Exercise 16.1

Chapter 16 Vector Calculus Exercise 16.1 1E

Consider the following vector field:

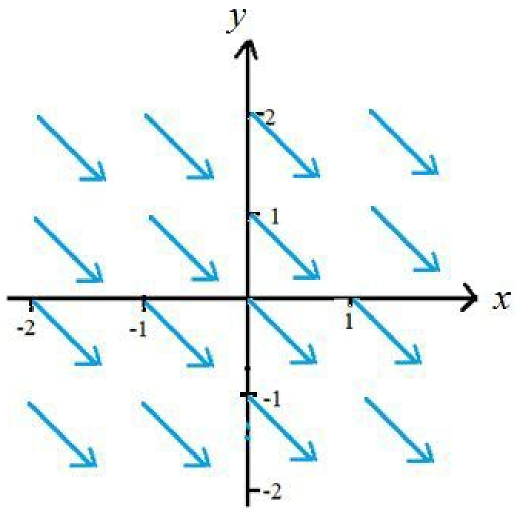
$\mathbf{F}(x, y) = 0.3\mathbf{i} - 0.4\mathbf{j}.$

Observe the vector field $\mathbf{F}(x, y)$, for every (x, y) on xy-plane represents the vector $\langle 0.3, -0.4 \rangle$.

The corresponding vectors represent a vector field.

A vector field on $\mathbb{R}^2$ is a function **F** that assigns to each point (x, y) in D a two-dimensional vector $\mathbf{F}(x, y)$.

Sketch the vector field of $\mathbf{F}(x, y) = 0.3\mathbf{i} - 0.4\mathbf{j}.$



$F(x, y) = \langle 0.3, -0.4 \rangle$

Chapter 16 Vector Calculus Exercise 16.1 2E

Consider the vector field,

$\mathbf{F}(x, y) = \frac{1}{2}x\mathbf{i} + y\mathbf{j}.$

The objective is to sketch the vector filed $\mathbf{F}$.

Observe the vector field $\mathbf{F}(x, y)$, such that every (x, y) point on xy-plane represents the vector $\left\langle \frac{1}{2}x, y \right\rangle$.

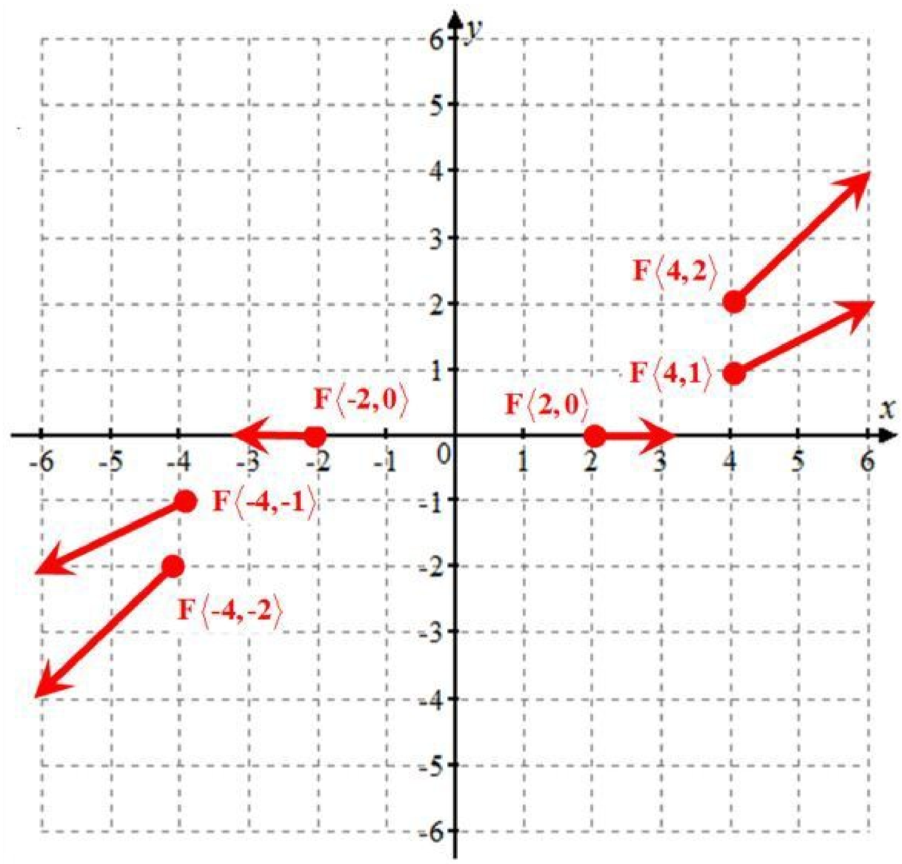
Since $\mathbf{F}(2, 0) = \frac{1}{2}(2)\mathbf{i} + (0)\mathbf{j} = \langle 1, 0 \rangle$, we draw the vector $\mathbf{i}$ starting at the point $(2, 0)$ in figure. Since $\mathbf{F}(0, 1) = \frac{1}{2}(0)\mathbf{i} + (1)\mathbf{j} = \langle 0, 1 \rangle$, we draw the vector $\mathbf{j}$ starting at the point $(0, 1)$

Proceed in the similar way to calculate several other representative values of $\mathbf{F}(x, y)$.

Tabulate and draw the corresponding vectors to represent the vector field.

(x,y)	$\mathbf{F}(x,y)$	(x,y)	$\mathbf{F}(x,y)$
$(2,0)$	$\langle 1,0\rangle$	$(-2,0)$	$\langle -1,0\rangle$
$(4,1)$	$\langle 2,1\rangle$	$(-4,-1)$	$\langle -2,-1\rangle$
$(4,2)$	$\langle 2,2\rangle$	$(-4,-2)$	$\langle -2,-2\rangle$

The above tabular vectors can be plotted in a rectangular coordinate system as follows:



Plot more number of tabular vectors, to obtain more precise vector field for the given vector function $\mathbf{F}(x, y) = \frac{1}{2}x\mathbf{i} + y\mathbf{j}$.

It is difficult to plot the vector field for more number of vectors by hand;

The vector field for the vector function plotted by using computer is shown below.

Type the following command in maple and then press ENTER.

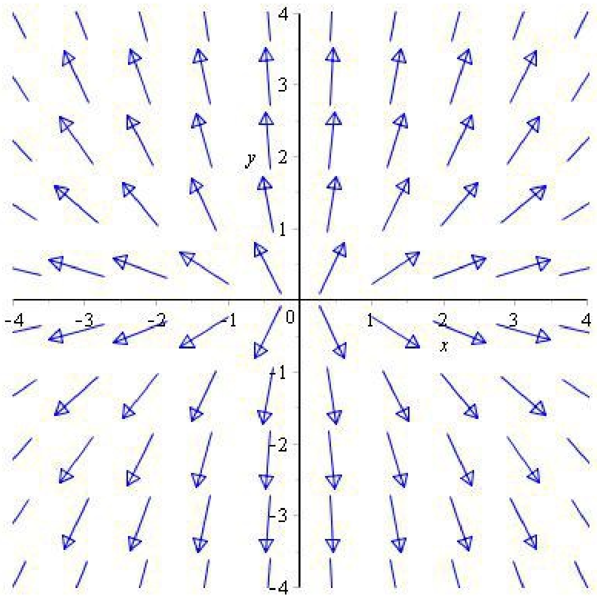
with(Student[VectorCalculus]) :

with(Student[VectorCalculus]):

$VectorField\left(\left\langle \frac{x}{2}, y\right\rangle, output = plot, view = [-4..4, -4..4], scaling = constrained, color = \text{"blue"}, fieldoptions = [fieldstrength = fixed, arrows = SLIM, grid = [10, 10]]\right);$

VectorField(`<,>`((1/2)*x, y), output = plot, view = [-4 .. 4, -4 .. 4], scaling = constrained, color = "blue", fieldoptions = [fieldstrength = fixed, arrows = SLIM, grid = [10, 10]]);

Press enter to view the output:



Chapter 16 Vector Calculus Exercise 16.1 3E

Consider the vector field

$$\mathbf{F}(x, y) = -\frac{1}{2}\mathbf{i} + (y - x)\mathbf{j}$$

Observe the vector field $\mathbf{F}(x,y)$, for every (x,y) on xy-plane represents the vector $\left\langle -\frac{1}{2}, y-x \right\rangle$.

Since $\mathbf{F}(0,0) = -\frac{1}{2}\mathbf{i}$, we draw the vector $\left\langle -\frac{1}{2}, 0 \right\rangle$ starting at the point $(0,0)$

Since $\mathbf{F}(1,0) = -\frac{1}{2}\mathbf{i} - \mathbf{j}$, we draw the vector $\left\langle -\frac{1}{2}, -1 \right\rangle$ starting at the point $(1,0)$

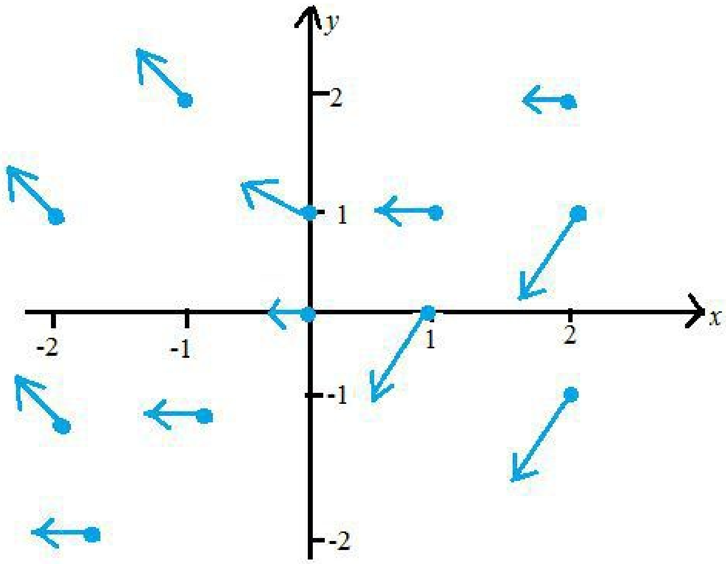
Continuing in this way, calculate several other representative values of $\mathbf{F}(x,y)$ in the table and draw the corresponding vectors to represent the vector field.

Table for the values of $\mathbf{F}(x,y)$

(x,y)	$\mathbf{F}(x,y)$
$(0,0)$	$\left\langle -\frac{1}{2}, 0 \right\rangle$
$(1,0)$	$\left\langle -\frac{1}{2}, -1 \right\rangle$
$(0,1)$	$\left\langle -\frac{1}{2}, 1 \right\rangle$
$(2,-1)$	$\left\langle -\frac{1}{2}, -3 \right\rangle$
$(-1,2)$	$\left\langle -\frac{1}{2}, 3 \right\rangle$
$(1,1)$	$\left\langle -\frac{1}{2}, 0 \right\rangle$

(x,y)	$\mathbf{F}(x,y)$
$(2,2)$	$\left\langle -\frac{1}{2}, 0 \right\rangle$
$(-2,-2)$	$\left\langle -\frac{1}{2}, 0 \right\rangle$
$(-1,-1)$	$\left\langle -\frac{1}{2}, 0 \right\rangle$
$(-2,-1)$	$\left\langle -\frac{1}{2}, 1 \right\rangle$
$(-2,1)$	$\left\langle -\frac{1}{2}, 3 \right\rangle$
$(2,1)$	$\left\langle -\frac{1}{2}, -1 \right\rangle$

Sketch the vectors corresponding to the points in xy-plane



Hence the vector filed for vector $\mathbf{F}(x, y) = -\frac{1}{2}\mathbf{i} + (y - x)\mathbf{j}$.

