}{2}\sqrt{x^2-4x+3} + \frac{1}{2} \log |x-2 + \sqrt{x^2-4x+3}| + C$$

### 18. Question

Evaluate the following integrals:

$$\int (x+2)\sqrt{x^2+x+1} dx$$

### Answer

$$\text{To Find : } \int (x+2)\sqrt{x^2+x+1} dx$$

Now, let  $x+2$  be written as  $\frac{1}{2}(2x+1) + \frac{3}{2}$  and split

Therefore ,

$$\begin{aligned} \int (x+2)\sqrt{x^2+x+1} dx &= \int \left\{ \frac{(2x+1)\sqrt{x^2+x+1}}{2} + \frac{3}{2}\sqrt{x^2+x+1} \right\} dx \\ &= \frac{1}{2} \int (2x+1)\sqrt{x^2+x+1} dx + \frac{3}{2} \int \sqrt{x^2+x+1} dx \end{aligned}$$

$$\text{Now solving, } \frac{1}{2} \int (2x+1)\sqrt{x^2+x+1} dx$$

$$\text{Let } x^2+x+1 = u \Rightarrow dx = \frac{du}{(2x+1)}$$

$$\text{Thus, } \frac{1}{2} \int (2x+1)\sqrt{x^2+x+1} dx \text{ becomes } \frac{1}{2} \int \sqrt{u} du$$

$$\text{Now, } \frac{1}{2} \int \sqrt{u} du = \frac{1}{2} \int u^{\frac{1}{2}} du = \frac{1}{2} \left( \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right) = \frac{1}{3} u^{\frac{3}{2}}$$

$$= \frac{1}{3} (x^2+x+1)^{\frac{3}{2}}$$

$$\text{Now solving, } \int \sqrt{x^2+x+1} dx$$

$$\text{Now, } \int \sqrt{x^2+x+1} dx \text{ can be written as } \int \sqrt{x^2+x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1} dx$$

$$\text{i.e., } \int \sqrt{\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}} dx$$

$$\text{Here, let } x + \frac{1}{2} = y \Rightarrow dx = dy$$

$$\text{Therefore, } \int \sqrt{\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}} dx \text{ can be written as } \int \sqrt{y^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dy$$

$$\text{Formula Used: } \int \sqrt{x^2+a^2} dx = \frac{x}{2} \sqrt{x^2+a^2} + \frac{a^2}{2} \log |x + \sqrt{x^2+a^2}| + C$$

Since  $\int \sqrt{y^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dy$  is of the form  $\int \sqrt{x^2+a^2} dx$  with change in variable.

$$\Rightarrow \int \sqrt{y^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dy = \frac{y}{2} \sqrt{y^2 + \left(\frac{\sqrt{3}}{2}\right)^2} + \frac{\left(\frac{\sqrt{3}}{2}\right)^2}{2} \log |y + \sqrt{y^2 + \left(\frac{\sqrt{3}}{2}\right)^2}| + C$$

$$= \frac{y}{2} \sqrt{y^2 + \frac{3}{4}} + \frac{3}{8} \log |y + \sqrt{y^2 + \frac{3}{4}}| + C$$

$$\text{Since, } x + \frac{1}{2} = y \text{ and } dx = dy$$

$$\Rightarrow \int \sqrt{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}} dx = \frac{1}{4}(2x + 1) \sqrt{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}} + \frac{1}{8} \log \left| \left(x + \frac{1}{2}\right) + \sqrt{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}} \right| + C$$

Therefore,

$$\int \sqrt{x^2 + x + 1} dx = \frac{1}{4}(2x + 1) \sqrt{x^2 + x + 1} + \frac{3}{8} \log \left| x + \frac{1}{2} + \sqrt{x^2 + x + 1} \right| + C$$

Hence ,

$$\frac{1}{2} \int (2x + 1) \sqrt{x^2 + x + 1} dx + \frac{3}{2} \int \sqrt{x^2 + x + 1} dx = \frac{1}{3} (x^2 + x + 1)^{\frac{3}{2}} + \frac{3}{8} (2x + 1) \sqrt{x^2 + x + 1} + \frac{9}{16} \log \left| \left(x + \frac{1}{2}\right) + \sqrt{x^2 + x + 1} \right| + C$$

$$\text{Therefore , } \int (x + 2) \sqrt{x^2 + x + 1} dx = \frac{1}{3} (x^2 + x + 1)^{\frac{3}{2}} + \frac{3}{8} (2x + 1) \sqrt{x^2 + x + 1} + \frac{9}{16} \log \left| \left(x + \frac{1}{2}\right) + \sqrt{x^2 + x + 1} \right| + C$$

### 19. Question

Evaluate the following integrals:

$$\int (x - 5) \sqrt{x^2 + x} dx$$

### Answer

$$\text{To Find : } \int (x - 5) \sqrt{x^2 + x} dx$$

Now, let  $x - 5$  be written as  $\frac{1}{2}(2x + 1) - \frac{11}{2}$  and split