Chapter 16 Vector Calculus Exercise 16.1 4E

Consider the vector field,

$\mathbf{F}(x, y) = y\mathbf{i} + (x + y)\mathbf{j}$

The objective is to sketch the vector filed $\mathbf{F}$.

Observe the vector field $\mathbf{F}(x, y)$, such that every (x, y) point on xy-plane represents the vector $\langle y, x+y \rangle$.

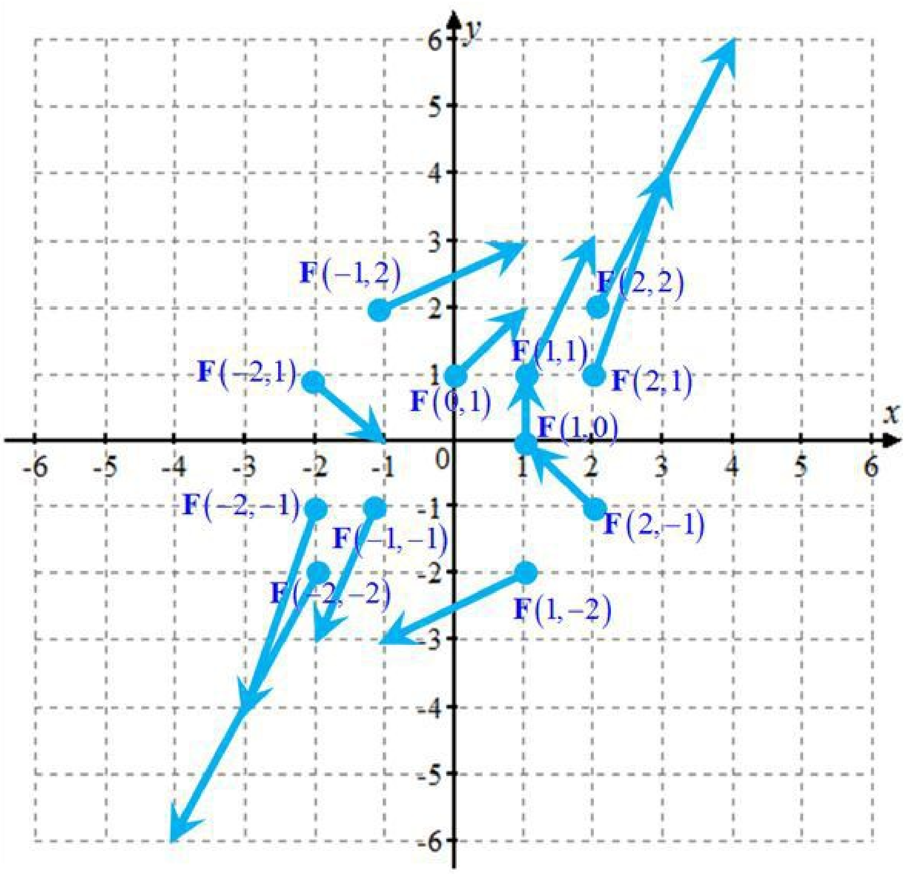
Since $\mathbf{F}(1,0) = \mathbf{j}$, we draw the vector $\mathbf{j} = \langle 0,1 \rangle$ starting at the point $(1,0)$ in figure. Since $\mathbf{F}(0,1) = \mathbf{i} + \mathbf{j}$, we draw the vector $\langle 1,1 \rangle$ starting at the point $(0,1)$

Proceed in the similar way to calculate several other representative values of $\mathbf{F}(x, y)$. Tabulate and draw the corresponding vectors to represent the vector field.

(x,y)	$\mathbf{F}(x,y)$
$(1,-2)$	$\langle -2,-1 \rangle$
$(1,0)$	$\langle 0,1 \rangle$
$(0,1)$	$\langle 1,1 \rangle$
$(2,-1)$	$\langle -1,1 \rangle$
$(-1,2)$	$\langle 2,1 \rangle$
$(1,1)$	$\langle 1,2 \rangle$

(x,y)	$\mathbf{F}(x,y)$
$(2,2)$	$\langle 2,4 \rangle$
$(-2,-2)$	$\langle -2,-4 \rangle$
$(-1,-1)$	$\langle -1,-2 \rangle$
$(-2,-1)$	$\langle -1,-3 \rangle$
$(-2,1)$	$\langle 1,-1 \rangle$
$(2,1)$	$\langle 1,3 \rangle$

The above tabular vectors can be plotted in a rectangular coordinate system as follows:



Plot more number of tabular vectors, to obtain more precise vector field for the given vector function $\mathbf{F}(x, y) = y\mathbf{i} + (x + y)\mathbf{j}$

It is difficult to plot the vector field for more number of vectors by hand;

The vector field for the vector function plotted by using computer is shown below.

Type the following command in maple and then press ENTER.

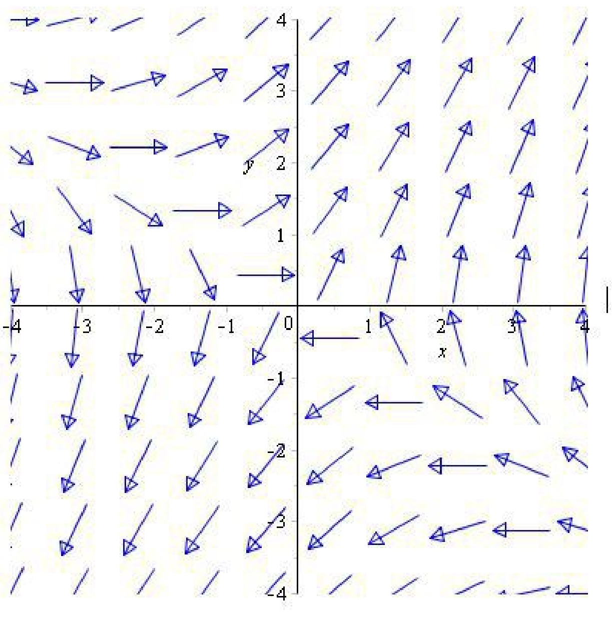
with(Student[VectorCalculus]) :

with(Student[VectorCalculus]):

VectorField(⟨*y*, *x* + *y*⟩, *output* = *plot*, *view* = [−4 ..4, −4 ..4], *scaling* = *constrained*, *color* = "blue", *fieldoptions* = [*fieldstrength* = *fixed*, *arrows* = *SLIM*, *grid* = [10, 10]]);

VectorField(`<,>`(y, x+y), output = plot, view = [-4 .. 4, -4 .. 4], scaling = constrained, color = "blue", fieldoptions = [fieldstrength = fixed, arrows = SLIM, grid = [10, 10]])

Press enter to view the output:



Chapter 16 Vector Calculus Exercise 16.1 5E

Consider the vector field: $\mathbf{F}(x,y)=\dfrac{y\,\mathbf{i}+x\,\mathbf{j}}{\sqrt{x^2+y^2}}$.

Need to sketch the vector field $\mathbf{F}$ by drawing a diagram.

Since $\mathbf{F}(1,0)=\mathbf{j}$, draw the vector $\mathbf{j}=\langle 0,1\rangle$ starting at the point $(1,0)$ and move one unit upwards along the positive y-direction as shown in the figure below.

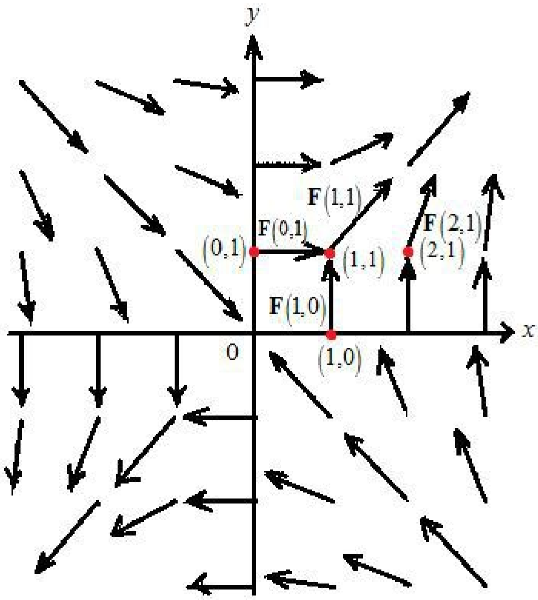
Since $\mathbf{F}(0,1)=\mathbf{i}$, draw the vector $\mathbf{i}=\langle 1,0\rangle$ starting at the point $(0,1)$ and move one unit right along the positive x-direction as shown in the figure below.

Since $\mathbf{F}(2,1)=0.447\mathbf{i}+0.894\mathbf{j}$, draw the vector $0.447\mathbf{i}+0.894\mathbf{j}=\langle 0.447,0.894\rangle$ starting at the point $(2,1)$ and move 0.447 units right along the positive x-direction and then move 0.894 units upwards along the positive y-direction as shown in the figure below.

Continuing in this way, calculate several other representative values of $\mathbf{F}(x,y)$ in the table and draw the corresponding vectors to represent the vector field in the below figure.

(x,y)	$\mathbf{F}(x,y)$	(x,y)	$\mathbf{F}(x,y)$
$(1,0)$	$\langle 0,1\rangle$	$(2,2)$	$\langle 0.7071,0.7071\rangle$
$(2,0)$	$\langle 0,1\rangle$	$(2,-1)$	$\langle -0.447,0.894\rangle$
$(-1,0)$	$\langle 0,-1\rangle$	$(2,-2)$	$\langle -0.7071,0.7071\rangle$
$(-2,0)$	$\langle 0,-1\rangle$	$(1,-1)$	$\langle -0.7071,0.7071\rangle$
$(0,1)$	$\langle 1,0\rangle$	$(1,-2)$	$\langle -0.894,0.447\rangle$
$(0,2)$	$\langle 1,0\rangle$	$(-1,-1)$	$\langle -0.7071,-0.7071\rangle$
$(0,-1)$	$\langle -1,0\rangle$	$(-1,-2)$	$\langle -0.894,-0.447\rangle$
$(0,-2)$	$\langle -1,0\rangle$	$(-2,-1)$	$\langle -0.447,-0.894\rangle$
$(1,1)$	$\langle 0.7071,0.7071\rangle$	$(-2,-2)$	$\langle -0.7071,0.7071\rangle$
$(1,2)$	$\langle 0.894,0.447\rangle$	$(-1,1)$	$\langle 0.7071,-0.7071\rangle$
$(2,1)$	$\langle 0.447,0.894\rangle$	$(-1,2)$	$\langle 0.894,-0.447\rangle$
$(-2,2)$	$\langle 0.7071,-0.7071\rangle$	$(-2,1)$	$\langle 0.447,-0.894\rangle$

Graph of the vector field $\mathbf{F}$ is shown below:



Chapter 16 Vector Calculus Exercise 16.1 6E

Consider $\mathbf{F}(x,y)=\frac{y\,\mathbf{i}-x\,\mathbf{j}}{\sqrt{x^2+y^2}}$

To draw the vector field

Since $\mathbf{F}(1,0)=-\mathbf{j}$, we draw the vector $\mathbf{j}=<0,-1>$ starting at the point $(1,0)$ and $\mathbf{F}(2,0)=-\mathbf{j}$, we draw the vector $\mathbf{j}=<0,-1>$ starting at the point. Continuing in this way, we calculate several other reprehensive values of $\mathbf{F}(x,y)$ in the below table and draw the corresponding vectors to represent the vector field.

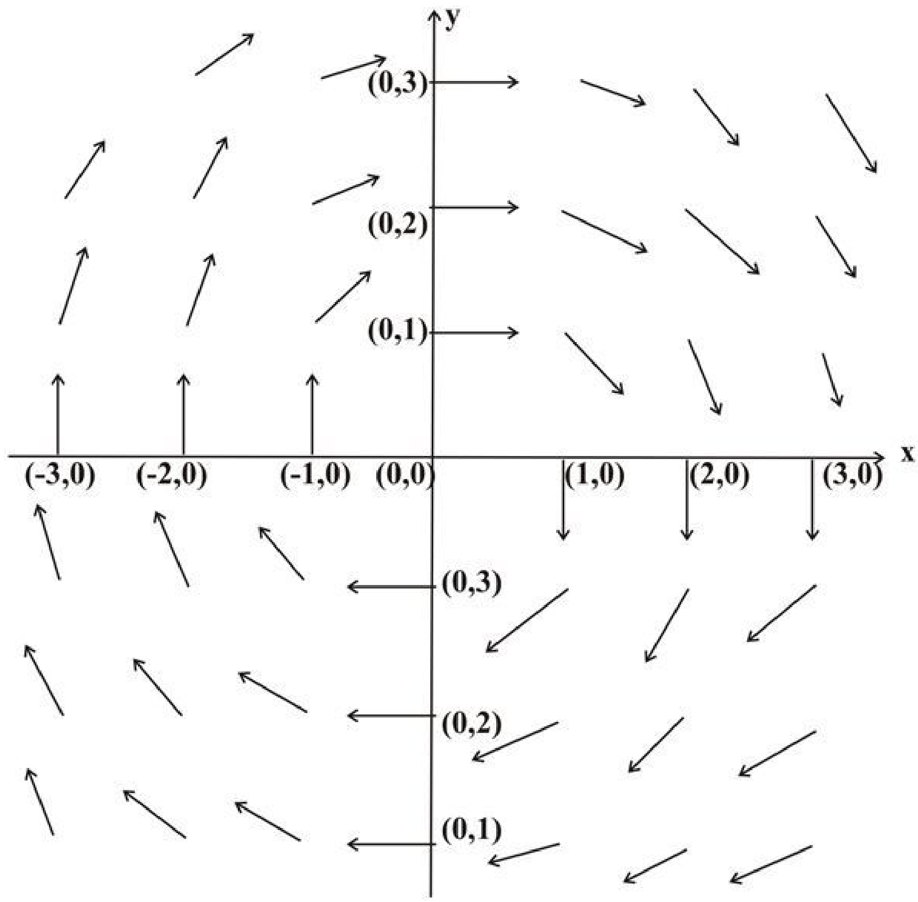
The table

(x,y)	$\mathbf{F}(x,y)$	(x,y)	$\mathbf{F}(x,y)$
$(1,0)$	$<0,-1>$	$(2,2)$	$<1/\sqrt{2},-1/\sqrt{2}>$
$(2,0)$	$<0,-1>$	$(2,-1)$	$<-1/\sqrt{5},-2/\sqrt{5}>$
$(-1,0)$	$<0,1>$	$(2,-2)$	$<-1/\sqrt{2},-1/\sqrt{2}>$
$(-2,0)$	$<0,1>$	$(1,-1)$	$<-1/\sqrt{2},-1/\sqrt{2}>$
$(0,1)$	$<1,0>$	$(1,-2)$	$<-2/\sqrt{5},-1/\sqrt{5}>$
$(0,2)$	$<1,0>$	$(-1,-1)$	$<-1/\sqrt{2},1/\sqrt{2}>$
$(0,-1)$	$<-1,0>$	$(-1,-2)$	$<-2/\sqrt{5},1/\sqrt{5}>$
$(0,-2)$	$<-1,0>$	$(-2,-1)$	$<-1/\sqrt{5},2/\sqrt{5}>$

Continue to the above table

$(1,1)$	$< 1/\sqrt{2}, -1/\sqrt{2} >$	$(-2,-2)$	$< -1/\sqrt{2}, 1/\sqrt{2} >$
$(1,2)$	$< 2/\sqrt{5}, -1/\sqrt{5} >$	$(-1,1)$	$< 1/\sqrt{2}, 1/\sqrt{2} >$
$(2,1)$	$< 1/\sqrt{5}, -2/\sqrt{5} >$	$(-1,2)$	$< 2/\sqrt{5}, 1/\sqrt{5} >$
$(-2,2)$	$< 1/\sqrt{2}, 1/\sqrt{2} >$	$(-2,1)$	$< 1/\sqrt{5}, 2/\sqrt{5} >$
$(3,1)$	$< 1/\sqrt{10}, -3/\sqrt{10} >$	$(-3,-1)$	$< -1/\sqrt{10}, 3/\sqrt{10} >$
$(3,2)$	$< 2/\sqrt{13}, -3/\sqrt{13} >$	$(-3,-2)$	$< -2/\sqrt{13}, 3/\sqrt{13} >$
$(3,3)$	$< 1/\sqrt{2}, -1/\sqrt{2} >$	$(-3,-3)$	$< -1/\sqrt{2}, 1/\sqrt{2} >$
$(-3,1)$	$< 1/\sqrt{10}, 3/\sqrt{10} >$	$(0,3)$	$< 1,0 >$
$(-3,2)$	$< 2/\sqrt{13}, 3/\sqrt{13} >$	$(0,-3)$	$< -1,0 >$
$(-3,3)$	$< 1/\sqrt{2}, 1/\sqrt{2} >$	$(3,0)$	$< 0,-1 >$
$(3,-1)$	$< -1/\sqrt{10}, 3/\sqrt{10} >$	$(-3,0)$	$< 0,1 >$
$(3,-2)$	$< -2/\sqrt{13}, -3/\sqrt{13} >$	$(1,3)$	$< 3/\sqrt{10}, -1/\sqrt{10} >$
$(3,-3)$	$< -1/\sqrt{2}, -1/\sqrt{2} >$	$(1,-3)$	$< -3/\sqrt{10}, -1/\sqrt{10} >$

The sketch of vector field by hand



Some computer algebra systems are capable of plotting vector fields in two or three dimensions. They give a better impression of the vector field than is possible by hand because the computer can plot a large number of representative vectors.

Maple software commands for plot the vector field of $\mathbf{F}(x,y) = \dfrac{y\,\mathbf{i} - x\,\mathbf{j}}{\sqrt{x^2+y^2}}$

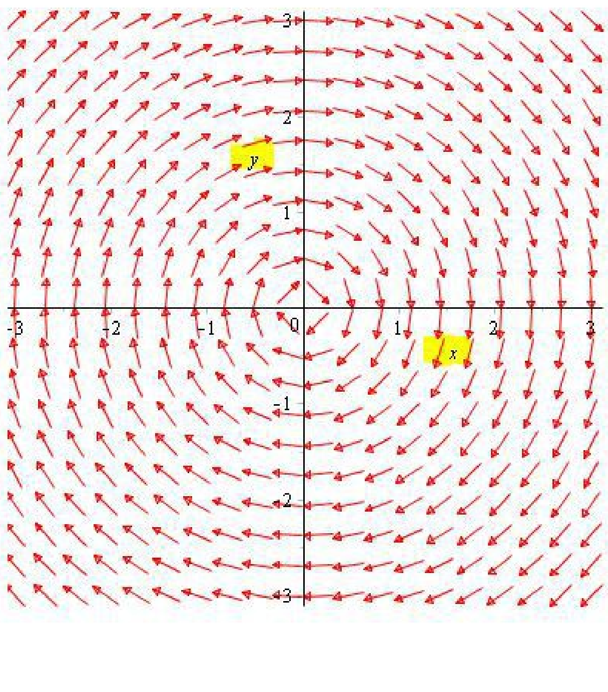
with(*plots*);

with(*VectorCalculus*);

$f := \left[\dfrac{y}{\mathrm{sqrt}(x^2+y^2)}, \dfrac{-x}{\mathrm{sqrt}(x^2+y^2)}\right];$

fieldplot(*f*, *x* = -3 ..3, *y* = -3 ..3, *arrows* = *slim*, *color* = *red*);

The sketch of the vector field using Maple software



Chapter 16 Vector Calculus Exercise 16.1 7E

Consider the vector field $\mathbf{F}(x,y,z)=\mathbf{k}$

Rewrite the vector field as $\mathbf{F}=\langle 0,0,1\rangle$

The vector field is independent of variables x, y and z.

The z component is 1(positive constant).

Thus, all vectors are vertical and pointing upward direction, with unit magnitude.

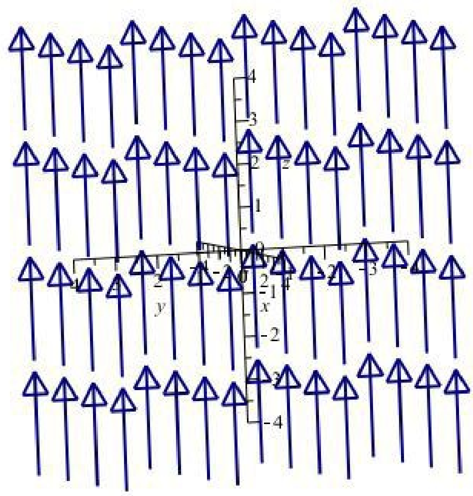
Use maple software to sketch the vector field $\mathbf{F}(x,y,z)=\mathbf{k}$.

Maple input:

VectorField(`<,>`(0, 0, 1), output = plot, view = [-4 .. 4, -4 .. 4, -4 .. 4], scaling = constrained, color = "NavyBlue", fieldoptions = [fieldstrength = fixed, arrows = SLIM, grid = [4, 4, 4]])

Maple output:

> *VectorField*(⟨0, 0, 1⟩, *output* = *plot*, *view* = [−4 ..4, −4 ..4, −4 ..4], *scaling* = *constrained*, *color* = "NavyBlue", *fieldoptions* = [*fieldstrength* = *fixed*, *arrows* = *SLIM*, *grid* = [4, 4, 4]]);



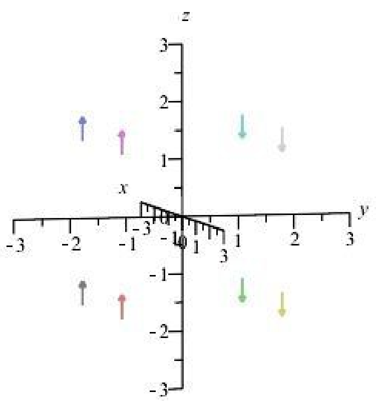
Chapter 16 Vector Calculus Exercise 16.1 8E

Consider the following vector field:

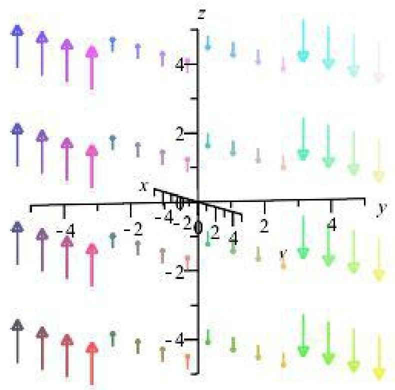
$\mathbf{F}(x,y,z)=-y\mathbf{k}$

A vector filed on $\mathbb{R}^3$ is a function $\mathbf{F}$, that assigns to each point (x,y,z) in E which is a three – dimensional vector $\mathbf{F}(x,y,z)$.

Sketch the diagram as shown below:



The sketch of the diagram with other intervals is shown below:



All vectors are vertical and the length of vector field $\mathbf{F}(x, y, z)$ is $|y|$.