Therefore ,

$$\int (x - 5) \sqrt{x^2 + x} dx = \int \left\{ \frac{(2x + 1) \sqrt{x^2 + x}}{2} - \frac{11}{2} \sqrt{x^2 + x} \right\} dx$$

$$= \frac{1}{2} \int (2x + 1) \sqrt{x^2 + x} dx - \frac{11}{2} \int \sqrt{x^2 + x} dx$$

$$\text{Now solving, } \frac{1}{2} \int (2x + 1) \sqrt{x^2 + x} dx$$

$$\text{Let } x^2 + x = u \Rightarrow dx = \frac{du}{(2x + 1)}$$

$$\text{Thus, } \frac{1}{2} \int (2x + 1) \sqrt{x^2 + x} dx \text{ becomes } \frac{1}{2} \int \sqrt{u} du$$

$$\text{Now , } \frac{1}{2} \int \sqrt{u} du = \frac{1}{2} \int u^{\frac{1}{2}} du = \frac{1}{2} \left( \frac{u^{\frac{1}{2} + 1}}{\frac{1}{2} + 1} \right) = \frac{1}{3} u^{\frac{3}{2}}$$

$$= \frac{1}{3} (x^2 + x)^{\frac{3}{2}}$$

$$\text{Now solving, } \int \sqrt{x^2 + x} dx$$

$$\text{Now, } \int \sqrt{x^2 + x} dx \text{ can be written as } \int \sqrt{x^2 + x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx$$

$$\text{i.e., } \int \sqrt{\left(x + \frac{1}{2}\right)^2 - \frac{1}{4}} dx$$

$$\text{Here , let } x + \frac{1}{2} = y \Rightarrow dx = dy$$

$$\text{Therefore, } \int \sqrt{\left(x + \frac{1}{2}\right)^2 - \frac{1}{4}} dx \text{ can be written as } \int \sqrt{y^2 - \left(\frac{1}{2}\right)^2} dy$$

Formula Used:  $\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + C$

Since  $\int \sqrt{y^2 - (\frac{1}{2})^2} dy$  is of the form  $\int \sqrt{x^2 - a^2} dx$  with change in variable.

$$\Rightarrow \int \sqrt{y^2 - (\frac{1}{2})^2} dy = \frac{y}{2} \sqrt{y^2 - (\frac{1}{2})^2} - \frac{(\frac{1}{2})^2}{2} \log |y + \sqrt{y^2 - (\frac{1}{2})^2}| + C$$

$$= \frac{y}{2} \sqrt{y^2 - \frac{1}{4}} - \frac{1}{8} \log |y + \sqrt{y^2 - \frac{1}{4}}| + C$$

Since,  $x + \frac{1}{2} = y$  and  $dx = dy$

$$\Rightarrow \int \sqrt{(x + \frac{1}{2})^2 - \frac{1}{4}} dx = \frac{1}{4} (2x + 1) \sqrt{(x + \frac{1}{2})^2 - \frac{1}{4}} - \frac{1}{8} \log |(x + \frac{1}{2}) + \sqrt{(x + \frac{1}{2})^2 - \frac{1}{4}}| + C$$

Therefore,

$$\int \sqrt{x^2 + x} dx = \frac{1}{4} (2x + 1) \sqrt{x^2 + x} - \frac{1}{8} \log |x + \frac{1}{2} + \sqrt{x^2 + x}| + C$$

Now ,

$$\frac{1}{2} \int (2x + 1) \sqrt{x^2 + x} dx - \frac{11}{2} \int \sqrt{x^2 + x} dx = \frac{1}{3} (x^2 + x)^{\frac{3}{2}} - \frac{11}{8} (2x + 1) \sqrt{x^2 + x} + \frac{11}{16} \log |x + \frac{1}{2} + \sqrt{x^2 + x}| + C$$

Therefore ,

$$\int (x - 5) \sqrt{x^2 + x} dx = \frac{1}{3} (x^2 + x)^{\frac{3}{2}} - \frac{11}{8} (2x + 1) \sqrt{x^2 + x} + \frac{11}{16} \log |x + \frac{1}{2} + \sqrt{x^2 + x}| + C$$

## 20. Question

Evaluate the following integrals:

$$\int (4x + 1) \sqrt{x^2 - x - 2} dx$$

### Answer

To Find :  $\int (4x + 1) \sqrt{x^2 - x - 2} dx$

Now, let  $4x + 1$  be written as  $2(2x - 1) + 3$  and split

Therefore ,

$$\int (4x + 1) \sqrt{x^2 - x - 2} dx = \int \{2(2x - 1) \sqrt{x^2 - x - 2} + 3 \sqrt{x^2 - x - 2}\} dx$$

$$= 2 \int (2x - 1) \sqrt{x^2 - x - 2} dx + 3 \int \sqrt{x^2 - x - 2} dx$$

Now solving,  $2 \int (2x - 1) \sqrt{x^2 - x - 2} dx$

$$\text{Let } x^2 - x - 2 = u \Rightarrow dx = \frac{du}{(2x-1)}$$

Thus,  $2 \int (2x - 1) \sqrt{x^2 - x - 2} dx$  becomes  $2 \int \sqrt{u} du$

$$\text{Now , } 2 \int \sqrt{u} du = 2 \int u^{\frac{1}{2}} du = 2 \left( \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right) = \frac{4}{3} u^{\frac{3}{2}}$$

$$= \frac{4}{3} (x^2 - x - 2)^{\frac{3}{2}}$$

Now solving,  $\int \sqrt{x^2 - x - 2} dx$

Now,  $\int \sqrt{x^2 - x - 2} dx$  can be written as  $\int \sqrt{x^2 - x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 2} dx$

$$\text{i.e., } \int \sqrt{\left(x - \frac{1}{2}\right)^2 - \frac{9}{4}} dx$$

Here, let  $x - \frac{1}{2} = y \Rightarrow dx = dy$

Therefore,  $\int \sqrt{\left(x - \frac{1}{2}\right)^2 - \frac{9}{4}} dx$  can be written as  $\int \sqrt{y^2 - \left(\frac{3}{2}\right)^2} dy$

Formula Used:  $\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + C$

Since  $\int \sqrt{y^2 - \left(\frac{3}{2}\right)^2} dy$  is of the form  $\int \sqrt{x^2 - a^2} dx$  with change in variable.

$$\Rightarrow \int \sqrt{y^2 - \left(\frac{3}{2}\right)^2} dy = \frac{y}{2} \sqrt{y^2 - \left(\frac{3}{2}\right)^2} - \frac{\left(\frac{3}{2}\right)^2}{2} \log |y + \sqrt{y^2 - \left(\frac{3}{2}\right)^2}| + C$$

$$= \frac{y}{2} \sqrt{y^2 - \frac{9}{4}} - \frac{9}{8} \log |y + \sqrt{y^2 - \frac{9}{4}}| + C$$

Since,  $x - \frac{1}{2} = y$  and  $dx = dy$

$$\Rightarrow \int \sqrt{\left(x - \frac{1}{2}\right)^2 - \frac{9}{4}} dx = \frac{1}{4}(2x - 1) \sqrt{\left(x - \frac{1}{2}\right)^2 - \frac{9}{4}} - \frac{9}{8} \log \left| \left(x - \frac{1}{2}\right) + \sqrt{\left(x - \frac{1}{2}\right)^2 - \frac{9}{4}} \right| + C$$

Therefore,

$$\int \sqrt{x^2 - x - 2} dx = \frac{1}{4}(2x - 1) \sqrt{x^2 - x - 2} - \frac{9}{8} \log \left| x - \frac{1}{2} + \sqrt{x^2 - x - 2} \right| + C$$

Hence,

$$2 \int (2x - 1) \sqrt{x^2 - x - 2} dx + 3 \int \sqrt{x^2 - x - 2} dx = \frac{4}{3}(x^2 - x - 2)^{\frac{3}{2}} + \frac{3}{4}(2x - 1) \sqrt{x^2 - x - 2} - \frac{27}{8} \log \left| x - \frac{1}{2} + \sqrt{x^2 - x - 2} \right| + C$$

Therefore,

$$\int (4x + 1) \sqrt{x^2 - x - 2} dx = \frac{4}{3}(x^2 - x - 2)^{\frac{3}{2}} + \frac{3}{4}(2x - 1) \sqrt{x^2 - x - 2} - \frac{27}{8} \log \left| x - \frac{1}{2} + \sqrt{x^2 - x - 2} \right| + C$$

## 21. Question

Evaluate the following integrals:

$$\int (x + 1) \sqrt{2x^2 + 3} dx$$

**Answer**

To Find :  $\int (x + 1) \sqrt{2x^2 + 3} dx$

Now,  $\int (x + 1) \sqrt{2x^2 + 3} dx$  can be written as

$$\int (x + 1) \sqrt{2x^2 + 3} dx = \int \{x \sqrt{2x^2 + 3} + \sqrt{2x^2 + 3}\} dx$$

$$= \int x \sqrt{2x^2 + 3} dx + \int \sqrt{2x^2 + 3} dx$$

Now solving,  $\int x \sqrt{2x^2 + 3} dx$

$$\text{Let } 2x^2 + 3 = u \Rightarrow dx = \frac{1 du}{4x}$$

Thus,  $\int x\sqrt{2x^2+3}dx$  becomes  $\frac{1}{4}\int \sqrt{u} du$

$$\begin{aligned}\text{Now, } \frac{1}{4}\int \sqrt{u} du &= \frac{1}{4}\int u^{\frac{1}{2}} du = \frac{1}{4}\left(\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right) = \frac{1}{6}u^{\frac{3}{2}} \\ &= \frac{1}{6}(2x^2+3)^{\frac{3}{2}}\end{aligned}$$