For $y > 0$, all points are moving downward direction and for $y < 0$, all points are moving upward direction.

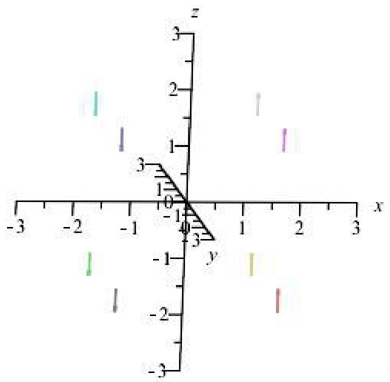
Chapter 16 Vector Calculus Exercise 16.1 9E

Consider the vector field $\mathbf{F}(x, y, z) = x\mathbf{k}$

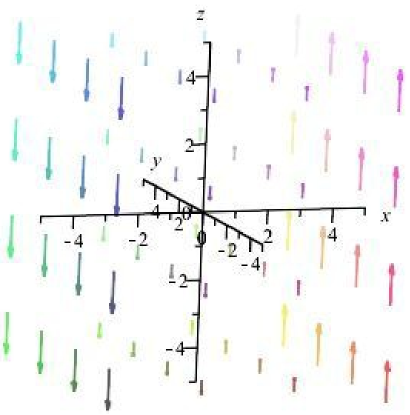
Sketch the vector filed $\mathbf{F}$.

A vector filed on $\mathbb{R}^3$ is a function $\mathbf{F}$ that assigns to each point (x, y, z) in E a three – dimensional vector $\mathbf{F}(x, y, z)$.

The sketch of the diagram is as shown below.



The sketch of the diagram with other intervals is shown below.



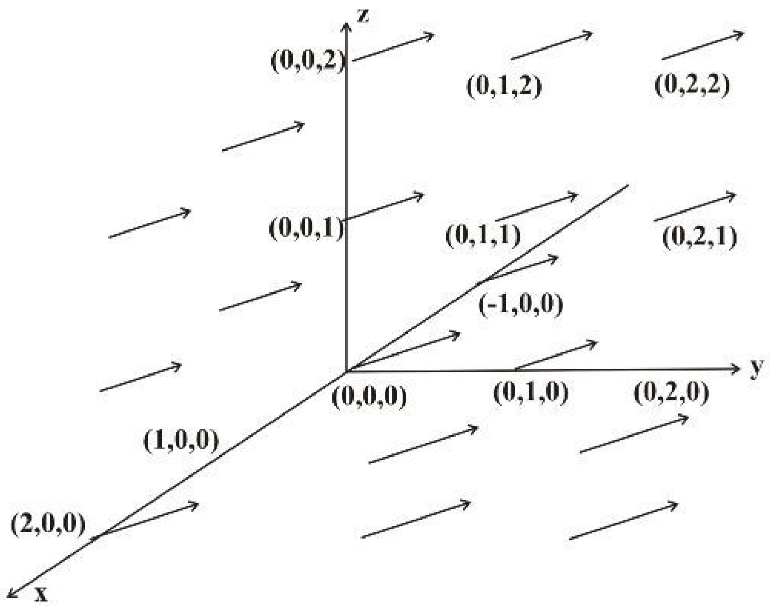
All vectors are vertical and the length of vector field $\mathbf{F}(x, y, z)$ is $|x|$.

For $x > 0$, all points are moving upward direction and for $x < 0$, all points are moving downward direction.

Chapter 16 Vector Calculus Exercise 16.1 10E

$\vec{F}(x,y,z) = \hat{j} - \hat{i} = -\hat{i} + \hat{j}$

(x,y,z)	$\vec{F}(x,y,z)$	(x,y,z)	$\vec{F}(x,y,z)$
$(0,0,0)$	$<-1,1,0>$	$(2,2,0)$	$<-1,1,0>$
$(1,0,0)$	$<-1,1,0>$	$(2,0,2)$	$<-1,1,0>$
$(0,1,0)$	$<-1,1,0>$	$(0,2,2)$	$<-1,1,0>$
$(0,0,1)$	$<-1,1,0>$	$(1,2,0)$	$<-1,1,0>$
$(2,0,0)$	$<-1,1,0>$	$(1,0,2)$	$<-1,1,0>$
$(0,2,0)$	$<-1,1,0>$	$(0,1,2)$	$<-1,1,0>$
$(0,0,2)$	$<-1,1,0>$	$(2,1,0)$	$<-1,1,0>$
$(1,1,0)$	$<-1,1,0>$	$(2,0,1)$	$<-1,1,0>$
$(0,1,1)$	$<-1,1,0>$	$(0,2,1)$	$<-1,1,0>$
$(1,0,1)$	$<-1,1,0>$	$(-1,0,0)$	$<-1,1,0>$



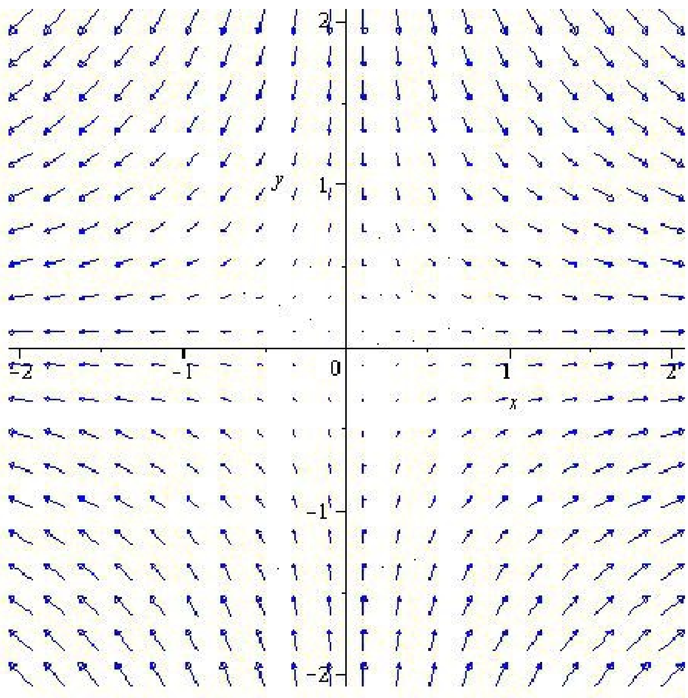
Chapter 16 Vector Calculus Exercise 16.1 11E

Given vector field is $\mathbf{F}(x,y) = \langle x, -y \rangle$

We have to match the given vector field with given plots

Let us start by plotting $\mathbf{F}(x, y) = \langle x, -y \rangle$.

The sketch of the vector field is as shown below



We note that the vector field obtained matches with the plot given in figure IV.

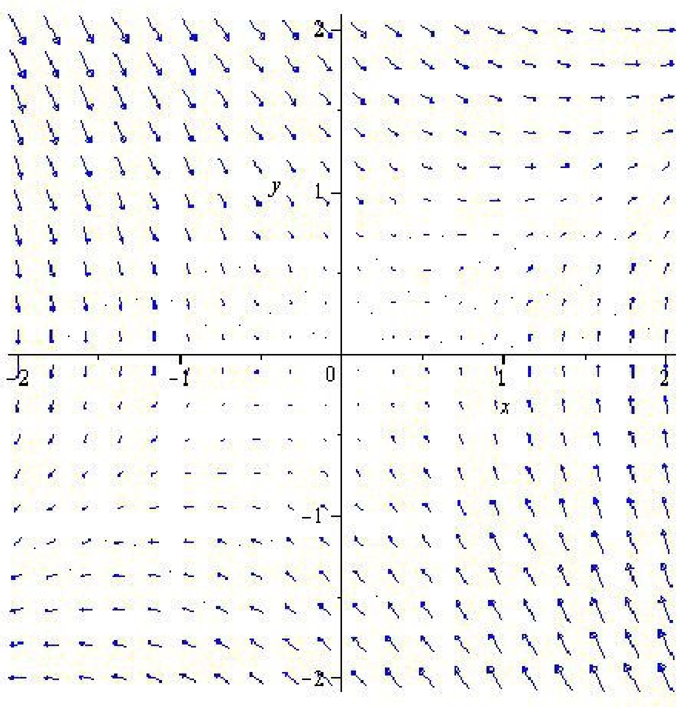
Chapter 16 Vector Calculus Exercise 16.1 12E

Given vector field is $\mathbf{F}(x, y) = \langle y, x - y \rangle$

We have to match the given vector field with the given plots

Let us start by plotting $\mathbf{F}(x, y) = \langle y, x - y \rangle$.

The sketch of the vector field is as shown below



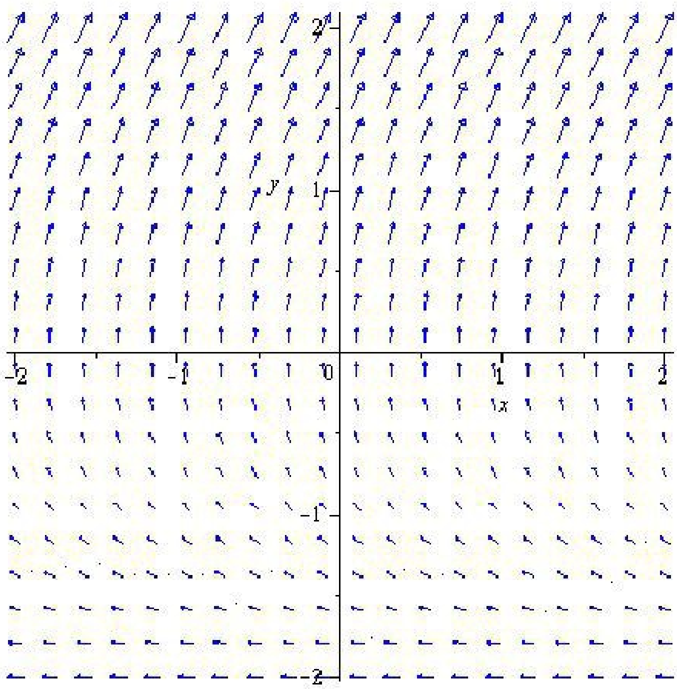
We note that the vector field obtained matches with the plot given in figure III.

Chapter 16 Vector Calculus Exercise 16.1 13E

Given vector field is $\mathbf{F}(x, y) = \langle y, y+2 \rangle$
We have to match the given vector field with the given plots.

Let us start by plotting $\mathbf{F}(x, y) = \langle y, y+2 \rangle$.

The sketch of the vector field is as shown below



We note that the vector field obtained matches with the plot given in figure I.

Chapter 16 Vector Calculus Exercise 16.1 14E

Consider the vector field,

$$\mathbf{F}(x, y) = \langle \cos(x+y), x \rangle$$

This can be written as,

$$F(x,y) = \langle \cos(x+y), x \rangle$$
$$= \cos(x+y)\mathbf{i} + (x)\mathbf{j}$$

The vector field has components as functions of variable either *x* or *y*. That means, the slope and magnitude of each of the vector will be depending on the coordinates (x, y).