Now solving,  $\int \sqrt{2x^2+3}dx$

Now,  $\int \sqrt{2x^2+3} dx$  can be written as  $\int \sqrt{(\sqrt{2}x)^2 + (\sqrt{3})^2} dx$

Formula Used:  $\int \sqrt{x^2+a^2} dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log|x+\sqrt{x^2+a^2}|+C$

Since  $\int \sqrt{2x^2+3} dx$  is of the form  $\int \sqrt{x^2+a^2} dx$ .

$$\begin{aligned}\Rightarrow \int \sqrt{2x^2+3} dx &= \frac{\sqrt{2}x}{2}\sqrt{(\sqrt{2}x)^2 + (\sqrt{3})^2} + \frac{(\sqrt{3})^2}{2}\log|\sqrt{2}x + \sqrt{(\sqrt{2}x)^2 + (\sqrt{3})^2}| + C \\ &= \frac{x}{2}\sqrt{2x^2+3} + \frac{3}{2\sqrt{2}}\log|\sqrt{2}x + \sqrt{2x^2+3}| + C\end{aligned}$$

Therefore,

$$\int x\sqrt{2x^2+3}dx + \int \sqrt{2x^2+3}dx = \frac{1}{6}(2x^2+3)^{\frac{3}{2}} + \frac{x}{2}\sqrt{2x^2+3} + \frac{3}{2\sqrt{2}}\log|\sqrt{2}x + \sqrt{2x^2+3}| + C$$

Hence ,

$$\int (x+1)\sqrt{2x^2+3} dx = \frac{1}{6}(2x^2+3)^{\frac{3}{2}} + \frac{x}{2}\sqrt{2x^2+3} + \frac{3}{2\sqrt{2}}\log|\sqrt{2}x + \sqrt{2x^2+3}| + C$$

## 22. Question

Evaluate the following integrals:

$$\int x\sqrt{1+x-x^2} dx$$

### Answer

To Find :  $\int x\sqrt{1+x-x^2} dx$

Now, let x be written as  $\frac{1}{2} - \frac{1}{2}(1-2x)$  and split

Therefore ,

$$\begin{aligned}\int x\sqrt{1+x-x^2} dx &= \int \left\{ \frac{\sqrt{-x^2+x+1}}{2} - \frac{(1-2x)\sqrt{-x^2+x+1}}{2} \right\} dx \\ &= \frac{1}{2} \int (2x-1)\sqrt{-x^2+x+1} dx + \frac{1}{2} \int \sqrt{-x^2+x+1} dx\end{aligned}$$

Now solving,  $\frac{1}{2} \int (2x-1)\sqrt{-x^2+x+1} dx$

$$\text{Let } -x^2+x+1 = u \Rightarrow dx = \frac{du}{(1-2x)}$$

Thus,  $\frac{1}{2} \int (2x-1)\sqrt{-x^2+x+1} dx$  becomes  $-\frac{1}{2} \int \sqrt{u} du$

$$\text{Now, } -\frac{1}{2} \int \sqrt{u} du = -\frac{1}{2} \int u^{\frac{1}{2}} du = -\frac{1}{2} \left( \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right) = -\frac{1}{3} u^{\frac{3}{2}}$$

$$= -\frac{1}{3}(-x^2 + x + 1)^{\frac{3}{2}}$$

Now solving,  $\int \sqrt{-x^2 + x + 1} dx$

$$\int \sqrt{-x^2 + x + 1} dx \text{ can be written as } \int \sqrt{-x^2 + x - \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + 1} dx$$

$$\text{i.e., } \int \sqrt{\frac{5}{4} - (x - \frac{1}{2})^2} dx = \frac{1}{2} \int \sqrt{5 - (2x - 1)^2} dx$$

$$\text{let } 2x - 1 = y \Rightarrow dx = \frac{1dy}{2}$$

$$\text{Therefore, } \frac{1}{4} \int \sqrt{5 - (2x - 1)^2} dx \text{ becomes } \frac{1}{4} \int \sqrt{(\sqrt{5})^2 - y^2} dy$$

$$\text{Formula Used: } \int \sqrt{a^2 - x^2} dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

Since  $\int \sqrt{(\sqrt{5})^2 - y^2} dy$  is of the form  $\int \sqrt{a^2 - x^2} dx$  with change in variable .

$$\text{Hence, } \int \sqrt{(\sqrt{5})^2 - y^2} dy = \frac{1}{2} y \sqrt{(\sqrt{5})^2 - y^2} + \frac{(\sqrt{5})^2}{2} \sin^{-1} \frac{y}{\sqrt{5}} + C$$

$$= \frac{1}{2} y \sqrt{5 - y^2} + \frac{5}{2} \sin^{-1} \frac{y}{\sqrt{5}} + C$$

$$\text{Since, } 2x - 1 = y \text{ and } dx = \frac{1dy}{2}$$

Therefore,

$$\frac{1}{4} \int \sqrt{5 - (2x - 1)^2} dx = \frac{1}{8} (2x - 1) \sqrt{5 - (2x - 1)^2} + \frac{5}{8} \sin^{-1} \frac{(2x-1)}{\sqrt{5}} + C$$

$$\text{i.e., } \int \sqrt{-x^2 + x + 1} dx = \frac{1}{8} (2x - 1) \sqrt{-x^2 + x + 1} + \frac{5}{8} \sin^{-1} \frac{(2x-1)}{\sqrt{5}} + C$$

$$\text{hence, } \int x \sqrt{1 + x - x^2} dx = \frac{1}{2} \int (2x - 1) \sqrt{-x^2 + x + 1} dx + \frac{1}{2} \int \sqrt{-x^2 + x + 1} dx = -\frac{1}{3} (-x^2 + x + 1)^{\frac{3}{2}} + \frac{1}{16} (2x - 1) \sqrt{-x^2 + x + 1} + \frac{5}{16} \sin^{-1} \left( \frac{2x-1}{\sqrt{5}} \right) + C$$

### 23. Question

Evaluate the following integrals:

**Answer**

$$\text{To Find : } \int (2x - 5) \sqrt{2 + 3x - x^2} dx \int (2x - 5) \sqrt{2 + 3x - x^2} dx$$

Now, let  $2x - 5$  be written as  $(2x - 3) - 2$  and split

Therefore ,

$$\int (2x - 5) \sqrt{2 + 3x - x^2} dx = \int \{(2x - 3) \sqrt{-x^2 + 3x + 2} - 2 \sqrt{-x^2 + 3x + 2}\} dx$$

$$= \int (2x - 3) \sqrt{-x^2 + 3x + 2} dx - 2 \int \sqrt{-x^2 + 3x + 2} dx$$

$$\text{Now solving, } \int (2x - 3) \sqrt{-x^2 + 3x + 2} dx$$

$$\text{Let } -x^2 + 3x + 2 = u \Rightarrow dx = \frac{du}{(3-2x)}$$

$$\text{Thus, } \int (2x - 3) \sqrt{-x^2 + 3x + 2} dx \text{ becomes } - \int \sqrt{u} du$$

$$\text{Now, } -\int \sqrt{u} \, du = -\int u^{\frac{1}{2}} \, du = -\left(\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right) = -\frac{2}{3} u^{\frac{3}{2}}$$

$$= -\frac{2}{3} (-x^2 + 3x + 2)^{\frac{3}{2}}$$

Now solving,  $\int \sqrt{-x^2 + 3x + 2} \, dx$

$$\int \sqrt{-x^2 + 3x + 2} \, dx \text{ can be written as } \int \sqrt{-x^2 + 3x - \left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^2 + 2} \, dx$$

$$\text{i.e., } \int \sqrt{\frac{17}{4} - \left(x - \frac{3}{2}\right)^2} \, dx$$

$$\text{let } x - \frac{3}{2} = y \Rightarrow dx = dy$$

$$\text{Therefore, } \int \sqrt{\frac{17}{4} - \left(x - \frac{3}{2}\right)^2} \, dx \text{ becomes } \int \sqrt{\left(\frac{\sqrt{17}}{2}\right)^2 - y^2} \, dy$$

$$\text{Formula Used: } \int \sqrt{a^2 - x^2} \, dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

Since  $\int \sqrt{\left(\frac{\sqrt{17}}{2}\right)^2 - y^2} \, dy$  is of the form  $\int \sqrt{a^2 - x^2} \, dx$  with change in variable .

$$\text{Hence, } \int \sqrt{\left(\frac{\sqrt{17}}{2}\right)^2 - y^2} \, dy = \frac{1}{2} y \sqrt{\left(\frac{\sqrt{17}}{2}\right)^2 - y^2} + \frac{\left(\frac{\sqrt{17}}{2}\right)^2}{2} \sin^{-1} \frac{y}{\frac{\sqrt{17}}{2}} + C$$

$$= \frac{1}{2} y \sqrt{\frac{17}{4} - y^2} + \frac{17}{8} \sin^{-1} \frac{y}{\frac{\sqrt{17}}{2}} + C$$

$$\text{Since, } x - \frac{3}{2} = y \text{ and } dx = dy$$

Therefore,

$$\int \sqrt{\frac{17}{4} - \left(x - \frac{3}{2}\right)^2} \, dx = \frac{1}{4} (2x - 3) \sqrt{\frac{17}{4} - \left(x - \frac{3}{2}\right)^2} + \frac{17}{8} \sin^{-1} \left(\frac{2x-3}{\sqrt{17}}\right) + C$$