To find the slope and the magnitude:

Let,

$$F(x,y) = Pi + Qj$$
$$= \cos(x+y)\ \mathrm{i} + x\ \mathrm{j}$$

Then,

$$P = \cos(x+y)$$
$$Q = x$$

$$\text{Slope} = \frac{Q}{P}$$
$$= \frac{x}{\cos(x+y)}$$

$$\text{Magnitude} = \sqrt{P^2 + Q^2}$$
$$= \sqrt{x^2 + \cos^2(x+y)}$$

>

Analysing the slope and the magnitude:

$$\text{Slope} = \frac{x}{\cos(x+y)}$$

$$\text{Magnitude} = \sqrt{x^2 + \cos^2(x+y)}$$

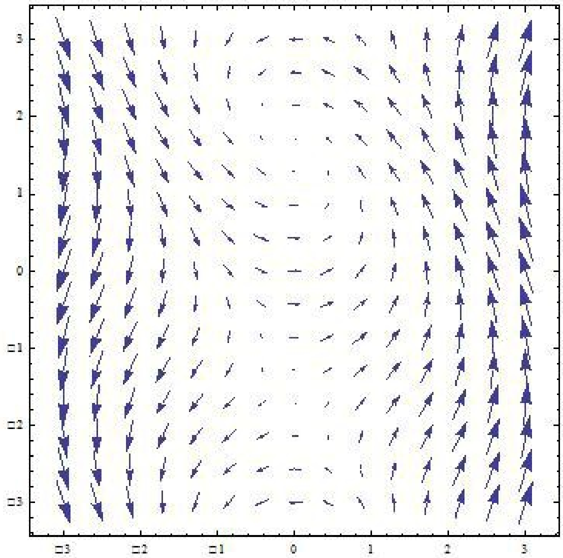
• Let x be positive constant, as y increases from $x = y$, the slope goes from zero infinity and then goes back to zero. (Vector turns 360 degree)

• For the same value of x, as y increases from $x = y$, the magnitude increases to a certain point and then decrease back to the original length.

• Let y be positive constant, as x increases from $x = y$, the slope goes from zero to infinity and the change of the slope gradually increases.

• For the same value of y, as x increases from $x=y$, the magnitude increase to infinity.

Based on the observation the vector fields should look similar to the following plot:



Checking with the slope and magnitude, each of the vector fields matches the observations, and it matches with the II plot.

Chapter 16 Vector Calculus Exercise 16.1 14E

img src="https://c8.staticflickr.com/1/725/31812811551_2a3279305d_o.jpg" width="324" height="460" alt="Stewart-Calculus-7e-Solutions-Chapter-16.1-Vector-Calculus-15E">

From the above figure all the vector are vertical and point upward above the $xy-$ plane.

So, the matched vector field is, IV.

Therefore, the matching vector field $\mathbf{F}$ with the plots is, $\boxed{\text{IV}}$.

Chapter 16 Vector Calculus Exercise 3.1 16E

$$\vec{F}(x,y,z) = \hat{i} + 2\hat{j} + z\hat{k}$$

Now x and y components of $\vec{F}$ are same for all (x, y, z). Then in xy – plane the tangents are parallel and as the z – component increases, $\vec{F}$ increases in magnitude. This pattern appears in plot I. Hence we can say that the plot I is the vector for $\vec{F}$.

Chapter 16 Vector Calculus Exercise 16.1 17E

$$\vec{F}(x,y,z) = x\hat{i} + y\hat{j} + 3\hat{k}$$

Now the z component is 3 for all (x, y, z) then the vector should point towards positive z – direction, and as x and y varies, the vectors are not parallel. This pattern appears in plot III. Hence we can say that the plot III is the vector field for $\vec{F}$

Chapter 16 Vector Calculus Exercise 16.1 18E

$\vec{F}(x,y,z) = x\hat{i} + y\hat{j} + z\hat{k}$

Now $\vec{F}(x,y,z) = < x, y, z >$

If we keep x and y constant, then the vector increases in magnitude with increase in z and it points in positive z – direction if z is positive and it points in negative z – direction if z is negative. Similarly if we keep y and z constant, then the vector increases in magnitude with increases in x and it points in positive x – direction if x is positive and points in negative x – direction if x is negative. A similar rule follows if we keep x and z constant and verifying y. This pattern appears in plot II. Hence we can say that plot II is the vector field for $\vec{F}$.

Chapter 16 Vector Calculus Exercise 16.1 19E

Consider the vector field $\mathbf{F}(x,y) = (y^2 - 2xy)\mathbf{i} + (3xy - 6x^2)\mathbf{j}$.

Plot the given vector field using computer algebra system.

Use Maple's fieldplot ( ) command to sketch the vector field.

First install the package **with (plots),**

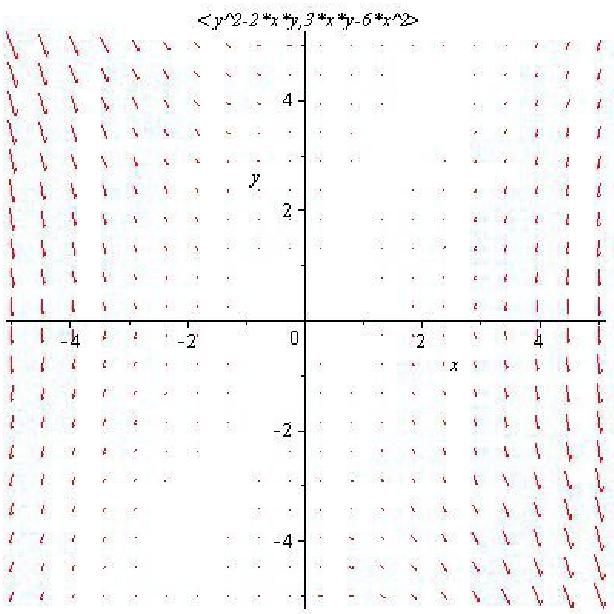
> *with*(*plots*);

[*animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot*]

Enter the following command,

fieldplot([y^2-2*x*y, 3*x*y-6*x^2], x=-5..5, y=-5..5, color=red, axes=framed,title=`< y^2-2*x*y,3*x*y-6*x^2>`);

> *fieldplot*([*y*^2 −2 * *x* * *y*, 3 * *x* * *y* −6 * *x*^2], *x* = −5 ..5, *y* = −5 ..5, *color* = *red*, *title* = `< y^2-2*x*y,3*x*y-6*x^2 >`);



>

> From the diagram, the vector field seems to have shot vectors near the line $y = 2x$.

Explain the appearance by find the set of points (x, y)such that $\mathbf{F}(x, y) = 0$.

The vector field $\mathbf{F}(x, y) = 0$ is valid if $y^2 - 2xy = 0$ and $3xy - 6x^2 = 0$ i.e.

$$y^2 - 2xy = 0 \qquad\qquad 3xy - 6x^2 = 0$$
$$y(y - 2x) = 0 \qquad\qquad 3x(y - 2x) = 0$$
$$y = 0 \text{ or } y = 2x \qquad\qquad x = 0 \text{ or } y = 2x$$

Hence, the statement $\mathbf{F}(x, y) = 0$is hold along the line $\boxed{y = 2x}$.

Chapter 16 Vector Calculus Exercise 16.1 20E

Consider the vector field $\mathbf{F} = (r^2 - 2r)\mathbf{x}$ where $\mathbf{x} = \langle x, y \rangle$ and $r = |\mathbf{x}|$.

Plot the vector field using computer algebra system.

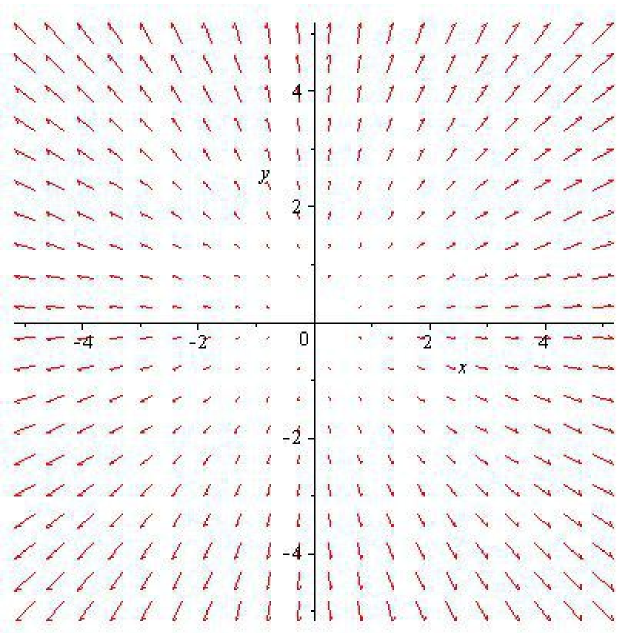
Replace $\mathbf{x}$ by $\langle x, y \rangle = x\mathbf{i} + y\mathbf{j}$ and r by $|\mathbf{x}| = \sqrt{x^2 + y^2}$,

$$\begin{aligned}\mathbf{F}(\mathbf{x}) &= (r^2 - 2r)\mathbf{x} \\ &= \left(x^2 + y^2 - 2\sqrt{x^2 + y^2}\right)(x\mathbf{i} + y\mathbf{j}) \\ &= x\left(x^2 + y^2 - 2\sqrt{x^2 + y^2}\right)\mathbf{i} + y\left(x^2 + y^2 - 2\sqrt{x^2 + y^2}\right)\mathbf{j}\end{aligned}$$

Construct a sketch for the vector field using Maple's fieldplot ( ) command,

First install the package **with (plots),**

```
> with(plots) :
> fieldplot([x(x^2 + y^2-2*sqrt(x^2 + y^2)), y(x^2 + y
    ^2-2*sqrt(x^2 + y^2))], x=-5..5, y=-5..5, color
    = red);
```



The vector field $\mathbf{F}(\mathbf{x}) = 0$ is,

$$(r^2 - 2r) = 0$$
$$r(r - 2) = (0, 0)$$
$$r = 0 \text{ or } r = 2$$
$$|x| = 0 \text{ or } |x| = 2$$

Hence, the vectors in the filed lie on the lines through the origin, and the vectors have small magnitudes near the circle $|x| = 2$ and near to the origin.

Chapter 16 Vector Calculus Exercise 16.1 21E

Gradient f(x,y) = (∂ f/∂ x) **i** + (∂ f/∂ y) **j**

(∂ f/∂ x) = xye(xy)+e(xy)

(∂ f/∂ y) = x2e(xy)

∴ **Gradient** f(x,y) = (xye(xy)+e(xy)) **i** + (x2e(xy)) **j**

Chapter 16 Vector Calculus Exercise 16.1 22E

We are given that

$$f(x, y) = \tan(3x - 4y)$$

We have to find the gradient vector field of f

The gradient vector field is given by

$$\nabla f(x, y) = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} \text{.......}(1)$$

Now

$$\frac{\partial f}{\partial x} = \sec^2(3x - 4y) \cdot 3$$

Or $\frac{\partial f}{\partial x} = 3\sec^2(3x - 4y)$

$$\frac{\partial f}{\partial y} = \sec^2(3x - 4y) \cdot (-4)$$

Or $\frac{\partial f}{\partial y} = -4\sec^2(3x - 4y)$

Putting $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ in the equation (1), we get

$$\nabla f(x, y) = 3\sec^2(3x - 4y)\hat{i} \; -4\sec^2(3x - 4y)\hat{j}$$

$$= \sec^2(3x - 4y)[3i - 4j].$$

$$\Rightarrow \nabla f(x, y) = \sec^2(3x - 4y)[3i - 4j]$$

Chapter 16 Vector Calculus Exercise 16.1 23E

$$f(x,y,z) = \sqrt{x^2 + y^2 + z^2}$$

The gradient vector field is given by

$$\vec{\nabla} f(x,y,z) = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

Now $\frac{\partial f}{\partial x} = \frac{2x}{2\sqrt{x^2+y^2+z^2}} = \frac{x}{\sqrt{x^2+y^2+z^2}}$

$\frac{\partial f}{\partial y} = \frac{2y}{2\sqrt{x^2+y^2+z^2}} = \frac{y}{\sqrt{x^2+y^2+z^2}}$

$\frac{\partial f}{\partial z} = \frac{2z}{2\sqrt{x^2+y^2+z^2}} = \frac{z}{\sqrt{x^2+y^2+z^2}}$

Then $\boxed{\vec{\nabla} f(x,y,z) = \frac{x}{\sqrt{x^2+y^2+z^2}}\hat{i} + \frac{y}{\sqrt{x^2+y^2+z^2}}\hat{j} + \frac{z}{\sqrt{x^2+y^2+z^2}}\hat{k}}$

Chapter 16 Vector Calculus Exercise 16.1 24E

Given function is $f(x,y,z) = x\ln(y-2z)$

We have to find the gradient vector field of the given function.