$$\text{i.e., } \int \sqrt{-x^2 + 3x + 2} \, dx = \frac{1}{4} (2x - 3) \sqrt{-x^2 + 3x + 2} + \frac{17}{8} \sin^{-1} \left(\frac{2x-3}{\sqrt{17}}\right) + C$$

hence ,

$$\int (2x - 5) \sqrt{2 + 3x - x^2} \, dx = \int (2x - 3) \sqrt{-x^2 + 3x + 2} \, dx - 2 \int \sqrt{-x^2 + 3x + 2} \, dx = -\frac{2}{3} (-x^2 + 3x + 2)^{\frac{3}{2}} - \frac{1}{2} (2x - 3) \sqrt{-x^2 + 3x + 2} - \frac{17}{4} \sin^{-1} \left(\frac{2x-3}{\sqrt{17}}\right) + C$$

## 24. Question

Evaluate the following integrals:

$$\int (6x + 5) \sqrt{6 + x - 2x^2} \, dx$$

**Answer**

$$\text{To Find : } \int (6x + 5) \sqrt{6 + x - 2x^2} \, dx$$

$$\text{Now, let } 6x + 5 \text{ be written as } \frac{13}{2} - \frac{3}{2}(1 - 4x) \text{ and split}$$

Therefore ,

$$\int (6x + 5) \sqrt{6 + x - 2x^2} \, dx = \int \left\{ \frac{13\sqrt{-2x^2+x+6}}{2} - \frac{3(1-4x)\sqrt{-2x^2+x+6}}{2} \right\} dx$$



$$= \frac{3}{2} \int (4x-1)\sqrt{-2x^2+x+6} dx + \frac{13}{2} \int \sqrt{-2x^2+x+6} dx$$

Now solving,  $\int (4x-1)\sqrt{-2x^2+x+6} dx$

$$\text{Let } -2x^2 + x + 6 = u \Rightarrow dx = \frac{du}{(1-4x)}$$

Thus,  $\int (4x-1)\sqrt{-2x^2+x+6} dx$  becomes  $-\int \sqrt{u} du$

$$\text{Now, } -\int \sqrt{u} du = -\int u^{\frac{1}{2}} du = -\left(\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right) = -\frac{2}{3} u^{\frac{3}{2}}$$

$$= -\frac{2}{3} (-2x^2 + x + 6)^{\frac{3}{2}}$$

Now solving,  $\int \sqrt{-2x^2+x+6} dx$

$$\int \sqrt{-2x^2+x+6} dx \text{ can be written as } \int \sqrt{-(\sqrt{2}x)^2 + x - \left(\frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{1}{2\sqrt{2}}\right)^2 + 6} dx$$

$$\text{i.e., } \int \sqrt{\frac{49}{8} - (\sqrt{2}x - \frac{1}{2\sqrt{2}})^2} dx$$

$$\text{let } \sqrt{2}x - \frac{1}{2\sqrt{2}} = y \Rightarrow dx = \frac{dy}{\sqrt{2}}$$

$$\text{Therefore, } \int \sqrt{\frac{49}{8} - (\sqrt{2}x - \frac{1}{2\sqrt{2}})^2} dx \text{ becomes } \int \sqrt{\left(\frac{7}{2\sqrt{2}}\right)^2 - y^2} dy$$

$$\text{Formula Used: } \int \sqrt{a^2 - x^2} dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

Since  $\int \sqrt{\left(\frac{7}{2\sqrt{2}}\right)^2 - y^2} dy$  is of the form  $\int \sqrt{a^2 - x^2} dx$  with change in variable .

$$\text{Hence, } \int \sqrt{\left(\frac{7}{2\sqrt{2}}\right)^2 - y^2} dy = \frac{1}{2} y \sqrt{\left(\frac{7}{2\sqrt{2}}\right)^2 - y^2} + \frac{\left(\frac{7}{2\sqrt{2}}\right)^2}{2} \sin^{-1} \frac{y}{\frac{7}{2\sqrt{2}}} + C$$

$$= \frac{1}{2} y \sqrt{\frac{49}{8} - y^2} + \frac{7}{16} \sin^{-1} \frac{y}{\frac{7}{2\sqrt{2}}} + C$$

$$\text{Since, } \sqrt{2}x - \frac{1}{2\sqrt{2}} = y \text{ and } dx = \frac{dy}{\sqrt{2}}$$

Therefore,

$$\int \sqrt{\frac{49}{8} - (\sqrt{2}x - \frac{1}{2\sqrt{2}})^2} dx = \frac{1}{4\sqrt{2}} (4x-1) \sqrt{\frac{49}{8} - (\sqrt{2}x - \frac{1}{2\sqrt{2}})^2} + \frac{49}{16} \sin^{-1} \left(\frac{4x-1}{7}\right) + C$$

$$\text{i.e., } \int \sqrt{-2x^2+x+6} dx = \frac{1}{4\sqrt{2}} (4x-1) \sqrt{-2x^2+x+6} + \frac{49}{16} \sin^{-1} \left(\frac{4x-1}{7}\right) + C$$

hence ,

$$\int (6x+5)\sqrt{6+x-2x^2} dx = \frac{3}{2} \int (4x-1)\sqrt{-2x^2+x+6} dx + \frac{13}{2} \int \sqrt{-2x^2+x+6} dx = -(-2x^2+x+6)^{\frac{3}{2}} + \frac{13}{16} (4x-1) \sqrt{-2x^2+x+6} + \frac{637}{32\sqrt{2}} \sin^{-1} \left(\frac{4x-1}{7}\right) + C$$

## 25. Question

Evaluate the following integrals:

$$\int (x+1)\sqrt{1-x-x^2} dx$$

**Answer**

To Find :  $\int (x+1)\sqrt{1-x-x^2} dx$

Now, let  $x+1$  be written as  $\frac{1}{2} - \frac{1}{2}(-2x-1)$  and split

Therefore ,

$$\begin{aligned}\int (x+1)\sqrt{1-x-x^2} dx &= \int \left\{ \frac{\sqrt{-x^2-x+1}}{2} - \frac{(-2x-1)\sqrt{-x^2-x+1}}{2} \right\} dx \\ &= \frac{1}{2} \int (2x-1)\sqrt{-x^2-x+1} dx + \frac{1}{2} \int \sqrt{-x^2-x+1} dx\end{aligned}$$

Now solving,  $\int (2x-1)\sqrt{-x^2-x+1} dx$

$$\text{Let } -x^2 - x + 1 = u \Rightarrow dx = \frac{du}{-2x-1}$$

Thus,  $\int (2x-1)\sqrt{-x^2-x+1} dx$  becomes  $-\int \sqrt{u} du$

$$\text{Now , } -\int \sqrt{u} du = -\int u^{\frac{1}{2}} du = -\left( \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right) = -\frac{2}{3} u^{\frac{3}{2}}$$

$$= -\frac{2}{3} (-x^2 - x + 1)^{\frac{3}{2}}$$

Now solving,  $\int \sqrt{-x^2-x+1} dx$

$$\int \sqrt{-x^2-x+1} dx \text{ can be written as } \int \sqrt{-x^2-x-\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + 1} dx$$

$$\text{i.e., } \int \sqrt{\frac{5}{4} - \left(x + \frac{1}{2}\right)^2} dx$$

$$\text{let } x + \frac{1}{2} = y \Rightarrow dx = dy$$

$$\text{Therefore , } \int \sqrt{\frac{5}{4} - \left(x + \frac{1}{2}\right)^2} dx \text{ becomes } \int \sqrt{\left(\frac{\sqrt{5}}{2}\right)^2 - y^2} dy$$

$$\text{Formula Used: } \int \sqrt{a^2 - x^2} dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

Since  $\int \sqrt{\left(\frac{\sqrt{5}}{2}\right)^2 - y^2} dy$  is of the form  $\int \sqrt{a^2 - x^2} dx$  with change in variable .

$$\text{Hence, } \int \sqrt{\left(\frac{\sqrt{5}}{2}\right)^2 - y^2} dy = \frac{1}{2} y \sqrt{\left(\frac{\sqrt{5}}{2}\right)^2 - y^2} + \frac{\left(\frac{\sqrt{5}}{2}\right)^2}{2} \sin^{-1} \frac{y}{\frac{\sqrt{5}}{2}} + C$$

$$= \frac{1}{2} y \sqrt{\frac{5}{4} - y^2} + \frac{5}{8} \sin^{-1} \frac{y}{\frac{\sqrt{5}}{2}} + C$$

$$\text{Since , } x + \frac{1}{2} = y \text{ and } dx = dy$$

Therefore,

$$\int \sqrt{\frac{5}{4} - \left(x + \frac{1}{2}\right)^2} dx = \frac{1}{4} (2x+1) \sqrt{\frac{5}{4} - \left(x + \frac{1}{2}\right)^2} + \frac{5}{8} \sin^{-1} \left( \frac{2x+1}{\sqrt{5}} \right) + C$$

$$\text{i.e., } \int \sqrt{-x^2-x+1} dx = \frac{1}{4} (2x+1) \sqrt{-x^2-x+1} + \frac{5}{8} \sin^{-1} \left( \frac{2x+1}{\sqrt{5}} \right) + C$$

hence ,

$$\int (x+1)\sqrt{1-x-x^2} dx = \frac{1}{2} \int (2x-1)\sqrt{-x^2-x+1} dx + \frac{1}{2} \int \sqrt{-x^2-x+1} dx = -\frac{1}{3} (-x^2-x+1)^{\frac{3}{2}} +$$

$$\frac{1}{8}(2x+1)\sqrt{-x^2-x+1} + \frac{5}{16} \sin^{-1}\left(\frac{2x+1}{\sqrt{5}}\right) + C$$