We know that if f is a scalar function of three variables, then its gradient vector field on $\mathbb{R}^3$ is given by $\nabla f(x,y,z) = f_x(x,y,z)\mathbf{i} + f_y(x,y,z)\mathbf{j} + f_z(x,y,z)\mathbf{k}$

Therefore, the gradient of f can be calculated as follows

$$\nabla f(x,y,z) = f_x(x,y,z)\mathbf{i} + f_y(x,y,z)\mathbf{j} + f_z(x,y,z)\mathbf{k}$$

$$= \ln(y-2z)\mathbf{i} + x\left(\frac{1}{y-2z}\right)\mathbf{j} + x\left(\frac{-2}{y-2z}\right)\mathbf{k}$$

$$= \ln(y-2z)\mathbf{i} + \frac{x}{y-2z}\mathbf{j} - \frac{2x}{y-2z}\mathbf{k}$$

Thus, the gradient vector field of the given function is

1″> $\boxed{\nabla f(x,y,z) = \ln(y-2z)\mathbf{i} + \frac{x}{y-2z}\mathbf{j} + \frac{-2x}{y-2z}\mathbf{k}}$

Chapter 16 Vector Calculus Exercise 16.1 25E

We are given that

$$f(x,y) = x^2 - y$$

We have to find the gradient vector field of f

$$\nabla f(x,y) = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} \text{ ....}(1)$$

$$\frac{\partial f}{\partial x} = 2x$$

And

$$\frac{\partial f}{\partial y} = -1$$

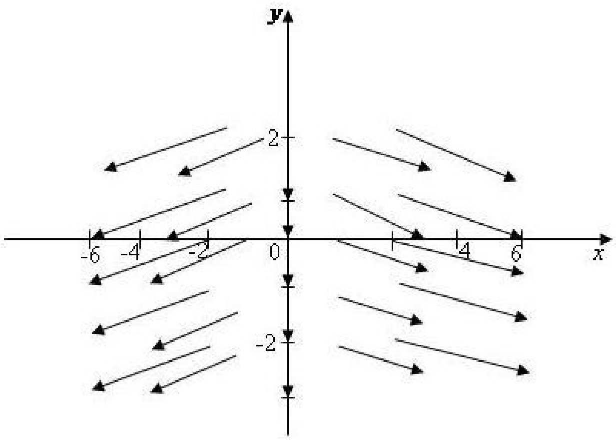
Putting $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ in the equation (1), we get

$$\nabla f(x,y) = 2x\hat{i} - \hat{j}$$

We calculate several other representative values of $\nabla f(x,y)$ in the table and draw the corresponding vectors to represent the vector field.

(x, y)	$\nabla f(x, y)$	(x, y)	$\nabla f(x, y)$
$(1, 0)$	$\langle 2, -1\rangle$	$(-1, 0)$	$\langle -2, -1\rangle$
$(2, 2)$	$\langle 4, -1\rangle$	$(-2, -2)$	$\langle -4, -1\rangle$
$(0, 1)$	$\langle 0, -1\rangle$	$(0, 4)$	$\langle 0, -1\rangle$

The sketch is



Chapter 16 Vector Calculus Exercise 16.1 26E

Find the gradient vector field and sketch it.

$$f\left(x, y\right) = \sqrt{x^2 + y^2}$$

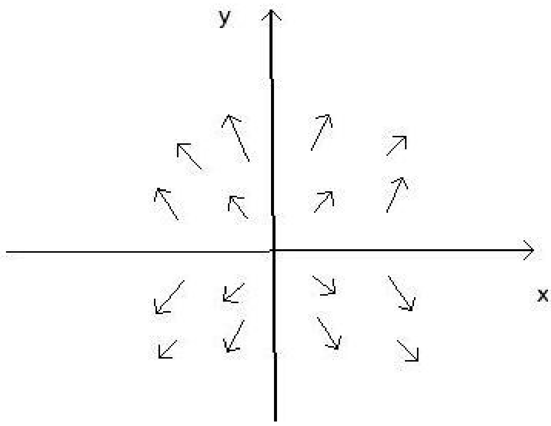
$$f_x = \frac{x}{\sqrt{x^2 + y^2}}, f_y = \frac{y}{\sqrt{x^2 + Y^2}}$$

$$\nabla f(x, y) = \frac{x}{\sqrt{x^2 + y^2}} i + \frac{y}{\sqrt{x^2 + Y^2}} j$$

Calculate several values and plot the vectors to represent the vector field.

(0, 0)	<0,0>	(-2, 1)	<-2/√5, 1/√5>
(1, 1)	<1/√2, 1/√2>	(2, -1)	<2/√5, -1/√5>
(-1, -1)	<-1/√2, -1/√2>	(-2, -1)	<-2/√5,- 1/√5>
(2, 1)	<2/√5, 1/√5>	(1, 2)	<1/√5, 2/√5>

(2, 2)	<1/√2, 1/√2>	(-2, 2)	<-1/√2, 1/√2>
(2, -2)	<1/√2, -1/√2>	(-2, -2)	<-1/√2, -1/√2>



Chapter 16 Vector Calculus Exercise 16.1 27E

Given function is $f(x,y) = \ln\left(1+x^2+2y^2\right)$

We have to plot the gradient vector field of the given function f together with a contour map of f.

We know that if f is a scalar function of two variables, then its gradient vector field on $\mathbb{R}^2$ is given by $\nabla f(x,y) = f_x(x,y)\mathbf{i} + f_y(x,y)\mathbf{j}$

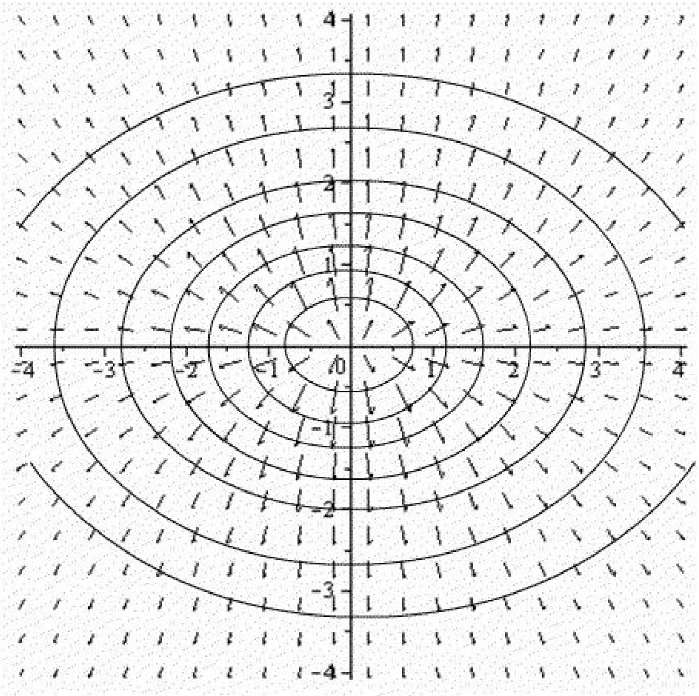
Therefore, the gradient of f can be calculated as follows.

$$\begin{aligned}\nabla f(x,y) &= f_x(x,y)\mathbf{i} + f_y(x,y)\mathbf{j} \\ &= \frac{\partial}{\partial x}\left[\ln\left(1+x^2+2y^2\right)\right]\mathbf{i} + \frac{\partial}{\partial y}\left[\ln\left(1+x^2+2y^2\right)\right]\mathbf{j} \\ &= \left(\frac{2x}{1+x^2+2y^2}\right)\mathbf{i} + \left(\frac{4y}{1+x^2+2y^2}\right)\mathbf{j}\end{aligned}$$

Thus, the gradient vector field of the given function is

$$\boxed{\nabla f(x,y) = \left(\frac{2x}{1+x^2+2y^2}\right)\mathbf{i} + \left(\frac{4y}{1+x^2+2y^2}\right)\mathbf{j}}$$

Now, the sketch of the contour map together with the gradient vector field is as below.



From the figure, we can say that the gradient vectors are long where the level curves are close to each other and are short where the level curves are far apart. This is because the length of the gradient vector is the value of the directional derivative of *f* and closely spaced level curves indicate a steep graph. We also note that the gradient vectors are perpendicular to the level curves.

Chapter 16 Vector Calculus Exercise 16.1 28E

Given function is $f(x,y)=\cos x-2\sin y$

We have to plot the gradient vector field of the given function together with a contour map of *f*.

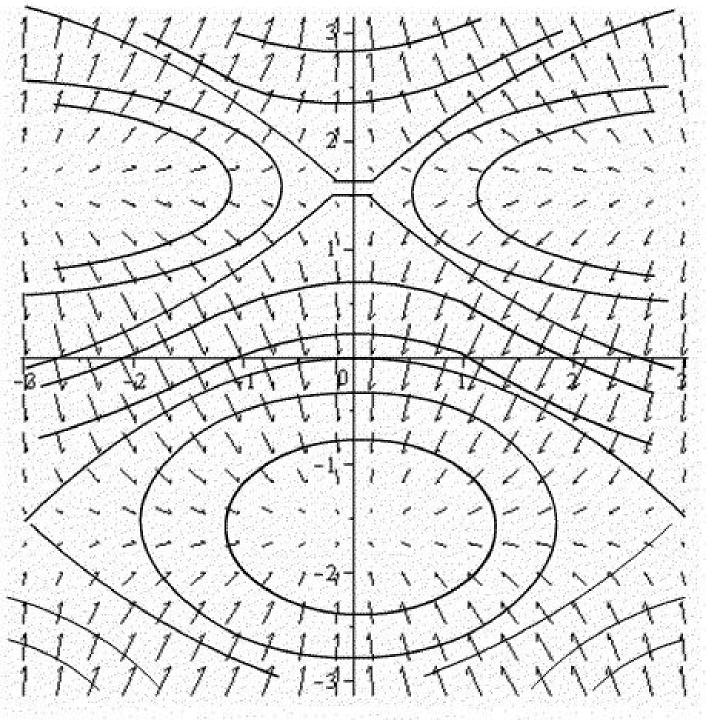
We know that if *f* is a scalar function of two variables, then its gradient vector field on $\mathbb{R}^2$ is given by $\nabla f(x,y)=f_x(x,y)\mathbf{i}+f_y(x,y)\mathbf{j}$

Therefore, the gradient of *f* can be calculated as follows.

$$\begin{aligned}\nabla f(x,y) &= f_x(x,y)\mathbf{i}+f_y(x,y)\mathbf{j}\\ &=\frac{\partial}{\partial x}(\cos x)\mathbf{i}+\frac{\partial}{\partial y}(-2\sin y)\mathbf{j}\\ &= -\sin x\mathbf{i}-2\cos y\mathbf{j}\end{aligned}$$

Thus, the gradient vector field of the given function is $\boxed{\nabla f(x,y)=-\sin x\mathbf{i}-2\cos y\mathbf{j}}$

Now, the sketch of the contour map together with the gradient vector field is as below.