## 26. Question

Evaluate the following integrals:

$$\int (x-3)\sqrt{x^2+3x-18} dx$$

## Answer

$$\text{To Find : } \int (x-3)\sqrt{x^2+3x-18} dx$$

Now, let  $x-3$  be written as  $\frac{1}{2}(2x+3) - \frac{9}{2}$  and split

Therefore ,

$$\int (x-3)\sqrt{x^2+3x-18} dx = \int \left\{ \frac{(2x+3)\sqrt{x^2+3x-18}}{2} - \frac{9\sqrt{x^2+3x-18}}{2} \right\} dx$$

$$= \frac{1}{2} \int (2x+3)\sqrt{x^2+3x-18} dx - \frac{9}{2} \int \sqrt{x^2+3x-18} dx$$

$$\text{Now solving, } \int (2x+3)\sqrt{x^2+3x-18} dx$$

$$\text{Let } x^2+3x-18 = u \Rightarrow dx = \frac{du}{2x+3}$$

$$\text{Thus, } \int (2x+3)\sqrt{x^2+3x-18} dx \text{ becomes } \int \sqrt{u} du$$

$$\text{Now, } \int \sqrt{u} du = \int u^{\frac{1}{2}} du = \left( \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right) = \frac{2}{3} u^{\frac{3}{2}}$$

$$= \frac{2}{3} (x^2+3x-18)^{\frac{3}{2}}$$

$$\text{Now solving, } \int \sqrt{x^2+3x-18} dx$$

$$\int \sqrt{x^2+3x-18} dx \text{ can be written as } \int \sqrt{x^2+3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 18} dx$$

$$\text{i.e., } \int \sqrt{\left(x+\frac{3}{2}\right)^2 - \frac{81}{4}} dx$$

$$\text{let } x+\frac{3}{2} = y \Rightarrow dx = dy$$

$$\text{Therefore, } \int \sqrt{\left(x+\frac{3}{2}\right)^2 - \frac{81}{4}} dx \text{ can be written as } \int \sqrt{y^2 - \left(\frac{9}{2}\right)^2} dy$$

$$\text{Formula Used: } \int \sqrt{x^2-a^2} dx = \frac{x}{2} \sqrt{x^2-a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2-a^2}| + C$$

Since  $\int \sqrt{y^2 - \left(\frac{9}{2}\right)^2} dy$  is of the form  $\int \sqrt{x^2 - a^2} dx$  with change in variable.

$$\Rightarrow \int \sqrt{y^2 - \left(\frac{9}{2}\right)^2} dy = \frac{y}{2} \sqrt{y^2 - \left(\frac{9}{2}\right)^2} - \frac{\left(\frac{9}{2}\right)^2}{2} \log |y + \sqrt{y^2 - \left(\frac{9}{2}\right)^2}| + C$$

$$= \frac{y}{2} \sqrt{y^2 - \frac{81}{4}} - \frac{81}{8} \log |y + \sqrt{y^2 - \frac{81}{4}}| + C$$

$$\text{Since, } x + \frac{3}{2} = y \text{ and } dx = dy$$

$$\Rightarrow \int \sqrt{\left(x + \frac{3}{2}\right)^2 - \frac{81}{4}} dx = \frac{1}{4}(2x + 3) \sqrt{\left(x + \frac{3}{2}\right)^2 - \frac{81}{4}} - \frac{81}{8} \log \left| \left(x + \frac{3}{2}\right) + \sqrt{\left(x + \frac{3}{2}\right)^2 - \frac{81}{4}} \right| + C$$

Therefore,

$$\int \sqrt{x^2 + 3x - 18} dx = \frac{1}{4}(2x + 3) \sqrt{x^2 + 3x - 18} - \frac{81}{8} \log \left| x + \frac{3}{2} + \sqrt{x^2 + 3x - 18} \right| + C$$

Hence ,

$$\begin{aligned} \int (x - 3) \sqrt{x^2 + 3x - 18} dx &= \frac{1}{2} \int (2x + 3) \sqrt{x^2 + 3x - 18} dx - \frac{9}{2} \int \sqrt{x^2 + 3x - 18} dx = \\ &= \frac{1}{3} (x^2 + 3x - 18)^{\frac{3}{2}} - \frac{9}{8} (2x + 3) \sqrt{x^2 + 3x - 18} + \frac{726}{16} \log \left| x + \frac{3}{2} + \sqrt{x^2 + 3x - 18} \right| + C \end{aligned}$$