From the figure, we can say that the gradient vectors are long where the level curves are close to each other and are short where the level curves are far apart. This is because the length of the gradient vector is the value of the directional derivative of *f* and closely spaced level curves indicate a steep graph.

Chapter 16 Vector Calculus Exercise 16.1 29E

$f(x,y) = x^2 + y^2$

The gradient vector field is given by

$$\vec{\nabla} f(x,y) = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j}$$

Now $\frac{\partial f}{\partial x} = 2x$ and $\frac{\partial f}{\partial y} = 2y$,

then $\vec{\nabla} f(x,y) = 2x\hat{i} + 2y\hat{j}$

Now $\vec{\nabla} f(x,y) = <2x, 2y>$. Then we keep y constant and only x – vectors, then $\vec{\nabla} f$ increases in magnitude and points in positive direction of x if x is positive and points in negative x – direction if x is negative. But if we keep x constant and very only y component, then $\vec{\nabla} f$ increases in magnitude and points in positive y – direction if y is positive and points in negative y – direction if y is negative. This pattern appears in plot III. Hence we find that the plot of the gradients vector field of f is plot III.

Chapter 16 Vector Calculus Exercise 16.1 30E

We are given that

$$f(x, y) = x(x + y) = x^2 + xy$$

We have to find the gradient vector field of f:

$$\nabla f(x, y) = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} \quad \text{.............................. (1)}$$

$$\frac{\partial f}{\partial x} = 2x + y \text{ and } \frac{\partial f}{\partial y} = x$$

Putting $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ in the equation (1), we get

$$\nabla f(x, y) = (2x + y)\hat{i} + x\hat{j}$$

We calculate several other representative values of $\nabla f(x, y)$ in the table

(x, y)	$\nabla f(x, y)$	(x, y)	$\nabla f(x, y)$
$(1, 0)$	$\langle 2, 2\rangle$	$(2, 2)$	$\langle 6, 2\rangle$
$\langle 1, 1\rangle$	$\langle 3, 1\rangle$	$(-2, 2)$	$\langle -2, -2\rangle$
$(0, 2)$	$\langle 2, 0\rangle$	$(0, -4)$	$\langle -4, 0\rangle$
$(-1, 0)$	$\langle -2, -2\rangle$		
$(0, -1)$	$\langle -1, 0\rangle$		

So, the option IV is right choice.

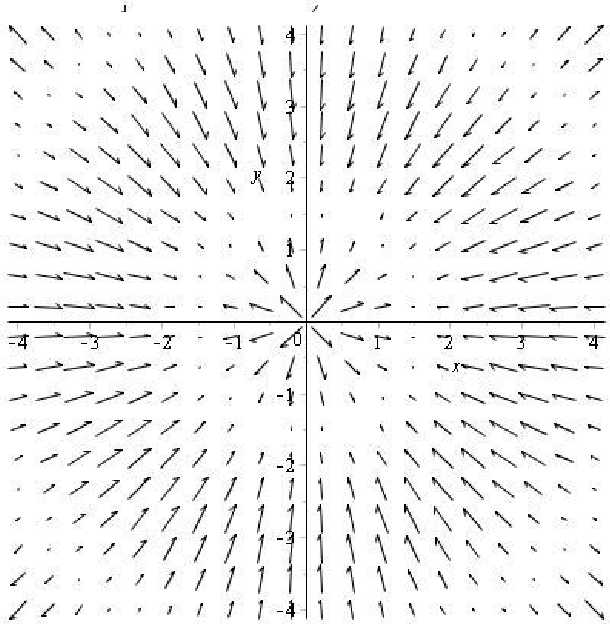
Chapter 16 Vector Calculus Exercise 16.1 31E

Consider the following plots of gradient vector fields labeled with I - IV to match with the function:

$$f(x,y)=(x+y)^2$$

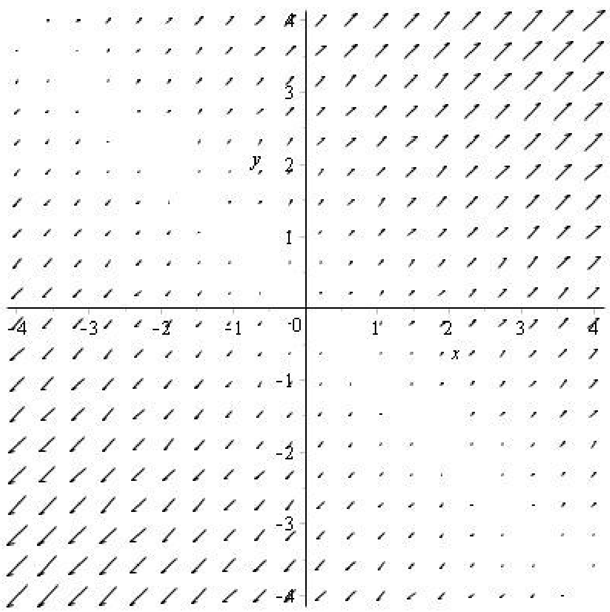
Sketch the gradient vector field of figure I:

Figure I



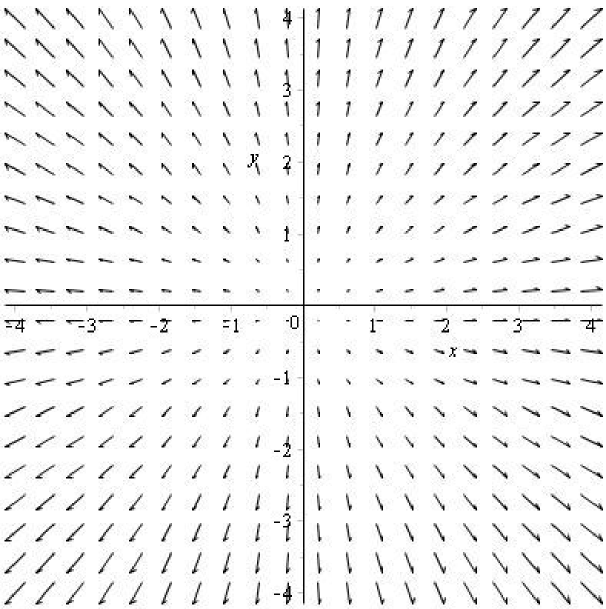
Sketch the gradient vector field of figure II:

Figure II



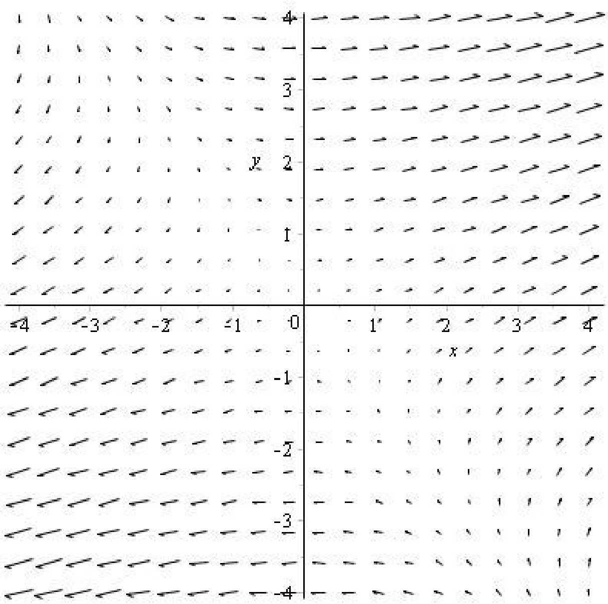
Sketch the gradient vector field of figure III:

Figure III



Sketch the gradient vector field of figure IV:

Figure IV



Observe that gradient vectors along the line $y=-x$ are all zero.

$f(x,y)=(x+y)^2$

Gradient ∇f (or grad f) is defined as follows:

$$\begin{aligned}\nabla f(x,y) &= f_x(x,y)\mathbf{i}+f_y(x,y)\mathbf{j}\\ &=(2x+2y)\mathbf{i}+(2y+2x)\mathbf{i}\end{aligned}$$

Since $2x+2y=0$ along the line $y=-x$,

So the gradient vectors along the line $y=-x$ are all zero.

Because they cancel out the value themselves, as can be seen that figure II have zero gradient vectors along the line $y=-x$.

Thus, the answer is figure II.

Hence, the figure II is represents the gradient vector field of function $f(x,y)=(x+y)^2$.

Chapter 16 Vector Calculus Exercise 16.1 32E

$f(x,y) = \sin\sqrt{x^2+y^2}$

The gradient vector field is given by

$$\vec{\nabla} f(x,y) = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j}$$

Now $\frac{\partial f}{\partial x} = \frac{x}{\sqrt{x^2+y^2}}\cos\sqrt{x^2+y^2}$

$\frac{\partial f}{\partial y} = \frac{y}{\sqrt{x^2+y^2}}\cos\sqrt{x^2+y^2}$

Then $\vec{\nabla} f(x,y) = \left\{\frac{x}{\sqrt{x^2+y^2}}\hat{i} + \frac{y}{\sqrt{x^2+y^2}}\hat{j}\right\}\cos\sqrt{x^2+y^2}$

(x,y)	$\vec{\nabla} f(x,y)$	(x,y)	$\vec{\nabla} f(x,y)$
$(1,0)$	$\langle 0.54,\ 0\rangle$	$(1,2)$	$\langle -0.28,\ -0.55\rangle$
$(2,0)$	$\langle -0.42,\ 0\rangle$	$(2,4)$	$\langle -0.11,\ -0.21\rangle$
$(3,0)$	$\langle -0.99,\ 0\rangle$	$(2,1)$	$\langle -0.55,\ -0.28\rangle$
$(-1,0)$	$\langle -0.54,\ 0\rangle$	$(4,2)$	$\langle -0.21,\ -0.11\rangle$
$(-2,0)$	$\langle 0.42,\ 0\rangle$	$(-1,2)$	$\langle 0.28,\ -0.55\rangle$
$(-3,0)$	$\langle 0.99,\ 0\rangle$	$(-2,4)$	$\langle 0.11,\ -0.21\rangle$
$(1,1)$	$\langle 0.11,\ 0.11\rangle$	$(1,-2)$	$\langle -0.28,\ 0.55\rangle$
$(2,2)$	$\langle -0.67,\ -0.67\rangle$	$(2,-4)$	$\langle -0.11,\ 0.21\rangle$
$(3,3)$	$\langle -0.32, -0.32\rangle$	$(1,-1)$	$\langle 0.11,\ -0.11\rangle$
$(-1,-1)$	$\langle -0.11,\ -0.11\rangle$	$(-1,1)$	$\langle -0.11,\ 0.11\rangle$
$(-2,-2)$	$\langle 0.67,\ 0.67\rangle$	$(2,-2)$	$\langle -0.67,\ 0.67\rangle$
$(-3,-3)$	$\langle 0.32, 0.32\rangle$	$(-2,2)$	$\langle 0.67,\ -0.67\rangle$

Referring to above obtained table we see that wherever x = y all the vectors are equal in magnitude along line y = x .

If x = - y then all the vectors are again equal in magnitude along line y = - x
This whole pattern appears in plot I. Hence we say that the plot for the gradient vector field of f is plot I.

Chapter 16 Vector Calculus Exercise 16.1 33E

Consider the velocity field $\mathbf{V}(x,y) = \langle x^2, x+y^2\rangle$.

A particle moves in the given velocity field and at time $t=3$ the particle is at position $(2,1)$.

The velocity is defined as the derivative of position with respect to time.

Therefore, the velocity is

$$\mathbf{V}(x,y) = \left\langle \frac{d}{dt}x(t), \frac{d}{dt}y(t)\right\rangle = \langle x^2, x+y^2\rangle.$$

And $\langle x(t), y(t)\rangle$ is the position vector of the particle.

That is at time $t=3$, the position of the particle is $\langle x(3), y(3)\rangle = \langle 2,1\rangle$.

And the velocity of the particle at time $t=3$ is

$$\mathbf{V}(x,y)=\langle x^2, x+y^2\rangle.$$
$$\mathbf{V}(2,1)=\langle 2^2, 2+1^2\rangle$$
$$=\langle 4,3\rangle$$

Need to estimate the particle location at time $t=3.01$.

The rate of change in time is

$$\Delta t = 3.01-3$$
$$=0.01$$

And the displacement of the particle is

$$\Delta r=[\mathbf{V}(2,1)]\cdot \Delta t$$
$$=\langle 4,3\rangle\cdot(0.01)$$
$$=\langle 0.04, 0.03\rangle$$

Let $\langle x_1(3.01), y_1(3.01)\rangle$ be the new position vector of the particle at time $t=3.01$.

Then

$$\langle x_1(3.01), y_1(3.01)\rangle=\langle x(3), y(3)\rangle+\Delta r$$
$$=\langle 2,1\rangle+\langle 0.04,0.03\rangle$$
$$=\langle 2.04, 1.03\rangle$$

Therefore, at time $t=3.01$ the particle is at position $\boxed{\langle 2.04, 1.03\rangle}$.

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Consider the velocity field $\mathbf{F}(x,y)=\langle xy-2, y^2-10\rangle$.

A particle moves in the given velocity field and at time $t=1$ the particle is at position $(1,3)$.

The velocity is defined as the derivative of position with respect to time.

Therefore, the velocity is

$$\mathbf{F}(x,y)=\left\langle \frac{d}{dt}x(t), \frac{d}{dt}y(t)\right\rangle=\langle xy-2, y^2-10\rangle.$$

And $\langle x(t), y(t)\rangle$ is the position vector of the particle.

That is at time $t=1$, the position of the particle is $\langle x(1), y(1)\rangle=\langle 1,3\rangle$.

And the velocity of the particle at time $t=1$ is

$$\mathbf{F}(x,y)=\langle xy-2, y^2-10\rangle.$$
$$\mathbf{F}(1,3)=\langle (1)(3)-2, (3)^2-10\rangle.$$
$$=\langle 1,-1\rangle$$

Need to estimate the particle location at time $t=1.05$.

The rate of change in time is

$$\Delta t = 1.05-1$$
$$=0.05$$

And the displacement of the particle is

$$\Delta r=[\mathbf{F}(1,3)]\cdot \Delta t$$
$$=\langle 1,-1\rangle\cdot(0.05)$$
$$=\langle 0.05, -0.05\rangle$$

Let $\langle x_1(1.05), y_1(1.05)\rangle$ be the new position vector of the particle at time $t = 1.05$.

Then

$$\begin{aligned}\langle x_1(1.05), y_1(1.05)\rangle &= \langle x(1), y(1)\rangle + \Delta r \\ &= \langle 1,3\rangle + \langle 0.05, -0.05\rangle \\ &= \langle 1.05, 2.95\rangle\end{aligned}$$

Therefore, at time $t = 1.05$ the particle is at position $\boxed{\langle 1.05, 2.95\rangle}$.

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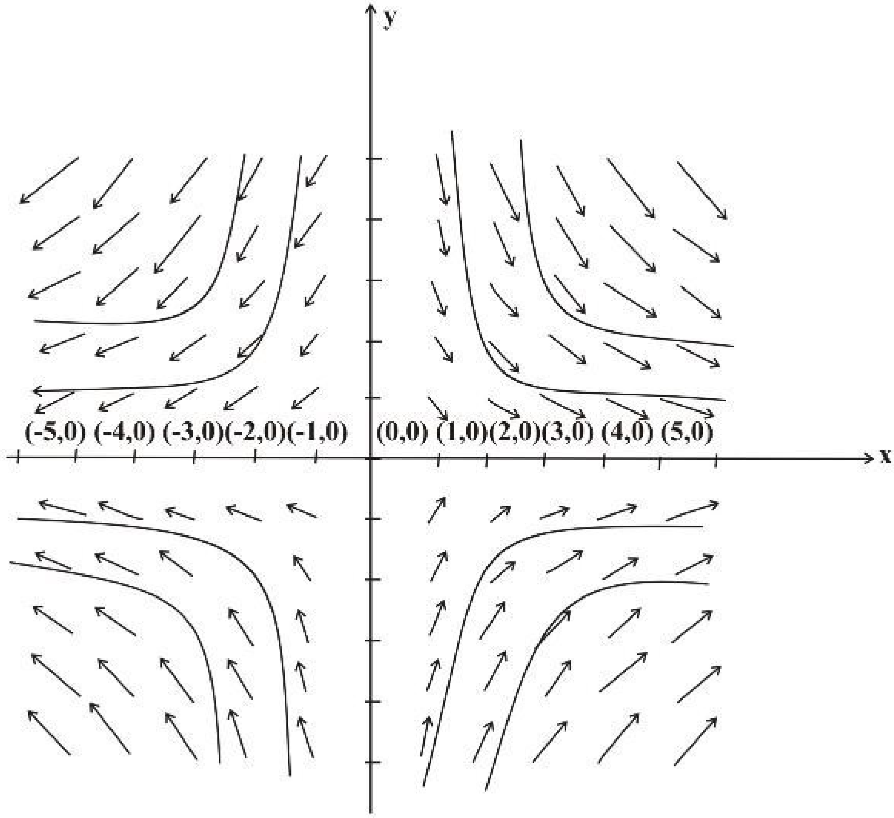
202-17.01-33E

(A)

$$\vec{F}(x,y) = x\hat{i} - y\hat{j}$$

(x,y)	$\vec{F}(x,y)$	(x,y)	$\vec{F}(x,y)$
$(1,0)$	$<1,0>$	$(-1,-4)$	$<-1,4>$
$(2,0)$	$<2,0>$	$(-1,-5)$	$<-1,5>$
$(3,0)$	$<3,0>$	$(2,1)$	$<2,-1>$
$(1,1)$	$<1,-1>$	$(2,2)$	$<2,-2>$
$(1,2)$	$<1,-2>$	$(2,3)$	$<2,-3>$
$(1,3)$	$<1,-3>$	$(2,4)$	$<2,-4>$
$(1,4)$	$<1,-4>$	$(-2,1)$	$<-2,-1>$
$(1,5)$	$<1,-4>$	$(-2,2)$	$<-2,-2>$
$(-1,1)$	$<-1,-1>$	$(-2,3)$	$<-2,-3>$
$(-1,2)$	$<-1,-2>$	$(-2,-3)$	$<-2,3>$
$(-1,3)$	$<-1,-3>$	$(-2,-1)$	$<-2,1>$
$(-1,4)$	$<-1,-4>$	$(-2,-2)$	$<-2,2>$
$(-1,5)$	$<-1,-5>$	$(-3,1)$	$<-3,-1>$
$(1,-1)$	$<1,1>$	$(3,1)$	$<3,-1>$
$(1,-2)$	$<1,2>$	$(3,-1)$	$<3,1>$
$(1,-3)$	$<1,3>$	$(-3,-3)$	$<-3,3>$
$(1,-4)$	$<1,4>$	$(4,4)$	$<4,-4>$
$(1,-5)$	$<1,5>$	$(4,3)$	$<4,-3>$
$(-1,-1)$	$<-1,1>$	$(-4,3)$	$<-4,-3>$
$(-1,-2)$	$<-1,2>$	$(-4,-1)$	$<-4,1>$
$(-1,-3)$	$<-1,3>$	$(-4,2)$	$<-4,-2>$



(B)

The parametric equation of flow lines are:

$x = x(t)$, $y = y(t)$

Since the flow lines are the path followed by a particle whose velocity field is given by given vector, then $\vec{v}(x,y) = \vec{F}(x,y)$

i.e. $< \frac{dx}{dt}, \frac{dy}{dt} > = < x, -y >$

i.e. $\frac{dx}{dt} = x$

And $\frac{dy}{dt} = -y$

Now consider $\frac{dx}{dt} = x$

Or $\frac{dx}{x} = dt$

Integrating both sides $\ln|x| = t + c_1$

Where c_1 is constant of integration

Or $x = e^t c_2$ ---------- (1)

Consider $\frac{dy}{dt} = -y$

Or $\frac{dy}{t} = -dt$

Integrating both sides

$\ln|y| = -t + c_3$

Where c_3 is constant of integration

Or $y = e^{-t} c_4$ -------------- (2)

Eliminating t from (1) and (2)

$$\frac{x}{c_2}=\frac{c_4}{y}$$

Or $\quad y=\frac{c_2c_4}{x}$

Or $\quad \boxed{y=\frac{C}{x}}$

Where $C=c_2c_4$

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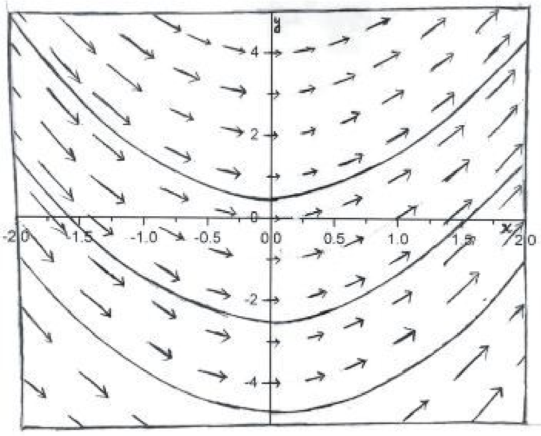

The sketch of the vector field $\vec{F}(x,y)=\hat{i}+x\hat{j}$ and the flow lines are as shown above. The flow lines represent parabolas.

(B) Vector field $\vec{F}(x,y)$ is given as

$$\vec{F}=\hat{i}+x\hat{j}$$

We know that the flow lines of a vector field are the paths followed by a particle whose velocity field is the given vector field.

Flow lines are $x=x(t)$, $y=y(t)$

Velocity of the particle $=\frac{dx}{dt}\hat{i}+\frac{dy}{dt}\hat{j}$

But, we have

$$\frac{dx}{dt}\hat{i}+\frac{dy}{dt}\hat{j}=\vec{F}$$

$$\Rightarrow \quad \frac{dx}{dt}\hat{i}+\frac{dy}{dt}\hat{j}=\hat{i}+x\hat{j}$$

Equating coefficients of $\hat{i}$ and $\hat{j}$ we get,

$\frac{dx}{dt}=1$ and $\frac{dy}{dt}=x$

Which are the differential equation satisfied by the flow lines.

Now,

$$\frac{dy}{dt}=\frac{dy/dt}{dx/dt}=\frac{x}{1}$$

$$\Rightarrow \quad \frac{dy}{dx}=x$$

(C) When particle moves in the velocity field

$\vec{F} = \hat{i} + x\hat{j}$ then the differential equation of motion of particle is

$$\frac{dy}{dx} = x$$

$\Rightarrow$ $dy = xdx$

Integrating both sides,

$$\int dy = \int x\,dx + k$$

$\Rightarrow$ $y = \frac{x^2}{2} + k$

Since the particle starts it's motion from the origin. Therefore, the equations of it's path will pass through (0, 0).

$$0 = \frac{0^2}{2} + k$$

$\Rightarrow$ $k = 0$

Hence,

Equation of the path followed by the particle is $y = \frac{x^2}{2}$